MATH203 Calculus

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Outline

- Ratio Test.
- Root Test.

The Ratio Test

Let $\sum_{n=1}^{\infty} a_n$ be a positive term series and suppose that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$, then

- If $L < 1$ the series $\sum_{n=1}^{\infty} a_n$ converges.
- If $L > 1$ the series $\sum_{n=1}^{\infty} a_n$ diverges.
- If $L = 1$ (fails), the series may converge or diverge.

Examples:

(1) $\sum_{n=1}^{\infty} n!$ (2) $\sum_{n=1}^{\infty} \frac{1}{(n+1)!}$ (3) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ (4) $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$.

The Root Test

Let $\sum_{n=1}^{\infty} a_n$ be a positive term series and suppose that $\lim_{n\to\infty} \sqrt[n]{a_n} = L$, then

- the series $\sum_{n=1}^{\infty} a_n$ converges if $L < 1$.
- the series $\sum_{n=1}^{\infty} a_n$ diverges if $L > 1$.
- If $L = 1$ (fails), the series may converge or diverge.

Examples:

(1) $\sum_{n=1}^{\infty} \frac{5^n}{n^n}$ (2) $\sum_{n=1}^{\infty} \left(\frac{8n^2-7}{n+1}\right)^n$ (3) $\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n^n}$