

9

Ch. 4 - Part 1

- Introduction.
- Sampel Spaces and Probability.

STAT.110

جمال السعدي
رياضيات - إحصاء



Ch. 4 Part. 1

• sample space	فراغ العينه	• Product	حاصل ضرب
• Experiment	تجربه	• Certain occur	حادثة مؤكدة
• probability	احتمال	• Never occur	حادثة مستحيلة
• Toss	القاء	• Complement	حادثة مكمل
• coin	قطعه نقود	• Outcomes	نواتج (عناصر)
• Roll	يتدحرج (القاء)	• Empirical	تجريبي (مبني على الملاحظة)
• Dice	حجر نرد	• Subjective	وهي (مبني على التخمين)
• Tree diagram	الشجرة البيانية	• Urn	صندوق
• Head	وجه القطعه النقدية	• Drawn	يسحب
• Tail	خلفيه القطعه النقدية	• Mutually exclusive	متنافية
• Event	حادثة	• Recent Study	دراسة حديثة
• Simple event	حادثة بسيطه	• Common	مشترك
• Odd number	عدد فردي	• Exactly	بالضبط
• Prime number	عدد اولي	• Contain	يحتوى على
• Even number	عدد زوجي	• Consists of	يتكون من
• Compound event	حدث مركب	• Select	يختار
• Random	عشوائي	• At least	على الأقل
• Gender	نوع	• At most	على الأكثر

Sample Spaces and Probability

A probability experiment

A chance leads to defined results called outcomes

An outcome

Is the result of a single trial of a probability experiment.

A sample space

Is the set of all possible outcomes.

An event

Consists of a set of outcomes of a probability experiment.

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Equally likely events

Are events that have the same probability of occurring.

Classical Probability

The probability of any event E

$$\text{Is } P(E) = \frac{\text{Number of outcomes in E}}{\text{Total number of outcomes in the sample space}} \quad P(E) = \frac{n(E)}{n(S)}$$

Empirical probability

Based on observation

$$P(E) = \frac{\text{Frequency for the class}}{\text{Total frequency in the distribution}}$$

Subjective probability

Based on estimate and inexact information

Simple event

Is an event contain one outcome.

Compound event

Is an event contain more than one outcome

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Coin

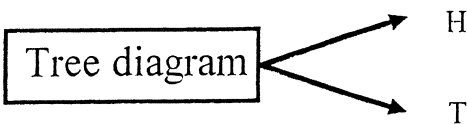
- The number of outcomes in the sample space:

$n(S) = 2^N$ \longrightarrow عدد الرميات أو عدد القطع

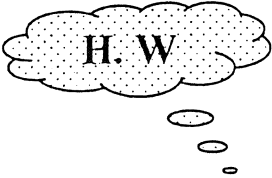
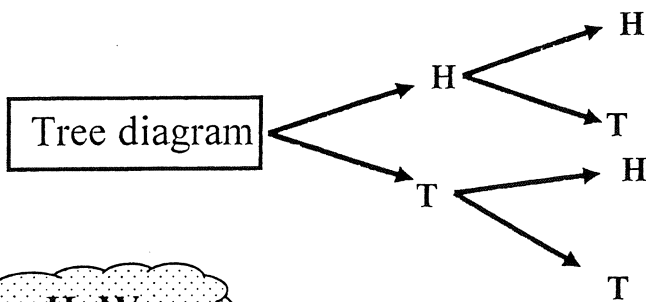
Find the sample space for:

(1) Toss one coin $\rightarrow S = \{H, T\}$ $\longrightarrow N(s) = 2^1 = 2$

\swarrow \searrow
 Head Tail



(2) Toss two coins $\rightarrow S = \{HH, HT, TH, TT\}$ $\longrightarrow N(s) = 2^2 = 2 \times 2 = 4$



(3) Toss 3 Coins

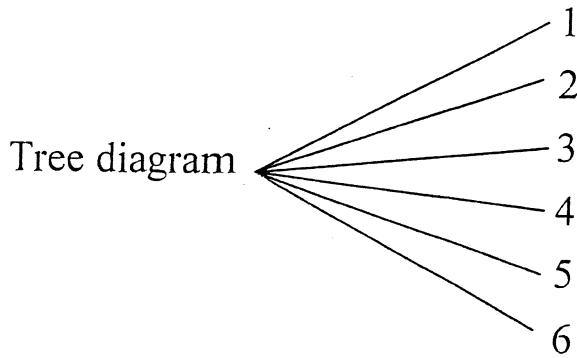
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Dice

The number of outcomes in the sample space.

