1 - Solve The following inequality :

a) $\frac{3}{x-1} - \frac{2}{x+1} \geq 2$

$$\frac{3}{x-1} - \frac{2}{x+1} - \frac{2}{1} \geq 0$$

$$\frac{3(x+1) - 2(x-1) - 2(x-1)(x+1)}{(x-1)(x+1)} \geq 0$$

$$\frac{3x+3-2x+2-2(x^2-1)}{(x-1)(x+1)} \geq 0$$

$$\frac{x+5-2x^2+2}{(x-1)(x+1)} \geq 0 \Longrightarrow \frac{(x+1.6)(x-2.1)}{(x-1)(x+1)} \geq 0$$

$$\frac{x+7-2x^2}{(x-1)(x+1)} \geq 0$$

$-$ ● $+$ ○ $-$ $-$ ○ $+$ ● $-$

-1.6 $\quad -1$ $\quad 1$ $\quad 2.1$

$\boxed{0}$

$(-\infty, -1.6]$	$[-1.6, -1)$	$(-1, 1)$	$(1, 2.1]$	$[2.1, \infty)$
$-$	$+$	$-$	$+$	$-$

S.S $= [-1.6, -1) \cup (1, 2.1]$

b) $\sqrt{(4-2x)^2} \leq 4$

لابد من الترتيب

$$|4-2x| \leq 4$$

$$|2x-4| \leq 4$$

$$-4 \leq 2x-4 \leq 4$$

$$0 \leq 2x \leq 8 \quad \div 2$$

$$0 \leq x \leq 4$$

S.S $= [0, 4]$

$\sqrt{x^2} = |x|$

$|x| < a$

$-a < x < a$

$|x| > a$

$x < -a$ | $x > a$

(1) (2)

$$\frac{1}{x+2} - \frac{1}{x-2} < 1$$

$$\frac{1}{x+2} - \frac{1}{x-2} - \frac{1}{1} < 0$$

$$\frac{1(x-2) - 1(x+2) - 1(x+2)(x-2)}{(x+2)(x-2)} < 0$$

$$\frac{\cancel{x} - 2 - \cancel{x} - 2 - (x^2-4)}{(x+2)(x-2)} < 0$$

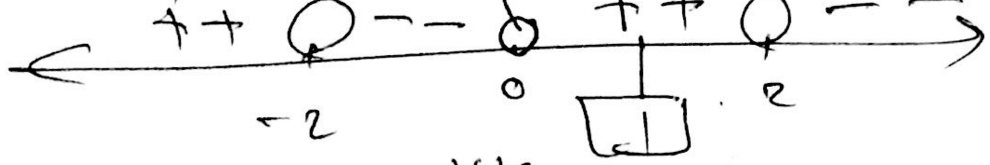
$$\frac{\cancel{-4} - x^2 + \cancel{4}}{(x+2)(x-2)} < 0$$

$$\frac{-x^2}{(x+2)(x-2)} < 0$$

اصفار م

$x = 0$ من البسط

$x = 2 , -2$



S.S = $(-2,0) \cup (2,\infty)$

$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
++	−−	++	−−

2

سري

2- Consider The function

$$f(x) = \frac{3x-6}{1-x}$$ and find

① The Domain ② The intercepts

③ $f^{-1}(x)$ ④ H.A , V.A.

⑤ Discuss The symmetry.

① Domain:

$1 - x \neq 0$

$x \neq 1$

$D_f = \mathbb{R} - \{1\}$

② x- Intercept $\boxed{y=0}$

$f(x) = 0 = \frac{3x-6}{1-x}$

$3x - 6 = 0$

$3x = 6 \longrightarrow \boxed{x = 2}$

y- intercept $\boxed{x=0}$

$y = \frac{0-6}{1-0} = \frac{-6}{1} = -6$

3) f^{-1}:

$$\frac{y}{1} = \frac{3x-6}{1-x}$$

$$y - xy = 3x - 6$$

$$y + 6 = 3x + xy$$

$$y + 6 = x(3+y)$$

$$\therefore x = \frac{y+6}{3+y}$$

$$f^{-1}(x) = \frac{x+6}{3+x}$$

حل آخر علشان فالمزعج

$$y = \frac{3x-6}{1-x}$$

$$\frac{x}{1} = \frac{3y-6}{1-y}$$

$$x - xy = 3y - 6$$

$$x + 6 = 3y + xy$$

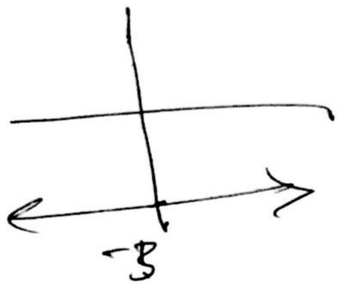
$$x + 6 = y(3+x)$$

$$y = f^{-1} = \frac{x+6}{3+x}$$

H.A:

$$Y = \lim_{x \to \infty} \frac{3x - 6}{1 - x}$$

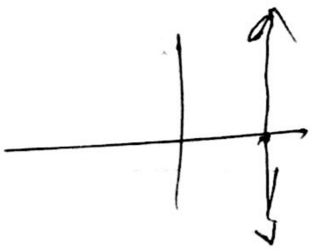
$$= \lim_{x \to \infty} \frac{\frac{3x}{x} - \frac{6}{x}}{\frac{1}{x} - \frac{x}{x}}$$



