Math 100

Mada Altiary

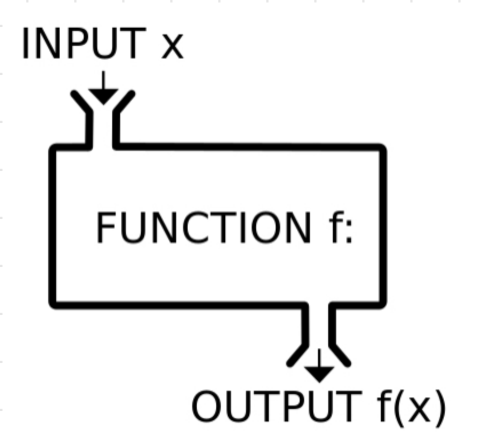
Functions

1. Definitions
2. Ways to representing functions
3. Evaluating functions
4. Domain of functions

Functions

1 What is a function?

A function is any mapp that takes an input and one out put.



2 A functions may be defined by:

- Arrow Diagram
- Set of ordered pairs
- An Equations
- Graph

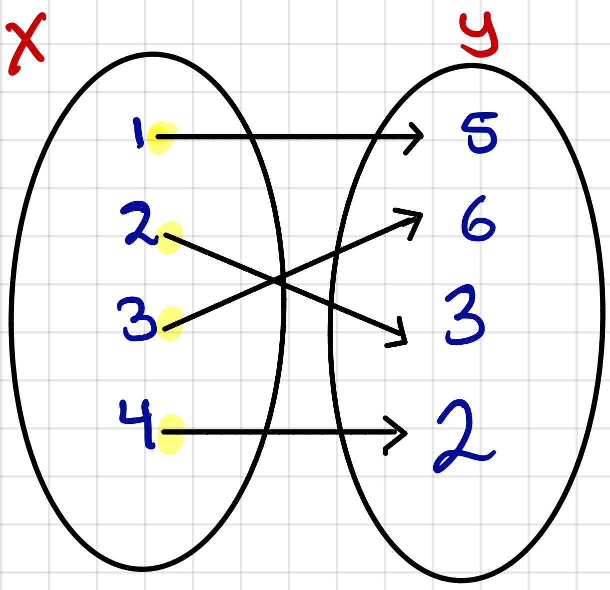
Functions

Functions Defined by Arrow Diagram

To be function: For each element in the first set there correspond one and only one element in the second test