$n(S) = 6^N$ → عدد الرميات أو عدد القطع

• Roll a dice → $S = \{1, 2, 3, 4, 5, 6\}$ → $N(s) = 6^1 = 6$



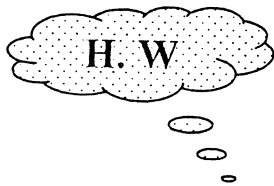
A is even number = {2, 4, 6}

B is odd number = {1, 3, 5}

C is prime number = {2, 3, 5}

D = {4} is simple event

A and B are mutually exclusive: where $A \cap B = \phi$



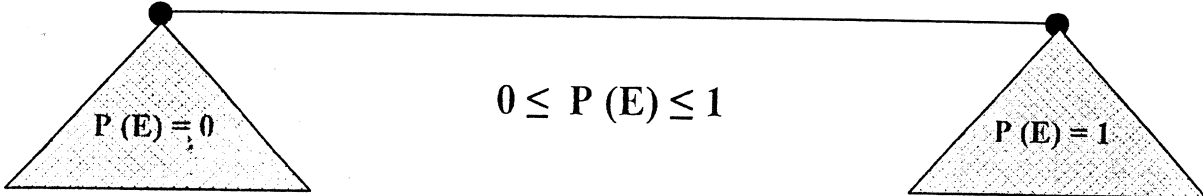
• Roll two dice → $S = \dots\dots\dots$, $n(s) = \dots\dots\dots$

Tree diagram.....

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Probability Rules

For any event E



- The Range of the values of the probability = $[0, 1]$ *****
- $P(E) = 0$ where E can never occur.
- $P(E) = 1$ where E certain occur.
- $\sum_{i=1}^n P(a_i) = 1$

The sum of the probabilities of all the outcomes in the sample space equal 1.

$$\bullet P(E') = 1 - P(E) \quad \rightarrow P(E) + P(E') = 1$$

Where E' is the complement of E

If $s = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3, 4\} \rightarrow A' = \{5, 6\}$

$$P(A) = \frac{4}{6} \quad \rightarrow \quad P(A') = \frac{2}{6}$$

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11. Classify each statement as an example of classical probability, empirical probability, or subjective probability.

a) The probability that a person will watch the 6 o' clock evening news is 0.15 → (Empirical)

b) The probability of winning at a Chuck-a-Luck game is $\frac{5}{36}$ → (Classical)

c) The probability that a bus will be in an accident on a specific run is about 6% → (Empirical)

d) The probability of getting a royal flush when five cards are selected at random is $\frac{1}{649.740}$ → (Classical)

e) The probability that a student will get a C or better in a statistics course is about 70% → (Empirical)

f) The probability that a new fast-food restaurant will be a success in Chicago is 35% → (Empirical)

g) The probability that interest rates will rise in the next 6 months is 0.50 → (Subjective)

Note

Empirical	Classical	Subjective
<ul style="list-style-type: none"> • نسبة مئوية • فاصلة عشرية 	<ul style="list-style-type: none"> • كسور 	<ul style="list-style-type: none"> • يبنى على التخمين ويشير إلى المستقبل
		Next.....

10. A probability experiment is conducted. Which of these cannot be considered a probability of an outcome?

a. $\frac{1}{3}$

b. $-\frac{1}{5}$

c. 0.80

d. -0.59

e. 0

f. 1.45

g. 1

h. 33%

i. 112%

Solution

(b) $-\frac{1}{5}$

(d) -0.59

(f) 1.45

(i) $112\% = 1.12$

Are can not be considered a probability of an outcome.