$$\underline{Y = \frac{3 - 0}{0 - 1} = \frac{3}{-1} = -3}$$

V.A

1) أصفار المقام

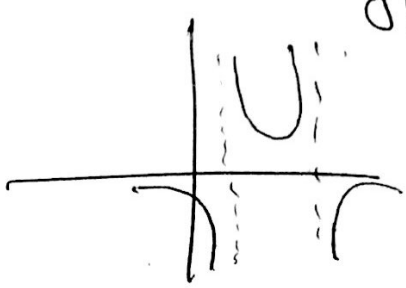


$$1 - x = 0$$

$$\boxed{x = 1}$$

Ex: Find V.A

$$g(x) = \frac{3}{x^2 - 4x + 3}$$



$$= \frac{3}{(x-3)(x-1)}$$

$$x = 3 \;,\quad x = 1$$

6- Use The graph of $y = f(x)$ to find The following

a) $\lim_{x\to 1} f(x)$

b) $f(1)$

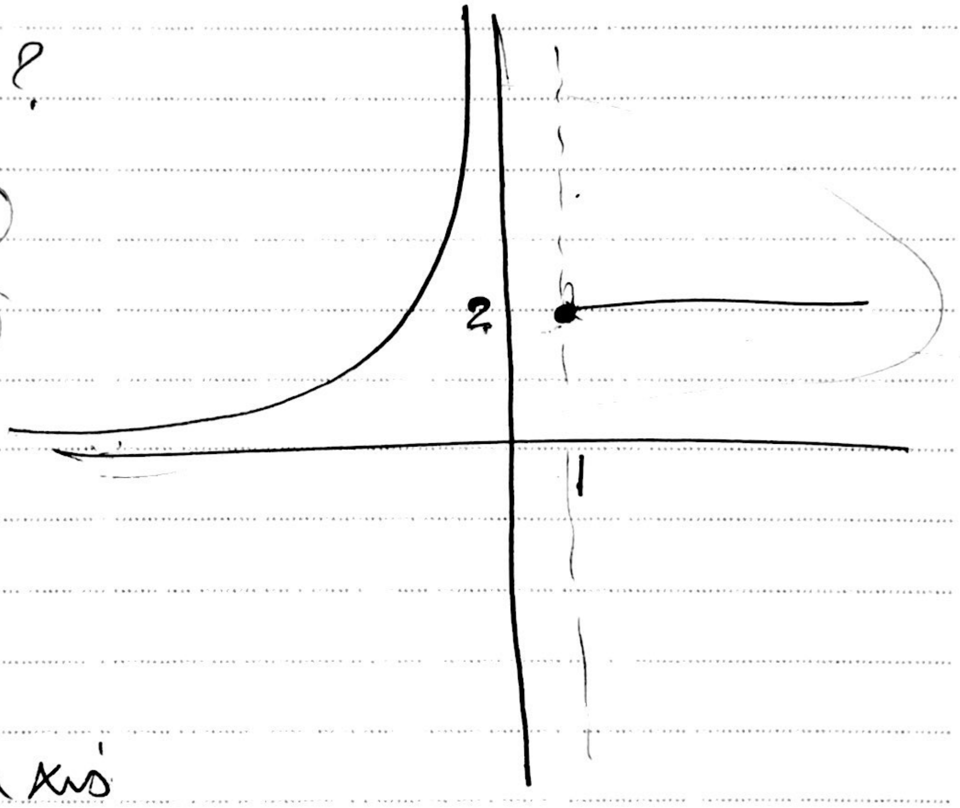
c) the horizontal assymptote of The graph.

$\lim_{x\to 1} f(x) = ?$

$\lim_{x\to 1^+} f(x) = 2$

$\lim_{x\to 1^-} f(x) = \infty$

D.N.E



b) $f(1) = 2$

c) H.A.

x - axis

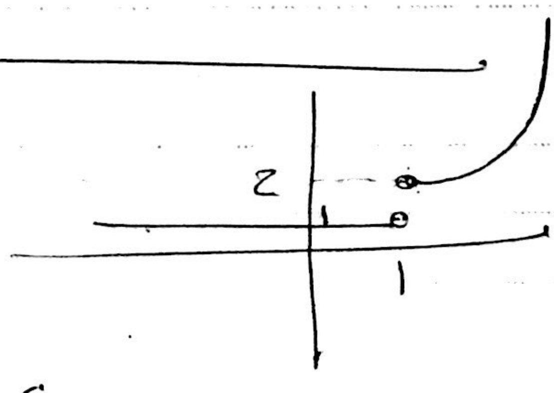
$y = 0$

$\lim_{x\to 1} f(x) =$

$\lim_{x\to 1^+} = 2$

$\lim_{x\to 1^-} = 1$

D.N.E



6

3-

If $f(x) = \sqrt{2x-4}$, $g(x) = 3x^2 - 6$

a) find $f \circ g(x)$ and $(g \circ f)^{-1}(x)$

b) find The Domain of The function $h(x) = \frac{g}{f}(x)$

$f \circ g = f(g)$

$= \sqrt{2g - 4}$

$= \sqrt{6x^2 - 12 - 4}$

$= \sqrt{6x^2 - 16}$

$g \circ f = g(f)$

$= 3f^2 - 6$

$= 3(2x-4) - 6$

$= 6x - 12 - 6$

$g \circ f = 6x - 18$

$y = 6x - 18$

$6x = y + 18$

$x = \frac{y+18}{6}$

$\cancel{f}\ (g \circ f)^{-1} = \frac{x+18}{6}$

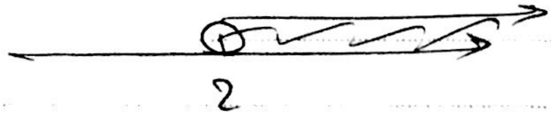
$h(x) = \frac{g}{f}(x)$

$= \frac{3x^2 - 6}{\sqrt{2x-4}}$

$2x - 4 > 0$

$2x > 4$

$x > 2$



2

$D_h = (2, \infty)$

$g \circ f^{-1}(x)$??

$f^{-1} = ??$ $\quad g(x) = 3x^2 - 6$

$y = \sqrt{2x-4} \longrightarrow y^2 = 2x - 4$

$2x = y^2 + 4$

$x = \frac{y^2 + 4}{2}$

$f^{-1}(x) = \frac{x^2 + 4}{2}$

$g \circ f^{-1}(x) = g(f^{-1})$

$= 3\left(\frac{x^2+4}{2}\right)^2 - 6$

$= 3\left(\frac{x^4 + 8x^2 + 16}{4}\right) - 6$

8

5-

Use The graph of The function g(x) to answer :-

A- Identify The intervals which makes g(x) inc. or dec.

