NAME:

Group Number:

244
First Midterm, March 2013

I) Choose the correct answer:

(a) If A, B, C are square matrices of the same size, then

$$(A-B)(C-A)+(C-B)(A-C)+(C-A)^2$$

equals

A-B | 0 | B-C | C-B

(b) If A and B are 3×3 invertible square matrices and

$$\det[2A^{-1}] = \det[A^3(B^{-1})^T] = -4,$$

then

det(A)=4 det(B)=4 | det(A)=-4 det(B)=4 | det(A)=2 det(B)=-2 | det(A)=-2 det(B)=2

(c) If $A^3 - 2B^T = \begin{bmatrix} 18 & -2 \\ -6 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 3 \\ 1 & 0 \end{bmatrix}$, then the matrix A is

$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} -4 & 0 \\ 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

II) Decide if the following statements are true (T) or false (F). Justify your answer.

(a) If A and B are two matrices, such that $A \cdot B = O$, then either $A = O$ or $B = O$.

T F

(b) If A and B are square matrices of the same size, such that $A + B$ is symmetric, then both A and B are symmetric.

T F

(c) If A is a $n \times n$ square matrix, $n > 1$ and $k \in \mathbb{R}$, $k \neq 0$, $k \neq \pm 1$, then $\det[kA] = k \cdot \det[A]$.

T F

III) Let $A = \begin{bmatrix} 2 & 3 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Find det$[A]$;

(b) Find adj $[A]$;

(c) Find A^{-1};

(d) Solve the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$.

IV) Find a, such that the matrix $A = \begin{bmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{bmatrix}$ is **not** invertible.