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MINISTRY OF EDUCATION

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هام

لكل المهتمين و المهتمات
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Bayes' Rule

Total Probability

Theorem of the total probability (rule of elimination)

If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A of S,

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{k} P(B_i)P(A|B_i).$$

Example:

Suppose that our sample space S is the population of adults in a small town who have completed the requirements for a college degree. We shall categorize them according to gender and employment status. The data are given in Table below

Table 2.1: Categorization of the Adults in a Small Town

	Employed	**Unemployed**	**Total**
Male	460	40	500
Female	140	260	400
Total	600	300	900

Suppose that we are now given the additional information that 36 of those employed and 12 of those unemployed are members of the Rotary Club. We wish to find the probability of the event A that the individual selected is a member of the Rotary Club.

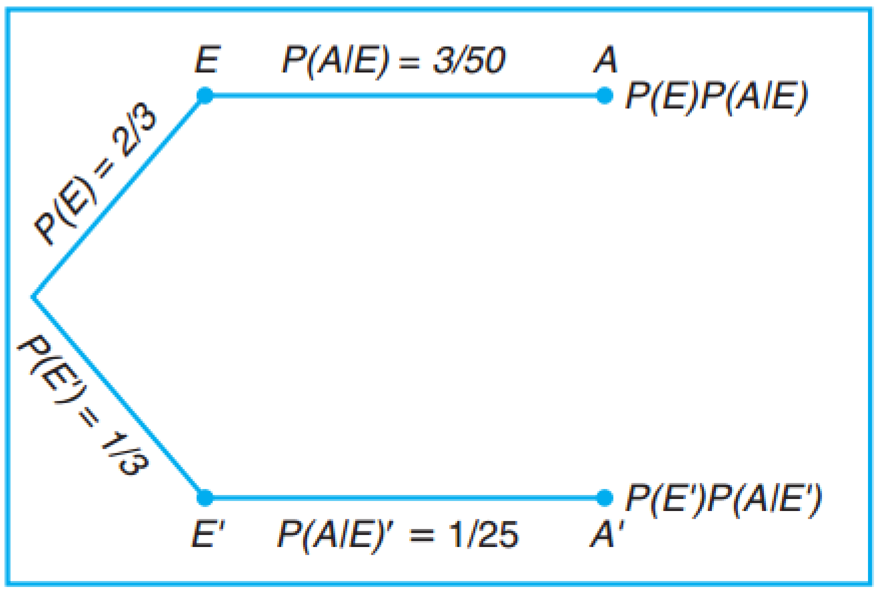
$$P(A) = P[(E \cap A) \cup (E' \cap A)] = P(E \cap A) + P(E' \cap A)$$

$$P(E) = \frac{600}{900} = \frac{2}{3}, \quad P(A|E) = \frac{36}{600} = \frac{3}{50},$$

and

$$P(E') = \frac{1}{3}, \quad P(A|E') = \frac{12}{300} = \frac{1}{25}.$$

$$P(A) = \left(\frac{2}{3}\right)\left(\frac{3}{50}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{25}\right) = \frac{4}{75}.$$


E
P(A|E) = 3/50
A
P(E) = 2/3
P(E)P(A|E)
P(E') = 1/3
P(E')P(A|E')
E'
P(A|E)' = 1/25
A'

Example 2.41: In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

***Solution*:** Consider the following events:

A: the product is defective,

B_1: the product is made by machine B_1,

B_2: the product is made by machine B_2,

B_3: the product is made by machine B_3.

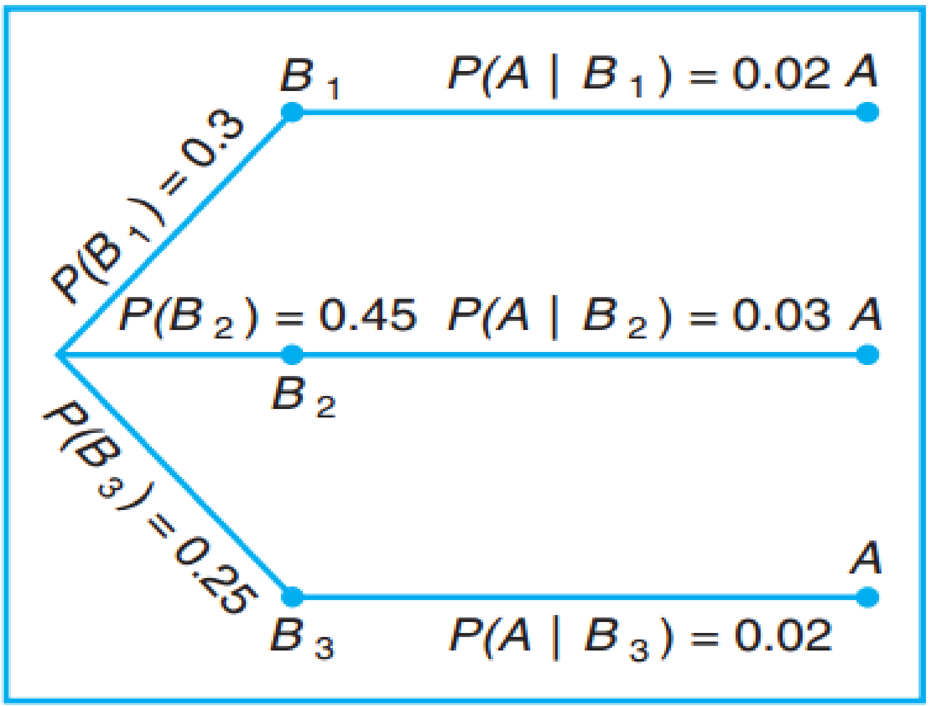
Applying the rule of elimination, we can write

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3).$$

$$P(B_1)P(A|B_1) = (0.3)(0.02) = 0.006,$$
$$P(B_2)P(A|B_2) = (0.45)(0.03) = 0.0135,$$
$$P(B_3)P(A|B_3) = (0.25)(0.02) = 0.005,$$

and hence

$$P(A) = 0.006 + 0.0135 + 0.005 = 0.0245.$$


B_1
$P(A \mid B_1) = 0.02$ A
$P(B_1) = 0.3$
$P(B_2) = 0.45$
$P(A \mid B_2) = 0.03$ A
B_2
$P(B_3) = 0.25$
A
B_3
$P(A \mid B_3) = 0.02$

Bayes' Rule

- Instead of asking for P(A) in the above example, by the rule of elimination, suppose that we now consider the problem of finding the conditional probability $P(B_i|A)$. In other words, suppose that a product was randomly selected and it is defective. What is the probability that this product was made by machine Bi? Questions of this type can be answered by using the following theorem, called Bayes' rule

Theorem:

(Bayes' Rule) If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A in S such that $P(A) \neq 0$,

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)} \quad \text{for } r = 1, 2, \ldots, k.$$

Example 2.42: With reference to Example 2.41, if a product was chosen randomly and found to be defective, what is the probability that it was made by machine B_3?

***Solution*:** Using Bayes' rule to write

$$P(B_3|A) = \frac{P(B_3)P(A|B_3)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)},$$

and then substituting the probabilities calculated in Example 2.41, we have

$$P(B_3|A) = \frac{0.005}{0.006 + 0.0135 + 0.005} = \frac{0.005}{0.0245} = \frac{10}{49}.$$

In view of the fact that a defective product was selected, this result suggests that it probably was not made by machine B_3.

Example 2.43: A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows:

$$P(D|P_1) = 0.01, \qquad P(D|P_2) = 0.03, \qquad P(D|P_3) = 0.02,$$

where $P(D|P_j)$ is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

***Solution*:** From the statement of the problem

$$P(P_1) = 0.30, \quad P(P_2) = 0.20, \quad \text{and} \quad P(P_3) = 0.50,$$

we must find $P(P_j|D)$ for $j = 1, 2, 3$. Bayes' rule (Theorem 2.14) shows

$$
\begin{aligned}
P(P_1|D) &= \frac{P(P_1)P(D|P_1)}{P(P_1)P(D|P_1) + P(P_2)P(D|P_2) + P(P_3)P(D|P_3)} \\
&= \frac{(0.30)(0.01)}{(0.3)(0.01) + (0.20)(0.03) + (0.50)(0.02)} = \frac{0.003}{0.019} = 0.158.
\end{aligned}
$$

Similarly,

$$
P(P_2|D) = \frac{(0.03)(0.20)}{0.019} = 0.316 \text{ and } P(P_3|D) = \frac{(0.02)(0.50)}{0.019} = 0.526.
$$

The conditional probability of a defect given plan 3 is the largest of the three; thus a defective for a random product is most likely the result of the use of plan 3. ◾

324 Stat
Lecture Notes

(1) Probability

(Chapter 2 of the book pg 35-71)

Definitions:

Sample Space:

Is the set of all possible outcomes of a statistical experiment, which is denoted by the symbol S •

Notes:

-Each outcome in a sample space is called an element or a member or a sample point.

- S is called the **sure event**

EX (1):

Consider the experiment of tossing a die. If we are interested in the number that shows on the top face, the sample space would be:

S= {1, 2, 3, 4, 5, 6}

If we are interested only in whether the number is even or odd, the sample space is simply:

S= {even, odd}

EX (2):

The sample space of tossing a coin **3** times:

S= {HHH, HHT, THH, HTH, HTT, TTH, THT, TTT}.

Note:

- you can use the tree diagram on pg 37 to find S in this example

EX (3):

$S = \{X | X+2=0\} = \{-2\}$.

See Ex 2.3 pg 37

Events:

An event A is a subset of a sample space ●

For instance, when a die is tossed, then the sample space is {1,2,3,4,5,6}.
If the event **A** is concerned with the elements that divisible by **3** such that

A={3,6}, then $A \subset S$.

EX (4):

The set which contains no elements at all is called the null set or the impossible event denoted by **Φ** .

Ex

B= {X|X is an even factor of 7}, then

B= **Φ= { }**

Note:

$$\Phi \subset S \,, S \subset S$$

Complement of an event A:

The complement of an event **A** with respect to **S** is the set of all elements of **S** that not in **A** denoted by

A' or A^C or $\bar{A}$.

See Ex 2.6 pg 39

EX (5):

S= {1, 2, 3, 4, 5, 6}.

Let **B** the event that has the number which greater than **3**

such that **B** = {4, 5, 6}, then B' = {1, 2, 3}.

Intersection:

The intersection of two events **A** and **B** denoted by

$$A \cap B$$

is the event containing all elements that are common to **A** and **B**.

<u>EX (6):</u>

Let M= {a, e, i, o, u ,r , s} and N={r, s, t},

then $M \cap N =$ {r, s}

Let **S**= {1,2,3,4,5,6}, **A**={1,2,3},**B**={3,6},

Then $A \cap B$ = {3}.

See Ex 2.7 pg 39

Mutually Exclusive (Disjoint) Events:

Two events **A** and **B** are mutually exclusive (disjoint) if and only if $A \cap B = \Phi$, that is if **A** and **B** have no elements in common.

See Ex 2.8 pg 40

EX (7):

S= {1, 2, 3, 4, 5, 6},

A= {1, 3, 5}, **B**= {2, 4, 6},

then $A \cap B = \Phi$

A and B are mutually exclusive or disjoint.

Union:

The union of the two events **A** and **B** denoted by

$$A \cup B$$

which is the event containing all the elements

that belong to **A** or **B** or both●

that is $A \cup B$ occurs if at least one of A or B occurs.

EX (8):

Let $M = \{X \mid 3 < X < 9\}$ $N = \{y \mid 5 < y < 12\}$

Then $M \cup N = \{z \mid 3 < z < 12\}$.

EX (9):

Let A= {a,b,c}, B={b,c,d,e}

then $A \cup B = \{a, b, c, d, e\}$

See Ex 2.11 and 2.12 pg 40

1.7 Definition:

If A contains B, then B is called a subset of A, that is $B \subset A$.

EX (10):

A= {1, 2, 3, 4}, **B**= {2, 3}. $\because B \subset A$

$$\therefore A \cap B = B, A \cup B = A, (A \cap B)^C = B^C, (A \cup B)^C = (A)^C$$

2.3 Counting Sampl Points (pg 44):

If an operation can be performed in $(\mathbf{n_1})$ ways and if for each of these a second operation can be performed in $(\mathbf{n_2})$ ways, then the two operations can be performed together in $(\mathbf{n_1 n_2})$ ways.

EX (11):

How many sample points are in the sample space when a pair of dice is thrown once?

Solution:

The first die can land in any one of $\mathbf{n_1}$=6 ways. For each of these 6 ways the second die can also land in $\mathbf{n_2}$=6 ways. Therefore, the pair of dice can land in $\mathbf{n_1 n_2}$= $(6)\times(6)$ **=36** possible ways.

EX (12):

How many lunches consisting of a soup, sandwich, dessert and a drink are possible if we can select from 4 soups, 3 kinds of sandwiches, 5 desserts and 4 drinks?

Solution:

Since $\mathbf{n_1}$=4, $\mathbf{n_2}$=3, $\mathbf{n_3}$=5, $\mathbf{n_4}$=4, then $(n_1)\ (n_2)\ (n_3)\ (n_4) = (4)\ (3)\ (5)\ (4) = 240$ different ways to choose lunch.

See Ex 2.15 and 2.16 pg 46

EX (13):

How many even three – digit numbers can be formed from the digits 1,2,5,6 and 9 if each digit can be used only once (without replacement)

3	4	2

$(n_1)(n_2)(n_3)= (2)\ (4)\ (3) =24$ even three – digit numbers.

Ex 2.17 pg 46

How many even four digit number can be formed from the digits 0,1,2,5,6 ,9 if each digit can be used only once?

3	4	5	1

or

4	3	4	2

$\downarrow$ if the unit position is 0

$\downarrow$ if the unit position is not 0

$(n_1)(n_2)(n_3)(n_4)+(n_1)(n_2)(n_3)(n_4)=$

$(1\times 5 \times 4 \times 3)+(2 \times 4 \times 4 \times 3)=60+96=165$

Definition:

A permutation is an arrangement of all or part of a set of objects ■

- The number of permutations of **n** distinct objects is **n!**.

EX (14):

How many words can be obtained using the three letters: a, b, c

Solution:

n=3, then n! =3! =6

In general n distinct objects can be arranged in

n! =n (n-1) (n-2)…3x2x1 ways.

By definition **1! =0! =1**

Permutations:

The number of permutations of **n** distinct objects taken **r** at a time is given by:

$$p_r^n = \frac{n!}{(n-r)!} \quad where \ \ n \geq r \qquad (1)$$

Where:

1. the order is important.
2. the chosen is without replacement

EX (15):

A president, treasurer and secretary all different are to be chosen from a club consisting of **10** – people. How many different choices of officers are possible?

Solution:

$$p_3^{10} = \frac{10!}{7!} = 720$$

■ See Ex 2.18 pg 48

EX (16):

How many ways can a local chapter of the American Chemical Society Schedules 3 speakers for 3 different meetings if they are available on any of 5 possible dates?

Solution:

The total number of possible schedules is:

$$p_3^5 = \frac{5!}{2!} = \frac{120}{2} = 60$$

Theorem:

The number of permutations of **n** distinct objects arranged in

a. a row is **n!**

b. a circle is **(n-1)!**

EX (17):

How many ways can **3** Arabic books, **2** Math books and **1** chemistry book arranged:

1. in a book shelf ?
2. in a rounded table?

Solution:

The number of books is **n=6** books, then:

1. **n!**=6!=720

2. (n-1)!=5!=120

Exercise:

In how many ways can 4 people sit in:

(a) a row.

(b) in a circle

(answer: 4!=24, 3!=6)

Theorem:

The number of distinct permutations of **n** things of which **n_1** are of one kind, n_2 of a second kind,…, **n_K** of a k^{th} kind is

$$\frac{n!}{n_1!\,n_2!\ldots n_K!} \qquad (2)$$

where $n = n_1 + n_2 + \ldots + n_K$

EX (18):

How many different ways can **3** red, **4** yellow and **2** blue bulbs be arranged in a string of Christmas tree light with **9** sockets?

Solution:

The total number of distinct arrangements is:

$$\frac{9!}{3!\ 4!\ 2!} = 1260$$

See Ex 2.20 pg 49

Exercise:

How many different ways can we arrange the letters in the word statistics?

Combinations:

The number of combinations of **n** distinct objects taken **r** at time is given by:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad where\ n \geq r \qquad (3)$$

where:

1. the order is not important
2. the chosen is without replacement

Note:

1. $\binom{n}{0} = 1$

2. $\binom{n}{1} = n$

3. $\binom{n}{n-1} = n$

Ex (19):

In how many ways can 5 starting positions on a basketball team be filled with 8 men who can play any of the positions?

Solution:

$$\binom{8}{5} = \frac{n!}{r!(n-r)!} = \frac{8!}{5!3!} = 56$$

Theorem:

The number of ways of partitioning a set of n objects into r cells with n_1 elements in the first cell, n_2 elements in the second , and so forth is

$$\binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{n_1!\; n_2!\; \ldots n_k!}$$

Where $n_1 + n_2 + \ldots + n_k = n$

Ex 2.21 pg 50

In how many ways can 7 graduate students be assigned to 1 triple and 2 double hotel rooms during a conference?

Solution

$$\binom{7}{3, \quad 2, \quad 2} = \frac{7!}{3!\,2!\,2!} = 210$$

Note

$$\binom{n}{r, \quad n-r} = \binom{n}{r} = \binom{n}{n-r}$$

See Ex 2022 pg 50

1.9 Probability of an Event:

The probability of an event **A**, P (A) is the sum of the weights of all sample points in **A.**

$$P(A) = \frac{n(A)}{n(S)} = \frac{number\ of\ ways\ event\ A\ occures}{total\ outcomes\ in\ S}$$

Therefore:

1. $P(\Phi) = \frac{n(\Phi)}{n(S)} = 0$

2. $P(S) = \frac{n(S)}{n(S)} = 1$

3. $0 \le P(A) \le 1,$

EX 2.24 pg 53:

A coin is tossed twice, what is the probability that at least one head occurs?

Solution:

S = {HH, HT, TH, TT},

n (A) = 3, n(S) = 4

4w=1, w=1/4,

A= {HH, HT, TH},

P (A) = n (A)/n(S) = 3/4

Ex 2.26 pg 54:

A die is loaded in such a way that an even number is twice as likely to occur as an odd number. Let **A** be the event tha even number turns up and **B** be the event that a number divisible by **3** occurs. Find $P(A \cap B)$ and $P(A \cup B)$.

Solution:

Let A: even number $\rightarrow$ A= {2, 4, 6},

B: number divisible by 3 $\rightarrow$ B= {3, 6},

That is P(1)=P(3)=P(5)= w , P(2)=P(4)=P(6)=2w. Then

3w+6w=9w=1, w =1/9.

$A \cap B = \{6\}$, $A \cup B = \{2,4,6,3\}$, $P(A \cap B)$ =2/9 , $P(A \cup B)$ =7/9

See Ex 2.25 pg 53

<u>Theorem:</u>

If an experiment can result in any one of **N** different equally likely outcomes and if exactly **n** of these outcomes correspond to event **A**, then the probability of an event **A** is:

$$P(A) = \frac{n}{N} = \frac{n(A)}{n(S)} \qquad (4)$$

where n(A) is the number of outcomes that satisfy the event A and n(S) is the total outcomes in the sample space S.

EX (25):

A mixture of candies has **6** mints, **4** toffees and **3** chocolates. If a person makes a random selection of one of these candies, find the probability of getting:

(a) a mint .

(b) a toffee or a chocolate.