Where $0 \leq P(E) \leq 1$

12. If a die is rolled one time, find these probabilities.

- Of getting a 4.
- Of getting an even number.
- Of getting a number greater than 4.
- Of getting a number less than 7.
- Of getting a number greater than 0.
- Of getting a number greater than 3 or an odd number.
- Of getting a number greater than 3 and an odd number.

Solution

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$(a) \quad a = \{4\} \rightarrow P(a) = \frac{1}{6}$$

$$(b) \quad b = \{2, 4, 6\} \rightarrow P(b) = \frac{3}{6} = \frac{1}{2}$$

$$(c) \quad c = \{5, 6\} \rightarrow P(c) = \frac{2}{6} = \frac{1}{3}$$

$$(d) \quad d = \{1, 2, 3, 4, 5, 6\} \rightarrow P(d) = \frac{6}{6} = 1 \rightarrow \text{certain occur}$$

$$(e) \quad e = \{1, 2, 3, 4, 5, 6\} \rightarrow P(e) = \frac{6}{6} = 1 \rightarrow \text{certain occur}$$

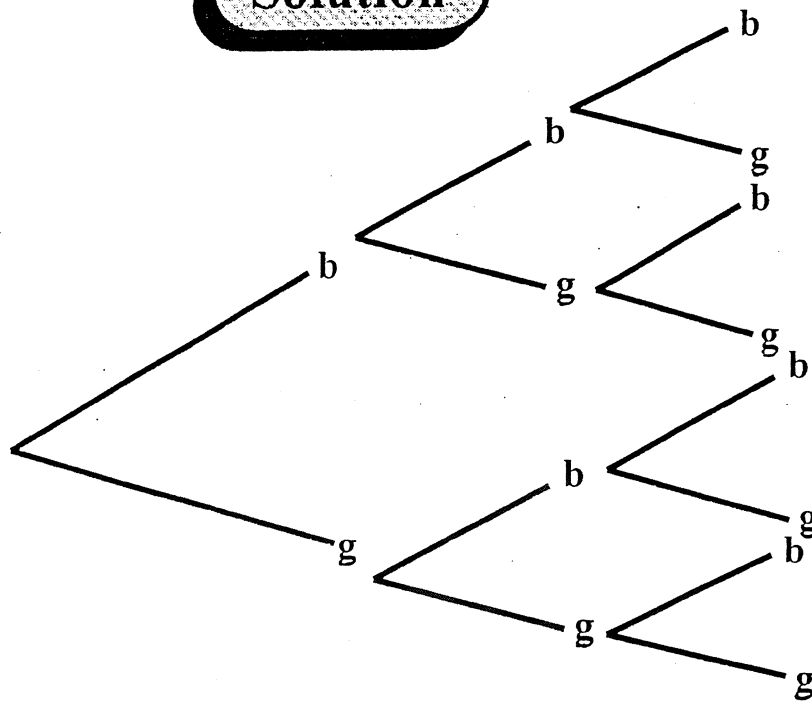
$$(f) \quad f = \{4, 5, 6, 1, 3\} \rightarrow P(f) = \frac{5}{6}$$

$$(g) \quad g = \{5\} \rightarrow p(g) = \frac{1}{6}$$

Example:

A couple has three children find each probability:

- All boys
- All girls or all boys
- Exactly two boys or two girls.
- At least one child of each gender.

Solution

$$S = \{ bbb , bbg , bgb , bgg , gbb , gbg , ggb , ggg \}$$

$$(a) \quad P(\text{all boys}) = \frac{1}{8}$$

$$(b) \quad P(\text{all girls or all boy}) = \frac{2}{8} = \frac{1}{4}$$

$$(c) \quad P(\text{Exactly two boys or two girls}) = \frac{6}{8} = \frac{3}{4}$$

$$(d) \quad P(\text{at least one child of each gender}) = \frac{6}{8} = \frac{3}{4}$$

13. if two dice are rolled one time, find the probability of getting these results.