B- find The range of The function

C- Determine whether even or odd.

D- Determine whether one-to-one or not.

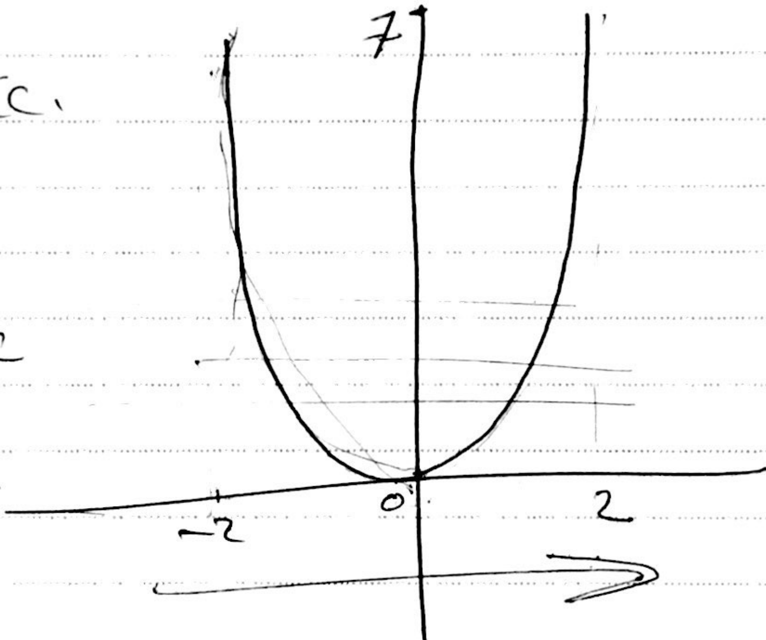
A:

~~(-2,0] dec.~~

~~[0,2)~~

[-2,0] dec

[0,2] inc.



B: Range المدى $= Y =$ من أقل Y إلى أكبر Y

$= [0,7]$

c: Symmetric about y-axis "even"

d: Using H. line Test f is not one-to-one

$f(x) = x^2 + 2x + 3 \quad , x \geq 1$

is one-to-one.

Let $x_1 , x_2 \in D = \mathbb{R}$

$f(x_1) = f(x_2)$

$x_1^2 + 2x_1 + \cancel{3} = x_2^2 + 2x_2 + \cancel{3}$

$x_1^2 + 2x_1 = x_2^2 + 2x_2$

جذر

نصف الأوسط

$(x_1 + 1)^2 \cancel{-1} = (x_2 + 1)^2 \cancel{-1}$

$\sqrt{(x_1 + 1)^2} = \sqrt{(x_2 + 1)^2}$

$|x_1 + 1| = |x_2 + 1| \quad , x \geq 1$

$x_1 + \cancel{1} = x_2 + \cancel{1}$

$x_1 = x_2$

is one-to-one.

حل آخر

(1) $x_1^2 + 2x_1 = x_2^2 + 2x_2$

$x_1^2 + 2x_1 - x_2^2 - 2x_2 = 0$

$x_1^2 - x_2^2 + 2x_1 - 2x_2 = 0$

$(x_1 - x_2)(x_1 + x_2) + 2(x_1 - x_2) = 0$

$(x_1 - x_2)[x_1 + x_2 + 2] = 0$

$(x_1 - x_2) = 0$ or $x_1 + x_2 + 2 = 0$

موجب
مرفوضة

$x_1 - x_2 = 0$

$x_1 = x_2$

4-

If $f(x) = \dfrac{x^2 - 4x + 3}{x^2 - 2x - 3}$

① find $\lim\limits_{x \to 3} f(x)$

② find H.A , V.A

① $\lim\limits_{x \to 3} \dfrac{x^2 - 4x + 3}{x^2 - 2x - 3} = \dfrac{0}{0}$

$$\lim_{x \to 3} \frac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x+1)} = \frac{2}{4} = \frac{1}{2}$$

② H.A : $y = \lim\limits_{x \to \infty} \dfrac{x^2 - 4x + 3}{x^2 - 2x - 3}$

$$= \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{4x}{x^2} + \frac{3}{x^2}}{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}}$$

$$y = \lim_{x \to \infty} \frac{1 - 0 + 0}{1 - 0 - 0} = 1$$

بعد الحل او الاختزال

V.A : اصفار المقام بعد الاختزال

$X = -1$

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13- Find The V.A of ① $f(x) = \dfrac{4x^2-2}{3x-6}$, f(any))

② $g(x) = \dfrac{x^2-x}{x^2-1}$

① $f(x) = \dfrac{4x^2-2}{3x-6} = \dfrac{2(2x^2-1)}{3(x-2)}$

V.A: $x = 2$

$3x-6=0$

$3x = 6$

$x = 2$

② $g(x) = \dfrac{x^2-x}{x^2-1}$

$= \dfrac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$

V.A : $x = -1$

تذكر V.A بعد الاختزال ،

(17) Discuss The Continuity

a) $f(x) = \dfrac{x^2-9}{x-3}$

(b) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ 4 & x = 3 \end{cases}$

c) $h(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ 6 & x = 3 \end{cases}$

$f(x) = \dfrac{x^2-9}{x-3}$

D: $R - \{3\}$

$f(3) =$ D.N.E.

not cont at $X = 3$

$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ 4 & x = 3 \end{cases}$

$\lim\limits_{x \to 3} \dfrac{x^2-9}{x-3} = \dfrac{0}{0}$

$\lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 6$

$f(3) = 4$

$\lim\limits_{x \to 3} \neq f(3)$

discontinuous

$h(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ 6 & x = 3 \end{cases}$

$f(3) = 6$

$\lim\limits_{x \to 3} f(x) = 6$

Continuous at $X = 3$.

