Estimation and Confidence Interval

Estimation and Confidence Interval: Single Mean:

To find the confidence intervals for a single mean:

1- $\bar{X} \pm \left(Z_{1-\frac{\alpha}{2}} \ \frac{\sigma}{\sqrt{n}} \right) \quad \sigma \ known$

2- $\bar{X} \pm \left(t_{\frac{\alpha}{2}, n-1} \ \frac{S}{\sqrt{n}} \right) \quad \sigma \ unknown$

Estimation and Confidence Interval: Two Means

To find the confidence intervals for two means:

1- $(\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right) \quad \sigma_1 \ and \ \sigma_2 \ known$

2- $(\bar{X}_1 - \bar{X}_2) \pm \left(t_{\frac{\alpha}{2}, n_1+n_2-2} \ Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right) \quad \sigma_1 \ and \ \sigma_2 \ unknown$

$$Sp^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

Estimation and Confidence Interval: Single Proportion

* *Point estimate for* P *is:* $\frac{x}{n}$

* *Interval estimate for* P *is:* $\hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$

Estimation and Confidence Interval: Two Proportions

* *Point estimate for* $P_1 - P_2 = \hat{p}_1 - \hat{p}_2 = \frac{x_1}{n_1} - \frac{x_2}{n_2}$

* *Interval estimate for* $P_1 - P_2$ *is:* $(\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \right)$

Q2. Suppose that we are interested in making some statistical inferences about the mean, μ, of a normal population with standard deviation σ=2.0. Suppose that a random sample of size *n*=49 from this population gave a sample mean $\bar{X}$ =4.5.

a. *Find the upper limit of 95% of the confident interval for* μ

$$\sigma = 2 \quad \& \quad \bar{X} = 4.5 \quad \& \quad n = 49$$

$$95\% \rightarrow \alpha = 0.05 \qquad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$\bar{X} + \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right) = 4.5 + \left(1.96 \times \frac{2}{7} \right) = 5.06$$

b. *Find the lower limit of 95% of the confident interval for* μ

$$\bar{X} - \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right) = 4.5 - \left(1.96 \times \frac{2}{7} \right) = 3.94$$

Q4. A researcher wants to estimate the mean lifespan of a certain light bulbs. Suppose that the distribution is normal with standard deviation of 5 hours.

Suppose that the researcher selected a random sample of 49 bulbs and found that the sample mean is 390 hours.

a. *find* $Z_{0.975}$ *:*

$$Z_{0.975} = 1.96$$

b. *find a point estimate for* μ

$$\hat{\mu} = \bar{X} = 390$$

c. *Find the upper limit of 95% of the confident interval for* μ

$$\bar{X} + \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right) = 390 + \left(1.96 \times \frac{5}{\sqrt{49}} \right) = 391.3$$

d. *Find the lower limit of 95% of the confident interval for* μ

$$\bar{X} - \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right) = 390 - \left(1.96 \times \frac{5}{\sqrt{49}} \right) = 388.6$$

➢ *A sample of 16 college students were asked about time they spent doing their homework. It was found that the average to be 4.5 hours. Assuming normal population with standard deviation 0.5 hours.*

(1) The point estimate for μ is:

(A) 0 hours *(B) 10 hours* *(C)0.5 hours* *<u>(D) 4.5 hours</u>*

(2) The standard error of $\overline{x}$ is:

<u>(A) 0.125 hours</u> *(B) 0.266 hours* *(C) 0.206 hours* *(D) 0.245hours*

(3) The correct formula for calculating $100(1-\alpha)\%$ confidence interval for μ is:

(A) $\bar{x} \mp t_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

<u>(B)</u> $\bar{x} \mp z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

(C) $\bar{x} \mp z_{1-\frac{\alpha}{2}} \frac{\sigma^2}{n}$

(D) $\bar{x} \mp t_{1-\frac{\alpha}{2}} \frac{\sigma^2}{n}$

(4) The upper limit of 95% confidence interval for μ is:

<u>(A)</u> 4.745 *(B) 4.531* *(C) 4.832* *(D) 4.891*

(5) The lower limit of 95% confidence interval for μ is:

(A) 5.531 *(B) 7.469* *(C) 3.632* *<u>(D) 4.255</u>*

(6) The length of the 95% confidence interval for μ is:

(A) 4.74 *<u>(B) 0.49</u>* *(C) 0.83* *(D) 0.89*

Estimation and Confidence Interval: Two Means

To find the confidence intervals for two means:

1- $(\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$

2- $(\bar{X}_1 - \bar{X}_2) \pm \left(t_{\frac{\alpha}{2}, n_1+n_2-2}\, Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$

$$Sp^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

Q1.(I) The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8

Q1.(II). The tensile strength of type II thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 25 pieces of the thread has an average tensile strength of 64.4 kilograms. Then for the 98% confidence interval of the difference in tensile strength means between type I and type II , we have:

$$Theard\ 1: \quad n_1 = 20, \bar{X}_1 = 72.8, \sigma_1 = 6.8$$

$$Thread\ 2: \quad n_2 = 25, \bar{X}_2 = 64.4, \sigma_2 = 6.8$$

$$98\% \rightarrow \alpha = 0.02 \rightarrow \quad Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.33$$

$$(\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$$

$$(72.8 - 64.4) \pm \left(2.33 \times \sqrt{\frac{6.8^2}{20} + \frac{6.8^2}{25}} \right)$$

$$(3.65 , 13.15)$$

(1): $The\ lower\ limit\ = 3.65$

(2): $The\ upper\ limit\ = 13.15$

	First sample	*Second sample*
Sample size (n)	*12*	*14*
Sample mean $(\bar{X})$	*10.5*	*10*
Sample variance (S^2)	*4*	*5*

(1) Estimate the difference $\mu_1 - \mu_1$*:*

$$E(\bar{X}_1 - \bar{X}_2) = \bar{X}_1 - \bar{X}_2 = 10.5 - 10 = 0.5$$

(2) Find the pooled estimator Sp *:*

$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{4(11)+5(13)}{24} = 4.54 \implies Sp = 2.13$$

(3) The upper limit of 95% confidence interval for μ is:

$$95\% \rightarrow \alpha = 0.05 \rightarrow t_{\frac{\alpha}{2},n_1+n_2-2} = t_{0.025,24} = 2.064,$$

$$(\bar{X}_1 - \bar{X}_2) + \left(t_{\frac{\alpha}{2},n_1+n_2-2} \times Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$(0.5) + \left(2.064 \times 2.13\sqrt{\frac{1}{12} + \frac{1}{14}}\right) = 2.23$$

(4) The lower limit of 95% confidence interval for μ is:

$$(\bar{X}_1 - \bar{X}_2) - \left(t_{\frac{\alpha}{2},n_1+n_2-2} \times Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$(0.5) - \left(2.064 \times 2.13\sqrt{\frac{1}{12} + \frac{1}{14}}\right) = -1.23$$

➢ *A researcher was interested in comparing the mean score of female students μ_1, with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:*

	Female	*Male*
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\bar{x}_1 = 82.63$	$\bar{x}_2 = 80.04$
Variance	$s_1^2 = 15.05$	$s_2^2 = 20.79$

(1) *The point estimate of $\boldsymbol{\mu_1 - \mu_2}$ is:*

(A) 2.63 *(B) -2.37* *(C) 2.59* *(D) 0.59*

(2) *The estimate of the pooled variance (s_p^2) is:*

(A) 17.994 *(B) 18.494* *(C) 17.794* *(D) 18.094*

(3) *The upper limit of the 95% confidence interval for $\boldsymbol{\mu_1 - \mu_2}$ is :*

(A) 26.717 *(B) 7.525* *(C) 7.153* *(D) 8.2*

(4) *The lower limit of the 95% confidence interval for $\boldsymbol{\mu_1 - \mu_2}$ is :*

(A) -21.54 *(B) - 2.345* *(C) - 3.02* *(D) -1.973*

<u>Estimation and Confidence Interval: Single Proportion</u>

$$\text{* Point estimate for } P \text{ is: } \frac{x}{n}$$

$$\text{* Interval estimate for } P \text{ is: } \hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$$

Q1. A random sample of 200 students from a certain school showed that 15 students smoke. Let p be the proportion of smokers in the school.