- A sum of 6.
- Doubles.
- A sum of 7 or 11.
- A sum greater than 9.
- A sum less than or equal to 4.

Solution

$$S = \{(1,1), (1,2), \dots, (1,6) \\ (2,1), (2,2), \dots, (2,6) \\ (6,1), (6,2), \dots, (6,6)\}$$

$$(a) a = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$$

$$P(a) = \frac{5}{36}$$

$$(b) b = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$P(b) = \frac{6}{36} = \frac{1}{6}$$

$$(c) c = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (5,6), (6,5)\}$$

$$P(c) = \frac{8}{36} = \frac{2}{9}$$

$$(d) d = \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}$$

$$P(d) = \frac{6}{36} = \frac{1}{6}$$

$$(e) e = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$$

$$P(e) = \frac{6}{36} = \frac{1}{6}$$

Roll two dice and multiply the number together.

- Write out the sample space.
- What is the probability that the product is a multiple of 6 ?
- What is the probability that the product is less than 10?

Solution

a) $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
 $(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
 $(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
 $(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
 $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
 $(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

b) $P(\text{Product is multiple of } 6) = \frac{15}{36}$

c) $P(\text{Product is less than } 10) = \frac{17}{36}$

19. For a recent year, 51% of the families in the United States had no children under the age of 18; 20% had one children 19% had two children; 7% had three children; and 3% had four or more children. If a family is selected at random, find the probability that the family has:

- Two or three children
- More than one child
- Less than three children
- Based on the answers to parts a, b, and c, which is most likely to occur? Explain why.

Solution

Information's:

$$* P(0 \text{ children}) = 0.51 \quad * P(1 \text{ children}) = 0.20$$

$$* P(2 \text{ children}) = 0.19 \quad * P(3 \text{ children}) = 0.07$$

$$* P(4 \text{ children or more}) = 0.03$$

$$(a) P(2 \text{ or } 3 \text{ children})$$

$$= P(2) + P(3)$$

$$= 0.19 + 0.07$$

$$= \underline{\underline{0.26}}$$

$$(b) P(\text{more than one children})$$

$$= P(2) + P(3) + P(4 \text{ children or more})$$

$$= 0.19 + 0.07 + 0.03$$

$$= \underline{\underline{0.29}}$$

$$(c) P(\text{less than three children})$$

$$= P(0) + P(1) + P(2)$$

$$= 0.51 + 0.20 + 0.19$$

$$= \underline{\underline{0.90}}$$

(d) In part c the event is most likely to occur.

Because the probability is greeter than any one.

الحادثه وكمليتها

An event and its complement are ...

- A) mutually exclusive B) not mutually exclusive C) independent D) dependent

الحادثه وكمليتها دائماً متنافيتاه

↓ mutually exclusive

Two events are ... if they cannot occur at the same time.

- A) not mutually exclusive C) dependent events
B) independent events D) mutually exclusive

تعريف
الحادثتاه
المتنافيتاه

Two events that can occur at the same time are called ...

- A) mutually exclusive. B) not mutually exclusive. C) independent. D) dependent.

لاحظ أنه :

* Can not occur at the same time \Rightarrow mutually exclu.

* can " " " " " \Rightarrow not mutually exclu.

• أن وجود not في احد طرفي الجملة وعدم وجودها في الطرف الآخر.

^{نكته}
The complement of guessing 10 incorrect answers on a 10-question true/false exam is

- A) guessing 10 incorrect answers C) guessing at least 1 correct answer
B) guessing at least 1 incorrect answer D) guessing 10 correct answers

Complement: guessing 10 incorrect answers
is \Rightarrow guessing at least 1 correct

When 10 adults are tested for high blood pressure, the complement of at least one of the results are positive.

- A) None of the adults have high blood pressure. C) All of the adults have high blood pressure.
B) Nine of the adults have high blood pressure. D) One of the adults have high blood pressure.

^{نكته واحد من الأهل على الإيجابي}
Complement of at least one are positive
^{لا أحد الإيجابي}
None of the adults have high blood pressure

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تأثر بـ

Two events are said to be ... if the probability of one event occurring is affected by the occurrence of the other event.