Continuous $\forall x \in \mathbb{R}$

(14) Discuss The Continuity of f

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x\sin x} & x \neq 0 \\ 2 & x = 0 \end{cases}$$

at $x=0$

$$\lim_{x\to 0} \frac{1-\cos x}{x \sin x} = \lim_{x\to 0} \frac{1-(1-2\sin^2\frac{x}{2})}{x\sin x}$$

$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}}{x \sin x}$$

$$= \lim_{x\to 0} \frac{2\sin\frac{x}{2}}{x} \cdot \frac{\sin\frac{x}{2}}{\sin x}$$

$$= \cancel{2}\left(\tfrac{1}{\cancel{2}}\right) \cdot \tfrac{1}{2}$$

$$= \tfrac{1}{2}$$

$f(0) = 2$

$$\lim_{x\to 0} \neq f(0)$$

discontinous

(18) Find The values of a and b where The function

$$f(x) = \begin{cases} ax^2+1 & x>2 \\ -11 & x=2 \\ x^3+b & x<2 \end{cases}$$

is Continuous on $(-\infty,\infty)$

$$\lim_{x \to 2^+} ax^2+1 = 4a+1$$

$$f(2) = -11$$

$$\lim_{x \to 2^-} x^3+b = 8+b$$

$\because$ f Continuous

$$4a+1=-11$$

$$4a=-12 \longrightarrow a=-3$$

$$8+b=-11 \longrightarrow b=-19.$$

(11) Use the definition of limit to show that

a) $\lim_{x \to 2^+} \sqrt{x-2} = 0$ (b) $\lim_{x \to 3^-} \sqrt{3-x} = 0$

d) $\lim_{x \to 1} \frac{3x+5}{8} = 1$

a) $\lim_{x \to 2^+} \sqrt{x-2} = 0$

$2 < x < 2+\delta$
$-2 \quad -2 \quad -2$

$0 < x-2 < \delta$

$0 < \underbrace{\sqrt{x-2}}_{f(x)} < \sqrt{\delta}$

$\varepsilon = \sqrt{\delta}$

$\delta = \varepsilon^2$

for all $\varepsilon > 0$ there exist $\delta = \varepsilon^2$

$2 < x < 2+\delta \rightarrow \sqrt{x-2} < \varepsilon$

b) $\lim_{x \to 3^-} \sqrt{3-x} = 0$

$3-\delta < x < 3$
$-3 \quad -3 \quad -3$

$-\delta < x-3 < 0$

(x−1)

$\delta > 3-x > 0$

(−1)

$\sqrt{\delta} > \sqrt{3-x} > 0$

Let $\sqrt{\delta} = \varepsilon$

$\delta = \varepsilon^2$

$\therefore$ for all $\varepsilon > 0$ there exist $\delta = \varepsilon^2$,

$3-\delta < x < 3 \Longrightarrow \sqrt{3-x} < \varepsilon$.

$$\lim_{x \to 1} \frac{3x+5}{8} = 1$$

$f(x) = \frac{3x+5}{8}$

$L = 1$

$a = 1$

$$\boxed{\lim_{x \to a} f(x) = L}$$

Let $\varepsilon > 0$,

$$|f(x) - L| < \varepsilon$$

$$\Rightarrow \left|\frac{3x+5}{8} - 1\right| < \varepsilon$$

بالضرب $\times 8$

$$|3x+5-8| < 8\varepsilon$$

$$|3x-3| < 8\varepsilon$$

$\div 3$

$$|x-1| < \frac{8\varepsilon}{3}$$

Let $\boxed{\delta = \frac{8\varepsilon}{3}}$

$\therefore$ for all $\varepsilon > 0$ there exist $\delta = \frac{8\varepsilon}{3}$,

$$|x-1| < \delta \Longrightarrow \left|\frac{3x+5}{8} - 1\right| < \varepsilon$$

12) Let $\lim_{x \to 2} f(x) = 4$ and $\lim_{x \to 2} g(x) = -3$

find ① $\lim_{x \to 2} \left[3f(x) + \frac{6}{g(x)}\right]$

② $\lim_{x \to 2} \frac{f(x)}{3 - 2g(x)}$

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① $\lim_{x \to 2} \left[3f(x) + \frac{6}{g(x)}\right]$

$= 3\lim_{x \to 2} f(x) + \frac{6}{\lim_{x \to 2} g(x)}$

$= 3(4) + \frac{6}{-3} = 12 - 2 = 10$

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② $\lim_{x \to 2} \frac{f(x)}{3 - 2g(x)}$

$= \frac{\lim_{x \to 2} f(x)}{3 - 2\lim_{x \to 2} g(x)} = \frac{4}{3 - 2(-3)}$

$= \frac{4}{3+6} = \frac{4}{9}$

③

(9)

Use sequeeze Theorem to evaluate

① $\lim\limits_{x \to 0^+} \sqrt{x}\,\sin\left(x + \frac{1}{x}\right)$

$$-1 \le \sin\left(x + \frac{1}{x}\right) \le 1 \qquad \times \sqrt{x}$$

$$-\sqrt{x} \le \sqrt{x}\,\sin\left(x + \frac{1}{x}\right) \le \sqrt{x}$$

$\lim\limits_{x \to 0^+} -\sqrt{x} = 0 \quad , \quad \lim\limits_{x \to 0^+} \sqrt{x} = 0$

$$\therefore \lim_{x \to 0^+} \sqrt{x}\,\sin\left(x + \frac{1}{x}\right) = 0$$

$-1 \le \sin \le 1$
$-1 \le \cos \le 1$
$-\frac{\pi}{2} < \tan^{-1} < \frac{\pi}{2}$

