Ministry of Higher Education
Kingdom of Saudi Arabia

College of Computing and Info.
Saudi Electronic University

Final Examination Cover Sheet

Final Examination 1435-1436/2014-2015

Course Instructor:		**Exam Date:**	$14^{th} Jan$, 2015
Course Title:	Discrete Mathematics	**Course Code:**	Math 150
Exam Duration:	2 Hours	**No. of Pages**	8

Answers Key

Question	Points	Score
1	10	
2	15	
3	5	
4	5	
5	5	
6	5	
7	5	
Total:	50	

1. State whether the following statements are true or false: [10]

   (a) $p \vee \sim p$ is a contradiction.

   (a) False

   (b) The value of $1.\overline{0} + 1.\overline{1} + 0.\overline{1}$ is 0.

   (b) False

   (c) $\backsim \forall x P(x) \equiv \exists x \backsim P(x)$.

   (c) True

   (d) The function $f(x) = \dfrac{x+1}{x+2}$ is not a bijection from $\mathbb{R} - \{-2\}$ to $\mathbb{R} - \{1\}$.

   (d) False

   (e) The value of $13520 \mod 10$ is 0.

   (e) True

   (f) Let a, b and c be integers where $a \neq 0$ then if $a|b$ and $a|c$, then $a|(b+c)$.

   (f) True

   (g) If adjacency matrix of a graph is an identity matrix then the graph is simple.

   (g) False

   (h) The recurrence relation $a_n = a_{n-1} + 2a_{n-2} + a_{n-4}$ is a linear homogeneous recurrence relation of degree 3.

   (h) False

   (i) If in a directed graph, the edges are only loops at each nodes then the corresponding relation is symmetric as well as anti symmetric.

   (i) True

   (j) A tree with 11 vertices has 10 edges.

   (j) True

2. Select one of the alternatives from the following questions as your answer. [15]

(a) $x \oplus y = 1$ when

A. $x = 0, \quad y = 0$

B. $x = 0, \quad y = 1$

C. $x = 1, \quad y = 1$

D. None

(b) The Boolean expression $C + CD$ is equal to

A. D

B. $C + D$

C. 1

D. C

(c) The sum of two positive integer is always positive. Its logical translation is

A. $\forall x \exists y((x > 0) \to (x + y > 0))$

B. $\forall x \exists y((x > 0) \wedge (y > 0) \to (x + y > 0))$

C. $\forall x \forall y((x > 0) \wedge (y > 0) \to (x + y > 0))$

D. None

(d) Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Then the set $\overline{A} \cup \overline{B}$ is expressed by

A. The set of students at your school who either are not sophomores or are not taking discrete mathematics.

B. The set of sophomores taking discrete mathematics in your school.

C. The set of sophomores at your school who are not taking discrete mathematics.

D. The set of students at your school who either are sophomores or are taking discrete mathematics.

(e) The value of $7 +_{11} 12$ is

A. 19

B. 9

C. 12

D. 8

(f) The inverse of $5 \mod 12$ is

A. 6

B. 0

C. 5

D. 10

(g) The sums of the first n positive odd integers are

A. n^2

B. $2n + 2$

C. $n^2 - 1$

D. $(2n - 1)n$

(h) The coefficient of $x^{12}y^{12}$ in the expression of $(3x - 2y)^{24}$ is

A. $\binom{24}{12}(-3)^{13}2^{12}$

B. $\binom{24}{12}3^{12}(-2)^{12}$

C. $3^{12}2^{12}$

D. $\binom{24}{12}$

(i) If $f(x) = x^2$ and $g(x) = 2x + 1$, then $f(g(x))$ is equal to

A. $2x + 1$

B. $(2x + 1)^2$

C. $2x^2 + 1$

D. None

(j) Relations R are defined on the set $\{1, 2, 3, 4\}$. Then, which one of the following is false:

A. $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ is reflexive, symmetric, antisymmetric and transitive.

B. $\{(2, 4), (4, 2)\}$ is symmetric only.

C. $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$ is reflexive, symmetric, antisymmetric and transitive.

D. $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$ is not reflexive, not symmetric, not anti-symmetric and not transitive.

(k) In K_4 graph, number of edges are

A. 6

B. 4

C. 8

D. 9

(l) Let $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x + 1$ be an isomorphism, then

A. $f^{-1}(y) = y + 1$

B. $f^{-1}(y) = y$

C. $f^{-1}(y) = y - 1$

D. f^{-1} does not exists.

(m) The value of $C(8, 3)$ is

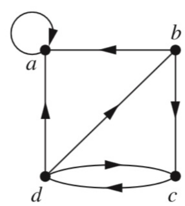
A. 336

B. 56

C. 48

D. 356

(n) For the following figure:



Choose the correct answer.

i. The $deg(a)$ is

A. 3

B. 4

C. 2

D. 5

ii. The value of $\sum_{v \in V} deg^{-}(v) =$

A. 6

B. 14

C. 7

D. 5

Attempt all the questions.

