1 Classical Probability

$$P(A) = \frac{|A|}{|\Omega|}$$

$P(A) + P(\bar{A}) = 1$ $\longrightarrow$ $P(\bar{A}) = 1 - P(A)$

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Complement of an Event:
is an event that occurs if A does not occur

$\bar{A} = \{w : w \in \Omega, w \notin A\}$

2 Addition Rule (U)(or):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

if A and B mutualy exclusive events $(A \cap B) = \phi$

Then, $P(A \cup B) = P(A) + P(B)$

Union of Two Events:
- occurrence of A or B or both.
- occurence of at least one of two events.
- occurrence of either two events

$A \cup B = \{w : w \in A \text{ or } w \in B\}$

3 Difference Between Two Events:

$$P(A \setminus B) = P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$P(B \setminus A) = P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$A \setminus B = A \cap \bar{B}$

→ A occurs and B does not occurs

→ only A must be occurs

$A \setminus B = A \cap \bar{B} = \{w : w \in A \text{ and } w \notin B\}$

4 De-Morgan's laws:

i) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

ii) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

5 Conditional Probability:

$$P(A | B) = \frac{P(A \cap B)}{P(B)} \qquad P(B) > 0$$

$$P(B | A) = \frac{P(A \cap B)}{P(A)} \qquad P(A) > 0$$

6 Exaclty one of events:

$$P(A \triangle B) = P((A \cap \bar{B}) \cup (\bar{A} \cap B))$$
$$= P(A \cup B) - P(A \cap B)$$

A or B occurs

$A \triangle B = \{w : w \in A \cap \bar{B} \text{ or } w \in \bar{A} \cap B\}$

7 Multiplication Rule (and) ($\cap$)

$$P(A \cap B) = P(A) \cdot P(B|A)$$
$$= P(B) \cdot P(A|B)$$

Intersection of two events:

- occurring both A and B togther
- Both the two events A and B occurs

$A \cap B = \{w : w \in A \text{ and } w \in B\}$

if A and B are independent; $P(B|A) = P(B)$, $P(A|B) = P(A)$ then;

$$P(A \cap B) = P(A) \cdot P(B)$$

* chain rule:

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \cdots P(A_n|A_1 \cap A_2 \cdots \cap A_{n-1})$$

8 *Total Probability:

$$P(B) = \sum_{i=1}^{n} P(A_i) P(B|A_i)$$

* Bayes' Theorem:

$$P(A_i|B) = \frac{P(A_i) \cdot P(B|A_i)}{P(B)}$$