$0 \le \sin^2 \le 1$

$0 \le \cos^2 \le 1$

② $\lim\limits_{x \to 0} \left(x^2 \cos\frac{1}{x}\right)$

$$-1 \le \cos\frac{1}{x} \le 1$$

بالضرب ب x^2

$$-x^2 \le x^2 \cos\frac{1}{x} \le x^2$$

$\lim\limits_{x \to 0} -x^2 = 0 \quad , \quad \lim\limits_{x \to 0} x^2 = 0$

$$\therefore \lim_{x \to 0} x^2 \cos\frac{1}{x} = 0$$

(20) $\lim_{x \to \infty} \frac{\tan^{-1} x}{x^2}$

$$\boxed{-90 < \tan^{-1} x < 90}$$

$$\frac{-90}{x^2} < \frac{\tan^{-1} x}{x^2} < \frac{90}{x^2}$$

$$\lim_{x \to \infty} \frac{-90}{x^2} = 0 = \lim_{x \to \infty} \frac{90}{x^2}$$

$$\therefore \lim_{x \to \infty} \frac{\tan^{-1} x}{x^2} = 0$$

$$\lim_{x \to \infty} \frac{\cos x}{x}$$

$$-1 \leq \cos x \leq 1 \quad \div x$$

$$\frac{-1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x}$$

$$\lim_{x \to \infty} \frac{-1}{x} = 0 = \lim_{x \to \infty} \frac{1}{x}$$

$$\therefore \lim_{x \to \infty} \frac{\cos x}{x} = 0$$

(5)

8- Evaluate The following limits "if exists"

a) $\lim\limits_{x \to 1} (2x^3 - 3x^2 - 4)$ (c) $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x^2}$

b) $\lim\limits_{x \to 0} \dfrac{\sin 5x + \tan 3x}{4x}$ (D) $\lim\limits_{x \to 0} x^2 \cos\left(x + \dfrac{3}{x}\right)$

في بداية الحل لابد من التعويض ، اذا كان عدد انتهت المسألة

واذا كان ∞ انتهت المسألة

واذا كان $\frac{\infty}{\infty}$ او $\infty - \infty$ او $\frac{0}{0}$ نكمل الحل.

(a) $\lim\limits_{x \to 1} 2x^3 - 3x^2 - 4 = 2 - 3 - 4 = -5$

b) $\lim\limits_{x \to 0} \dfrac{\sin 5x + \tan 3x}{4x} = \lim\limits_{x \to 0} \dfrac{\dfrac{\sin 5x}{x} + \dfrac{\tan 3x}{x}}{\dfrac{4x}{x}}$

نقسم على x ذات أكبر أُس مع sin او tan

$= \dfrac{5 + 3}{4} = \dfrac{8}{4} = 2$

c) $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x^2} = \lim\limits_{x \to 0} \dfrac{1 - (1 - 2\sin^2 x)}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{1} \cancel{-1} + 2\sin^2 x}{x^2} = \lim\limits_{x \to 0} \dfrac{2\sin^2 x}{x^2}$

$= \dfrac{2\sin x}{x} \cdot \dfrac{\sin x}{x}$

$= 2 \cdot 1 = 2$

D :

$$\lim_{x\to 0} x^2 \cos\left(x+\frac{3}{x}\right)$$

$$-1 \le \cos\left(x+\frac{3}{x}\right) \le 1$$

$$-x^2 \le x^2\cos\left(x+\frac{3}{x}\right) \le x^2$$

$$\lim_{x\to 0} -x^2 = 0 = \lim_{x\to 0} x^2 = 0$$

$$\therefore \lim_{x\to 0} x^2\cos\left(x+\frac{3}{x}\right) = 0$$

$$\lim_{x\to 0} \sqrt{x^3+x^2}\,\sin\left(x+\frac{2}{x}\right) = 0$$

$$-1 \le \sin\left(x+\frac{2}{x}\right) \le 1$$

$$-\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\,\sin\left(x+\frac{2}{x}\right) \le \sqrt{x^3+x^2}$$

$$\lim_{x\to 0} -\sqrt{x^3+x^2} = 0 = \lim_{x\to 0} \sqrt{x^3+x^2}$$

$$\therefore \lim_{x\to 0} \sqrt{x^3+x^2}\,\sin\left(x+\frac{2}{x}\right) = 0$$

7

9) Evaluate the following limits "if exists"

a) $\lim_{x \to 2} \frac{\sqrt{4-x} - \sqrt{x}}{2-x}$ (b) $\lim_{x \to 1} \frac{x^2 - x}{x-1}$

c) $\lim_{x \to \infty} \sqrt{x^2+2} - x$ (d) $\lim_{x \to \infty} \frac{\cos x}{x}$

$$\lim_{x \to 2} \frac{\sqrt{4-x} - \sqrt{x}}{2-x}$$

$$= \frac{0}{0}$$

$$\lim_{x \to 2} \frac{\sqrt{4-x} - \sqrt{x}}{2-x} \cdot \frac{\sqrt{4-x} + \sqrt{x}}{\sqrt{4-x} + \sqrt{x}}$$

$$= \lim_{x \to 2} \frac{4-x-x}{(2-x)(\sqrt{4-x} + \sqrt{x})}$$

$$= \lim_{x \to 2} \frac{4-2x}{(2-x)(\sqrt{4-x} + \sqrt{x})}$$

$$= \lim_{x \to 2} \frac{2\cancel{(2-x)}}{\cancel{(2-x)}(\sqrt{4-x} + \sqrt{x})}$$

$$= \frac{2}{\sqrt{4-x} + \sqrt{x}}$$

$$= \frac{2}{\sqrt{2} + \sqrt{2}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

b) $\lim_{x \to 1} \frac{x^2 - x}{x-1} = \frac{0}{0}$

$$\lim_{x \to 1} \frac{x\cancel{(x-1)}}{\cancel{(x-1)}} = 1$$

c) $\lim_{x \to \infty} \sqrt{x^2+2} - x = \infty - \infty$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2+2} - x}{1} \cdot \frac{\sqrt{x^2+2} + x}{\sqrt{x^2+2} + x}$$

$$= \lim_{x \to \infty} \frac{\cancel{x^2} + 2 - \cancel{x^2}}{\sqrt{x^2+2} + x}$$

$$= \lim_{x \to \infty} \frac{2}{\sqrt{x^2+2} + x}$$

$$= \frac{2}{\infty} = 0$$

10) Evaluate The following limits: if exists,,

a) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin(2x-\pi)}{x-\frac{\pi}{2}}$ (b) $\lim\limits_{x \to \infty} \dfrac{(x+3)(x-2)}{(x+2)(2x+1)}$