A) not mutually exclusive B) independent C) dependent D) mutually exclusive

إذا كان احتمال حدوث حادثة تأثر بـ حدوث حادثة أخرى .

تعريف

فإنه يقال أنه الحادتان مرتبطتان
(أي غير مستقلتان)

→ dependent.

84) probability uses a frequency distribution to compute probabilities.
A) Classical B) Empirical C) Subjective D) continuous

التوزيع التكراري
(Use frequency distribution)

→ تجريبي Empirical

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Consider this table to answer the following two questions:

Cookie Types	Number Sold
Chocolate Chip	20
Peanut Butter	15
Oatmeal	30
Sugar	10
	$\Sigma = 75$

243

What is the level of measurement for the cookie type

- A) Discrete B) Ordinal C) Nominal D) Continuous

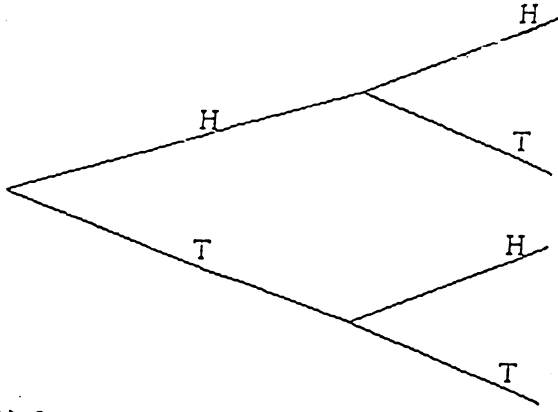
نوع الكيك
مقياس أسماء

What is the probability of selecting a chocolate chip cookie?

- A) 4/15 B) 1/2 C) 1/15 D) 1/4

$$P(\text{chocolate chip}) = \frac{20}{75} = \frac{4}{15}$$

How many times was the coin tossed in the figure below?



- A) 3 B) 6 C) 4 D) 2

من الشكل
كم عدد مرات
القاء قطعة النقود

« مرتين »

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A student and a professor each choose a number between 1 and 5 (1 and 5 are both possible choices). What is the probability that the two choose the same odd number?

- A) 0.04 B) 0.03 C) 0.12 D) 0.16

		Professor				
		1	2	3	4	5
Student	1	(1,1)				
	2					
	3			(3,3)		
	4					
	5					(5,5)

اختيار نفس العدد الفردي

$$P = \frac{n(A)}{n(S)} = \frac{3}{25} = 0.12$$

عشر واحد

Consider the experiment of selecting one item at random from a box containing an equal number of defective (D) and non-defective (N) items. The sample space for this experiment is

- A) 2 B) $S = \{D, N\}$ C) 4 D) $S = \{DD, DN, ND, NN\}$

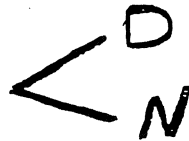
صندوق يحتوي على defective (D)
 عيب



non-defective (N)
 غير عيب

حسب عشر واحد فقط اما $N \cup D$

$$S = \{D, N\}$$



The complement of guessing at least 5 correct answers on a 6-question true/false exam is

- A) guessing 6 incorrect answers
 B) guessing at least 2 incorrect answer
 C) guessing at least 2 correct answer
 D) guessing 6 correct answers

البادئة المكملة لتخمين 5 إجابات صحيحة على الأقل من ضمن
 6 إجابة .

على الأقل 2 خطأ
 guessing at least 2 incorrect

at least 5 correct (الكادئة) → at least 2 incorrect (المكمل)

$$S = \{ (6\checkmark, 0X), (5\checkmark, 1X), (4\checkmark, 2X), (3\checkmark, 3X), (2\checkmark, 4X), (1\checkmark, 5X), 6X \}$$

The complement of guessing 4 correct answers on a 4-question true/false exam is

- A) guessing 4 incorrect answers
 B) guessing at least 1 incorrect answer
 C) guessing at least 1 correct answer
 D) guessing no incorrect answers

Complement "All correct"

= At least 1 incorrect

$$S = \{ (4 \text{ correct}), (3 \text{ correct and } 1 \text{ incorrect}), (2 \text{ correct and } 2 \text{ incorrect}), (1 \text{ correct and } 3 \text{ incorrect}), (4 \text{ incorrect}) \}$$

Use the following to answer questions

Two dice are rolled. Let X represents the summation of the two faces that will appear.

