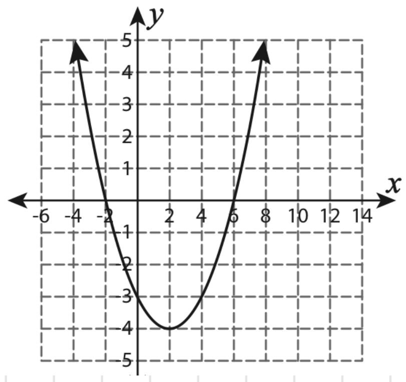
Math 101

Nada Altiary

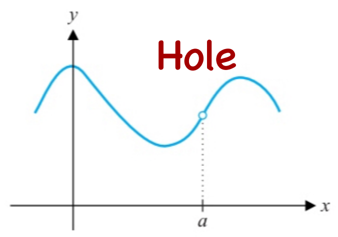
Continuity

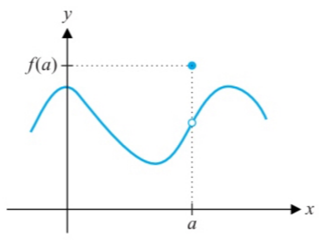
Continuity at point

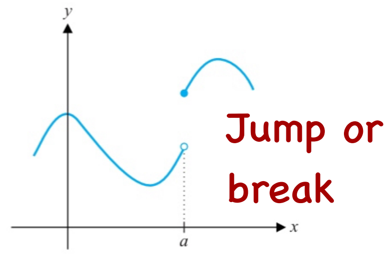
Graphically : A function f is continuous at a if its graph has no hole or break at a. Otherwise, we say that f is discontinuous.

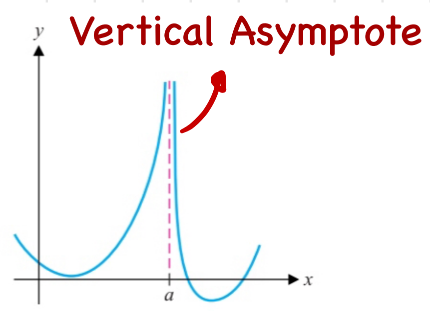


- We will classify such discontinuities as : point, Jump and infinite.









f (a) undefined

Point discontinuity

- f(a) is defined.
- $\lim_{x\to a} f(x)$ exsit.

