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(1)	Two engines operate independently, if the probability that an engine will start is 0.4, and the probability that other engine will start is 0.6, then the probability that both will start is:							
	(A)	1	(B)	<u>0.24</u>	(C)	0.2	(D)	0.5

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(2)	If $P(B) = 0.3$ and $P(A B) = 0.4$, then $P(A \cap B)$ equal to;							
	(A)	0.67	(B)	<u>0.12</u>	(C)	0.75	(D)	0.3

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(3)	The probability that a computer system has an electrical failure is 0.15, and the probability that it has a virus is 0.25, and the probability that it has both problems is 0.20, then the probability that the computer system has the electrical failure or the virus is:							
	(A)	1.15	(B)	<u>0.2</u>	(C)	0.15	(D)	0.35

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Two brothers, Ahmad and Mohammad, are the owners and operators of a small restaurant. Ahmad and Mohammad alternate between the jobs of cooking and dish washing, so that at any time, the probability that Ahmad is washing the dishes is 0.50, and Mohammad is also 0.5. The probability that Mohammad breaks a dish is 0.40. On the other hand, the probability that Ahmad breaks a dish is only 0.10. Then,

(4)	the probability that a dish will be broken is:							
	(A)	0.667	(B)	<u>0.25</u>	(C)	0.8	(D)	0.5
(5)	If there is a broken dish in the kitchen of the restaurant. The probability that it was washed by Mohammad is:							
	(A)	0.667	(B)	0.25	(C)	<u>0.8</u>	(D)	0.5

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(6)	From a box containing 4 black balls and 2 green balls, 3 balls are drawn in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:							
	(A)	<u>6/27</u>	(B)	2/27	(C)	12/27	(D)	4/27

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(7)	The value of k, that makes the function $f(x) = k \binom{2}{x} \binom{3}{3-x} \text{ For } x=0,1,2$ serve as a probability distribution of the discrete random variable X;							
	(A)	<u>1/10</u>	(B)	1/9	(C)	1	(D)	1/7

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The cumulative distribution of a discrete random variable, X, is given below:

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/16 & \text{for } 0 \leq x < 1 \\ 5/16 & \text{for } 1 \leq x < 2 \\ 11/16 & \text{for } 2 \leq x < 3 \\ 15/16 & \text{for } 3 \leq x < 4 \\ 1 & \text{for } x \geq 4. \end{cases}$$

(8)	the $P(X = 2)$ is equal to:							
	(A)	<u>3/8</u>	(B)	11/16	(C)	10/16	(D)	5/16
(9)	the $P(2 \leq X < 4)$ is equal to:							
	(A)	20/16	(B)	11/16	(C)	<u>10/16</u>	(D)	5/16

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(10)	The proportion of people who respond to a certain mail-order is a continuous random variable X that has the density function							
	$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$							
	Then, the probability that more than $\frac{1}{4}$ but less than $\frac{1}{2}$ of the people contacted will respond to the mail-order is:							
	(A)	<u>19/80</u>	(B)	1/2	(C)	1/4	(D)	81/400

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Suppose the failure time (in hours) of a specific type of electrical device is distributed with a probability density function:

$$f(x) = \frac{1}{50}x, \quad 0 < x < 10$$

then,

(11)	the average failure time of such device is:							
	(A)	<u>6.667</u>	(B)	1.00	(C)	2.00	(D)	5.00
(12)	the variance of the failure time of such device is:							
	(A)	0	(B)	50	(C)	<u>5.55</u>	(D)	10

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A random variable X has a mean of 10 and a variance of 4, then, the random variable $Y = 2X - 2$,

(13)	has a mean of:							
	(A)	10	(B)	<u>18</u>	(C)	20	(D)	22
(14)	and a standard deviation of:							
	(A)	6	(B)	2	(C)	<u>4</u>	(D)	16

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(15)	The probability distribution of X, the number of typing errors committed by a typist is:																
	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$f(x)$</td> <td>0.41</td> <td>0.37</td> <td>0.16</td> <td>0.05</td> <td>0.01</td> </tr> </table>					x	0	1	2	3	4	$f(x)$	0.41	0.37	0.16	0.05	0.01
x	0	1	2	3	4												
$f(x)$	0.41	0.37	0.16	0.05	0.01												
	Then the average number of errors for this typist is:																
	(A)	2	(B)	0.88	(C)	1.28	(D)	4									

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If the random variable X has an exponential distribution with the mean 4, then

(16)	$P(X < 8)$ equals to:							
	(A)	0.2647	(B)	0.4647	(C)	<u>0.8647</u>	(D)	0.6647
(17)	the variance of X is:							
	(A)	4	(B)	<u>16</u>	(C)	2	(D)	1/4

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If the random variable X has a normal distribution with the mean 10 and the variance 36, then

(18)	the value of X above which an area of 0.2296 lie is:							
	(A)	<u>14.44</u>	(B)	16.44	(C)	10.44	(D)	18.44
(19)	the probability that the value of X is greater than 16 is:							
	(A)	0.9587	(B)	<u>0.1587</u>	(C)	0.7587	(D)	0.0587

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(20)	Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 65 and the variance 16. A student fails the exam if he obtains a mark less than 60. Then the percentage of students who fail the exam is:							
	(A)	20.56%	(B)	90.56%	(C)	50.56%	(D)	<u>10.56%</u>

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In a certain industrial facility accidents occur infrequently. If the probability of an accident on a given day is p , and accidents are independent of each other. **If $p = 0.2$** , then

(21)	probability that within seven days there will be at most two accidents will occur is:							
	(A)	<u>0.7865</u>	(B)	<u>0.4233</u>	(C)	0.5767	(D)	0.6647
(22)	probability that within seven days there will be at least three accidents will occur is:							
	(A)	0.7865	(B)	0.2135	(C)	0.5767	(D)	0.1039
(23)	the expected number of accidents to occur within this week is:							
	(A)	<u>1.4</u>	(B)	0.2135	(C)	2.57	(D)	0.59

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The number of traffic accidents per week in a small city has a Poisson distribution with mean equal to 1.3. Then,

(24)	the probability of at least two accidents in 2 weeks is:							
	(A)	0.2510	(B)	0.3732	(C)	0.5184	(D)	<u>0.7326</u>
(25)	the standar diviation of traffic accidents per week in the small city is:							
	(A)	<u>1.14</u>	(B)	1.30	(C)	1.69	(D)	3.2

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A study was made by a taxi company to decide whether the use of new tires (A) instead of the present tires (B) improves fuel economy. Six cars were equipped with tires (A) and driven over a prescribed test course. Without changing drivers and cars, test course was made with tires (B). The gasoline consumption, in kilometers per liter (km/L), was recorded as follows: (assume the population to be normally distributed with unknown variances and are equal)

Car	1	2	3	4	5	6
Type (A)	4.5	4.8	6.6	7.0	6.7	4.6
Type (B)	3.9	4.9	6.2	6.5	6.8	4.1

(26)	A 95% confidence interval for the true mean gasoline brand A consumption is:					
	(A)	$4.462 \leq \mu_A \leq 6.938$			(B)	$2.642 \leq \mu_A \leq 4.930$
	(C)	$5.2 \leq \mu_A \leq 9.7$			(D)	$6.154 \leq \mu_A \leq 6.938$
(27)	A 99% confidence interval for the difference between the true mean of type (A) and type (B) ($\mu_A - \mu_B$) is:					
	(A)	$-1.939 \leq \mu_A - \mu_B \leq 2.539$			(B)	$-2.939 \leq \mu_A - \mu_B \leq 1.539$
	(C)	$0.939 \leq \mu_A - \mu_B \leq 1.539$			(D)	$-1.939 \leq \mu_A - \mu_B \leq 0.539$

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A food company distributes two brands of milk. If it is found that 80 of 200 consumers prefer brand A and that 90 of 300 consumers prefer brand B,

(28)	96% confidence interval for the true proportion of brand (A) is:					
	(A)	$0.328 \leq p_A \leq 0.375$			(B)	$0.228 \leq p_A \leq 0.675$
	(C)	$0.328 \leq p_A \leq 0.475$			(D)	$0.518 \leq p_A \leq 0.875$
(29)	A 99% confidence interval for the true difference in the proportion of brand (A) and (b), is:					
	(A)	$0.0123 \leq p_A - p_B \leq 0.212$			(B)	$-0.2313 \leq p_A - p_B \leq 0.3612$
	(C)	$-0.0023 \leq p_A - p_B \leq 0.012$			(D)	$-0.0123 \leq p_A - p_B \leq 0.212$
(30)	If the value of α decrease (get smaller), then the interval estimate will decrease (get smaller);					
	(A)	Yes	(B)	No	(C)	No change

Department of Statistics and Operations Research
College of Science
King Saud University



First-term 1424/1425
Second Mid-term Exam

Probability and Statistics for Engineering
Time: 2 hours Total 30 Marks

Student name:

Student number:

Serial number ()

Instructor: Dr.

Section Number:

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Question 1

I. Given a standard normal distribution Z ,

- 1) the area under the curve to the left of $z = 1.43$ is:
(A) 0.0764 (B) 0.9236 (C) 0 (D) 0.8133
- 2) the area under the curve to the right of $z = -0.89$ is:
(A) 0.7815 (B) 0.8133 (C) 0.1867 (D) 0.0154
- 3) the area under the curve between $z = -2.16$ and $z = -0.65$ is:
(A) 0.7576 (B) 0.8665 (C) 0.0154 (D) 0.2424
- 4) the value of k such that $P(0.93 < Z < k) = 0.0427$ is:
(A) 0.8665 (B) -1.11 (C) 1.11 (D) 1.00

II. The finished inside diameter of a piston ring is normally distributed with a mean 12 centimeters and a standard deviation of 0.03 centimeter. Then,

- 5) the proportion of rings that will have inside diameter less than 12.05 centimeters is:
(A) 0.0475 (B) 0.9525 (C) 0.7257 (D) 0.8413
- 6) the proportion of rings that will have inside diameter exceeding 11.97 centimeters is:
(A) 0.0475 (B) 0.8413 (C) 0.1587 (D) 0.4514
- 7) the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters is:
(A) 0.905 (B) -0.905 (C) 0.4514 (D) 0.7257

Question 2

- 8) The average life of a certain type of small motor is 10 years with a standard deviation of 2 years. Assume the live of the motor is normally distributed. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 1.5% of the motors that fail, then he should give a guarantee of :
(A) 10.03 years (B) 8 years (C) 5.66 years (D) 3 years

Question 3

Suppose the failure time (in hours) of a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress is distributed with a probability density function:

$$f(x) = \frac{1}{70} e^{-x/70}, \quad x > 0,$$

Then,

- 9) the probability that a randomly selected insulation will fail within the first 50 hours is:
 (A) 0.4995 (B) 0.7001 (C) 0.5105 (D) 0.2999
- 10) the probability that a randomly selected insulation will last more than 150 hours is:
 (A) 0.8827 (B) 0.2788 (C) 0.1173 (D) 0.8827
- 11) the average failure time of the electrical insulation is:
 (A) 1/70 (B) 70 (C) 140 (D) 35
- 12) the variance of the failure time of the electrical insulation is:
 (A) 4900 (B) 1/49000 (C) 70 (D) 1225

Question 4

The average life of a manufacturer's blender is 5 years, with a standard deviation of 1 year. Assuming the live of the battery follows approximately a normal distribution. So,

- 13) If a random sample of 5 batteries (to be selected from this production) has a mean of 3 years with a standard deviation of one year, then: the random variable \bar{X} (the mean of all possible samples of size 5 batteries) has a mean $\mu_{\bar{X}}$ equal to:
 (A) 0.2 (B) 5 (C) 3 (D) None of these
- 14) the variance $\sigma_{\bar{X}}^2$ of this random sample is equal to:
 (A) 0.2 (B) 5 (C) 3 (D) None of these

- 15) the probability that the mean life of a random sample of size 16 of such batteries will be between 4.5 and 5.4 years is:
(A) 0.1039 (B) 0.2135 (C) 0.7865 (D) None of these
- 16) the probability that the mean life of a random sample of size 16 of such batteries will be less than 5.5 years is:
(A) 0.9772 (B) 0.0228 (C) 0.9223 (D) None of these
- 17) the probability that the mean life of a random sample of size 16 of such batteries will be not less than 4.75 years is:
(A) 0.8413 (B) 0.1587 (C) 0.9452 (D) None of these
- 18) If $P(\bar{X} > a) = 0.1492$ where \bar{X} represents the sample mean for a random sample of size 9 to be selected from the production lot, then the numerical value of a is:
(A) 4.653 (B) 6.5 (C) 5.347 (D) None of these

Question 5

I. A machine is producing metal pieces that are cylindrical in shape. A sample is taken and the diameters are 1.70, 2.11, 2.20, 2.31 and 2.28 centimeters. Then,

- 19) The sample mean is:
(A) 2.22 (B) 2.32 (C) 2.90 (D) 2.20 (E) None of these
- 20) The sample variance is:
(A) 0.597 (B) 0.285 (C) 0.356 (D) 0.534 (E) None of these

II. The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

- 21) To be 95% confident that the error of our estimate of the mean of tensile strength will be less than 3.4 kilograms, the minimum sample size should be:
(A) 4 (B) 16 (C) 20 (D) 18 (E) None of these
- 22) For 98% confidence interval for the mean of tensile strength we have the lower bound equal to:
(A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these

- 23) For 98% confidence interval for the mean of tensile strength we have the upper bound equal to:
- (A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

III. The tensile strength of type II thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 25 pieces of the thread has an average tensile strength of 64.4 kilograms. Then for the 98% confidence interval of the difference in tensile strength means between type I and type II , we have:

- 24) the lower bound equal to:
- (A) 2.90 (B) 4.21 (C) 3.65 (D) 6.58 (E) None of these
- 25) the upper bound equal to:
- (A) 13.90 (B) 13.15 (C) 12.59 (D) 10.22 (E) None of these

Question 6

I. In a study involved 1200 car drivers it was found that 50 car drivers do not use seat belt. Then,

- 26) A point estimate for the proportion of car drivers that do not use seat built is:
- (A) 50 (B) 0.0417 (C) 0.9583 (D) 1150 (E) None of these

II. Using part (I), the 95% confidence interval of the proportion of car drivers that do not use seat built has

- 27) the lower bound equal to:
- (A) 0.0322 (B) 0.0416 (C) 0 .0304 (D) -0.3500 (E) None of these
- 28) the upper bound equal to:
- (A) 0.0417 (B) 0.0530 (C) 0.0512 (D) 0.4333 (E) None of these

Question No. 1:

Suppose a fair die is thrown twice, then

(1) the probability that the sum of numbers of two dice is less than or equal to 4 is;

- (A) 0.1667
- (B) 0.6667
- (C) 0.8333
- (D) 0.1389

(2) the probability that at least one of the die shows 4 is;

- (A) 0.6667
- (B) 0.3056
- (C) 0.8333
- (D) 0.1389

(3) the probability that the sum of two dice is 4 and one of them shows 1;

- (A) 0.0556
- (B) 0.6667
- (C) 0.8333
- (D) 0.1389

(4) the event $A = \{\text{the sum of two dice is 4}\}$ and the event $B = \{\text{At least one die shows 2}\}$, then $P(B|A)$ equal to,

- (A) 0.8333
- (B) 0.6667
- (C) 0.3333
- (D) 0.1389

(5) the event $A = \{\text{the sum of two dice is 4}\}$ and the event $B = \{\text{exactly one die shows 2}\}$ are,

- (A) Independent
- (B) Dependent
- (C) Disjoint
- (D) None of these.

Question No. 2:

A man wants to paint his house in 3 colors. He can choose out of 6 colors. Then,

(6) the number of color settings he can make is,

- (A) 216
- (B) 20
- (C) 120
- (D) 10

(7) If he selected one color, then the number of color settings he can make is,

- (A) 216
- (B) 20
- (C) 120
- (D) 10

Question No. 3:

A random sample of 200 adults is classified according to sex and their level of education in the following table:

<i>Education</i>	<i>Male</i>	<i>Female</i>
<i>Elementary</i>	28	50
<i>Secondary</i>	38	45
<i>College</i>	22	17

If a person is selected at random from this group, then:

(8) the probability that he is a male is:

- (A) 0.3182
- (B) 0.44
- (C) 0.66
- (D) 88

(9) The probability that the person is male given that the person has a secondary education is:

- (A) 0.4318
- (B) 0.19
- (C) 0.66
- (D) 0.4578

(10) The probability that the person does not have a college degree given that the person is a female is:

- (A) 0.8482
- (B) 0.1518
- (C) 0.475
- (D) 0.085

Question No. 4:

Two brothers, Ed and Jim, are the owners and operators of a small restaurant. Ed and Jim alternate between the jobs of cooking and dish washing, so that at any time, the probability that Ed is washing the dishes is



0.50. Jim, the younger of the two brothers, is a bit clumsy. When Jim is washing the dishes, the probability that Jim breaks a dish he is washing is 0.40. Ed, on the other hand, is very careful and the probability that Ed breaks a dish he is washing is only 0.10.

(11) The probability that a dish will be broken is

- (A) 0.667
- (B) 0.25
- (C) 0.8
- (D) 0.5

(12) There is a broken dish in the kitchen of the restaurant. The probability that it was washed by Jim is;

- (A) 0.667
- (B) 0.25
- (C) 0.8
- (D) 0.5

(13) Suppose Ed and Jim want the probability of a broken dish to equal 0.20. then, the probability that Ed washes the dishes is,

- (A) 0.667
- (B) 0.25
- (C) 0.8
- (D) 0.5

Question No. 5:

(14) Two engines operate independently, if the probability that an engine will start is 0.4, and the probability that other engine will start is 0.6, then the probability that both will start is:

- (A) 1
- (B) 0.24
- (C) 0.2
- (D) 0.5

(15) If $P(B) = 0.3$ and $P(A|B) = 0.4$, then $P(A \cap B)$ equal to;

- (A) 0.67
- (B) 0.12
- (C) 0.75
- (D) 0.3

Question No. 6:

A random variable X takes the values 0, 1, 2. Assume that $E(X) = \frac{3}{2}$ and $\sigma = \frac{1}{2}$, then

(16) $E(X^2) =$

- (A) 1/4
- (B) 10/4
- (C) 9/4
- (D) 2

(17) $E(2X + 3) =$

- (A) 6
- (B) 5
- (C) 3
- (D) 1/2

(18) $E(5X^2 - 2X) =$

- (A) 50/4
- (B) 19/2
- (C) 41/3
- (D) 1/3

(19) $Var(X + 1) =$

- (A) 5/4
- (B) 3/4
- (C) 1/4
- (D) 5/2

(20) $Var(2 - 3X) =$

- (A) 1/4
- (B) 10/4
- (C) 9/4
- (D) 10/3

(21) $P(X = 0) =$

- (A) 1/4
- (B) 1/2
- (C) 1/3
- (D) 0

(22) $P(0 < X < 2) =$

- (A) 1/4
- (B) 1/3
- (C) 1/2
- (D) 0

(23) $P(X \leq 1) =$

- (A) 1/4
- (B) 5/4
- (C) 1/2
- (D) 1/3

Question No. 7:

Let X be a continuous random variable with probability density function is given by

$$f(x) = \begin{cases} c(1-x), & 0 < x < 1, \\ 0, & \text{otherwise} \end{cases}$$

(24) The values of c is

- (A) 1/4
- (B) 2
- (C) 1/2
- (D) 1

(25) $E(X) =$

- (A) 1/4
- (B) 1/3
- (C) 9/4
- (D) 1/2

(26) $Var(X) = \sigma^2 =$

- (A) 1/18
- (B) 1/9
- (C) 1/27
- (D) 1/3

(27) $P(X = 0) =$

- (A) 1
- (B) 0
- (C) 1/2
- (D) 1/6

(28) $P(1/5 < X < 1) =$

- (A) 1/24
- (B) 10/24
- (C) 15/25
- (D) 16/25

(29) $P(|X - \mu| < 2\sigma) =$

- (A) 0.76
- (B) 0.96
- (C) 0.90
- (D) 0.82

(30) By using Chebyshev's theorem, then

$P(|X - \mu| < 2\sigma) =$

- (A) $\leq 1/4$
- (B) $\geq 10/4$
- (C) $\leq 3/4$
- (D) $\geq 3/4$

 THE END



Question No. 1.

The following data show the number of traffic accidents at a particular intersection for five months: 8, 5, 2, 9, 4.

(1). The sample mean \bar{X} equals:

- (A) 28.0
- (B) 5.60
- (C) 7.00
- (D) 5.00

(2). The sample variance S^2 equals:

- (A) 8.30
- (B) 6.64
- (C) 44.09
- (D) 68.89

Question No. 2.

In a photographic process, the developing time of prints may be considered as a random variable having the normal distribution with a mean of 16.28 second and a standard deviation of 0.12 second. Then, the probability that the developing time to develop one of the prints will be:

(3). anywhere from 16 to 16.5 second equals:

- (A) 0.0435
- (B) 0.1762
- (C) 0.9565
- (D) 0.2018

(4). at least 16.20 second equals:

- (A) 0.7486
- (B) 0.34221
- (C) 0.6502
- (D) 0.2514

(5). at most 16.35 second equals:

- (A) 0.3101
- (B) 0.7190
- (C) 0.2810
- (D) 0.4053

Question No. 3.

(6). If $Z \sim N(0, 1)$, then $P(-1.33 \leq Z \leq 2.42)$ equals:

- (A) 0.4521
- (B) 0.9004
- (C) 0.2315
- (D) 0.4009

(7). Suppose that $Z \sim N(0, 1)$. The value of k such that $P(Z \leq k) = 0.0207$ equals:

- (A) -2.04
- (B) 2.04
- (C) 4.02
- (D) -4.02

(8). The t - value with degree of freedom $\nu = 14$ that leaves an area of 0.95 to the left equals:

- (A) 1.671
- (B) 1.215
- (C) 1.761
- (D) 2.312

Question No. 4.

The life of a certain tire brand lives is a random variable X that follows the exponential distribution with a mean of 2 years.

(9). For $x > 0$, the cumulative distribution function (CDF) for the random variable X is:

- (A) e^{-2}
- (B) $1 - e^{-2}$
- (C) e^{-x}
- (D) $1 - e^{-\frac{x}{2}}$

(10). The probability that a tire of this brand will live less than 1.5 years is:

- (A) 0.9534
- (B) 0.3935
- (C) 0.6065
- (D) 0.5276

(11). The probability that a tire of this brand will live at least 3 years is:

- (A) 0.6358
- (B) 0.2231
- (C) 0.4905
- (D) 0.3679

Question No. 5.

Suppose that the number of traffic violation tickets issued by a policeman has a Poisson distribution with an average of 2.5 tickets per day.

(12). The average number of tickets issued by this policeman for a period of two days is:

- (A) 2.00
- (B) 1.25
- (C) 2.50
- (D) 5.00

(13). The probability that this policeman will issue 2 tickets in a period of two days is:

- (A) 0.1404
- (B) 0.2565
- (C) 0.0842
- (D) 0.1755

Question No. 6.

Suppose that 25% of the products of a manufacturing process are defective. Three items are selected at random, inspected, and classified as defective (D) or non-defective (N).

(14). The expected number of defective items equals:

- (A) 0.75
- (B) 0.70
- (C) 0.25
- (D) 1.20

(15). The variance of the number of defective items equals:

- (A) 0.3425
- (B) 0.6525
- (C) 0.2556
- (D) 0.5625

(16). The probability of getting at least two defective items equals:

- (A) $\frac{10}{64}$
- (B) $\frac{63}{64}$
- (C) $\frac{5}{64}$
- (D) $\frac{1}{64}$

(17). The probability of getting at most two defective items equals:

- (A) $\frac{63}{64}$
- (B) $\frac{5}{64}$
- (C) $\frac{1}{64}$
- (D) $\frac{10}{64}$

Question No. 7.

If X_1, X_2, \dots, X_n is a random sample of size n from any population with mean μ and finite variance σ^2 . Denote the sample mean by \bar{X} .

(18). If the sample size $n \geq 30$ then \bar{X} has approximately a normal distribution with *mean* and *variance* respectively equals:

- (A) μ and σ
- (B) μ and σ^2
- (C) μ and σ/\sqrt{n}
- (D) μ and σ^2/n

Question No. 8.

Suppose that we have two populations. The first with mean μ_1 and variance σ_1^2 and the second with mean μ_2 and variance σ_2^2 . We select a random sample of size n_1 from the first population and another sample of size n_2 from the second population (assume that these two samples are independent). Let \bar{X}_1 be the sample mean of the first sample and \bar{X}_2 be the sample mean of the second sample. If n_1 and n_2 are large then:

(19). The expected value (mean) of $\bar{X}_1 - \bar{X}_2$ is:

- (A) $\mu_1 + \mu_2$
- (B) $\sqrt{\mu_1 - \mu_2}$
- (C) $\mu_1 - \mu_2$
- (D) $\sqrt{\mu_1} - \sqrt{\mu_2}$

(20). The variance of $\bar{X}_1 - \bar{X}_2$ is:

- (A) $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
- (B) $\sqrt{\frac{\sigma_1^2}{n_1}} + \sqrt{\frac{\sigma_2^2}{n_2}}$
- (C) $\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$
- (D) $\frac{\sigma_1}{\sqrt{n_1}} + \frac{\sigma_2}{\sqrt{n_2}}$

Question No. 9.

Lots of 40 components each are called acceptable if they contain no more than 3 defectives. The procedure for sampling the lot is to select 5 components at random (without replacement) and to reject the lot if a defective is found.

(21). the probability that exactly one defective is found in the sample if there are 3 defectives in the entire lot equals:

- (A) 0.1103
- (B) 0.3011
- (C) 0.1013
- (D) 0.3110

(22). the expected value (mean) of the number of defectives in the sample equals:

- (A) 0.375
- (B) 0.213
- (C) 0.821
- (D) 0.735

(23). the variance of the number of defectives in the sample equals:

- (A) 0.113298
- (B) 0.311298
- (C) 0.251471
- (D) 0.174251

Question No. 10.

Suppose that the hemoglobin level for healthy adults males has a normal distribution with mean $\mu = 16$ and variance $\sigma^2 = 0.81$.

(24). the probability that a randomly chosen healthy adult male has hemoglobin level less than 14 equals:

- (A) 0.1032
- (B) 0.2013
- (C) 0.0132
- (D) 0.3210

(25). the percentage of healthy adult males who have hemoglobin level less than 14 equals:

- (A) 2.45 %
- (B) 4.21 %
- (C) 1.25 %
- (D) 1.32 %

Question No. 11.

A machine which manufactures a part for a car engine was observed over a period of time before a random sample of 300 parts was selected from those produced by this machine. Out of the 300 parts, 15 were defective. Then

(26). The proportion of defective parts in the sample equals:

- (A) 15.0
- (B) 0.15
- (C) 0.05
- (D) 300

(27). The standard deviation of the proportion of defective parts in the sample equals:

- (A) 0.01258
- (B) 0.26314
- (C) 0.02136
- (D) other value

Question No. 12.

(28). Suppose that X_1, X_2, \dots, X_n is a small random sample of size n from a normal distribution with mean μ and unknown variance σ^2 . Let S^2 denote the sample variance, then the statistic $\frac{(\bar{X}-\mu)}{S/\sqrt{n}}$ has a:

- (A) normal distribution
- (B) standard normal distribution
- (C) t - distribution with $n-1$ degree of freedom
- (D) other distribution

Question No. 13.

From the same population, we independently select a random sample of size n_1 and another sample of size n_2 . Let \bar{X}_1 be the sample mean of the first sample and \bar{X}_2 be the sample mean of the second sample. Let $n_1 < n_2$, then:

(29). the relation between the expected value of the two sample mean is:

- (A) $E(\bar{X}_1) < E(\bar{X}_2)$
- (B) $E(\bar{X}_1) > E(\bar{X}_2)$
- (C) $E(\bar{X}_1) = E(\bar{X}_2)$
- (D) can not compare

(30). the relation between the variance of the two sample mean is:

- (A) $Var(\bar{X}_1) > Var(\bar{X}_2)$
- (B) $Var(\bar{X}_1) < Var(\bar{X}_2)$
- (C) $Var(\bar{X}_1) = Var(\bar{X}_2)$
- (D) can not compare



**Department of Statistics
& Operations Research
College of Science, King Saud University**



STAT 324
Second midterm Examination
Second Semester
1430 – 1431 H

Student Name			
Student No.		Group No.	
Attendance No		Teacher's Name	

- Mobile Telephones are not allowed in the classrooms.
- Time allowed is **90 minutes**.
- Answer all questions.
- Choose the nearest number to your answer.
- **WARNING:** Do not copy answers from your neighbors. They have different question forms.
- For each question, put the code of the correct answer in the following table beneath the question number.

1	2	3	4	5	6	7	8	9	10
A	A	D	B	C	D	A	C	C	D

11	12	13	14	15	16	17	18	19	20
B	D	A	D	C	A	B	A	D	B

21	22	23	24	25
C	A	B	C	B

*

① In testing a certain kind of truck tire over a rugged terrain, it was found that 75% of the trucks completed the test without a blowout. If five trucks are selected at random, then:

- 1) The probability that no truck will complete the test without a blowout is :
(A) **0.0010** (B) 0.9990 (C) 0.2373 (D) 0.7627
 - 2) The probability that at least four of them will complete the test without a blowout is:
(A) **0.6328** (B) 0.3627 (C) 0.3955 (D) 0.2763
 - 3) The expected number of trucks that will complete the test without a blowout is equals to :
(A) 1.00 (B) 0.00 (C) 3.00 (D) **3.75**
-

② Suppose that on average 0.6 persons out of 1000 make a numerical error in preparing his or her income tax return. Assuming that the number of persons making numerical errors follows Poisson distribution, then:

- 4) The probability that at most one person out of 1000 will make a numerical error is:
(A) 0.8601 (B) **0.8781** (C) 0.9769 (D) 0.2351
- 5) The probability that two persons out of 500 make a numerical error is:
(A) 0.0988 (B) 0.0451 (C) **0.0333** (D) 0.5000

③ From a lot of 20 missiles, five missiles will not fire. If 10 are selected at random, then:

- 6) The probability that two missiles will not fire is:
(A) 0.2140 (B) 0.9314 (C) 0.6517 (D) **0.3483**
 - 7) The probability that at most one missile will not fire is:
(A) **0.1571** (B) 0.2614 (C) 0.8483 (D) 0.9998
 - 8) The expected number of missiles that will not fire is:
(A) 1.00 (B) 2.00 (C) **2.50** (D) 3.50
-

④ The lifetime of an HP laserprinter model (in months) is assumed to have an exponential distribution with mean lifetime equal to 70 months, then

- 9) The probability that a randomly selected HP laserprinter model will last more than 150 months is:
(A) 0.8827 (B) 0.2788 (C) **0.1173** (D) 0.7212

- 10) The probability that a randomly selected HP laserprinter model will last exactly 150 months is:
(A) 0.8827 (B) 0.2788 (C) 0.1173 (D) **0.0000**
- 11) The variance lifetime of HP laserprinters model in months is:
(A) 1/70 (B) **4900** (C) 140 (D) 35
-

- 5** A controlled satellite is known to have an error (distance from target) that is normally distributed with average error distance equals to 40 cm and a standard deviation equals to 4 cm, then:
- 12) The probability that the error distance from the target is greater than 42 cm is:
(A) 0.1269 (B) 0.2167 (C) 0.4257 (D) **0.3085**
- 13) The probability that the error distance from the target is greater than 42 cm and less than 44 cm is
(A) **0.1498** (B) 0.2599 (C) 0.3599 (D) 0.4599
- 14) If the probability that the error distance from the target is greater than the distance C is equal to 0.9732, then the distance C is equal to:
(A) 4.50 cm (B) 10.20 cm (C) 20.50 cm (D) **32.28 cm**
-

- 6** A certain machine makes electrical resistors that have an average resistance of 100 ohms and a standard deviation of 36 ohms. If a random sample of size, 36 resistors are drawn from the product of this machine, then:
- 15) The probability that the average resistance of the 36 resistors will be less than 91 ohms is:
(A) 0.1549 (B) 0.0753 (C) **0.0668** (D) 0.0875
- 16) The probability that the average resistance of the 36 resistors will be between 95 and 105 ohms is:
(A) **0.5934** (B) 0.6174 (C) 0.8432 (D) 0.7647
-

- 7** A random sample of 10 automobile owners shows that in a small village an automobile is driven on average 7.575 km in one hour with a standard deviation of 1.724 km. If μ is the average distance automobiles is driven in the village in one hour, assuming that the population follows a normal distribution, then:
- 17) The point estimate for μ is
(A) 1.233 (B) **7.575** (C) 0.9772 (D) 0.5793
- 18) The maximum error of estimating μ with a 95% confident is:
(A) **1.233** (B) 12.33 (C) 0.9772 (D) 0.5793
-

19) The lower bound of the 95% confidence interval for estimating μ is

- (A) 0.5793 (B) 12.33 (C) 0.9772 (D) **6.342**
-

8) Students may choose between a 3-semester hour course on physics without labs and a 4-semester hour course with labs. The final examination is the same for each section. If 10 students in the section with labs made an average grade of 70 with standard deviation 5, and 8 students in the section without labs made an average grade of 65 with standard deviation 3. Assuming that two populations to be normally distributed with equal variances, then:

20) The point estimate of the difference between the true grade means ($\mu_1 - \mu_2$) is:

- (A) 4 (B) **5** (C) 6 (D) 7

21) The maximum value of the error of estimating a 95% confidence interval of the difference between the true grade means ($\mu_1 - \mu_2$) is:

- (A) 1.2584 (B) 2.2536 (C) **4.2664** (D) -4.1258

22) The lower bound of the 95% confidence interval for the difference between the true grade means ($\mu_1 - \mu_2$) is equal to:

- (A) **0.7334** (B) 7.65 (C) 2.35 (D) 7.02

23) the upper bound of the 95% confidence interval for the difference between the true grade means ($\mu_1 - \mu_2$) is equal to:

- (A) 2.99 (B) **9.2665** (C) 2.35 (D) 7.02
-

9) A random sample of 1000 voters is selected and 250 are found to be in favor of the Republicans, then:

24) The point estimate for the true proportion of the voters who are in favor of the Republicans is:

- (A) 0.96 (B) 0.75 (C) **0.25** (D) 0.42

25) The lower bound of the 95% confidence interval for the true proportion of the voters who are in favor of the Republicans is:

- (A) 0.217 (B) **0.223** (C) 0.285 (D) 0.567

Good Luck....



