بسم الله الرحمن الرحيم

الحمد لله رب العالمين والصلاة والسلام على أشرف الأنبياء والمرسلين

حل تمارين المقرر 101 ريض differential calculus

Chapter 2

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أعزائي طلاب وطالبات السنة الأولى المشتركة ؛ إن رأيتم أن هذا العمل مفيد فالرجاء إخبار زملائكم فالدال على الخير كفاعله

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لا تنسوني من دعائكم بارك الله فيكم

المحتويات رقم الصفحة

Section 2.1

حل تمارين (2.1) EXERCISES صفحة 159 و 160 و 161 في الكتاب

In Exercises 1 – 9 find the derivative of the given function using the limit definition of the derivative

في التمارين 1 – 9 أوجد مشتقة (derivative) الدالة المعطاة مستخدماً تعريف النهاية للمشتقة

the limit definition of the derivative:-

The derivative of a function f is the function f′ which value at any number x is

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

,provided the limit exists

1. $f(x) = 2x + 1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2(x+h) + 1 - 2x - 1}{h}$$

$$= \lim_{h\to 0} \frac{2x + 2h + 1 - 2x - 1}{h} = \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 = 2$$

2. $f(x) = x^2 - 3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 3 - x^2 + 3}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3 - x^2 + 3}{h} = \lim_{h\to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} (2x+h) = 2x + 0 = 2x$$

3. $f(x) = x^2 + 2x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 + 2(x+h) - x^2 - 2x}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 + 2h}{h} = \lim_{h\to 0} \frac{h(2x+h+2)}{h}$$

$$= \lim_{h\to 0} \quad 2x + h + 2 = 2x + 0 + 2 = 2x + 2$$

4. $f(x) = (x-2)^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h-2)^2 - (x-2)^2}{h}$$

نحسب $(x + h - 2)(x + h - 2)$

$$
\begin{array}{l}
x + h - 2 \\
x + h - 2 \\
\hline
x^2 + hx - 2x \\
\quad +hx + h^2 - 2h \\
\quad -2x - 2h + 4 \\
\hline
x^2 + 2hx - 4x - 4h + h^2 + 4
\end{array}
$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx - 4x - 4h + h^2 + 4 - x^2 + 4x - 4}{h}$$

$$\lim_{h \to 0} \frac{2hx - 4h + h^2}{h} = \lim_{h \to 0} \frac{h(2x - 4 + h)}{h}$$

$$= \lim_{h \to 0} 2x - 4 + h = 2x - 4 + 0 = 2x - 4 = 2(x - 2)$$

5. $f(x) = \dfrac{1}{\sqrt{x}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}(\sqrt{x+h})}}{h} = \lim_{h \to 0} \frac{\frac{(\sqrt{x} - \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}{\sqrt{x}(\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}}{\frac{h}{1}}$$

بالضرب في مرافق البسط والقسمة عليه للتخلص من الجذور

$$= \lim_{h\to 0} \frac{\frac{x-x-h}{\sqrt{x}\left(\sqrt{x+h}\,\right)(\sqrt{x}+\sqrt{x+h}\,)}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{-h}{\sqrt{x}\left(\sqrt{x+h}\,\right)(\sqrt{x}+\sqrt{x+h}\,)}\,\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{x}\left(\sqrt{x+h}\,\right)(\sqrt{x}+\sqrt{x+h}\,)}$$

$$= \frac{-1}{\sqrt{x}\left(\sqrt{x+0}\,\right)(\sqrt{x}+\sqrt{x+0}\,)} = \frac{-1}{\sqrt{x}\left(\sqrt{x}\,\right)(\sqrt{x}+\sqrt{x}\,)}$$

$$= \frac{-1}{x(2\sqrt{x}\,)} = -\frac{1}{2x^1x^{\frac{1}{2}}} = -\frac{1}{2x^{\frac{3}{2}}} =$$

6. $f(t) = \sqrt{1-t}$

$$f'(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \frac{\sqrt{1-t-h}-\sqrt{1-t}}{h}$$

بالضرب في مرافق البسط والقسمة عليه للتخلص من الجذور

$$= \lim_{h\to 0} \frac{(\sqrt{1-t-h}-\sqrt{1-t})(\sqrt{1-t-h}+\sqrt{1-t})}{h(\sqrt{1-t-h}+\sqrt{1-t})}$$

$$= \lim_{h\to 0} \frac{1-t-h-1+t}{h(\sqrt{1-t-h}+\sqrt{1-t})}$$

$$= \lim_{h\to 0} \frac{-h}{h(\sqrt{1-t-h}+\sqrt{1-t})} = \lim_{h\to 0} \frac{-1}{(\sqrt{1-t-h}+\sqrt{1-t})}$$

$$= \frac{-1}{(\sqrt{1-t-0}+\sqrt{1-t})} = \frac{-1}{\sqrt{1-t}+\sqrt{1-t}} = -\frac{1}{2\sqrt{1-t}}$$

