Student ID #: [redacted]

Question 5 (13 + 12 = 25 points):

Evaluate both sides of the stoke's theorem for the magnetic field $\overline{H} = 6xy\overline{a}_x - 3y^2\overline{a}_y$ **A/m** and the rectangular path around the **region**, $2 \le x \le 5, -1 \le y \le 1, z = 0$. Let the positive direction of $d\overline{S}$ be a_z.

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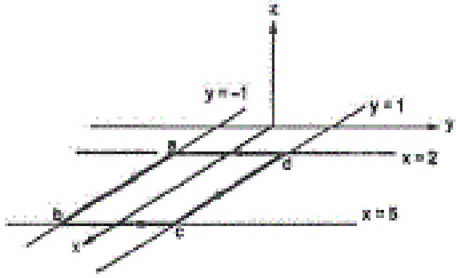
Solution:



$$\oint_L \overline{H} \cdot dL = \int_S (\nabla \times \overline{H}) \cdot ds$$

$H = 6xy a_x - 3y^2 a_y$

$dl = dx\, a_x + dy\, a_y + dz\, a_z$

$\overline{H} \cdot d\overline{l} = 6xy\, dx - 3y^2 dy + 0$

$$\int_2^5 6xy\, dx - \int_{-1}^{1} 3y^2 dy = \frac{6x^2 y}{2}\Big]_2^5 - \frac{3y^3}{3}\Big]_{-1}^{1}$$

4

$= [3(5)^2 y - 3(2)^2 y] - [1+1]$

$= 75y - 12y - 2$

when $y = 1$ $75 - 12 - 2 = 61$ ✗ -63. when $y = -1$ $75(-1) - 12(-1) - 2$ $= -65$? -63.

Mistakes!! & Incomplete

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & -3y^2 & 0 \end{vmatrix} = a_x(0-0) - a_y(0-0) + a_z(0 - 6x)$$

13

$= -6x\, a_z$ $ds = dx\, dy\, a_z$

$$\int_{-1}^{1}\int_2^5 -6x\, dx\, dy = -6\left(\frac{x^2}{2}\right]_2^5 \left[y\right]_{-1}^{1} = -6\left(\frac{5^2}{2} - \frac{2^2}{2}\right)(1+1)$$

$\boxed{= -126}$

Student ID #: [redacted]

Question 4 (10 + 10 = 20 points):

10 + 9

a. **Calculate** the value of **current density** at point **(3, 2, -4)**, if $\bar{H}$ is given as, $H = x^3 y a_x - xy^2 z^2 a_y + xy^2 z a_z$ **A/m.** J

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b. Given that the general vector $\bar{A}$ is, $H = 2.5 a_\theta + 5 a_\phi$ **in spherical coordinates. Find** the **curl** of $\bar{H}$ at **(2, π/6, 0).**

Solution:

a) $J = \nabla \times H$ 10 a

← Solution on back

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3 y & -xy^2 z^2 & xy^2 z \end{vmatrix} = a_x(2xyz + 2xy^2 z) - a_y(y^2 z - 0) + a_z(-y^2 z^2 - x^3)$$

$$J\Big|_{(3,2,-4)} = a_x[(2)(3)(2)(-4) + (2)(3)(2)^2(-4)] - a_y[(2)^2(-4) - 0] + a_z[-(2)^2(-4)^2 - (3)^3]$$

$$\boxed{= -144 a_x + 16 a_y - 91 a_z}$$

b) $H = 2.5 a_\theta + 5 a_\phi$ spherical curl H at $(2, \pi/6, 0)$

$$\nabla \times H = \frac{1}{r \sin\theta} \begin{vmatrix} a_r & a_\theta & a_\phi \\ \frac{\partial}{\partial} & & \end{vmatrix}$$

$$a \quad a_\theta \quad a$$
$$\frac{1}{r}\frac{\partial}{\partial r}(r) \quad \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta} \sin \quad \frac{\partial}{\partial \phi}$$

← a_r a_θ a_ϕ

Student ID #: [redacted]

Question 3 (15 points):

Apply Biot-Savart law and **Calculate** the **differential magnetic field intensity** at **point A (2, 3, -2)** due to a **differential length** of conductor, **2π (-10a$_x$ + 7a$_y$ – 3a$_z$) m**, carrying a **current** of **3.37μA**, if the **differential length** is placed at **point B (1, 3, 2)**.