but

$\lim_{x\to a} f(x) \neq f(a)$

Point discontinuity

removable discontinuity

$\lim_{x\to a} f(x)$ DNE

$\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$

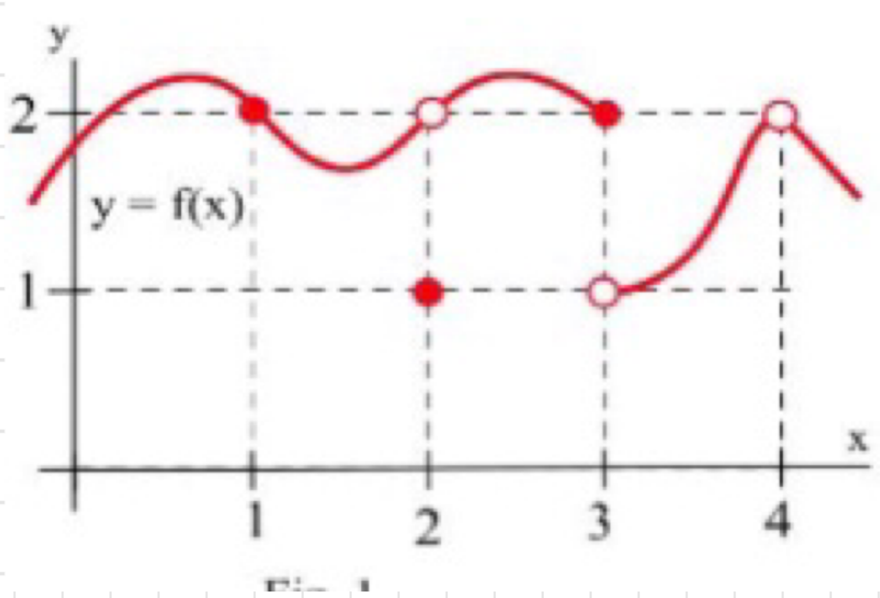
Jump discontinuity

- F(a) undefined
- $\lim_{x\to a} f(x)$ DNE

Infinite discontinuity

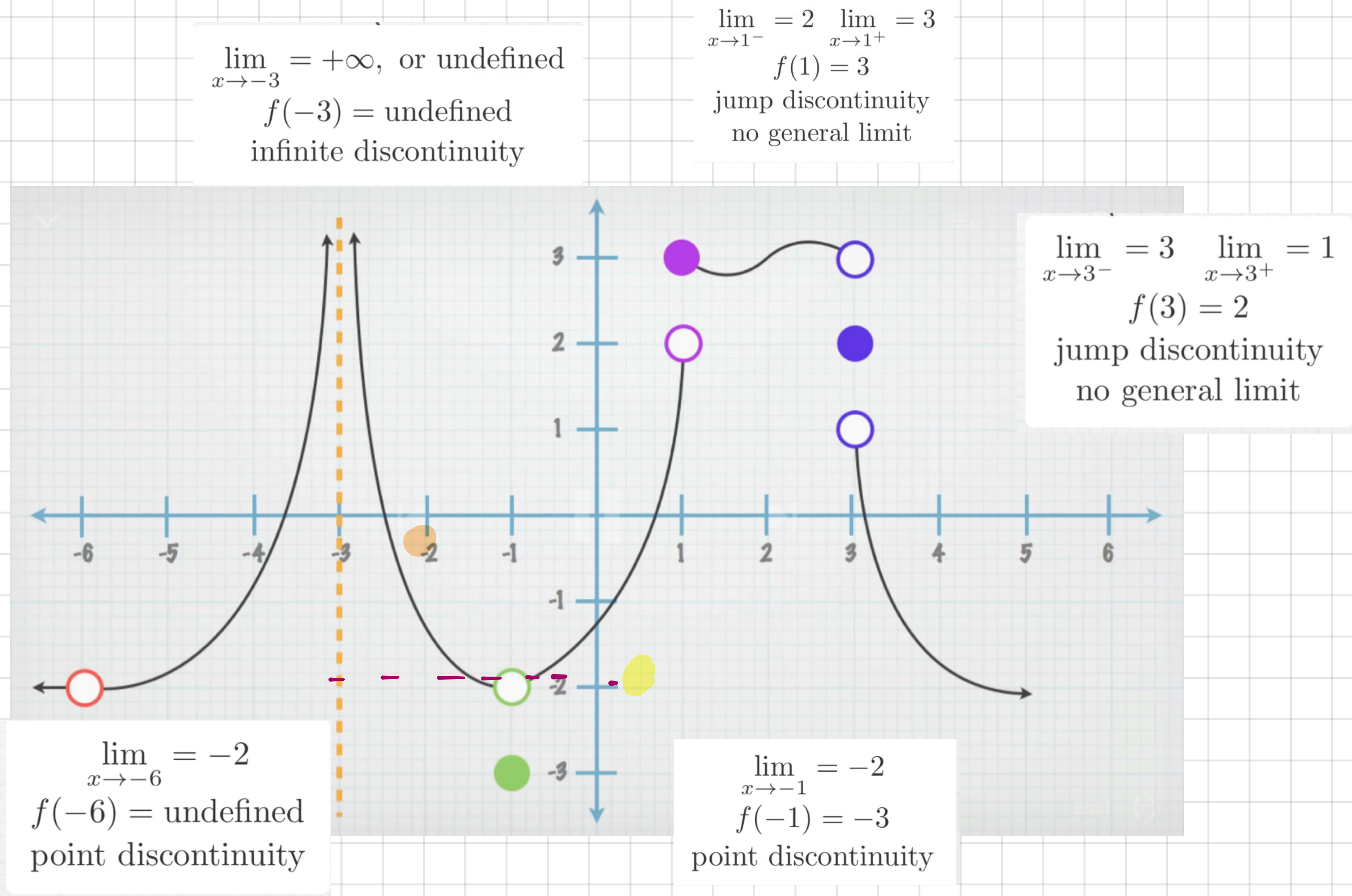
Non-removable discontinuity

Example 1: Use the following graph to fill the table



a	f(a)	$\lim_{x\to a} f(x)$
1	2	2
2	1	2
3	2	does not exist
4	undefined	2

Example 2:



Algebraically A function f is continuous at a iff

1. $f(a)$ is defined
2. $\lim_{x\to a} f(x)$ exist
3. $\lim_{x\to a} f(x) = f(a)$

Discontinuity:

A function f fails to be continuous at a if one or more of the following conditions holds:

إنفصال قابل للإزالة — 1. If $f(a)$ is not defined.