Department of Statistics and Operations Research
College of Science, King Saud University

Summer Semester 1426-1427

STAT – 324

Probability and Statistics for Engineers

EXERCISES

A Collection of Questions Selected from
Midterm and Final Examinations' Papers
for the Years from 1422 to 1427

PREPARED BY:

Dr. Abdullah Al-Shiha

1. COMBINATIONS

$\binom{n}{r}$ = The number of combinations of n distinct objects taken r at a time (r objects in each combination)

= The number of different selections of r objects from n distinct objects.

= The number of different ways to select r objects from n distinct objects.

= The number of different ways to divide a set of n distinct objects into 2 subsets; one subset contains r objects and the other subset contains the rest.

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

$$0! = 1$$

Q1. Compute:

(a) $\binom{6}{2}$ (b) $\binom{6}{4}$.

Q2. Show that $\binom{n}{x} = \binom{n}{n-x}$.

Q3. Compute:

(a) $\binom{n}{0}$, (b) $\binom{n}{1}$, (c) $\binom{n}{n}$

Q4. A man wants to paint his house in 3 colors. If he can choose 3 colors out of 6 colors, how many different color settings can he make?

- (A) 216 (B) 20 (C) 18 (D) 120

Q5. The number of ways in which we can select two students among a group of 5 students is

- (A) 120 (B) 10 (C) 60 (D) 20 (E) 110

Q6. The number of ways in which we can select a president and a secretary among a group of 5 students is

- (A) 120 (B) 10 (C) 60 (D) 20

2. PROBABILITY, CONDITIONAL PROBABILITY, AND INDEPENDENCE

Q1. Let A, B, and C be three events such that: $P(A)=0.5$, $P(B)=0.4$, $P(C \cap A^c)=0.6$, $P(C \cap A)=0.2$, and $P(A \cup B)=0.9$. Then

- (a) $P(C) =$
 (A) 0.1 (B) 0.6 (C) 0.8 (D) 0.2 (E) 0.5
- (b) $P(B \cap A) =$
 (A) 0.0 (B) 0.9 (C) 0.1 (D) 1.0 (E) 0.3
- (c) $P(C|A) =$
 (A) 0.4 (B) 0.8 (C) 0.1 (D) 1.0 (E) 0.7
- (d) $P(B^c \cap A^c) =$
 (A) 0.3 (B) 0.1 (C) 0.2 (D) 1.1 (E) 0.8

Q2. Consider the experiment of flipping a balanced coin three times independently.

- (a) The number of points in the sample space is
 (A) 2 (B) 6 (C) 8 (D) 3 (E) 9
- (b) The probability of getting exactly two heads is
 (A) 0.125 (B) 0.375 (C) 0.667 (D) 0.333 (E) 0.451
- (c) The events 'exactly two heads' and 'exactly three heads' are
 (A) Independent (B) disjoint (C) equally (D) identical (E) None likely
- (d) The events 'the first coin is head' and 'the second and the third coins are tails' are
 (A) Independent (B) disjoint (C) equally (D) identical (E) None likely

Q3. Suppose that a fair die is thrown twice independently, then

- the probability that the sum of numbers of the two dice is less than or equal to 4 is;
 (A) 0.1667 (B) 0.6667 (C) 0.8333 (D) 0.1389
- the probability that at least one of the die shows 4 is;
 (A) 0.6667 (B) 0.3056 (C) 0.8333 (D) 0.1389
- the probability that one die shows one and the sum of the two dice is four is;
 (A) 0.0556 (B) 0.6667 (C) 0.3056 (D) 0.1389
- the event $A = \{\text{the sum of two dice is 4}\}$ and the event $B = \{\text{exactly one die shows two}\}$ are,
 (A) Independent (B) Dependent (C) Joint (D) None of these.

Q4. Assume that $P(A) = 0.3$, $P(B) = 0.4$, $P(A \cap B \cap C) = 0.03$, and $P(\overline{A \cap B}) = 0.88$, then

- the events A and B are,
 (A) Independent (B) Dependent (C) Disjoint (D) None of these.
- $P(C|A \cap B)$ is equal to,
 (A) 0.65 (B) 0.25 (C) 0.35 (D) 0.14

Q5. If the probability that it will rain tomorrow is 0.23, then the probability that it will not rain tomorrow is:

- (A) -0.23 (B) 0.77 (C) -0.77 (D) 0.23

Q6. The probability that a factory will open a branch in Riyadh is 0.7, the probability that it will open a branch in Jeddah is 0.4, and the probability that it will open a branch in either Riyadh or Jeddah or both is 0.8. Then, the probability that it will open a branch:

- 1) in both cities is:
 - (A) 0.1
 - (B) 0.9
 - (C) 0.3
 - (D) 0.8
- 2) in neither city is:
 - (A) 0.4
 - (B) 0.7
 - (C) 0.3
 - (D) 0.2

Q7. The probability that a lab specimen is contaminated is 0.10. Three independent specimen are checked.

- 1) the probability that none is contaminated is:
 - (A) 0.0475
 - (B) 0.001
 - (C) 0.729
 - (D) 0.3
- 2) the probability that exactly one sample is contaminated is:
 - (A) 0.243
 - (B) 0.027
 - (C) 0.729
 - (D) 0.3

Q8. 200 adults are classified according to sex and their level of education in the following table:

Sex	Male (M)	Female (F)
Elementary (E)	28	50
Secondary (S)	38	45
College (C)	22	17

If a person is selected at random from this group, then:

- 1) the probability that he is a male is:
 - (A) 0.3182
 - (B) 0.44
 - (C) 0.28
 - (D) 78
- 2) The probability that the person is male given that the person has a secondary education is:
 - (A) 0.4318
 - (B) 0.4578
 - (C) 0.19
 - (D) 0.44
- 3) The probability that the person does not have a college degree given that the person is a female is:
 - (A) 0.8482
 - (B) 0.1518
 - (C) 0.475
 - (D) 0.085
- 4) Are the events M and E independent? Why? $[P(M)=0.44 \neq P(M|E)=0.359 \Rightarrow \text{dependent}]$

Q9. 1000 individuals are classified below by sex and smoking habit.

		SEX	
		Male (M)	Female (F)
SMOKING HABIT	Daily (D)	300	50
	Occasionally (O)	200	50
	Not at all (N)	100	300

A person is selected randomly from this group.

1. Find the probability that the person is female. $[P(F)=0.4]$
2. Find the probability that the person is female and smokes daily. $[P(F \cap D)=0.05]$
3. Find the probability that the person is female, given that the person smokes daily. $[P(F|D)=0.1429]$
4. Are the events F and D independent? Why? $[P(F)=0.4 \neq P(F|D)=0.1429 \Rightarrow \text{dependent}]$

Q10. Two engines operate independently, if the probability that an engine will start is 0.4, and the probability that the other engine will start is 0.6, then the probability that both will start is:

- (A) 1
- (B) 0.24
- (C) 0.2
- (D) 0.5

Q11. If $P(B) = 0.3$ and $P(A|B) = 0.4$, then $P(A \cap B)$ equals to;

- (A) 0.67 (B) 0.12 (C) 0.75 (D) 0.3

Q12. The probability that a computer system has an electrical failure is 0.15, and the probability that it has a virus is 0.25, and the probability that it has both problems is 0.10, then the probability that the computer system has the electrical failure or the virus is:

- (A) 1.15 (B) 0.2 (C) 0.15 (D) 0.30

Q13. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:

- (A) 6/27 (B) 2/27 (C) 12/27 (D) 4/27

Q14. 80 students are enrolled in STAT-324 class. 60 students are from engineering college and the rest are from computer science college. 10% of the engineering college students have taken this course before, and 5% of the computer science college students have taken this course before. If one student from this class is randomly selected, then:

1) the probability that he has taken this course before is:

- (A) 0.25 (B) 0.0875 (C) 0.80 (D) 0.75

2) If the selected student has taken this course before then the probability that he is from the computer science college is:

- (A) 0.14 (B) 0.375 (C) 0.80 (D) 0.25

Q15. Two machines A and B make 80% and 20%, respectively, of the products in a certain factory. It is known that 5% and 10% of the products made by each machine, respectively, are defective. A finished product is randomly selected.

1. Find the probability that the product is defective. [$P(D) = 0.06$]

2. If the product were found to be defective, what is the probability that it was made by machine B. [$P(B|D) = 0.3333$]

Q16. If $P(A_1) = 0.4$, $P(A_1 \cap A_2) = 0.2$, and $P(A_3|A_1 \cap A_2) = 0.75$, then

(1) $P(A_2|A_1)$ equals to

- (A) 0.00 (B) 0.20 (C) 0.08 (D) 0.50

(2) $P(A_1 \cap A_2 \cap A_3)$ equals to

- (A) 0.06 (B) 0.35 (C) 0.15 (D) 0.08

Q17. If $P(A) = 0.9$, $P(B) = 0.6$, and $P(A \cap B) = 0.5$, then:

(1) $P(A \cap B^c)$ equals to

- (A) 0.4 (B) 0.1 (C) 0.5 (D) 0.3

(2) $P(A^c \cap B^c)$ equals to

- (A) 0.2 (B) 0.6 (C) 0.0 (D) 0.5

(3) $P(B|A)$ equals to

- (A) 0.5556 (B) 0.8333 (C) 0.6000 (D) 0.0

(4) The events A and B are

- (A) independent (B) disjoint (C) joint (D) none

(5) The events A and B are

- (A) disjoint (B) dependent (C) independent (D) none

Q18. Suppose that the experiment is to randomly select with replacement 2 children and register their gender (B=boy, G=girl) from a family having 2 boys and 6 girls.

- (1) The number of outcomes (elements of the sample space) of this experiment equals to
(A) 4 (B) 6 (C) 5 (D) 125
- (2) The event that represents registering at most one boy is
(A) {GG, GB, BG} (B) {GB, BG} (C) {GB}^C (D) {GB, BG, BB}
- (3) The probability of registering no girls equals to
(A) 0.2500 (B) 0.0625 (C) 0.4219 (D) 0.1780
- (4) The probability of registering exactly one boy equals to
(A) 0.1406 (B) 0.3750 (C) 0.0141 (D) 0.0423
- (5) The probability of registering at most one boy equals to
(A) 0.0156 (B) 0.5000 (C) 0.4219 (D) 0.9375

3. BAYES RULE:

Q1. 80 students are enrolled in STAT-324 class. 60 students are from engineering college and the rest are from computer science college. 10% of the engineering college students have taken this course before, and 5% of the computer science college students have taken this course before. If one student from this class is randomly selected, then:

- 1) the probability that he has taken this course before is:
(A) 0.25 (B) 0.0875 (C) 0.80 (D) 0.75
- 2) If the selected student has taken this course before then the probability that he is from the computer science college is:
(A) 0.14 (B) 0.375 (C) 0.80 (D) 0.25

Q2. Two machines A and B make 80% and 20%, respectively, of the products in a certain factory. It is known that 5% and 10% of the products made by each machine, respectively, are defective. A finished product is randomly selected.

- (a) Find the probability that the product is defective. [$P(D)=0.06$]
- (b) If the product were found to be defective, what is the probability that it was made by machine B. [$P(B|D)=0.3333$]

Q3. Dates' factory has three assembly lines, A, B, and C. Suppose that the assembly lines A, B, and C account for 50%, 30%, and 20% of the total product of the factory. Quality control records show that 4% of the dates packed by line A, 6% of the dates packed by line B, and 12% of the dates packed by line C are improperly sealed. If a pack is randomly selected, then:

- (a) the probability that the pack is from line B and it is improperly sealed is
(A) 0.018 (B) 0.30 (C) 0.06 (D) 0.36 (E) 0.53
- (b) the probability that the pack is improperly sealed is
(A) 0.62 (B) 0.022 (C) 0.062 (D) 0.22 (E) 0.25
- (c) if it is found that the pack is improperly sealed, what is the probability that it is from line B?
(A) 0.0623 (B) 0.0223 (C) 0.6203 (D) 0.2203 (E) 0.2903

Q4. Two brothers, Mohammad and Ahmad own and operate a small restaurant. Mohammad washes 50% of the dishes and Ahmad washes 50% of the dishes. When Mohammad washes a dish, he might break it with probability 0.40. On the other hand, when Ahmad washes a dish, he might break it with probability 0.10. Then,

- (a) the probability that a dish will be broken during washing is:
(A) 0.667 (B) 0.25 (C) 0.8 (D) 0.5
- (b) If a broken dish was found in the washing machine, the probability that it was washed by Mohammad is:
(A) 0.667 (B) 0.25 (C) 0.8 (D) 0.5

Q5. A vocational institute offers two training programs (A) and (B). In the last semester, 100 and 300 trainees were enrolled for programs (A) and (B), respectively. From the past experience it is known that the passing probabilities are 0.9 for program (A) and 0.7 for program (B). Suppose that at the end of the last semester, we selected a trainee at random from this institute.

- (1) The probability that the selected trainee passed the program equals to
(A) 0.80 (B) 0.75 (C) 0.85 (D) 0.79
- (2) If it is known that the selected trainee passed the program, then the probability that he has been enrolled in program (A) equals to
(A) 0.8 (B) 0.9 (C) 0.3 (D) 0.7

**4. RANDOM VARIABLES, DISTRIBUTIONS, EXPECTATIONS
AND CHEBYSHEV'S THEOREM:**

4.1. DISCRETE DISTRIBUTIONS:

Q1. Consider the experiment of flipping a balanced coin three times independently.

Let $X = \text{Number of heads} - \text{Number of tails}$.

- (a) List the elements of the sample space S .
- (b) Assign a value x of X to each sample point.
- (c) Find the probability distribution function of X .
- (d) Find $P(X \leq 1)$
- (e) Find $P(X < 1)$
- (f) Find $\mu = E(X)$
- (g) Find $\sigma^2 = \text{Var}(X)$

Q2. It is known that 20% of the people in a certain human population are female. The experiment is to select a committee consisting of two individuals at random. Let X be a random variable giving the number of females in the committee.

1. List the elements of the sample space S .
2. Assign a value x of X to each sample point.
3. Find the probability distribution function of X .
4. Find the probability that there will be at least one female in the committee.
5. Find the probability that there will be at most one female in the committee.
6. Find $\mu = E(X)$
7. Find $\sigma^2 = \text{Var}(X)$

Q3. A box contains 100 cards; 40 of which are labeled with the number 5 and the other cards are labeled with the number 10. Two cards were selected randomly with replacement and the number appeared on each card was observed. Let X be a random variable giving the total sum of the two numbers.

- (i) List the elements of the sample space S .
- (ii) To each element of S assign a value x of X .
- (iii) Find the probability mass function (probability distribution function) of X .
- (iv) Find $P(X=0)$.
- (v) Find $P(X>10)$.
- (vi) Find $\mu = E(X)$
- (vii) Find $\sigma^2 = \text{Var}(X)$

Q4. Let X be a random variable with the following probability distribution:

x	-3	6	9
$f(x)$	0.1	0.5	0.4

- 1) Find the mean (expected value) of X , $\mu = E(X)$.
- 2) Find $E(X^2)$.
- 3) Find the variance of X , $\text{Var}(X) = \sigma_X^2$.
- 4) Find the mean of $2X+1$, $E(2X+1) = \mu_{2X+1}$.

5) Find the variance of $2X+1$, $\text{Var}(2X+1) = \sigma_{2X+1}^2$.

Q5. Which of the following is a probability distribution function:

- (A) $f(x) = \frac{x+1}{10}$; $x=0,1,2,3,4$ (B) $f(x) = \frac{x-1}{5}$; $x=0,1,2,3,4$
 (C) $f(x) = \frac{1}{5}$; $x=0,1,2,3,4$ (D) $f(x) = \frac{5-x^2}{6}$; $x=0,1,2,3$

Q6. Let the random variable X have a discrete uniform with parameter $k=3$ and with values $0,1$, and 2 . The probability distribution function is:

$$f(x) = P(X=x) = 1/3; \quad x=0, 1, 2.$$

- (1) The mean of X is
 (A) 1.0 (B) 2.0 (C) 1.5 (D) 0.0
 (2) The variance of X is
 (A) 0.0 (B) 1.0 (C) 0.67 (D) 1.33

Q7. Let X be a discrete random variable with the probability distribution function:

$$f(x) = kx \text{ for } x=1, 2, \text{ and } 3.$$

- (i) Find the value of k .
 (ii) Find the cumulative distribution function (CDF), $F(x)$.
 (iii) Using the CDF, $F(x)$, find $P(0.5 < X \leq 2.5)$.

Q8. Let X be a random variable with cumulative distribution function (CDF) given by:

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.25, & 0 \leq x < 1 \\ 0.6, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

- (a) Find the probability distribution function of X , $f(x)$.
 (b) Find $P(1 \leq X < 2)$. (using both $f(x)$ and $F(x)$)
 (c) Find $P(X > 2)$. (using both $f(x)$ and $F(x)$)

Q9. Consider the random variable X with the following probability distribution function:

X	0	1	2	3
f(x)	0.4	c	0.3	0.1

The value of c is

- (A) 0.125 (B) 0.2 (C) 0.1 (D) 0.125 (E) -0.2

Q10. Consider the random variable X with the following probability distribution function:

X	-1	0	1	2
f(x)	0.2	0.3	0.2	c

Find the following:

- (a) The value of c .
 (b) $P(0 < X \leq 2)$
 (c) $\mu = E(X)$
 (d) $E(X^2)$

(e) $\sigma^2 = \text{Var}(X)$

Q11. Find the value of k that makes the function

$$f(x) = k \binom{2}{x} \binom{3}{3-x} \text{ for } x=0,1,2$$

serve as a probability distribution function of the discrete random variable X.

Q12. The cumulative distribution function (CDF) of a discrete random variable, X, is given below:

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/16 & \text{for } 0 \leq x < 1 \\ 5/16 & \text{for } 1 \leq x < 2 \\ 11/16 & \text{for } 2 \leq x < 3 \\ 15/16 & \text{for } 3 \leq x < 4 \\ 1 & \text{for } x \geq 4. \end{cases}$$

(a) the $P(X = 2)$ is equal to:

- (A) 3/8 (B) 11/16 (C) 10/16 (D) 5/16

(b) the $P(2 \leq X < 4)$ is equal to:

- (A) 20/16 (B) 11/16 (C) 10/16 (D) 5/16

Q13. If a random variable X has a mean of 10 and a variance of 4, then, the random variable $Y = 2X - 2$,

(a) has a mean of:

- (A) 10 (B) 18 (C) 20 (D) 22

(b) and a standard deviation of:

- (A) 6 (B) 2 (C) 4 (D) 16

Q14. Let X be the number of typing errors per page committed by a particular typist. The probability distribution function of X is given by:

x	0	1	2	3	4
f(x)	3k	3k	2k	k	k

(1) Find the numerical value of k.

(2) Find the average (mean) number of errors for this typist.

(3) Find the variance of the number of errors for this typist.

(4) Find the cumulative distribution function (CDF) of X.

(5) Find the probability that this typist will commit at least 2 errors per page.

Q15. Suppose that the discrete random variable X has the following probability function: $f(-1)=0.05$, $f(0)=0.25$, $f(1)=0.25$, $f(2)=0.45$, then:

(1) $P(X < 1)$ equals to

- (A) 0.30 (B) 0.05 (C) 0.55 (D) 0.50

(2) $P(X \leq 1)$ equals to

- (A) 0.05 (B) 0.55 (C) 0.30 (D) 0.45

(3) The mean $\mu = E(X)$ equals to

- (A) 1.1 (B) 0.0 (C) 1.2 (D) 0.5

(4) $E(X^2)$ equals to

- (A) 2.00 (B) 2.10 (C) 1.50 (D) 0.75

- (5) The variance $\sigma^2 = \text{Var}(X)$ equals to
 (A) 1.00 (B) 3.31 (C) 0.89 (D) 2.10
 (6) If $F(x)$ is the cumulative distribution function (CDF) of X , then $F(1)$ equals to
 (A) 0.50 (B) 0.25 (C) 0.45 (D) 0.55

4.2. CONTINUOUS DISTRIBUTIONS:

Q1. If the continuous random variable X has mean $\mu=16$ and variance $\sigma^2=5$, then $P(X = 16)$ is
 (A) 0.0625 (B) 0.5 (C) 0.0 (D) None of these.

Q2. Consider the probability density function:

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

- 1) The value of k is:
 (A) 1 (B) 0.5 (C) 1.5 (D) 0.667
 2) The probability $P(0.3 < X \leq 0.6)$ is,
 (A) 0.4647 (B) 0.3004 (C) 0.1643 (D) 0.4500
 3) The expected value of X , $E(X)$ is,
 (A) 0.6 (B) 1.5 (C) 1 (D) 0.667

[Hint: $\int \sqrt{x} dx = \frac{x^{3/2}}{(3/2)} + c$]

Q3. Let X be a continuous random variable with the probability density function $f(x)=k(x+1)$ for $0 < x < 2$.
 (i) Find the value of k .
 (ii) Find $P(0 < X \leq 1)$.
 (iii) Find the cumulative distribution function (CDF) of X , $F(x)$.
 (iv) Using $F(x)$, find $P(0 < X \leq 1)$.

Q4. Let X be a continuous random variable with the probability density function $f(x)=3x^2/2$ for $-1 < x < 1$.

1. $P(0 < X < 1) = \dots\dots\dots$
2. $E(X) = \dots\dots\dots$
3. $\text{Var}(X) = \dots\dots\dots$
4. $E(2X+3) = \dots\dots\dots$
5. $\text{Var}(2X+3) = \dots\dots\dots$

Q5. Suppose that the random variable X has the probability density function:

$$f(x) = \begin{cases} kx; & 0 < x < 2 \\ 0; & \text{elsewhere.} \end{cases}$$

1. Evaluate k .
2. Find the cumulative distribution function (CDF) of X , $F(x)$.
3. Find $P(0 < X < 1)$.
4. Find $P(X = 1)$ and $P(2 < X < 3)$.

Q6. Let X be a random variable with the probability density function:

$$f(x) = \begin{cases} 6x(1-x); & 0 < x < 1 \\ 0; & \text{elsewhere.} \end{cases}$$

1. Find $\mu = E(X)$.
2. Find $\sigma^2 = \text{Var}(X)$.
3. Find $E(4X+5)$.
4. Find $\text{Var}(4X+5)$.

Q7. If the random variable X has a uniform distribution on the interval $(0,10)$ with the probability density function given by:

$$f(x) = \begin{cases} \frac{1}{10}; & 0 < x < 10 \\ 0; & \text{elsewhere.} \end{cases},$$

1. Find $P(X < 6)$.
2. Find the mean of X .
3. Find $E(X^2)$.
4. Find the variance of X .
5. Find the cumulative distribution function (CDF) of X , $F(x)$.
6. Use the cumulative distribution function, $F(x)$, to find $P(1 < X \leq 5)$.

Q8. Suppose that the failure time (in months) of a specific type of electrical device is distributed with the probability density function:

$$f(x) = \begin{cases} \frac{1}{50}x & , 0 < x < 10 \\ 0 & , \text{otherwise} \end{cases}$$

- (a) the average failure time of such device is:
 (A) 6.667 (B) 1.00 (C) 2.00 (D) 5.00
- (b) the variance of the failure time of such device is:
 (A) 0 (B) 50 (C) 5.55 (D) 10
- (c) $P(X > 5) =$
 (A) 0.25 (B) 0.55 (C) 0.65 (D) 0.75

Q9. If the cumulative distribution function of the random variable X having the form:

$$P(X \leq x) = F(x) = \begin{cases} 0 & ; x < 0 \\ x/(x+1) & ; x \geq 0 \end{cases}$$

Then

- (1) $P(0 < X < 2)$ equals to
 (a) 0.555 (b) 0.333 (c) 0.667 (d) none of these.
- (2) If $P(X \leq k) = 0.5$, then k equals to
 (a) 5 (b) 0.5 (c) 1 (d) 1.5

Q10. If the diameter of a certain electrical cable is a continuous random variable X (in cm) having the probability density function :

$$f(x) = \begin{cases} 20x^3(1-x) & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (1) $P(X > 0.5)$ is:
 a) 0.8125 b) 0.1875 c) 0.9844 d) 0.4445
- (2) $P(0.25 < X < 1.75)$ is:
 a) 0.8125 b) 0.1875 c) 0.9844 d) 0.4445
- (3) $\mu = E(X)$ is:
 a) 0.667 b) 0.333 c) 0.555 d) none of these.
- (4) $\sigma^2 = \text{Var}(X)$ is:
 a) 0.3175 b) 3.175 c) 0.0317 d) 2.3175
- (5) For this random variable, $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ will have an exact value equals to:
 a) 0.3175 b) 0.750 c) 0.965 d) 0.250
- (6) For this random variable, $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ will have a lower bound value according to chebyshev's theory equals to:
 a) 0.3175 b) 0.750 c) 0.965 d) 0.250
- (7) If $Y = 3X - 1.5$, then $E(Y)$ is:
 a) -0.5 b) -0.3335 c) 0.5 d) none of these.
- (8) If $Y = 3X - 1.5$, then $\text{Var}(Y)$ is:
 a) 2.8575 b) 0.951 c) 0.2853 d) 6.9525

4.3. CHEBYSHEV'S THEOREM :

Q1. According to Chebyshev's theorem, for any random variable X with mean μ and variance σ^2 , a lower bound for $P(\mu - 2\sigma < X < \mu + 2\sigma)$ is,

- (A) 0.3175 (B) 0.750 (C) 0.965 (D) 0.250

Note: $P(\mu - 2\sigma < X < \mu + 2\sigma) = P(|X - \mu| < 2\sigma)$

Q2. Suppose that X is a random variable with mean $\mu = 12$, variance $\sigma^2 = 9$, and unknown probability distribution. Using Chebyshev's theorem, $P(3 < X < 21)$ is at least equal to,

- (A) 8/9 (B) 3/4 (C) 1/4 (D) 1/16

Q3. Suppose that $E(X) = 5$ and $\text{Var}(X) = 4$. Using Chebyshev's Theorem,

- (i) find an approximated value of $P(1 < X < 9)$.
 (ii) find some constants a and b ($a < b$) such that $P(a < X < b) \approx 15/16$.

Q4. Suppose that the random variable X is distributed according to the probability density function given by:

$$f(x) = \begin{cases} \frac{1}{10}; & 0 < x < 10 \\ 0; & \text{elsewhere.} \end{cases}$$

Assuming $\mu = 5$ and $\sigma = 2.89$,

- Find the exact value of $P(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$.
- Using Chebyshev's Theorem, find an approximate value of $P(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$.
- Compare the values in (1) and (2).

Q5. Suppose that X and Y are two independent random variables with $E(X) = 30$, $\text{Var}(X) = 4$, $E(Y) = 10$, and $\text{Var}(Y) = 2$. Then:

- (1) $E(2X - 3Y - 10)$ equals to
 (A) 40 (B) 20 (C) 30 (D) 90

- (2) $\text{Var}(2X-3Y-10)$ equals to
(A) 34 (B) 24 (C) 2.0 (D) 14
- (3) Using Chebyshev's theorem, a lower bound of $P(24 < X < 36)$ equals to
(A) 0.3333 (B) 0.6666 (C) 0.8888 (D) 0.1111

DISCRETE UNIFORM DISTRIBUTION:

Q1. Let the random variable X have a discrete uniform with parameter $k=3$ and with values 0,1, and 2. Then:

- (a) $P(X=1)$ is
(A) 1.0 (B) $1/3$ (C) 0.3 (D) 0.1 (E) None
- (b) The mean of X is:
(A) 1.0 (B) 2.0 (C) 1.5 (D) 0.0 (E) None
- (c) The variance of X is:
(A) $0/3=0.0$ (B) $3/3=1.0$ (C) $2/3=0.67$ (D) $4/3=1.33$ (E) None

5. BINOMIAL DISTRIBUTION:

Q1. Suppose that 4 out of 12 buildings in a certain city violate the building code. A building engineer randomly inspects a sample of 3 new buildings in the city.

- (a) Find the probability distribution function of the random variable X representing the number of buildings that violate the building code in the sample.
- (b) Find the probability that:
(i) none of the buildings in the sample violating the building code.
(ii) one building in the sample violating the building code.
(iii) at least one building in the sample violating the building code.
- (c) Find the expected number of buildings in the sample that violate the building code ($E(X)$).
- (d) Find $\sigma^2 = \text{Var}(X)$.

Q2. A missile detection system has a probability of 0.90 of detecting a missile attack. If 4 detection systems are installed in the same area and operate independently, then

- (a) The probability that at least two systems detect an attack is
(A) 0.9963 (B) 0.9477 (C) 0.0037 (D) 0.0523 (E) 0.5477
- (b) The average (mean) number of systems detect an attack is
(A) 3.6 (B) 2.0 (C) 0.36 (D) 2.5 (E) 4.0

Q3. Suppose that the probability that a person dies when he or she contracts a certain disease is 0.4. A sample of 10 persons who contracted this disease is randomly chosen.

- (1) What is the expected number of persons who will die in this sample?
(2) What is the variance of the number of persons who will die in this sample?
(3) What is the probability that exactly 4 persons will die among this sample?
(4) What is the probability that less than 3 persons will die among this sample?
(5) What is the probability that more than 8 persons will die among this sample?

Q4. Suppose that the percentage of females in a certain population is 50%. A sample of 3 people is selected randomly from this population.