(c) $\lim\limits_{x \to 3} \dfrac{x^2+x-12}{x^2-3x}$ (d) $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^4+3}}{3x^2-x+1}$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin(2x-\pi)}{x-\frac{\pi}{2}} = \lim_{x \to 90} \frac{\sin(2x-180)}{x-90}$$

$y = x-90$

$$\lim_{y \to 0} \frac{\sin(2(x-90))}{(x-90)} = \lim_{y \to 0} \frac{\sin 2y}{y} = 2$$

b) $$\lim_{x \to \infty} \frac{(x+3)(x-2)}{(x+2)(2x+1)} = \lim_{x \to \infty} \frac{x^2-2x+3x-6}{2x^2+x+4x+2}$$

$$= \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{3x}{x^2} - \frac{6}{x^2}}{\frac{2x^2}{x^2} + \frac{x}{x^2} + \frac{4x}{x^2} + \frac{2}{x^2}}$$

$$= \lim_{x \to \infty} \frac{1 - \cancel{\frac{2}{x}} + \cancel{\frac{3}{x}} - \cancel{\frac{6}{x^2}}}{2 + \cancel{\frac{1}{x}} + \cancel{\frac{4}{x}} + \cancel{\frac{2}{x^2}}}$$

$= \frac{1}{2}$

$$\lim_{x\to\infty} \frac{\sqrt{4x^4+3}}{3x^2-x+1}$$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{4\cancel{x^4}}{\cancel{x^4}}+\frac{3}{x^4}}}{\frac{3x^2}{x^2}-\frac{x}{x^2}+\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{4+\cancel{\frac{3}{x^4}}}}{3-\cancel{\frac{1}{x}}+\cancel{\frac{1}{x^2}}} = \frac{2}{3}$$

$$\lim_{x\to 3} \frac{x^2+x-12}{x^2-3x}$$

عامل مشترك

$$\lim_{x\to 3} \frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}\,x}$$

$$= \frac{7}{3}$$

حل آخر

$$\lim_{x\to 3} \frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}\,x} = \frac{7}{3}$$

$$\lim_{x\to 2} \frac{x^3 - 8}{x^2 - 3x + 2} = \frac{0}{0}$$

ربع اضرب اعكس ربع

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}(x-1)}$$

$$= \frac{12}{1} = 12$$

$$\lim_{x\to -1} \frac{x^2 - 1}{x^3 + 1} = \frac{0}{0}$$

$$\lim_{x\to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(x^2 - x + 1)}$$

$$= \frac{-2}{1-(-1)+1} = \frac{-2}{3}$$

4-

If $f(x) = \dfrac{x^2-4x+3}{x^2-2x-3}$

① find $\lim\limits_{x\to 3} f(x)$

② find H.A , V.A

---

① $\lim\limits_{x\to 3} \dfrac{x^2-4x+3}{x^2-2x-3} = \dfrac{0}{0}$

$\lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x+1)} = \dfrac{2}{4}$

$= \dfrac{1}{2}$

---

② Horizontal

$y = \lim\limits_{x\to\infty} f(x)$

$= \lim\limits_{x\to\infty} \dfrac{x^2-4x+3}{x^2-2x-3}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{x^2}{x^2} - \frac{4x}{x^2} + \frac{3}{x^2}}{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}}$

$y = 1$

في حال V.A لابد من الاختزال أولاً.

$$f(x) = \frac{x^2-4x+3}{x^2-2x-3}$$

$a = 1$
$b = -4$
$c = 3$

$$= \frac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x+1)}$$

V. Asy.

$$\boxed{x = -1}$$

أصفار المقام بعد الاختزال.

(14) Discuss The Continuity of f

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x\sin x} & x \neq 0 \\ 2 & x = 0 \end{cases}$$

at $x=0$

(1) $$\lim_{x\to 0} \frac{1-\cos x}{x \sin x} = \lim_{x\to 0} \frac{1-(1-2\sin^2\frac{x}{2})}{x\sin x}$$ ← قانون ضعف الزاوية

$$= \lim_{x\to 0} \frac{\cancel{1}-\cancel{1}+2\sin^2\frac{x}{2}}{x\sin x} = \lim_{x\to 0} \frac{2\sin\frac{x}{2}}{x} \cdot \frac{\sin\frac{x}{2}}{\sin x}$$

$$= \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{2} = \frac{1}{2}$$

(2) $f(0) = 2$

$$\because f(0) \neq \lim_{x\to 0} f(x)$$

$\therefore$ dis Continuous.

في المثال السابق اذا طلب redefine اعد التعريف

$f(0) = \frac{1}{2}$

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x\sin x} & x \neq 0 \\ \frac{1}{2} & x = 0 \end{cases}$$

(17) Discuss The Continuity

a) $f(x) = \frac{x^2-9}{x-3}$

b) $f(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 4 & x = 3 \end{cases}$

c) $h(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 6 & x = 3 \end{cases}$

a) $f(x) = \frac{x^2-9}{x-3}$

$x \neq 3$

continuous , $x \in \mathbb{R} - \{3\}$

b) $f(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 4 & x = 3 \end{cases}$

$\lim_{x \to 3} \frac{x^2-9}{x-3} = \frac{0}{0}$

$\lim_{x \to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = 6$

$f(3) = 4$

dis cont.