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

The probability of $X \geq 10$ is ...

- A) 0.167 B) 0 C) 0.028 D) 0.083

X : is sum. of two faces

$$X \geq 10 = \{ (4,6), (5,5), (5,6), (6,4), (6,5), (6,6) \}$$

$$P(X \geq 10) = \frac{\text{عدد الأزواج المرتبة السابقة}}{\text{العدد الكلي للأزواج المرتبة}} = \frac{6}{36} = 0.166 \approx \boxed{0.167}$$

The probability of $X \leq 12$ is ...

- A) 0 B) 0.083 C) 0.056 D) 1

$X \leq 12$ مجموع الوجوه من أقل من أو يساوي 12

عدد الأزواج المرتبة التي تحقق هذا الشرط 36

$$\therefore P(X \leq 12) = \frac{36}{36} = \boxed{1}$$

جاريته يؤكدته certain occur

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Use the following to answer questions

مجموع الوجوه

If two dice are rolled and X is the random variable represents the summation of two faces that will appear

		Die 1					
		Sums	1	2	3	4	5
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

7) The probability of $X=4$ is

- A) 0.083 B) 0.139 C) 0.861 D) 0.917

$$P(X=4) = \frac{3}{36} = 0.083$$

مجموع الوجوه
4

8) The probability of $X=12$ is

- A) 0.972 B) 0.917 C) 0.028 D) 0

$$P(X=12) = \frac{1}{36} = 0.028$$

مجموع الوجوه
12

When 2 dice (6-sided each) are rolled, find the probability of getting a sum of 13

- A) 0.25 B) 0.72 C) 0.17 D) 0

مجموع الوجوه

13

ساده مستحيله

لانه اكبر مجموع ممكن الحصول عليه هو 12 من (6,6)

$$P(\text{sum of } 13) = P(\emptyset) = 0$$

عدد زوجي

If a die were rolled, the event of getting an even number would be called a

- A) compound event B) simple event C) sample space D) sample size

حدث الحصول على عدد زوجي { 2, 4, 6 }
تكون من أكثر من عنصر

Compound event
حدث مركب.

If there is a 20% chance that it will rain tomorrow, what is the probability that it will not rain tomorrow?

تتطر غدًا

لا تتطر

- A) 0.08 B) 0.20 C) 0.80 D) 0

$$P(\text{rain}) = 20\% = 0.20$$

$$P(\text{not rain}) = 1 - P(\text{rain}) = 1 - 0.20$$

متساوية

If there are 20 equally likely events, then the probability of the second one occurring is

- A) 1 B) 1/10 C) 1/20 D) 0

يوجد 20 حدثه متساوية (لها نفس احتمالية الحدوث)

∴ احتمال حدوث امر حادثه منهم

$$\frac{1}{20}$$

* لو كانت عدد الحوادث 50 كانت الاجابة $\frac{1}{50}$

The number of outcomes in a compound event can be ...

- A) $E=\{2, 4, 6\}$ B) 3 C) 1 D) $E=\{3\}$

عدد عناصر الحادثة المركبة (أكثر من عنصر)
ليوليس على شكل مجموع { }

What type of probability uses sample spaces to determine the numerical probability that an event occurs?

- A) Empirical probability C) Subjective probability
B) Classical probability D) Conditional probability

تعريف

Uses sample spaces

to determine numerical probability

↳ Classical probability

A child gets 22 heads out of 30 tosses of a coin. This would be an example of ... type of probability.

- A) empirical B) subjective C) classical D) empirical and classical

عدد محاولات التجربة 30
حصل منهم الطفل على 22 صورة
∴ احتمال تجربتها

empirical