7. $f(x) = 2-\sqrt{x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \frac{2-\sqrt{x+h}-2+\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{-\sqrt{x+h}+\sqrt{x}}{h} = \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h}$$

بالضرب في مرافق البسط والقسمة عليه للتخلص من الجذور

$$= \lim_{h\to 0} \frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{h(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h\to 0} \frac{x-x-h}{h(\sqrt{x}+\sqrt{x+h})} = \lim_{h\to 0} \frac{-1}{(\sqrt{x}+\sqrt{x+h})}$$

$$= -\frac{1}{\sqrt{x}+\sqrt{x+0}} = -\frac{1}{2\sqrt{x}} = -\frac{\sqrt{x}}{2x}$$

8. $f(t) = \dfrac{t}{t+2}$

$$f'(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h\to 0} \frac{\frac{t+h}{t+h+2}-\frac{t}{t+2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(t+h)(t+2)-t(t+h+2)}{(t+2)(t+h+2)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{t^2+ht+2t+2h-t^2-ht-2t}{(t+2)(t+h+2)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2h}{(t+2)(t+h+2)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{2h}{(t+2)(t+h+2)}\frac{1}{h}$$

$$\lim_{h\to 0} \frac{2}{(t+2)(t+h+2)} = \frac{2}{(t+2)(t+0+2)}$$

$$= \frac{2}{(t+2)(t+2)} = \frac{2}{(t+2)^2}$$

9. $f(x) = 1 - \frac{1}{x-1} = \frac{x-1-1}{x-1} = \frac{x-2}{x-1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{x+h-2}{x+h-1} - \frac{x-2}{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x-1)(x+h-2)-(x-2)(x+h-1)}{(x-1)(x+h-1)}}{h}$$

$$\lim_{h\to 0} \frac{\frac{h}{(x-1)(x+h-1)}}{\frac{h}{1}} = \frac{h}{(x-1)(x+h-1)}\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{1}{(x-1)(x+h-1)} = \frac{1}{(x-1)(x+0-1)} = \frac{1}{(x-1)^2}$$

In Exercises 10 – 15 find an equation of the tangent line to the graph $y = f(x)$ at the point x=a

في التمارين 10 – 15 أوجد معادلة الخط المماس (tangent line) لرسم $y = f(x)$ عند النقطة x=a

الميل m لخط المماس لرسم (منحنى) الدالة f عند النقطة $P(\,a, f(a)\,)$ هو:-

$$m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

أي أن ميل المماس عند النقطة $P(\,a, f(a)\,)$ لرسم الدالة هو مشتقة الدالة عند x=a أي عند الإحداثي السيني لنقطة التماس

النقطة $P(\,a, f(a)\,)$ تقع على المماس ولنفرض أن النقطة (x,y) تقع على المماس لذلك فإن معادلة المماس بدلالة الميل هي:-

$$m = \frac{y - f(a)}{x - a} \quad , \quad m(x-a) = y - f(a)$$

أي أن معادلة المماس هي(تحفظ) :-

$$\boxed{y = m(x-a) + f(a)}$$

10. $f(x) = x^2 - 1 \ , a = 1$

خطوات ايجاد معادلة المماس (tangent line)

1) نوجد ميل المماس عند النقطة المعطاة هنا a=1

$$m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{(1+h)^2 - 1 - 1^2 + 1}{h}$$

$$= \lim_{h\to 0} \frac{1 + 2h + h^2 - 1 - 1 + 1}{h} = \lim_{h\to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2+h)}{h} = \lim_{h\to 0} 2 + h = 2 + 0 = 2$$