1. Find a point Estimate for p.
2. Find 95% confidence interval for p.

<u>Solution</u>

$$n = 200 \quad \& \quad x = 15$$

(1):

$$\hat{p} = \frac{x}{n} = \frac{15}{200} = 0.075 \rightarrow \hat{q} = 0.925$$

(2):

$$95\% \rightarrow \alpha = 0.05 \quad \rightarrow \quad Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$$

$$\hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}} \right) = 0.075 \pm \left(1.96 \times \sqrt{\frac{0.075 \times 0.925}{200}} \right)$$

The 95% confidence interval is: $(0.038, \ 0.112)$

➢ ***A researchers group has perfected a new treatment of a disease which they claim is very efficient. As evidence, they say that they have used the new treatment on 50 patients with the disease and cured 25 of them. To calculate a 95% confidence interval for the proportion of the cured.***

1. ***The point estimate of p is equal to:***

(A) 0.25 *<u>(B)0.5</u>* *(C) 0.01* *(D) 0.33*

2. ***The reliability coefficient*** $(z_{1-\frac{\alpha}{2}})$ ***is equal is :***

<u>(A) 1.96</u> *(B) 1.645* *(C) 2.02* *(D) 1.35*

3. ***The 95% confidence interval is equal to:***

(A) (0.1114, 0.3886) (B) (0.3837, 0.6163) (C) (0.1614, 0.6386) <u>(D) (0.3614, 0.6386)</u>

Estimation and Confidence Interval: Two Proportions

$$* \textit{ Point estimate for } P_1 - P_2 = \hat{p}_1 - \hat{p}_2 = \frac{x_1}{n_1} - \frac{x_2}{n_2}$$

$$* \textit{ Interval estimate for } P_1 - P_2 \textit{ is: } (\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

Q3. A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200 students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smokers in school "A" and p_2 is the proportion of smokers in school "B".

(1) Find a point Estimate for $p_1 - p_2$.

(2) Find 95% confidence interval for $p_1 - p_2$.

Solution

$$n_1 = 100 \quad x_1 = 15 \rightarrow \hat{p}_1 = \frac{15}{100} = \boxed{0.15} \implies \hat{q}_1 = 1 - 0.15 = \boxed{0.85}$$

$$n_2 = 200 \quad x_2 = 20 \rightarrow \hat{p}_2 = \frac{20}{200} = \boxed{0.10} \implies \hat{q}_1 = 1 - 0.10 = \boxed{0.90}$$

(1)

$$\hat{p}_1 - \hat{p}_2 = 0.15 - 0.1 = 0.05$$

(2)

$$95\% \rightarrow \alpha = 0.05 \rightarrow Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$(\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$= (0.05) \pm \left(1.96 \times \sqrt{\frac{(0.15)(0.85)}{100} + \frac{(0.1)(0.9)}{200}}\right)$$

$$= 0.05 \pm \left(1.96 \times \sqrt{0.001725}\right)$$

The 95% confidence interval is: $(-0.031,\ 0.131)$

➢ *In a first sample of 100 store customers, 43 used a MasterCard. In a second sample of 100 store customers, 58 used a Visa card. To find the 95% confidence interval for difference in the proportion (* $\boldsymbol{P_1 - P_2}$ *) of people who use each type of credit card?*

1. ***The value of α is :***

   (A) 0.95 *(B) 0.5* *<u>(C) 0.05</u>* *(D) 0.025*

2. ***The upper limit of 95% confidence interval for the proportion difference is:***

$$n_1 = 100 \quad x_1 = 43 \quad \rightarrow \hat{p}_1 = \frac{43}{100} = \boxed{0.43} \Longrightarrow \hat{q}_1 = 1 - 0.43 = \boxed{0.57}$$

$$n_2 = 100 \quad x_2 = 58 \quad \rightarrow \hat{p}_2 = \frac{58}{100} = \boxed{0.58} \Longrightarrow \hat{q}_2 = 1 - 0.58 = \boxed{0.42}$$

$$(\hat{p}_1 - \hat{p}_2) + \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$= (0.43 - 0.58) + \left(1.96 \times \sqrt{\frac{(0.43)(0.57)}{100} + \frac{(0.58)(0.42)}{100}}\right)$$

(A) 0.137 *<u>(B) -0.013</u>* *(C) 0.518* *(D) 0.150*

3. ***The lower limit of 95% confidence interval for the proportion difference is:***

$$(\hat{p}_1 - \hat{p}_2) - \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$= (0.05) - \left(1.96 \times \sqrt{\frac{(0.15)(0.85)}{100} + \frac{(0.1)(0.9)}{200}}\right)$$

<u>(A) – 0.278</u> *(B) 1.547* *(C) 0.421* *(D) -0.129*