15

$dH = ?$ $\quad A(2,3,-2)$ $\quad dL = 2\pi(-10a_x + 7a_y - 3a_z)\ m$

$I = 3.37\ \mu A$ $\quad B(1,3,2)$

$$dH = \frac{I\,dL \times a_{R_{12}}}{4\pi R_{12}^2}$$

$$\therefore dH = (I\,dL \times a_{R_{12}})\left(\frac{1}{4\pi R^2}\right)$$

$$= \left(\frac{188.72\pi}{\sqrt{17}}a_x + \frac{289.82\pi}{\sqrt{17}}a_y + \frac{47.18\pi}{\sqrt{17}}a_z\right)\mu \cdot \left(\frac{1}{(4\pi)(17)}\right)$$

~~214.365a_x +~~

$$\boxed{= (673.108a_x + 1033.701a_y + 168.277a_z)\ nA/m^2}$$

$R_{AB} = (-1, 0, 4)$ $\quad |R_{AB}| = \sqrt{1+0+16} = \sqrt{17}$

$$a_{R_{AB}} = \left(\frac{-1, 0, 4}{\sqrt{17}}\right)$$

$$I\,dL = (3.37\mu)(-20\pi a_x + 14\pi a_y - 6\pi a_z)$$

$$= (-67.4\pi a_x + 47.18\pi a_y - 20.22\pi a_z)\ \mu$$

$$I\,dL \times a_{R_{12}} = \begin{vmatrix} a_x & a_y & a_z \\ -67.4\pi & 47.18\pi & -20.22\pi \\ -\frac{1}{\sqrt{17}} & 0 & \frac{4}{\sqrt{17}} \end{vmatrix}$$

$$= a_x\left(\frac{188.72\pi}{\sqrt{17}} - 0\right) - a_y\left(\frac{-269.6\pi}{\sqrt{17}} - \frac{20.22\pi}{\sqrt{17}}\right) + a_z\left(0 + \frac{47.18\pi}{\sqrt{17}}\right)$$

$$\boxed{= \frac{188.72\pi}{\sqrt{17}}a_x + \frac{289.82\pi}{\sqrt{17}}a_y + \frac{47.18\pi}{\sqrt{17}}a_z\ \mu}$$

Islamic University in Madinah, Faculty of Engineering, Electrical Engineering Department
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Question 2 (10 + 10) = 20 points:

20

a. **Find the magnitude of $\overline{D}$ and $\overline{P}$ for a dielectric material** in which $|\overline{E}| = 7.95$ **mV/m** and $\chi_e = 11.35$.

b. **Find the polarization in dielectric material with $\varepsilon_R = 5.99$ if $\overline{D} = 5.87\text{x}10^{-7}$** C/m^2.

Solution:

a) $D = E\varepsilon_0(1+\chi_e) = (7.95m)(8.85\times10^{-12})(1+11.35)$

$\boxed{= 8.689\times10^{-13}}$ C/m^2

$P = E\varepsilon_0\chi_e = (7.95m)(8.85\times10^{-12})(11.35) = \boxed{7.986\times10^{-13}}$ C/m^2

b) = P = ? $\varepsilon_R = 5.99$ $\bar{D} = 5.87\times10^{-7}$ C/m^2

$P = E\varepsilon_0\chi_e$

$\boxed{D = \varepsilon_0\bar{E} + \bar{P}}$

$D = \varepsilon_0\varepsilon_R\bar{E} \Rightarrow \boxed{E = \frac{D}{\varepsilon_0\varepsilon_R}}$

$E = \frac{5.87\times10^{-7}}{(8.85\times10^{-12})(5.99)} = 11.073\times10^{3}$ V/m

$\bar{P} = D - \varepsilon_0\bar{E} = (5.87\times10^{-7}) - (8.85\times10^{-12})(11.073\times10^{3})$

$\boxed{= 4.89\times10^{-7}}$ C/m^2

Student ID #: [redacted]

Question 1 (10 + 10) = 20 points:

a. Consider a **cylinder** of **length** L and **radius** R as shown in **Fig. 1**. **Find** its **volume** by **integration**.