إذاً $m = 2$

2) نوجد قيمة f(a) هنا نوجد f(1)

$$f(1) = (1)^2 - 1 = 1 - 1 = 0$$

3) نعوض عن كل من x=1 و f(1)=0 و m=2 في معادلة المماس أي

$$y = m(x-a) + f(a)$$

$$y = 2(x-1) + 0 \ , \qquad y = 2x - 2$$

$y = 2x - 2$ معادلة المماس

11. $f(x) = 2x^3 + 1 \ , \qquad a = -1$

1) نوجد ميل المماس عند النقطة المعطاة هنا a=-1

$$m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \frac{2(-1+h)^3 + 1 - (2(-1)^3 + 1)}{h}$$

$$= \lim_{h \to 0} \frac{2(-1 + 2h - h^2 + h - 2h^2 + h^3 - (-2 + 1)}{h}$$

$$= \lim_{h \to 0} \frac{-2 + 4h - 2h^2 + 2h - 4h^2 + 2h^3 + 1 + 2 - 1)}{h}$$

$$\lim_{h \to 0} \frac{6h - 6h^2 + 2h^3}{h} = \lim_{h \to 0} \frac{h(6 - 6h + 2h^2)}{h}$$

$$\lim_{h \to 0} 6 - 6h + 2h^2 = 6 - (6)(0) + 2(0) = 6$$

إذاً $m = 2$

2) نوجد قيمة f(a) هنا نوجد f(-1)

$$f(-1) = 2(-1)^3 + 1 = -2 + 1 = -1$$

3) نعوض عن كل من x=-1 و f(-1)=-1 و m=6 في معادلة المماس أي

$$y = m(x - a) + f(a)$$

$$y = 6(x - (-1)) - 1 , y = 6(x + 1) - 1 , y = 6x + 6 - 1$$

$y = 6x + 5$ معادلة المماس

12. $f(x) = 7x - x^2 \quad , \quad a = 2$

1) نوجد ميل المماس عند النقطة المعطاة هنا a=2

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \frac{7(2 + h) - \ (2 + h)^2 - 7x + x^2}{h}$$

$$= \lim_{h \to 0} \frac{14 + 7h - 4 - 4h - h^2 - 14 + 4}{h}$$

$$\lim_{h\to 0}\frac{3h-h^2}{h}=\lim_{h\to 0}\frac{h(3-h)}{h}=\lim_{h\to 0}3-h=3-0=3$$

إذاً $m=3$

2) نوجد قيمة f(a) هنا نوجد f(2)

$$f(2)=7(2)-2^2=14-4=10$$

3) نعوض عن كل من x=2 و f(2)=10 و m=3 في معادلة المماس أي

$$y=m(x-a)+f(a)$$

$$y=3(x-2)+10\,,\qquad y=3x-6+10\,,\;\; y=3x+4$$

$y=3x+4$ معادلة المماس

13. $f(x)=\dfrac{8}{x+4}\quad ,\qquad a=-2$

1) نوجد ميل المماس عند النقطة المعطاة هنا a= -2

$$m=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\frac{8}{(-2+h)+4}-\frac{8}{-2+4}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{8}{h+2}-\frac{8}{2}}{h}=\lim_{h\to 0}\frac{\frac{16-8h-16}{2h+4}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{-8h}{2h+4}}{\frac{h}{1}}=\lim_{h\to 0}\frac{-8h}{2h+4}\,\frac{1}{h}=\lim_{h\to 0}\frac{-8}{2h+4}$$

$$= \frac{-8}{2(0)+4} = \frac{-8}{4} = -2$$

إذاً $m = -2$

2) نوجد قيمة f(a) هنا نوجد f(-2)

$$f(-2) = \frac{8}{-2+4} = \frac{8}{3} = 4$$

3) نعوض عن كل من x=-2 و f(-2)=4 و m=-2 في معادلة المماس أي

$$y = m(x-a) + f(a)$$

$$y = -2(x-(-2)\,) + 4 = \ -2x - 4 + 4 = -2x$$

$y = -2x$ معادلة المماس

14. $f(x) = \dfrac{1}{x^2}$, $a = 3$

1) نوجد ميل المماس عند النقطة المعطاة هنا a= 3

$$m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \frac{\frac{1}{(3+h)^2} - \frac{1}{3^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{9-(3+h)^2}{9(3+h)^2}}{h} = \lim_{h\to 0} \frac{\frac{9-9-6h-h^2}{9(3+h)^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-6h-h^2}{9(3+h)^2}}{h} = \lim_{h\to 0} \frac{\frac{h(-6-h)}{9(3+h)^2}}{\frac{h}{1}} = \lim_{h\to 0} \frac{h(-6-h)}{9(3+h)^2}\,\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-6-h}{9(3+h)^2} = \frac{-6-0}{9(3+0)^2} = \frac{-6}{9(9)} = -\frac{2}{27}$$