Domain: First set

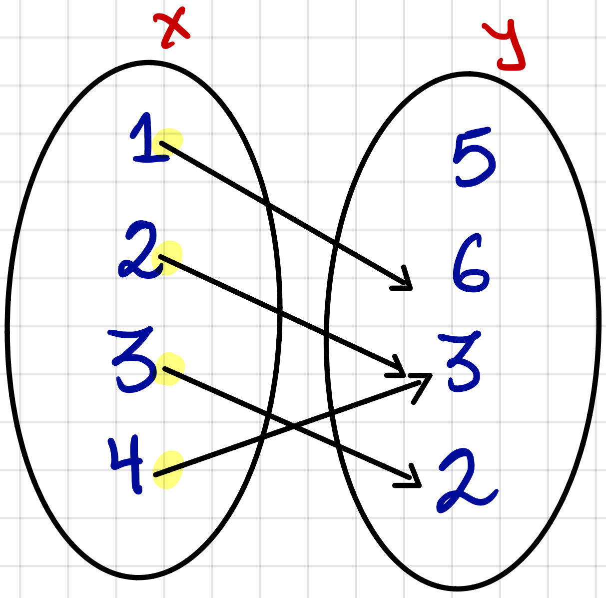
Range: Second Set



Function: Yes

Domain: {1,2,3,4}

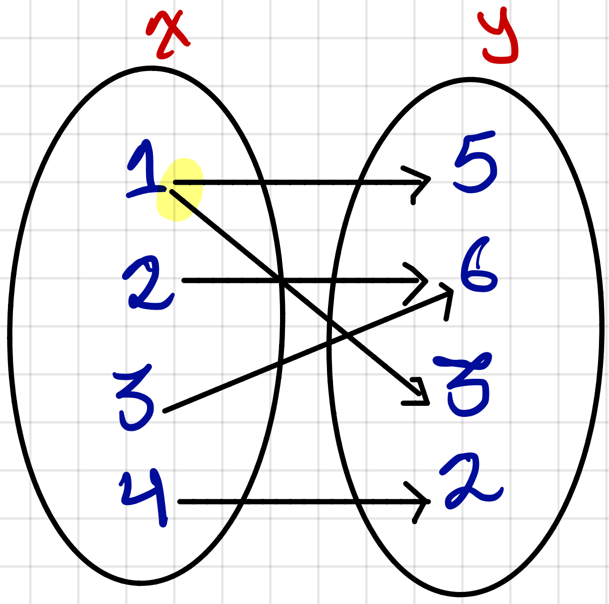
Range: {5,6,3,2}



Function: Yes

Domain: {1,2,3,4}

Range: {6,3,2}



Function: No

Domain:

Range:

Functions Defined by Set of Ordered Pairs

To be function: No ordered pairs have the same first component and different second component.

Domain: First component

Range: Second component

Determine whether each set specifies a function. If it does, then state the domain and range.

(A) $S = \{(1, 4), (2, 3), (3, 2), (4, 3), (5, 4)\}$

(B) $T = \{(1, 4), (2, 3), (3, 2), (2, 4), (1, 5)\}$

A)

Function: Yes

Domain: $\{1,2,3,4,5\}$

Range $\{2,3,4\}$

B)

Function: No

Domain:

Range

Functions Defined by an Equations

To be function: For each value of independent variable x there correspond exactly one value of dependent variable. y

Domain: Set of all possible real x-value which will make the function "work" or "defined"

Range: Set of all y-value corresponding to domain value.

Example:

$y = x^2 + 2x$

x	y
-2	0
-1	-1
0	0
1	3
2	8

Function: Yes

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

Function: Yes

$x = y^2$

x	y
4	-2
1	-1
0	0
1	1
4	2

Function: No

Note: It is very easy to determine whether an equation defines a function or not if we have the graph of the equation.

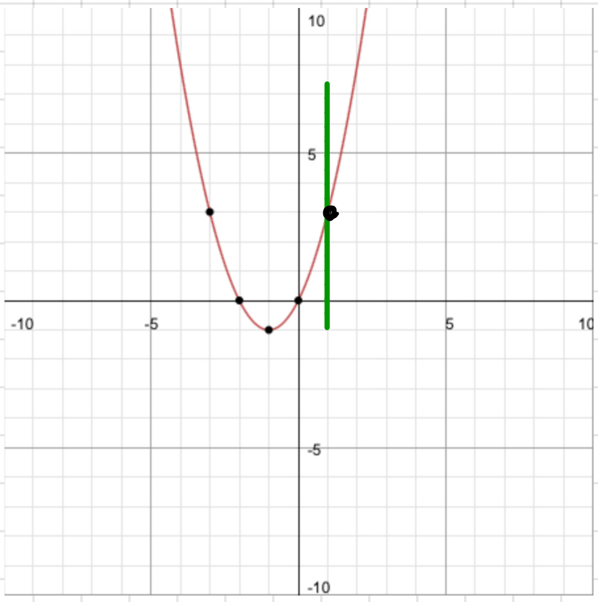
Functions Defined by Graph

To be function: Vertical Line Test (VLT):

Function: if each VL pass through at most one point on graph.

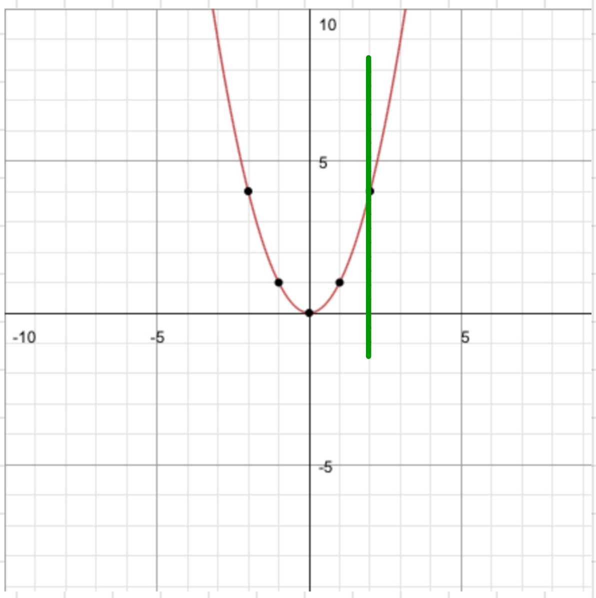
Not function: if any VL pass through two or more points on the graph.