$h(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 6 & x = 3 \end{cases}$

$\lim_{x \to 3} = 6$

$f(3) = 6$

$\therefore$ Cont.

ABU OMAR MASRAWY 0536260004

(18) Find The values of a and b where The function

$$f(x) = \begin{cases} ax^2 + 1 & x > 2 \\ -11 & x = 2 \\ x^3 + b & x < 2 \end{cases}$$

is Continuous on $(-\infty, \infty)$

$\lim_{x \to 2^+} ax^2 + 1 = 4a + 1 \longrightarrow (1)$

$\lim_{x \to 2^-} x^3 + b = 8 + b \longrightarrow (2)$

$\boxed{f(2) = -11} \longrightarrow (3)$

from (1, 3) $\quad 4a + 1 = -11$

$4a = -12$

$\boxed{a = -3}$

from (2, 3) $\quad 8 + b = -11$

$\boxed{b = -19}$

9) Evaluate the following limits « if exists »

a) $\lim_{x\to 2} \frac{\sqrt{4-x}-\sqrt{x}}{2-x}$ (b) $\lim_{x\to 1} \frac{x^2-x}{x-1}$

c) $\lim_{x\to\infty} \sqrt{x^2+2}-x$ (d) $\lim_{x\to\infty} \frac{\cos x}{x}$

a) $\lim_{x\to 2} \frac{\sqrt{4-x}-\sqrt{x}}{2-x} = \frac{\sqrt{2}-\sqrt{2}}{2-2} = \frac{0}{0}$

$= \lim_{x\to 2} \frac{\sqrt{4-x}-\sqrt{x}}{2-x} \cdot \frac{\sqrt{4-x}+\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}$

$= \lim_{x\to 2} \frac{4-x-x}{(2-x)(\sqrt{4-x}+\sqrt{x})} = \lim_{x\to 2} \frac{4-2x}{(2-x)(\sqrt{4-x}+\sqrt{x})}$

$= \lim_{x\to 2} \frac{2\cancel{(2-x)}}{\cancel{(2-x)}(\sqrt{4-x}+\sqrt{x})} = \frac{2}{\sqrt{2}+\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\lim_{x\to\infty} \frac{\sqrt{x^2+2}-x}{1} \cdot \frac{\sqrt{x^2+2}+x}{\sqrt{x^2+2}+x}$

$= \lim_{x\to\infty} \frac{\cancel{x^2}+2-\cancel{x^2}}{\sqrt{x^2+2}+x} = \frac{2}{\sqrt{x^2+2}+x} = \frac{2}{\infty} = 0$

حل آخر

$\lim_{x\to\infty} \frac{\frac{2}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{2}{x^2}}+\frac{x}{x}} = \frac{0}{\sqrt{1+0}+1} = \frac{0}{2} = 0$

10) Evaluate the following limits, if exists.

a) $\lim_{x \to \frac{\pi}{2}} \frac{\sin(2x - \pi)}{x - \frac{\pi}{2}}$

b) $\lim_{x \to \infty} \frac{(x+3)(x-2)}{(x+2)(2x+1)}$

c) $\lim_{x \to 3} \frac{x^2 + x - 12}{x^2 - 3x}$

d) $\lim_{x \to \infty} \frac{\sqrt{4x^4 + 3}}{3x^2 - x + 1}$

a) $\lim_{x \to 90} \frac{\sin(2x - 180)}{x - 90}$

$\theta = x - 90$

$\lim_{x \to 90} \frac{\sin 2(x - 90)}{x - 90} = \lim_{\theta \to 0} \frac{\sin 2\theta}{\theta} = \frac{2}{1} = 2$

b) $\lim_{x \to \infty} \frac{(x+3)(x-2)}{(x+2)(2x+1)} = \lim_{x \to \infty} \frac{(\frac{x}{x} + \frac{3}{x})(\frac{x}{x} - \frac{2}{x})}{(\frac{x}{x} + \frac{2}{x})(\frac{2x}{x} + \frac{1}{x})}$

$= \lim_{x \to \infty} \frac{(1 + \cancel{\frac{3}{x}})(1 - \cancel{\frac{2}{x}})}{(1 + \cancel{\frac{2}{x}})(2 + \cancel{\frac{1}{x}})} = \frac{1(1)}{1(2)} =$

$= \frac{1}{2}$

c) $\lim_{x \to 3} \frac{x^2 + x - 12}{x^2 - 3x} = \frac{0}{0}$

$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}x} = \frac{7}{3}$

d) $\lim_{x \to \infty} \frac{\sqrt{4x^4 + 3}}{3x^2 - x + 1} = \lim_{x \to \infty} \frac{\sqrt{4\frac{x^4}{x^4} + \frac{3}{x^4}}}{\frac{3x^2}{x^2} - \frac{x}{x^2} + \frac{1}{x^2}}$

$= \lim \frac{\sqrt{4 + \cancel{\frac{3}{x^4}}}}{3 - \cancel{\frac{1}{x}} + \cancel{\frac{1}{x^2}}} = \frac{\sqrt{4}}{3} = \frac{2}{3}$

(15) Use the definition of the slope to Evaluate the equation of the tangent to the curve

$$f(x) = x^2 - x + 3 \quad \text{at } x = 1$$

* $x_0 = 1$, $y_0 = (1)^2 - 1 + 3 = 3$

point $(1, 3)$

* $m = \lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0} \dfrac{(1+h)^2 - (1+h) + 3 - 3}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{1} + 2h + h^2 - \cancel{1} - h \cancel{+3} \cancel{-3}}{h}$

$= \lim_{h \to 0} \dfrac{h + h^2}{h} = \lim_{h \to 0} \dfrac{\cancel{h}(1+h)}{\cancel{h}}$