Solution:

n (S)=13, n(M)=6, n(T)=4, n(C)=3

Let **M**, **T** and **C** represent the events that the person selects respectively a mint, a toffee or chocolate candy:

$$P(M) = \frac{n(M)}{n(S)} = \frac{6}{13}\ ,\ P(T) = \frac{n(T)}{n(S)} = \frac{4}{13}\ ,\ P(C) = \frac{n(C)}{n(S)} = \frac{3}{13}$$

a. P (getting a mint) = $P(M) = \frac{6}{13}$

b. P(getting a toffee or a chocolate) =

$$P(T\ or\ C) = P(T \cup C) = P(T) + P(C) = \frac{4}{13} + \frac{3}{13} = \frac{7}{13}$$

See Ex 2.27 pg 54

2.5 Additive Rule:

Theorem:

If **A** and **B** are any two events, then:

$$P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B) \qquad (5)$$

Corollary:

If **A** and **B** are mutually exclusive, then:

$$P(A \bigcup B) = P(A) + P(B) \qquad (6)$$

Since $A \bigcap B = \Phi$, P (Φ) =0

Corollary:

If A_1, A_2… A_n are mutually exclusive (disjoint), then

$$P(A_1 \cup A_2 \cup \ldots A_n) = P(A_1) + P(A_2) + \ldots + P(A_n) = P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) \qquad (7)$$

Corollary:

$$P(A_1 \cup A_2 \cup \ldots A_n) = P(A_1) + P(A_2) + \ldots + P(A_n) = P(S) = 1 \qquad (8)$$

EX (26):

The probability that Paula passes mathematics is 2/3 and the probability that she passes English is 4/9. If the probability of passing both courses is 1/4, what is the probability that Paula will pass at least one of these courses?

Solution:

Let M be the event "passing mathematics" and E be the event "passing English", then

$$P(M) = 2/3 \ , \ P(E) = 4/9 \ , \ P(M \cap E) = 1/4$$

$$P(M \cup E) = P(M) + P(E) - P(M \cap E) = \frac{2}{3} + \frac{4}{9} - \frac{1}{4} = 31/36$$

<u>Ex 2.30 pg 57</u>

What is the probability of getting a total of **7**or **11**when a pair of dice are tossed?

<u>Solution:</u>

Let A be the event that **7**occures and B the event **11** comes up, then:

A={(1,6),(6,1),(3,4),(4,3),(2,5),(5,2)}

B= {(5,6),(6,5)}

$\because A \cap B = \Phi, P(A) = 6/36, P(B) = 2/36, P(A \cap B) = 0$,

$\therefore P(A \cup B) = P(A) + P(B) = 6/36 + 2/36 = 8/36$

See Ex2.29 & 2.31 pg 57-58

Theorem:

If A and A' are complementary events, then

$$P(A) + P(A') = 1$$

$$\Rightarrow P(A') = 1 - P(A) \qquad (9)$$

EX (2.23 pg 58):

If the probabilities that an automobile mechanic will service **3,4,5,6,7,8** cars on any given working day are respectively **0.12,0.19,0.28,0.24,0.1** and **0.07**, what is the probability that will service at least **5** cars on his next day at work?

Solution:

Let E be the event that at least 5 cars are serviced,

then

$$P(E) = P(X \geq 5) = P(5) + P(6) + P(7) + P(8),$$

$$P(E) = 1 - P(E')$$

$$P(E') = P(X = 3 \; or \; 4) = .12 + .19 = .31,$$

$$P(E) = 1 - P(E') = 1 - .31 = .69$$

Conditional Probability:

The conditional probability of occurring an event B when knowing that an event A is happened, that is B given **A** denoted by P (B|A) which is defined by:

$$P(B|A)=\frac{P(B\bigcap A)}{P(A)}=\frac{n(B\bigcap A)}{n(A)}=\frac{n(B\bigcap A)/n(S)}{n(A)/n(S)} \quad if \quad P(A)>0$$

$$P(A|B)=\frac{P(A\bigcap B)}{P(B)}=\frac{n(A\bigcap B)}{n(B)}=\frac{n(A\bigcap B)/n(S)}{n(B)/n(S)} \quad if \quad P(B)>0 \qquad (9)$$

EX (29):

	(E)Employed	(E^C) Unemployed	Total
(M) Male		40	
(M^C) Female	140		400
Total	600	300	900

Complete the table, then answer the questions:

What is the probability of:

1. getting a male
2. getting a male given he is an employed
3. getting an unemployed female
4. getting an employed or male

Solution:

1. $$P(M)=\frac{n(M)}{n(S)}=\frac{500}{900}=.555$$

2. $$P(M\mid E)=\frac{P(M\cap E)}{P(E)}=\frac{n(M\cap E)}{n(E)}=\frac{460/900}{600/900}=460/600=.766$$

3. $$P(E^C\cap M^C)=260/900=0.289$$

4. $$\begin{aligned}P(E\cup M)&=P(E)+P(M)-P(E\cap M)\\&=600/900+500/900-460/900\\&=640/900=0.711\end{aligned}$$

See Ex 2.34 pg 63

Independent Events:

Two events **A** and **B** are independent if and only if:

$P(B|A) = P(B)$ and $P(A|B) = P(A)$

Theorem:

Two events **A** and **B** are independent if and only if:

$$P(A \cap B) = P(A)P(B)$$

EX (30):

Let A and B are independent events as follows:

$P(A) = 0.5$, $P(B) = 0.6$, find $P(A|B)$, $P(B|A)$.

Solution:

$\because$ A and B are independent

$\therefore P(A|B) = P(A) = 0.5$, $P(B|A) = P(B) = 0.6$

Multiplicative Rules:

Theorem:

If in experiment the events **A** and **B** can both occur**,**

then

$$P(A \cap B) = P(A)P(B|A) \quad or \quad P(A \cap B) = P(B)P(A|B)$$

EX 2.37 pg 66

One bag contains **4** white balls and **3** black balls and a second bag contains **3** white balls and **5** black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?

Solution:

Let $B_1, B_2 \; and \, W_1$ represent respectively the drawing of a black ball from bag **1**, a black ball from bag **2** and a white ball from bag **1**. We are interested in the union of $B_1 \cap B_2$ and $W_1 \cap B_2$.

$$P[(B_1 \cap B_2) \cup P(W_1 \cap B_2)] = P(B_1 \cap B_2) + P(W_1 \cap B_2)$$

$$= P(B_1)P(B_2 \mid B_1) + P(W_1)P(B_2 \mid W_1) = (3/7)(6/9) + (4/7)(5/9) = 38/63$$

EX (32):

If A and B are independent events, $P(A)=0.4, P(A\cup B)=0.6$,

find $P(B)$.

Solution:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A)P(B)$$

$$0.6 \quad = 0.4 + P(B) - 0.4P(A)$$

$$0.2 \quad = 0.6P(B) \Rightarrow P(B) = 0.2/0.6 = 0.33$$

Theorem:

If in an experiment the events $A_1 \ldots A_K$ can occur,

then:

$$P(A_1 \cap \ldots \cap A_K) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_2 \cap A_1)\ldots P(A_K \mid A_1 \cap \ldots \cap A_{K-1})$$

If $A_1 \ldots A_K$ are independent, then

$$P(A_1 \cap \ldots \cap A_K) = P(A_1)P(A_2)P(A_3)\ldots P(A_K)$$

EX (33):

A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed **3** times, what is the probability of getting **(2)** tails and **(1)** head?

Solution:

H : T

2w : w

$2w+w=3w=1 \Rightarrow w=1/3$

P (H) =2/3, P (T) =1/3

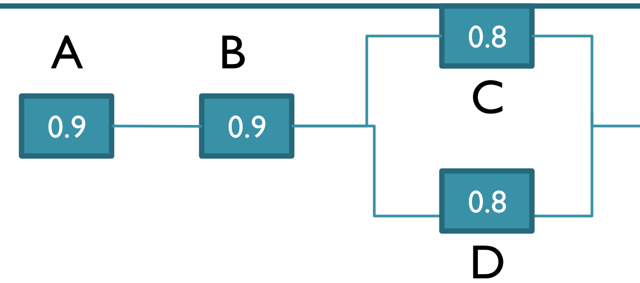
S= {HHH, HHT, THH, HTH, HTT, TTH, THT, TTT}

P (H) = 2/3, P (T) = 1/3

A = {TTH, HTT, THT}

P (TTH) = (1/3) (2/3) (1/3) =2/27

P (A) =3(2/27) =6/27



Ex 2.39 pg 67

Find the probability that

a) The entire system works? b) the component C does not work given that the entire system work?

If the 4 components work independently

P (entire system works) = $P[A \cap B \cap (C \cup D)] = P(A)\ P(B)\ P(CUD) =$
$(0.9)\ (0.9)[\ (0.8) + (0.8) - (0.8)\ (0.8)\ ] = 0.7779$

Or P (entire system works)= $P[A \cap B \cap (C \cup D)] = P(A)\ P(B)\ [1-P(\ \bar{C} \cap \bar{D})]\ \ )$

b) P (C does not work| the entire system work)=$P(A \cap B \cap \bar{C} \cap D)$ / P (entire system works)

$= (0.9)\ (0.9)[\ 1-(0.8)]\ (0.8)\ /\ 0.7776\ \ =0.1667$

EX (21): (Reading)

A box contains **4** white balls, **2** red balls and **3** green balls. Two balls are drawn without replacement, find the probability that:

1. the two balls are white
2. the two balls are red
3. the two balls are green
4. one ball is red and one ball is white
5. the two balls are not green
6. the two balls are the same colour

Solution:

1. $$P(two\ balls\ are\ white) = \frac{\binom{4}{2}\binom{5}{0}}{\binom{9}{2}} = \frac{(6)(1)}{(36)} = 0.1667$$

2. $$P(two\ balls\ are\ red) = \frac{\binom{2}{2}\binom{7}{0}}{\binom{9}{2}} = \frac{(1)}{(36)} = 0.028$$

3. $$P(\text{two balls are green}) = \frac{\binom{3}{2}\binom{6}{0}}{\binom{9}{2}} = \frac{(3)(1)}{(36)} = 0.083$$

4. $$P(\text{one ball is red and one ball is white}) = \frac{\binom{4}{1}\binom{2}{1}\binom{3}{0}}{\binom{9}{2}} = \frac{(4)(2)(1)}{(36)} = 0.222$$

5. $$P(\text{the two balls are not green}) = \frac{\binom{2}{2}\binom{4}{0}\binom{3}{0}}{\binom{9}{2}} + \frac{\binom{4}{2}\binom{2}{0}\binom{3}{0}}{\binom{9}{2}} = 0.028 + 0.167 = 0.195$$

6. $$P(\text{the two balls are the same colour}) = \frac{\binom{4}{2}\binom{5}{0}}{\binom{9}{2}} + \frac{\binom{2}{2}\binom{7}{0}}{\binom{9}{2}} + \frac{\binom{3}{2}\binom{6}{0}}{\binom{9}{2}}$$

$$= 0.25 + 0.028 + 0.083 = 0.361$$

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(2) Random Variable and probability Distribution

(Chapter 3 of the book pg 81-94)

2.1 Definition: A Random Variable:

A random variable is a function that associates a real number with each element in the sample space.

<u>EX (3.1 pg 82):</u>

Two balls are drawn in succession without replacement from an urn containing **4** red balls and **3** black balls.

The possible outcomes and the values **y** of the random variable **Y** where is the number of red balls, are

Sample space	y
RR	2
RB	1
BR	1
BB	0

Y = number of red balls

Possible values of the random variable is y = 0,1,2

Definition: Discrete Sample Space:

If a sample space contains a finite number **n** of different values $x_1, x_2, \ldots, x_n$ or countably infinite number of different values $x_1, x_2, \ldots$ it is called a discrete sample space.

Examples of discrete random variables are;

*The number of bacteria per unit area in the study of drug control on bacterial growth.

* The number of defective television sets in a shipment of **100**.

Definition: Continuous Sample Space:

If **X** can take an infinite number of possibilities equal to the number of points on a line segment, then **X** has a continuous sample space. Examples of the continuous random variable; heights, weights, temperature, distances or life periods

`Discrete Probability Distributions:

Definition:

The set of ordered pairs **(x, f(x))** is a probability function, probability mass function or probability distribution of the discrete random variable **X** if for each possible outcome **x,**

1. $f(x) \geq 0$

2. $\sum_{\forall x} f(x) = 1$

3. $P(X = x) = f(x)$

EX (2):

A shipment of **8** similar microcomputers to a retail outlet contains **3** that are defective. If a school makes a random purchase of **2** of these computers, let **X**= # of defective in the sample. find:

1. the different values of r.v. X
2. the probability distribution of x ; f(x)
3. $P(1 \le x \le 2), P(x \le 1), P(0 < x \le 2), f(2), f(5)$

Solution:

(1)The possible values of X is: **X** =0, 1, 2

(2) $$f(0) = P(X = 0) = \frac{\binom{3}{0}\binom{5}{2}}{\binom{8}{2}} = \frac{10}{28}$$

$$f(1) = P(X = 1) = \frac{\binom{3}{1}\binom{5}{1}}{\binom{8}{2}} = \frac{15}{28}$$

$$f(2) = P(X = 2) = \frac{\binom{3}{2}\binom{5}{0}}{\binom{8}{2}} = \frac{3}{28}$$

X	0	1	2
f(x)	10/28	15/28	3/28

(3) $$P(1 \le x \le 2) = P(x = 1) + P(x = 2)$$

$$= 15/28 + 3/28 = 18/28$$

$$P(x \le 1) = P(x = 0) + P(x = 1)$$

$$= 10/28 + 15/28 = 25/28$$

$$P(0 < x \le 2) = P(x = 1) + P(x = 2) = 18/28$$

$$f(2) = P(x = 2) = 15/28$$

$$f(5) = P(x = 5) = 0$$

See Ex 3.8 pg 84

2.5 Definition: The Cumulative distribution Function:

The cumulative distribution function, denoted by **F(x)** of a discrete random variable **X** with probability distribution **f(x)** is given by:

$$F(x) = P(X \le x) = \sum_{X<x} f(x) \qquad for \ -\infty < x < \infty \qquad (1)$$

*For example $F(2) = P(x \le 2)$

$$P(a \le X \le b) = F(b) - F(a-1) \qquad (2)$$

* For example $F(3 \le x \le 7) = F(7) - F(2)$

EX (3):

For the given data find :(a) F (1)   (b) $P(1 \leq x \leq 2)$

X	0	1	2
f(x)	10/28	15/28	3/28

Solution:

X	0	1	2
f(x)	10/28	15/28	3/28
F(x)	10/28	25/28	28/28

See Ex 3.10

a. F (1) = 25/28

b. $P(1 \leq X \leq 2) = F(2) - F(0) = 28/28 - 10/28 = 18/28$

2.6 Continuous Probability Distributions:

The function **f(x)** is a probability density function for the continuous random variable **X** defined over the set of real numbers **R**, if:

1. $f(x) \geq 0 \qquad for\ all\ \ x \in R$

2. $$\int_{-\infty}^{\infty} f(x)\ dx = 1$$

3. $$P(a < X < b) = \int_{a}^{b} f(x)\,dx$$

EX (3.11) pg 89:

Suppose that the error in the reaction temperature in ${}^{0}C$ for a controlled laboratory experiment is a continuous random variable **X** having the probability density function:

$$f(x) = \frac{x^2}{3} \quad , \quad -1 < x < 2$$
$$= 0 \qquad otherwise$$

a. Show that $\int_{-\infty}^{\infty} f(x)\,dx = 1$ b. Find $P(0 < X \le 1)$

c. find $P(0 < x < 3), P(x = 2), F(x), F(0.5)$

Solution:

a. $$\int_{-1}^{2} \frac{x^2}{3}\, dx = \frac{x^3}{9}\Big|_{-1}^{2} = \frac{1}{9}[8-(-1)^3] = \frac{8+1}{9} = 1$$

b. $$P(0 < X \le 1) = \int_{0}^{1} \frac{x^2}{3} dx = \frac{x^3}{9}\Big|_{0}^{1} = \frac{1}{9}$$

c. $$P(0 < x < 3) = \int_{0}^{2} \frac{x^2}{3} dx = \frac{x^3}{9}\Big|_{0}^{2} = \frac{(2)^3}{9} = 8/9$$

$$P(x = 2) = 0$$

$$F(x) = \int_{-1}^{x} \frac{x^2}{3} dx = \frac{x^3}{9}\Big|_{-1}^{x} = \frac{x^3-(-1)^3}{9} = \frac{x^3+1}{9}$$

$$F(0.5) = \frac{(0.5)^3+1}{9} = 0.139$$

2.7 Definition:

The cumulative distribution **F(x)** of a continuous random variable **X** with density function **f(x)** is given by:

$$P(x) = P(X \le x) = \int_{-\infty}^{X} x\,dx \quad for \;\; -\infty < x < \infty$$

$$P(a \le X \le b) = P(a < X < b) = F(b) - F(a)$$

Ex 3.12 pg 90

For example (4): find $P(0 < x < 1)$

$$\because F(x) = \frac{x^3 + 1}{9}$$

$$\therefore P(0 < x < 1) = F(1) - F(0) = \frac{1+1}{9} - \frac{0+1}{9} = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

Therefore

$$F(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x^3+1}{9} & -1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

See Ex 3.13 pg 90

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(3) Mathematical Expectation

(Book: Chapter 4 ,pg 111-137)

Mean of a Random Variable:

Definition:

Let **X** be a random variable with probability distribution **f(x)**.

The mean or expected value of **X** is:

$$\mu = E(X) = \sum_{\forall X} X\ f(X) \qquad if\ \ X\ \ is\ \ \ discrete$$

$$\mu = E(X) = \int_{-\infty}^{\infty} X\ f(X)\, dX \qquad if\ \ X\ \ is\ \ continuous \qquad (1)$$

Properties of the Expectation:

1. $E(a) = \mathbf{a}$, where **a** is a constant
2. $E(aX) = a\,E(X)$
3. $E(aX + b) = a\,E(X) + b$

<u>Ex (1):</u>

Find the expected number of chemists on a committee of **3** selected at random from **4** chemists and **3** biologists.

Find: E (5), E (3x), E (2x-1)

Solution:

Let **X** represent the number of chemists on the committee.

The probability distribution of **X** is given by:

$$f(x) = \frac{\binom{4}{x}\binom{3}{3-x}}{\binom{7}{3}} \quad , \quad X = 0,1,2,3$$

$$f(0)=\frac{\binom{4}{0}\binom{3}{3}}{\binom{7}{3}}=\frac{1}{35}\quad ,\qquad f(1)=\frac{\binom{4}{1}\binom{3}{2}}{\binom{7}{3}}=\frac{12}{35}\quad ,$$

$$f(2)=\frac{\binom{4}{2}\binom{3}{1}}{\binom{7}{3}}=\frac{18}{35}\quad ,\qquad f(3)=\frac{\binom{4}{3}\binom{3}{0}}{\binom{7}{3}}=\frac{4}{35}$$

X	0	1	2	3	$\sum$
f(x)	1/35	12/35	18/35	4/35	1
x f(x)	0	12/35	36/35	12/35	60/35=1.71

$$E(X) = \mu_X = \sum x f(x) = 60/35 = 1.71$$

$$E(5) = 5$$

See Ex 4.1 pg 113

$$E(3x) = 3E(x) = 3(60/35) = 5.143$$

$$E(2x - 1) = 2E(x) - 1 = 2(60/35) - 1 = 2.429$$

Ex 4.3 pg114:

Let **X** be a random variable that denotes the life in hours of a certain electronic device. The probability density function is given by:

$$f(x) = \begin{cases} \dfrac{20000}{X^3} & , \quad X > 100 \\ 0 & \quad otherwise \end{cases}$$

Find the expected life of this type of device

Solution:

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{100}^{\infty} X\left(\frac{20000}{X^{3}}\right) dX$$

$$= \int_{100}^{\infty} \frac{2000}{X^{2}} dX = 20000 \int_{100}^{\infty} X^{-2} dX$$

$$= 20000\left(\frac{X^{-1}}{-1}\right)_{100} \Big|^{\infty} = 20000[(100)^{-1} - (\infty)^{-1}]$$

$$= \frac{20000}{100} - \frac{20000}{\infty} = 200 - 0 = 200$$

EX 4.4 pg 115:

Suppose that the number of cars **X** that pass through a car wash between **4** P.M. and **5** P.M. on any sunny Friday has the following probability distribution:

X	4	5	6	7	8	9
f(X)	1/12	1/12	1/4	1/4	1/6	1/6

Let g(x) = 2 x-1 represent the amount of money in dollars, paid to the attendant by the manager. Find the attendant's expected earning for this particular time period.