$y = x^2 + 2x$



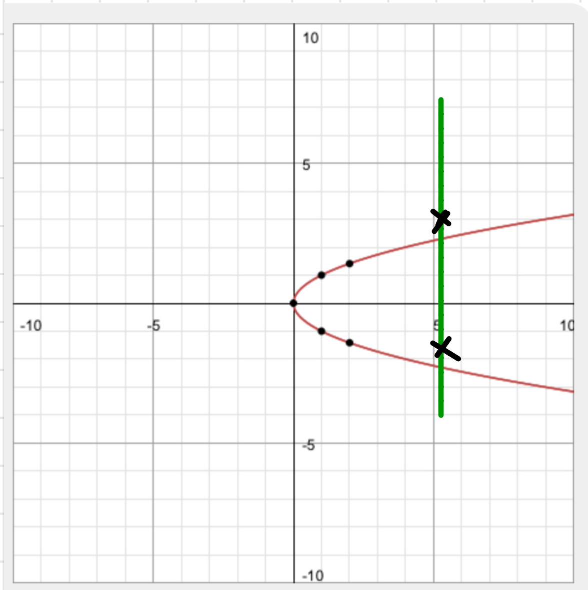
Function by VLT

$y = x^2$



Function by VLT

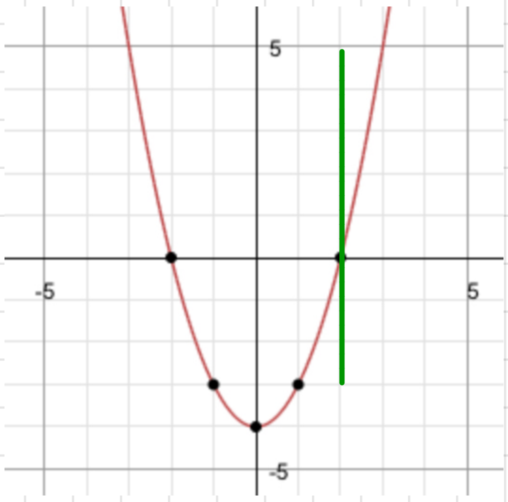
$x = y^2$



Not function by VLT

Example: **Determine if each equation defines a function with independent variable x**

