مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

Inverse Functions

One to one function : A one-to-one function is a function where each input (x-value) has a unique output (y-value)

Example : **Determine if each the following function is one to one**

$f = \{(7, 3), (8, -5), (-2, 11), (-6, 4)\}$ is one-to-one

$h = \{(-3, 8), (-11, -9), (5, 4), (6, -9)\}$ is not one-to-one

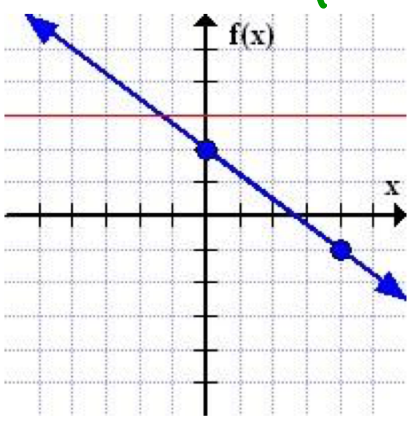
Is the Function a One-to-One Function?

Horizontal Line Test (HLT):

One-to-one: if each HL pass through at most one point on graph.

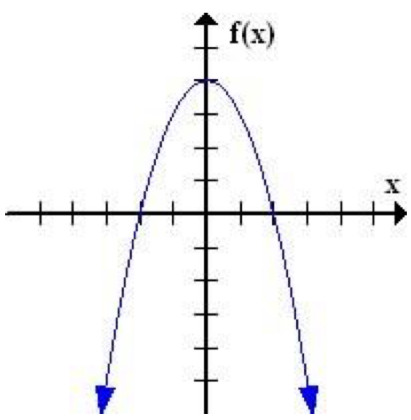
Example Determine if the function $f(x) = -\frac{3}{4}x + 2$ is a one-to-one function.

$f(x) = -\frac{3}{4}x + 2$



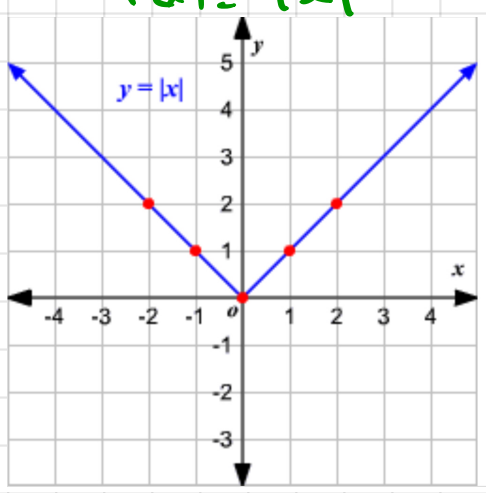
one-to-one

$f(x) = -x^4 + 4$



not one-to-one

$f(x) = |x|$



not one-to-one

2 Finding the inverse of a function

A) Inverse of order pairs function

If f is a one-to-one $\Rightarrow$ $f^{-1} = \{ (y,x) : (x,y) \text{ is in } f \}$

If f is not one-to-one $\Rightarrow$ f^{-1} does not exist.

Example: For each of the following function find f^{-1}.

$F = \{ (-3,9), (0,0), (3,9) \}$

F is not one-to-one, f^{-1} does not exist.

$F = \{ (1,2), (2,4), (3,9) \}$

F is one-to-one, $f^{-1} = \{ (2,1), (4,2), (9,3) \}$

Domain f^{-1} = {2, 4, 9} = Range F.

Range f^{-1} = {1, 2, 4} = Domain F.

B) Inverse of the equation function

- **Method 1:**

Step 1: Change f(x) to y.

Step 2: Switch x and y.

Step 3: Solve for y.

Step 4: Change y back to $f^{-1}(x)$.

$f(x) = 2x - 5$

$y = 2x - 5$

$x = 2y - 5$

$x + 5 = 2y$

$y = \frac{x+5}{2}$

$f^{-1}(x) = \frac{x+5}{2}$

- **Method 2 :**

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$F(x) = 3x + 2$

x		$(x-2)/3$	$\rightarrow f^{-1}(x)$
$\downarrow$	$\times 3$	$\uparrow$	$\div 3$
$3x$		$x-2$	
$\downarrow$	$+2$	$\uparrow$	-2
$3x+2$		x	

$$f^{-1}(x) = \frac{x-2}{3}.$$

Remark: Domain of f^{-1} = Range of f.
Range of f^{-1} = Domain of f.

Example: Find f^{-1} for $f(x) = \sqrt{x-1}$

Method 1:

$y = \sqrt{x-1}$

$x = \sqrt{y-1}$

$x^2 = y-1$

$x^2 + 1 = y$

$\therefore f^{-1}(x) = x^2 + 1$

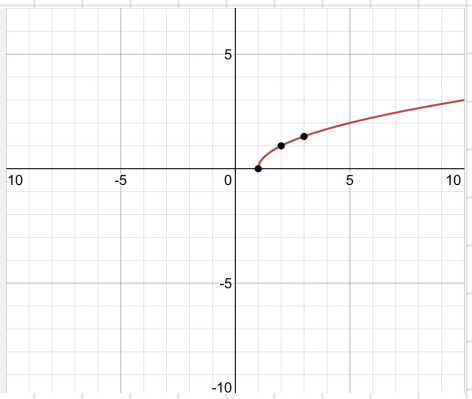
- Domain f^{-1} = Rang f.
  = $[0, \infty)$

Method 2:

$x \xrightarrow{-1} x-1 \xrightarrow{\text{Squar root}} \sqrt{x-1}$

$x \xleftarrow{\text{square}} x^2 \xleftarrow{+1} x^2+1 \rightarrow f^{-1}(x)$

$\therefore f^{-1}(x) = x^2 + 1$



3 Deciding If Two Functions are Inverses

Remark: • If f^{-1} exists then

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x$$

• If $f(g(x)) = x$ and $g(f(x)) = x$ then

f and g are inverses to each other.

Example: Are two function inverses

$$f(x) = 3x - 7 \qquad g(x) = \frac{x+7}{3}$$

- $f(g(x)) = \cancel{3}\left(\frac{x+7}{\cancel{3}}\right) - 7$

$$= x + 7 - 7 = x$$

- $g(f(x)) = \frac{3x \cancel{-7} \cancel{+7}}{3}$

$$= \frac{3x}{3} = x$$

∴ f and g are inverses.