إذاً $m = -\frac{2}{27}$

2) نوجد قيمة f(a) هنا نوجد f(3)

$$f(3) = \frac{1}{3^2} = \frac{1}{9}$$

3) نعوض عن كل من x=3 , $f(3) = \frac{1}{9}$, $m = -\frac{2}{27}$ في معادلة المماس أي

$$y = m(x-a) + f(a)$$

$$y = -\frac{2}{27}(x-3) + \frac{1}{9} = \frac{-2x+6+3}{27} = \frac{-2x+9}{27}$$

$y = \frac{-2x+9}{27}$ معادلة المماس

$$15.\ f(x) = \begin{cases} 4x+1 & ,x<2 \\ & & ,a=2 \\ x^2+5 & ,x\geq 2 \end{cases}$$

1) نوجد ميل المماس عند النقطة المعطاة هنا a= 2 لاحظ أن 2 في مجال الدالة x^2+5

لذلك نستخدم هذه الدالة أي تصبح المسألة

$f(x) = x^2 + 5 \qquad , a = 2$

$$m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{(2+h)^2 + 5 - 2^2 - 5}{h}$$

$$= \lim_{h\to 0} \frac{4 + 4h + h^2 + 5 - 4 - 5}{h} = \lim_{h\to 0} \frac{h(4+h)}{h}$$

$$= \lim_{h\to 0} 4 + h$$

$$= 4 + 0 = 4$$

$m = 4$ إذاً

2) نوجد قيمة f(a) هنا نوجد f(2)

$f(2) = 2^2 + 5 = 9$

3) نعوض عن كل من x=2 و f(2)=9 و m=-2 في معادلة المماس أي

$y = m(x-a) + f(a)$

$y = 4(x-2) + 9 = 4x - 8 + 9 = \ 4x + 1$

$y = 4x + 1$ معادلة المماس

In Exercises 16 – 19 .the given limit is derivative , find the function and the point

في التمارين 16 – 19 النهاية المعطاة هي المشتقة , أوجد الدالة والنقطة

هذه التمارين عكس السابقة

16. $\lim_{h\to 0} \frac{3(2+h)^2 - 3(2)^2}{h}$

كنا نعوض عن كل x بالمقدار a+h والآن نعوض عن كل a+h بالمتغير x

$f(x) = 3(x)^2 = 3x^2 \quad , \quad a = 2$

17. $\lim_{h \to 0} \dfrac{(3+h)^3 + 2(3+h) - 33}{h}$

كنا نعوض عن كل x بالمقدار a+h والآن نعوض عن كل a+h بالمتغير x

$f(x) = (x)^3 + 2(x) = x^3 + 2x \quad , \quad a = 3$

لم أفهم المطلوب من التمارين 18 و 19 ؟؟؟

20. $Let\ \ f(x) = x^2 - 7x \ \ , find$

$a.\ \ f'(x)$

$b.\ The\ \ equation\ \ of\ \ the\ \ tangent\ \ line\ to\ the\ graph\ of\ f\ at\ x = 4$

معادلة الخط المماس لرسم f عند x=4

$c.\ \ f'(5)$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 7(x+h) - x^2 + 7x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 7x - 7h - x^2 + 7x}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2 - 7h}{h} = \lim_{h \to 0} \frac{h(2x + h - 7)}{h}$$

$$\lim_{h\to 0}\ 2x+h-7=2x+0-7=2x-7$$

$a.\ f'(x)=2x-7$

$m=f'(4)=2(4)-7=1$

$f(4)=4^2-7(4)=16-28=-12$

نعوض عن كل من x=4 و f(4)=-12 و m=1 في معادلة المماس أي

$y=m(x-a)+f(a)$

$b.\ y=(1)(x-4)-12=x-16$

$c.\ f'(5)=2(5)-7=3$

$21.\ Let\ \ f(x)=x^3-7\ \ ,find$

$a.\ f'(x)$

$b.\ The\ slope\ of\ the\ tangent\ line\ to\ the\ graph\ of f at\ x=1$

$c.\ f'(2)$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^3-7\ -x^2+7}{h}$$

$$=\lim_{h\to 0}\frac{x^3+3hx^2+3xh^2+h^3-7-x^3+7}{h}$$

$$=\lim_{h\to 0}\frac{3hx^2+3xh^2+h^3}{h}=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}$$

