

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

نَدْرِيَاتٌ (٣) لِلْامْتِحَانِ النَّهَائِيِّ

Question 1: (30 points)

Choose the correct answer, write your answer in the table below:

1. The product of two negative numbers is:

- a) negative b) positive c) zero d) either positive or negative
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2. If $\frac{a}{b}$, is a rational number then the conditions on a and b are:

- a) $\begin{cases} a > 0 \\ b > 0 \end{cases}$ b) $\begin{cases} a \neq 0 \\ b \neq 0 \end{cases}$ c) $\begin{cases} a > 0 \\ b \neq 0 \end{cases}$ d) $\begin{cases} a \text{ and } b \text{ integers} \\ b \neq 0 \end{cases}$
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3. The solution set of the equation $\sqrt{x^2} = 6$ is:

- a) $\{-6, 6\}$ b) $\{6\}$ c) $\{-6\}$ d) $\{-36, 36\}$
-

4. Any two parallel lines have the same:

- a) x -intercept b) y -intercept c) slope d) equation

5. The equation of the line perpendicular to the line $y = \frac{1}{2}x + 5$ and containing the point $(2, 1)$ is:

- a) $y = \frac{1}{2}x$ b) $y = -2x - 3$ c) $y = -\frac{1}{2}x - 5$ d) $y = -2x + 5$
-

6. $\sqrt{-64} =$

- a) 8 b) -8 c) $-8i$ d) $8i$
-

7. The interval notation for the set $\{x : -3 \leq x < 5\}$ is:

- a) $(-3, 5)$ b) $[-3, 5]$ c) $(-3, 5]$ d) $[-3, 5)$
-

8. $LCM(x^2 - x, x^2 - 1) =$

- a) $x^3 - x$ b) $x - 1$ c) $x^4 - x$ d) $x(x+1)$

9. The factorization of $x(x-5) - 3(x-5)$ is:

- a) $-3x(x-5)$ b) $-3x(x-5)^2$ c) $(x-5)(x-3)$ d) $(x-5)(x+3)$
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10. The result of the division $\frac{\sqrt[3]{16x^5y^6}}{\sqrt[3]{2x^2y^3}}$ is:

- a) $2xy$ b) $2x^2y^2$ c) $4x^3y^3$ d) $8x^3y^3$
-

11. The slope of the horizontal line $y = 2$ is:

- a) 2 b) -2 c) 0 d) undefined
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12. The result of the multiplication $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$ is:

- a) $x+y$ b) $x-y$ c) $-2xy$ d) $\sqrt{2xy}$

13. The simplification of $\left| \frac{-24x^3}{8x^2} \right|$ is:

a) $-3|x|$

b) $3|x|$

c) $-3x$

d) $3x$

14. The solution set of the quadratic equation $x^2 - 5x + 6 = 0$ is:

a) $\{-2, 3\}$

b) $\{-2, -3\}$

c) $\{2, -3\}$

d) $\{2, 3\}$

15. The subtraction $(3x^2 - x + 3) - (-x^2 + x + 1)$ is equal to:

a) $2x^2 + 2$

b) $4x^2 - 2x + 2$

c) $4x^2 + 2x + 2$

d) $2x^2 + 4$

16. The solution set of the equation $|x - 1| = 5$ is:

a) $\{6\}$

b) $\{-6\}$

c) $\{-4, 6\}$

d) $\{-5, 5\}$

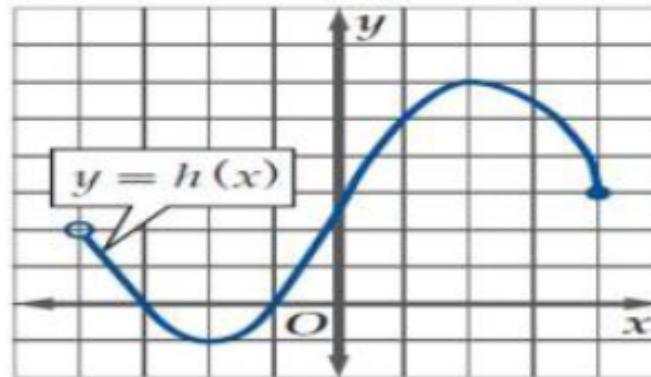
17. The result of $\frac{a^{-1} \cdot b^3}{a^2 \cdot b^{-2}}$ is:

- a) $a^{-3}b^5$ b) ab c) a^3b^{-5} d) $a^{-1}b^{-1}$
-

18. The result of $\frac{x}{3y} \div \frac{5}{6y}$ is:

- a) $\frac{5x}{18y^2}$ b) $\frac{18y^2}{5x}$ c) $\frac{x+5}{9y}$ d) $\frac{2x}{5}$
-

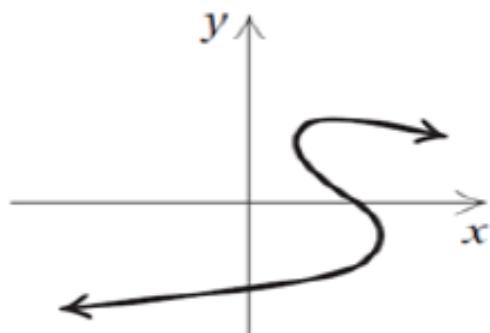
19. The domain of the function graphed below is:



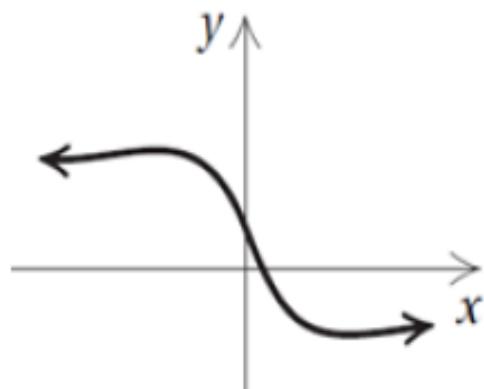
- a) $[-4, 4]$ b) $[-1, 6]$ c) $(-4, 4]$ d) $(2, 3]$

20. Which of the following is a graph of a function:

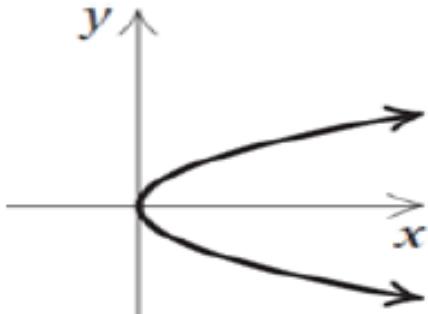
a)



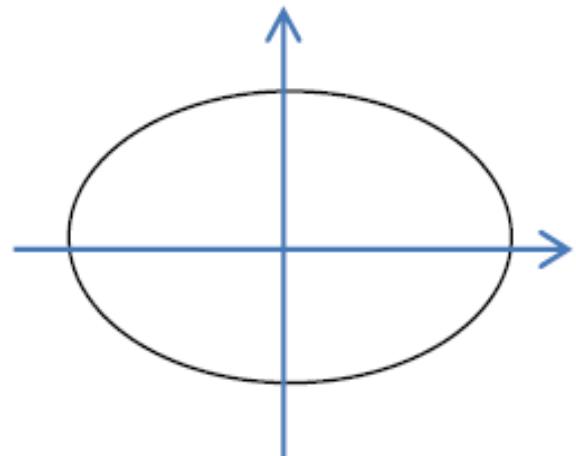
b)



c)



d)



Question 2: Perform and simplify the following:

Question 2-1

$$(2x - 3)^2 - 3x(x^2 + 5x - 2)$$

Answer 2-1

$$\begin{aligned}(2x - 3)^2 - 3x(x^2 + 5x - 2) &= 4x^2 - 12x + 9 - 3x^3 - 15x^2 + 6x \\ &= -3x^3 - 11x^2 - 6x + 9\end{aligned}$$

Question2-2

$$\frac{x+2}{(x-1)^2} \cdot \frac{(x-3)^2}{x^2 - 4} \cdot \frac{3x-3}{3-x}$$

Answer 2-2

$$\begin{aligned}\frac{x+2}{(x-1)^2} \cdot \frac{(x-3)^2}{x^2 - 4} \cdot \frac{3x-3}{3-x} &= \frac{(x+2)(x-3)^2(3x-3)}{(x-1)^2(x^2 - 4)(3-x)} \\ &= \frac{-3(x+2)(x-3)(x-3)(x-1)}{(x-1)(x-1)(x-2)(x+2)(x-3)} \\ &= \frac{-3(x-3)}{(x-1)(x-2)}\end{aligned}$$

Q 3-1 Solve the following equations and inequalities:

$$\sqrt{x+5} = x+3$$

A 3-1

$$\begin{aligned}\sqrt{x+5} &= x+3 \Rightarrow x+5 = (x+3)^2 \\ &\Rightarrow x+5 = x^2 + 6x + 9 \\ &\Rightarrow x^2 + 5x + 4 = 0\end{aligned}$$

$\Delta = 9 \Rightarrow$ the quadratic equation has 2 real solutions:

$$x_1 = \frac{-5+3}{2} = -1 \quad \text{and} \quad x_2 = \frac{-5-3}{2} = -4$$

We check the solutions by replacing them in equation (1):

$$\sqrt{-1+5} = -1+3 \Rightarrow \sqrt{4} = 2 \text{ which is always true} \Rightarrow \text{this solution is then acceptable.}$$

$$\sqrt{-4+5} = -4+3 \Rightarrow \sqrt{1} = -1 \text{ which is impossible} \Rightarrow \text{this solution is then refused.}$$

The solution set is $S = \{-1\}$.