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b. **Calculate** the **total surface area** (**for** S_1, S_2 **and** S_3) of the cylinder as shown in **Fig. 1** having **length** L and **radius** R by the method of **integration**.

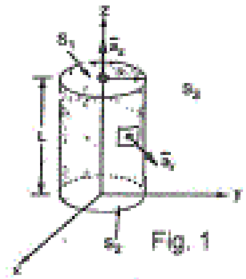

Fig. 1

Solution:

a) $dv = r\,dr\,d\phi\,dz$

$$\int dv = \int_0^L \int_0^{2\pi} \int_0^R r\,dr\,d\phi\,dz$$

$$V = \frac{r^2}{2}\Big]_0^R \phi\Big]_0^{2\pi} z\Big]_0^L = \left(\frac{R^2}{2}\right)(2\pi)(L) = \boxed{\pi R^2 L}$$

b) Top region $\quad ds = r\,dr\,d\phi\,a_z$

$$S_1 = \int_0^{2\pi}\int_0^R r\,dr\,d\phi = \frac{r^2}{2}\Big]_0^R \phi\Big]_0^{2\pi} = \left(\frac{R^2}{2}\right)(2\pi) = \boxed{\pi R^2}$$

Side region $\quad ds = r\,d\phi\,dz\,a_r \quad ds\Big|_{r=R} = R\,d\phi\,dz\,a_r$

$$S_2 = \int_0^L\int_0^{2\pi} R\,d\phi\,dz = R\,(\phi\Big]_0^{2\pi} z\Big]_0^L = \boxed{2\pi RL}$$

bottom region $\quad ds = -r\,dr\,d\phi\,a_z$

$$S_3 = \int_0^{2\pi}\int_0^R -r\,dr\,d\phi = -\frac{r^2}{2}\Big]_0^R \phi\Big]_0^{2\pi} = \left(-\frac{R^2}{2}\right)(2\pi) = \boxed{-R^2\pi}$$

should be same as

$$S_{total} = \pi R^2 + 2\pi RL - \pi R^2 = \boxed{2\pi RL} \quad \times$$

Mistakes !

$\pi R^2 + \pi R^2 + 2\pi RL$

$S_{total} = 2\pi R(R+L)$!!

Islamic University Faculty of Engineering Department of Electrical Engineering		الجامعة الإسلامية كلية الهندسة قسم الهندسة الكهربائية

EE 282 – ELECTROMAGNETIC FIELD THEORY

Fall Semester 2017-2018

Final Exam

Date: January 03rd, 2018; Duration: 120 minutes (2 Hours)

Student's Full Name: [redacted]

Student ID #: [redacted] **Section #:1052** **Signature:** [redacted]

Instructions:

- Write your student ID number on the top of each page
- Write the solution in the space provided under each question
- Show all the steps of your calculations / derivations
- Exchange of Calculators are strictly NOT allowed
- Formula sheet (if applicable) will be provided with the exam paper

Question No.	Points Assigned	Points Awarded
1. [CO_1, PI_1_62, SO_1]	20	17.
2. [CO_5, PI_5_24, SO_5]	20	20
3. [CO_6, PI_5_25, SO_5]	15	15
4. [CO_7, PI_1_63, SO_1]	20	19.
5. [CO_8, PI_5_74, SO_5]	25	17.
Total	100	88/100

Instructor's Full Name	Dr. Khawaja Bilal Mahmood
Signature	