$$=\lim_{h\to 0}\ 3x^2+3xh+h^2=3x^2+3x(0)+0=3x^2$$

$a.\ f'(x)=3x^2$

$b.\ m = f'(1) = 3$

$c.\ f'(2) = 3(2^2) = 12$

22. $f(x) = \sqrt{x-1}$, $find$

$a.\ \ f'(x)$

$b.\ D_{f'}$

$c.\ \ f'(3)$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{x+h-1} - \sqrt{x-1}\right)\left(\sqrt{x+h-1} + \sqrt{x-1}\right)}{h\left(\sqrt{x+h-1} + \sqrt{x-1}\right)}$$

$$= \lim_{h\to 0} \frac{x+h-1-x+1}{h\left(\sqrt{x+h-1} + \sqrt{x-1}\right)}$$

$$= \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h-1} + \sqrt{x-1}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h-1} + \sqrt{x-1}} = \frac{1}{\sqrt{x+0-1} + \sqrt{x-1}}$$

$$= \frac{1}{2\sqrt{x-1}}$$

$a.\ \ f'(x) = \dfrac{1}{2\sqrt{x-1}}$

$b.\ D_{f'} = domain\ \ f'(x)$

$$f'(x) = \frac{1}{2\sqrt{x-1}} \;, \qquad x \neq 1 = (-\infty, 1) \cup (1, \infty)$$

$b.\ D_{f'} = domain \;\; f'(x) = (1, \infty)$

$$c.\ \ f'(3) = \frac{1}{2\sqrt{3-1}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\frac{dy}{dx} = f'(x) = y'$$ ملاحظة

$23.\ f(x) = \sqrt{x+2} \quad , \qquad find$

$a.\ \ f'(x)$

$b.\ D_{f'}$

$c.\ \ f'(2)$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{x+h+2} - \sqrt{x+2}\right)\left(\sqrt{x+h+2} + \sqrt{x+2}\,\right)}{h\left(\sqrt{x+h+2} + \sqrt{x+2}\,\right)}$$

$$\lim_{h\to 0} \frac{x+h+2-x-2}{h\left(\sqrt{x+h+2} + \sqrt{x+2}\,\right)} \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h+2} + \sqrt{x+2}\right)}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{x+h+2} + \sqrt{x+2}} = \frac{1}{\sqrt{x+0+2} + \sqrt{x+2)}}$$

$$= \frac{1}{2\sqrt{x+2}}$$

$a.\quad f'(x) = \dfrac{1}{2\sqrt{x+2}}$

$b. D_{f'} = domain \quad f'(x)\,, \qquad x > -2 = (-2, \infty)$

$c.\quad f'(2) = \dfrac{1}{2\sqrt{2+2}} = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

$$24.\ Given\ the\ function\ f(x) = \begin{cases} 2x+1 & ,x \le 3 \\ x+4 & ,x > 3 \end{cases} \quad show\ that$$

$a.\ f\ is\ continuous\ at = 3$

$b.\ f\ is\ not\ differentiable\ at\ x = 3$

$a.\ f(3) = 2(3) + 1 = 7$

$$\lim_{x\to 3^-} 2x + 1 = 2(3) + 1 = 7 \qquad ,x < 3$$

$$\lim_{x\to 3^+} x + 4 = 3 + 4 = 7 \qquad ,x > 3$$

$$\lim_{x\to 3^-} = \lim_{x\to 3^+} = f(3) = 7 \quad so\ \ f\ is\ continuous\ at = 3$$

$$b.\ \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0}\ 2(h) + 1 = 0 + 1 = 1\,, x \le 3$$

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0}\ h + 4 = 0 + 4 = 4 \qquad ,x > 3$$

$$so\ \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = 1 \ \ and\ \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = 4$$

$$since\ \ 1 \neq 4\ \ \ so\ \ \ \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = d.n.e\ \ \ at\ x = 3$$

$$since\ \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = d.n.e$$

$so\ \ f\ is\ not\ differentiable\ at\ x = 3$

$25.$ $Given\ \ the\ function\ f(x) = |x-5|\ \ \ \ show\ that$

$a.\ \ f\ is\ continuous\ at\ =5$

$b.\ \ f\ is\ not\ differentiable\ at\ x\ =5$

$$f(x) = \begin{cases} x-5 & ,x \geq 5 \\ \\ -x+5 & ,x < 5 \end{cases}$$

$a.\ \ f(5) = x-5 = 5-5 = 0$

$$\lim_{x\to 5^-}\ -x+5 = (-5)+5 = -5+5 = 0 \qquad ,x < 5$$

$$\lim_{x\to 5^+}\ \ x-5 = 5-5 = 0\ \ \ ,x > 5$$

$$\lim_{x\to 5^-}\ =\ \lim_{x\to 5^+}\ = f(5) = 0\ \ \ \ so\ \ f\ is\ continuous\ at\ x = 5$$