إنفصال غير قابل للإزالة — 2. If $\lim_{x\to a} f(x)$ does not exist

إنفصال قفزي — • $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$ or

إنفصال لانهائي — • $\lim_{x\to a} f(x) = \pm\infty$

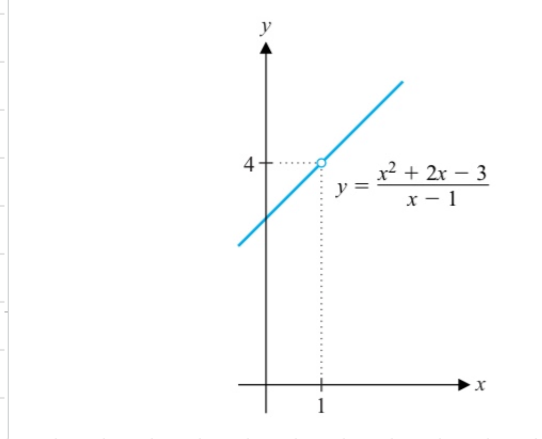
إنفصال قابل للإزالة — 3. If $\lim_{x\to a} f(x) \neq f(a)$

Example 1:

Determine whether the given functions are continuous at $a=1$

$$f(x) = \frac{x^2+2x-3}{x-1}$$

$$f(1) = \frac{(1)^2+2(1)-3}{1-1} = \frac{0}{0} \quad \text{undefined.}$$



f is discontinuous at $a=1$

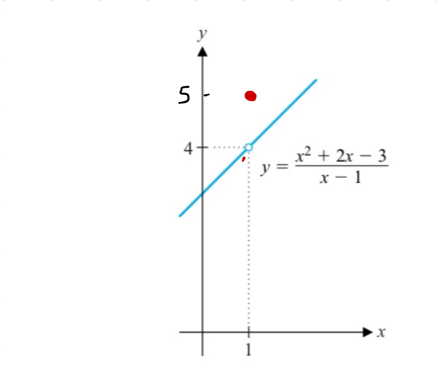
f has removable discontinuity at $a=1$

$$g(x) = \begin{cases} \frac{x^2+2x-3}{x-1} & , x \neq 1 \\ 5 & x = 1 \end{cases}$$

1)- $f(1) = 5$

2)- $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \frac{x^2+2x-3}{x-1} =$

$= \lim\limits_{x\to 1} \frac{(\cancel{x-1})(x+3)}{\cancel{x-1}}$



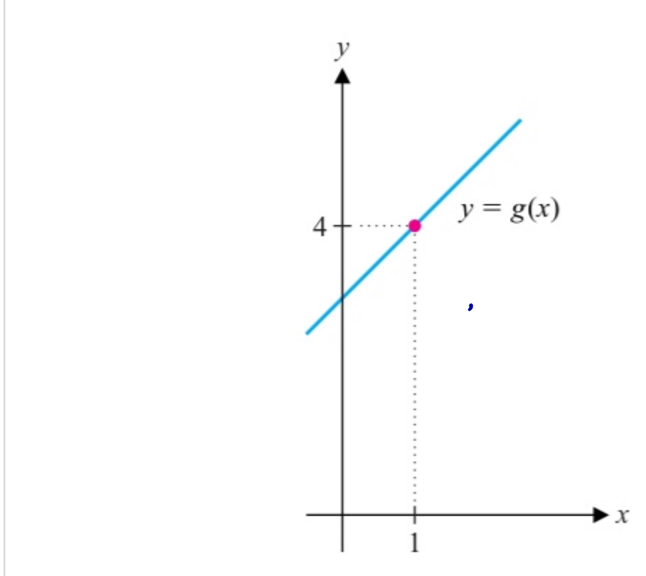
$= \lim\limits_{x\to 1} x+3 = 4$

3)- $\lim\limits_{x\to 1} f(x) \neq f(1)$

$\therefore$ f is discontinuous at $a=1$

f has removable discontinuity at $a=1$

$$h(x) = \begin{cases} \dfrac{x^2+2x-3}{x-1} & , x \neq 1 \\ 4 & x = 1 \end{cases}$$



1)- $F(1) = 4$

2)- $$\lim_{x\to 1} F(x) = \lim_{x\to 1} \frac{x^2+2x-3}{x-1}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+3)}{\cancel{x-1}}$$

$$= \lim_{x\to 1} x+3 = 4$$

3)- $$\lim_{x\to 1} F(x) = F(1)$$

$\therefore$ F is continuous at $x = 1$

$$h(x) = \begin{cases} \dfrac{x^3-1}{x-1} & x<1 \\ -x^2+2x+2 & x \geq 1 \end{cases}$$

$$h(1) = (-1)^2 + 2(1) + 2 = 3$$

$$\lim_{x\to 1^+} h(x) = \lim_{x\to 1^-} -x^2 + 2(1) + 2 = 3.$$

$$\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-} \frac{x^3-1}{x-1} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x\to 1^-} \frac{(x^2+x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= \lim_{x\to 1^-} x^2+x+1 = 3$$

$$\Rightarrow \lim_{x\to 1} h(x) = 3$$

$\because h(1) = \lim_{x\to 1} h(x)$

$\therefore$ F is continuous at $x = 1$

حل case 0/0
بالقسمة المطولة

$$\begin{array}{r|l} & x^2 + x + 1 \\ \hline x-1 & x^3+1 \\ & -x^3 \pm x^2 \\ \hline & x^2 + 1 \\ & -x^2 \pm x \\ \hline & x + 1 \\ & -x \mp 1 \\ \hline & 0 \quad 0 \end{array}$$