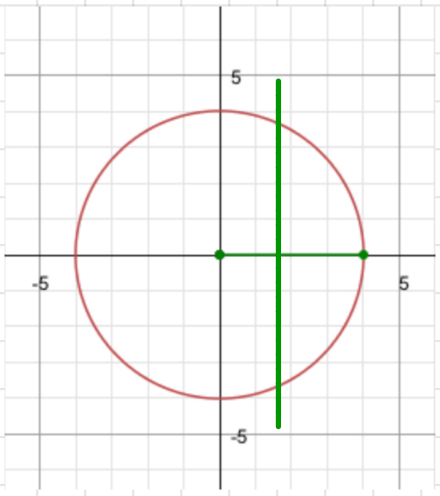
A) $y = x^2 + 4$



B) $x^2 + y^2 = 16$

$y^2 = 16 - x^2$

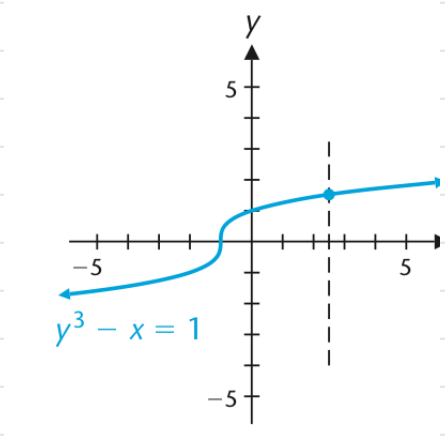
$y = \pm\sqrt{16 - x^2}$



C) $y^3 - x = 1$

$y^3 = 1 + x$

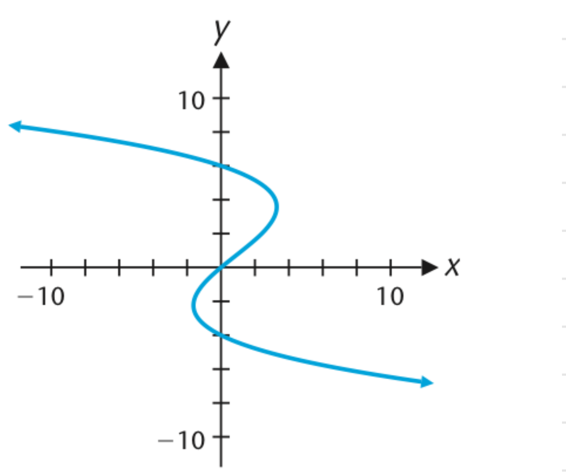
$y = \sqrt[3]{1 + x}$



(a)

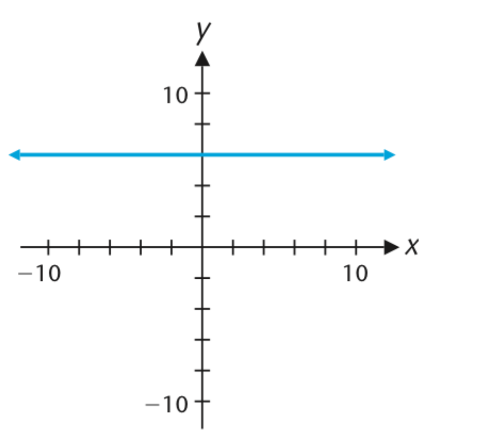
Example: **Determine if each graph defines a function**

15.



Not function

16.

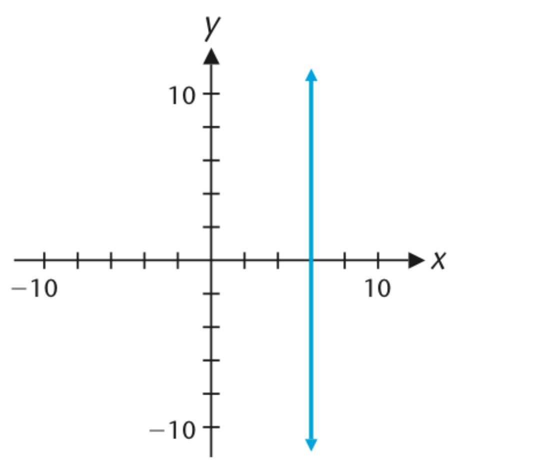


Function

Domain = R

Range = 3

17.



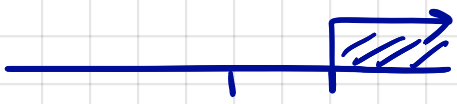
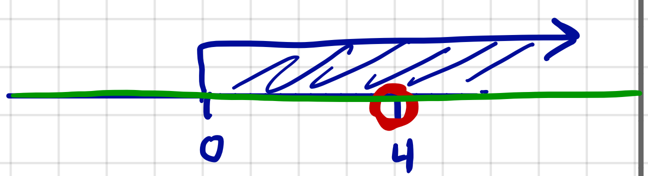
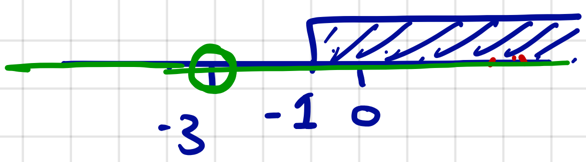
Not function

ملاحظة: سيتم دراسة إيجاد المجال والمدى من التمثيل البياني في الدرس اللا حق.

