BMT-222 (September 2018)

Chapter 12 Techniques and applications of differentiation (Summary)

The product rule: If $y(x)=u(x)v(x)$, then $\dfrac{dy}{dx}=\dfrac{du}{dx}v+u\dfrac{dv}{dx}$ or $y'=u'v+uv'$

Block 1 **Exercises (Product Rule) (page 644)**

Problem # 1 Find $\dfrac{dy}{dx}$ where y is given by:

(a) $x\cos x$ (b) xe^{x} (c) $\sin x\cos 2x$ (d) $x^{3}e^{2x}$ (e) $x^{4}\sin 2x$

Problem # 2. Calculate y' where y is given by:

(a) $\left(t^{2}+1\right)\sin 4t$ (b) $(3t+7)e^{-2t}$ (c) $\left(e^{x}+e^{-2x}\right)\left(3x^{2}-2x\right)$ (d) $\sqrt{x}\,e^{x}$ (e) $\dfrac{t^{2}+1}{e^{t}}$

Problem # 3 Find the second derivative of the function in question 1:

(a) $x\cos x$ (b) xe^{x} (c) $\sin x\cos 2x$ (d) $x^{3}e^{2x}$ (e) $x^{4}\sin 2x$

HW: Problem # 1 (a) (b), and (e), Problem # 2 (b), (d), Problem # 3 (a) (b), and (e).

(Problems solved in class # 1 (c), (d), # 2 (a), (c), and (e) # 3 (c), (d))

The quotient rule: If $y(x)=\dfrac{u(x)}{v(x)}$, then $\dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^{2}}$ or $y'=\dfrac{u'v-uv'}{v^{2}}$

Exercises (Quotient Rule) (page 646)

Problem # 1 Find $\dfrac{dy}{dx}$ where y is given by: (a) $\dfrac{e^{x}}{x}$ (b) $\dfrac{x}{e^{x}+1}$

(c) $\dfrac{\cos x}{\sin x}$ (d) $\dfrac{1-x}{1+x}$ (e) $\dfrac{\ln x}{x^{2}}$

Problem # 2. Calculate y' where y is given by: (a) $\dfrac{t^{2}-1}{t^{2}+1}$ (b) $\dfrac{e^{2t}+t}{e^{t}-1}$

(c) $\dfrac{\sin 3t}{\cos t+t}$ (d) $\dfrac{z+\sin z}{z+\cos z}$ (e) $\dfrac{1+x+x^{2}}{1+x^{3}}$

HW: Problem # 1 (a), (c)and (d), Problem # 2 (b), and (c).
(Problems solved in class # 1 (b) and (e), # 2 (a), (d) and (e))

Block 3 **Exercises (Implicit Differentiation) (page 658)**

Problem # 1 Find $\dfrac{dy}{dx}$ given: (a) $x^{2}+y^{2}=1$, (d) $x^{3}-y^{3}=x+y$

Problem # 2. Find $\dfrac{dx}{dt}$ given: (b) $3\cos 2x-t^{2}=20$, (e) $\tan 2t-x^{2}=\sin 2x$

Problem # 3. Find $\dfrac{dy}{dx}$ given: (d) $e^{2x+3y}+2x^{3}-y^{5}=0$, (e) $2e^{y-x}=3e^{x}+y^{2}$

Problem # 4 Find $\dfrac{d^{2}y}{dx^{2}}$ given: $x^{2}+y^{2}=1$ Problem # 5 Find y'' given: $\sin x+\cos y=1$

Problem # 6 Use implicit differentiation to find $\dfrac{d^{2}y}{dx^{2}}$ if $4x^{2}-2y^{2}=9$ (answer: $-\dfrac{9}{y^{3}}$)

(Problems solved in class # 1 (a), 2 (b), 3 (e), 5), (HW # 1 (d), 2 (e), 3 (d), 4, 6)