Solution:

X	4	5	6	7	8	9	∑
f(X)	1/12	1/12	1/4	1/4	1/6	1/6	1
X f(x)	4/12	5/12	6/4	7/4	8/6	9/6	164/24

$$E(g(x)) = E(2x-1) = 2E(X)-1 =$$

$$= 2\left(\frac{164}{24}\right)-1 = 12.67$$

Ex (4.5 pg 115):

Let **X** be a random variable with density function:

$$f(x) = \begin{cases} \dfrac{X^2}{3}, & -1 < x < 2 \\ 0 & otherwise \end{cases}$$

Find the expected value of g(x) = 4x+3

Solution:

$$E(X)=\int_{-1}^{2} x(\frac{x^2}{3})\,dx=\int_{-1}^{2}(\frac{x^3}{3})dx=(\frac{x^4}{12})_{-1}\,|^2$$

$$=\frac{1}{12}[2^4-(-1)^4]=\frac{1}{12}(16-1)=\frac{15}{12}$$

$$E(g(x))=E(4x+3)=4E(X)+3=4(\frac{15}{12})+3=8$$

Variance:

Definition:

Let **X** be a random variable with probability distribution **f(x)** and mean μ. The variance of **X** is denoted by V(x) or σ_X^2 :

$$V(x) = \sigma^2 = E(x-\mu)^2 = \sum_{\forall x}(x-\mu)^2 f(x) = E(X^2) - (E(X))^2 \quad if\ x\ is\ discrete\ (2)$$

$$V(x) = \sigma^2 = E(x-\mu)^2 = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx = E(X^2) - (E(X))^2\ if\ x\ is\ continuous\ (3$$

where:

$$E(x^2) = \begin{Bmatrix} \sum x^2 f(x) & if\ x\ is\ discrete \\ \int_{-\infty}^{\infty} x^2 f(x)dx & if\ x\ is\ continuous \end{Bmatrix}$$

Properties of the variance:

1. $V(a)=0$ where **a** is a constant

2. $V(aX)=a^2V(X)$

3. $V(aX+b)=a^2V(X)+0$

The Standard Deviation:

The positive square root of the variance, σ is called the standard deviation of **X** which is given by:

$$\sigma_X=\sqrt{V(x)}=\sqrt{E(x-\mu_X)^2}$$

Ex (4.8 pg 120):

The probability distribution for company **A** is given by:

X	1	2	3
f(x)	0.3	0.4	0.3

and for company **B** is given by:

Y	0	1	2	3	4
f(y)	0.2	0.1	0.3	0.3	0.1

Show that the variance of the probability distribution for company **B** is greater than that of company **A**.

Solution:

X	1	2	3	$\sum$
f(x)	0.3	0.4	0.3	1
x f(x)	0.3	0.8	0.9	2
f(x)x^2	0.3	1.6	2.7	4.6

$$\sigma^2 = E(x^2) - (E(x))^2 = 4.6 - 4 = 0.6, \sigma = .77$$

Y	0	1	2	3	4	$\sum$
f(y)	0.2	0.1	0.3	0.3	0.1	1
Y f(y)	0	0.1.	0.6	0.9	0.4	2
y^2 f(y)	0	0.1	1.2	2.7	1.6	5.6

$$\sigma^2 = E(y^2) - (E(y))^2 = 5.6 - 4 = 1.6, \sigma = 1.26$$

Note that σ_y^2 is greater than σ_x^2.

Ex (4.10 pg 121):

The weekly demand for a drinking-water product, in thousands of liters from a local chain of efficiency stores having the probability density:

$$f(x) = \begin{cases} 2(x-1) \quad , & 1 < X < 2 \\ 0 & otherwise \end{cases}$$

Find the mean and variance of x .

Solution:

$$\mu = \int_1^2 2x(x-1)\ dx = 2\int_1^2 (x^2 - x)\ dx = 2\left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_1^2 = 2[(\frac{8}{3} - 2) - (\frac{1}{3} - \frac{1}{2})]$$

$$= 2\left(\frac{8-6}{3} - \frac{2-3}{6}\right) = 2\left(\frac{2}{3} + \frac{1}{6}\right) = \frac{5}{3}$$

$$E(X^2) = \int_1^2 2x^2(x-1)\,dx = 2\int_1^2 (x^3 - x^2)\,dx = 2\left(\frac{x^4}{4} - \frac{x^3}{3}\right)_1 |^2$$

$$= 2[(4 - \frac{2}{8}) - (\frac{1}{4} - \frac{1}{3})] = 17/6$$

$$\sigma^2 = E(x^2) - (E(x))^2 = \frac{17}{6} - (\frac{5}{3})^2 = 1/18$$

<u>Ex 4.18 pg 129:</u>

Let **X** be a random variable having the density function:

$$f(x) = \begin{cases} \dfrac{x^2}{3} & , \quad -1 < x < 2 \\ 0 & \quad otherwise \end{cases}$$

Find the variance of the random variable **<u>g(x) = 4x+3</u>**.

Solution:

$$V(g(x)) = V(4x+3) = 16\,V(x) = 16[E(x^2)-(E(x))^2]$$

$$E(x) = \int_{-1}^{2} X\frac{X^2}{3}dx = \frac{X^4}{12}\Big|_{-1}^{2} = \frac{16}{12} - \frac{1}{12} = \frac{15}{12} = \frac{5}{4}$$

$$E(x^2) = \int_{-1}^{2} x^2\left(\frac{x^2}{3}\right)dx = \frac{x^5}{15}\Big|_{-1}^{2} = \frac{32}{15} - \frac{(-1)}{15} = \frac{11}{5}$$

$$V(x) = E(x^2) - (E(x))^2 = \frac{11}{5} - \left(\frac{5}{4}\right)^2 = \frac{11}{5} - \frac{25}{16} = \frac{176-125}{80} = 0.6375$$

$$V(g(x)) = V(4x+3) = 16V(x) + 0 = 16(0.6375) = 10.2$$

4.3 Means and Variance of Linear Combinations of Random Variables (pg 128):

The expected value of the sum or difference of two or more functions of a random variable **X** is the sum or difference of the expected values of the functions. That is

$$E(g(x) \pm h(x)) = E(g(x)) \pm E(h(x)) \qquad (6)$$

Ex4.19 pg 129 :

Let **X** be a random variable with probability distribution as follows:

X	0	1	2	3
f(x)	1/3	1/2	0	1/6

Find the expected value of $y=(x-1)^2$.

Solution:

$$E(y) = E(x-1)^2 = E(x^2 - 2x + 1) = E(x^2) - 2E(x) + 1$$

X	0	1	2	3	$\sum$
f(x)	1/3	1/2	0	1/6	1
X f(x)	0	1/2	0	3/6	1
X^2 f(x)	0	1/2	0	9/6	2

$$E(y) = 2 - 2(1) + 1 = 1$$

See ex 4.17 pg 128

<u>Ex 4.20 pg 130:</u>

Find the expected value for $g(x) = x^2+x-2$,

where **X** has the density function:

$$f(x) = \begin{cases} 2(x-1) & , \quad 1 < x < 2 \\ 0 & \quad otherwise \end{cases}$$

Solution:

$$E(x)=\int_1^2 x\,2(x-1)\,dx=2\int_1^2 (x^2-x)dx=2\left(\frac{x^3}{3}-\frac{x^2}{2}\right)_1\Big|^2=$$

$$=2\left(\frac{2}{3}+\frac{1}{6}\right)=\frac{5}{3}$$

$$E(x^2)=\int_1^2 2x^2(x-1)\,dx=2\int_1^2 (x^3-x^2)\,dx=2\left(\frac{x^4}{4}-\frac{x^3}{3}\right)_1\Big|^2$$

$$=2\left[\left(4-\frac{8}{3}\right)-\left(\frac{1}{4}-\frac{1}{3}\right)\right]=2\left(\frac{12-8}{3}-\frac{3-4}{12}\right)=2\left(\frac{4}{3}+\frac{1}{12}\right)=\frac{17}{6}$$

$$E(x^2+x-2)=E(x^2)+E(x)-2=\frac{17}{6}+\frac{5}{3}-2=\frac{5}{2}$$

4.4 Chebyshev's Theorem (pg 135):

The probability that any random variable **X** will assume a value within **K** standard deviations of the mean μ_X is at least $(1-\frac{1}{K^2})$.

That is:

$$P(\mu - K\sigma < X < \mu + K\sigma) \geq 1 - \frac{1}{K^2} \qquad (7)$$

Ex (4.27 pg 137):

A random variable **X** has a mean $\mu = 8$, a variance $\sigma^2 = 9$ and an unknown probability distribution. Find:

(a) $P(-4 < X < 20)$

(b) $P(|X - 8| \geq 6)$

Solution:

$$(a)\,P(-4 < X < 20) = P[8-(k)(3) < X < 8+(k)(3)] \rightarrow$$

$$8-3k = -4 \rightarrow 8+4 = 3k \rightarrow 12 = 3k \rightarrow k = 4$$

$$P(-4 < X < 20) \ge 1-\frac{1}{16} \rightarrow P(-4 < X < 20) \ge \frac{15}{16}$$

$$(b)\,P(|X-8| \ge 6) = 1-P(|x-8| < 6) = 1-P(-6 < (X-8) < 6)$$

$$= 1-P(-6+8 < X < 6+8) = 1-P(2 < X < 14)$$

$$1-P(2 < X < 14) \ge 1-\frac{1}{k^2} \rightarrow 1-1+\frac{1}{k^2} \ge P(2 < X < 14)$$

$$\rightarrow \frac{1}{k^2} \ge P(2 < X < 14) \rightarrow P(2 < X < 14) \le \frac{1}{k^2}$$

$$2 = 8-3K \rightarrow 3K = 8-2 = 6 \rightarrow K = 2 \quad or$$

$$14 = 8+3k \rightarrow 14-8 = 3K \rightarrow 6 = 3K \rightarrow K = 2$$

$$P(2 < x < 14) \le \frac{1}{4}$$

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(4) Some Discrete Probability Distributions

(Book: Chapter 5 ,pg 143)

4.1 Discrete Uniform Distribution:

Discrete Uniform is not in the book, it should be studied from the notes

- If the random variable **X** assume the values with equal probabilities, then the discrete uniform distribution is given by:

$$P(X,K)=\frac{1}{K} \quad , \quad X=x_1,x_2,\ldots,x_K$$
$$= 0 \qquad elsewhere \qquad (1)$$

Theorem:

- The mean and variance of the discrete uniform distribution $P(X, K)$ are:

$$\mu = \frac{\sum_{i=1}^{K} X_i}{K} \quad , \quad \sigma^2 = \frac{\sum_{i=1}^{K} (X-\mu)^2}{K} \qquad (2)$$

EX (1):

- When a die is tossed once, each element of the sample space $S = \{1,2,3,4,5,6\}$ occurs with probability 1/6. Therefore we have a uniform distribution with:

$$P(X,6) = \frac{1}{6}, \; X = 1,2,3,4,5,6$$

- Find:
- 1. $P(1 \le x < 4)$
- 2. $P(x < 3)$
- 3. $P(3 < x < 6)$

Solution:

- 1. $P(1 \le x < 4)$

$$= P(x = 1) + P(x = 2) + P(x = 3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

- 2. $P(x < 3) = P(x = 1) + P(x = 2) = \frac{2}{6}$

- 3. $P(3 < x < 6) = P(x = 4) + P(x = 5) = \frac{2}{6}$

EX (2):

- For example (1): Find μ *and* σ^2

Solution:

$$\mu = \frac{\sum_{i=1}^{k} X_i}{k} = \frac{1+2+3+4+5+6}{6} = 3.5$$

$$\sigma^2 = \frac{\sum_{i=1}^{k} (X_i - \mu)^2}{k}$$

$$= \frac{(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2}{6} = \frac{35}{12}$$

The Bernoulli Process:

- Bernoulli trials are an experiment with:

1) Only two possible outcomes.
2) Labeled as success (S), failure (F).
3) Probability of success $= p$, probability of failure $= q = 1 - p$ $(p + q = 1)$.

Binomial Distribution:

- * The **Binomial** trials must possess the following properties:

1) The experiment consists of **n** repeated trials.
2) Each trial results in an outcome that may be classified as a success or a failure.
3) The probability of success denoted by **p** remains constant from trial to trial.
4) The repeated trials are independent.
5) The parameters of binomial are n,p.

Binomial Distribution:

- The probability distribution of the binomial random variable **X**, the number of successes in **n** independent trials is:

$$b(x, n, p) = \binom{n}{x} p^{X} q^{n-X} \quad , \quad x = 0,1,2,\ldots.,n \qquad (5)$$

Theorem:

- The mean and the variance of the **binomial** distribution b(x, n, p) are:

$$\mu = np \qquad and \qquad \sigma^2 = npq \tag{6}$$

EX (4):

- According to a survey by the Administrative Management society, **1/3** of U.S. companies give employees four weeks of vacation after they have been with the company for **15** years. Find the probability that among **6** companies surveyed at random, the number that gives employees **4** weeks of vacation after **15** years of employment is:

a) anywhere from **2** to **5**;

b) fewer than **3**;

c) at most 1;

d) at least **5**;

e) greater than **2**;

f) calculate μ *and* σ^2

Solution:

$$n=6, p=1/3, q=2/3$$

$$(a)\,P(2\le x\le 5)=P(x=2)+P(x=3)+P(x=4)+P(x=5)$$

$$=\binom{6}{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{4}+\binom{6}{3}\left(\frac{1}{3}\right)^{3}\left(\frac{2}{3}\right)^{3}+\binom{6}{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{2}+\binom{6}{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{1}$$

$$=\frac{240}{729}+\frac{160}{729}+\frac{60}{729}+\frac{12}{729}=\frac{472}{729}=0.647$$

$$(b)\,P(x<3)=P(x=0)+P(x=1)+P(x=2)$$

$$=\binom{6}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{6}+\binom{6}{1}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{5}+\binom{6}{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{4}$$

$$=\frac{64}{729}+\frac{192}{729}+\frac{240}{729}=\frac{496}{729}=0.68$$

$$(c)\, P(x \le 1) = P(x = 0) + p(x = 1)$$

$$= \binom{6}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{6} + \binom{6}{1}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{5}$$

$$= \frac{64}{729} + \frac{192}{729} = \frac{256}{729} = 0.351$$

$$(d)\, P(x \ge 5) = P(x = 5) + P(x = 6)$$

$$= \binom{6}{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{1} + \binom{6}{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{0}$$

$$= \frac{12}{729} + \frac{1}{729} = \frac{13}{729} = 0.018$$

$$(e)\,P(x>2)=1-P(x\le 2)=P(x=0)+P(x=1)+P(x=2)$$

$$=1-\left\{\binom{6}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{6}+\binom{6}{1}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{5}+\binom{6}{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{4}\right\}$$

$$=1-\left\{\frac{64}{729}+\frac{192}{729}+\frac{240}{729}\right\}=1-\frac{496}{729}=\frac{233}{729}=0.319$$

$$(f\,)\,\mu=np=(6)(\frac{1}{3})=\frac{6}{3}=2$$

$$\sigma^{2}=npq=(6)(\frac{1}{3})(\frac{2}{3})=\frac{12}{9}=1.333$$

EX (5):

- The probability that a person suffering from headache will obtain relief with a particular drug is **0.9**. **Three** randomly selected sufferers from headache are given the drug. Find the probability that the number obtaining relief will be:

a) exactly zero;
b) at most one;
c) more than one;
d) two or fewer;
e) Calculate μ *and* σ^2

See Ex 5.2 pg 146

Solution:

$$n = 3, p = 0.9, q = 0.1$$

$$(a)\, P(X = 0) = \binom{3}{0}(0.9)^0(0.1)^3 = 0.001$$

$$(b)\, P(x \le 1) = P(x = 0) + P(x = 1)$$

$$= \binom{3}{0}(0.9)^0(0.1)^3 + \binom{3}{1}(0.9)^1(0.1)^2$$

$$= 0.001 + 0.027 = 0.028$$

$$(c)\,P(X>1)=1-p(x\le 1)=1-[P(x=0)+P(x=1)]$$

$$=1-[\binom{3}{0}(0.9)^0(0.1)^3+\binom{3}{1}(0.9)^1(0.1)^2$$

$$=1-(0.001+0.027)=1-0.028=0.972$$

$$(d)\,P(x\le 2)=P(x=0)+P(x=1)+P(x=2)$$

$$=\binom{3}{0}(0.9)^0(0.1)^3+\binom{3}{1}(0.9)^1(0.1)^2+\binom{3}{2}(0.9)^2(0.1)^1$$

$$=0.001+0.027+0.243=0.271$$

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Lecture Notes

(5) Some Continuous Probability Distributions

(Book*: Chapter 6 ,pg171)

Probability& Statistics for Engineers & Scientists

By Walpole, Myers, Myers, Ye

5.1 Normal Distribution:

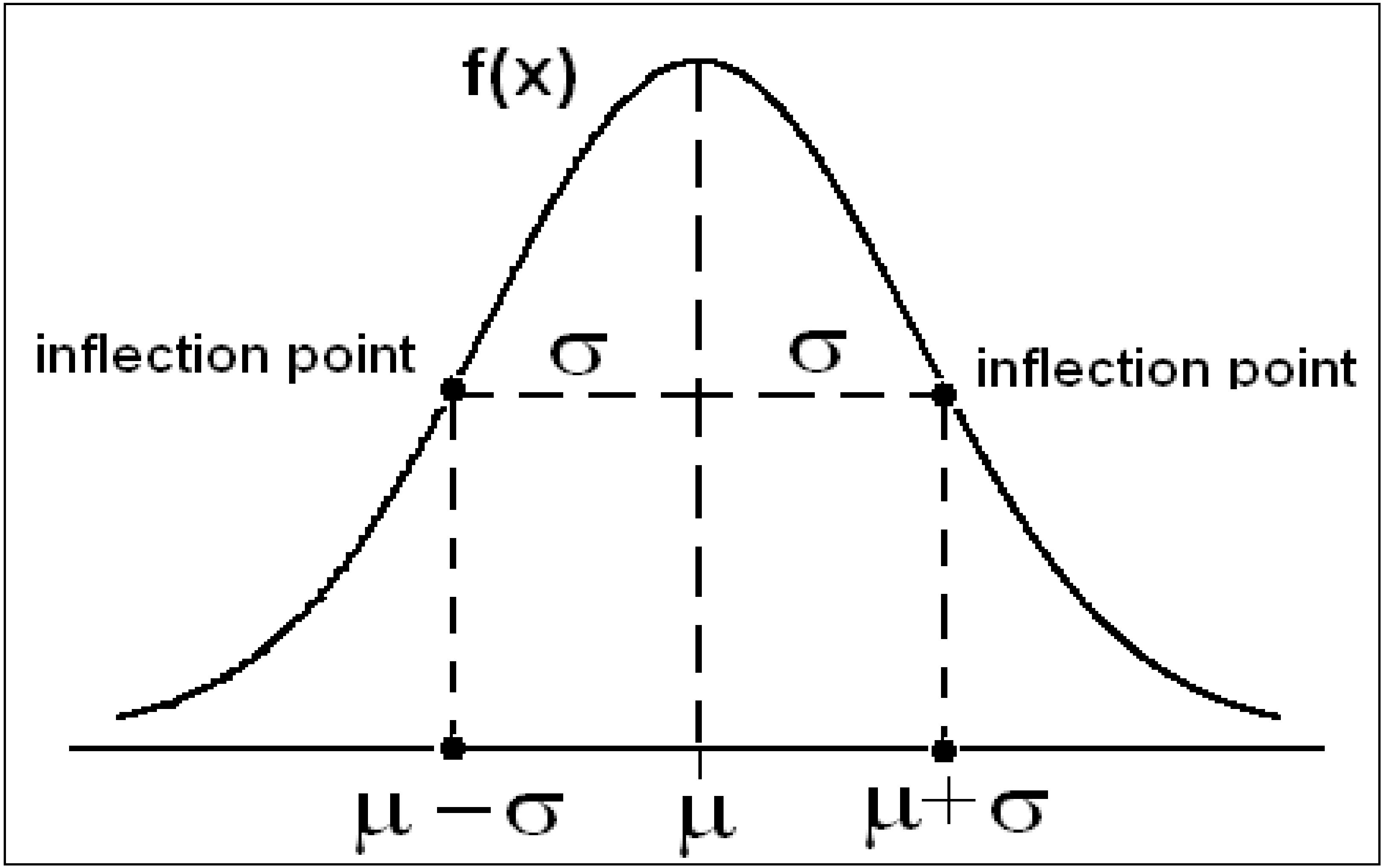
- The probability density function of the normal random variable **X,** with mean μ and variance σ^2 is given by:

$$f(x,\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\Pi}} \; e^{-\frac{1}{2}(\frac{X-\mu}{\sigma})^2}, \quad -\infty < x < \infty$$

5.1.1 The Normal Curve has the Following Properties:

- The mode, which is the point on the horizontal axis where the curve is a maximum, occurs at X= μ , (Mode = Median = Mean).
- The curve is symmetric about a vertical axis through the mean μ .