- (a) The probability that no females are selected is
(A) 0.000 (B) 0.500 (C) 0.375 (D) 0.125
- (b) The probability that at most two females are selected is
(A) 0.000 (B) 0.500 (C) 0.875 (D) 0.125
- (c) The expected number of females in the sample is
(A) 3.0 (B) 1.5 (C) 0.0 (D) 0.50

- (d) The variance of the number of females in the sample is
 (A) 3.75 (B) 2.75 (C) 1.75 (D) 0.75

Q5. 20% of the trainees in a certain program fail to complete the program. If 5 trainees of this program are selected randomly,

- (i) Find the probability distribution function of the random variable X , where:
 X = number of the trainees who fail to complete the program.
- (ii) Find the probability that all trainees fail to complete the program.
- (iii) Find the probability that at least one trainee will fail to complete the program.
- (iv) How many trainees are expected to fail completing the program?
- (v) Find the variance of the number of trainees who fail completing the program.

Q6. In a certain industrial factory, there are 7 workers working independently. The probability of accruing accidents for any worker on a given day is 0.2, and accidents are independent from worker to worker.

- (a) The probability that at most two workers will have accidents during the day is
 (A) 0.7865 (B) 0.4233 (C) 0.5767 (D) 0.6647
- (b) The probability that at least three workers will have accidents during the day is:
 (A) 0.7865 (B) 0.2135 (C) 0.5767 (D) 0.1039
- (c) The expected number workers who will have accidents during the day is
 (A) 1.4 (B) 0.2135 (C) 2.57 (D) 0.59

Q7. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:

- (A) $6/27$ (B) $2/27$ (C) $12/27$ (D) $4/27$

Q8. The probability that a lab specimen is contaminated is 0.10. Three independent samples are checked.

- 1) the probability that none is contaminated is:
 (A) 0.0475 (B) 0.001 (C) 0.729 (D) 0.3
- 2) the probability that exactly one sample is contaminated is:
 (A) 0.243 (B) 0.027 (C) 0.729 (D) 0.3

Q9. If $X \sim \text{Binomial}(n, p)$, $E(X)=1$, and $\text{Var}(X)=0.75$, find $P(X=1)$.

Q10. Suppose that $X \sim \text{Binomial}(3, 0.2)$. Find the cumulative distribution function (CDF) of X .

Q11. A traffic control engineer reports that 75% of the cars passing through a checkpoint are from Riyadh city. If at this checkpoint, five cars are selected at random.

- (1) The probability that none of them is from Riyadh city equals to:
 (A) 0.00098 (B) 0.9990 (C) 0.2373 (D) 0.7627
- (2) The probability that four of them are from Riyadh city equals to:
 (A) 0.3955 (B) 0.6045 (C) 0 (D) 0.1249
- (3) The probability that at least four of them are from Riyadh city equals to:
 (A) 0.3627 (B) 0.6328 (C) 0.3955 (D) 0.2763
- (4) The expected number of cars that are from Riyadh city equals to:
 (A) 1 (B) 3.75 (C) 3 (D) 0

6. HYPERGEOMETRIC DISTRIBUTION:

Q1. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets.

- (i) Find the probability distribution function of the random variable X representing the number of defective sets purchased by the hotel.
- (ii) Find the probability that the hotel purchased no defective television sets.
- (iii) What is the expected number of defective television sets purchased by the hotel?
- (iv) Find the variance of X .

Q2. Suppose that a family has 5 children, 3 of them are girls and the rest are boys. A sample of 2 children is selected randomly and without replacement.

- (a) The probability that no girls are selected is
(A) 0.0 (B) 0.3 (C) 0.6 (D) 0.1
- (b) The probability that at most one girls are selected is
(A) 0.7 (B) 0.3 (C) 0.6 (D) 0.1
- (c) The expected number of girls in the sample is
(A) 2.2 (B) 1.2 (C) 0.2 (D) 3.2
- (d) The variance of the number of girls in the sample is
(A) 36.0 (B) 3.6 (C) 0.36 (D) 0.63

Q3. A random committee of size 4 is selected from 2 chemical engineers and 8 industrial engineers.

- (i) Write a formula for the probability distribution function of the random variable X representing the number of chemical engineers in the committee.
- (ii) Find the probability that there will be no chemical engineers in the committee.
- (iii) Find the probability that there will be at least one chemical engineer in the committee.
- (iv) What is the expected number of chemical engineers in the committee?
- (v) What is the variance of the number of chemical engineers in the committee?

Q4. A box contains 2 red balls and 4 black balls. Suppose that a sample of 3 balls were selected randomly and without replacement. Find,

1. The probability that there will be 2 red balls in the sample.
2. The probability that there will be 3 red balls in the sample.
3. The expected number of the red balls in the sample.

Q5. From a lot of 8 missiles, 3 are selected at random and fired. The lot contains 2 defective missiles that will not fire. Let X be a random variable giving the number of defective missiles selected.

1. Find the probability distribution function of X .
2. What is the probability that at most one missile will not fire?
3. Find $\mu = E(X)$ and $\sigma^2 = \text{Var}(X)$.

Q6. A particular industrial product is shipped in lots of 20 items. Testing to determine whether an item is defective is costly; hence, the manufacturer samples production rather than using 100% inspection plan. A sampling plan constructed to minimize the number of defectives shipped to consumers calls for sampling 5 items from each lot and rejecting the lot if more than one defective is observed. (If the lot is rejected, each item in the lot is then tested.) If a lot contains 4 defectives, what is the probability that it will be accepted.

Q7. Suppose that $X \sim h(x; 100, 2, 60)$; i.e., X has a hypergeometric distribution with parameters $N=100$, $n=2$, and $K=60$. Calculate the probabilities $P(X=0)$, $P(X=1)$, and $P(X=2)$ as follows:

- exact probabilities using hypergeometric distribution.
- approximated probabilities using binomial distribution.

Q8. A particular industrial product is shipped in lots of 1000 items. Testing to determine whether an item is defective is costly; hence, the manufacturer samples production rather than using 100% inspection plan. A sampling plan constructed to minimize the number of defectives shipped to consumers calls for sampling 5 items from each lot and rejecting the lot if more than one defective is observed. (If the lot is rejected, each item in the lot is then tested.) If a lot contains 100 defectives, calculate the probability that the lot will be accepted using:

- hypergeometric distribution (exact probability.)
- binomial distribution (approximated probability.)

Q9. A shipment of 20 digital voice recorders contains 5 that are defective. If 10 of them are randomly chosen (without replacement) for inspection, then:

- The probability that 2 will be defective is:
(A) 0.2140 (B) 0.9314 (C) 0.6517 (D) 0.3483
- The probability that at most 1 will be defective is:
(A) 0.9998 (B) 0.2614 (C) 0.8483 (D) 0.1517
- The expected number of defective recorders in the sample is:
(A) 1 (B) 2 (C) 3.5 (D) 2.5
- The variance of the number of defective recorders in the sample is:
(A) 0.9868 (B) 2.5 (C) 0.1875 (D) 1.875

Q10. A box contains 4 red balls and 6 green balls. The experiment is to select 3 balls at random. Find the probability that all balls are red for the following cases:

- If selection is without replacement
(A) 0.216 (B) 0.1667 (C) 0.6671 (D) 0.0333
- If selection is with replacement
(A) 0.4600 (B) 0.2000 (C) 0.4000 (D) 0.0640

7. POISSON DISTRIBUTION:

Q1. On average, a certain intersection results in 3 traffic accidents per day. Assuming Poisson distribution,

- (i) what is the probability that at this intersection:
- (1) no accidents will occur in a given day?
 - (2) More than 3 accidents will occur in a given day?
 - (3) Exactly 5 accidents will occur in a period of two days?
- (ii) what is the average number of traffic accidents in a period of 4 days?

Q2. At a checkout counter, customers arrive at an average of 1.5 per minute. Assuming Poisson distribution, then

- (1) The probability of no arrival in two minutes is
(A) 0.0 (B) 0.2231 (C) 0.4463 (D) 0.0498 (E) 0.2498
- (2) The variance of the number of arrivals in two minutes is
(A) 1.5 (B) 2.25 (C) 3.0 (D) 9.0 (E) 4.5

Q3. Suppose that the number of telephone calls received per day has a Poisson distribution with mean of 4 calls per day.

- (a) The probability that 2 calls will be received in a given day is
(A) 0.546525 (B) 0.646525 (C) 0.146525 (D) 0.746525
- (b) The expected number of telephone calls received in a given week is
(A) 4 (B) 7 (C) 28 (D) 14
- (c) The probability that at least 2 calls will be received in a period of 12 hours is
(A) 0.59399 (B) 0.19399 (C) 0.09399 (D) 0.29399

Q4. The average number of car accidents at a specific traffic signal is 2 per a week. Assuming Poisson distribution, find the probability that:

- (i) there will be no accident in a given week.
(ii) there will be at least two accidents in a period of two weeks.

Q5. The average number of airplane accidents at an airport is two per a year. Assuming Poisson distribution, find

1. the probability that there will be no accident in a year.
2. the average number of airplane accidents at this airport in a period of two years.
3. the probability that there will be at least two accidents in a period of 18 months.

Q6. Suppose that $X \sim \text{Binomial}(1000, 0.002)$. By using Poisson approximation, $P(X=3)$ is approximately equal to (choose the nearest number to your answer):

- (A) 0.62511 (B) 0.72511 (C) 0.82511 (D) 0.92511 (E) 0.18045

Q7. The probability that a person dies when he or she contracts a certain disease is 0.005. A sample of 1000 persons who contracted this disease is randomly chosen.

- (1) What is the expected number of persons who will die in this sample?
- (2) What is the probability that exactly 4 persons will die among this sample?

Q8. The number of faults in a fiber optic cable follows a Poisson distribution with an average of 0.6 per 100 feet.

- (1) The probability of 2 faults per 100 feet of such cable is:
(A) 0.0988 (B) 0.9012 (C) 0.3210 (D) 0.5
- (2) The probability of less than 2 faults per 100 feet of such cable is:
(A) 0.2351 (B) 0.9769 (C) 0.8781 (D) 0.8601
- (3) The probability of 4 faults per 200 feet of such cable is:
(A) 0.02602 (B) 0.1976 (C) 0.8024 (D) 0.9739

CONTINUOUS UNIFORM DISTRIBUTION:

Q1. If the random variable X has a uniform distribution on the interval (0,10), then

1. $P(X < 6)$ equals to
 (A) 0.4 (B) 0.6 (C) 0.8 (D) 0.2 (E) 0.1
2. The mean of X is
 (A) 5 (B) 10 (C) 2 (D) 8 (E) 6
2. The variance X is
 (A) 33.33 (B) 28.33 (C) 8.33 (D) 25 (E) None

Q2. Suppose that the random variable X has the following uniform distribution:

$$f(x) = \begin{cases} 3 & , \frac{2}{3} < x < 1 \\ 0 & , otherwise \end{cases}$$

- (1) $P(0.33 < X < 0.5) =$
 (A) 0.49 (B) 0.51 (C) 0 (D) 3
- (2) $P(X > 1.25) =$
 (A) 0 (B) 1 (C) 0.5 (D) 0.33
- (3) The variance of X is
 (A) 0.00926 (B) 0.333 (C) 9 (D) 0.6944

Q3. Suppose that the continuous random variable X has the following probability density function (pdf): $f(x)=0.2$ for $0 < x < 5$. Then

- (1) $P(X > 1)$ equals to
 (A) 0.4 (B) 0.2 (C) 0.1 (D) 0.8
- (2) $P(X \geq 1)$ equals to
 (A) 0.05 (B) 0.8 (C) 0.15 (D) 0.4
- (3) The mean $\mu = E(X)$ equals to
 (A) 2.0 (B) 2.5 (C) 3.0 (D) 3.5
- (4) $E(X^2)$ equals to
 (A) 8.3333 (B) 7.3333 (C) 9.3333 (D) 6.3333
- (5) $\text{Var}(X)$ equals to
 (A) 8.3333 (B) 69.444 (C) 5.8333 (D) 2.0833
- (6) If $F(x)$ is the cumulative distribution function (CDF) of X, then $F(1)$ equals to
 (A) 0.75 (B) 0.25 (C) 0.8 (D) 0.2

8. NORMAL DISTRIBUTION:

Q1. (A) Suppose that Z is distributed according to the standard normal distribution.

- 1) the area under the curve to the left of $z = 1.43$ is:
(A) 0.0764 (B) 0.9236 (C) 0 (D) 0.8133
- 2) the area under the curve to the left of $z = 1.39$ is:
(A) 0.7268 (B) 0.9177 (C) .2732 (D) 0.0832
- 3) the area under the curve to the right of $z = -0.89$ is:
(A) 0.7815 (B) 0.8133 (C) 0.1867 (D) 0.0154
- 4) the area under the curve between $z = -2.16$ and $z = -0.65$ is:
(A) 0.7576 (B) 0.8665 (C) 0.0154 (D) 0.2424
- 5) the value of k such that $P(0.93 < Z < k) = 0.0427$ is:
(A) 0.8665 (B) -1.11 (C) 1.11 (D) 1.00

(B) Suppose that Z is distributed according to the standard normal distribution. Find:

- 1) $P(Z < -3.9)$
- 2) $P(Z > 4.5)$
- 1) $P(Z < 3.7)$
- 2) $P(Z > -4.1)$

Q2. The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimeters and a standard deviation of 0.03 centimeter. Then,

- 1) the proportion of rings that will have inside diameter less than 12.05 centimeters is:
(A) 0.0475 (B) 0.9525 (C) 0.7257 (D) 0.8413
- 2) the proportion of rings that will have inside diameter exceeding 11.97 centimeters is:
(A) 0.0475 (B) 0.8413 (C) 0.1587 (D) 0.4514
- 3) the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters is:
(A) 0.905 (B) -0.905 (C) 0.4514 (D) 0.7257

Q3. The average life of a certain type of small motor is 10 years with a standard deviation of 2 years. Assume the live of the motor is normally distributed. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 1.5% of the motors that fail, then he should give a guarantee of :

- (A) 10.03 years (B) 8 years (C) 5.66 years (D) 3 years

Q4. A machine makes bolts (that are used in the construction of an electric transformer). It produces bolts with diameters (X) following a normal distribution with a mean of 0.060 inches and a standard deviation of 0.001 inches. Any bolt with diameter less than 0.058 inches or greater than 0.062 inches must be scrapped. Then

- (1) The proportion of bolts that must be scrapped is equal to
(A) 0.0456 (B) 0.0228 (C) 0.9772 (D) 0.3333 (E) 0.1667
- (2) If $P(X > a) = 0.1949$, then a equals to:
(A) 0.0629 (B) 0.0659 (C) 0.0649 (D) 0.0669 (E) 0.0609

Q5. The diameters of ball bearings manufactured by an industrial process are normally distributed with a mean $\mu = 3.0$ cm and a standard deviation $\sigma = 0.005$ cm. All ball bearings with diameters not within the specifications $\mu \pm d$ cm ($d > 0$) will be scrapped.

- (1) Determine the value of d such that 90% of ball bearings manufactured by this process will not be scrapped.
- (2) If $d = 0.005$, what is the percentage of manufactured ball bearings that will be scrapped?

Q6. The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128 kg and a standard deviation of 9 kg.

- (1) The percentage of fat persons with weights at most 110 kg is
(A) 0.09 % (B) 90.3 % (C) 99.82 % (D) 2.28 %
- (2) The percentage of fat persons with weights more than 149 kg is
(A) 0.09 % (B) 0.99 % (C) 9.7 % (D) 99.82 %
- (3) The weight x above which 86% of those persons will be
(A) 118.28 (B) 128.28 (C) 154.82 (D) 81.28
- (4) The weight x below which 50% of those persons will be
(A) 101.18 (B) 128 (C) 154.82 (D) 81

Q7. The random variable X , representing the lifespan of a certain electronic device, is normally distributed with a mean of 40 months and a standard deviation of 2 months. Find

1. $P(X < 38)$. (0.1587)
2. $P(38 < X < 40)$. (0.3413)
3. $P(X = 38)$. (0.0000)
4. The value of x such that $P(X < x) = 0.7324$. (41.24)

Q8. If the random variable X has a normal distribution with the mean μ and the variance σ^2 , then $P(X < \mu + 2\sigma)$ equals to

- (A) 0.8772 (B) 0.4772 (C) 0.5772 (D) 0.7772 (E) 0.9772

Q9. If the random variable X has a normal distribution with the mean μ and the variance 1, and if $P(X < 3) = 0.877$, then μ equals to

- (A) 3.84 (B) 2.84 (C) 1.84 (D) 4.84 (E) 8.84

Q10. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 70 and the variance 25. If it is known that 33% of the student failed the exam, then the passing mark x is

- (A) 67.8 (B) 60.8 (C) 57.8 (D) 50.8 (E) 70.8

Q11. If the random variable X has a normal distribution with the mean 10 and the variance 36, then

1. The value of X above which an area of 0.2296 lie is
(A) 14.44 (B) 16.44 (C) 10.44 (D) 18.44 (E) 11.44
2. The probability that the value of X is greater than 16 is
(A) 0.9587 (B) 0.1587 (C) 0.7587 (D) 0.0587 (E) 0.5587

Q12. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 65 and the variance 16. A student fails the exam if he obtains a mark less than 60. Then the percentage of students who fail the exam is

- (A) 20.56% (B) 90.56% (C) 50.56% (D) 10.56% (E) 40.56%

Q13. The average rainfall in a certain city for the month of March is 9.22 centimeters. Assuming a normal distribution with a standard deviation of 2.83 centimeters, then the probability that next March, this city will receive:

- (1) less than 11.84 centimeters of rain is:
(A) 0.8238 (B) 0.1762 (C) 0.5 (D) 0.2018
- (2) more than 5 centimeters but less than 7 centimeters of rain is:
(A) 0.8504 (B) 0.1496 (C) 0.6502 (D) 0.34221
- (3) more than 13.8 centimeters of rain is:
(A) 0.0526 (B) 0.9474 (C) 0.3101 (D) 0.4053

9. EXPONENTIAL DISTRIBUTION

Q1. If the random variable X has an exponential distribution with the mean 4, then:

1. $P(X < 8)$ equals to
 (A) 0.2647 (B) 0.4647 (C) 0.8647 (D) 0.6647 (E) 0.0647
2. The variance of X is
 (A) 4 (B) 16 (C) 2 (D) 1/4 (E) 1/2

Q2. Suppose that the failure time (in hours) of a certain electrical device is distributed with a probability density function given by:

$$f(x) = \frac{1}{70} e^{-x/70}, \quad x > 0,$$

- 1) the probability that a randomly selected device will fail within the first 50 hours is:
 (A) 0.4995 (B) 0.7001 (C) 0.5105 (D) 0.2999
- 2) the probability that a randomly selected device will last more than 150 hours is:
 (A) 0.8827 (B) 0.2788 (C) 0.1173 (D) 0.8827
- 3) the average failure time of the electrical device is:
 (A) 1/70 (B) 70 (C) 140 (D) 35
- 4) the variance of the failure time of the electrical device is:
 (A) 4900 (B) 1/49000 (C) 70 (D) 1225

[Hint: $\int e^{-ax} dx = -\frac{1}{a} e^{-ax} + c$]

Q3. The lifetime of a specific battery is a random variable X with probability density function given by:

$$f(x) = \frac{1}{200} e^{-x/200}, \quad x > 0$$

- (1) The mean life time of the battery equals to
 (A) 200 (B) 1/200 (C) 100 (D) 1/100 (E) Non of these
- (2) $P(X > 100) =$
 (A) 0.5 (B) 0.6065 (C) 0.3945 (D) 0.3679 (E) 0.6321
- (3) $P(X = 200) =$
 (A) 0.5 (B) 0.0 (C) 0.3945 (D) 0.3679 (E) 1.0

Q4. Suppose that the lifetime of a certain electrical device is given by T. The random variable T is modeled nicely by an exponential distribution with mean of 6 years. A random sample of four of these devices are installed in different systems. Assuming that these devices work independently, then:

- (1) the variance of the random variable T is
 (A) 136 (B) $(36)^2$ (C) 6 (D) 36
- (2) the probability that at most one of the devices in the sample will be functioning more than 6 years is
 (A) 0.4689 (B) 0.6321 (C) 0.5311 (D) 0.3679
- (3) the probability that at least two of the devices in the sample will be functioning more than 6 years is
 (A) 0.4689 (B) 0.6321 (C) 0.5311 (D) 0.3679
- (4) the expected number of devices in the sample which will be functioning more than 6 years is approximately equal to
 (A) 3.47 (B) 1.47 (C) 4.47 (D) 1.47

Q5. Assume the length (in minutes) of a particular type of a telephone conversation is a random variable with a probability density function of the form:

$$f(x) = \begin{cases} 0.2 e^{-0.2x} & ; x \geq 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

1. $P(3 < x < 10)$ is:
(a) 0.587 (b) -0.413 (c) 0.413 (d) 0.758
2. For this random variable, $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ will have an exact value equals:
(a) 0.250 (b) 0.750 (c) 0.950 (d) 0.3175
3. For this random variable, $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ will have a lower bound valued according to chebyshev's theory equals:
(a) 0.750 (b) 0.250 (c) 0.950 (d) 0.3175

Q6. The length of time for one customer to be served at a bank is a random variable X that follows the exponential distribution with a mean of 4 minutes.

- (1) The probability that a customer will be served in less than 2 minutes is:
(A) 0.9534 (B) 0.2123 (C) 0.6065 (D) 0.3935
- (2) The probability that a customer will be served in more than 4 minutes is:
(A) 0.6321 (B) 0.3679 (C) 0.4905 (D) 0.0012
- (3) The probability that a customer will be served in more than 2 but less than 5 minutes is:
(A) 0.6799 (B) 0.32 (C) 0.4018 (D) 0.5523
- (4) The variance of service time at this bank is
(A) 2 (B) 4 (C) 8 (D) 16

10. SAMPLING DISTRIBUTIONS**10.1. Single Mean:**

Q1. A machine is producing metal pieces that are cylindrical in shape. A random sample of size 5 is taken and the diameters are 1.70, 2.11, 2.20, 2.31 and 2.28 centimeters. Then,

- 1) The sample mean is:
 (A) 2.12 (B) 2.32 (C) 2.90 (D) 2.20 (E) 2.22
- 2) The sample variance is:
 (A) 0.59757 (B) 0.28555 (C) 0.35633 (D) 0.06115 (E) 0.53400

Q2. The average life of a certain battery is 5 years, with a standard deviation of 1 year. Assume that the live of the battery approximately follows a normal distribution.

1) The sample mean \bar{X} of a random sample of 5 batteries selected from this product has a mean

$E(\bar{X}) = \mu_{\bar{x}}$ equal to:

- (A) 0.2 (B) 5 (C) 3 (D) None of these
- 2) The variance $Var(\bar{X}) = \sigma_{\bar{x}}^2$ of the sample mean \bar{X} of a random sample of 5 batteries selected from this product is equal to:
 (A) 0.2 (B) 5 (C) 3 (D) None of these
- 3) The probability that the average life of a random sample of size 16 of such batteries will be between 4.5 and 5.4 years is:
 (A) 0.1039 (B) 0.2135 (C) 0.7865 (D) 0.9224
- 4) The probability that the average life of a random sample of size 16 of such batteries will be less than 5.5 years is:
 (A) 0.9772 (B) 0.0228 (C) 0.9223 (D) None of these
- 5) The probability that the average life of a random sample of size 16 of such batteries will be more than 4.75 years is:
 (A) 0.8413 (B) 0.1587 (C) 0.9452 (D) None of these
- 6) If $P(\bar{X} > a) = 0.1492$ where \bar{X} represents the sample mean for a random sample of size 9 of such batteries, then the numerical value of a is:
 (A) 4.653 (B) 6.5 (C) 5.347 (D) None of these

Q3. The random variable X, representing the lifespan of a certain light bulb, is distributed normally with a mean of 400 hours and a standard deviation of 10 hours.

- What is the probability that a particular light bulb will last for more than 380 hours?
- Light bulbs with lifespan less than 380 hours are rejected. Find the percentage of light bulbs that will be rejected.
- If 9 light bulbs are selected randomly, find the probability that their average lifespan will be less than 405.

Q4. Suppose that you take a random sample of size $n=64$ from a distribution with mean $\mu=55$ and standard deviation $\sigma=10$. Let $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ be the sample mean.

- What is the approximated sampling distribution of \bar{X} ?
- What is the mean of \bar{X} ?
- What is the standard error (standard deviation) of \bar{X} ?

(d) Find the probability that the sample mean \bar{x} exceeds 52.

Q5. The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with the mean 3.0 minutes and the standard deviation of 1.4 minutes. If a random sample of 49 customers is observed, then

- (1) the probability that their mean time will be at least 2.8 minutes is
 (A) 1.0 (B) 0.8413 (C) 0.3274 (D) 0.4468
- (2) the probability that their mean time will be between 2.7 and 3.2 minutes is
 (A) 0.7745 (B) 0.2784 (C) 0.9973 (D) 0.0236

Q6. The average life of an industrial machine is 6 years, with a standard deviation of 1 year. Assume the life of such machines follows approximately a normal distribution. A random sample of 4 of such machines is selected. The sample mean life of the machines in the sample is \bar{x} .

- (1) The sample mean has a mean $\mu_{\bar{x}} = E(\bar{x})$ equals to:
 (A) 5 (B) 6 (C) 7 (D) 8
- (2) The sample mean has a variance $\sigma_{\bar{x}}^2 = \text{Var}(\bar{x})$ equals to:
 (A) 1 (B) 0.5 (C) 0.25 (D) 0.75
- (3) $P(\bar{x} < 5.5) =$
 (A) 0.4602 (B) 0.8413 (C) 0.1587 (D) 0.5398
- (4) If $P(\bar{x} > a) = 0.1492$, then the numerical value of a is:
 (A) 0.8508 (B) 1.04 (C) 6.52 (D) 0.2

10.2. Two Means:

Q1. A random sample of size $n_1 = 36$ is taken from a normal population with a mean $\mu_1 = 70$ and a standard deviation $\sigma_1 = 4$. A second independent random sample of size $n_2 = 49$ is taken from a normal population with a mean $\mu_2 = 85$ and a standard deviation $\sigma_2 = 5$. Let \bar{X}_1 and \bar{X}_2 be the averages of the first and second samples, respectively.

- a) Find $E(\bar{X}_1)$ and $\text{Var}(\bar{X}_1)$.
- b) Find $E(\bar{X}_1 - \bar{X}_2)$ and $\text{Var}(\bar{X}_1 - \bar{X}_2)$.
- c) Find $P(70 < \bar{X}_1 < 71)$.
- d) Find $P(\bar{X}_1 - \bar{X}_2 > -16)$.

Q2. A random sample of size 25 is taken from a normal population (first population) having a mean of 100 and a standard deviation of 6. A second random sample of size 36 is taken from a different normal population (second population) having a mean of 97 and a standard deviation of 5. Assume that these two samples are independent.

- (1) the probability that the sample mean of the first sample will exceed the sample mean of the second sample by at least 6 is
 (A) 0.0013 (B) 0.9147 (C) 0.0202 (D) 0.9832
- (2) the probability that the difference between the two sample means will be less than 2 is
 (A) 0.099 (B) 0.2480 (C) 0.8499 (D) 0.9499

10.3. Single Proportion:

Q1. Suppose that 20% of the students in a certain university smoke cigarettes. A random sample of 5 students is taken from this university. Let \hat{p} be the proportion of smokers in the sample.

- (1) Find $E(\hat{p}) = \mu_{\hat{p}}$, the mean \hat{p} .
- (2) Find $Var(\hat{p}) = \sigma_{\hat{p}}^2$, the variance of \hat{p} .
- (3) Find an approximate distribution of \hat{p} .
- (4) Find $P(\hat{p} > 0.25)$.

Q2: Suppose that you take a random sample of size $n=100$ from a binomial population with parameter $p=0.25$ (proportion of successes). Let $\hat{p} = X/n$ be the sample proportion of successes, where X is the number of successes in the sample.

- (a) What is the approximated sampling distribution of \hat{p} ?
- (b) What is the mean of \hat{p} ?
- (c) What is the standard error (standard deviation) of \hat{p} ?
- (d) Find the probability that the sample proportion \hat{p} is less than 0.2.

10.4. Two Proportions:

Q1. Suppose that 25% of the male students and 20% of the female students in a certain university smoke cigarettes. A random sample of 5 male students is taken. Another random sample of 10 female students is independently taken from this university. Let \hat{p}_1 and \hat{p}_2 be the proportions of smokers in the two samples, respectively.

- (1) Find $E(\hat{p}_1 - \hat{p}_2) = \mu_{\hat{p}_1 - \hat{p}_2}$, the mean of $\hat{p}_1 - \hat{p}_2$.
- (2) Find $Var(\hat{p}_1 - \hat{p}_2) = \sigma_{\hat{p}_1 - \hat{p}_2}^2$, the variance of $\hat{p}_1 - \hat{p}_2$.
- (3) Find an approximate distribution of $\hat{p}_1 - \hat{p}_2$.
- (4) Find $P(0.10 < \hat{p}_1 - \hat{p}_2 < 0.20)$.

10.5 t-distribution:

Q1. Using t-table with degrees of freedom $df=14$, find $t_{0.02}$, $t_{0.985}$.

Q2. From the table of t-distribution with degrees of freedom $\nu = 15$, the value of $t_{0.025}$ equals to

- (A) 2.131 (B) 1.753 (C) 3.268 (D) 0.0

11. ESTIMATION AND CONFIDENCE INTERVALS:**11.1. Single Mean:**

Q1. An electrical firm manufacturing light bulbs that have a length of life that is normally distributed with a standard deviation of 30 hours. A sample of 50 bulbs were selected randomly and found to have an average of 750 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm.

- (1) Find a point estimate for μ .
- (2) Construct a 94% confidence interval for μ .

Q2. Suppose that we are interested in making some statistical inferences about the mean, μ , of a normal population with standard deviation $\sigma=2.0$. Suppose that a random sample of size $n=49$ from this population gave a sample mean $\bar{X}=4.5$.

- (1) The distribution of \bar{X} is
 (A) $N(0,1)$ (B) $t(48)$ (C) $N(\mu, 0.2857)$ (D) $N(\mu, 2.0)$ (E) $N(\mu, 0.3333)$
- (2) A good point estimate of μ is
 (A) 4.50 (B) 2.00 (C) 2.50 (D) 7.00 (E) 1.125
- (3) The standard error of \bar{X} is
 (A) 0.0816 (B) 2.0 (C) 0.0408 (D) 0.5714 (E) 0.2857
- (4) A 95% confidence interval for μ is
 (A) (3.44,5.56) (B) (3.34,5.66) (C) (3.54,5.46) (D) (3.94,5.06) (E) (3.04,5.96)
- (5) If the upper confidence limit of a confidence interval is 5.2, then the lower confidence limit is
 (A) 3.6 (B) 3.8 (C) 4.0 (D) 3.5 (E) 4.1
- (6) The confidence level of the confidence interval (3.88, 5.12) is
 (A) 90.74% (B) 95.74% (C) 97.74% (D) 94.74% (E) 92.74%
- (7) If we use \bar{X} to estimate μ , then we are 95% confident that our estimation error will not exceed
 (A) $e=0.50$ (B) $E=0.59$ (C) $e=0.58$ (D) $e=0.56$ (E) $e=0.51$
- (8) If we want to be 95% confident that the estimation error will not exceed $e=0.1$ when we use \bar{X} to estimate μ , then the sample size n must be equal to
 (A) 1529 (B) 1531 (C) 1537 (D) 1534 (E) 1530

Q3. The following measurements were recorded for lifetime, in years, of certain type of machine: 3.4, 4.8, 3.6, 3.3, 5.6, 3.7, 4.4, 5.2, and 4.8. Assuming that the measurements represent a random sample from a normal population, then a 99% confidence interval for the mean life time of the machine is

- (A) $-5.37 \leq \mu \leq 3.25$ (B) $4.72 \leq \mu \leq 9.1$
- (C) $4.01 \leq \mu \leq 5.99$ (D) $3.37 \leq \mu \leq 5.25$

Q4. A researcher wants to estimate the mean lifespan of a certain light bulbs. Suppose that the distribution is normal with standard deviation of 5 hours.