$\therefore x^3+1 = (x^2+x+1)(x-1)$

Try to use

$(x^3 - a^3) =$

$(x-a)(x^2+ax+a^2)$

or 2- حل case 0/0 بطريقة أخرى تعرف بطريقة لوبيتال سوف ندرسها لاحقا

تفاضل البسط

$$\lim_{x\to 1^-} h(x) = \lim_{x\to 1} \frac{x^3-1}{x-1} = \lim_{x\to 1} \frac{3x^2}{1} = 3(1)^2 = 3$$

تفاضل المقام

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Example 2:

Discuss the continuity of the following function at the given number

$$g(x) = \begin{cases} x+1 & , x < 2 \\ 2x-1 & , x > 2 \end{cases}$$

$f(2)$ undefined.

$\therefore$ f is discontinuous at $x=2$.

f has removable discontinuity

$$h(x) = \begin{cases} x+1 & , x < 2 \\ 2x-1 & , x \geq 2 \end{cases}$$

1) $F(2) = 2(2) - 1 = 3$

2) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x+1 = 3$

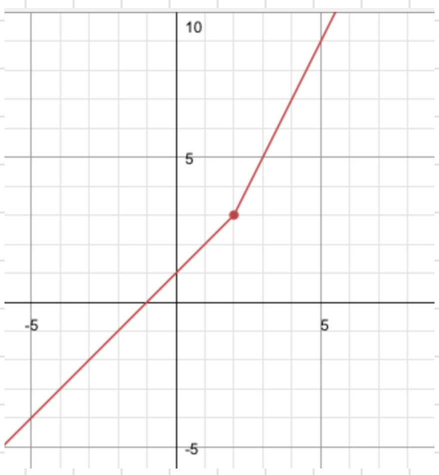
Exist and equal

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2x-1 = 3$

$\therefore \lim_{x \to 2} f(x) = 3$

3) $\lim_{x \to 2} f(x) = 3 = f(2)$

$\therefore$ F is discontinuous at $a=2$



$$f(x) = \begin{cases} x+1 & , x < 2 \\ 2x+1 & , x \geq 2 \end{cases}$$

1) $F(2) = 2(2) - 1 = 3$

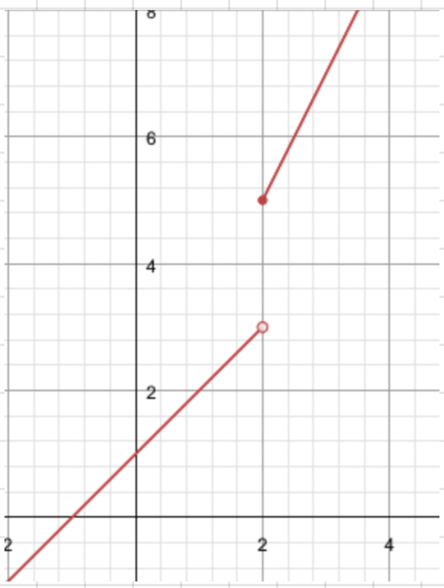
2) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x+1 = 3$

Exist but not equal

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2x+1 = 5$

$\therefore \lim_{x \to 2} f(x)$ DNE

$\therefore$ f has Jump discontinuity



Example 3: Find the value of a that makes the given function continuous

1)- $h(x) = \begin{cases} \dfrac{x^2-a^2}{x-a} & , x \neq a \\ 6 & , x = a \end{cases}$

$$h(a) = 6 = \lim_{x \to a} h(x)$$

$$= \lim_{x \to a} \frac{x^2-a^2}{x-a} \quad \left[\frac{0}{0} \text{ , factor}\right]$$

$$= \lim_{x \to a} \frac{(\cancel{x-a})(x+a)}{\cancel{x-a}}$$

$$= \lim_{x \to a} x+a = 2a$$

$\Rightarrow 6 = 2a \Rightarrow a = 3$

1)- $f(x) = \begin{cases} 5x-2 & , x \geq 2 \\ ax^2+2 & , x < 2 \end{cases}$

- f is continuous at $x=2$ iff $\lim_{x \to 2} f(x) = f(a)$

1)- $f(2) = 5(2) - 2 = 8$

2)- $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (5x-2) = 5(2) - 2 = 8$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} ax^2+2 = a(2)^2 + 2 = 4a+2$

- $\lim_{x \to 2} f(x)$ exist iff $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$

$$4a+2 = 8 \Leftrightarrow 4a = 6 \Leftrightarrow a = \frac{6}{4} = \frac{3}{2}$$