- The total area under the curve and above the horizontal axis is equal to **1**.


f(x)
inflection point
σ
σ
inflection point
μ − σ
μ
μ+σ

Definition: Standard Normal Distribution:

- The distribution of a normal random variable with mean **zero** and variance **one** is called a standard normal distribution denoted by $Z \approx N(0,1)$

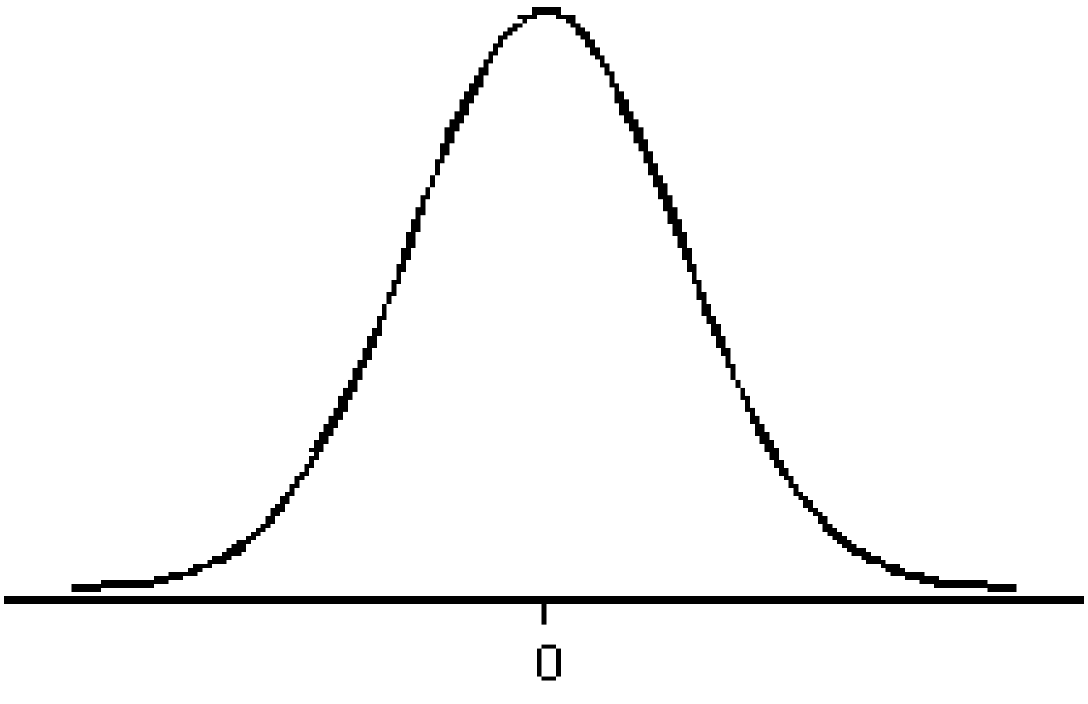
- **Areas under the Normal Curve:**

$$X \approx N(\mu, \sigma)$$

$$Z = \frac{X - \mu}{\sigma} \approx N(0,1)$$

- Using the standard normal tables to find the areas under the curve.

The pdf of Z~N(0,1) is given by:



EX (1):

Using the tables of the standard normal distribution, find:

$$(a)\,P(Z < 2.11)$$

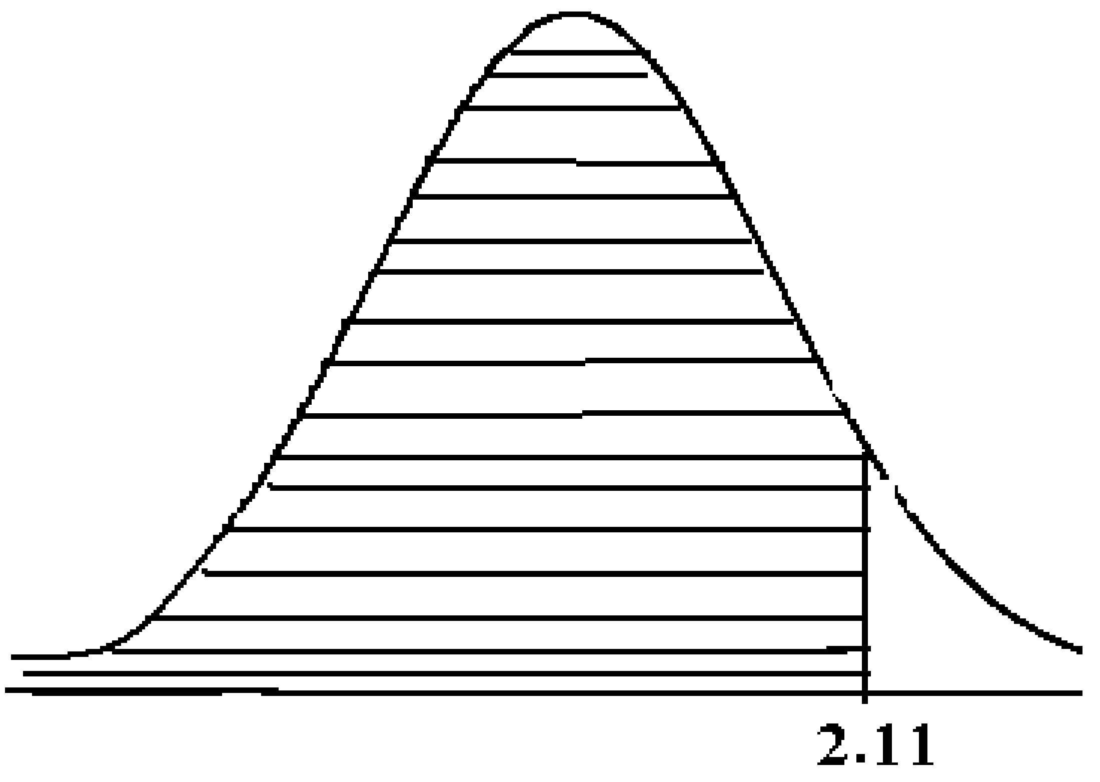
$$(b)\,P(Z > -1.33)$$

$$(c)\,P(Z = 3)$$

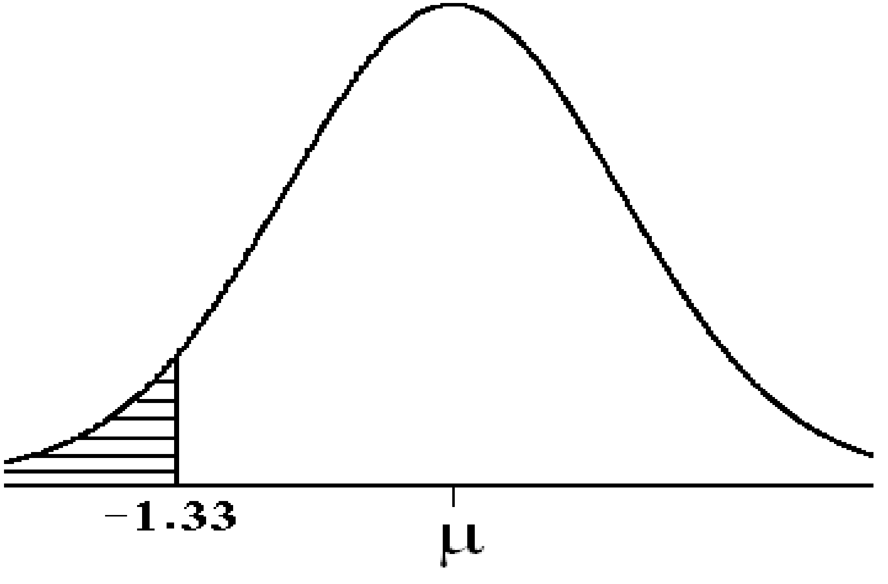
$$(d)\,P(-1.2 < Z < 2.1)$$

Solution:

$$(a)\, P(Z < 2.11) = 0.9826$$

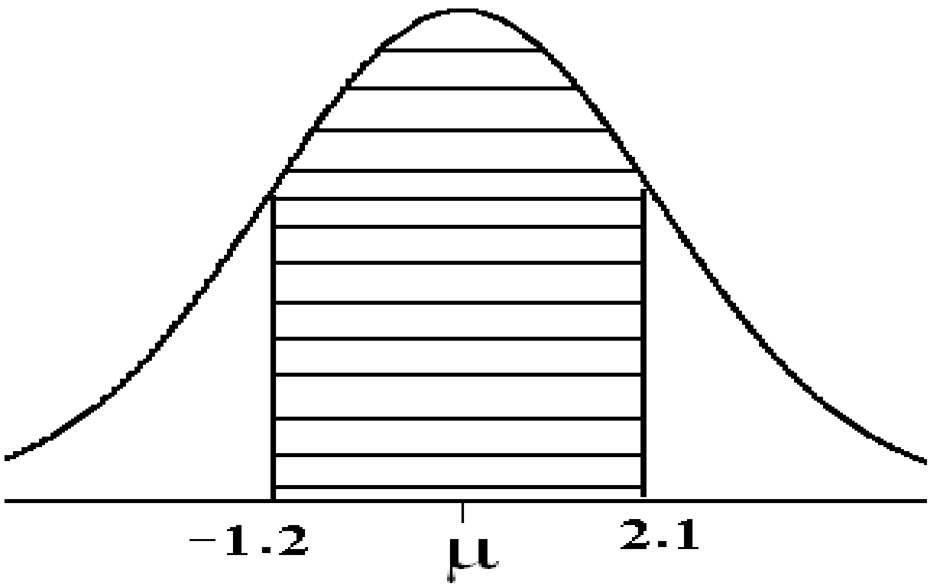


$$(b)\, P(Z > -1.33) = 1 - 0.0918 = 0.9082$$



$$(c)\, P(Z = 3) = 0$$

$$(d)\, P(-1.2 < Z < 2.1) = 0.9821 - 0.1151 = 0.867$$



See Ex: 6.2, 6.3, pg 178-179

EX (6.2 pg 178):

Using the standard normal tables, find the area under the curve that lies:

A. to the right of $Z=1.84$

B. to the left of $z=2.51$

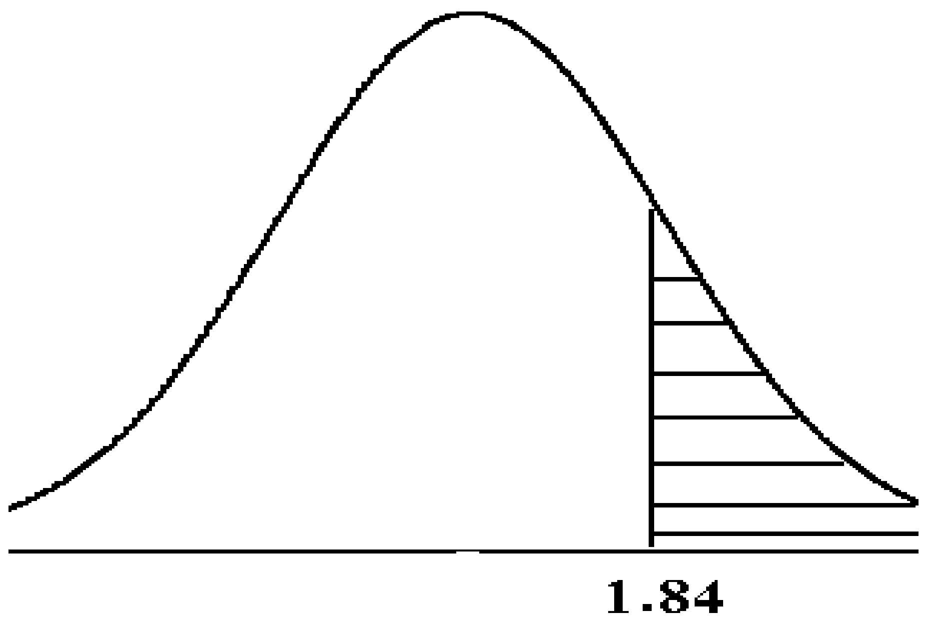
C. between $z=-1.97$ and $z=0.86$

A. at the point $z= -2.15$

Solution:

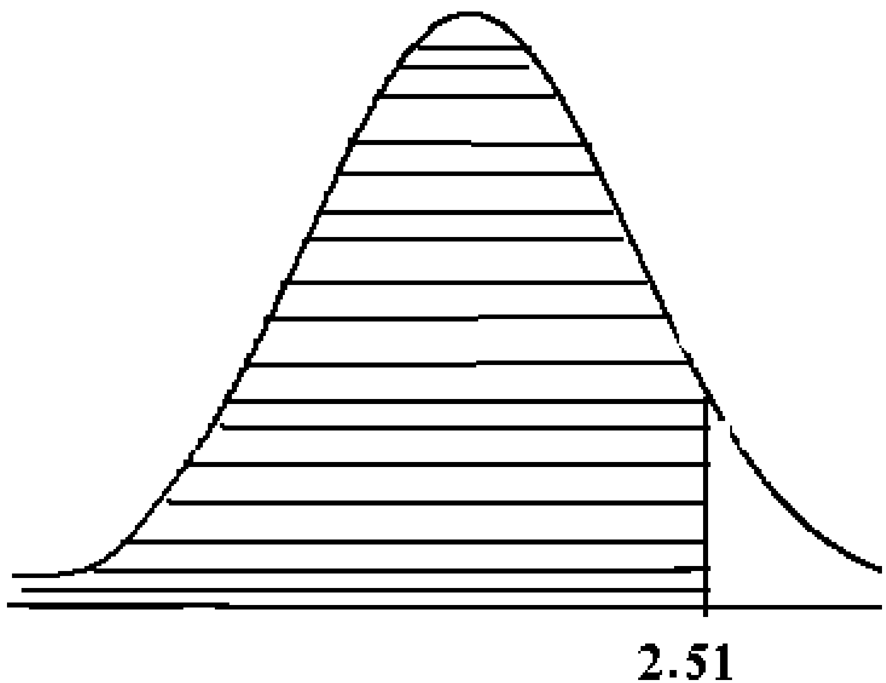
A. to the right of Z=1.84

$$P(Z > 1.84) = 1 - 0.9671 = 0.0329$$



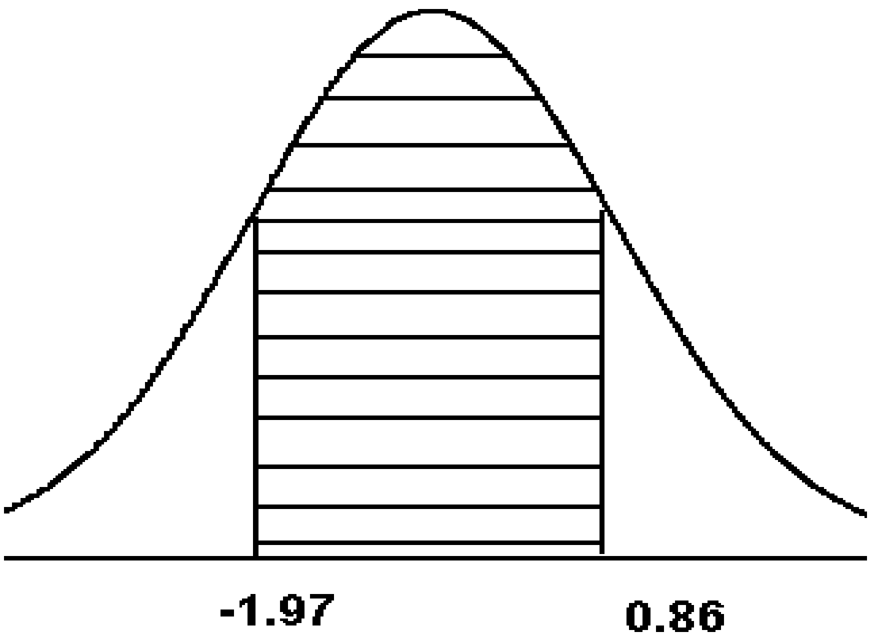
B. to the left of z=2.51

$$P(Z < 2.51) = 0.9940$$



C. between z=-1.97 and z=0.86

$$P(-1.97 < Z < 0.86) = 0.8051 - 0.0244 = 0.7807$$



D. at the point z= -2. 15

$$P(Z = -2.15) = 0$$

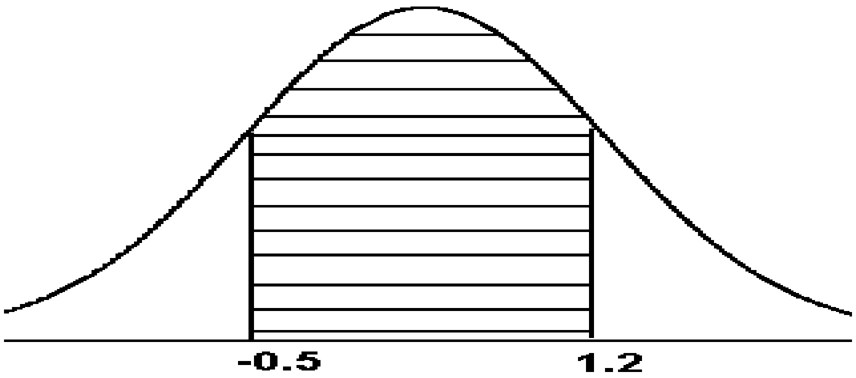
EX (6.4 pg 179):

Given a normal distribution with $\mu=50$, $\sigma=10$. Find the probability that **X** assumes a value between **45** and **62**.