1. Determine the sample size needed on order that the researcher will be 90% confident that the error will not exceed 2 hours when he uses the sample mean as a point estimate for the true mean.
2. Suppose that the researcher selected a random sample of 49 bulbs and found that the sample mean is 390 hours.
 - (i) Find a good point estimate for the true mean μ .
 - (ii) Find a 95% confidence interval for the true mean μ .

Q5. The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with a standard deviation of 1.4 minutes. If we wish to estimate the population mean μ by the sample mean \bar{X} , and if we want to be 96% confident that the sample mean will be within 0.3 minutes of the population mean, then the sample size needed is

- (A) 98 (B) 100 (C) 92 (D) 85

Q6: A random sample of size $n=36$ from a normal quantitative population produced a mean $\bar{X}=15.2$ and a variance $s^2=9$.

- (a) Give a point estimate for the population mean μ .
 (b) Find a 95% confidence interval for the population mean μ .

Q7. A group of 10 college students were asked to report the number of hours that they spent doing their homework during the previous weekend and the following results were obtained:

$$7.25, 8.5, 5.0, 6.75, 8.0, 5.25, 10.5, 8.5, 6.75, 9.25$$

$$\sum X = 75.75, \sum X^2 = 600.563 \}$$

It is assumed that this sample is a random sample from a normal distribution with unknown variance σ^2 . Let μ be the mean of the number of hours that the college student spend doing his/her homework during the weekend.

- (a) Find the sample mean and the sample variance.
 (b) Find a point estimate for μ .
 (c) Construct a 80% confidence interval for μ .

Q8. An electronics company wanted to estimate its monthly operating expenses in thousands riyals (μ). It is assumed that the monthly operating expenses (in thousands riyals) are distributed according to a normal distribution with variance $\sigma^2=0.584$.

- (I) Suppose that a random sample of 49 months produced a sample mean $\bar{X}=5.47$.
 (a) Find a point estimate for μ .
 (b) Find the standard error of \bar{X} .
 (c) Find a 90% confidence interval for μ .
 (II) Suppose that they want to estimate μ by \bar{X} . Find the sample size (n) required if they want their estimate to be within 0.15 of the actual mean with probability equals to 0.95.

Q9. The tensile strength of a certain type of thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

- (a) A point estimate of the population mean of the tensile strength (μ) is:
 (A) 72.8 (B) 20 (C) 6.8 (D) 46.24 (E) None of these
 (b) Suppose that we want to estimate the population mean (μ) by the sample mean (\bar{X}). To be 95% confident that the error of our estimate of the mean of tensile strength will be less than 3.4 kilograms, the minimum sample size should be at least:
 (A) 4 (B) 16 (C) 20 (D) 18 (E) None of these
 (c) For a 98% confidence interval for the mean of tensile strength, we have the lower bound equal to:
 (A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these
 (d) For a 98% confidence interval for the mean of tensile strength, we have the upper bound equal to:

- (A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

11.2. Two Means:

Q1.(I) The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

- 1) To be 95% confident that the error of estimating the mean of tensile strength by the sample mean will be less than 3.4 kilograms, the minimum sample size should be:
 (A) 4 (B) 16 (C) 20 (D) 18 (E) None of these
- 2) The lower limit of a 98% confidence interval for the mean of tensile strength is
 (A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these
- 3) The upper limit of a 98% confidence interval for the mean of tensile strength is
 (A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

Q1.(II). The tensile strength of type II thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 25 pieces of the thread has an average tensile strength of 64.4 kilograms. Then for the 98% confidence interval of the difference in tensile strength means between type I and type II , we have:

- 1) the lower bound equals to:
 (A) 2.90 (B) 4.21 (C) 3.65 (D) 6.58 (E) None of these
- 2) the upper bound equals to:
 (A) 13.90 (B) 13.15 (C) 12.59 (D) 10.22 (E) None of these

Q2. Two random samples were independently selected from two normal populations with equal variances. The results are summarized as follows.

	First Sample	Second Sample
sample size (n)	12	14
sample mean (\bar{X})	10.5	10.0
sample variance (S^2)	4	5

Let μ_1 and μ_2 be the true means of the first and second populations, respectively.

1. Find a point estimate for $\mu_1 - \mu_2$.
2. Find 95% confidence interval for $\mu_1 - \mu_2$.

Q3. A researcher was interested in comparing the mean score of female students, μ_f , with the mean score of male students, μ_m , in a certain test. Two independent samples gave the following results:

Sample	Observations							mean	Variance
Scores of Females	89.2	81.6	79.6	80.0	82.8			82.63	15.05
Scores of Males	83.2	83.2	84.8	81.4	78.6	71.5	77.6	80.04	20.79

Assume the populations are normal with equal variances.

- (1) The pooled estimate of the variance s_p^2 is
 (A) 17.994 (B) 17.794 (C) 18.094 (D) 18.294 (E) 18.494
- (2) A point estimate of $\mu_f - \mu_m$ is
 (A) 2.63 (B) -2.59 (C) 2.59 (D) 0.00 (E) 0.59
- (3) The lower limit of a 90% confidence interval for $\mu_f - \mu_m$ is
 (A) -1.97 (B) -1.67 (C) 1.97 (D) 1.67 (E) -1.57
- (4) The upper limit of a 90% confidence interval for $\mu_f - \mu_m$ is
 (A) 6.95 (B) 7.45 (C) -7.55 (D) 7.15 (E) 7.55

Q4. A study was conducted to compare to brands of tires A and B. 10 tires of brand A and 12 tires of brand B were selected randomly. The tires were run until they wear out. The results are:

Brand A: $\bar{X}_A = 37000$ kilometers $S_A = 5100$

Brand B: $\bar{X}_B = 38000$ kilometers $S_B = 6000$

Assuming the populations are normally distributed with equal variances,

- (1) Find a point estimate for $\mu_A - \mu_B$.
- (2) Construct a 90% confidence interval for $\mu_A - \mu_B$.

Q5. The following data show the number of defects of code of particular type of software program made in two different countries (assuming normal populations with unknown equal variances)

Country	observations							mean	standard dev.
A	48	39	42	52	40	48	54	46.143	5.900
B	50	40	43	45	50	38	36	43.143	5.551

- (a) A point estimate of $\mu_A - \mu_B$ is
 (A) 3.0 (B) -3.0 (C) 2.0 (D) -2.0 (E) None of these
- (b) A 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$ is
 (A) $-2.46 \leq \mu_A - \mu_B \leq 8.46$ (B) $1.42 \leq \mu_A - \mu_B \leq 6.42$
 (C) $-1.42 \leq \mu_A - \mu_B \leq -0.42$ (D) $2.42 \leq \mu_A - \mu_B \leq 10.42$

Q6. A study was made by a taxi company to decide whether the use of new tires (A) instead of the present tires (B) improves fuel economy. Six cars were equipped with tires (A) and driven over a prescribed test course. Without changing drivers and cares, test course was made with tires (B). The gasoline consumption, in kilometers per liter (km/L), was recorded as follows: (assume the populations are normal with equal unknown variances)

Car	1	2	3	4	5	6
Type (A)	4.5	4.8	6.6	7.0	6.7	4.6
Type (B)	3.9	4.9	6.2	6.5	6.8	4.1

- (a) A 95% confidence interval for the true mean gasoline consumption for brand A is:
 (A) $4.462 \leq \mu_A \leq 6.938$ (B) $2.642 \leq \mu_A \leq 4.930$
 (C) $5.2 \leq \mu_A \leq 9.7$ (D) $6.154 \leq \mu_A \leq 6.938$
- (b) A 99% confidence interval for the difference between the true means consumption of type (A) and type (B) ($\mu_A - \mu_B$) is:
 (A) $-1.939 \leq \mu_A - \mu_B \leq 2.539$ (B) $-2.939 \leq \mu_A - \mu_B \leq 1.539$
 (C) $0.939 \leq \mu_A - \mu_B \leq 1.539$ (D) $-1.939 \leq \mu_A - \mu_B \leq 0.539$

Q7. A geologist collected 20 different ore samples, all of the same weight, and randomly divided them into two groups. The titanium contents of the samples, found using two different methods, are listed in the table:

Method (A)					Method (B)				
1.1	1.3	1.3	1.5	1.4	1.1	1.6	1.3	1.2	1.5
1.3	1.0	1.3	1.1	1.2	1.2	1.7	1.3	1.4	1.5
$\bar{X}_1 = 1.25, S_1 = 0.1509$					$\bar{X}_2 = 1.38, S_2 = 0.1932$				

- (a) Find a point estimate of $\mu_A - \mu_B$ is
 (b) Find a 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$. (Assume two normal populations with equal variances).

11.3. Single Proportion:

Q1. A random sample of 200 students from a certain school showed that 15 students smoke. Let p be the proportion of smokers in the school.

- Find a point Estimate for p .
- Find 95% confidence interval for p .

Q2. A researcher was interested in making some statistical inferences about the proportion of females (p) among the students of a certain university. A random sample of 500 students showed that 150 students are female.

- A good point estimate for p is
 (A) 0.31 (B) 0.30 (C) 0.29 (D) 0.25 (E) 0.27
- The lower limit of a 90% confidence interval for p is
 (A) 0.2363 (B) 0.2463 (C) 0.2963 (D) 0.2063 (E) 0.2663
- The upper limit of a 90% confidence interval for p is
 (A) 0.3337 (B) 0.3137 (C) 0.3637 (D) 0.2937 (E) 0.3537

Q3. In a random sample of 500 homes in a certain city, it is found that 114 are heated by oil. Let p be the proportion of homes in this city that are heated by oil.

- Find a point estimate for p .
- Construct a 98% confidence interval for p .

Q4. In a study involved 1200 car drivers, it was found that 50 car drivers do not use seat belt.

- A point estimate for the proportion of car drivers who do not use seat belt is:
 (A) 50 (B) 0.0417 (C) 0.9583 (D) 1150 (E) None of these
- The lower limit of a 95% confidence interval of the proportion of car drivers not using seat belt is
 (A) 0.0322 (B) 0.0416 (C) 0.0304 (D) -0.3500 (E) None of these
- The upper limit of a 95% confidence interval of the proportion of car drivers not using seat belt is
 (A) 0.0417 (B) 0.0530 (C) 0.0512 (D) 0.4333 (E) None of these

Q5. A study was conducted to make some inferences about the proportion of female employees (p) in a certain hospital. A random sample gave the following data:

Sample size	250
Number of females	120

- Calculate a point estimate (\hat{p}) for the proportion of female employees (p).
- Construct a 90% confidence interval for p .

Q6. In a certain city, the traffic police was interested in knowing the proportion of car drivers who do not use seat built. In a study involved 1200 car drivers it was found that 50 car drivers do not use seat belt.

- A point estimate for the proportion of car drivers who do not use seat built is:

- (A) 50 (B) 0.0417 (C) 0.9583 (D) 1150 (E) None of these
- (b) A 95% confidence interval of the proportion of car drivers who do not use seat built has the lower bound equal to:
 (A) 0.0322 (B) 0.0416 (C) 0.0304 (D) -0.3500 (E) None of these
- (c) A 95% confidence interval of the proportion of car drivers who do not use seat built has the upper bound equal to:
 (A) 0.0417 (B) 0.0530 (C) 0.0512 (D) 0.4333 (E) None of these

11.4. Two Proportions:

Q1. A survey of 500 students from a college of science shows that 275 students own computers. In another independent survey of 400 students from a college of engineering shows that 240 students own computers.

- (a) a 99% confidence interval for the true proportion of college of science's student who own computers is
 (A) $-0.59 \leq p_1 \leq 0.71$ (B) $0.49 \leq p_1 \leq 0.61$
 (C) $2.49 \leq p_1 \leq 6.61$ (D) $0.3 \leq p_1 \leq 0.7$
- (29) a 95% confidence interval for the difference between the proportions of students owning computers in the two colleges is
 (A) $0.015 \leq p_1 - p_2 \leq 0.215$ (B) $-0.515 \leq p_1 - p_2 \leq 0.215$
 (C) $-0.450 \leq p_1 - p_2 \leq -0.015$ (D) $-0.115 \leq p_1 - p_2 \leq 0.015$

Q2. A food company distributes "smiley cow" brand of milk. A random sample of 200 consumers in the city (A) showed that 80 consumers prefer the "smiley cow" brand of milk. Another independent random sample of 300 consumers in the city (B) showed that 90 consumers prefer "smiley cow" brand of milk. Define:

p_A = the true proportion of consumers in the city (A) preferring "smiley cow" brand.

p_B = the true proportion of consumers in the city (B) preferring "smiley cow" brand.

- (a) A 96% confidence interval for the true proportion of consumers preferring brand (A) is:
 (A) $0.328 \leq p_A \leq 0.375$ (B) $0.228 \leq p_A \leq 0.675$
 (C) $0.328 \leq p_A \leq 0.475$ (D) $0.518 \leq p_A \leq 0.875$
- (b) A 99% confidence interval for the difference between proportions of consumers preferring brand (A) and (B) is:
 (A) $0.0123 \leq p_A - p_B \leq 0.212$ (B) $-0.2313 \leq p_A - p_B \leq 0.3612$
 (C) $-0.0023 \leq p_A - p_B \leq 0.012$ (D) $-0.0123 \leq p_A - p_B \leq 0.212$

Q3. A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200 students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smokers in school "A" and p_2 is the proportion of smokers in school "B".

- (1) Find a point Estimate for $p_1 - p_2$.
 (2) Find 95% confidence interval for $p_1 - p_2$.

12. HYPOTHESES TESTING:**12.1. Single Mean:**

Q1. Suppose that we are interested in making some statistical inferences about the mean, μ , of a normal population with standard deviation $\sigma=2.0$. Suppose that a random sample of size $n=49$ from this population gave a sample mean $\bar{X} = 4.5$.

- (1) If we want to test $H_0: \mu=5.0$ against $H_1: \mu \neq 5.0$, then the test statistic equals to
 (A) $Z = -1.75$ (B) $Z = 1.75$ (C) $T = -1.70$ (D) $T = 1.70$ (E) $Z = -1.65$
- (2) If we want to test $H_0: \mu=5.0$ against $H_1: \mu > 5.0$ at $\alpha=0.05$, then the Rejection Region of H_0 is
 (A) $(1.96, \infty)$ (B) $(2.325, \infty)$ (C) $(-\infty, -1.645)$ (D) $(-\infty, -1.96)$ (E) $(1.645, \infty)$
- (3) If we want to test $H_0: \mu=5.0$ against $H_1: \mu > 5.0$ at $\alpha=0.05$, then we
 (A) Accept H_0 (B) Reject H_0 (C) (D) (E)

Q2. An electrical firm manufactures light bulbs that have a length of life that is normally distributed with a standard deviation of 30 hours. A sample of 50 bulbs were selected randomly and found to have an average of 750 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm. Test $H_0: \mu = 740$ against $H_1: \mu < 740$? Use a 0.05 level of significance.

Q3. An electrical firm manufactures light bulbs that have a length of life that is normally distributed. A sample of 20 bulbs were selected randomly and found to have an average of 655 hours and a standard deviation of 27 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm. Test $H_0: \mu = 660$ against $H_1: \mu \neq 660$? Use a 0.02 level of significance.

Q4: A random sample of size $n=36$ from a normal quantitative population produced a mean $\bar{X} = 15.2$ and a variance $s^2 = 9$. Test $H_0: \mu = 15$ against $H_a: \mu \neq 15$, use $\alpha=0.05$.

Q5. A group of 10 college students were asked to report the number of hours that they spent doing their homework during the previous weekend and the following results were obtained:

$$7.25, 8.5, 5.0, 6.75, 8.0, 5.25, 10.5, 8.5, 6.75, 9.25$$

$$\sum X = 75.75, \sum X^2 = 600.563 \}$$

It is assumed that this sample is a random sample from a normal distribution with unknown variance σ^2 . Let μ be the mean of the number of hours that the college student spend doing his/her homework during the weekend. Test $H_0: \mu = 7.5$ against $H_a: \mu > 7.5$, use $\alpha=0.2$.

Q6. An electronics company wanted to estimate its monthly operating expenses in thousands riyals (μ). It is assumed that the monthly operating expenses (in thousands riyals) are distributed according to a normal distribution with variance $\sigma^2=0.584$. Suppose that a random sample of 49 months produced a sample mean $\bar{X} = 5.47$. Test $H_0: \mu=5.5$ against $H_a: \mu \neq 5.5$. Use $\alpha=0.01$.

Q7. The tensile strength of a certain type of thread is approximately normally distributed with standard deviation of 6.8 kilograms. The manufacturer claims that the mean of the tensile strength of this type of thread equals to 70.0 kilogram. Do you agree with this claim if a sample of 20 pieces of the thread had an average tensile strength of 72.8 kilograms? Use $\alpha=0.05$.

12.2. Two Means:

Q1. Two random samples were independently selected from two normal populations with equal variances. The results are summarized as follows.

	First Sample	Second Sample
sample size (n)	12	14
sample mean (\bar{X})	10.5	10.0
sample variance (S^2)	4	5

Let μ_1 and μ_2 be the true means of the first and second populations, respectively. Test $H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$. (use $\alpha=0.05$)

Q2. A researcher was interested in comparing the mean score of female students, μ_f , with the mean score of male students, μ_m , in a certain test. Two independent samples gave the following results:

Sample	Observations						mean	variance	
Scores of Females	89.2	81.6	79.6	80.0	82.8		82.63	15.05	
Scores of Males	83.2	83.2	84.8	81.4	78.6	71.5	77.6	80.04	20.79

Assume that the populations are normal with equal variances.

- (1) The pooled estimate of the variance S_p^2 is
 (A) 17.994 (B) 17.794 (C) 18.094 (D) 18.294 (E) 18.494
- (2) If we want to test $H_0: \mu_f = \mu_m$ against $H_1: \mu_f \neq \mu_m$ then the test statistic equals to
 (A) $Z=1.129$ (B) $T=-1.029$ (C) $T=1.029$ (D) $T=1.329$ (E) $T=-1.329$
- (3) If we want to test $H_0: \mu_f = \mu_m$ against $H_1: \mu_f \neq \mu_m$ at $\alpha=0.1$, then the Acceptance Region of H_0 is
 (A) $(-\infty, 1.812)$ (B) $(-1.812, 1.812)$ (C) $(-1.372, \infty)$ (D) $(-1.372, 1.372)$ (E) $(-1.812, \infty)$
- (4) If we want to test $H_0: \mu_f = \mu_m$ against $H_1: \mu_f \neq \mu_m$ at $\alpha=0.1$, then we
 (A) Reject H_0 (B) Accept H_0 (C) (D) (E)

Q3. A study was conducted to compare to brands of tires A and B. 10 tires of brand A and 12 tires of brand B were selected randomly. The tires were run until they wear out. The results are:

$$\text{Brand A: } \bar{X}_A = 37000 \text{ kilometers } S_A = 5100$$

$$\text{Brand B: } \bar{X}_B = 38000 \text{ kilometers } S_B = 6000$$

Assuming the populations are normally distributed with equal variances. Test $H_0: \mu_A = \mu_B$ against $H_1: \mu_A < \mu_B$. Use a 0.1 level of significance.

Q4. A study was made by a taxi company to decide whether the use of new tires (A) instead of the present tires (B) improves fuel economy. Six cars were equipped with tires (A) and driven over a prescribed test course. Without changing drivers and cares, test course was made with tires (B). The gasoline consumption, in kilometers per liter (km/L), was recorded as follows: (assume the populations are normal with equal unknown variances)

Car	1	2	3	4	5	6
Type (A)	4.5	4.8	6.6	7.0	6.7	4.6
Type (B)	3.9	4.9	6.2	6.5	6.8	4.1

- (a) Test $H_0: \mu_A \leq 5.6$ against $H_1: \mu_A > 5.6$. Use a 0.1 level of significance.
 (b) Test $H_0: \mu_A \geq \mu_B$ against $H_1: \mu_A < \mu_B$. Use a 0.1 level of significance.

Q5. To determine whether car ownership affects a student's academic achievement, two independent random samples of 100 male students were each drawn from the students' body. The first sample is for non-owners of cars and the second sample is for owners of cars. The grade point average for the 100

non-owners of cars had an average equals to 2.70, while the grade point average for the 100 owners of cars had an average equals to 2.54. Do data present sufficient evidence to indicate a difference in the mean achievement between car owners and nonowners of cars? Test using $\alpha=0.05$. Assume that the two populations have variances $\sigma_{non-owner}^2 = 0.36$ and $\sigma_{owner}^2 = 0.40$

Q6. A geologist collected 20 different ore samples, all of the same weight, and randomly divided them into two groups. The titanium contents of the samples, found using two different methods, are listed in the table:

Method 1	Method 2
1.1 1.3 1.3 1.5 1.4	1.1 1.6 1.3 1.2 1.5
1.3 1.0 1.3 1.1 1.2	1.2 1.7 1.3 1.4 1.5
$\bar{X}_1 = 1.25$, $S_1 = 0.1509$	$\bar{X}_2 = 1.38$, $S_2 = 0.1932$

Does this data provide sufficient statistical evidence to indicate that there is a difference between the mean titanium contents using the two different methods? Test using $\alpha=0.05$. (Assume two normal populations with equal variances).

12.3. Single Proportion:

Q1. A researcher was interested in making some statistical inferences about the proportion of smokers (p) among the students of a certain university. A random sample of 500 students showed that 150 students smoke.

- (1) If we want to test $H_0: p=0.25$ against $H_1: p \neq 0.25$ then the test statistic equals to
 (A) $z=2.2398$ (B) $T=-2.2398$ (C) $z=-2.4398$ (D) $Z=2.582$ (E) $T=2.2398$
- (2) If we want to test $H_0: p=0.25$ against $H_1: p \neq 0.25$ at $\alpha=0.1$, then the Acceptance Region of H_0 is
 (A) $(-1.645, \infty)$ (B) $(-\infty, 1.645)$ (C) $(-1.645, 1.645)$ (D) $(-1.285, \infty)$ (E) $(-1.285, 1.285)$
- (3) If we want to test $H_0: p=0.25$ against $H_1: p \neq 0.25$ at $\alpha=0.1$, then we
 (A) Accept H_0 (B) Reject H_0 (C) (D) (E)

Q2. In a random sample of 500 homes in a certain city, it is found that 114 are heated by oil. Let p be the proportion of homes in this city that are heated by oil. A builder claims that less than 20% of the homes in this city are heated by oil. Would you agree with this claim? Use a 0.01 level of significance.

Q3. The Humane Society in the USA reports that 40% of all U.S. households own at least one dog. In a random sample of 300 households, 114 households said that they owned at least one dog. Does this data provide sufficient evidence to indicate that the proportion of households with at least one dog is different from that reported by the Humane Society? Test using $\alpha=0.05$.

Q4. A study was conducted to make some inferences about the proportion of female employees (p) in a certain hospital. A random sample gave the following data:

Sample size	250
Number of females	120

Does this data provide sufficient statistical evidence to indicate that the percentage of female employees in this hospital differs from the percentage of male employees? Test using $\alpha=0.1$. Justify your answer statistically. {Hint: Consider testing $H_0: p=0.5$ against $H_a: p \neq 0.5$ }

Q5. In a certain city, the traffic police was interested in knowing the proportion of car drivers who do not use seat belt. In a study involved 1200 car drivers it was found that 50 car drivers do not use seat

belt. Does this data provide sufficient statistical evidence to indicate that the percentage of car drivers who do not use seat belt is less than 5%. Test using $\alpha=0.1$.

12.4. Two Proportions:

Q1. A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200 students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smokers in school "A" and p_2 is the proportion of smokers in school "B". Test $H_0: p_1 = p_2$ against $H_1: p_1 > p_2$. (use $\alpha=0.05$)

Q2. A food company distributes "smiley cow" brand of milk. A random sample of 200 consumers in the city (A) showed that 80 consumers prefer the "smiley cow" brand of milk. Another independent random sample of 300 consumers in the city (B) showed that 90 consumers prefer "smiley cow" brand of milk. Define:

p_A = the true proportion of consumers in the city (A) preferring "smiley cow" brand.

p_B = the true proportion of consumers in the city (B) preferring "smiley cow" brand.

(a) Test $H_0: p_A = 0.33$ against $H_1: p_A \neq 0.33$. Use a 0.05 level of significance.

(b) Test $H_0: p_A \geq p_B$ against $H_1: p_A < p_B$. Use a 0.05 level of significance.

Q3: Independent random samples of $n_1 = 1500$ and $n_2 = 1000$ observations were selected from binomial populations 1 and 2, and $X_1 = 1200$ and $X_2 = 700$ successes were observed. Let p_1 and p_2 be the population proportions.

(a) Test $H_0: p_1 = p_2$ against $H_1: p_1 \neq p_2$. Use a 0.1 level of significance.

(b) Find a 90% confidence interval for $(p_1 - p_2)$.

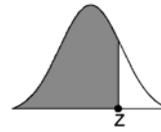
(c) Based on the 90% confidence interval in part (b), can you conclude that there is a difference between the two binomial proportions? Explain.

Q4. A study was conducted to compare between the proportions of smokers in two universities. Two independent random samples gave the following data:

	Univ. (1)	Univ. (2)
Sample size	200	300
Number of smokers	80	111

Does this data provide sufficient statistical evidence to indicate that the percentage of students who smoke differs for these two universities? Test using $\alpha=0.01$.

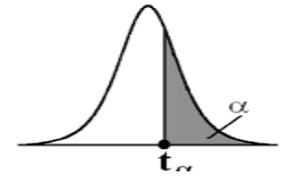
Areas under the Standard Normal Curve



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Percentage Points of the t Distribution; $t_{v, \alpha}$ $\{P(T > t_{v, \alpha}) = \alpha\}$

v	α													
	0.40	0.30	0.20	0.15	0.10	0.05	0.025	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326	2.432	2.576	2.807	3.291



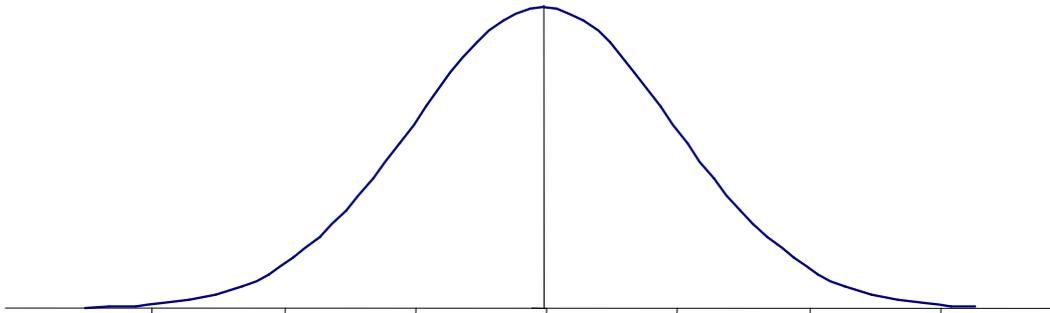
Summary of Confidence Interval Procedures

Problem Type	Point Estimate	Two-Sided 100(1-α)% Confidence Interval
Mean μ variance σ ² known, normal distribution, or any distribution with n>30	\bar{X}	$\bar{X} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
Mean μ normal distribution, variance σ ² unknown	\bar{X}	$\bar{X} - t_{\alpha/2} \frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2} \frac{S}{\sqrt{n}}$ or $\bar{X} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}$ (df: v=n-1)
Difference in two means μ ₁ and μ ₂ variances σ ₁ ² and σ ₂ ² are known, normal distributions, or any distributions with n ₁ , n ₂ >30	$\bar{X}_1 - \bar{X}_2$	$(\bar{X}_1 - \bar{X}_2) - Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ or $(\bar{X}_1 - \bar{X}_2) \pm Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
Difference in means μ ₁ and μ ₂ normal distributions, variances σ ₁ ² = σ ₂ ² and unknown	$\bar{X}_1 - \bar{X}_2$	$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ or $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$; (df: v=n ₁ +n ₂ -2)
Proportion p (or parameter of a binomial distribution)	\hat{p}	$\hat{p} - Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$ or $\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$; $\hat{q} = 1 - \hat{p}$
Difference in two proportions p ₁ - p ₂ (or difference in two binomial parameters)	$\hat{p}_1 - \hat{p}_2$	$(\hat{p}_1 - \hat{p}_2) - Z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + Z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$ or $(\hat{p}_1 - \hat{p}_2) \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$

Summary of Hypotheses Testing Procedures

Null Hypothesis	Test Statistic	Alternative Hypothesis	Critical Region (Rejection Region)
$H_0: \mu = \mu_0$ variance σ^2 is known, Normal distribution, or any distribution with $n > 30$	$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$	$H_1: \mu \neq \mu_0$	$ Z > Z_{\alpha/2}$
		$H_1: \mu > \mu_0$	$Z > Z_{\alpha}$
		$H_1: \mu < \mu_0$	$Z < -Z_{\alpha}$
$H_0: \mu = \mu_0$ Normal distribution, variance σ^2 is unknown	$T = \frac{\bar{X} - \mu_0}{S / \sqrt{n}} ; \text{ df: } v=n-1$	$H_1: \mu \neq \mu_0$	$ T > t_{\alpha/2}$
		$H_1: \mu > \mu_0$	$T > t_{\alpha}$
		$H_1: \mu < \mu_0$	$T < -t_{\alpha}$
$H_0: \mu_1 = \mu_2$ Variances σ_1^2 and σ_2^2 are known, Normal distributions, or any distributions with $n_1, n_2 > 30$	$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$H_1: \mu_1 \neq \mu_2$	$ Z > Z_{\alpha/2}$
		$H_1: \mu_1 > \mu_2$	$Z > Z_{\alpha}$
		$H_1: \mu_1 < \mu_2$	$Z < -Z_{\alpha}$
$H_0: \mu_1 = \mu_2$ Normal distributions, variances $\sigma_1^2 = \sigma_2^2$ and unknown	$T = \frac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} ; \text{ df: } v=n_1+n_2-2$ $S_p^2 = [(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2] / (n_1 + n_2 - 2)$	$H_1: \mu_1 \neq \mu_2$	$ T > t_{\alpha/2}$
		$H_1: \mu_1 > \mu_2$	$T > t_{\alpha}$
		$H_1: \mu_1 < \mu_2$	$T < -t_{\alpha}$
$H_0: p = p_0$ Proportion or parameter of a binomial distribution p ($q=1-p$)	$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{X - np_0}{\sqrt{np_0 q_0}}$	$H_1: p \neq p_0$	$ Z > Z_{\alpha/2}$
		$H_1: p > p_0$	$Z > Z_{\alpha}$
		$H_1: p < p_0$	$Z < -Z_{\alpha}$
$H_0: p_1 = p_2$ Difference in two proportions or two binomial parameters $p_1 - p_2$	$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ $\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$	$H_1: p_1 \neq p_2$	$ Z > Z_{\alpha/2}$
		$H_1: p_1 > p_2$	$Z > Z_{\alpha}$
		$H_1: p_1 < p_2$	$Z < -Z_{\alpha}$

Confidence Interval and Hypothesis Testing: Exercises and Solutions



You can use the graphical representation of the normal distribution to solve the problems.

Exercise 1: Confidence Interval

A machine is set up such that the average content of juice per bottle equals μ .
A sample of 100 bottles yields an average content of 48cl.
Calculate a 90% and a 95% confidence interval for the average content.

Assume that the population standard deviation $\sigma = 5cl$.

Exercise 2: Sample size

What sample size is required to estimate the average contents to within 0.5cl at the 95% confidence level? (= + or - 0.5 cl)

Assume that the population standard deviation $\sigma = 5cl$.