$$b.\ \ \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\ \frac{5+h-5-5+5}{h}$$

$$= \lim_{h\to 0} \frac{h}{h} = 1 \ , x \geq 5$$

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{-5-h+5+5-5}{h} = \lim_{h\to 0} \frac{-h}{h}$$

$$= \lim_{h\to 0} \frac{-1}{1} = -1 \, , x < 5$$

$$so \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = 1 \ \ and \ \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = -1$$

$$since \ \ 1 \neq -1 \ \ so \ \ \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = d.n.e \quad at \ x = 5$$

$$since \ \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = d.n.e$$

$so \ \ f \ is \ not \ differentiable \ at \ x$ =5

In Exercises 20 – 29 .Find all point(s) on the graph of the function f(x) where the slope m of the tangent line has the indicated value.

في التمارين 20 – 29 أوجد كل النقاط التي على رسم الدالة f(x) حيث الميل m لخط التماس له القيمة المشار إليها في التمرين

ميل الخط المماس لرسم الدالة = مشتقة الدالة = m

26. $f(x) = 2x^2 - 4x \ \ , m = 0$

$$m = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h)^2 - 4(x+h) - 2x^2 + 4x}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2 + 2xh + h^2) - 4x - 4h - 2x^2 + 4x}{h}$$

$$= \lim_{h\to 0} \frac{(2x^2 + 4xh + 2h^2) - 4x - 4h - 2x^2 + 4x}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 - 4h}{h} = \lim_{h\to 0} \frac{h(4x + 2h - 4)}{h}$$

$$= \lim_{h\to 0} \quad 4x + 2h - 4 = 4x + 0 - 4 = 4x - 4$$

$m = 4x - 4$, $\quad$ $when\ m = 0$,$4x - 4 = 0\ so\ x = 1$

$f(1) = 2(1)^2 - 4(1) = 2 - 4 = -2$

so the slope of the tangent line *at point* $(1, -2) = 0$

27. $f(x) = x^2 - 6x$, $m = 0$

$$m = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - 6(x+h) - x^2 + 6x}{h}$$

$$= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2) - 6x - 6h - x^2 + 6x}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 6h}{h} = \lim_{h\to 0} \frac{h(2x + h - 6)}{h}$$

$$= \lim_{h\to 0} 2x + h - 6 = 2x + 0 - 6 = 2x - 6$$

$m = 2x - 6\ ,\qquad when\ m = 0\ , 2x = 6\ , x = 3$

$f(3) = (3)^2 - 6(3) = 9 - 18 = -9$

so the slope of the tangent line $at\ point\ (3, -9) = 0$

28. $f(x) = 2x^2 + 2x\ , m = 6$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 + 2(x+h) - 2x^2 - 2x}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + 2x + 2h - 2x^2 - 2x}{h}$$

$$= \lim_{h \to 0} \frac{(2x^2 + 4xh + 2h^2) + 2x + 2h - 2x^2 - 2x}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + 2h}{h} = \lim_{h \to 0} \frac{h(4x + 2h + 2)}{h}$$

$$= \lim_{h \to 0}\ 4x + 2h + 2 = 4x + 0 + 2 = 4x + 2$$

$m = 4x + 2\ ,\qquad when\ m = 6\ , 4x + 2 = 6\ so\ x = 1$

$f(1) = 2(1)^2 + 2(1) = 2 + 2 = 4$

so the slope of the tangent line $at\ point\ (1,4) = 6$

29. $f(x) = 2x^2 - 4x \ , m = 4$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 - 4(x+h) - 2x^2 + 4x}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 4x - 4h - 2x^2 + 4x}{h}$$

$$= \lim_{h \to 0} \frac{(2x^2 + \ 4xh + 2h^2) - 4x - 4h - 2x^2 + 4x}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 4h}{h} = \lim_{h \to 0} \frac{h(4x + 2h - 4)}{h}$$

$$= \lim_{h \to 0} \quad 4x + 2h - 4 = 4x + 0 - 4 = 4x - 4$$

$m = 4x - 4 \ , \qquad when\ m = 4 \ , 4x - 4 = 4\ so\ x = 2$

$f(2) = 2(2)^2 - 4(2) = 8 - 8 = 0$

so the slope of the tangent line $at\ point\ (2{,}0) = 4$

Each figure in Exercises 30 – 31 shows the graph of a function f. List the points on the graph at which the function is not differentiable

