KING SAUD UNIVERSITY
PREPARATORY YEAR DEANSHIP
BASIC SCIENCE DEPARTMENT

MATH 150 FINAL EXAM / WINTER 2014-2015 DATE: 04/01/2015

INSTRUCTOR: ..... SECTION: ..... ST. NAME: .....

TIME ALLOWED: 3 Hours ST. ID: .....

⋆ *This exam consists of* **9** *essay questions pointed in two pages for a total of* **50** *marks.*

**QUESTION ONE**: Find the limit if exists.

(A) $\lim_{x\to 1} \dfrac{2x^2+5}{x^3+2}$ (B) $\lim_{x\to 3} \dfrac{x^2-9}{x^2-4x+3}$ (C) $\lim_{x\to 0} \dfrac{\tan(9x)}{\sin(6x)}$ (D) $\lim_{x\to 3^-} \dfrac{x+2}{x-3}$

(E) $\lim_{x\to 0^+} x^2 \sin\left(\dfrac{1}{x}\right)$ (Use the Squeeze Theorem).

a b c d e

(8 Marks: 1 + 2 + 1 + 2 + 2)

**QUESTION TWO:**

(A) Find the horizontal asymptotes for the function $f(x) = \dfrac{x-4x^2}{2x^2-1}$.

(B) Find the value of the constant $c$ such that the function

$$f(x) = \begin{cases} cx^2, & \text{if } x < 2 \\ 5cx - 2, & \text{if } x \geq 2 \end{cases}$$

is continuous on $(-\infty, \infty)$.

(5 Marks: 2 + 3)

**QUESTION THREE**: Find the first derivative $\dfrac{dy}{dx}$ for the following functions:

(1) $y = x^7 - 4x^{-3}$

(2) $y = \sqrt{x^3 - x + 5}$

(3) $y = \cot^2(7x)$

(4) $y = e^{x-\cos x}$

(5) $y = \log_2(x^3 + 3x - 4)$

(6) $y^2 + xy = x^3 + 2$

1 2 3 4 5 6

(11 Marks: 1 + 1 + 2 + 2 + 2 + 3)

**QUESTION FOUR:**

(A) Let $y = t^2 - t$ and $t = x^2$. Find $\dfrac{dy}{dx}$.

(B) Find the equation of the tangent line to the curve of $f(x) = x^3 - 2x + 1$ at the point $x = 1$.

(5 Marks: 2 + 3)

QUESTION FIVE: Verify that the function $f(x) = x^2 - 4x + 3$ satisfies the three hypotheses of **Rolle's Theorem** on the interval [1,3]. Then find all possible values of $c$ that satisfy the conclusion of the theorem.

(3 Marks)

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QUESTION SIX: Evaluate the following limits using **L'Hopital's rule**:

(2) $\lim_{x \to 0} \dfrac{e^x - 1}{\sin x}$

(2) $\lim_{x \to \infty} x \tan\left(\dfrac{1}{x}\right)$

(5 Marks: 2 + 3)

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QUESTION SEVEN: Given $f(x) = x^3 - 12x$. Find:

(1) The critical numbers of $f$.

(2) The intervals on which $f$ is increasing.

(3) The relative extrema of $f$.

(4) The intervals on which $f$ is concave down.

(5) The $x-$coordinates of inflection points of $f$.

1 2 3 4 5

(7 Marks: 2 + 1 + 1 + 2 + 1)

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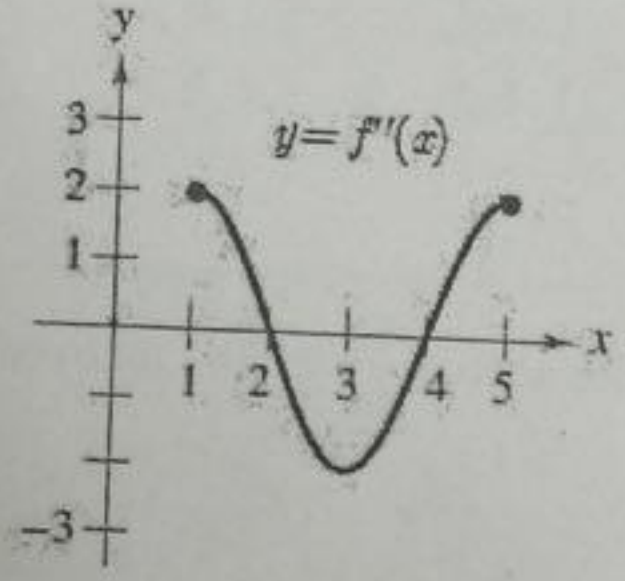
QUESTION EIGHT: Use the graph of $y = f''(x)$ below to find the following:




(1) The intervals on which $f$ is concave up.

(2) The $x-$coordinates of inflection points of $f$.

(3 Marks: 1 + 2)

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QUESTION NINE: Find two nonnegative real numbers whose sum is 12 with the property that the product of them is largest.

(3 Marks)

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GOOD LUCK