3 Finding the domain of Functions

Polynomial	Fraction only	Square root only
$f(x) = x^2 + 2x + 1$ **Domain=** All real numbers $\mathbb{R}$ or $(-\infty, \infty)$	$f(x) = \frac{2}{x-4}$ **Domain=** bottom expression $\neq 0$ R-{أصفار المقام}	$F(x) = \sqrt{x+1}$ **Domain=** expression under root ≥ 0
square root on bottom	**Square root on bottom on x only**	**square root on top**
$F(x) = \frac{5}{\sqrt{x+1}}$ **Domain=** expression under root > 0	$F(x) = \frac{x}{\sqrt{x} - 2}$ • $x \geq 0$ • Bottom expression $\neq 0$ • Take intersection	$F(x) = \frac{\sqrt{x+1}}{x^2-4}$ • under root ≥ 0 • Bottom $\neq 0$ • Take intersection.

Example

Polynomial	Fraction only	Square root only
$f(x) = 16 + 3x - x^2$ Domain $= \mathbb{R} = (-\infty, \infty)$	$f(x) = \frac{15}{x-3}$ $x - 3 \neq 0 \Rightarrow x \neq 3$ $\therefore$ Domain $= \mathbb{R} - \{3\}$ or $(-\infty, 3) \cup (3, \infty)$	$F(x) = \sqrt{x-3}$ $x - 3 \geq 0$ $x \geq 3$ $\therefore$ Domain $= [3, \infty)$
square root on bottom	**Square root on bottom on x only**	**square root on top**
$F(x) = \frac{x}{\sqrt{x-2}}$ $x - 2 > 0 \Rightarrow x > 2$  Domain $= (2, \infty)$	$F(x) = \frac{x}{\sqrt{x} - 2}$ • $x \geq 0$ • $\sqrt{x} - 2 \neq 0 \Rightarrow \sqrt{x} \neq 2$ $\Rightarrow x \neq 4$ •  Domain $= [0, 4) \cup (4, \infty)$	$F(x) = \frac{\sqrt{x+1}}{x+3}$ • $x + 1 \geq 0 \Rightarrow x \geq -1$ • $x + 3 \neq 0 \Rightarrow x \neq -3$  Domain $= [-1, \infty)$

Example: **Find the domain of each of the following function**

$f(x) = x^2 + 16$

Domain $= \mathbb{R} = (-\infty, \infty)$

$f(x) = \dfrac{x}{x^2+16}$

$x^2 + 16 = 0$. There is no such x.

$\therefore$ Domain $= \mathbb{R} = (-\infty, \infty)$

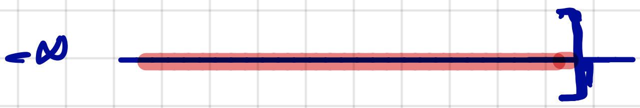
$g(x) = \sqrt{10-2x}$

$10 - 2x \geq 0 \Rightarrow 10 \geq 2x$

$\Rightarrow 5 \geq x$

$x \leq 5$

$\therefore$ Domain $= (-\infty, 5]$



$h(x) = \dfrac{x}{x^3+27}$

$x^3 + 27 \neq 0$

$\Rightarrow x^3 \neq -27 \Rightarrow x \neq \sqrt[3]{-27}$

$\Rightarrow x \neq -3$

$\therefore$ Domain $= \mathbb{R} - \{-3\}$

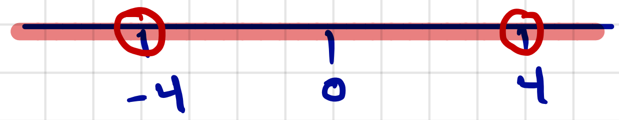
$(-\infty, -3) \cup (-3, \infty)$

$f(x) = \dfrac{x}{x^2-16}$

$x^2 - 16 \neq 0$

$(x-4)(x+4) \neq 0$

$x \neq 4$ or $x \neq -4$



$\therefore$ Domain $\mathbb{R}$, $x \neq \mp 4$

or $\mathbb{R} - \{+4, -4\}$

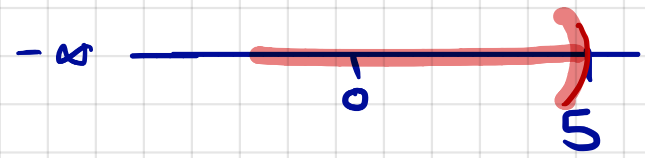
$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