كل شكل في التمارين 30 – 31 يوضح رسم دالة f. أسرد النقاط التي على الشكل والتي عندها تكون الدالة غير قابلة للاشتقاق(أي المشتقة غير موجودة)

تكون الدالة غير قابلة للاشتقاق (not differentiable)عند $x=a$ (أي المشتقة غير موجودة غند $x=a$) إذ

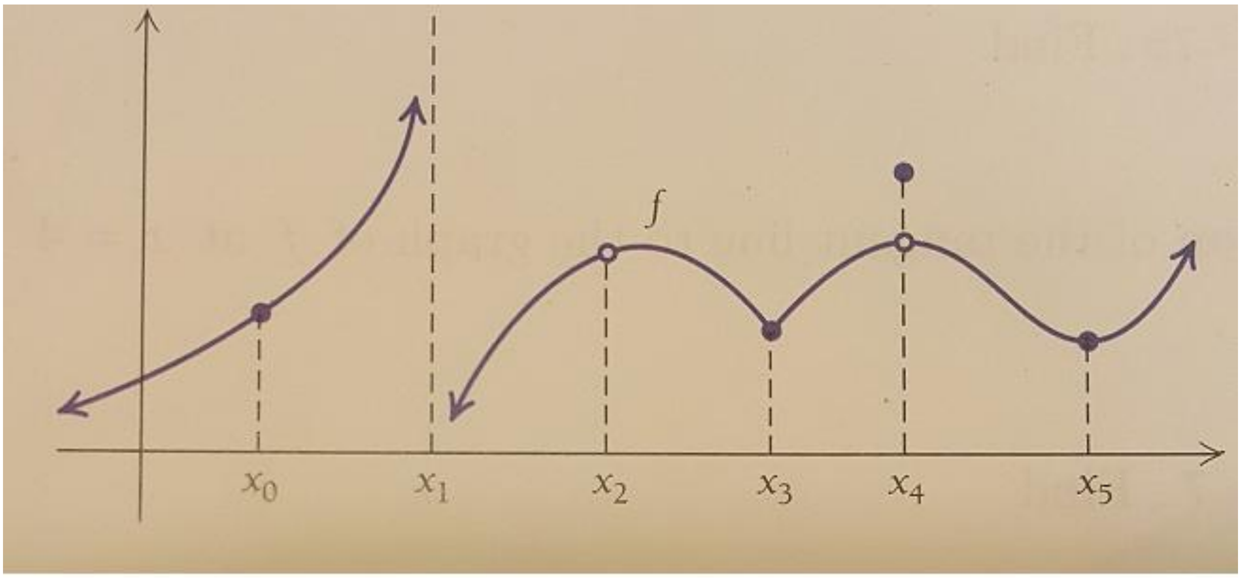
1) إذا كانت الدالة غير متصلة (discontinuous) عند $x=a$

2) رسم الدالة يكون على شكل زاوية (corner)عند النقطة (a,f(a))

أي اتجاه الرسم يتغير بصورة حادة عند النقطة (a,f(a))

3) المماس (tangent) لرسم الدالة يكون خط عمودي (vertical line) عند $x=a$

30.



X_1 لأن المماس عندها خط عمودي (vertical line)