Solution:

$$P(45 < X < 62) = P\left(\frac{45-50}{10} < Z < \frac{62-50}{10}\right) = P(-0.5 < Z < 1.2)$$

$$= 0.8849 - 0.3085 = 0.5764$$



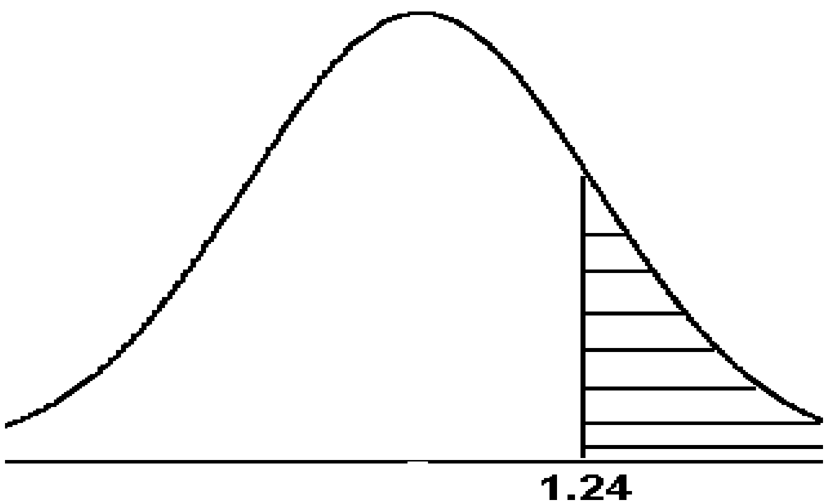
EX(6.5 pg 180) :

Given a normal distribution with $\mu=300$, $\sigma=50$, find the probability that **X** assumes a value greater than **362**.

Solution:

$$P(X > 362) = P(Z > \frac{362-300}{50}) = P(Z > 1.24)$$

$$= 1 - 0.8925 = 0.1075$$



Applications of the Normal Distribution:

EX (1):

The reaction time of a driver to visual stimulus is normally distributed with a mean of **0.4** second and a standard deviation of **0.05** second.

(a) What is the probability that a reaction requires more than **0.5** second?

(b) What is the probability that a reaction requires between **0.4** and **0.5** second?

(c) Find mean and variance.

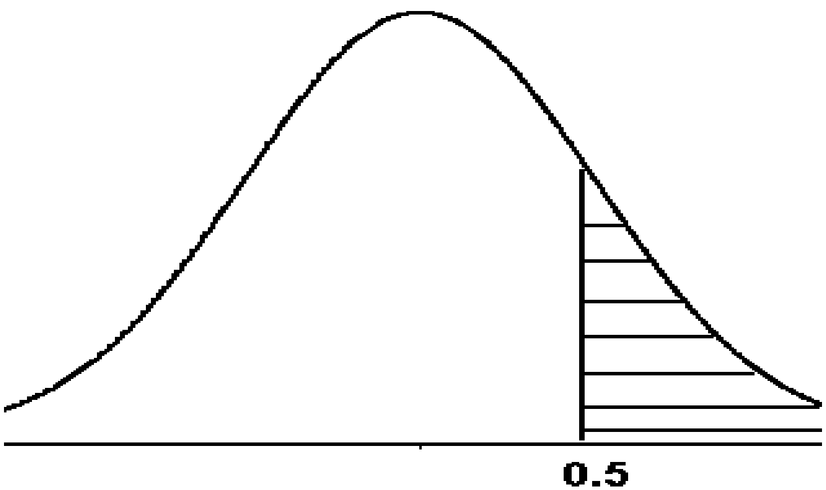
Solution:

$$\mu_X = 0.4, \sigma_X = 0.05$$

(a) What is the probability that a reaction requires more than **0.5** second?

$$(a) P(X > 0.5) = P(Z > \frac{0.5 - 0.4}{0.05}) = P(Z > 2)$$

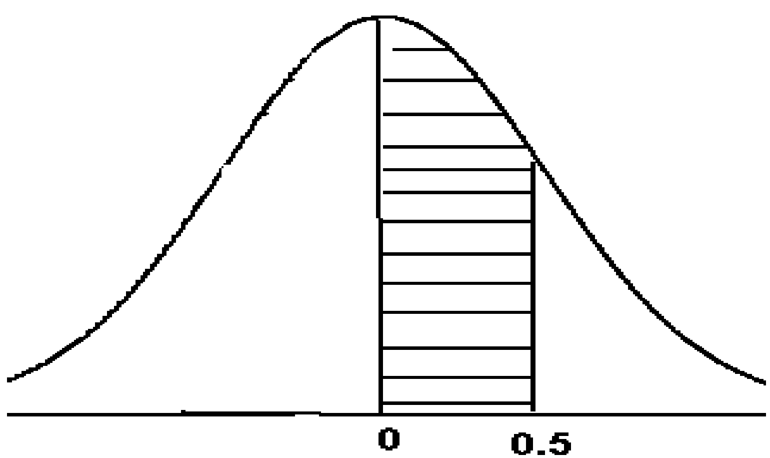
$$= 1 - 0.9772 = 0.0228$$



(b) What is the probability that a reaction requires between 0.4 and 0.5 second?

$$(b)\, P(0.4 < X < 0.5) = P\left(\frac{0.4-0.4}{0.05} < Z < \frac{0.5-0.4}{0.05}\right)$$

$$= P(0 < Z < 2) = 0.9772 - 0.5 = 0.4772$$



(c) Find mean and variance.

$$(c)\ \mu = 0.4, \sigma^2 = 0.0025$$

EX (2):

The line width of a tool used for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.

(a) What is the probability that a line width is greater than 0.62 micrometer?

(b) What is the probability that a line width is between 0.47 and 0.63 micrometer?

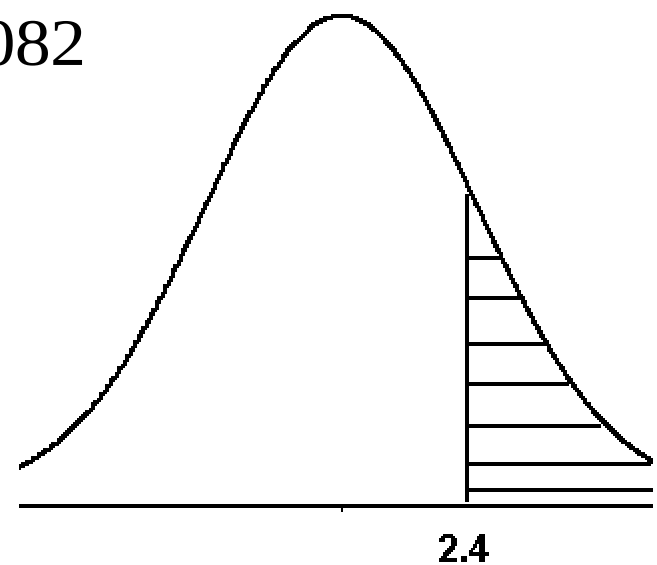
Solution:

(a) What is the probability that a line width is greater than 0.62 micrometer?

$$\mu_X = 0.5, \quad \sigma_X = 0.05$$

$$(a)\,P(x > 0.62) = P(Z > \frac{0.62 - 0.5}{0.05}) = P(Z > 2.4)$$

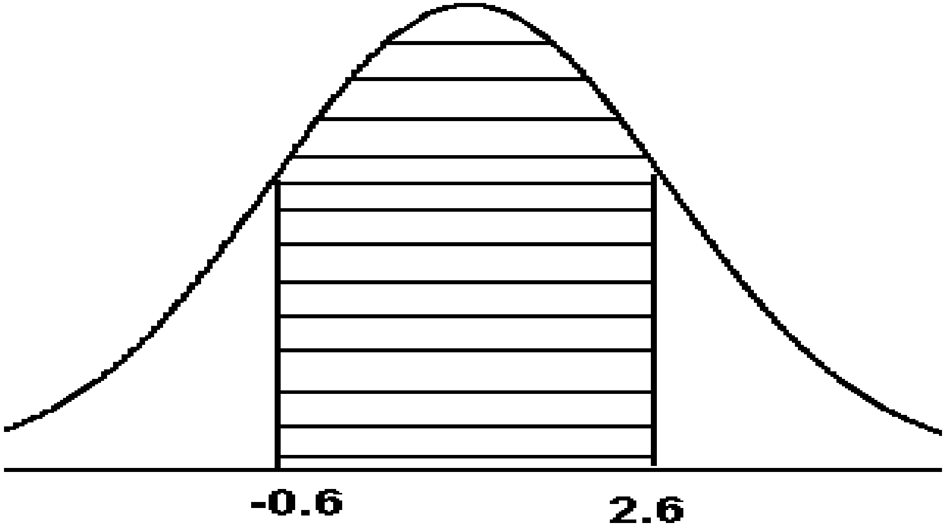
$$= 1 - 0.9918 = 0.0082$$



(b) What is the probability that a line width is between 0.47 and 0.63 micrometer?

$$(b)\, P(0.47 < X < 0.63) = P\left(\frac{0.47-0.5}{0.05} < Z < \frac{0.63-0.5}{0.05}\right)$$

$$= P(-0.6 < Z < 2.6) = 0.9953 - 0.2743 = 0.721$$



See Ex 6.7, 6.8 pg 182

Normal Approximation to the Binomial(Reading):

Theorem:

If **X** is a binomial random variable with mean $\mu = n\,p$ and variance $\sigma^2 = n\,p\,q$, then the limiting form of the distribution of

$$Z = \frac{X - n\,p}{\sqrt{n\,p\,q}} \quad as\ n \to \infty$$

is the standard normal distribution $N(0,1)$.

EX

The probability that a patient recovers from rare blood disease is **0.4**. If **100** people are known to have contracted this disease, what is the probability that less than **30** survive?

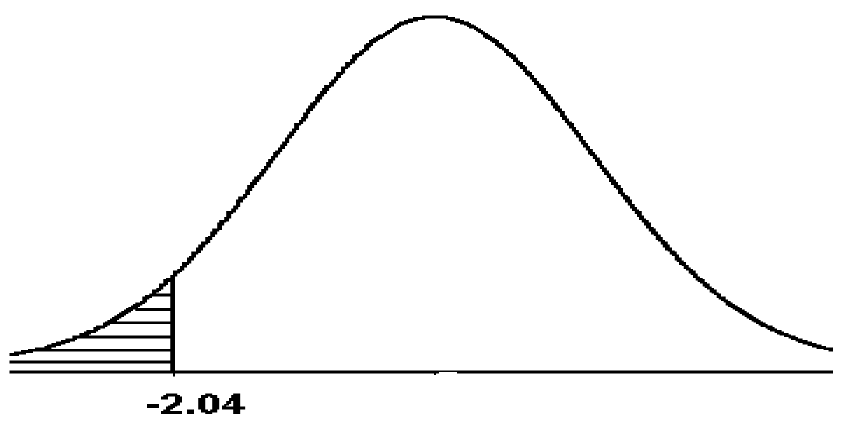
Solution:

$$n = 100 \quad , \quad p = 0.4 \quad , \quad q = 0.6$$

$$\mu = np = (100)(0.4) = 40 \quad ,$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(0.4)(0.6)} = 4.899$$

$$P(X < 30) = P(Z < \frac{30-40}{4.899}) = P(Z < -2.04) = 0.0207$$



EX (6.16, pg 192)

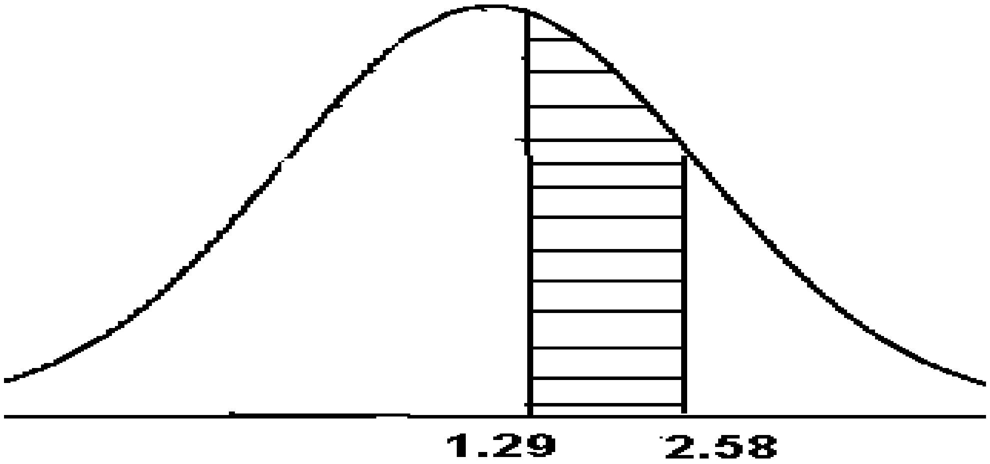
A multiple – choice quiz has **200** questions each with **4** possible answers of which only **1** is the correct answer. What is the probability that sheer guess – work yields from **25** to **30** correct answers for **80** of the **200** problems about which the student has no knowledge?

Solution:

$$p = np = (80)(0.25) = 20 \quad ,$$

$$\sigma = \sqrt{npq} = \sqrt{(80)(0.25)(0.75)} = 3.873$$

$$P(25 < X < 30) = P\left(\frac{25-20}{3.873} < Z < \frac{30-20}{3.873}\right) = P(1.29 < Z < 2.58) = 0.9951 - 0.9015 = 0.0936$$



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Lecture Notes

(6) Fundamental Sampling Distribution and Data Discription

(Book*: Chapter 8 ,pg225)

Probability& Statistics for Engineers & Scientists
By Walpole, Myers, Myers, Ye

8.1 Random Sampling:

Population:

A population consists of the total observations with which we are concerned at a particular time.

Sample:

A sample is a subset of a population.

Random Sample

Let $X_1, X_2, ..X_n$ be n independent random variable; having the same probability distribution f(X), then $X_1, X_2, ..X_n$ is defined to be a random sample of size n from the population f(x)

Statistic :

Any function of the random sample is called a statistic.

8.2 Some important Statistics:

1- Location Measures of a Sample: The Sample Mean ,Median and Mode

(a) Sample Mean:

If $X_1, X_2, \ldots X_n$ represent a random sample of size **n**, then the sample mean is defined by the statistic:

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \qquad (1)$$

Properties of the Mean (Reading):

1. The mean is the most commonly used measure of certain location in statistics.
2. It employs all available information.
3. The mean is affected by extreme values.
4. It is easy to calculate and to understand.
5. It has a unique value given a set of data.

EX 1:

The length of time, in minutes, that **10** patients waited in a doctor's office before receiving treatment were recorded as follows: **5, 11, 9, 5, 10, 15, 6, 10, 5** and **10.** Find the mean.

Solution:

$$n = 10, \quad \sum x_i = 5 + 11 + \ldots + 10 = 86$$

$$\bar{X} = \frac{\sum x_i}{n} = \frac{86}{10} = 8.6$$

See Ex 8.4 pg 228

(b) Sample Median:

- If $X_1 \ldots X_n$ represent a random sample of size **n** , arranged in increasing order of magnitude, then the sample median, which is denoted by Q_2 is defined by the statistic:

$$Q_2 = \begin{cases} X_{\frac{(n+1)}{2}} & if \quad n \quad is \quad odd \\ \dfrac{X_{n/2} + X_{(n/2)+1}}{2} & if \quad n \quad is \quad even \end{cases} \qquad (2)$$

Properties of the Median(Reading):

1. The median is easy to compute if the number of observations is relatively small.
2. It is not affected by extreme values.

EX 2:

The number of foreign ships arriving at an east cost port on **7** randomly selected days were **8, 3, 9, 5, 6, 8** and **5.**

Find the sample median.

Solution:

The arranged values are: **3 5 5 6 8 8 9**

$$n = 7 \quad , \quad \frac{n+1}{2} = \frac{7+1}{2} = 4$$

$$Q_2 = 6$$

EX 3:

The nicotine contents for a random sample of **6** cigarettes of a certain brand are found to be **2.3, 2.7, 2.5, 2.9, 3.1** and **1.9** milligrams. Find the median.

Solution:

The arranged values are: **1.9 ,2.3 ,2.5 ,2.7, 2.9, 3.1 .**

$$n = 6 \quad , \quad \frac{n}{2} = \frac{6}{2} = 3 \ , \qquad \frac{n}{2} + 1 = \frac{6}{2} + 1 = 3 + 1 = 4$$

$$Q_2 = \frac{2.5 + 2.7}{2} = 2.6$$

(c)Sample Mode:

The sample mode is the value of the sample that occurs most often.

Properties of the Mode(Reading):

1. The value of the mode for small sets of data is almost useless.
2. It requires no calculation.

EX(4):

The numbers of incorrect answers on a true – false test for a random sample of **14** students were recorded as follows: **2, 1, 3, 0, 1, 3, 6, 0, 3, 3, 2, 1, 4, and 2**, find the mode.

Solution:

mode=3

H.w.

Find the mean and median

Notes:

1. The sample means usually will not vary as much from sample to sample as will the median.
2. The median (when the data is ordered) and the mode can be used for qualitative as well as quantitative data.

2- Variability Measures of a Sample: The Sample Variance, Standard deviation and Range:

6.3.1 The Range:

(a)The range of a random sample $X_1 \dots X_n$ is defined by the statistic $X_{(n)} - X_{(1)}$, where $X_{(n)}$ *and* $X_{(1)}$ respectively the largest and the smallest observations which is denoted by **R**, then:

$$R = Max - Min = x_{(n)} - x_{(1)}$$

EX 5:

Let a random sample of five members of a sorority are **108,112,127,118** and **113**. Find the range.

Solution:

$$\mathbf{R}=127-108=19$$

(b) Sample Variance:

If $X_1 \ldots X_n$ represent a random sample of size **n**, then the sample variance, which is denoted by S^2 is defined by the statistic:

$$S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1} = \frac{1}{n-1} [\sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}]$$

$$= \frac{1}{n-1} [\sum_{i=1}^{n} X_i^2 - n\bar{X}^2]$$

<u>Ex 8.2 pg 229</u>

A comparison of coffee prices at **4** randomly selected grocery stores in San Diego showed increases from the previous month of **12, 15, 17, 20,** cents for a **200** gram jar. Find the variance of this random sample of price increases.

<u>Solution:</u>

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{12+15+17+20}{4} = 16$$

$$S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1} = \frac{(12-16)^2 + (15-16)^2 + (17-16)^2 + (20-16)^2}{3}$$

$$= \frac{34}{3}$$

(b) Sample Standard Deviation:

The sample standard deviation s is given by:

$$S = \sqrt{S^2}$$

Where S^2 is the sample variance.

EX 6 :

The grade – point average of **20** college seniors selected at random from a graduating class are as follows:

3.2, 1.9, 2.7, 2.4, 2.8, 2.9, 3.8, 3.0, 2.5, 3.3, 1.8, 2.5, 3.7, 2.8, 2.0, 3.2, 2.3, 2.1, 2.5, 1.9. Calculate the variance and the standard deviation.