Exercise 3: Hypothesis Testing

A machine is set up such that the average content of juice per bottle equals μ .
A sample of 36 bottles yields an average content of 48.5cl.
Test the hypothesis that the average content per bottle is 50cl at the 5% significance level.

Assume that the population standard deviation $\sigma = 5cl$.

Exercise 4: The impact of sample size

A machine is set up such that the average content of juice per bottle equals μ .
A sample of 100 bottles yields an average content of 48.8cl.
Test the hypothesis that the average content per bottle is 50cl at the 5% significance level.

Compare the conclusion to that based on the 36 bottles sample.

Assume that the population standard deviation $\sigma = 5\text{cl}$

Exercise 5: One-tailed tests

A machine is set up such that the average content of juice per bottle equals μ .

A sample of 36 bottles yields an average content of 48.5cl.

Can you reject the hypothesis that the average content per bottle is less than or equal to 45cl in favour of the alternative that it exceeds 45cl (5% significance level)?

Assume that the population standard deviation $\sigma = 5\text{cl}$.

Exercise 6: Formulating H0

The manager claims that the average content of juice per bottle is less than 50cl. The machine operator disagrees. A sample of 100 bottles yields an average content of 49cl per bottle.

Does this sample allow the manager to claim he is right (5% significance level)?

Assume that the population standard deviation $\sigma = 5\text{cl}$.

Exercise 7: CI for proportions

Sample of 80 customers

60 reply they are satisfied with the service they received

Calculate a 95% confidence interval for the proportion of satisfied customers

Observed value p (from the sample):

Standard deviation of p:

Distribution of p:

95% confidence interval for the true proportion p

Exercise 8: Confidence level of interval

The latest poll (1,100 respondents) reveals that 54% of the population supports the government's budgetary decisions. The margin of error is $\pm 3\%$.

==> Point estimate: 54%

Margin of error: $\pm 3\%$

==> Confidence interval: [51%, 57%]

Observed value p (from the sample):

Standard deviation of p:

Confidence level of interval:

Solution 1

$$\bar{x} = 48$$

$$n=100$$

$$\sigma = 5$$

$$\implies SD(\bar{x}) = \sigma / \sqrt{n} = 5 / \sqrt{100} = 5 / 10 = 0.50$$

$$\implies 90\% \text{ confidence interval: } [\bar{x} - 1.64 SD(\bar{x}), \bar{x} + 1.64 SD(\bar{x})] \\ = [48 - 0.82, 48 + 0.82] = [47.18, 48.82]$$

$$95\% \text{ confidence interval: } [\bar{x} - 1.96 SD(\bar{x}), \bar{x} + 1.96 SD(\bar{x})] \\ = [48 - 0.98, 48 + 0.98] = [47.02, 48.98]$$

Note: 95% confidence interval: It is common practice to round 1.96 up to 2
 $[\bar{x} - 2 SD(\bar{x}), \bar{x} + 2 SD(\bar{x})]$
 $= [48 - 1, 48 + 1] = [47, 49]$

Solution 2

$$SD(\bar{x}) = \sigma / \sqrt{n} = 5 / \sqrt{n}$$

$$2SD(\bar{x}) = 0.5$$

$$\implies 2 \cdot 5 / \sqrt{n} = 0.5$$

$$\implies 10 = 0.5 \sqrt{n}$$

$$\implies 20 = \sqrt{n}$$

$$\implies n = 400$$

Solution 3

$$SD(\bar{x}) = 5 / \sqrt{36} = .83 \implies 2SD(\bar{x}) = 1.66$$

$$\implies \text{Acceptance region} = [\mu - 2SD(\bar{x}), \mu + 2SD(\bar{x})] \\ = [48.34, 51.66]$$

and... $\bar{x} = 48.5$

\implies cannot reject hypothesis $\mu=50$

Solution 4

$$SD(\bar{x}) = 5 / \sqrt{100} = .50 \implies 2SD(\bar{x}) = 1$$

$$\implies \text{Acceptance region} = [\mu - 2SD(\bar{x}), \mu + 2SD(\bar{x})] = [49, 51]$$

$$x_b = 48.8$$

\implies Reject hypothesis $\mu=5$

(if $\mu=50$ is true, we have 95% ch to have \bar{x} between 49 and 51...)

Or,

$$(50 - 48.8) / 0.5 = 1.2 / 0.5 = 2.4 > 2$$

\implies Reject H_0

// Previous case

Here: reject $\mu=50$ on basis of observing 48.8
Before: not reject $\mu=50$ despite observing 48.5

Solution 5

$$SD(\bar{x}) = 5/\sqrt{36} = .83$$

One-tailed test, 5% significance level \implies 5% in right tail
 \implies 1.64 standard deviations

$$\implies \text{Acceptance region} = [-\infty, \mu + 1.64sd(xb)] = [-\infty, 45 + 1.64 * 0.83] = [-\infty, 46.36]$$

$$\bar{x} = 48.5$$

\implies Reject hypothesis $H_0: \mu \leq 45$ in favour of $H_1: \mu > 45$

Note: z-score: $(48.5 - 45) / 0.83 = 4.2 > 1.64 \implies$ Reject

Solution 6

Objective of manager: show that $\mu < 50$

\implies Need to reject : $\mu \geq 50$ (remember:: Cannot 'prove' a hypothesis, only 'disprove' i.e. reject)

$\implies H_0: \mu \geq 50$

$H_0: \mu \geq 50$

$H_1: \mu < 50$

\implies 1-tailed test!

\implies 5% in 1 tail \implies 1.64SD(\bar{x})

$$SD(xb) = 5/\sqrt{100} = .50 \implies 1.64SD(\bar{x}) = 0.82$$

\implies Acceptance region = $[m - 1.64SD(\bar{x}), \infty] = [49.18, \infty]$

$$xb = 49$$

\implies Reject hypothesis $\mu \geq 50$

Or,

$$(49-50)/0.5 = -2 < -1.64$$

\implies Reject H_0

Solution 7

Observed value $\hat{p} : 60/80 = 0.75 = 75\%$

Standard deviation of \hat{p} :

p unknown \implies use p^{\wedge} instead

Proportion of satisfied customers: $\text{Var}(X/n) = \text{Var}(X)/n^2 = \hat{p} (1 - \hat{p})/n = 0.0023$

$\implies N(\hat{p}, \sqrt{\hat{p} (1 - \hat{p})/n}) = N(0.75, 0.048)$

Distribution of \hat{p}

Satisfied or not with probability p, independent opinions ==> Binomial
Approximation of Binomial by Normal distribution: required conditions
 $n\hat{p} = 60 > 5$, $n(1-\hat{p}) = 20 > 5$, $0.1 < \hat{p} = 0.75 < 0.9$ ==> OK

95% Confidence interval for the true proportion p:

$$[\hat{p} - 2\sqrt{\hat{p}(1-\hat{p})/n}, \hat{p} + 2\sqrt{\hat{p}(1-\hat{p})/n}]$$
$$= [0.75 - 2*0.048, 0.75 + 2*0.048] = [0.654, 0.846]$$

Solution 8

Observed value $\hat{p} : 0.54$

Standard deviation of $\hat{p} : \sqrt{\hat{p}(1-\hat{p})/n} = .54*.46/1,100 = 0.015$

==> "error margin of $\pm 3\%$ " corresponds to $\approx 2SD(\hat{p})$

=> 95% confidence level ($3\%/1.5\% = 2\dots$ and The critical value 2 corresponds to a 95% CI)

Question:

Two different types of bulbs (A and B) were being tested to test the claim stating that Type B has a longer lifetime (in hours), on average, than that of Type A. A random sample of Type A and a random sample of Type B were tested; the following statistics were obtained from the two samples:

Type	Sample Size	Mean
A	16	175
B	26	200

Assume that the lifespan of the two types are normally distributed with standard deviations of 7 and 9, respectively. Test the claim using $\alpha=0.05$.

Solution:

$$H_0: \mu_A = \mu_B$$

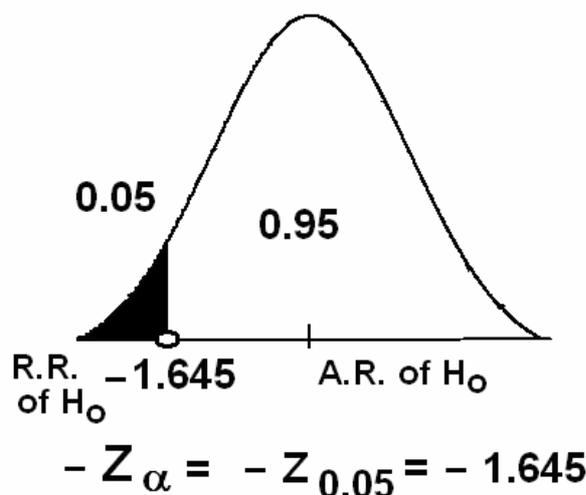
$$H_1: \mu_A < \mu_B$$

$$H_0: \mu_A - \mu_B = 0$$

$$H_1: \mu_A - \mu_B < 0$$

$$\alpha=0.05$$

$$Z = \frac{\bar{X}_A - \bar{X}_B}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} = \frac{175 - 200}{\sqrt{\frac{49}{16} + \frac{81}{26}}} = -10.06$$



Since $Z=-10.06 \in \text{R.R.}$, we reject H_0 at $\alpha=0.05$ and accept $H_1: \mu_A < \mu_B$. Therefore, we conclude that the Type B has a longer lifetime (in hours), on average, than that of Type A.

Answer the following questions

Question No. 1.

The cumulative distribution of a discrete random variable X , is given below:

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{4} & \text{for } 1 \leq x < 3 \\ \frac{1}{2} & \text{for } 3 \leq x < 5 \\ \frac{3}{4} & \text{for } 5 \leq x < 7 \\ 1 & \text{for } x \geq 7 \end{cases}$$

(1) The $P(X = 5)$ equals:

- (A) 0.5
- (B) **0.25**
- (C) 0.75
- (D) 0.0

(2). $P(X > 3)$ equals:

- (A) **0.5**
- (B) 0.25
- (C) 1
- (D) 0.75

(3). $P(1.4 < X < 6)$ equals:

- (A) 0.25
- (B) **0.50**
- (C) 0.30
- (D) 0.0

Question No. 2.

The life of a certain tire brand lives is a random variable X that follows the exponential distribution with a mean of 2 years.

(4). For $x > 0$, the cumulative distribution function (CDF) for the random variable X is:

- (A) e^{-2}
- (B) $1 - e^{-2}$
- (C) e^{-x}
- (D) $1 - e^{-\frac{x}{2}}$

(5). The probability that a tire of this brand will live less than 1.5 years is:

- (A) 0.9534
- (B) 0.3935
- (C) 0.6065
- (D) **0.5276**

(6). The probability that a tire of this brand will live at least 3 years is:

- (A) 0.6358
- (B) **0.2231**
- (C) 0.4905
- (D) 0.3679

Question No. 3.

Let X represents the outcome when a balanced die is tossed.

(7). The mean of $g(X) = 3X^2 + 4$ is

- (A) 45.5
- (B) 14.5
- (C) **49.5**
- (D) 12.5.

(8). The variance of X is:

- (A) 3.641
- (B) **2.916**
- (C) 5.751
- (D) 6.254

(9). The variance of $g(X) = 3X^2 + 4$ is:

- (A) 36.64
- (B) **1342.25**
- (C) 2275
- (D) 254.3.

(10). According to Chebyshev's theorem, for any random variable X with mean μ and variance σ^2 , a lower bound for $P(\mu - 2\sigma < X < \mu + 2\sigma)$ is:

- (A) 0.267
- (B) 0.3175
- (C) **0.750**
- (D) 0.250

Question No. 4.

Let X be a continuous random variable with the probability density function

$$f(x) = \frac{3}{2}x^2, \text{ for } -1 < x < 1.$$

(11). $P(0 < X < 1) = \dots$

- (A) **0.5**
- (B) 0.3
- (C) 0.7
- (D) 0.2

(12). $E(X) = \dots$

- (A) 0.9
- (B) **0.0**
- (C) 0.8
- (D) 0.1

(13). $Var(X) = \dots$

- (A) 0.12
- (B) **0.60**
- (C) 0.40
- (D) 0.18

Question No. 5.

Suppose that the percentage of females in a certain population is 50%. A random sample of 3 people is selected from this population. Let X be the number of females in the sample.

(14). The probability that no females are selected is:

- (A) 0.375
- (B) 0.112
- (C) 0.240
- (D) **0.125**

(15). The probability that at most two females are selected is:

- (A) 0.624
- (B) 0.245
- (C) **0.875**
- (D) 0.821

(16). The expected number of females in the sample is:

- (A) **1.5**
- (B) 2.3
- (C) 5.8
- (D) 0.0

(17). The variance of the number of females in the sample is:

- (A) **0.75**
- (B) 0.30
- (C) 2.1
- (D) 3.25

Question No. 6.

Lots of 40 components each are called acceptable if they contain no more than 3 defectives. The procedure for sampling the lot is to select 5 components at random (without replacement) and to reject the lot if a defective is found. If there are 3 defectives in the entire lot:

(18). the probability that exactly one defective is found in the sample equals:

- (A) 0.1103
- (B) **0.3011**
- (C) 0.1013
- (D) 0.3110

(19). the expected value (mean) of the number of defectives in the sample equals:

- (A) **0.375**
- (B) 0.213
- (C) 0.821
- (D) 0.735

(20). the variance of the number of defectives in the sample equals:

- (A) 0.113298
- (B) **0.311298**
- (C) 0.251471
- (D) 0.174251

Question No. 7.

Suppose that the number of traffic violation tickets issued by a policeman has a Poisson distribution with an average of 2.5 tickets per day.

(21). The average number of tickets issued by this policeman for a period of two days is:

- (A) 2.00
- (B) 1.25
- (C) 2.50
- (D) **5.00**

(22). The probability that this policeman will issue 2 tickets in a period of two days is:

- (A) 0.1404
- (B) 0.2565
- (C) **0.0842**
- (D) 0.1755

Question No. 8.

In a photographic process, the developing time of prints may be considered as a random variable having the normal distribution with a mean of 16.28 second and a standard deviation of 0.12 second. Then, the probability that the developing time to develop one of the prints will be:

(23). anywhere from 16 to 16.5 seconds equals:

- (A) 0.0435
- (B) 0.1762
- (C) **0.9565**
- (D) 0.2018

(24). at least 16.20 seconds equals:

- (A) **0.7454**
- (B) 0.34221
- (C) 0.6502
- (D) 0.2514

(25). at most 16.35 second equals:

- (A) 0.3101
- (B) **0.7190**
- (C) 0.2810
- (D) 0.4053

Question No. 9.

The average life of a certain battery is 5 years, with a standard deviation of 1 year. Assume that the life of the battery approximately follows a normal distribution.

(26). The *sample mean* of a random sample of 5 batteries selected from this product has a mean [i.e $E(\bar{X})$], equal to:

- (A) 0.2
- (B) 5
- (C) 3
- (D) 1

(27). The *variance* of the *sample mean* [i.e $Var(\bar{X})$] of 5 batteries selected from this product is equal to:

- (A) 0.2
- (B) 5
- (C) 3
- (D) 1

(28). The probability that the average life of a random sample of size 16 of such batteries will be less than 5.5 years, is:

- (A) 0.9223
- (B) 0.0228
- (C) 0.9772
- (D) 0.5321

(29). The probability that the average life of a random sample of size 16 of such batteries will be more than 4.75 years is:

- (A) 0.8103
- (B) 0.1587
- (C) 0.9452
- (D) 0.8413

(30). If $P(\bar{X} > a) = 0.1492$ where \bar{X} represents the sample mean for a random sample of size 9 of such batteries, then the numerical value of the constant a is:

- (A) 4.6532
- (B) 6.510
- (C) 5.3466
- (D) 2.8713

Good Luck.



Question No. 1:

A box contains 7 red balls and 3 green balls. A sample of 4 balls was selected randomly in succession and with replacement (each ball being replaced in the box before the next draw is made). Let 'R' denotes a red ball and 'G' denotes a green ball, and suppose that the random variable X represents the number of red balls in the sample.

- (1) The number of elements of the sample space 'S' is:
 - (A) 6
 - (B) 8
 - (C) 27
 - (D) 16**
- (2) The probability $P(\{RRRR\})$ equals:
 - (A) 0.2401**
 - (B) 0.5401
 - (C) 0.3401
 - (D) 0.6401
- (3) The event $(X=1)$ is equivalent to the event:
 - (A) {RGGG}
 - (B) {GGGR}
 - (C) ϕ
 - (D) {RGGG, GRGG, GGRG, GGGR}**
- (4) The probability $P(X = 1)$ equals:
 - (A) 0.3756
 - (B) 0.6756
 - (C) 0.0756**
 - (D) 0.4756

Question No. 2:

500 adult people are classified according to their gender and size as follows:

Size	Male (M)	Female (F)
(A) Large	150	50
(B) Medium	100	60
(C) Small	50	90

Suppose that one person is randomly selected from this group of people.

- (5) The probability that the size of the selected person is large is:
 - (A) 0.7
 - (B) 0.4**
 - (C) 0.1
 - (D) 0.5
- (6) If it is known that the selected person is male, then the probability that his size is

large equals:

- (A) **0.5**
 - (B) 0.3
 - (C) 0.6
 - (D) 0.4
- (7) The events (B) and (F) are:
- (A) Disjoint
 - (B) Independent
 - (C) Not independent**
 - (D) Mutually exclusive

Question No. 3:

Suppose that the events A and B are disjoint (mutually exclusive) and that $P(A)=0.5$ and $P(B \cap A^C)=0.3$. Then:

- (8) $P(A \cap B^C)$ equals:
 - (A) 0
 - (B) 0.3
 - (C) 0.2
 - (D) 0.5**
- (9) $P(A|B)$ equals:
 - (A) 0.5
 - (B) 0**
 - (C) 0.2
 - (D) 1.0

Question No. 4:

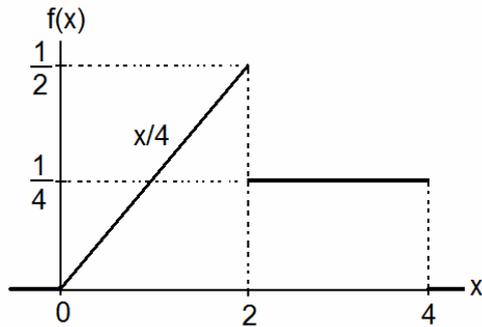
Suppose that the random variable X has a mean $\mu = 12$ and a standard deviation $\sigma = 2$, then:

- (10) The approximated value of $P(2 < X < 22)$ is:
 - (A) 0.96**
 - (B) 0.9375
 - (C) 0.8889
 - (D) 0.75
- (11) $Var(7X - 2)$ equals:
 - (A) 194
 - (B) 192
 - (C) 196**
 - (D) 12

Question No. 5:

If the probability density function of the random variable X is given by:





$$f(x) = \begin{cases} \frac{x}{4}, & 0 < x \leq 2 \\ \frac{1}{4} & ; 2 < x \leq 4 \\ 0 & ; \text{elsewhere} \end{cases}$$

(12) $P(X > 1)$ equals:

- (A) 0.575
(B) 0.875
 (C) 0.175
 (D) 0.775

(13) If $F(x)$ is the cumulative distribution function (CDF) of X , then $F(3)$ equals:

- (A) **0.75**
 (B) 0.55
 (C) 0.95
 (D) 0.65

(14) The mean of X is:

- (A) 1.1667
 (B) 2.0000
 (C) 1.5667
(D) 2.1667

Question No. 6:

The probability function of the random variable X is given by:

$$f(x) = \begin{cases} \frac{k}{x} & ; x = 1, 2, 3, 4 \\ 0 & ; \text{elsewhere} \end{cases}$$

(15) The value of k equals:

- (A) 0.88
 (B) 1.88
(C) 0.48
 (D) 1.48

(16) The mean of X is:

- (A) 2.50
(B) 1.92
 (C) 2.92
 (D) 1.50

(17) The Variance of X is:

- (A) 2.1136
 (B) 3.1136
 (C) 0.1136
(D) 1.1136

Question No. 7:

From the past experience, the owner of a fashion store noticed that the percentages of the customers with large size, medium size, and small size are 25%, 40%, and 35%, respectively. He also noticed that 20% of the large-sized customer are females, 50% of the medium-sized customer are females, and 60% of the small-sized customer are females. If a new customer has arrived to the store, then:

(18) The probability that the customer is female is:

- (A) 0.36
 (B) 0.16
(C) 0.46
 (D) 0.26

(19) If it is known that the customer is female, then the probability that her size is large is:

- (A) 0.7087
(B) 0.1087
 (C) 0.4087
 (D) 0.2087

Question No. 8:

Suppose that X has a binomial distribution with mean $\mu=2$ and variance $\sigma^2=1.96$, then:

(20) The values of n (number of trials) and p (the probability of success) are respectively:

- (A) 10 and 0.3
(B) 100 and 0.02
 (C) 30 and 0.1
 (D) 75 and 0.05

(21) Using Poisson approximation to binomial distribution, the approximated value of $P(X=3)$ is:

- (A) **0.1804**
 (B) 0.2804
 (C) 0.3804
 (D) 0.4804





Question No. 9:

A box contains 8 black balls and 2 white balls. A sample of 4 balls was selected randomly and without replacement. Suppose that the random variable X represents the number of black balls in the sample.

(22) The set of possible values of X is:

- (A) {2, 3, 4}
- (B) {0, 1, 2, 3, 4}
- (C) {1, 2, 3, 4}
- (D) {3, 4}

(23) The probability of getting at least 3 black balls is:

- (A) 0.7667
- (B) 0.0667
- (C) 0.5667
- (D) **0.8667**

Question No. 10:

(24) Suppose that the number of telephone calls received every hour by the secretary in certain company has a Poisson distribution with an average of 8 calls per hour. The probability that the secretary will receive 5 calls during a period of 30 minutes is:

- (A) 0.9563
- (B) **0.1563**
- (C) 0.4563
- (D) 0.2463

(25) Suppose that the random variable X has an exponential distribution with a mean of 4, and suppose that $F(x)$ is the CDF of X . Then $F(8)$ equals:

- (A) 0.7647
- (B) 0.1647
- (C) **0.8647**
- (D) 0.5647

(26) If $Z \sim N(0,1)$ and $P(0 < Z < a) = 0.437$, then the value of a equals:

- (A) 0.53
- (B) -1.53
- (C) -0.53
- (D) **1.53**

(27) Suppose that $X \sim \text{Uniform}(5, 10)$. The mean and the variance of X are, respectively:

- (A) **7.5 and 2.0833**
- (B) 7.0 and 2.5833

(C) 8.0 and 1.0833

(D) 7.8 and 1.5833

Question No. 11:

Suppose that the random variable X , representing the lifespan of a certain electronic device, is normally distributed with a mean of 18 months and a standard deviation of 4 months. Then:

(28) $P(X < 11)$ equals:

- (A) 0.9599
- (B) 0.3485
- (C) **0.0401**
- (D) 0.1973

(29) The percentage of electronic devices that will last for more than 11 months is:

- (A) **95.99%**
- (B) 34.85%
- (C) 4.01%
- (D) 19.73%

(30) If the manufacture is willing to replace no more that 2.5% of the devices in case of malfunctioning, then the warranty time (in months) must be no more than:

- (A) 7.16
- (B) 8.16
- (C) 9.16
- (D) **10.16**

THE END





Questions 1 – 2 refer to the following:

Below is the probability distribution function for the number of high school years that students at a local high school play on a sports team.

X	$P(X=x)$
0	0.32
1	0.12
2	?
3	0.18
4	0.14

Ex. 1

What is the probability that $X=2$?

- A. 0.24
- B. 0.76
- C. 0.32
- D. Cannot determine

Solution

A

Ex. 2

Over the long run, the average number of years that we would expect students at this high school to play on a sports team is:

- A. 0
- B. 1.7
- C. 2
- D. 2.6

Solution

B

Ex. 3

According to the 2000 United States Census, 12.3% of the population is Black or African American. The probability that a randomly selected U. S. resident is NOT Black or African American is:

- A. 0.123
- B. 0.877
- C. 0.754
- D. Cannot determine

Solution

B

Ex. 4

Assume the statistics final is a multiple choice test with 40 questions. Each question has four choices with one correct answer per question. If you were to randomly guess on each of the questions, what is the probability of getting exactly the expected number of correct answers?

- A. 0.5839
- B. 0.5605
- C. 0.25

- D. 0.1444

Solution

B

Ex. 5

In an exponential distribution, the mean is larger than the median.

- A. true
- B. false

Solution

B

Ex. 6

In Fall 1999, students in one Math 10 section determined that the length of movies at the cinema was normally distributed with a mean of 148 minutes and a standard deviation of 19 minutes. Find the third quartile and interpret it.

- A. 75 minutes; Three-fourths of the movie lengths fall below 75 minutes.
- B. 160.8 minutes; Three-fourths of the movie lengths fall below 160.8 minutes.
- C. 160.8; Three-fourths of the movies last 160.8 minutes.
- D. 75 minutes; Three-fourths of the movies last 75 minutes.

Solution

B

Ex. 7

Which of the following is FALSE about data that follows the normal distribution?

- A. The mean is the same as the mode.
- B. The standard deviation is the same as the mean.
- C. The median is the same as the mode.
- D. Most data is within 3 standard deviations of the median.

Solution

B

Ex. 8

The graph showing the age of getting a driver's license in California starts and peaks at age 16, and decreases from there. This shape most closely resembles what type of distribution?

- a. Normal
- b. Binomial
- c. Uniform
- d. Exponential

Solution

D

Use the following information for questions 9 and 10:

The amount of time that a randomly chosen 6th grade student spends on homework per week is uniformly distributed from 30 to 120 minutes.

Ex. 9

What is the probability that a randomly chosen 6th grade student spends at least 60 minutes per week on homework knowing that he/she will spend at most 80 minutes per week on homework?

- a. 1.20
- b. 0.6667
- c. 0.2222
- d. 0.4

Solution

D

Ex. 10

What is the expected amount of time that a randomly chosen 6th grade student spends on homework per week?

- a. 45 minutes
- b. 60 minutes
- c. 30 minutes
- d. 75 minutes

Solution

D

Use the following information for questions 11 and 12:

The length of time a randomly chosen 9-year old child spends playing video games per day is approximately exponentially distributed with a mean equal to 2 hours.

Ex. 11

Find the probability that a randomly chosen 9-year old will play video games at most 3 hours.

- a. 0.7769
- b. 0.9975
- c. 0.0025
- d. 0.2231

Solution

A

Ex. 12

70% of 9-year old children will play video games per day for at most how long?

- a. 0.60 hours
- b. 2.41 hours
- c. 0.71 hours
- d. Cannot determine

Solution

B

Use the following information for questions 13 and 14:

Research has shown that studying improves a student's chances to 80% of selecting the correct answer to a multiple choice question. A multiple choice test has 15 questions. Each question has 4 choices.

Ex. 13

What is the distribution for the number of questions answered correctly when a student studies?

- a. $B(15, 0.80)$
- b. $B(15, 0.25)$

- c. $P(15)$
- d. $P(6)$

Solution

A

Ex. 14

Suppose that a student does not study for the test but randomly guesses the answers. What is the probability that the student will answer 7 or 8 questions correctly?

- a. 0.2951
- b. 0.0524
- c. 0.0131
- d. Cannot determine

Solution

B

Ex. 15

A downtown hotel determined the probability of finding X taxicabs waiting outside the hotel anytime between 5 PM and midnight. The information is shown in the table.

X	$P(X)$
1	0.0667
2	0.1331
3	0.2000
4	0.2667
5	0.3333

What is the average number of taxicabs that are expected to be waiting outside the hotel anytime between 5 PM and midnight?

- a. 3.7
- b. 3
- c. 0
- d. 15

Solution

A

Ex. 16

. During the registration period for a new quarter, the De Anza College Registrar’s Office processes approximately 75 applications per hour, on the average. What is the probability that it will process more than 80 applications for a randomly chosen hour? (This is a Poisson problem. If you did not cover the Poisson Distribution, then skip this problem.)

- a. 0.0379
- b. 0.2589
- c. 0.7411
- d. 0.0248

Solution

B

Questions 17 - 19 refer to the following:

$$P(T) = 0.69 \quad P(S) = 0.5, \quad P(S|T) = 0.5$$

Ex. 17

Events S and T are:

- a. mutually exclusive
- b. independent
- c. mutually exclusive and independent
- d. neither mutually exclusive nor independent

Solution

B

Ex. 18

Find $P(S \text{ AND } T)$

- a. 0.3450
- b. 0.2500
- c. 0.6900
- d. 1

Solution

A

Ex. 19

Find $P(S \text{ OR } T)$

- a. 0.6900
- b. 1.19
- c. 0.8450
- d. 0

Solution

C

Ex. 20

Based on data from the US Census Bureau the average age of US residents is 36.31 with a standard deviation of 21.99. The data is normally distributed. The notation for the distribution is:

- a. $X \sim N(36.31, 21.99)$
- b. $X \sim N(21.99, 36.31)$
- c. $X \sim B(36.31, 22)$
- d. $X \sim U(0, 36.31)$

Solution

A

Ex. 21

In a binomial distribution we:

- a. count the number of successes until a failure is obtained
- b. count the number of trials until a success is obtained
- c. count the number of successes in a finite number of trials
- d. count the number of trials until the number of successes equals the number of failures

Solution

C

Ex. 22

Certain stocks have a probability of 0.6 of returning a \$100 profit. They also have a probability of 0.4 of having a loss of \$300. Over the long run, what is the best thing to do to maximize your profit, and why?

- a. Invest in the stocks because there is a greater probability of making money than losing money.
- b. Do not invest in the stocks because the dollar amount for each loss is greater than the dollar amount for each gain.
- c. Invest in the stocks because making \$100 per stock is preferred to losing \$300 per stock.
- d. Do not invest in the stocks because the expected value is a loss.

Solution

D

**SOLVED PROBLEMS – STAT 324,
Dr. M. Kayid**

Questions 23 - 27 refer to the following table :

	American Indian	Asian/Pacific Islander	Black	Hispanic	White	Undeclared	Total
Administrator	0	3	5	5	21	0	34
Staff	1	35	21	30	201	16	304
Faculty	3	58	14	45	141	17	278
Total	4	96	40	80	363	33	616

Suppose that one De Anza College employee is randomly selected.

Ex. 23

Find P (the employee is an Administrator)

- **a:** 278/34
- **b:** 304/616
- **c:** 34/616
- **d:** 80/616

Solution

C

Ex. 24

Find P (the employee is Faculty AND American Indian)

- **a:** 382/616
- **b:** 3/616
- **c:** 3/4
- **d:** 3/278

Solution

B

Ex. 25

Find P (employee is Staff OR Hispanic)

- a. 384/616
- b. 80/616
- c. 304/616
- d. 354/616

Solution

D

Ex. 26

Find P (employee is an Administrator GIVEN the employee is Black)

- a. 40/616
- b. 5/34
- c. 5/616
- d. 5/40

Solution

D

Exercise 27

Being an Administrator and an American Indian are

- a. mutually exclusive events
- b. independent events
- c. mutually exclusive and independent events

- d. neither mutually exclusive nor independent events

Solution

A

Questions 28 - 31 refer to the following:

When a customer calls the "Help Line" at ABC Computer Software Co., the amount of time that a customer must wait "on hold" until somebody answers the line and helps the customer follows an exponential distribution with mean of 7.5 minutes.

Ex. 28

What is the probability that a customer waits more than 10 minutes to receive help?

- a. 0.2636
- b. 0.75
- c. 0.7364
- d. 0

Solution

A

Ex. 29

What is the 40th percentile of wait times for customers calling the help line?