$m = 1$

$y - y_0 = m(x - x_0)$

$y - 3 = 1(x - 1)$

$y = x - 1 + 3 = x + 2$

16) نفس المسألة السابقة

Do the same with $f(x) = \sqrt{x-3}$ at $x = 4$

$x_0 = 4 \longrightarrow y_0 = \sqrt{4-3} = 1$

point $(4,1)$

$$m = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{4+h-3} - 1}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1}$$

$$= \lim_{h\to 0} \frac{\cancel{1} + \cancel{h} - \cancel{1}}{\cancel{h}(\sqrt{1+h} + 1)} = \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

$m = \frac{1}{2}$

$$y - 1 = \frac{1}{2}(x - 4)$$

$$y = \frac{1}{2}(x-4) + 1$$

$$= \frac{1}{2}x - 2 + 1$$

$$y = \frac{1}{2}x - 1$$

مع العلم ان الطلب $\frac{dy}{dx}$ هي مثل ال $\boxed{m}$ بالضبط.

(24)

Use The Trigonometry to show That

$$\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x = 1$$

L.H.S: $(\sin^2 x + \cos^2 x)(\sin^2 x + \cos^2 x)$

$= 1 \cdot 1 = 1$

---

prove That $1 + \cot^2 x = \csc^2 x$

$$\frac{1}{1} + \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$= \csc^2 x.$$

---

(25) Find The av. velocity and inst. velocity of a particle moving according to

$r(t) = \sqrt{2t+3}$ at $t = 3$,

$$S_{av} = \frac{s(t+h) - s(t)}{h} = \frac{\sqrt{2t+2h+3} -}{}$$

$$S_{av} = \frac{s(t+h) - s(t)}{h}$$

$$= \frac{s(3+h) - s(3)}{h}$$

$$= \frac{\sqrt{2(3+h)+3} - 3}{h}$$

$$S_{av} = \frac{\sqrt{6+2h+3} - 3}{h} = \frac{\sqrt{9+2h} - 3}{h}$$

S_{ins}

السرعة اللحظية

$$= \lim_{h \to 0} \frac{\sqrt{9+2h} - 3}{h} \cdot \frac{\sqrt{9+2h}+3}{\sqrt{9+2h}+3}$$

$$= \lim_{h \to 0} \frac{\cancel{9}+2h-\cancel{9}}{h(\sqrt{9+2h}+3)}$$

$$= \frac{2}{6} = \frac{1}{3}$$

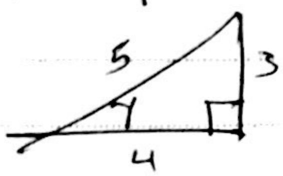
21

Find the exact value of

$$\cos\left(\tan^{-1}\frac{3}{4}\right) + \sin\left(\cos^{-1}\frac{2}{\sqrt{5}}\right)$$

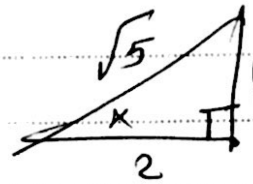
$y = \tan^{-1}\frac{3}{4}$

$\tan y = \frac{3}{4}$



$x = \cos^{-1}\frac{2}{\sqrt{5}}$

$\cos x = \frac{2}{\sqrt{5}}$

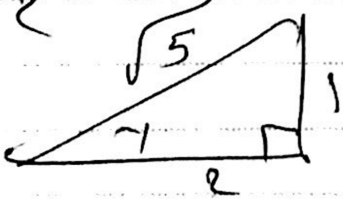


$\cos y + \sin x$

$= \frac{4}{5} + \frac{1}{\sqrt{5}}$

$\sec\left(\tan^{-1}\frac{1}{2}\right)$

$y = \tan^{-1}\frac{1}{2} \Longrightarrow \tan y = \frac{1}{2}$

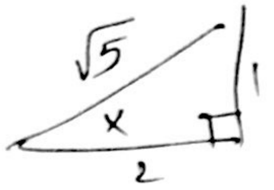


$\sec y = \frac{1}{\cos y} = \frac{\sqrt{5}}{2}$

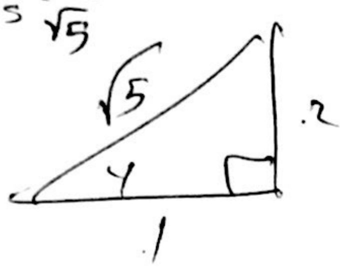
(23) If $\tan x = \frac{1}{2}$, $\sin y = \frac{2}{\sqrt{5}}$ Then find

a) $\sin(x+y)$ b) $\sin 2x$ c) $\tan 2y$

$\tan x = 1/2$



$\sin y = \frac{2}{\sqrt{5}}$



$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}}$$

$$= \frac{1}{5} + \frac{4}{5} = 1$$

$$\sin 2x = 2 \sin x \cos x$$

$$= 2 \cdot \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} = \frac{4}{5}$$

$$\tan 2y = \frac{2\tan y}{1-\tan^2 y} = \frac{2 \cdot \frac{2}{1}}{1-4}$$

$$= \frac{4}{1-4} = \frac{4}{-3}$$

$$= -\frac{4}{3}$$

(22)

Solve The equation

$4\sin^2 x = 1$, $x \in [0, 2\pi]$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm \frac{1}{2}$

احسب الربع الأول
بدون سالب
shift $\sin \frac{1}{2} = 30$

1st $x = 30^\circ$

2nd $180^\circ - 30^\circ = 150^\circ$

S $180 - \theta$	A θ
T $180 + \theta$	C $360 - \theta$

3ed $180 + 30^\circ = 210^\circ$

4Th $360 - 30^\circ = 330^\circ$

$\tan x = -\frac{1}{\sqrt{2}}$

اهمل الاشارة
مؤقتا

shift $\tan \frac{1}{\sqrt{2}}$

$x = 35^\circ$

S	A +
+ T	C

2nd $180 - 35$

4Th $360 - 35$