3. (a) Ali can choose a picture from one of four lists. The four lists contain 18,6,12 and 4 possible pictures, respectively. No picture is on more than one list. How many possible ways are there to choose the picture. [2]

   Solution: Ali can choose a picture by selecting it from the first list, the second list, the third list or the fourth list. Because no picture is on more than one list, by the sum rule there are 18+6+12+4=40 ways to choose a picture.

   (b) From a group of 7 men, 3 men are to be selected to form a committee. In how many ways can it be done. [3]

   Solution: Since there is no ordering involve, so this is the case of combination. Just find
   $C(7,3) = \dfrac{7!}{3!(7-4)!} = 35.$

4. Solve the recurrence relation [5]
$a_n = 2a_{n-1}, \quad \text{for } n \geq 1, \; a_0 = 3.$

Solution: Put $a_n = r^n$ and $a_{n-1} = r^{n-1}$ in the given recurrence relation and simplify it, we get
$r = 2$, which is the characteristic root of the given recurrence relation.
The general solution of the recurrence relation is given by
$a_n = cr^n = c2^n$, where c is constant. Now using initial condition $a_0 = 3$ in this. Put $n = 0$, we get $c = 3$
Therefore its general solution is given by
$a_n = 3.2^n$.

5. Let m be an integer with $m > 1$. Show that the congruence modulo m relation $R = \{(a,b) | a \equiv b \; (\mod m)\}$ is an equivalence relation on the set of integers. [5]

Solution: Recall that $a \equiv b(\mod m)$ if and only if m divides $a - b$.
Reflexivity: $a \equiv a(\mod m)$ since $a - a = 0$ is divisible by m since $0 = 0m$.

Symmetry: Suppose that $a \equiv b(\mod m)$. Then $a - b$ is divisible by m, and so $a - b = km$, where k is an integer. It follows that $b - a = (-k)m$, so $b \equiv a(\mod m)$.

Transitivity: Suppose that $a \equiv b(\mod m)$ and $b \equiv c(\mod m)$. Then m divides both $a - b$ and $b - c$. Hence, there are integers k and l with $ab = km$ and $bc = lm$. We obtain by adding the equations:

$$\begin{aligned} a - c &= (a - b) + (b - c) \\ &= km + lm = (k + l)m \\ \therefore \quad a &\equiv c(\mod m). \end{aligned}$$

OR

Show that the sum of two rational numbers is rational.

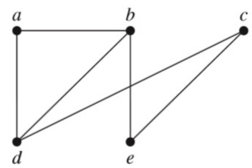
Solution: Let r and s be two rational numbers. Then by the definition of rational numbers

$$r = \frac{p}{q}, \; q \neq 0, \quad s = \frac{t}{u}, \; u \neq 0$$

Now,

$$\begin{aligned} r+s &= \frac{p}{q}+\frac{t}{u} \\ &= \frac{pu+qt}{qu}, \qquad qu \neq 0 \\ \Rightarrow \quad & r+s \text{ is a rational number.} \end{aligned}$$

6. State the Handshaking theorem and verify it for the following graph: [5]



Solution: The Handshaking Theorem: Let $G=(V,E)$ be an undirected graph with m edges. Then

$$2m = \sum_{v\in V} deg(v).$$

From figure we have
$deg(a)=2,\ deg(b)=3,\ deg(c)=2,\ deg(d)=3,\ deg(e)=2$
So,

$$\begin{aligned} \sum_{v\in V} deg(v) &= 2+3+2+3+2 \\ &= 12 \\ &= 2\times 6 \\ &= 2\times \text{No. of edges} \end{aligned}$$

Therefore, $\sum_{v\in V} deg(v) = 2m$, where m is the number edges.

7. Show that if n is an integer and $3n+2$ is odd, then n is odd. [5]

Solution: We will prove it by contraposition. (Recall that if $p \rightarrow q$ then its contraposition is $\neg Q \rightarrow \neg p$.) Suppose that n is not odd *i.e.* n is even. Then $n=2k$, where k is some integer. Now

$$3n+2 = 3(2k)+2 = 6k+2 = 2(3k+1) = 2S, \quad \text{where } S=3k+1 \text{ is an integer}$$

Therefore, $3n+2$ is even. *i.e.* $3n+2$ is not odd. Thus, if $3n+2$ is odd, then n is odd.

 End of exam.