$g(x) = \dfrac{2}{\sqrt{10-2x}}$

$10 - 2x > 0 \Rightarrow 10 > 2x$

$5 > x$ or $x < 5$

Domain $= (-\infty, 5)$

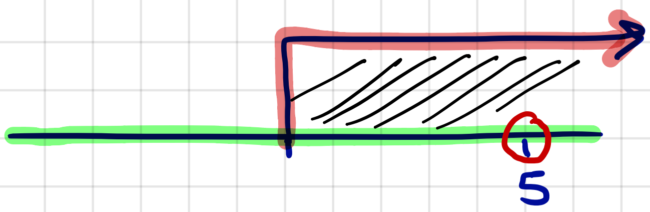


$g(x) = \dfrac{2}{10 - \sqrt{2x}}$

- $2x \geq 0 \Rightarrow x \geq 0$
- $10 - \sqrt{2x} \neq 0$

$10 \neq \sqrt{2x}$

$5 \neq x$



Domain $= [0, 5) \cup (5, \infty)$

4 Evaluating Function

(A) Find $f(6)$, $f(a)$, and $f(6 + a)$ for $f(x) = \dfrac{15}{x - 3}$.

(B) Find $g(7)$, $g(h)$, and $g(7 + h)$ for $g(x) = 16 + 3x - x^2$.

(C) Find $k(9)$, $4k(a)$, and $k(4a)$ for $k(x) = \dfrac{2}{\sqrt{x} - 2}$.

SOLUTIONS

(A) $f(6) = \dfrac{15}{6 - 3} = \dfrac{15}{3} = 5$

$$f(a) = \frac{15}{a - 3}$$

$$f(6 + a) = \frac{15}{6 + a - 3} = \frac{15}{3 + a}$$

(B) $g(7) = 16 + 3(7) - (7)^2 = 16 + 21 - 49 = -12$

$$g(h) = 16 + 3h - h^2$$

$$g(7 + h) = 16 + 3(7 + h) - (7 + h)^2$$ Remove the first set of parentheses and square the binomial.

$$= 16 + 21 + 3h - (49 + 14h + h^2)$$ Combine like terms and remove the parentheses.

$$= 37 + 3h - 49 - 14h - h^2$$ Combine like terms.

$$= -12 - 11h - h^2$$

(C) $k(9) = \dfrac{2}{\sqrt{9} - 2} = \dfrac{2}{3 - 2} = 2$ $\sqrt{9} = 3$, not ± 3.

$$4k(a) = 4\frac{2}{\sqrt{a} - 2} = \frac{8}{\sqrt{a} - 2}$$

$$k(4a) = \frac{2}{\sqrt{4a} - 2}$$ $\sqrt{4a} = \sqrt{4}\sqrt{a} = 2\sqrt{a}$.

$$= \frac{2}{2\sqrt{a} - 2}$$ Divide numerator and denominator by 2.

$$= \frac{1}{\sqrt{a} - 1}$$

Evaluating and Simplifying a Difference Quotient

For $f(x) = x^2 + 4x + 5$, find and simplify:

(A) $f(x + h)$ (B) $f(x + h) - f(x)$ (C) $\frac{f(x + h) - f(x)}{h}, h \neq 0$

SOLUTIONS

(A) To find $f(x + h)$, we replace x with $x + h$ everywhere it appears in the equation that defines f and simplify:

$$\begin{aligned} f(x + h) &= (x + h)^2 + 4(x + h) + 5 \\ &= x^2 + 2xh + h^2 + 4x + 4h + 5 \end{aligned}$$

(B) Using the result of part A, we get

$$\begin{aligned} f(x + h) - f(x) &= x^2 + 2xh + h^2 + 4x + 4h + 5 - (x^2 + 4x + 5) \\ &= x^2 + 2xh + h^2 + 4x + 4h + 5 - x^2 - 4x - 5 \\ &= 2xh + h^2 + 4h \end{aligned}$$

(C)

$$\begin{aligned} \frac{f(x + h) - f(x)}{h} &= \frac{2xh + h^2 + 4h}{h} = \frac{h(2x + h + 4)}{h} \\ &= 2x + h + 4 \end{aligned}$$

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(graphing
function)
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