- a. 6.87 minutes
- b. 3.83 minutes
- c. 0.68 minutes
- d. 0.122 minutes

Solution

B

Ex. 30

The customer wait time that is 1 standard deviation above the mean is:

- a. 2.17 minutes
- b. 7.5 minutes
- c. 9.67 minutes
- d. 15 minutes

Solution

D

Ex. 31

The probability that a customer calling the help line waits exactly 6 minutes for help:

- a. 0
- b. 0.45
- c. 0.55
- d. 0.8

Solution

A

Questions 32 – 34 refer to the following:

ABC Delivery Service offers next day delivery of packages weighing between 2 and 20 pounds in a certain city. They have found that the weights of the packages they deliver are uniformly distributed between 2 and 20 pounds.

Ex. 32

What is the probability that a package weighs between 10 and 15 pounds?

- a. 0.2778
- b. 0.5556
- c. 0.2500
- d. 0.8333

Solution

A

Ex. 33

Given that a package weighs less than 10 pounds, what is the probability that it weighs less than 5 pounds?

- a. 0.1667
- b. 0.6250
- c. 0.3750
- d. 0.5000

Solution

C

Ex. 34

35% of packages weigh less than how many pounds?

- a. 7.8 pounds
- b. 8.3 pounds
- c. 11.7 pounds
- d. 13.7 pounds

Solution

B

Ex. 35

Suppose that the probability that an adult in California will watch a Giant's World Series game is 65%. Each person is considered independent. Of interest, is the number of adults in California we must survey until we find one who will watch a Giant's World Series game. What is the probability that you must ask 2 or 3 people? (This is a geometric problem. If you did not cover the geometric distribution, then skip this problem.)

- a. 0.6500
- b. 0.3071
- c. 0.2275
- d. 0.0796

Solution

B

Questions 36– 38 refer to the following:

The amount of time De Anza students work per week is approximately normally distributed with mean of 18.17 hours and a standard deviation of 12.92 hours.

Ex. 36

The median is:

- a: Not enough information
- b: 12.92
- c: 2.0
- d: 18.17

Solution

D

Ex. 37

The 90th percentile for the amount of time De Anza students work per week is:

- a. 1.61
- b. 18.17
- c. 90
- d. 34.7

Solution

D

Ex. 38

Which of the following is NOT TRUE about the normal distribution?

- a. the mean, median and mode are equal
- b. the curve is skewed to the right
- c. the curve never touches the x-axis

- d. the area under the curve is one.

Solution

B

Ex. 39

We use the z-score to:

- a. compare normal distributions with different averages and standard deviations
- b. drive statistics students nuts
- c. compare exponential distributions with the same average
- d. compare uniform distributions with different minimum and maximum numbers

Solution

A

With My Best Regards

Dr. M. Kayid



**SOLVED PROBLEMS – STAT 324,
Dr. M. Kayid**



Question No. 1:

400 people are classified according to their monthly salary as follows:

Monthly salary	No. People
(L) Less than 3000	50
(B) Between 3000 and 8000	200
(M) More than 8000	150

The percentage of male in each salary group are 40%, 60% and 80% respectively. A person was chosen randomly.

(1) The probability that the person is a male =

(A) 0.45
(B) 0.65
(C) 0.18
(D) 1.00

(2) If it is known that the person is a male, then the probability that his salary less than 3000:

(A) 0.0769
(B) 0.1769
(C) 0.2769
(D) 0.5769

Question No. 2:

A continuous random variable X has a cumulative distribution function F(x) as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ x/4 & 0 \leq x < 1 \\ x^2/4 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

(3) P(X>1)

(A) 0.25
(B) 0.50
(C) 0.75
(D) 1.00

(4) P(X=2)=

(A) 1.0
(B) 0.0
(C) 0.5
(D) 2.0

(5) P(1.0<X<2.0)=

(A) 1.00
(B) 0.75
(C) 0.50
(D) 0.25

Question No. 3:

In a certain Class of STAT 324, it is known that 60% of the students are from engineering college. A random sample of 4 students is selected at random. Let X represents the number of engineering students in the sample.

(6) The probability that there will be exactly one engineering student in the sample is

(A) 0.6352
(B) 0.2736
(C) 0.2536
(D) 0.1536

(7) The expected number (mean) of engineering students in the sample is

(A) 0.6
(B) 0.4
(C) 2.4
(D) 2.0





Question No. 4:

If the probability density function is given by $f(x) = 3x^2$ for $0 < x < 1$, then:

(8) $P(X > 0.5)$ equals:

(A)	0.975
(B)	0.875
(C)	0.775
(D)	0.675

(9) $E(X^2)$ equals:

(A)	0.6
(B)	0.5
(C)	0.4
(D)	0.3

(10) If $F(x)$ is the cumulative distribution function (CDF) of X , then $F(0.5)$ equals:

(A)	0.125
(B)	0.225
(C)	0.325
(D)	0.425

Question No. 5:

A random committee of size 3 is selected from 2 chemical engineers and 4 industrial engineers. Let X representing the number of chemical engineers in the committee.

(11) The number of possible committees are

(A)	5
(B)	12
(C)	6
(D)	20

(12) The probability that there will be no industrial engineers in the selected committee is

(A)	0.65
(B)	0.45
(C)	0.05
(D)	0.0

(13) the mean of the random variable X will be

(A)	2.0
(B)	1.5
(C)	1.0
(D)	0.5

Question No. 6:

The random variable, X , representing the number of patients arriving to the emergency department in a certain hospital has a Poisson distribution with an average of 2 patient per an hour.

(14) The probability that exactly 3 patients will arrive during a period of two hours to this emergency department is:

(A)	0.1954
(B)	0.2954
(C)	0.3954
(D)	0.4954

(15) The probability that exactly 2 patients will arrive during an hour to this emergency department is:

(A)	0.2707
(B)	0.2804
(C)	0.3804
(D)	1.0

(16) The variance of the number of patients arriving to this emergency department during a day is:

(A)	48
(B)	24
(C)	12
(D)	6





Question No. 7:

Suppose that the events A and B are defined on the same sample space such that: $P(A)=0.3$ and $P(B)=0.6$,

(17) If A and B are independent then $P(B|A)=$

(A)	0.0
(B)	0.3
(C)	0.6
(D)	0.18

(18) If A and B are disjoint then $P(A|B)=$

(A)	0.0
(B)	0.3
(C)	0.6
(D)	0.18

(19) If $A \subset B$ then $P(A|B)=$

(A)	0.12
(B)	0.50
(C)	0.22
(D)	0.30

Question No. 8:

Suppose that $X \sim \text{Binomial}(5,0.4)$ and $Y \sim \text{Poisson}(4)$ are independent random variables. Then

(20) $E(X^2)$

(A)	2.2
(B)	3.2
(C)	4.2
(D)	5.2

(21) $\text{Var}(2X-Y)$

(A)	20.8
(B)	8.8
(C)	2.8
(D)	0.8

Question No. 9:

60 people are classified according to their nationality and monthly salary as follows:

	Nationality	
	(S) Saudi	(N) Non-Saudi
Monthly salary		
(L) Less than 3000	5	8
(B) Between 3000 and 8000	20	5
(M) More than 8000	15	7

Suppose that one person is randomly selected from this group of people.

(22) The probability that the salary of the selected person is less than 3000 equals:

(A)	0.1167
(B)	0.9167
(C)	0.5167
(D)	0.2167

(23) If it is known that the selected person is Saudi, then the probability that his salary is less than 3000 equals:

(A)	0.825
(B)	0.225
(C)	0.125
(D)	0.025

Question No. 10:

Suppose that the mean and the variance of a random variable, X, are: $\mu = 50$ and $\sigma^2 = 16$, then:

(24) The approximated value of $P(38 < X < 62)$ is:

(A)	0.9999
(B)	0.8889
(C)	0.7778
(D)	0.6667

(25) $\text{Var}(10X + 100)$ equals:

(A)	16
(B)	160
(C)	1600
(D)	16000

THE END





Question No. 1:

A random sample of size 9 from a normal distribution with mean μ gave the following sample mean and sample standard deviation $\bar{X} = 12$, $S = 0.3$.

1. A point estimate of μ is

(A)	1.50
(B)	12.00
(C)	1.20
(D)	1.33

2. The estimated variance of \bar{X}

(A)	0.01
(B)	0.09
(C)	0.30
(D)	0.90

3. The lower bound of the 96% confidence for μ is

(A)	9.87
(B)	10.45
(C)	9.23
(D)	11.76

4. The upper bound of the 96% confidence for μ is

(A)	14.23
(B)	13.55
(C)	12.24
(D)	14.77

Question No. 2:

Suppose that a random sample of size $n_1 = 16$ is taken from a normal population with a mean $\mu_1 = 2.5$ and a standard deviation $\sigma_1 = 2.0$. A second independent random sample of size $n_2 = 25$ is taken from a normal population with a mean $\mu_2 = 3.6$ and a standard deviation $\sigma_2 = 2.5$. Let \bar{X}_1 and \bar{X}_2 be the means of the first and second samples, respectively. The observed values of \bar{X}_1 and \bar{X}_2 are 2.6 and 3.4 respectively.

5. The mean of $\bar{X}_1 - \bar{X}_2$

(A)	6.6
(B)	-0.8
(C)	6.1
(D)	-1.1

6. The variance of $\bar{X}_1 - \bar{X}_2$,

(A)	4.5
(B)	0.5
(C)	0.3
(D)	0.18

7. $P(\bar{X}_1 - \bar{X}_2 < 0)$

(A)	0.8406
(B)	0.9406
(C)	0.5406
(D)	0.2406

8. The lower bound of the 95% confidence interval for $\mu_1 - \mu_2$ is

(A)	-2.1859
(B)	-0.1459
(C)	-1.2059
(D)	-3.1259

9. The upper bound of the 95% confidence interval for $\mu_1 - \mu_2$ is

(A)	0.5859
(B)	2.1859
(C)	1.2259
(D)	0.3059



Question No. 3:

A random variable X has a normal distribution with mean $\mu = 40$ and standard deviation $\sigma = 4$.

10. $P(X > 42) =$

(A)	0.1269
(B)	0.3085
(C)	0.4257
(D)	0.2167

11. $P(42 < X < 44) =$

(A)	0.1498
(B)	0.2598
(C)	0.3598
(D)	0.4598

12. The value of c for which $P(X > c) = 0.9732$

(A)	31.63
(B)	32.83
(C)	31.28
(D)	32.28

Question No. 4:

The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution:

$$F(x) = 1 - e^{-10x}, x > 0.$$

13. This distribution is

(A)	uniform
(B)	Poisson
(C)	normal
(D)	exponential

14. Find the probability of waiting less than 12 minutes between successive speeders:

(A)	0.8647
(B)	0.7482
(C)	0.9423
(D)	0.6121

15. The mean waiting time between successive speeders spotted is

(A)	0.12
(B)	0.01
(C)	0.10
(D)	0.05

Question No. 5:

For the sample 1.6, 1.1, 1.2, 1.7 and 1.9,

16. the mean =

(A)	1.30
(B)	1.50
(C)	1.41
(D)	1.28

17. the variance =

(A)	0.415
(B)	0.315
(C)	0.115
(D)	0.215



Question No. 6:

A population has a mean μ and a variance σ^2 . Ahmed, Fahad, and Saleh have been asked to draw samples of sizes 60, 60 and 50 respectively. Denote by \bar{X}_A , \bar{X}_F and \bar{X}_S to their sample averages. Then

18.

(A)	$\bar{X}_A = \bar{X}_F$ always
(B)	$\bar{X}_A > \bar{X}_F$ always
(C)	$\bar{X}_A < \bar{X}_F$ always
(D)	None of the above

19.

(A)	$E(\bar{X}_F) = E(\bar{X}_S)$ always
(B)	$E(\bar{X}_F) > E(\bar{X}_S)$ always
(C)	$E(\bar{X}_F) < E(\bar{X}_S)$ always
(D)	None of the above

20.

(A)	$\text{Var}(\bar{X}_F) = \text{Var}(\bar{X}_S)$ always
(B)	$\text{Var}(\bar{X}_F) > \text{Var}(\bar{X}_S)$ always
(C)	$\text{Var}(\bar{X}_F) < \text{Var}(\bar{X}_S)$ always
(D)	None of the above

Question No. 7:

In order to estimate the proportion of homes with DSL connection in Riyadh, a survey of 500 homes found 350 with DSL connections. Let \hat{p} be the sample proportion for the homes with DSL connections. Then

21. $\hat{p} =$

(A)	.35
(B)	.45
(C)	.50
(D)	.70

22. The estimated standard deviation of $\hat{p} =$

(A)	0.0205
(B)	0.2050
(C)	0.0250
(D)	0.2105

23. The estimated standard error for $\hat{p} =$

(A)	0.0205
(B)	0.2050
(C)	0.0250
(D)	0.2105

24. The length of 95% confidence interval for p=

(A)	.4020
(B)	0.8030
(C)	0.0803
(D)	0.0402

25. For 95% confidence interval, the maximum error will not exceed

(A)	0.0803
(B)	0.0402
(C)	0.8030
(D)	0.4020

H. W. [1]

Answer the Following Questions:

→ If the probability that it will rain tomorrow is 0.23, then:

(1) the probability that it will not rain tomorrow is:

(A) -0.23	(B) 0.77	(C) -0.77	(D) 0.23
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→ The probability that a factory will open a branch in Riyadh is 0.7, probability that it will open a branch in Jeddah is 0.4, and the probability that it will open a branch in either Riyadh or Jeddah or both is 0.8, then the probability that it will open a branch:

(2) In both cities is:

(A) 1.1	(B) 1.9	(C) 0.3	(D) 0.8
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(3) In neither cities is:

(A) 0.4	(B) 0.7	(C) 0.3	(D) 0.2
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→ A random sample of 200 adults are classified according to sex and their level of education in the following table:

Education	Male	Female
Elementary	28	50
Secondary	38	45
College	22	17

If a person is selected at random from this group, then:

(4) the probability that he is a male is:

(A) 0.3182	(B) 0.44	(C) 28	(D) 78
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(5) The probability that the person is male given that the person has a secondary education is:

(A) 0.4318	(B) 0.4578	(C) 0.19	(D) 0.44
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(6) The probability that the person does not have a college degree given that the person is a female is:

(A) 0.8482	(B) 0.1518	(C) 0.475	(D) 0.085
------------	------------	-----------	-----------

➔ If in a class of 324-Stat. of 80 students , 60 are from engineering college and the rest are from computer science college, 10% of the engineering college students have taken this course before, and 5% of computer science college students have taken this course before. If one student from this class is randomly selected, then:

(7) the probability that he has taken this course before is:

(A) 0.25	(B) 0.20	(C) 0.80	(D) 0.75
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(8) If the selected student has taken this course before then the probability that he is from the computer science college is:

(A) 0.625	(B) 0.375	(C) 0.80	(D) 0.20
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(9) Two engines operate independently, if the probability that an engine will start is 0.3, and the probability that other engine will start is 0.5, then the probability that both will start is:

(A) 1	(B) 0.15	(C) 0.24	(D) 0.5
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(10) Assume that $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cap B \cap C) = 0.05$, and $P(A \cap B) = 0.92$, then the event A and B are,

(A) Independent	(B) Dependent	(C) Disjoint	(D) None of these.
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(11) Using question (2), $P(C|A \cap B)$ is equal to,

(A) 0.604	(B) 0.625	(C) 0.054	(D) -0.925
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(12) A company in Riyadh has 100 cars, 0.46 are white. Of the cars that aren't white, 0.40 are green. How many company cars are neither white nor green (not white or not green)?

(A) 67	(B) 54	(C) 14	(D) 20
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Suppose there is a factory with three assembly lines (A, B, and C) that each makes the same part. 50% of parts produced by the factory come off of assembly line A, 30% come off of assembly line B, and 20% come off of assembly line C. Finished parts can be categorized as either defective or not. It is known that 0.4% of the parts from line A are defective, 0.6% of the parts from line B are defective, and 1.2% of the parts from line C are defective.

(13) The probability of selecting a defective part is equal to:

(A) 0.323	(B) 0.290	(C) 0.387	(D) 0.006
-----------	-----------	-----------	-----------

(14) Suppose that we are holding a defective part in our hand, the probability that it came from assembly line A?

(A) 0.323	(B) 0.290	(C) 0.387	(D) 0.006
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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ
 Department of Statistics
 & Operations Research
 College of Science, King Saud University



STAT 324
Supplementary Examination
Second Semester
1427 – 1428

- Mobile Telephones are not allowed in the classrooms.
- Time allowed is 2 hours.
- Answer all questions.
- Choose the nearest number to your answer.
- **WARNING:** Do not copy answers from your neighbors. They have different questions forms.
- For each question, put the code of the correct answer in the following table beneath the question number:

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

►►

Let the random variable X have a discrete uniform with parameter $k=3$ and with values 0, 1, and 2.

(1)	The mean of X is							
	(A)	1.0	(B)	2.0	(C)	1.5	(D)	0.0
(2)	The variance of X is							
	(A)	0.0	(B)	1.0	(C)	0.67	(D)	1.33

►►

Suppose that the percentage of females in a certain population is 50%. A sample of 3 people is selected randomly from this population.

(3)	The probability that no females are selected is							
	(A)	0.000	(B)	0.500	(C)	0.375	(D)	0.125
(4)	The expected number of females in the sample is							
	(A)	3.0	(B)	1.5	(C)	0.0	(D)	0.50
(5)	The variance of the number of females in the sample is							
	(A)	3.75	(B)	2.75	(C)	1.75	(D)	0.75

►►

Suppose that a family has 5 children, 3 of them are girls and the rest are boys. A sample of 2 children is selected randomly and without replacement.

(6)	The probability that no girls are selected is							
	(A)	0.0	(B)	0.3	(C)	0.6	(D)	0.1
(7)	The expected number of girls in the sample is							
	(A)	2.2	(B)	1.2	(C)	0.2	(D)	3.2
(8)	The variance of the number of girls in the sample is							
	(A)	36.0	(B)	3.6	(C)	0.36	(D)	0.63

►►

Suppose that the number of telephone calls received per day has a Poisson distribution with mean of 4 calls per day.

(9)	The probability that 2 calls will be received in a given day is							
	(A)	0.546525	(B)	0.646525	(C)	0.146525	(D)	0.746525
(10)	The expected number of telephone calls received in a given week is							
	(A)	4	(B)	7	(C)	28	(D)	14
(11)	The probability that at least 2 calls will be received in a period of 12 hours is							
	(A)	0.59399	(B)	0.19399	(C)	0.09399	(D)	0.29399

►►

Given a standard normal distribution. The area under the curve which lies:

(12)	to the left of $Z = 1.39$ (Hint: $Z \leq 1.39$) is							
	(A)	0.7268	(B)	0.9177	(C)	0.2732	(D)	0.0832
(13)	between $Z = -2.16$ and $Z = 0.65$ is							
	(A)	0.9177	(B)	0.2732	(C)	0.0294	(D)	0.7268

►►►

The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128 kg and a standard deviation of 9 kg.

(14)	The percentage of those fat persons with weights at most 110 kg is							
	(A)	0.09 %	(B)	90.3 %	(C)	99.82 %	(D)	2.28 %
(15)	The percentage of those fat persons with weights more than 149 kg is							
	(A)	0.09 %	(B)	0.99 %	(C)	9.7 %	(D)	99.82 %
(16)	The weight x above which 86% of those persons will be							
	(A)	118.28	(B)	128.28	(C)	137.72	(D)	81.28

►►►

Suppose that a system contains a certain type of components whose lifetime is given by T . The random variable T is modeled nicely by an exponential distribution with mean of 6 years. If a random sample of four of these components are installed in different systems. Then,

(17)	the variance of the random variable T is							
	(A)	136	(B)	$(36)^2$	(C)	6	(D)	36
(18)	the probability that at most one of the components in the sample will be functioning more than 6 years is							
	(A)	0.4689	(B)	0.6321	(C)	0.5311	(D)	0.3679
(19)	the probability that at least two of the components in the sample will be functioning more than 6 years is							
	(A)	0.4689	(B)	0.6321	(C)	0.5311	(D)	0.3679
(20)	the expected number of components in the sample which will be functioning more than 6 years is approximately							
	(A)	3.47	(B)	1.47	(C)	4.47	(D)	3

►►►

The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with the mean 3.0 minutes and the standard deviation of 1.4 minutes. If a random sample of 49 customers is observed, then

(21)	the probability that their mean time will be at least 3 minutes is							
	(A)	1.0	(B)	0.8413	(C)	0.50	(D)	0.4468
(22)	the probability that their mean time will be between 2.7 and 3.2 minutes is							
	(A)	0.7745	(B)	0.2784	(C)	0.9973	(D)	0.0236
(23)	if we wish to be 96% confident that the sample mean will be within 0.3 minutes of the population mean, then the sample size needed is							
	(A)	98	(B)	100	(C)	92	(D)	85

►►►

A random sample of size 25 is taken from a normal population (first population) having a mean of 100 and a standard deviation of 6. A second random sample of size 36 is taken from a different normal population (second population) having a mean of 97 and a standard deviation of 5.

(24)	the probability that the sample mean of the first population will exceed the sample mean of the second population by at least 6 is							
	(A)	0.0013	(B)	0.9147	(C)	0.0202	(D)	0.9832
(25)	the probability that the difference between the two sample means will be less than 2 is							
	(A)	0.099	(B)	0.2483	(C)	0.8499	(D)	0.9499

►►

(26)	From the table of t-distribution with degrees of freedom $\nu = 15$, the value of $t_{0.025}$ equals to							
	(A)	2.131	(B)	1.753	(C)	3.268	(D)	0.0

►►

(27)	The following measurements were recorded for lifetime, in years, of certain type of machine: 3.4, 4.8, 3.6, 3.3, 5.6, 3.7, 4.4, 5.2, and 4.8. Assuming that the measurements represent a random sample from a normal population, then 99% confidence interval for the mean life time of the machine is						
	(A)	$-5.37 \leq \mu \leq 3.25$			(B)	$4.72 \leq \mu \leq 9.1$	
	(C)	$4.01 \leq \mu \leq 5.99$			(D)	$3.37 \leq \mu \leq 5.25$	

►►

A survey of 500 students from a college of science shows that 275 students own computer of type A. In another survey of 400 students from a college of engineering shows that 240 students own the same type of computer.

(28)	a 99% confidence interval for the true proportion of the first population is						
	(A)	$-0.59 \leq p_1 \leq 0.71$			(B)	$0.49 \leq p_1 \leq 0.61$	
	(C)	$2.49 \leq p_1 \leq 6.61$			(D)	$0.3 \leq p_1 \leq 0.7$	
(29)	a 95% confidence interval for the difference between the proportion of students owning type A computers						
	(A)	$0.015 \leq p_1 - p_2 \leq 0.215$			(B)	$-0.515 \leq p_1 - p_2 \leq 0.215$	
	(C)	$-0.450 \leq p_1 - p_2 \leq -0.015$			(D)	$-0.115 \leq p_1 - p_2 \leq 0.015$	

►►

The following data show the number of defects of code of particular type of software program made in two different countries (assuming normal populations)

Country A	48	39	42	52	40	48	54
Country B	50	40	43	45	50	38	36

(30)	a 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$ is						
	(A)	$-2.46 \leq \mu_A - \mu_B \leq 8.46$			(B)	$1.42 \leq \mu_A - \mu_B \leq 6.42$	
	(C)	$-1.42 \leq \mu_A - \mu_B \leq -0.42$			(D)	$2.42 \leq \mu_A - \mu_B \leq 10.42$	

بسم الله الرحمن الرحيم

Department of Statistics
& Operations Research
College of Science
King Saud University



STAT 324
Supplementary Examination
Second Semester
1424 - 1425

Student Name:			
Student Number:		Section Number:	
Teacher Name:		Serial Number:	

- ▶▶ Mobile Telephones are not allowed in the classrooms
- ▶▶ Time allowed is 2 hours
- ▶▶ Attempt all questions
- ▶▶ Choose the nearest number to your answer
- ▶▶ For each question, put the code of the correct answer in the following table beneath the question number:

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

▶▶

(1)	Two engines operate independently, if the probability that an engine will start is 0.4, and the probability that other engine will start is 0.6, then the probability that both will start is:								
	(A)	1	(B)	<u>0.24</u>	(C)	0.2	(D)	0.5	

▶▶

(2)	If $P(B) = 0.3$ and $P(A B) = 0.4$, then $P(A \cap B)$ equal to;								
	(A)	0.67	(B)	<u>0.12</u>	(C)	0.75	(D)	0.3	

▶▶

(3)	The probability that a computer system has an electrical failure is 0.15, and the probability that it has a virus is 0.25, and the probability that it has both problems is 0.20, then the probability that the computer system has the electrical failure or the virus is:								
	(A)	1.15	(B)	<u>0.2</u>	(C)	0.15	(D)	0.35	

▶▶▶

Two brothers, Ahmad and Mohammad, are the owners and operators of a small restaurant. Ahmad and Mohammad alternate between the jobs of cooking and dish washing, so that at any time, the probability that Ahmad is washing the dishes is 0.50, and Mohammad is also 0.5. The probability that Mohammad breaks a dish is 0.40. On the other hand, the probability that Ahmad breaks a dish is only 0.10. Then,

(4)	the probability that a dish will be broken is:							
	(A)	0.667	(B)	<u>0.25</u>	(C)	0.8	(D)	0.5
(5)	If there is a broken dish in the kitchen of the restaurant. The probability that it was washed by Mohammad is:							
	(A)	0.667	(B)	0.25	(C)	<u>0.8</u>	(D)	0.5

▶▶▶

(6)	From a box containing 4 black balls and 2 green balls, 3 balls are drawn in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:							
	(A)	<u>6/27</u>	(B)	2/27	(C)	12/27	(D)	4/27

▶▶▶

(7)	The value of k, that makes the function $f(x) = k \binom{2}{x} \binom{3}{3-x} \text{ For } x=0,1,2$ serve as a probability distribution of the discrete random variable X;							
	(A)	<u>1/10</u>	(B)	1/9	(C)	1	(D)	1/7

▶▶▶

The cumulative distribution of a discrete random variable, X , is given below:

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/16 & \text{for } 0 \leq x < 1 \\ 5/16 & \text{for } 1 \leq x < 2 \\ 11/16 & \text{for } 2 \leq x < 3 \\ 15/16 & \text{for } 3 \leq x < 4 \\ 1 & \text{for } x \geq 4. \end{cases}$$

(8)	the $P(X = 2)$ is equal to:							
	(A)	<u>3/8</u>	(B)	11/16	(C)	10/16	(D)	5/16
(9)	the $P(2 \leq X < 4)$ is equal to:							
	(A)	20/16	(B)	11/16	(C)	<u>10/16</u>	(D)	5/16

▶▶▶

(10)	The proportion of people who respond to a certain mail-order is a continuous random variable X that has the density function							
	$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$							
	Then, the probability that more than $\frac{1}{4}$ but less than $\frac{1}{2}$ of the people contacted will respond to the mail-order is:							
	(A)	<u>19/80</u>	(B)	1/2	(C)	1/4	(D)	81/400

▶▶▶

Suppose the failure time (in hours) of a specific type of electrical device is distributed with a probability density function:

$$f(x) = \frac{1}{50}x, \quad 0 < x < 10$$

then,

(11)	the average failure time of such device is:							
	(A)	<u>6.667</u>	(B)	1.00	(C)	2.00	(D)	5.00
(12)	the variance of the failure time of such device is:							

	(A)	0	(B)	50	(C)	<u>5.55</u>	(D)	10
--	-----	---	-----	----	-----	-------------	-----	----

▶▶▶

A random variable X has a mean of 10 and a variance of 4, then, the random variable $Y = 2X - 2$,

(13)	has a mean of:							
	(A)	10	(B)	<u>18</u>	(C)	20	(D)	22
(14)	and a standard deviation of:							
	(A)	6	(B)	2	(C)	<u>4</u>	(D)	16

▶▶▶

(15)	The probability distribution of X , the number of typing errors committed by a typist is:																			
<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$f(x)$</td> <td>0.41</td> <td>0.37</td> <td>0.16</td> <td>0.05</td> <td>0.01</td> </tr> </table>									x	0	1	2	3	4	$f(x)$	0.41	0.37	0.16	0.05	0.01
x	0	1	2	3	4															
$f(x)$	0.41	0.37	0.16	0.05	0.01															
Then the average number of errors for this typist is:																				
	(A)	2	(B)	0.88	(C)	1.28	(D)	4												

▶▶▶

If the random variable X has an exponential distribution with the mean 4, then

(16)	$P(X < 8)$ equals to:							
	(A)	0.2647	(B)	0.4647	(C)	<u>0.8647</u>	(D)	0.6647
(17)	the variance of X is:							
	(A)	4	(B)	<u>16</u>	(C)	2	(D)	1/4

▶▶▶

If the random variable X has a normal distribution with the mean 10 and the variance 36, then

(18)	the value of X above which an area of 0.2296 lie is:							
	(A)	<u>14.44</u>	(B)	16.44	(C)	10.44	(D)	18.44
(19)	the probability that the value of X is greater than 16 is:							

	(A)	0.9587	(B)	<u>0.1587</u>	(C)	0.7587	(D)	0.0587
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▶▶▶

(20)	Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 65 and the variance 16. A student fails the exam if he obtains a mark less than 60. Then the percentage of students who fail the exam is:							
	(A)	20.56%	(B)	90.56%	(C)	50.56%	(D)	<u>10.56%</u>

▶▶▶

In a certain industrial facility accidents occur infrequently. If the probability of an accident on a given day is p , and accidents are independent of each other. If $p = 0.2$, then

(21)	probability that within seven days there will be at most two accidents will occur is:							
	(A)	<u>0.7865</u>	(B)	<u>0.4233</u>	(C)	0.5767	(D)	0.6647
(22)	probability that within seven days there will be at least three accidents will occur is:							
	(A)	0.7865	(B)	0.2135	(C)	0.5767	(D)	0.1039
(23)	the expected number of accidents to occur within this week is:							
	(A)	<u>1.4</u>	(B)	0.2135	(C)	2.57	(D)	0.59

▶▶▶

The number of traffic accidents per week in a small city has a Poisson distribution with mean equal to 1.3. Then,

(24)	the probability of at least two accidents in 2 weeks is:							
	(A)	0.2510	(B)	0.3732	(C)	0.5184	(D)	<u>0.7326</u>
(25)	the standar diviation of traffic accidents per week in the small city is:							
	(A)	<u>1.14</u>	(B)	1.30	(C)	1.69	(D)	3.2

▶▶▶

A study was made by a taxi company to decide whether the use of new tires (A) instead of the present tires (B) improves fuel economy. Six cars were equipped with tires (A) and driven over a prescribed test course. Without changing drivers and cares, test course was made with tires (B). The gasoline consumption, in kilometers per liter (km/L), was

recorded as follows: (assume the population to be normally distributed with unknown variances and are equals)

Car	1	2	3	4	5	6
Type (A)	4.5	4.8	6.6	7.0	6.7	4.6
Type (B)	3.9	4.9	6.2	6.5	6.8	4.1

(26)	A 95% confidence interval for the true mean gasoline brand A consumption is:					
	(A)	$4.462 \leq \mu_A \leq 6.938$			(B)	$2.642 \leq \mu_A \leq 4.930$
	(C)	$5.2 \leq \mu_A \leq 9.7$			(D)	$6.154 \leq \mu_A \leq 6.938$
(27)	A 99% confidence interval for the difference between the true mean of type (A) and type (B) ($\mu_A - \mu_B$) is:					
	(A)	$-1.939 \leq \mu_A - \mu_B \leq 2.539$			(B)	$-2.939 \leq \mu_A - \mu_B \leq 1.539$
	(C)	$0.939 \leq \mu_A - \mu_B \leq 1.539$			(D)	$-1.939 \leq \mu_A - \mu_B \leq 0.539$

►►

A food company distributes two brands of milk. If it is found that 80 of 200 consumers prefer brand A and that 90 of 300 consumers prefer brand B,

(28)	96% confidence interval for the true proportion of brand (A) is:					
	(A)	$0.328 \leq p_A \leq 0.375$			(B)	$0.228 \leq p_A \leq 0.675$
	(C)	$0.328 \leq p_A \leq 0.475$			(D)	$0.518 \leq p_A \leq 0.875$
(29)	A 99% confidence interval for the true difference in the proportion of brand (A) and (b), is:					
	(A)	$0.0123 \leq p_A - p_B \leq 0.212$			(B)	$-0.2313 \leq p_A - p_B \leq 0.3612$
	(C)	$-0.0023 \leq p_A - p_B \leq 0.012$			(D)	$-0.0123 \leq p_A - p_B \leq 0.212$
(30)	If the value of α decrease (get smaller), then the interval estimate will decrease (get smaller);					
	(A)	Yes	(B)	No	(C)	No change



Department of Statistics & Operations Research
College of Science, King Saud University



STAT 324
Final Examination
Second Semester 1431 – 1432 H

			اسم الطالب
	رقم التحضير		الرقم الجامعي
	اسم الدكتور		رقم الشعبة

INSTRUCTIONS:

- Answer all questions.
- Do not copy answers from your neighbors; they have different question forms.
- Mobile Telephones are not allowed in the classroom.
- Time allowed is 3 Hours
- For each question, put the code of the correct answer in the following table beneath the question number. Please use capital letters: A, B, C, and D.