Q 3-2

$$x^2 - 2x + 5 = 0$$

A 3-2

$\Delta = -16$; $\Delta < 0 \Rightarrow$ the quadratic equation has 2 complex solutions:

$$x_1 = \frac{2+i\sqrt{|\Delta|}}{2} = \frac{2+i\sqrt{16}}{2} = \frac{2+4i}{2} = 1+2i,$$

$$x_2 = \frac{2-i\sqrt{|\Delta|}}{2} = \frac{2-i\sqrt{16}}{2} = \frac{2-4i}{2} = 1-2i.$$

The solution set is $S = \{(1+2i, 1-2i)\}.$

Q 4-1 Given $f(x) = 6 + 3x$ and $g(x) = 2x - 1$, find $f(g(-3))$

A 4-1

$$g(-3) = 2(-3) - 1 = -6 - 1 = -7$$

$$f(-7) = 6 + 3(-7)^2 = 6 + 3 \times 49 = 6 + 147 = 153$$

Then $f[g(-3)] = 153$.

Q 4-2

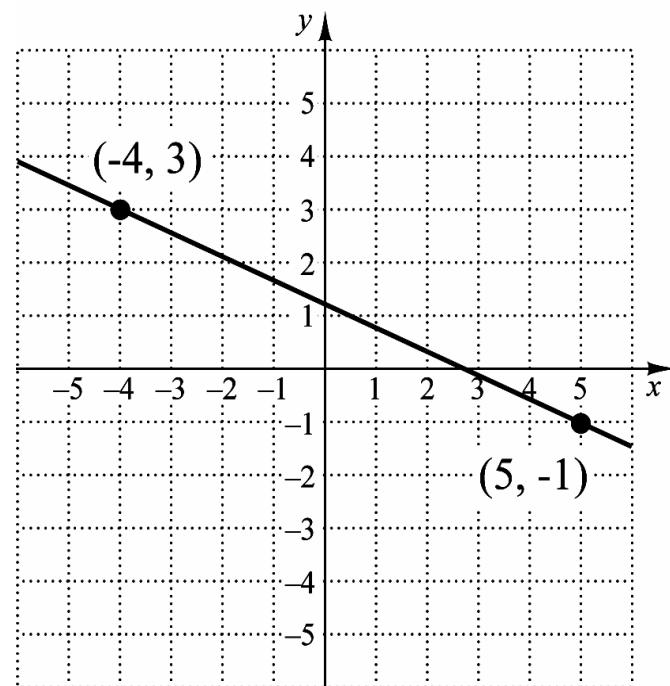
Write an equation for the line shown in the graph bellow:

A 4-2

The equation of the line is of the form

$$y = ax + b.$$

The line passes through the points $(-4, 3)$ and $(5, -1)$.



$$a = -\frac{4}{9} \quad \text{and} \quad b = \frac{11}{9}.$$

Then the equation of the line is

$$y = -\frac{4}{9}x + \frac{11}{9}.$$

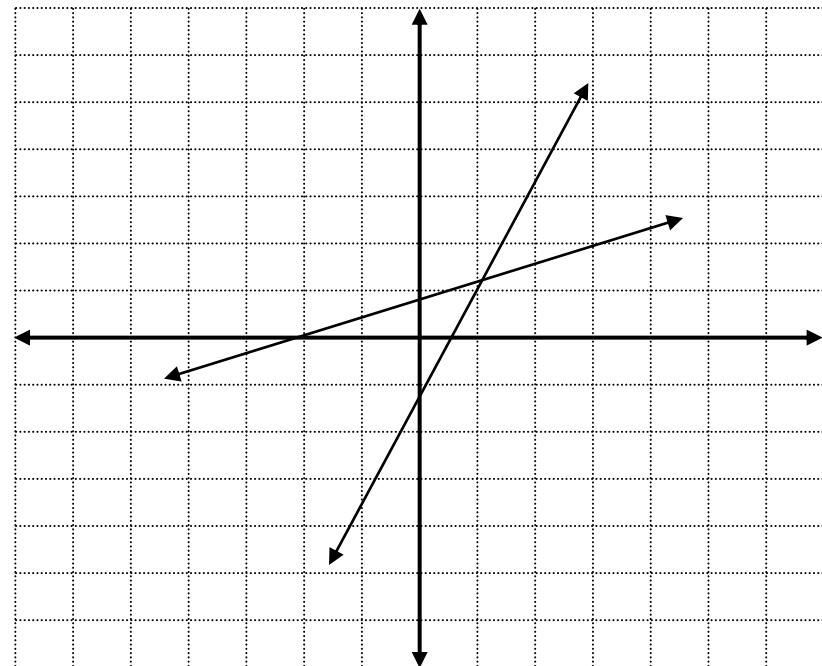
Q 5-1 Solve the system

$$\begin{cases} 2x - y = 1 \\ -x + 3y = 2 \end{cases}$$

graphically.

A 5-1

The solution point is (1 , 1).



Q5-2 Solve the following system using the elimination method:

$$\begin{cases} 18x - 75y = 2 \\ 12x - 45y = 4 \end{cases}$$

A 5-2

$$-2(18x - 75y = 2)$$

$$3(12x - 45y = 4)$$

$$-36x + 150y = -4$$

$$36x - 135y = 12$$

$$15y = 8y = \frac{8}{15}$$

$$18x - 75\left(\frac{8}{15}\right) = 2$$

$$x = \frac{7}{3}$$

$$S = \left\{ \left(\frac{7}{3}, \frac{8}{15} \right) \right\}.$$