Solution: $$\sum_{i=1}^{n} X_i = 53.3, \sum_{i=1}^{n} X_i^2 = 148.55$$

See Ex 8.3 pg 230

(answer: $\bar{X} = 2.665, S = 0.585, S^2 = 0.342$)

8.3 Sampling Distributions

Definition:

The probability distribution of a statistic is called a sampling distribution.

8.4 Sampling Distributions of Means and the Central Limit Theorem

Sampling Distributions of Means:

Suppose that a random sample of size n observation is taken from normal distribution then

$$\bar{X} = \frac{1}{n}(X_1 + X_2 + \cdots . + X_n)$$

Has a normal distribution with mean and variance

$$E(\bar{X}) = \mu_{\bar{X}} = \mu \quad , \quad V(\bar{X}) = \sigma^2_{\bar{X}} = \frac{\sigma^2}{n} \qquad (3)$$

Central Limit Theorem:

If $\bar{X}$ is the mean of a random sample of size **n** taken from a population with mean μ and finite variance σ^2, then the limiting form of the distribution of:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \quad as \quad n \to \infty \qquad (4)$$

is approximately the standard normal distribution■

Notes

- $f(Z) \sim N(0,1) \Rightarrow E(\bar{X}) = \mu \quad , \quad V_{\bar{X}}(\bar{X}) = \dfrac{\sigma^2}{n} \quad , \quad \sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}}$

$$\Rightarrow \bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}}) \qquad (5)$$

- The approximation of will generally be good if $n \geq 30$.

EX(8.4 pg 234):

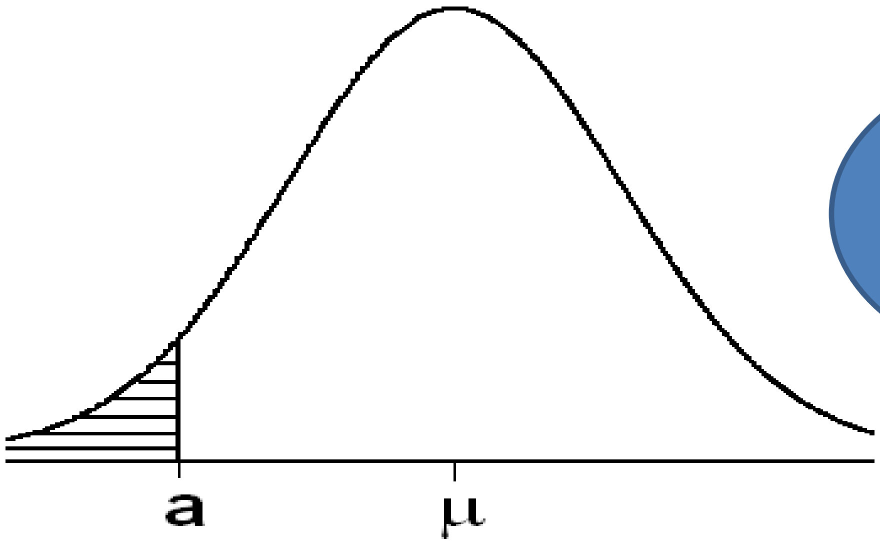
An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with mean equal to **800** hours and a standard deviation of **40** hours. Find the probability that a random sample of **16** bulbs will have an average life of less than **775** hours.

Solution:

Let **X** be the length of life and $\bar{X}$ is the average life;

$$n = 16, \mu = 800, \; \sigma = 40$$

$$P(\bar{X} < 775) = P\left(Z < \frac{775 - 800}{40/\sqrt{16}}\right) = P(Z < -2.5) = 0.0062$$



See Ex 8.5 pg 237

Sampling distribution of the difference between two means

Theorem If independent samples of size **n_1** and **n_2** are drawn at random from populations, discrete or continuous with means μ_1 *and* μ_2 and variances σ_1^2 *and* σ_2^2 respectively, then the sampling distribution of the difference of means $\bar{X}_1 - \bar{X}_2$ is approximately normally distributed with mean and variance given by:

$$E(\bar{X}_1 - \bar{X}_2) = \mu_{(\bar{X}_1 - \bar{X}_2)} = \mu_1 - \mu_2 \quad , \quad \sigma^2_{(\bar{X}_1 - \bar{X}_2)} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} \qquad (6)$$

Hence

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad (7)$$

is approximately a standard normal variable.

EX(10):

A sample of size **n_1=5** is drawn at random from a population that is normally distributed with mean $\mu_1 = 50$ and variance $\sigma_1^2 = 9$ and the sample mean $\overline{X}_1$ is recorded. A second random sample of size **n_2=4** is selected independent of the first sample from a different population that is also normally distributed with mean $\mu_2 = 40$ and variance $\sigma_2^2 = 4$ and the sample mean $\overline{X}_2$ is recorded.

Find $P(\overline{X}_1 - \overline{X}_2 < 8.2)$

Solution:

$$n_1 = 5 \qquad n_2 = 4$$

$$\mu_1 = 50 \qquad \mu_2 = 40$$

$$\sigma_1^2 = 9 \qquad \sigma_2^2 = 4$$

See Case Study 8.2 pg 238

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 = 50 - 40 = 10$$

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{9}{5} + \frac{4}{4} = 2.8$$

$$P(\bar{X}_1 - \bar{X}_2 < 8.2) = P(Z < \frac{8.2 - 10}{1.673}) = P(Z < -1.08) = 0.1401$$

<u>EX(11): (H.W)</u>

The television picture tubes of manufacturer **A** have a mean lifetime of **6.5** years and a standard deviation of **0.9** year, while those of manufacturer **B** have a mean lifetime of **6** years and a standard deviation of **0.8** year. What is the probability that a random sample of **36** tubes from manufacture **A** will have a mean lifetime that at least **1** year more than the mean lifetime of a sample of **49** from tubes manufacturer **B**?

Solution:

Population 1	Population 2
$\mu_1 = 6.5$	$\mu_2 = 6.0$
$\sigma_1 = 0.9$	$\sigma_2 = 0.8$
$n_1 = 36$	$n_2 = 49$

$$\mu_{\bar{X}_1 - \bar{X}_2} = 6.5 - 6 = 0.5$$

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{0.81}{36} + \frac{0.64}{49}} = 0.189$$

$$P[(\bar{X}_1 - \bar{X}_2) > 1) = P(Z > \frac{1 - 0.5}{0.189}) = P(Z > 2.65)$$

$$= 1 - P(Z < 2.65) = 1 - 0.996 = 0.004$$

Sampling Distribution of the sample Proportion (Reading):

Let **X**= no. of elements of type **A** in the sample

P= population proportion = no. of elements of type A in the population / **N**

$\hat{p}$ **=** sample proportion **=** no. of elements of type A in the sample / n = x/n

$$\because x \sim binomial\,(n, p) \rightarrow E(x) = np, V(x) = npq$$

$$\therefore 1. E(\hat{p}) = E\left(\frac{x}{n}\right) = p$$

$$2. V(\hat{p}) = V\left(\frac{x}{n}\right) = \frac{pq}{n},\; q = 1 - p$$

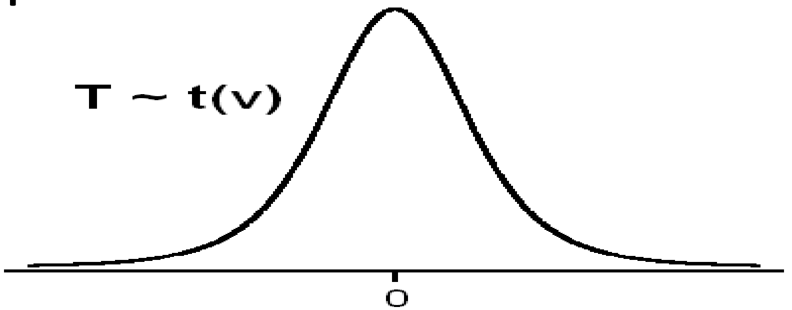
3. For large n, we have:

$$\hat{p} \sim N\left(p, \sqrt{\frac{pq}{n}}\right)$$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \sim N(0,1)$$

<u>8.6 *t* – Distribution (pg 246):</u>

* ***t*** distribution has the following properties:



1. It has mean of zero.
2. It is symmetric about the mean.
3. It ranges from $-\infty \; to \; \infty$.
4. Compared to the normal distribution, the ***t*** distribution is less peaked in the center and has higher tails.
5. It depends on the degrees of freedom **(n-1)**.
6. The t distribution approaches the normal distribution as **(n-1)**

   approaches ∞.

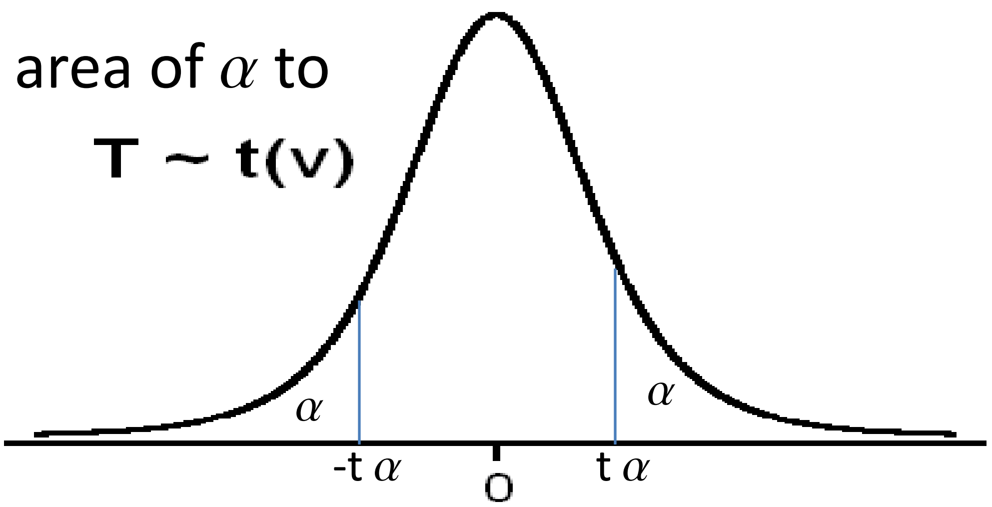
Notes

- Since the t-distribution is symmetric about zero we have

$t_{1-\alpha} = -t_{\alpha}$

- Table A4 pg 737-738 represent the critical values of t-distribution

Where t_{α} leaving an area of α to The right.



Corollary 8.1

- Let X_1, X_2, ...X_n be independent random variables from normal with mean μ and standard deviation σ. Let
- $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})$

Then the random variable $\mathrm{T} = \frac{\bar{X}-\mu}{s/\sqrt{n}}$ has a t-distribution with υ =n-1 degrees of freedom.

EX(12):

Find:

$$(a)\ t_{0.025} \quad when \quad v = 14$$

$$(b)\ t_{0.01} \quad when \quad v = 10$$

$$(c)\ t_{0.995} \quad when \quad v = 7$$

(a) $t_{0.025}$ at $v = 14 \rightarrow t = 2.1448$

(b) $t_{0.01}$ at $v = 10 \rightarrow t = 2.764$

$(c) t_{0.995}$ at $v = 7 \rightarrow$

$$t = -t_{0.005} = -3.499$$

EX(13):

Find:

$(a)\, P(T < 2.365) \quad when \quad v = 7$

$(b)\, P(T > 1.318) \quad when \quad v = 24$

$(c)\, P(-t_{0.025} < T < t_{0.05})$

$(d)\, P(T > -2.567) \quad when \quad v = 17$

solution

(a) $P(T<2.356)=1-0.025=$ **0.975** at $\nu = 7$

(b) $P(T>1.318)= 0.1$ at $\nu =24$

(c) $P(-t_{0.025}<T<t_{0.05})$

$t_{0.05}$ leaves an area of 0.05 to the right and $-t_{0.025}$ leaves an area 0.025 to the left so the total area is

1-0.05 -0.025=0.925.

(d) $P(T>-2.567)= 1-0.01=0.99$ at $\nu =17$

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(7 One- and Two-Sample Estimation Problem)

(Book*: Chapter 8 ,pg265)

Probability& Statistics for Engineers & Scientists
By Walpole, Myers, Myers, Ye

Estimation

- <u>Point estimate:</u>

- Is a single numerical value to estimate parameter
- <u>Example :</u>

$$\bar{X} = \sum \frac{X_i}{n}$$

$$\hat{P} = \frac{X}{n}$$

$$S^2 = \frac{\sum (X_i - \bar{X})}{n-1}$$

- <u>Interval estimate</u>

- Is two numerical values to estimate parameter
- <u>It means to be in an interval s.a</u>

$$(\theta_L < \theta < \theta_U)$$

- Lower bound (L) upper bound (U) •

Interval estimation

- An interval estimate of a population parameter θ is an interval of the form $\hat{\theta}_L < \theta < \hat{\theta}_U$ where $\hat{\theta}_L$ and $\hat{\theta}_U$ depend on the value of the statistic $\hat{\Theta}$ for a particular sample and also on the sampling distribution of $\hat{\Theta}$. Since different samples will generally yield different values of $\hat{\Theta}$ and therefore different values of $\hat{\theta}_L$ and $\hat{\theta}_U$. From the sampling distribution of $\hat{\Theta}$ we shall be able to determine $\hat{\theta}_L$ and $\hat{\theta}_U$ such that the $P(\hat{\theta}_L < \theta < \hat{\theta}_U)$ is equal to any positive fractional value we care to specify. If for instance we find $\hat{\theta}_L$ and $\hat{\theta}_U$ such that:

$$P(\hat{\theta}_L < \theta < \hat{\theta}_U) = 1 - \alpha \qquad for \quad 0 < \alpha < 1$$

then we have a probability of $(1-\alpha)$ of selecting a random sample that will produce an interval containing θ .

- The interval $\hat{\theta}_L < \theta < \hat{\theta}_U$ computed from the selected sample, is then called a $(1-\alpha)100\%$ confidence interval.

- The fraction $(1-\alpha)$ is called confidence coefficient or the degree of confidence.

- The end points $\hat{\theta}_L$ and $\hat{\theta}_U$ are called the lower and upper confidence limits.

- For Example:

when $\alpha = 0.05$ we have a 95% confidence interval and so on, that we are 95% confident that θ is between $\hat{\theta}_L$, $\hat{\theta}_U$

9.4 Single Sample: Estimating the Mean:

1-Confidence Interval on μ (σ^2 Known):

If $\bar{X}$ is the mean of a random sample of size **n** from a population with known variance σ^2, a $(1-\alpha)100\%$ confidence interval for μ is given by:

$$\bar{X} - Z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} \qquad (1)$$

where $Z_{1-\alpha/2}$ is the -value leaving an area of $\frac{\alpha}{2}$ to the right■

$$\hat{\theta}_L = \bar{X} - Z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} \quad , \quad \hat{\theta}_U = \bar{X} + Z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} \qquad (2)$$

EX (1):

The mean of the quality point averages of a random sample of **36** college seniors is calculated to be **2.6**.

Find the 95% confidence intervals for the mean of the entire senior class. Assume that the population standard deviation is **0.3**.

Solution:

$$n = 36 \ , \bar{X} = 2.6 \ , \ \sigma = 0.3$$

95% confidence interval for the mean μ :

See Ex 9.2 pg 271

$$1-\alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \rightarrow Z_{1-\frac{\alpha}{2}} = 1.96$$

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$2.6 \pm 1.96 \left(\frac{0.3}{\sqrt{36}}\right) \rightarrow 2.6 \pm 0.098$$

$$2.502 < \mu < 2.698$$

Thus , we have 95% confident that μ lies between 2.502 and 2.698

Theorem 9.1:

If $\bar{X}$ is used as an estimate of μ , we can be $(1-\alpha)100\%$ confident that the error will not be exceed $Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$

For example (1): e= (1.96) (0.3/6)=0.098 or

Theorem (2):

If $\bar{X}$ is used as an estimate of μ , we can be $(1-\alpha)100\%$ confident that the error will not exceed a specified amount, e when the sample size is:

$$n=\left(\frac{Z_{1-\frac{\alpha}{2}}\sigma}{e}\right)^2 \qquad (3)$$

The fraction of **n** is rounded up to next whole number.

EX (2):

How large a sample is required in Ex. (1) if we want to be 95% confident that our estimate of μ is off by less than **0.05?**

Solution

$$\alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \rightarrow Z_{1-\frac{\alpha}{2}} = 1.96,$$

$$\sigma = 0.3 \ , \quad e = 0.05$$

$$n = \left(\frac{(1.96)(0.3)}{0.05} \right)^2 = 138.2976 \approx 138$$

n is rounded up to whole number.

2- Confidence Interval of when σ^2 is Unknown n<30 :

If $\overline{X}$ and s are the mean and standard deviation of a random sample from a normal population with unknown variance σ^2, a $(1-\alpha)100\ \%$ confidence interval for μ is given by:

$$\bar{X} - t_{\frac{\alpha}{2},\, n-1} \frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{\frac{\alpha}{2},\, n-1} \frac{S}{\sqrt{n}} \qquad (4)$$

where $t_{\frac{\alpha}{2}}$ is the t-value with **n-1** degrees of freedom leaving an area of $\frac{\alpha}{2}$ to the right.

Ex 9.5 pg 275

The contents of 7 similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10, 10.2, 9.6 liters. Find a 95%confidence interval for the mean of all such containers assuming an approximate normal distribution.

Solution:

confidence interval for the mean :

$$n = 7 \quad , \quad \bar{X} = 10 \quad , \quad S = 0.283 \quad ,$$

$$at : 1-\alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \rightarrow t_{\frac{\alpha}{2}, n-1} = t_{0.025,6} = 2.447$$

$$\bar{X} \pm t_{\frac{\alpha}{2}, n-1} \left(\frac{S}{\sqrt{n}}\right) \Rightarrow 10 \pm (2.447)\left(\frac{0.283}{\sqrt{7}}\right) \Rightarrow 10 \pm 0.262$$

$$9.738 < \mu < 10.262 \rightarrow P(9.738 < \mu < 10.262) = 0.95$$

9:4 Two Samples: Estimating the Difference between Two Means:

1- Confidence Interval for $\mu_1 - \mu_2$ when σ_1^2 and σ_2^2 Known:

If $\bar{X}_1$ *and* $\bar{X}_2$ are the means of independent random samples of size **n_1** and **n_2** from populations with known variances σ_1^2 *and* σ_2^2 respectively, a $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ is given by:

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \qquad (5)$$

where $Z_{1-\frac{\alpha}{2}}$ is the -value leaving an area of $\frac{\alpha}{2}$ to the right .

EX (4):

A standardized chemistry test was given to **50** girls and **75** boys. The girls made an average grade of **76**, while the boys made an average grade of **82**. Find a 96% confidence interval for the difference $\mu_1 - \mu_2$ where μ_1 is the mean score of all boys and μ_2 is the mean score of all girls who might take this test. Assume that the population standard deviations are **6** and **8** for girls and boys respectively.