1	2	3	4	5	6	7	8	9	10

11	12	13	14	51	61	71	81	91	20

21	22	32	42	52	62	72	28	92	30

31	32	33	34	35	36	73	38	39	40

41	24	34	44	45	64	47	84	94	50

Term Marks	Final Exam. Marks	Total Marks

QUESTION (1)

Let X be a continuous random variable with probability density function given by:

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(1) The expected value of X [$\mu = E(X)$] equals:

(A) 0.25	(B) 2.25	(C) 0.33	(D) 0.50
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(2) The variance of X [$\sigma^2 = \text{Var}(X)$] equals

(A) 0.056	(B) 0.113	(C) 0.037	(D) 0.333
-----------	-----------	-----------	-----------

(3) The value of the probability $P(X = 0.5)$ equals:

(A) 1	(B) 0.5	(C) 0.1	(D) 0
-------	---------	---------	-------

(4) The value of the probability $P(X < 0.5)$ equals:

(A) 0.25	(B) 0.75	(C) 0.50	(D) 1.25
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(5) The value of the probability $P(0.5 < X < 1)$ equals:

(A) 0.64	(B) 0.45	(C) 0.25	(D) 0.75
----------	----------	----------	----------

(6) The cumulative distribution function [$F(x)$] for $0 < x < 1$, equals:

(A) $(2-x)$	(B) $x(2-x)$	(C) $x-2$	(D) $x(x^2-1)$
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QUESTION (2)

Suppose that a survey of 500 parents was conducted. The survey asked questions about whether or not the person had a child in college and about the cost of attending college. Results are shown in the table below.

	Cost Too Much (M)	Cost Just Right (R)	Cost Too Low (L)
Child in College (A)	150	65	5
Child not in College (B)	100	125	55

Suppose one person is chosen at random, then

(7) The probability that the person thinks college cost is just right given that he has a child in college equals:

(A) 0.512	(B) 0.295	(C) 0.384	(D) 0.842
-----------	-----------	-----------	-----------

(8) the probability that the person does not have a child in college and he thinks that the college cost is too low equals:

(A) 0.11	(B) 0.20	(C) 0.917	(D) 0.25
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(9) the probability that the person thinks college cost is too low given that he does not have a child in college equals:

(A) 0.242	(B) 0.38	(C) 0.57	(D) 0.38
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QUESTION (3)

Suppose that a factory has two machines: machine A and machine B. These machines make widgets. Machine A makes 800 per day and 1% of these are defective. Machine B makes 200 per day of which 2% are defective. If we select a widget product by the factory, then:

(10) the probability that a widget produced by the factory will be defective equals:

(A) 0.02	(B) 0.012	(C) 0.8	(D) 0.03
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(11) the probability that the widget is produced by machine A, given that it is defective equals:

(A) 0.9	(B) 0.333	(C) 0.03	(D) 0.667
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QUESTION (4)

Consider the following probability function:

x	0	1	2	3
f(x)= P(X=x)	0.216	0.432	0.288	0.064

(12) The mean (expected value) equals:

(A) 1.5	(B) 0.25	(C) 1.2	(D) 1.8
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(13) The variance equals:

(A) 0.72	(B) 2	(C) 2.16	(D) 1.25
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(14) The P(X <2) equals:

(A) 0.288	(B) 0.432	(C) 0.648	(D) 0.936
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QUESTION (5)

Let X and Y be two independent random variables such that $\mu_x = 1$, $\sigma_x^2 = 2$, $\mu_y = -2$, and $\sigma_y^2 = 1$

(15) The value of E(X-3Y+1) is equals:

(A) 11	(B) 8	(C) 38	(D) 40
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(16) The value of Var(X-3Y+1) is equals:

(A) 11	(B) 8	(C) 20	(D) 6
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(17) The value of E(Y²) is equals:

(A) 1	(B) 2	(C) 0.8	(D) 5
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(18) The highest lower bound for P(-4 < Y < 0) is:

(A) 1.0	(B) 0.25	(C) 0.5	(D) 0.75
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QUESTION (6)

Suppose that 15 % of new residential central air conditioning units installed by a supplier need additional adjustments requiring a service call. Assume that a recent sample of 7 such units constitutes a Bernoulli process. Let X be the number of units among these 7 that need additional adjustments.

(19) The probability that exactly 2 units need additional adjustments equals:

(A) 0.156	(B) 1.05	(C) 0.209	(D) 0.16
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(20) The probability that at least one units need additional adjustments equals:

(A) 0.152	(B) 0.679	(C) 1.052	(D) 0.163
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(21) The mean number of units that need additional adjustments equals:

(A) 0.15	(B) 0.32	(C) 1.05	(D) 0.16
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(22) the variance of X equals:

(A) 0.5892	(B) 0.2598	(C) 0.5298	(D) 0.8925
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QUESTION (7)

In a certain communications system, there is an average of one transmission error per 10 seconds. Let the distribution of transmission errors be Poisson.

(23) the probability that there is exactly 2 transmission error per 10 seconds equals:

(A) 0.184	(B) 0.285	(C) 0.124	(D) 0.247
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(24) the probability that there is at least two transmission errors per 10 seconds equals:

(A) 0.814	(B) 0.264	(C) 0.352	(D) 0.514
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(25) the probability of more than one error in a communication per half minute in duration equals:

(A) 0.950	(B) 0.262	(C) 0.738	(D) 0.199
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(26) the average number of transmission errors per one minute equals:

(A) 7	(B) 8	(C) 6	(D) 4
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QUESTION (8)

The diameters of steel disks produced in a plant are normally distributed with a mean of 2.5 cm and standard deviation of 0.02 cm.

(27) The probability that a disk picked at random has a diameter greater than 2.54 cm equals:

(A) 0.5080	(B) 0.2000	(C) 0.1587	(D) 0.0228
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(28) The probability that a disk picked at random has a diameter less than 3.2 cm equals:

(A) 0.8413	(B) 0.3148	(C) 0.2716	(D) 0.4138
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(29) The probability that a disk picked at random has a diameter less than 2.54 cm and greater than 2.52 equals:

(A) 0.6843	(B) 0.1359	(C) 0.3871	(D) 0.9124
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QUESTION (9)

In an investigation on toxins produced by molds that infect corn crops, a biochemist prepares extracts of the mold culture and then measures the amount of the toxic substance per gram of solution. From six extracts of the mold culture the following information are obtained:

n	Mean (\bar{X})	Standard deviation (S)
6	0.950	0.251

Assuming the data follows approximately a normal distribution,

(30) The standard error of the sample mean equals:

(A) 0.25	(B) 0.102	(C) 4.59	(D) 28.67
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(31) A good approximation value to the population mean equals:

(A) 1.87	(B) 4.59	(C) 0.950	(D) 0.25
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(32) The lower bound of 90% confidence interval for the population mean equals:

(A) 2.412	(B) 1.48	(C) 0.744	(D) 0.12
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(33) The upper bound of 90% confidence interval for the population mean equals:

(A) 3.145	(B) 1.990	(C) 0.88	(D) 1.157
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QUESTION (10)

A study conducted by Saudi airline showed that a random sample of nine of its passengers at the King Khaled airport, took an average of 24.1 minutes to claim their luggage. From a previous survey it was willing to assume that time to claim luggage is normally distributed with standard deviation of 4.24 minutes.

(34) The 99% lower confidence limit for the sample mean equals:

(A) 0.5000	(B) 1.6587	(C) 20.458	(D) 0.0221
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(35) The 99% upper confidence limit for the sample mean equals:

(A) 27.742	(B) 2.531	(C) 0.022	(D) 0.814
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QUESTION (11)

A sample of size 100 is taken from a population having a proportion $p_1 = 0.8$. Another independent sample of size 400 is taken from a population having a proportion $p_2 = 0.5$.

(36) The sampling distribution for the difference in sample proportions has a mean equals:

(A) 0.3	(B) 1.3	(C) 0	(D) 0.8
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(37) The sampling distribution for the difference in sample proportions has a standard error equals:

(A) 0.015	(B) 0.0022	(C) 0.047	(D) 0.1239
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(38) $P(\hat{p}_1 - \hat{p}_2 < 0.2)$ equals:

(A) 0.4423	(B) 0.993	(C) 0.0166	(D) 0.2415
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QUESTION (12)

A researcher wishes to compare the resistance of two types of wires. In a sample of 81 resistance readings of type A, the mean and standard deviation are $\bar{X}_A = 27$ ohm. In a sample of 90 resistance readings of type B, the mean and standard deviation are $\bar{X}_B = 24$ ohm. Assuming the two populations follow approximately two different normal distributions with standard deviations $\sigma_A = 6.9$ ohm and $\sigma_B = 6.2$ ohm.

(39) The point estimate for the difference between the two populations means ($\mu_A - \mu_B$):

(A) 27	(B) 24	(C) 6.2	(D) 3
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(40) The standard error for the difference between the two sample means ($\bar{X}_A - \bar{X}_B$):

(A) 6.9	(B) 6.2	(C) 1.007	(D) 3
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(41) A lower limit of a 95% C.I. for the difference between the two population means ($\mu_A - \mu_B$):

(A) 1.0263	(B) 4.9745	(C) 5.9120	(D) 1.2354
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(42) A width of the 95% C.I. for the difference between the two population means ($\mu_A - \mu_B$):

(A) ****	(B) ****	(C) ****	(D) ****
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QUESTION (13)

A sample of 25 freshman students made a mean score of 77 on a test designed to measure the attitude toward colleges. The sample standard deviation was 10. Assuming the data comes from a normal population,

(43) The statistical hypothesis for testing the hypothesis that the mean score is different than 80 is:

(A) $H_0 : \mu = 80$ vs $H_1 : \mu \neq 80$	(B) $H_0 : \mu = 80$ vs $H_1 : \mu < 80$
(C) $H_0 : \mu = 80$ vs $H_1 : \mu > 80$	(D) $H_0 : \mu = 77$ vs $H_1 : \mu < 77$

(44) The test statistic for this statistical hypothesis is:

(A) -1.500	(B) -2.025	(C) 3.258	(D) 0
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(45) At the 5% significance level, the rejection region (R.R) is

(A) (3.052, 3.861)	(B) (5.821, 6.972)
(C) (3.847, 4.512)	(D) (-2.069, 2.064)

(46) At the 5% significance level we are able to :

(A) Reject H_0	(B) Don't Reject H_0	(C) Decision is not possible
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QUESTION (14)

In the 2000 Census, 2.4 percent of the King dome Saudi Arabia population reported being two or more races. However, the percent varies tremendously from state to state. Suppose that two random surveys are conducted. In the first random survey, out of 1000 Riyadh, only 9 people reported being of two or more races. In the second random survey, out of 500 Gada, 17 people reported being of two or more races. We wish to conduct a hypothesis test to determine if the population percents are the same for the two states or if the percent for Gada is statistically higher than for Riyadh.

(47) the null and alternative hypotheses is:

(A) $H_0: P_G = P_R, H_1: P_G > P_R$	(B) $H_0: P_G = P_R, H_1: P_G < P_R$
(C) $H_0: P_G = P_R, H_1: P_G \neq P_R$	(D) $H_0: P_G < P_R, H_1: P_G < P_R$

(48) The distribution would we use for this hypothesis test is:

(A) T- distribution	(B) Normal distribution	(C) Poisson distribution	(D) Binomial distribution
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(49) The value of the test statistic equal:

(A) 4.91	(B) - 2.61	(C) 5.30	(D) 3.50
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(3.50)

(50) The decision is:

(A) Reject H_0	(B) Don't Reject H_0	(C) Decision is not possible
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Question 1

I. Suppose a fair die is thrown twice, then

- 1) the probability that the sum of numbers of the two dice is less than or equal to 4 is;
(A) 0.1667 (B) 0.6667 (C) 0.8333 (D) 0.1389
- 2) the probability that at least one of the die shows 4 is;
(A) 0.6667 (B) 0.3056 (C) 0.8333 (D) 0.1389
- 3) the probability that one die shows one and the sum of the two dice is four is;
(A) 0.0556 (B) 0.6667 (C) 0.3056 (D) 0.1389
- 4) the event $A = \{\text{the sum of two dice is 4}\}$ and the event $B = \{\text{exactly one die shows two}\}$ are,
(A) Independent (B) Dependent (C) Disjoint (D) None of these.

II. 5) Assume that $P(A) = 0.3$, $P(B) = 0.4$, $P(A \cap B \cap C) = 0.03$, and $P(\overline{A \cap B}) = 0.88$, then the event A and B are,
(A) Independent (B) Dependent (C) Disjoint (D) None of these.

6) Using question (5), $P(C|A \cap B)$ is equal to,

- (A) 0.65 (B) 0.25 (C) 0.35 (D) 0.14

7) If the probability that it will rain tomorrow is 0.23, then the probability that it will not rain tomorrow is:

- (A) -0.23 (B) 0.77 (C) -0.77 (D) 0.23

Question 2

I. The probability that a factory will open a branch in Riyadh is 0.7, probability that it will open a branch in Jeddah is 0.4, and the probability that it will open a branch in either Riyadh or Jeddah or both is 0.8, then the probability that it will open a branch:

8) In both cities is:

- (A) 0.1 (B) 0.9 (C) 0.3 (D) 0.8

9) In neither cities is:

- (A) 0.4 (B) 0.7 (C) 0.3 (D) 0.2

II. The probability that a lab specimen contains high levels of contamination is 0.10. Three samples are checked, and the samples are independent, then:

10) the probability that none contains high levels of contamination is:

- (A) 0.0475 (B) 0.001 (C) 0.729 (D) 0.3

11) the probability that exactly one contains high levels of contamination is:

- (A) 0.243 (B) 0.081 (C) 0.757 (D) 0.3

Question 3

I. If in a class of 324-Stat. of 80 students, 60 are from engineering college and the rest are from computer science college, 10% of the engineering college students have taken this course before, and 5% of computer science college students have taken this course before. If one student from this class is randomly selected, then:

12) the probability that he has taken this course before is:

- (A) 0.25 (B) 0.0875 (C) 0.8021 (D) 0.75

13) If the selected student has taken this course before then the probability that he is from the computer science college is:

- (A) 0.1429 (B) 0.375 (C) 0.80 (D) 0.25

II. A random sample of 200 adults are classified according to sex and their level of education in the following table:

<i>Education</i>	<i>Male</i>	<i>Female</i>
<i>Elementary</i>	28	50
<i>Secondary</i>	38	45
<i>College</i>	22	17

If a person is selected at random from this group, then:

14) the probability that he is a male is:

- (A) 0.3182 (B) 0.44 (C) 0.28 (D) 78

15) The probability that the person is male given that the person has a secondary education is:

- (A) 0.4318 (B) 0.4578 (C) 0.19 (D) 0.44

- 16) The probability that the person does not have a college degree given that the person is a female is:
 (A) 0.8482 (B) 0.1518 (C) 0.475 (D) 0.085

III.

- 17) A man wants to paint his house in 3 colors. He can choose out of 6 colors. How many different color settings can he make?
 (A) 216 (B) 20 (C) 18 (D) 120
- 18) If continuous random variable X has a mean $\mu=16$, a variance $\sigma^2=5$, then $P(X = 16)$ is,
 (A) 0.0625 (B) 0.5 (C) 0.0 (D) None of these.
- 19) A lower bound valued according to Chebyshev's theory for $P(\mu - 2\sigma < X < \mu + 2\sigma)$ is,
 (A) 0.3175 (B) 0.750 (C) 0.965 (D) 0.250
- 20) A random variable X has a mean $\mu=12$, a variance $\sigma^2=9$, and unknown probability distribution. Using Chebyshev's theorem, $P(3 < X < 21)$ is at least equal to,
 (A) 8/9 (B) 3/4 (C) 1/4 (D) 1/16

Question 4

- I. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets.

- 21) If x is the number of defective sets purchased by the hotel, the probability of no defective television set, $P(X = 0)$ is,
 (A) 0.57 (B) 0.14 (C) 0.29 (D) 0
- 22) $P(0 < X \leq 2)$ is,
 (A) 0.29 (B) 0.43 (C) 0.71 (D) 1

- II. Consider the density function

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

- 23) The value of k is:
 (A) 1 (B) 0.5 (C) 1.5 (D) 0.667

- 24) The probability $P(0.3 < X \leq 0.6)$ is,
 (A) 0.4647 (B) 0.3004 (C) 0.1643 (D) 0.4500
- 25) The expected value of X, $E(X)$ is,
 (A) 0.6 (B) 1.5 (C) 1 (D) 0.667

Question 5

Let X be a random variable with the following probability distribution:

x	-3	6	9
f(x)	0.167	0.5	0.333

- 26) $E(X)$ is,
 (A) 4.0 (B) 5.5 (C) 6.5 (D) 6.0
- 27) $E(X^2)$ is,
 (A) 30.25 (B) 36.0 (C) 46.5 (D) 126.0
- 28) $\text{Var}(X) = \sigma_X^2$ is,
 (A) 13.25 (B) 16.25 (C) 90.25 (D) 95.75
- 29) $E[(2X + 1)]$ is,
 (A) 8 (B) 11 (C) 12 (D) 13
- 30) Variance of $(2X+1) = \sigma_{2X+1}^2$ is,
 (A) 65 (B) 66 (C) 16.25 (D) 95.75

Department of Statistics
& Operations Research
College of Science
King Saud University



STAT 324
Final Examination
First Semester
1425 – 1426

Student Name:			
Student Number:		Section Number:	
Teacher Name:		Serial Number	

- ▶▶ Mobile Telephones are not allowed in the classrooms
- ▶▶ Time allowed is 3 hours
- ▶▶ Answer 40 questions only from 44 questions
- ▶▶ Choose the nearest number to your answer
- ▶▶ For each question, put the code of the correct answer in the following table beneath the question number:

1	2	3	4	5	6	7	8	9	10	11
A	E	D	D	C	C	A	B	D	D	D

12	13	14	15	16	17	18	19	20	21	22
D	A	D	D	C	A	D	E	B	B	B

23	24	25	26	27	28	29	30	31	32	33
A	D	C	A	B	C	B	E	B	A	B

34	35	36	37	38	39	40	41	42	43	44
D	E	C	A	D	B	A	C	B	A	D

1. Consider the sample space $S = \{\text{White, Black, Red, Blue, Yellow, Violet, Green}\}$ and the events $A = \{\text{White, Black, Green}\}$, $B = \{\text{Black, Red, Blue}\}$, $C = \{\text{Violet}\}$. Then the list of the elements of the set corresponding to the event $(A \cap B') \cup C'$ is:

- (A) $\{\text{White, Black, Red, Blue, Yellow, Green}\}$, (B) $\{\text{Black, Violet}\}$, (C) $\{\text{White, Black, Red, Blue, Yellow, Violet, Green}\}$, (D) $\{\text{White, Red, Blue, Yellow, Green}\}$, (E) $\{\text{White, Green, Violet}\}$.

2. The number of ways to select 2 computers from 9 computers of the same brand is

- (A) 18 (B) 72 (C) 81 (D) 512 (E) 36

3. Among the 500 first year students of a college, 270 students study computer science, 345 students study mathematics, and 175 students study both computer science and mathematics. If one student is selected at random, then the probability that the student did not take either of these subjects is

- (A) 0.88 (B) 0.65 (C) 0.77 (D) 0.12 (E) 0.35

⇒ **Use the following data to answer questions 4 and 5.** A random sample of 2000 computers are classified according to brands A, B and C and levels of satisfaction of the owner, Good, Average and Poor as shown in the following table:

Level of Satisfaction	Brands		
	A	B	C
Good	200	100	100
Average	400	300	300
Poor	100	200	300

4. If a computer is selected randomly, then the probability that the satisfaction level of the owner of the computer is poor, is:

- (A) 0.428 (B) 0.1429 (C) 0.3333 (D) 0.300 (E) 0.1667

5. If the randomly selected computer gives a satisfaction level good to the owner, then the probability that the computer is of brand C, is:

- (A) 0.35 (B) 0.20 (C) 0.25 (D) 0.10 (E) 0.2858

6. The probability that you will leave home to attend your class on time (A) is 0.75; the probability that you will arrive at your class on time (L) is 0.60

and the probability that you will leave home and arrive on time is 0.45. Then the two events, A and L are:

- (A) Disjoint (B) Dependent (C) Independent
(D) Mutually Exclusive

7. A certain defect (D) is present in about 1 out of 1000 cars during production [$P(D)=0.001$], and a program of testing is to be carried out using a detection device which gives a positive reading with probability 0.99 for a defective car [$P(+/D)=0.99$] and with probability 0.05 for a non-defective car [$P(+/ND)=0.05$]. Then if a randomly selected car has a positive reading then the probability that it actually does have the defect [$P(D/+)$] is:

- (A) 0.0194 (B) 0.05094 (C) 0.00099 (D) 0.04995 (E) 0.99

8. If 50% of the automobiles sold by an agency for a certain car are equipped with diesel engines, let X represent the number of diesel models among the next 5 cars sold by this agency, then the probability distribution of X is

- (A) $f(x) = \frac{\binom{5}{x}}{16}, x = 0,1,2,3,4$. (B) $f(x) = \frac{\binom{5}{x}}{32}, x = 0,1,2,3,4,5$.
(C) $f(x) = \frac{\binom{4}{x}}{16}, x = 0,1,2,3,4$. (D) $f(x) = \frac{\binom{5}{x}}{32}, x = 1,2,3,4$.
(E) $f(x) = \frac{\binom{5}{x}}{64}, x = 0,1,2,3,4$.

⇒ **For question 9, 10 and 11** . Let X be a random variable with the following probability distribution function

x	-1	0	1	2
P(X=x)	0.3	0.35	0.1	0.25

9. the mean of X ($E(X)$) =

- (A) 1.31 (B) 0.2 (C) 1.4 (D) 0.3 (E) 2.0

10. the variance of X =

- (A) 1.4 (B) 0.2 (C) 2.4 (D) 1.3 1 (E) 2.0

11. $P(X > -1) =$

- (A) 0.2 (B) 0.75 (C) 0.65 (D) 0.7 (E) 0.12

⇒ **For question 12, 13 and 14** . Consider a continuous random variables X with the following probability density function

$$f(x) = \frac{x^2}{9}, 0 < x < k,$$

12. the value of k is :

- (A) 0.33 (B) 3.112 (C) 3.334 (D) 3.0 (E) 0.22

13. $P(X < 1) =$
 (A) 0.03704 (B) 0.02223 (C) 0.6555 (D) 0.254 (E) 0.9
14. $E(X) =$
 (A) 2.20 (B) 2.22 (C) 2.65 (D) 2.25 (E) 1.0
- \Rightarrow **Answer for question 15, 16 and 17.** A random variables X has a mean $\mu = 6$ and a variance $\sigma^2 = 4$.
15. $E(3X+4) =$
 (A) 24.0 (B) 14.0 (C) 36.0 (D) 22.0 (E) 25.0
16. $\text{Var}(3X+5) = \sigma_{3X+5}^2 =$
 (A) 41.0 (B) 17.0 (C) 36.0 (D) 22.0 (E) 14.0
17. $P(-4 < X < 16)$
 (A) $\geq 24/25$ (B) $< 24/25$ (C) $\geq 1/5$ (D) $< 1/5$ (E) $\geq 15/16$

\Rightarrow **Answer for question 18, 19, 20 and 21.** A manufacture plant received a shipment of circuit boards from a manufacturer, 5 boards randomly chosen for inspection and determine whether they are defective or not. It is known that 8% of the boards in the shipment are defective.

18. The probability of no defective circuit boards is :
 (A) 0.0544 (B) 0.3409 (C) 0.9456 (D) 0.6591 (E) 0.5
19. The probability that more than 1 of the circuit boards is defective is:
 (A) 0.7865 (B) 0.9456 (C) 0.6591 (D) 0.1039 (E) 0.0543
20. The variance of the number of defective boards is:
 (A) 0.2135 (B) 0.368 (C) 0.5767 (D) 0.2052 (E) 0.2152
21. Suppose in this experiment, a shop receives 20 circuit boards out of which 6 are defective boards. If we buy 4 boards, then the probability that we will find 2 defectives boards is:
 (A) 0.2135 (B) 0.2817 (C) 0.5858 (D) 0.1039 (E) 0.1139

- \Rightarrow **Answer for question 22 and 23.** In a certain industrial facility accidents occur. If the average number of accident per a month is 2, then
22. probability that within a month there will be at most two accidents is:
 (A) 0.7865 (B) 0.6767 (C) 0.4060 (D) 0.3233 (E) 0.334

23. probability that within a month there will be at least one accidents is:
 (A) 0.8647 (B) 0.2135 (C) 0.5767 (D) 0.1353 (E) 0.11
24. Given a standard normal distribution, then $Z_{0.75}$ is
 (A) 0.75 (B) 0.7734 (C) 0.25 (D) 0.675 (E) 0.251
25. If the variable X has normal with $\mu = 10$ and $\sigma^2 = 25$, then the probability that X-values exceeds 8 is:
 (A) 0.80 (B) 0.20 (C) 0.6554 (D) 0.10 (E) 0.3446

⇒ **Answer for question 26 and 27.** Suppose that X has the exponential density
 $f(x) = 0.25 e^{-0.25x}$, then

26. the mean of X ($E(X)$) is:
 (A) 4 (B) 0.25 (C) 1.0 (D) 0.25 (E) 2.0
27. the probability $P(X \leq 4)$ is:
 (A) 0.3679 (B) 0.6321 (C) 0.5 (D) 0.0 (E) 0.71
28. If the mean and the variance of weights of students in certain school is 35 kg and 25 kg^2 . Then the probability that the average mean of weights will be greater than 34 kg in a sample of size 64 students:
 (A) 0.0548 (B) 0.9542 (C) 0.9452 (D) 0.0450 (E) 0.123

⇒ **For question 29, 30, 31 and 32.** Consider following information. A sample of 25 men has a mean weight of 65 kg with a standard deviation of 8 kg. Suppose that the weight of men follows normal distribution with a standard deviation of 10 kg.

29. The population variance of sample means ($\sigma_{\bar{X}}^2$) is:
 (A) 2 kg^2 (B) 4 kg^2 (C) 1.6 kg^2 (D) 2.56 kg^2 (E) 0.4 kg^2
30. The point estimate of μ is
 (A) 10 kg (B) 8 kg (C) 25 kg (D) 9 kg (E) 65 kg
31. The lower 95% confidence limit for a population mean μ is
 (A) 64.671 kg (B) 61.08 kg (C) 61.864 kg (D) 61.698 kg (E) 63.0 kg
32. The upper 95% confidence limit for a population mean μ is
 (A) 68.92 kg (B) 68.3024 kg (C) 68.29 kg (D) 72.84 kg (E) 70.1 kg
33. In a sample of 100 items, 8 were found defective. The lower 98% confidence limit for the population proportion is:
 (A) 0.08 (B) 0.0168 (C) 0.024 (D) 0.075 (E) 0.089

⇒ **For question 34 ,35 and 36.** Consider following

Let $n_1 = 100$, $\bar{X}_1 = 12.2$ and $S_1 = 1.1$ for sample 1 and $n_2 = 200$, $\bar{X}_2 = 9.1$ and $S_2 = 0.9$ for sample 2. Suppose that the samples are drawn from two independent populations.

34. The point estimate of the difference of two population means ($\mu_1 - \mu_2$) is
(A) 12.2 (B) 9.1 (C) 0.2 (D) 3.1 (E) 4.1

35. The lower 90% confidence limit for the difference of two population means ($\mu_1 - \mu_2$) is:
(A) 3.973 (B) 3.084 (C) 3.948 (D) 2.904 (E) 2.8909

36. The upper 90% confidence limit for the difference of two population means ($\mu_1 - \mu_2$) is:
(A) 3.227 (B) 3.116 (C) 3.3091 (D) 3.252 (E) 3.296

⇒ **For answering Questions 37, 38, 39 and 40** .Use the following information.

It is claimed that an automobile is driven on the average more than 20,000 kilometres per year. To test this claim at the 0.05 level of significance, a random sample of 10 automobile owners are asked to keep a record of the kilometres they travel. If the sample mean and standard deviation of automobile driven are 23,500 kilometres and 3900 kilometres respectively, assuming that the kilometres they travel is normally distributed. Then to test for $H_0 : \mu = 20,000$ kilometres against $H_1 : \mu > 20,000$ kilometres:

37. Test statistic is:

(A) $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$ (B) $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ (C) $t = \frac{\bar{d}}{s_d / \sqrt{n}}$ (D) $t = \frac{\bar{x} - \mu}{s}$ (E) $z = \frac{\bar{x} - \mu}{\sigma}$

38. Critical region (rejection area) is:
(A) $z < -1.645$ (B) $z > 1.645$ (C) $t < -1.833$ (D) $t > 1.833$ (E) $z > 1.96$

39. Computed value of test statistic is:
(A) 2.6923 (B) 2.838 (C) 0.8974 (D) 177.2295 (E) 3.12

40. Your decision is:
(A) reject H_0 (B) accept H_0 (C) reject H_1 (D) no decision

⇒ **For answering Questions 41, 42 and 43** . Consider the following

Two samples are drawn from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ giving the following information: First sample: $\bar{x}_1 = 21$, $s_1 = 11$ and $n_1 = 36$ and

Second sample: $\bar{x}_2 = 31$, $s_2 = 16$ and $n_2 = 32$.

Then to test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$ at $\alpha = 0.05$

41. The critical region (rejection area) is :

- (A) $z > 1.645$ and $z < -1.645$ (B) $z < 1.645$ (C) $z < -1.96$ and $z > 1.96$
(D) $z < -1.645$ (E) $t < -1.833$ and $t > 1.833$

42. The Computed value of test statistic is:
(A) 2.6923 (B) -2.9668 (C) 0.8974 (D) -17.2295 (E) 0.992
43. Your decision is:
(A) reject H_0 (B) reject H_1 (C) no decision
44. A certain geneticist is interested in the proportion of males and females in the population that have a certain minor blood disorder. In a random sample of 1500 males, 75 are found to have disorder, whereas 80 of 2000 females appear to have disorder. Then the 90% confidence intervals for the difference between the proportions of males and females that have blood disorder is :
(A) (0.001, 0.0445) (B) (-0.0167, 0.0223) (C) (0.0112, 0.0223)
(D) (-0.0017, 0.0217)



Question No. 1:

Suppose a fair die is thrown twice, then

(1) the probability that the sum of numbers of two dice is less than or equal to 4 is;

- (A) 0.1667
- (B) 0.6667
- (C) 0.8333
- (D) 0.1389

(2) the probability that at least one of the die shows 4 is;

- (A) 0.6667
- (B) 0.3056
- (C) 0.8333
- (D) 0.1389

(3) the probability that the sum of two dice is 4 one of them shows 1;

- (A) 0.0556
- (B) 0.6667
- (C) 0.8333
- (D) 0.1389

(4) the event $A = \{\text{the sum of two dice is 4}\}$ and the event $B = \{\text{exactly one die shows 2}\}$, then $P(B|A)$ equal to,

- (A) 0.8333
- (B) 0.6667
- (C) 0.3333
- (D) 0.1389

(5) the event $A = \{\text{the sum of two dice is 4}\}$ and the event $B = \{\text{exactly one die shows 2}\}$ are,

- (A) Independent
- (B) Dependent
- (C) Disjoint
- (D) None of these.