Solution:

girls	Boys
$n_1 = 50$	$n_2 = 75$
$\bar{X}_1 = 76$	$\bar{X}_2 = 82$
$\sigma_1 = 6$	$\sigma_2 = 8$

96% confidence interval for the mean $\mu_1 - \mu_2$:

$$1-\alpha = 0.94 \rightarrow \alpha = 0.04 \rightarrow \frac{\alpha}{2} = 0.02 \rightarrow Z_{1-\frac{\alpha}{2}} = 2.05$$

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(82-76) \pm (2.05)\sqrt{\frac{36}{50} + \frac{64}{75}} \Rightarrow 6 \pm 2.571$$

$$3.429 < \mu_1 - \mu_2 < 8.571 \rightarrow P(3.429 < \mu_1 - \mu_2 < 8.571) = 0.96$$

See Ex 9.10 pg 286

2-Confidence Interval for $\mu_1 - \mu_2$ when σ_1^2 and σ_2^2 Unknown but equal variances:

If $\bar{X}_1$ and $\bar{X}_2$ are the means of independent random samples of size $\mathbf{n_1}$ and $\mathbf{n_2}$ respectively from approximate normal populations with unknown but equal variances, a $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ is given by:

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2},v} \, S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \qquad (6)$$

, where

$$S_P = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} \qquad (7)$$

is the pooled estimate of the population standard deviation and $t_{\frac{\alpha}{2},v}$ is the **t** – value with $v = n_1 + n_2 - 2$ degrees of freedom leaving an area of $\frac{\alpha}{2}$ to the right.

EX (9.11- pg 288):

The independent sampling stations were chosen for this study, one located down stream from the acid mine discharge point and the other located upstream. For **12** monthly samples collected at the down stream station the species diversity index had a mean value $\bar{X}_1 = 3.11$ and a standard deviation $S_1 = 0.771$ while **10** monthly samples had a mean index value $\bar{X}_2 = 2.04$ and a standard deviation $S_2 = 0.448$. Find a 90% confidence interval for the difference between the population means for the two locations, assuming that the populations are approximately normally distributed with equal variances.

Solution:

Station 1	Station 2
$n_1 = 12$	$n_2 = 10$
$\bar{X}_1 = 3.11$	$\bar{X}_2 = 2.04$
$S_1 = 0.771$	$S_2 = 0.448$

90% confidence interval for the mean $\mu_1 - \mu_2$:

$$S_P = \sqrt{\frac{11(0.771)^2 + 9(0.448)^2}{12+10-2}} = 0.646$$

$$at\ \ 1-\alpha = 0.90 \rightarrow \alpha = 0.1 \rightarrow \frac{\alpha}{2} = 0.05 \rightarrow t_{\frac{\alpha}{2}, n_1+n_2-2} \rightarrow t_{0.05,20} = 1.725$$

$$(3.11 - 2.04) \pm (1.725)(0.646)\sqrt{\frac{1}{12} + \frac{1}{10}} \Rightarrow 1.07 \pm 0.477$$

$$0.593 < \mu_1 - \mu_2 < 1.547 \rightarrow P(0.593 < \mu_1 - \mu_2 < 1.547) = 0.90$$

9:10 Single Sample Estimating a Proportion:

Large – Sample Confidence Interval for P:

If $\hat{p}$ is the proportion of successes in a random sample of size n and an approximate $(1-\alpha)100\%$ confidence interval for the binomial parameter is given by:

$$\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} \qquad (8)$$

$$\hat{q} = 1 - \hat{p}$$

Where $Z_{1-\frac{\alpha}{2}}$ is the Z- value leaving an area of $\frac{\alpha}{2}$ to the right.

EX(6):

A new rocket – launching system is being considered for deployment of small, short – rang rockets. The existing system has $p=0.8$ as the probability of a successful launch. A sample of **40** experimental launches is made with the new system and **34** are successful. Construct a 95% confidence interval for p

Solution:

a 95% confidence interval for *p* .

$$n = 40 \quad , \quad \hat{p} = \frac{34}{40} = 0.85 \quad , \quad \hat{q} = 0.15$$

$$at \quad 1-\alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \rightarrow Z_{1-\frac{\alpha}{2}} = 1.96$$

$$\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} \rightarrow 0.85 \pm (1.96)\sqrt{\frac{(0.85)(0.15)}{40}} \rightarrow 0.85 \pm (0.111)$$

$$0.739 < p < 0.961 \rightarrow P(0.739 < p < 0.961) = 0.95$$

See Ex 9.14
pg 297

Theorem 3:

If $\hat{p}$ is used as an estimate of *p* we can be $(1-\alpha)100\%$

confident that the error will not exceed $e = Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$.

EX 7:

In Ex. 7, find the error of p .

Solution:

The error will not exceed the following value:

$$e = Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} = (1.96)\sqrt{\frac{(0.85)(0.15)}{40}} = 0.111$$

Theorem :

If $\hat{p}$ is used as an estimate of p we can be $(1-\alpha)100\%$ confident that the error will be less than a specified amount **e** when the sample size is approximately:

$$n = \frac{Z^2_{1-\alpha/2} \quad \hat{p}\hat{q}}{e^2} \qquad (9)$$

Then the fraction of **n** is rounded up.

EX(8):

How large a sample is required in Ex. 7 if we want to be

95% confident that our estimate of *p* is within **0.02?**

Solution:

$$e = 0.02 \quad , \quad Z_{1-\frac{\alpha}{2}} = 1.96 \quad , \quad \hat{p} = 0.85 \quad , \quad \hat{q} = 0.15$$

$$n = \frac{(1.96)^2(0.85)(0.15)}{(0.02)^2} = 1224.51 \approx 1225$$

Two Samples: Estimating the difference between two proportions

Large- Sample confidence interval for p1-p2

If $\hat{p}_1$ and $\hat{p}_2$ are the two proportions of successes in random samples of sizes n1 and n2 respectively, $\hat{q}_1$=1-$\hat{p}_1$ and $\hat{q}_2$=1-$\hat{p}_2$, an approximate $(1-\alpha)100\%$ confidence interval for the difference of two binomial parameters , *p1-p2* is given by

$$(\hat{p}_1\text{-}\hat{p}_2) \pm z_{1-\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

where $Z_{1-\frac{\alpha}{2}}$ is the -value leaving an area of $\alpha/2$ to the right .

- Ex 9.17 pg 301

A certain change n the process for manufacturing component parts is being considered. Samples are taken under both the existing and the new process results in an improvement.

If **75 of 1500** items from the **existing process** are found to be defective, and **80 of 2000** items from the **new process** found to be defective, find a 90% confidence interval for the rue difference in the proportions of defectives for the existing and new process respectively.

- Solution

Let p1 and p2 be the true proportion of defectives for the existing and new process respectively.

$\hat{p}_1$=75/1500=0.05

$\hat{p}_2$=80/2000=0.04

$\hat{p}_1$-$\hat{p}_2$=0.05-0.04=0.01

$Z_{1-\frac{\alpha}{2}}$ =1.645

The 90% confidence interval is

$$0.01 \pm (1.645)\sqrt{\frac{(0.05)(0.95)}{1500} + \frac{(0.04)(0.96)}{2000}}$$

(-0.0017, 0.0217)

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(8 One- and Two-Sample Test Of Hypothesis)

(Book*: Chapter 10 ,pg319)

Probability& Statistics for Engineers & Scientists
By Walpole, Myers, Myers, Ye

Definition:

A statistical hypothesis is a statement concerning one population or more.

The Null and The Alternative Hypotheses:

The structure of hypothesis testing will be formulated with the use of the term null hypothesis. This refers to any hypothesis we wish to test that called . The rejection of H_0 leads to the acceptance of an alternative hypothesis H_1

A null hypothesis concerning a population parameter, will denoted by H_0 always be stated so as to specify an exact value of the parameter, θ whereas the alternative hypothesis allows for the possibility of several values. We usually test the null hypothesis: $H_0 : \theta = \theta_0$ against one of the following alternative hypothesis:

$$H_1 : \begin{Bmatrix} \theta \neq \theta_0 \\ \theta > \theta_0 \\ \theta < \theta_0 \end{Bmatrix}$$

Two Types of Errors:

	H_0 is true	H_0 is false
Accept H_0	Correct decision	Type II error, β
Reject H_0	Type I error, α	Correct decision

type I error: rejecting H_0 when H_0 is true.

Type II error: accepting H_0 when H_0 is false.

P (Type I error) =P (rejecting $H_0 \mid H_0$ is true) = α .

P (Type II error) = P (accepting $H_0 \mid H_0$ is false) =β .

Ideally we like to use a test procedure for which both the type I and type II errors are small.

* It is noticed that a reduction in β is always possible by increasing the size of the critical region, α .

* For a fixed sample size, decrease in the probability of one error will usually result in an increase in the probability of the other error.

* Fortunately the probability of committing both types of errors can be reduced by increasing the sample size.

Definition: Power of the Test:

The power of a test is the probability of rejecting H_0 given that a specific alternative hypothesis H_1 is true. The power of a test can be computed as $(1-\beta)$.

One – Tailed and Two – Tailed test:

A test of any statistical hypothesis where the alternative is one – sided such as:

$$H_0 : \theta = \theta_0 \quad vs \quad H_1 : \theta > \theta_0$$

$$or \quad H_1 : \theta < \theta_0$$

is called a one – tailed test

The critical region for the alternative hypothesis $H_1 : \theta > \theta_0$ lies entirely in the right tail of the distribution while the critical region for the alternative hypothesis $H_1 : \theta < \theta_0$ lies entirely in the left tail.

A test of any statistical hypothesis where the alternative is two – sided, such as: $H_0 : \theta = \theta_0 \quad vs \quad H_1 : \theta \neq \theta_0$ is called two – tailed test since the critical region is split into two parts having equal probabilities placed in each tail of the distribution of the test statistic.

The Use of P – Values in Decision Making:

Definition:

A p – value is the lowest level (of significance) at which the observed value of the test statistic is significant.

$P-value = 2P(Z > |Z_{obs}|)$ when H_1 is as follows: $H_1 : \theta \neq \theta_0$

$p-value = P(Z > Z_{obs})$ when H_1 is as follows: $H_1 : \theta > \theta_0$

$p-value = P(Z < Z_{obs})$ when H_1 is as follows: $H_1 : \theta < \theta_0$

H_0 is rejected if $p\text{-}value \leq \alpha$ otherwise H_0 is accepted.

EX(1):

$$H_0 : \mu = 10 \qquad vs \qquad H_1 : \mu \neq 10 \quad , \quad \alpha = 0.05 \Rightarrow Z = 1.87$$

$$P - value = 2P(Z > |1.87|) = 2P(Z > 1.87) = 2[1 - P(Z \leq 1.87)]$$

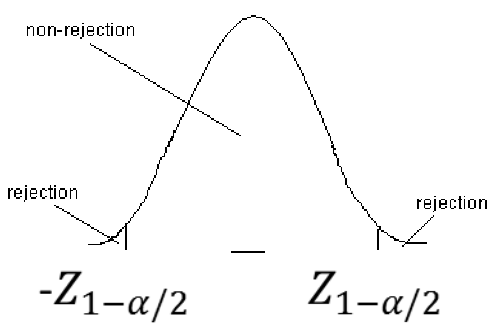
$$= 2[1 - 0.9693] = 2(0.0307) = 0.0614$$

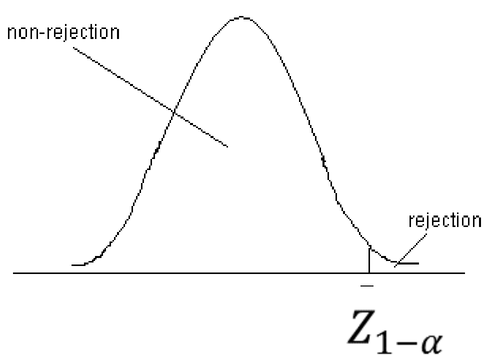
Since $P - value > \alpha$ then H_0 is accepted.

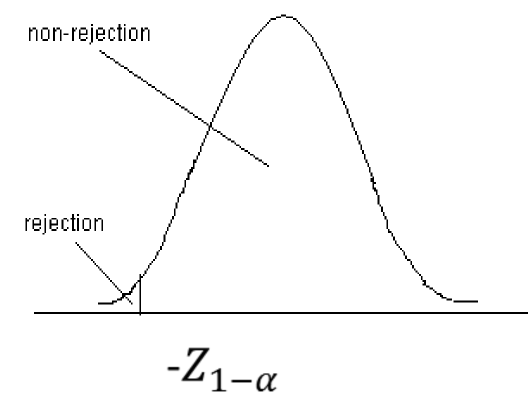
The Steps for testing a Hypothesis Concerning a Population Parameter θ (Reading):

1. Stating the null hypothesis H_0 that $\theta = \theta_0$.
2. Choosing an appropriate alternative hypothesis from one of the alternatives . $H_1 : \theta < \theta_0$ or $\theta > \theta_0$ or $\theta \neq \theta_0$
3. Choosing a significance level of size $\alpha = 0.01, 0.025, 0.05$ *or* 0.1

4. Determining the rejection or critical region (R.R.) and the acceptance region (A.R.) of H_0.







5- Selecting the appropriate test statistic and establish the critical region. If the decision is to be based on a p – value it is not necessary to state the critical region.

6. Computing the value of the test statistic from the sample data.

7. Decision rule:

A. rejecting H_0 if the value of the test statistic in the critical region or also $p - value \leq \alpha$

B. accepting H_0 if the value of the test statistic in the A.R. or if

$$p - value > \alpha$$

EX(2):

The manufacturer of a certain brand of cigarettes claims that the average nicotine content does not exceed **2.5** milligrams. State the null and alternative hypotheses to be used in testing this claim and determine where the critical region is located.

Solution:

$$H_0 : \mu = 2.5 \;\; against \;\; H_1 : \mu < 2.5$$

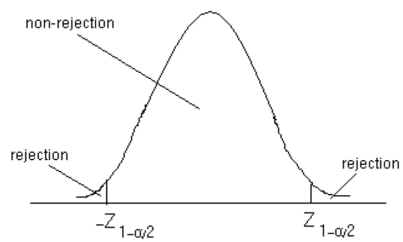
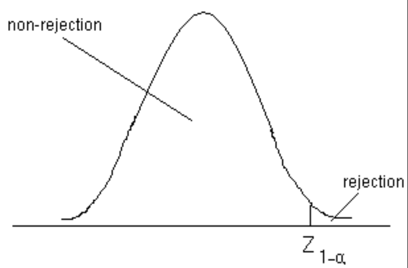
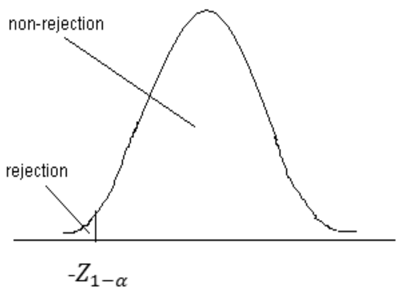
EX(3) H.w.:

A real state agent claims that 60% of all private residence being built today are **3** – bed room homes. To test this claim, a large sample of new residence is inspected, the proportion of the homes with **3** – bed rooms is recorded and used as our test statistic. State the null and alternative hypotheses to be used in this test and determine the location of the critical region.

Solution:

$$H_0 : p = 0.6 \qquad vs \qquad H_1 : p \neq 0.6$$

Tests Concerning a Single Mean

Hypothesis	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$
Test statistic (T.S.)	$Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}, \sigma \text{ known or } n \geq 30$		
R.R. and A.R. of H_0			
Decision	Reject H_0 (and accept H_1) at α the significance level if:		
	$Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$ Two − Sided Test	$Z > Z_{1-\alpha}$ One − Sided Test	$Z < -Z_{1-\alpha}$ One − Sided Test

EX 10.3 pg 338

A random sample of **100** recorded deaths in the United States during the past year showed an average life span of **71.8** years with a standard deviation of **8.9** years. Dose this seem to indicate that the average life span today is greater than **70** years? Use a **0.05** level of significance.

Solution:

$$H_0 : \mu = 70 \quad vs \quad H_1 : \mu > 70 \quad , \quad \alpha = 0.05$$

$$n = 100, \bar{X} = 71.8, \sigma = 8.9$$

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{71.8 - 70}{8.9 / \sqrt{100}} = 2.02$$

$$Z_{1-\alpha} = Z_{0.05} = Z_{0.95} = 1.645 \rightarrow R.R.: Z > 1.645$$

$$Z = 2.02 > Z_{1-\alpha} = 1.645$$

$$Since\ Z = 2.02 \in R.R. \rightarrow we\ reject\ H_0\ at\ \alpha = 0.05$$

Reject H_0 since the value of the test statistic is in the critical region (R.R.) or

$$P-value = P(Z > 2.02) = 1 - P(Z \leq 2.02) = 1 - 0.9783 = 0.0217$$

Reject H_0 since $\alpha > p-value$

EX 10.4 pg 338

A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has a standard deviation of **0.5** kilogram. Test the hypothesis that $\mu=8$ kilograms against the alternative that $\mu\neq8$ kilograms if a random sample of **50** lines is tested and found to have a mean breaking strength of **7.8** kilograms. Use a **0.01** level of significance.

Solution:

$$H_0 : \mu = 8 \quad vs \quad H_1 : \mu \neq 8 \quad , \quad \alpha = 0.01$$

$$n = 50, \bar{X} = 7.8, \sigma = 0.5$$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.8 - 8}{0.5/\sqrt{50}} = -2.82$$

$$Z_{1-\alpha/2} = Z_{0.995} = 2.575 \ and \ -Z_{1-\alpha/2} = -Z_{0.995} = -2.575$$

$$R.R.: Z > 2.575 \ or \ Z < -2.575$$

$$Since \ Z = -2.83 \in R.R. \rightarrow we \ reject \ H_0 \ at \ \alpha = 0.01$$

Reject H_0 since the value of **Z** is in the critical region (R.R.)

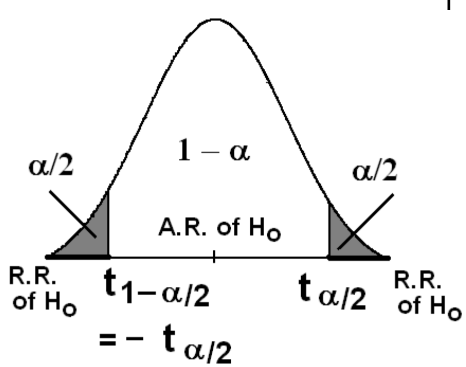
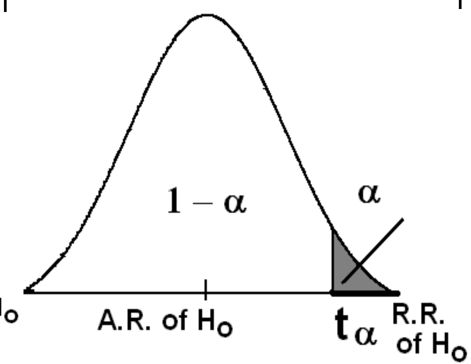
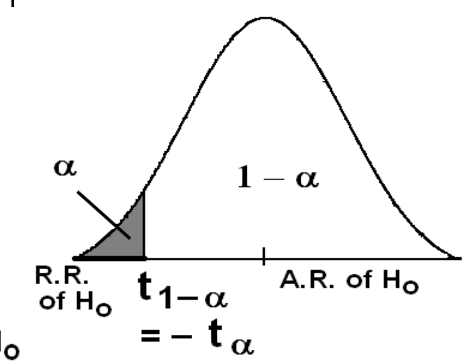
$$p - value = 2P(Z > |-2.83|) = 2P(Z > 2.83)$$

$$= 2(1 - P(Z \leq 2.83) = 2(1 - 0.9977)$$

$$= 2(0.0023) = 0.0046$$

H_0 is reject since $p - value \leq \alpha$

Tests Concerning a Single Mean (Variance Unknown)

<table>
<tr><td>Hypothesis</td><td>$H_0 : \mu = \mu_0$
$H_1 : \mu \neq \mu_0$</td><td>$H_0 : \mu = \mu_0$
$H_1 : \mu > \mu_0$</td><td>$H_0 : \mu = \mu_0$
$H_1 : \mu < \mu_0$</td></tr>
<tr><td>Test statistic
(T.S.)</td><td colspan="3">$T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}, \sigma \; unknown \; or \;\; n < 30$</td></tr>
<tr><td>R.R. and A.R. of H_0</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Decision</td><td colspan="3">Reject H_0 (and accept H_1) at the significance level α if:</td></tr>
<tr><td></td><td>$T > t_{\alpha/2}$ or
$T < -t_{\alpha/2}$
(Two- Sided Test)</td><td>$T > t_{\alpha}$
(One- Sided Test)</td><td>$T < -t_{\alpha}$
(One- Sided Test)</td></tr>
</table>

EX(10.5):

If a random sample of **12** homes with a mean $\bar{X} = 42$ included in a planned study indicates that vacuum cleaners expend an average of **42** kilowatt – hours per year with standard deviation of **11.9** kilowatt hours dose this suggest at the **0.05** level of significance that vacuum cleaners expend on the average less than **46** kilowatt hours annually, assume the population of kilowatt - hours to be normal?