Question No. 2:

A man wants to paint his house in 3 colors. He can choose out of 6 colors. Then,

(6) the number of color settings he can make is,

- (A) 216
- (B) 20
- (C) 120
- (D) 10

(7) If he selected one color, then the number of color settings he can make is,

- (A) 216
- (B) 20
- (C) 120
- (D) 10

Question No. 3:

A random sample of 200 adults is classified according to sex and their level of education in the following table:

<i>Education</i>	<i>Male</i>	<i>Female</i>
<i>Elementary</i>	28	50
<i>Secondary</i>	38	45
<i>College</i>	22	17

If a person is selected at random from this group, then:

(8) the probability that he is a male is:

- (A) 0.3182
- (B) 0.44
- (C) 0.66
- (D) 88

(9) The probability that the person is male given that the person has a secondary education is:

- (A) 0.4318
- (B) 0.19
- (C) 0.66
- (D) 0.4578

(10) The probability that the person does not have a college degree given that the person is a female is:

- (A) 0.8482
- (B) 0.1518
- (C) 0.475
- (D) 0.085

Question No. 4:

Two brothers, Ed and Jim, are the owners and operators of a small restaurant. Ed and Jim alternate between the jobs of cooking and dish washing, so that at any time, the probability that Ed is washing the dishes is





0.50. Jim, the younger of the two brothers, is a bit clumsy. When Jim is washing the dishes, the probability that Jim breaks a dish he is washing is 0.40. Ed, on the other hand, is very careful and the probability that Ed breaks a dish he is washing is only 0.10.

(11) The probability that a dish will be broken is

- (A) 0.667
- (B) 0.25
- (C) 0.8
- (D) 0.5

(12) There is a broken dish in the kitchen of the restaurant. The probability that it was washed by Jim is;

- (A) 0.667
- (B) 0.25
- (C) 0.8
- (D) 0.5

(13) Suppose Ed and Jim want the probability of a broken dish to equal 0.20. then, the probability that Ed washes the dishes is,

- (A) 0.667
- (B) 0.25
- (C) 0.8
- (D) 0.5

Question No. 5:

(14) Two engines operate independently, if the probability that an engine will start is 0.4, and the probability that other engine will start is 0.6, then the probability that both will start is:

- (A) 1
- (B) 0.24
- (C) 0.2
- (D) 0.5

(15) If $P(B) = 0.3$ and $P(A|B) = 0.4$, then $P(A \cap B)$ equal to;

- (A) 0.67
- (B) 0.12
- (C) 0.75
- (D) 0.3

Question No. 6:

A random variable X takes the values 0, 1, 2. Assume that $E(X) = \frac{3}{2}$ and $\sigma = \frac{1}{2}$, then

(16) $E(X^2) =$

- (A) 1/4
- (B) 10/4
- (C) 9/4
- (D) 2

(17) $E(2X + 3) =$

- (A) 6
- (B) 5
- (C) 3
- (D) 1/2

(18) $E(5X^2 - 2X) =$

- (A) 50/4
- (B) 19/2
- (C) 41/3
- (D) 1/3

(19) $Var(X + 1) =$

- (A) 1/4
- (B) 3/4
- (C) 1/4
- (D) 5/2

(20) $Var(2 - 3X) =$

- (A) 9/4
- (B) 10/4
- (C) 9/4
- (D) 10/3

(21) $P(X = 0) =$

- (A) 1/4
- (B) 1/2
- (C) 1/3
- (D) 0

(22) $P(0 < X < 2) =$

- (A) 1/4
- (B) 1/3
- (C) 1/2
- (D) 0





- (23) $E(X \leq 1) =$
(A) $1/4$
(B) $5/4$
(C) $\underline{1/2}$
(D) $1/3$

Question No. 7:

Let X be a continuous random variable with probability density function is given by

$$f(x) = \begin{cases} c(1-x), & 0 < x < 1, \\ 0, & \text{otherwise} \end{cases}$$

- (24) The values of c is

- (A) $1/4$
(B) $\underline{2}$
(C) $1/2$
(D) 1

- (25) $E(X) =$
(A) $1/4$
(B) $\underline{1/3}$
(C) $9/4$
(D) $1/2$

- (26) $Var(X) = \sigma^2 =$
(A) $\underline{1/18}$
(B) $1/9$
(C) $1/27$
(D) $1/3$

- (27) $P(X = 0) =$
(A) 1
(B) $\underline{0}$
(C) $1/2$
(D) $1/6$

- (28) $P(1/5 < X < 1) =$
(A) $1/24$
(B) $10/24$
(C) $15/25$
(D) $\underline{16/25}$

- (29) $P(|X - \mu| < 2\sigma) =$
(A) 0.76
(B) $\underline{0.96}$
(C) 0.90
(D) 0.82

- (30) By using Chebyshev's theorem, then $P(|X - \mu| < 2\sigma) =$
(A) $\leq 1/4$
(B) $\geq 10/4$
(C) $\leq 3/4$
(D) $\underline{\geq 3/4}$

THE END





Q1. Let A and B be independent events defined on the same sample space such that $P(A \cup B) = 0.6$ and $P(B) = 0.2$. Then

(1) $P(A) =$

- (A) 0.400
- (B) **0.500**
- (C) 0.050
- (D) 0.200

(2) $P(B | A) =$

- (A) **0.200**
- (B) 0.900
- (C) 0.500
- (D) 0.050

(3) $P(B^c | A) =$

- (A) 0.400
- (B) **0.800**
- (C) 0.0200
- (D) 0.0.120

Q2. The following table shows the population of mentally weak children classified by the level of weakness and how often they misunderstand an activity.

Level of weakness	Misunderstands an activity		
	Rarely (R)	Sometimes (S)	Often (O)
Slight (G)	65	75	10
Moderate (M)	98	68	84
Severe (V)	12	32	56

If one child is randomly selected from the population then,

(4) the probability that the child has moderate weakness if we know that he or she rarely misunderstands an activity is:

- (A) 0.46
- (B) **0.56**
- (C) 0.350
- (D) 0.70

(5) if we know that the child has not been weak, then the probability that the child rarely misunderstands an activity is

- (A) 0.0437
- (B) 0.0475
- (C) 0.4075
- (D) 0.4375

(6) $P(M | O) =$

- (A) **0.5600**
- (B) 0.1680
- (C) 0.3360
- (D) 0.6320

(7) the events G and O are :

- (A) disjoint
- (B) independent
- (C) dependent
- (D) Non of these

Q3. A factory has three assembly lines A, B, and C. Each line makes the same part. 50% of parts produced by the factory come off of assembly line A, 30% come off of assembly line B, and 20% come off of assembly line C. Finished parts can be either defective or not. It is known that 0.4% of the parts from line A are defective, 0.6% of the parts from line B are defective, and 1.2% of the parts from line C are defective.

(8) The probability of selecting a defective part is equal to:

- (A) 0.323
- (B) 0.290
- (C) 0.387
- (D) **0.006**

(9) Suppose that we are holding a defective part in our hand the probability that it came from assembly line A

- (A) 0.323
- (B) 0.290
- (C) 0.387
- (D) **0.006**





Q4. A discrete random variable X has a cumulative distribution function (CDF), F(x) as:

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/6 & \text{for } 0 \leq x < 1 \\ 1/2 & \text{for } 1 \leq x < 2 \\ 5/6 & \text{for } 2 \leq x < 3 \\ 1 & \text{for } x \geq 3 \end{cases} \quad \text{So,}$$

(10) $P(X > 2.5)$ is

- (A) 4/6
- (B) 3/6
- (C) 1/6
- (D) **2/6**

(11) $f(2) = P(X = 2)$ is

- (A) **2/6**
- (B) 4/6
- (C) 1/6
- (D) **3/6**

(12) $P(0 < X < 3)$ is

- (A) 1/3
- (B) 1/2
- (C) 1/6
- (D) **2/3**

Q5. If the pdf of the continuous random variable X having the form :

$$f(x) = \begin{cases} 0.25x^3 & 0 < X < 2 \\ 0 & \text{otherwise} \end{cases}$$

Then:

(13) $P(X \leq 0.50) = \dots$

- (A) 0.1250
- (B) 0.8300
- (C) 0.2500
- (D) **0.0039**

(14) $P(0.3 < X \leq 1.5) = \dots$

- (A) 0.1250
- (B) 0.2500
- (C) **0.8300**
- (D) 0.0039

(15) the mean value of the random variable X is

- (A) 0.800
- (B) 3.200
- (C) 1.100
- (D) **1.600**

(16) and $E(X^2)$ will be

- (A) 0.800
- (B) 1.100
- (C) 2.512
- (D) **3.200**

(17) while the standard deviation of the random variable X will be:

- (A) **0.800**
- (B) 3.200
- (C) 1.100
- (D) 1.600

Q6. If the pdf of the random variable X is

given by $f(x) = \begin{cases} \frac{3}{2}\sqrt{x} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$, and

$E(X) = 0.6$ and $\sigma^2 = 0.06857$, and let us to define the random $Y = 5X - 0.5$

(18) the expected value of Y will be

- (A) 70.50
- (B) 3.500
- (C) 3
- (D) **2.500**

(19) $E(Y^2)$ is

- (A) 70.50
- (B) 13.9643
- (C) 13.7143
- (D) **7.9643**

(20) the standard deviation Y will be

- (A) 1.7143
- (B) 6.0465
- (C) 6.5465
- (D) **1.3093**





Q7. Question No. 7:

Let X be a continuous random variable with probability density function is given

by $f(x) \begin{cases} c(1-x), & 0 < x < 1, \\ 0, & \text{otherwise} \end{cases}$

(21) The values of c is

- (A) $1/4$
- (B) 2
- (C) $1/2$
- (D) 1

(22) $E(X) =$

- (A) $1/4$
- (B) $1/3$
- (C) $9/4$
- (D) $1/2$

(23) $Var(X) = \sigma^2 =$

- (A) $1/18$
- (B) $1/9$
- (C) $1/27$
- (D) $1/3$

(24) $P(X = 0) =$

- (A) 1
- (B) 0
- (C) $1/2$
- (D) $1/6$

(25) $P(1/5 < X < 1) =$

- (A) $1/24$
- (B) $10/24$
- (C) $15/25$
- (D) $16/25$

(26) $P(|X - \mu| < 2\rho) =$

- (A) 0.76
- (B) 0.96
- (C) 0.90
- (D) 0.82

(27) By using Chebyshev's theorem, then

$P(|X - \mu| < 2\rho) =$

- (A) $\leq 1/4$
- (B) $\geq 10/4$
- (C) $\leq 3/4$
- (D) $\geq 3/4$

Q8. If X and Y are two **independent** random variables defined such that :

$E(X) = 10, E(Y) = 15, V(X) = 9, \sigma_y = 4$

if $U = 3X + 2Y - 5$ Then

(28) $E(U) =$

- (A) 145
- (B) 109
- (C) **65**
- (D) 2500

(29) $V(U) =$

- (A) **145**
- (B) 109
- (C) 65
- (D) 2500

(30) $E(X^2) =$

- (A) 0.1250
- (B) **109**
- (C) 65
- (D) 2500

THE END





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STAT 324
First Midterm Exam
Second Semester
1430 – 1431 H

- Mobile Telephones are not allowed in the classrooms.
- Time allowed is 90 minutes.
- Answer all questions.
- Choose the nearest number to your answer.
- **WARNING:** Do not copy answers from your neighbors. They have different questions forms.
- For each question, put the code of the correct answer in the following table beneath the question number:

1	2	3	4	5	6	7	8	9	10
A	C	C	D	B	A	D	B	A	D

11	12	13	14	15	16	17	18	19	20
B	B	D	C	A	C	A	B	B	C

21	22	23	24	25	26	27	28	29	30
A	A	C	B	D	C	A	D	D	A

►►

Suppose that the error in the reaction temperature, in $^{\circ}C$, for a controlled laboratory experiment is a continuous random variable X having the function

$$f(x) = \begin{cases} \frac{8}{x^3}, & x > 2 \\ 0, & \text{elsewhere,} \end{cases}$$

then:

(1)	$P(X < 4)$							
	(A)	0.75	(B)	3.0	(C)	0.50	(D)	0.15
(2)	$P(-1 < X < 4)$							
	(A)	3.0	(B)	0.15	(C)	0.75	(D)	0.5
(3)	$P(X \geq 5)$							
	(A)	1.0	(B)	0.15	(C)	0.16	(D)	0.5
(4)	The expected value of X ; $E(X)$ equals							
	(A)	2.0	(B)	1.0	(C)	8.0	(D)	4.0

►►

An investment firm offers its customers municipal bonds that mature after different numbers of years. Given that cumulative distribution function of X , the number of years to maturity for a randomly selected bond is:

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.24 & 1 \leq x < 3 \\ 0.56, & 3 \leq x < 5 \\ 1, & x \geq 5 \end{cases}$$

(5)	$P(X = 5)$ equals to							
	(A)	0.76	(B)	0.44	(C)	0.56	(D)	0.20
(6)	$P(X > 2)$							
	(A)	0.76	(B)	0.56	(C)	0.50	(D)	0.20
(7)	$P(1.5 < X < 5)$							
	(A)	0.2	(B)	0.76	(C)	0.56	(D)	0.32

►►

Suppose that $P(A_1) = 0.4$, $P(A_1 \cap A_2) = 0.2$, $P(A_3|A_1 \cap A_2) = 0.75$, then:

(8)	$P(A_2 A_1)$ equals to							
	(A)	0.00	(B)	0.50	(C)	0.1	(D)	0.2
(9)	$P(A_1 \cap A_2 \cap A_3)$ equals to							
	(A)	0.15	(B)	0.75	(C)	1.0	(D)	0.2

►►

A certain group of adults are classified according to sex and their level of education as given by the following table:

Sex Education	Female	Male
College	17	22
Secondary	45	38
Elementary	50	28

If a person is selected at random from this group, then

(10)	The probability that the person is female is:							
	(A)	0.44	(B)	0.50	(C)	0.28	(D)	0.56
(11)	The probability that the person is female and has an elementary education is:							
	(A)	0.64	(B)	0.25	(C)	0.45	(D)	0.50

►►

Suppose that a certain institute offers two training programs T_1 and T_2 . In the last year, 100 and 200 trainees were enrolled for programs T_1 and T_2 , respectively. From the past experience it is known that the passing probabilities are 0.75 for the program T_1 and 0.80 for the program T_2 . Assume that at the end of the last year we selected a trainee at random from this institute.

(12)	The probability that the selected trainee passed the program equals to							
	(A)	0.53	(B)	0.78	(C)	0.50	(D)	0.25
(13)	What is the probability that the selected trainee has been enrolled in the program T_2 given that he passed the program							
	(A)	0.80	(B)	0.32	(C)	0.78	(D)	0.68

►►

If $P(A) = 0.9$, $P(B) = 0.6$, and $P(A^c \cap B) = 0.1$, then:

(14)	$P(A \cap B)$ equals to							
	(A)	0.30	(B)	0.40	(C)	0.50	(D)	0.20
(15)	$P(A \cup B)^c$ equals to							
	(A)	0.00	(B)	1.00	(C)	0.50	(D)	0.15
(16)	$P(A^c B)$ equals to							
	(A)	0.10	(B)	0.50	(C)	0.17	(D)	0.011
(17)	$P(B A^c)$ equals to							
	(A)	1.00	(B)	0.011	(C)	0.50	(D)	0.017

►►

If $P(A) = 0.8$, $P(B) = 0.5$, and $P(A \cup B) = 0.9$, then:

(18)	The two events A and B are							
	(A)	dependent	(B)	independent	(C)	disjoint	(D)	Mutually exclusive

►►►

If the function $f(x) = C(x^2 + 3)$ for $x = 0, 1, 2$ can serve as a probability distribution of the discrete random variable X .

(19)	The value of C equals to			
	(A) 14	(B) 0.071	(C) 12	(D) 0.032

►►►

Suppose that we have probability function $f(x) = 0.1x$, for $x = 1, 2, 3, 4$. Then

(20)	P($X > 2$) equals to			
	(A) 0.3	(B) 0.1	(C) 0.7	(D) 0.9
(21)	The expected value of X equals			
	(A) 3.0	(B) 2.5	(C) 0.25	(D) 0.5
(22)	The Variance of X equals			
	(A) 1.0	(B) 3.54	(C) 1.25	(D) 0.5

►►►

If the random variable X has probability density

$$f(x) = \begin{cases} \frac{x^2}{3}, & k < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

(23)	Then the value of k equals			
	(A) 0.44	(B) 0.40	(C) -1.0	(D) 0.23

►►►

If the random variable X has probability density

$$f(x) = \begin{cases} 1+x, & -1 < x < 0 \\ 1-x & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(24)	P($X < 0.5$) equals to			
	(A) 0.5	(B) 0.875	(C) 0.375	(D) 0.75
(25)	P($X = 0.2$) equals to			
	(A) 1.2	(B) 0.5	(C) 0.8	(D) 0

▶▶▶

The cumulative distribution function $F(x)$ of a continuous random variable X is as follows:

$$F(x) = \begin{cases} 0, & x \leq -1 \\ \frac{x^3 + 1}{9}, & -1 < x < 2 \\ 1 & x \geq 2 \end{cases}$$

(26)	$P(-0.5 < X < 1.5)$ equals to							
	(A)	0.30	(B)	0.40	(C)	0.39	(D)	0.20
(27)	$P(X \geq 0.6)$ equals to							
	(A)	0.86	(B)	0.14	(C)	0.50	(D)	0.15

▶▶▶

A random variable 'X' has $E(X) = 2$ and $E(X^2) = 8$. Another random variable 'Y' is related with X as follows:

$$Y = (3X + 5) / 2.$$

(28)	The mean of Y is:							
	(A)	2.0	(B)	6.0	(C)	8.5	(D)	5.5
(29)	The Variance of Y is:							
	(A)	4.0	(B)	8.5	(C)	6.0	(D)	9.0

▶▶▶

A random variable 'X' has $E(X) = 2$, and variance = 4.

(30)	Then by Chebychev theorem, $P(-1 < X < 5)$ is							
	(A)	$\geq 5/9$	(B)	$\geq 4/9$	(C)	$\leq 5/9$	(D)	$\leq 4/9$

►►

- 1) Two engines operate independently, if the probability that an engine will start is 0.3, and the probability that other engine will start is 0.5, then the probability that both will start is:

(A) 1	(B) <u>0.15</u>	(C) 0.24	(D) 0.5
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►►

- 2) Assume that $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cap B \cap C) = 0.05$, and $P(\overline{A \cap B}) = 0.92$, then the event A and B are,

(A) <u>Independent</u>	(B) Dependent	(C) Disjoint	(D) None of these.
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- 3) Using question (2), $P(C|A \cap B)$ is equal to,

(A) 0.604	(B) <u>0.625</u>	(C) 0.054	(D) -0.925
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►►

- 4) A company in Riyadh has 100 cars, 0.46 are white. Of the cars that aren't white, 0.40 are green. How many company cars are neither white nor green (not white or not green)?

(A) 67	(B) 54	(C) <u>14</u>	(D) 20
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►►

- Suppose there is a factory with three assembly lines (A, B, and C) that each make the same part. 50% of parts produced by the factory come off of assembly line A, 30% come off of assembly line B, and 20% come off of assembly line C. Finished parts can be categorized as either defective or not. It is known that 0.4% of the parts from line A are defective, 0.6% of the parts from line B are defective, and 1.2% of the parts from line C are defective.

- 5) The probability of selecting a defective part is equal to:

(A) 0.323	(B) 0.290	(C) 0.387	(D) <u>0.006</u>
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- 6) Suppose that we are holding a defective part in our hand, the probability that it came from assembly line A?

(A) 0.323	(B) 0.290	(C) 0.387	(D) <u>0.006</u>
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►►

- 7) A lower bound valued according to chebyshev's theory for $P(\mu - 3\sigma \leq X \leq \mu + 3\sigma)$ equal to:

(A) 0.3175	(B) 0.750	(C) <u>0.889</u>	(D) 0.250
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- 8) If $Y = 3X - 1.5$ and $E(X) = 0.5$, then $E(Y)$ is:

(A) -0.5	(B) 0.5	(C) <u>0.0</u>	(D) None of these.
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- 9) If $Y = 3X - 1.5$ and $V(X) = 0.4$, then $V(Y)$ is:

(A) 0.6	(B) <u>3.6</u>	(C) 0.3	(D) -0.9
---------	----------------	---------	----------

►►

- 10) Consider the probability function $f(x) = kx$, $0 < x < 1$. Then the value of k is equal to:

(A) <u>2</u>	(B) 1	(C) 0.1	(D) 0.2
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►►

- 11) Consider the probability function $f(x) = cx$, $x = 1, 2, 3, 4$. Then the value of c is equal to:

(A) 2	(B) 1	(C) <u>0.1</u>	(D) 0.2
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►►►

Let X be a random variable have a discrete uniform with parameter $k=3$ and with values 0,1, and 2.

12) The mean of X is:

(A) <u>1.0</u>	(B) 2.0	(C) 1.5	(D) 0.0
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13) The variance of X is

(A) 0.0	(B) 1.0	(C) 0.67	(D) 1.33
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►►►

A study by the traffic police claims that only 20% of the drivers in Riyadh fasten their seat belts. A sample of 10 drivers in Riyadh has been taken. (Hint: X is a binomial variable.)

14) the probability of observing 2 or less drivers in Riyadh fasten their seat belts is equal to:

(A) 0	(B) 0.107	(C) 0.376	(D) 0.678
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15) the probability of observing more than 2 drivers in Riyadh fasten their seat belts is equal to:

(A) 0.322	(B) 0.107	(C) 0.376	(D) 0.678
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16) the probability of observing exactly 2 drivers in Riyadh fasten their seat belts is equal to:

(A) 0.322	(B) 0.302	(C) 0.376	(D) 0.268
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17) the probability of observing between 2 and 8 drivers in Riyadh fasten their seat belts is equal to:

(A) 0.678	(B) 0.624	(C) 0.376	(D) 0.322
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18) the expected number of drivers in Riyadh that fasten their seat belts is equal to:

(A) 2	(B) 4	(C) 10	(D) 1.8
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19) the variance of the number of drivers in Riyadh that fasten their seat belts is equal to:

(A) 2	(B) 4	(C) 10	(D) 1.8
-------	-------	--------	---------

►►►

Suppose that the number of telephone calls received per day has a Poisson distribution with mean of 3 calls per day.

20) The probability that 2 calls will be received in a given day is

(A) 0.546	(B) 0.646	(C) 0.149	(D) <u>0.224</u>
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21) The expected number of telephone calls received in a given week is

(A) 3	(B) <u>21</u>	(C) 18	(D) 15
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22) The probability that at least 2 calls will be received in a period of 12 hours is

(A) 0.594	(B) 0.191	(C) 0.809	(D) <u>0.442</u>
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►►►

Suppose the failure time (in hours) of a specific type of electrical device is distributed with a probability density function:

$$f(x) = \frac{1}{50}x, \quad 0 < x < 10$$

then,

23) the average failure time of such device is:

(A) <u>6.667</u>	(B) 1.00	(C) 2.00	(D) 5.00
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24) the variance of the failure time of such device is:

(A) 0	(B) 50	(C) <u>5.55</u>	(D) 10
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►►

Suppose the failure time (in hours) of a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress is distributed with a probability density function:

$$f(x) = \frac{1}{70} e^{-x/70}, \quad x > 0,$$

then,

25) the probability that a randomly selected insulation will be less than 50 hours is:

(A) 0.4995	(B) 0.7001	(C) <u>0.5105</u>	(D) 0.2999
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26) the probability that a randomly selected insulation will last more than 150 hours is:

(A) 0.8827	(B) 0.2788	(C) <u>0.1173</u>	(D) 0.8827
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27) the average failure time of the electrical insulation is:

(A) 1/70	(B) <u>70</u>	(C) 140	(D) 35
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28) the variance of the failure time of the electrical insulation is:

(A) <u>4900</u>	(B) 1/49000	(C) 70	(D) 1225
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►►

The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimeters and a standard deviation of 0.03 centimeter. Then,

29) the probability of rings that will have inside diameter less than 12.05 centimeters is:

(A) 0.0475	(B) <u>0.9525</u>	(C) 0.7257	(D) 0.8413
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30) the probability of rings that will have inside diameter exceeding 11.97 centimeters is:

(A) 0.0475	(B) <u>0.8413</u>	(C) 0.1587	(D) 0.4514
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31) the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters is:

(A) <u>0.905</u>	(B) -0.905	(C) 0.4514	(D) 0.7257
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►►

A machine is producing metal pieces that are cylindrical in shape. A sample is taken and the diameters are 1.70, 2.11, 2.20, 2.31 and 2.28 centimeters. Then,

32) The sample mean is:

(A) 2.22	(B) 2.32	(C) 2.90	(D) <u>2.12</u>
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33) The sample variance is:

(A) 0.597	(B) 0.285	(C) <u>0.061</u>	(D) 0.534
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▶▶▶

A certain engineer is interested in the proportion of defective items in the population. In a random sample of 1000 items 250 are found to be defective.

34) The point estimate for the true proportion of homes in the population with a VCR is:

(A) 250	(B) 1000	(C) 0.25	(D) 4
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35) The upper bound of the 95% confidence interval estimate for the true proportion is:

(A) 0.226	(B) <u>0.277</u>	(C) 0.295	(D) 0.567
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36) The lower bound of the 95% confidence interval estimate for the true proportion is:

(A) 0.217	(B) <u>0.223</u>	(C) 0.285	(D) 0.567
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37) If the value of α decrease (get smaller), then the interval estimate will decrease (get smaller).

(A) Yes	(B) <u>No</u>	(C) No change
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▶▶▶

A company surveyed television viewers in an effort to estimate the proportion of homes with a video cassette recorder (VCR). A survey of 600 homes found 470 with a VCR. Representatives of the VCR industry claim that the true proportion of homes with a VCR is 0.80. Test this hypotheses using $\alpha=0.05$, we get:

38) the following hypotheses:

(A) $H_0 : p=0.80$ VS $H_1:p \neq 0.8$	(B) $H_0 : p=0.80$ VS $H_1:p > 0.80$	(C) $H_0 : p=0.80$ VS $H_1:p < 0.80$	(D) $H_0 : p \neq 0.80$ VS $H_1:p = 0.80$
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39) Our decision will be:

(A) Accept H_0 (Don't reject H_0)	(B) Reject H_0	(C) Don't reject H_1	(D) Reject H_0 and H_1
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40) Suppose the company wish to have a sampling error of plus or minus 0.1 in estimating the proportion of homes with a VCR at the 95% confidence level. The sample size required is equal to:

(A) 600	(B) <u>66</u>	(C) 660	(D) 60
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▶▶▶

An investigation of a photocopying equipment show that 60 failures of equipment took on the average 84.2 minutes to repair with a standard deviation of 19.4 minutes. Find 95% confidence interval for the true mean μ of repairing the equipment?

41) The point estimate for the true mean μ of repairing the equipment is equal to:

(A) 60	(B) <u>84.2</u>	(C) 19.4	(D) 376.36
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42) The upper bound of the 95% confidence interval for the true mean μ of repairing the equipment is equal to:

(A) 89.1	(B) <u>79.3</u>	(C) 82.6	(D) 85.9
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(A) <u>89.1</u>	(B) 79.3	(C) 82.6	(D) 85.9
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▶▶▶

A random sample of size $n_1=31$ is taken from a normal population with standard deviation $\sigma_1=5$ has mean 80. A second random sample of size $n_2=36$ is taken from a different normal population with standard deviation $\sigma_2=3$ has mean equal to 75.

44) How large the sample size of the second population is needed if we want to be 95% confident that our sample mean will be within one (1) unit of the true mean μ_2 ?

(A) 6	(B) 35	(C) 138	(D) 85
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45) The upper bound of the 99% confidence interval for the difference $\mu_1-\mu_2$ is equal to:

(A) 2.99	(B) 7.65	(C) 2.35	(D) 7.02
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(A) 1	(B) <u>0.15</u>	(C) 0.24	(D) 0.5
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►►

2) Assume that $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cap B \cap C) = 0.05$, and $P(\overline{A \cap B}) = 0.92$, then the event A and B are,

(A) <u>Independent</u>	(B) Dependent	(C) Disjoint	(D) None of these.
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(A) 0.604	(B) <u>0.625</u>	(C) 0.054	(D) -0.925
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►►

Suppose there is a factory with three assembly lines (A, B, and C) that each make the same part. 50% of parts produced by the factory come off of assembly line A, 30% come off of assembly line B, and 20% come off of assembly line C. Finished parts can be categorized as either defective or not. It is known that 0.4% of the parts from line A are defective, 0.6% of the parts from line B are defective, and 1.2% of the parts from line C are defective.

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(A) 0.6	(B) <u>3.6</u>	(C) 0.3	(D) -0.9
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►►

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(A) <u>2</u>	(B) 1	(C) 0.1	(D) 0.2
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►►

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►►►

Let X be a random variable have a discrete uniform with parameter $k=3$ and with values 0,1, and 2.

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13) The variance of X is

(A) 0.0	(B) 1.0	(C) 0.67	(D) 1.33
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A study by the traffic police claims that only 20% of the drivers in Riyadh fasten their seat belts. A sample of 10 drivers in Riyadh has been taken. (Hint: X is a binomial variable.)

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22) The probability that at least 2 calls will be received in a period of 12 hours is

(A) 0.594	(B) 0.191	(C) 0.809	(D) <u>0.442</u>
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►►►

Suppose the failure time (in hours) of a specific type of electrical device is distributed with a probability density function:

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then,

23) the average failure time of such device is:

(A) <u>6.667</u>	(B) 1.00	(C) 2.00	(D) 5.00
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(A) 0	(B) 50	(C) <u>5.55</u>	(D) 10
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Suppose the failure time (in hours) of a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress is distributed with a probability density function:

$$f(x) = \frac{1}{70} e^{-x/70}, \quad x > 0,$$

then,

25) the probability that a randomly selected insulation will be less than 50 hours is:

(A) 0.4995	(B) 0.7001	(C) <u>0.5105</u>	(D) 0.2999
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(A) 2.22	(B) 2.32	(C) 2.90	(D) <u>2.12</u>
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33) The sample variance is:

(A) 0.597	(B) 0.285	(C) <u>0.061</u>	(D) 0.534
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►►►

A certain engineer is interested in the proportion of defective items in the population. In a random sample of 1000 items 250 are found to be defective.

34) The point estimate for the true proportion of homes in the population with a VCR is:

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38) the following hypotheses:

(A) $H_0 : p=0.80$ VS $H_1 : p \neq 0.80$	(B) $H_0 : p=0.80$ VS $H_1 : p > 0.80$	(C) $H_0 : p=0.80$ VS $H_1 : p < 0.80$	(D) $H_0 : p \neq 0.80$ VS $H_1 : p = 0.80$
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39) Our decision will be:

(A) Accept H_0 (Don't reject H_0)	(B) Reject H_0	(C) Don't reject H_1	(D) Reject H_0 and H_1
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(A) 600	(B) <u>66</u>	(C) 660	(D) 60
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(A) 6	(B) 35	(C) 138	(D) 85
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(A) 2.99	(B) 7.65	(C) 2.35	(D) 7.02
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46) The lower bound of the 99% confidence interval for the difference $\mu_1-\mu_2$ is equal to:

(A) 2.99	(B) 7.65	(C) 2.35	(D) 7.02
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