Solution:

$$H_0 : \mu = 46 \quad vs \quad H_1 : \mu < 46 \quad , \quad \alpha = 0.05$$

$$n = 12, \bar{X} = 42, S = 11.9$$

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{42 - 46}{11.9/\sqrt{12}} = -1.16$$

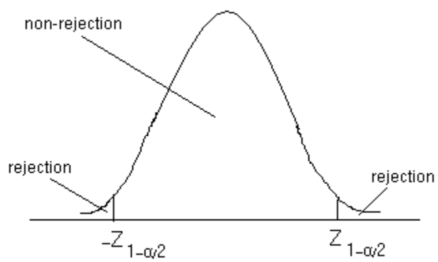
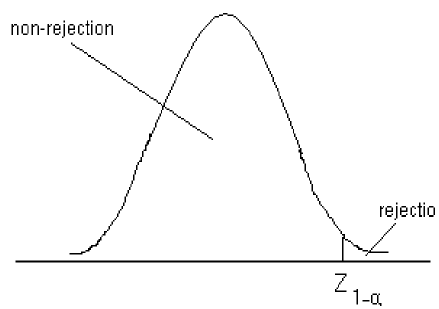
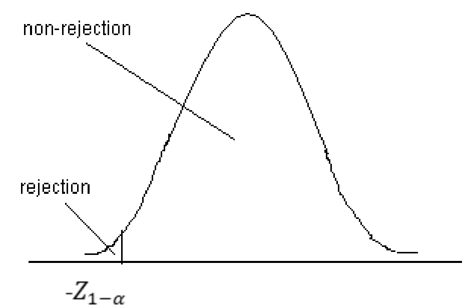
$$v = n - 1 = 11, -t_{-\alpha} = -\ t_{0.05,11=}\ -1.796$$

$$R.R.:T < -1.796$$

$$Since\ T = -1.16 \in A.R. \rightarrow we\ \ accept\ H_0\ \ at\ \ \alpha = 0.05$$

accept H_0 since the value of **t** is in the acceptance region (A.R.)

Tests Concerning Two Means

Hypothesis	$H_0: \mu_1 - \mu_2 = d$ $H_1: \mu_1 - \mu_2 \neq d$	$H_0: \mu_1 - \mu_2 = d$ $H_1: \mu_1 - \mu_2 > d$	$H_0: \mu_1 - \mu_2 = d$ $H_1: \mu_1 - \mu_2 < d$
Test statistic (T.S.)	$Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ $(if\ \sigma_1^2\ and\ \sigma_2^2\ are\ known)$ $or\ T = \dfrac{(\bar{X}_1 - \bar{X}_2) - d}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}, S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$ $(if\ \sigma_1^2 = \sigma_2^2 = \sigma^2\ is\ unknown\ but\ equal)$		
R.R. and A.R. of H_0			
Decision	Reject H_0 (and accept H_1) at the significance level α		
	$T.S. \in R.R.$ $Two - Sided\ Test$	$T.S. \in R.R.$ $One - Sided\ Test$	$T.S. \in R.R.$ $One - Sided\ Test$

EX(10.6 pg 344):

An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material **1** was tested, by exposing each piece to a machine measuring wear. Ten pieces of material **2** were similarly tested. In each case the depth of wear was observed. The samples of material **1** gave an average coded wear of **85** units with a standard deviation of **4** while the samples of material **2** gave an average coded wear of **81** and a standard deviation of **5.** Can we conclude at the **0.05** level of significance that the abrasive wear of material **1** exceeds that a material **2** by more than **2**units? Assume the population to be approximately normal with equal variances.

Solution:

Material 1	Material 2
$n_1 = 12$	$n_1 = 10$
$\bar{X}_1 = 85$	$\bar{X}_1 = 81$
$S_1 = 4$	$S_1 = 5$

$$H_0 : \mu_1 - \mu_2 = 2 \quad vs \quad H_1 : \mu_1 - \mu_2 > 2 \quad , \quad \alpha = 0.05 \quad ,$$

$$S_P = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{11(14) + 9(25)}{12 + 10 - 2}} = 4.478$$

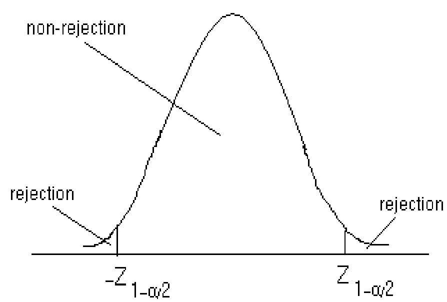
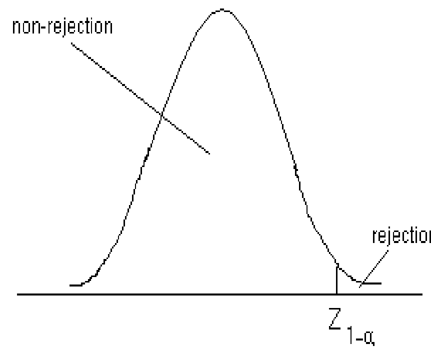
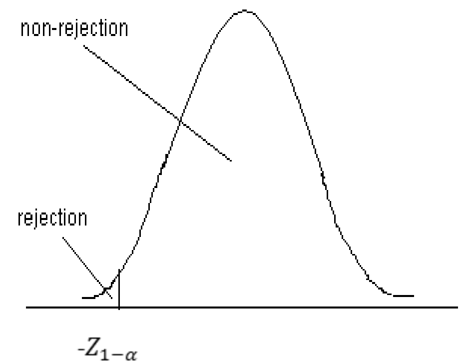
$$T = \frac{(\bar{X}_1 - \bar{X}_2) - d_0}{S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(85 - 81) - 2}{4.478\sqrt{\frac{1}{12} + \frac{1}{10}}} = 1.04$$

$$v = n_1 + n_2 - 2 = 20, t_{1-0.05,20} = t_{0.95,20} = 1.725 \rightarrow R.R.: T > 1.725$$

$$Since\ T = 1.04 \in A.R. \rightarrow we\ accept\ H_0\ at\ \alpha = 0.05$$

Accept H_0 since the value of **t** is in the acceptance region.

Tests Concerning Proportions

Hypothesis	$H_0: p = p_0$ $H_1: p \neq p_0$	$H_0: p = p_0$ $H_1: p > p_0$	$H_0: p = p_0$ $H_1: p < p_0$
Test statistic (T.S.)	$Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$ $\quad (q_0 = 1 - p_0)$		
R.R. and A.R. of H_0			
Decision	Reject H_0 (and accept H_1) at the significance level α if:		
	$Z > Z_{1-\alpha/2}$ $or\ Z < -Z_{1-\alpha/2}$ $Two - Sided\ Test$	$Z > Z_{1-\alpha}$ $One - Sided\ Test$	$Z < -Z_{1-\alpha}$ $One - Sided\ Test$

EX 10.9 pg 362

A builder claims that heat pumps are installed in 70% of all homes being constructed today in the city of Richmond. Would you agree with this claim if a random survey of new homes in this city shows that **8** out of **15** had heat significance? (α=0.1)

Solution:

$$H_0 : p = 0.7 \quad vs \quad H_1 : p \neq 0.7 \quad , \quad \alpha = 0.1$$

$$n = 15, X = 8 \quad , \hat{p} = \frac{X}{n} = \frac{8}{15} = 0.533$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.533 - 0.7}{\sqrt{\frac{(0.7)(0.3)}{15}}} = -1.42$$

$$Z_{1-\alpha/2} = Z_{1-0.05} = Z_{0.95} = 1.645, -Z_{1-0.05} = -1.645$$

$$R.R.: Z > 1.645 \quad or \quad Z < -1.645$$

$$Since \quad Z = -1.42 \in \text{A.R} \rightarrow we \ \ accept \ \ H_0 \ \ at \ \ \alpha = 0.1$$

$$p - value = 2P(Z > |-1.42|) = 2P(Z > 1.42)$$

$$= 2P(1 - P(Z \le 1.42)) = 2(1 - 0.9222)$$

$$= 2(0.0778) = 0.1556$$

Accept H_0 since $\alpha < p - value$

Testing for the two populations Proportions

If we have two independent samples of size and with proportions and respectively. Thus, we will use the following steps:

1-data needed: x_1 , $\hat{p}_1 = \frac{a_1}{n_1}$ and x_2 , $\hat{p}_2 = \frac{a_2}{n_2}$

2- the hypothesis: $H_0 : P_1 = P_2 \longrightarrow P_1 - P_2 = 0$

$$H_1 : \begin{cases} P_1 < P_2 \longrightarrow P_1 - P_2 < 0 \\ P_1 > P_2 \longrightarrow P_1 - P_2 > 0 \\ P_1 \neq P_2 \longrightarrow P_1 - P_2 \neq 0 \end{cases}$$

3- the statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where} \quad \hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_1}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$

4- Determining the rejection of H_0, that is:

i)if $H_1 : P_1 > P_2 \longrightarrow P_1 - P_2 > 0$,reject H_0 if $Z > Z_{1-\alpha}$

ii) if $H_1 : P_1 < P_2 \longrightarrow P_1 - P_2$,reject H_0 if $Z < Z_\alpha$

iii) if $H_1 : P_1 \neq P_2 \longrightarrow P_1 - P_2 \neq 0$, reject H_0 if $Z > Z_{1-\frac{\alpha}{z}}$ or $Z < Z_{\frac{\alpha}{z}}$

Ex (9):

Two machine A and B, a random sample of size 300 units from machine A with defective proportion 8% and another sample of size 200 units from machine B with defective proportion 4%. The manager think that the defective proportion from machine A is differ from the defective proportion from machine B, is he right?. use $\alpha = 0.05$

Solu.

1-data needed: $n_1 = 300$, $x_1 = 0.08$ and $n_2 = 200$, $x_2 = 0.04, \alpha = 0.05$

2- the hypothesis: $H_0: P_1 = P_2 \longrightarrow P_1 - P_2 = 0$

$H_1: P_1 \neq P_2 \longrightarrow P_1 - P_2 \neq 0$

3- the statistic:

$$Z = \frac{0.08 - 0.04}{\sqrt{0.064(1-0.064)\left(\frac{1}{300} + \frac{1}{200}\right)}} = 0.895$$

where $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_1}{n_1 + n_2} = \frac{300(0.08)+200(0.04)}{500} = 0.064$

4- reject H_0 if $Z < Z_{\frac{\alpha}{2}} = Z_{0.025} = -1.96$ or $Z \rangle Z_{1-\frac{\alpha}{z}} = 1.96$

Thus , we accept H_0 and reject H_1 that says there is a difference between the defective proportions from machines A and B.

Discrete Distributions :Chapter 4 -4(2)

Name	Probability mass function	Values of X	mean	variance	parameters
Binomail distribution	$f(x) = \binom{n}{x} p^x q^{n-x}$,q=1-p	X=0,1,2,..,n	$\boldsymbol{\mu = np}$	$\sigma^2 = npq$	n, p
Poisson distribution	$f(x) = \frac{e^{-\mu}\mu^x}{x!}$ $,\mu = \lambda t$	X=0,1,2,….	$\boldsymbol{\mu} = \lambda t$	$\sigma^2 = \lambda t$	λ
Discrete Uniform distribution	$f(x) = \frac{1}{k}$	X=x_1,x_2,….,x_k	$\boldsymbol{\mu = \frac{\sum x_i}{k}}$	$\sigma^2 = \frac{\sum(x_i - \mu)^2}{k}$	-
Hypergeometric distribution	$f(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$	X=0,1,2,..,n	$\boldsymbol{\mu = \frac{nk}{N}}$	$\sigma^2 = \frac{N-n}{N-1} n \frac{k}{N}(1 - \frac{k}{N})$	N, K, n

Continuous Distributions :Chapter 5 -5(2)

Name	Probability density function	Values of X	mean	variance	parameters
Continuous Uniform distribution	$f(x) = \frac{1}{B-A}$	$A \le x \le B$	$\boldsymbol{\mu} = \frac{A+B}{2}$	$\sigma^2 = \frac{(B-A)^2}{12}$	$-$
Exponential distribution	$f(x) = \frac{1}{\beta} e^{-\frac{x}{B}}$	$x > 0$	$\boldsymbol{\mu} = \beta$	$\sigma^2 = \beta^2$	β
Normal distribution	Transform to Z and use tables	$-\infty < x < \infty$	$\boldsymbol{\mu}$	σ^2	μ, σ^2

$How\ to\ use\ Z - table$

المطلوب ايجاد قيمة الاحتمال

الحالة الأولى اقل من

الحالة الأولى اقل من
استخدم الجدول مباشرة

$$P(Z \le \underbrace{1.5}_{\text{column}}\underbrace{0}_{\text{row}}) =$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	**0.00**
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	**0.10**
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	**0.20**
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	**0.30**
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	**0.40**
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	**0.50**
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	**0.60**
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	**0.70**
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	**0.80**
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	**0.90**
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	**1.00**
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	**1.10**
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	**1.20**
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	**1.30**
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	**1.40**
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	**1.50**
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	**1.60**
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	**1.70**
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	**1.80**
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	**1.90**

أ /خلود باسالم

الحالة الثانية اكبر من

احول الى اقل من بستخدام المكملة

$$P(Z \ge 0.98) = 1 - P(Z \le \underbrace{0.9}_{\text{column}}\underbrace{8}_{\text{row}})$$

= 1 - 0.83646

=0.16354

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	0.00
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	0.10
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	0.20
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	0.30
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	0.40
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	0.50
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	0.60
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	0.70
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	0.80
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	0.90
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	1.00
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	1.10

طريقة اخرى للحل

احول الى اقل من والرقم باشارة مخالفة

$$P(Z \ge 0.98) = P(Z \le -0.98) = 0.16354$$

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	-0.90
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	-0.80
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	-0.70
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	-0.60

الحالة الثالثة المحصورة بين قيمتين

القاعدة

$$P(a \le Z \le b) =$$

$$P(Z \le b) - P(Z \le a)$$

$$P(-1.33 \le Z \le 2.42) =$$

$$P(Z \le 2.42) - P(Z \le -1.33)$$

colum row colum row

= 0.99224 - 0.09176

= 0.90048

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	[illegible]	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	0.00
0.10	0.53983	0.54380	0.[illegible]4776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	0.10
0.20	0.57926	0.58317	0.[illegible]8706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	0.20
0.30	0.61791	0.62172	0.[illegible]2552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	0.30
0.40	0.65542	0.65910	0.[illegible]6276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	0.40
0.50	0.69146	0.69497	0.[illegible]9847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	0.50
0.60	0.72575	0.72907	0.[illegible]3237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	0.60
0.70	0.75804	0.76115	0.[illegible]6424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	0.70
0.80	0.78814	0.79103	0.[illegible]9389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	0.80
0.90	0.81594	0.81859	0.[illegible]2121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	0.90
1.00	0.84134	0.84375	0.[illegible]614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	1.00
1.10	0.86433	0.86650	0.[illegible]864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	1.10
1.20	0.88493	0.88686	0.[illegible]877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	1.20
1.30	0.90320	0.90490	0.[illegible]658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	1.30
1.40	0.91924	0.92073	0.[illegible]220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	1.40
1.50	0.93319	0.93448	0.[illegible]574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	1.50
1.60	0.94520	0.94630	0.[illegible]738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	1.60
1.70	0.95543	0.95637	0.[illegible]728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	1.70
1.80	0.96407	0.96485	0.[illegible]562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	1.80
1.90	0.97128	0.97193	0.[illegible]257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	1.90
2.00	0.97725	0.97778	0.[illegible]831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	2.00
2.10	0.98214	0.98257	0.[illegible]300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	2.10
2.20	0.98610	0.98645	0.[illegible]679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	2.20
2.30	0.98928	0.98956	0.[illegible]983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	2.30
2.40	[illegible]	[illegible]	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	2.40

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	[illegible]	0.00022	0.00022	0.00023	-3.50
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00[illegible]0	0.00031	0.00032	0.00034	-3.40
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00[illegible]3	0.00045	0.00047	0.00048	-3.30
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00[illegible]2	0.00064	0.00066	0.00069	-3.20
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00[illegible]7	0.00090	0.00094	0.00097	-3.10
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00[illegible]2	0.00126	0.00131	0.00135	-3.00
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00[illegible]9	0.00175	0.00181	0.00187	-2.90
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00[illegible]3	0.00240	0.00248	0.00256	-2.80
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00[illegible]7	0.00326	0.00336	0.00347	-2.70
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.004[illegible]7	0.00440	0.00453	0.00466	-2.60
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00[illegible]0	0.00587	0.00604	0.00621	-2.50
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00[illegible]5	0.00776	0.00798	0.00820	-2.40
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00[illegible]0	0.01017	0.01044	0.01072	-2.30
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01[illegible]7	0.01321	0.01355	0.01390	-2.20
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01[illegible]9	0.01700	0.01743	0.01786	-2.10
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02[illegible]8	0.02169	0.02222	0.02275	-2.00
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02[illegible]0	0.02743	0.02807	0.02872	-1.90
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03[illegible]2	0.03438	0.03515	0.03593	-1.80
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04[illegible]2	0.04272	0.04363	0.04457	-1.70
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05[illegible]5	0.05262	0.05370	0.05480	-1.60
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06[illegible]1	0.06426	0.06552	0.06681	-1.50
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07[illegible]6	0.07780	0.07927	0.08076	-1.40
-1.30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.09176	0.09342	0.09510	0.09680	-1.30
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	-1.20

الحالة الرابعة المساواة دائماً الناتج صفر

$$P(Z = 1.50) = 0$$

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$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$	$\bar{X} \pm t_{1-\frac{\alpha}{2}} . \frac{S}{\sqrt{n}}$
$Z = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$(\bar{X}_1 - \bar{X}_1) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
$T = \frac{(\bar{X}_1 - \bar{X}_2) - d}{S_P . \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$	$(\bar{X}_1 - \bar{X}_1) \pm t_{1-\frac{\alpha}{2}} . S_P . \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
$Z = \frac{\hat{P} - P_0}{\sqrt{\frac{p_0 q_0}{n}}}$	$S_P^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$
$n = \frac{Z_{1-\frac{\alpha}{2}}{}^2 \hat{p}\hat{q}}{e^2}$	$\hat{P} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$n = (\frac{Z_{1-\alpha/2}\sigma}{e})^2$	$e = Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$,	$e = Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

Values of Z	
$Z_{0.90}$	**1.285**
$Z_{0.95}$	**1.645**
$Z_{0.97}$	**1.885**
$Z_{0.975}$	**1.96**
$Z_{0.98}$	**2.055**
$Z_{0.99}$	**2.325**
$Z_{0.995}$	**2.575**