X_2 لأنها غير معرفة أي غير موجودة لذلك الدالة غير متصلة عندها

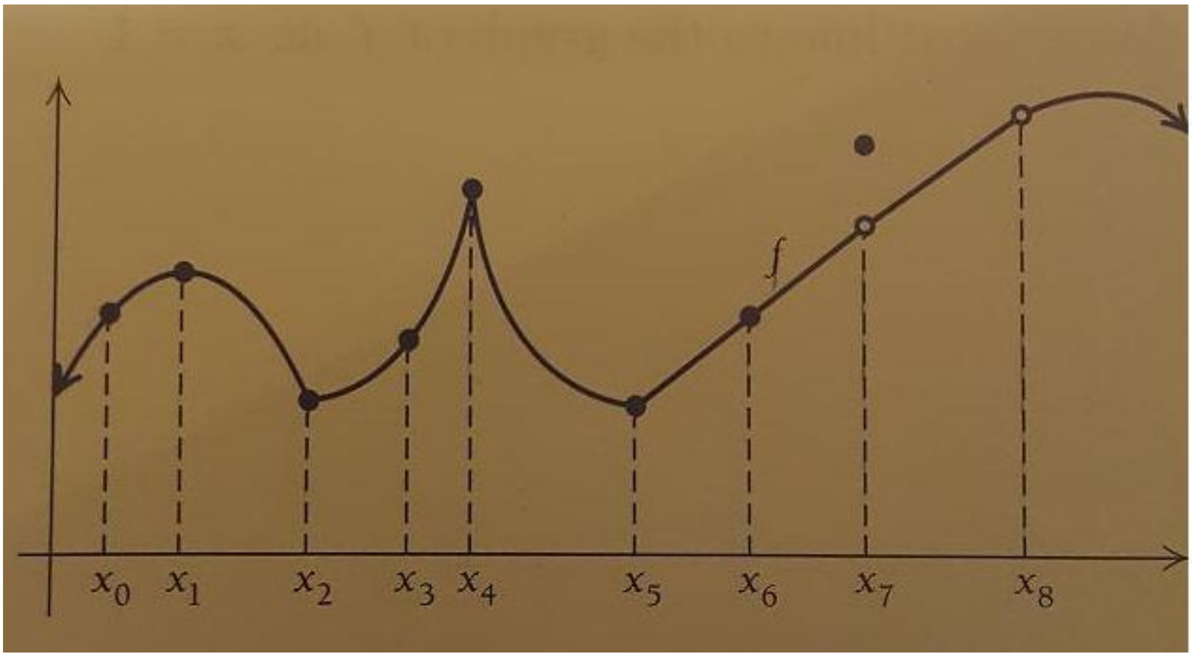
X_3 عندها رسم الدالة على شكل زاوية

X_4 عندها نهاية الدالة لا تساوي $f(x_4)$ لذلك الدالة غير متصلة عندها

يكون الجواب

the function is not differentiable at x_1 , x_2 , x_3 , x_4

31.



x_2 عندها رسم الدالة على شكل زاوية

x_4 لأن المماس عندها خط عمودي (vertical line)

X_5 عندها رسم الدالة على شكل زاوية

x_7 عندها نهاية الدالة لا تساوي $f(x_4)$ لذلك الدالة غير متصلة عندها

x_8 لأنها غير معرفة أي غير موجودة لذلك الدالة غير متصلة عندها

يكون الجواب

the function is not differentiable at x_2 , x_4 ,x_5,x_7,x_8

$32. A\ particle\ moves\ along\ the\ graph\ of\ the\ function$

$s(t) = t^2 + 3t\ , where\ s\ is\ measured\ in\ meters\ \ and\ t$

$in\ seconds\ . Find\ the\ instantaneuos\ velocity\ of\ the$

$particle\ when\ t = 1.$

32. جزئ يتحرك على رسم الدالة $s(t) = t^2 + 3t$, حيث s مقاسة بالأمتار و t بالثواني . أوجد السرعة اللحظية للجزئ عندما t=1

السرعة = مشتقة المسافة

$$v(t) = s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

$$= \lim_{h \to 0} \frac{(t+h)^2 + 3(t+h) - t^2 - 3t}{h}$$

$$= \lim_{h \to 0} \frac{(t^2 + 2th + h^2) + 3t + 3h - t^2 - 3t}{h}$$

$$= \lim_{h \to 0} \frac{2th + h^2 + 3h}{h} = \lim_{h \to 0} \frac{h(2t + h + 3)}{h}$$

$$= \lim_{h \to 0} 2t + h + 3 \ = 2t + 0 + 3 = 2t + 3$$

$$v(t) = 2t + 3$$

$the\ instantaneuos\ velocity\ of\ the\ particle\ when\ t = 1\ is$

$$v(1) = 2(1) + 3 = 5\ \ m/s$$

$33.$ *A particle moves along the graph of the function* $s(t) = 2t^3 + t + 1$ *, where s is measured in meters and t in seconds . Find the instantaneous velocity of the particle when* $t = 2.$

32. جزئ يتحرك على رسم الدالة $s(t) = 2t^3 + t + 1$, حيث s مقاسة بالأمتار و t بالثواني . أوجد السرعة اللحظية للجزئ عندما t=2

$$v(t) = s'(t) = \lim_{h\to 0} \frac{s(t+h) - s(t)}{h}$$

$$= \lim_{h\to 0} \frac{2(t+h)^3 + (t+h) + 1 - 2t^3 - t - 1}{h}$$

$$= \lim_{h\to 0} \frac{2(t^3 + 3ht^2 + 3th^2 + h^3) + t + h + 1 - 2t^3 - t - 1}{\mathrm{h}}$$

$$= \lim_{h\to 0} \frac{2t^3 + 6ht^2 + 6th^2 + 2h^3 + t + h + 1 - 2t^3 - t - 1}{h}$$

$$= \lim_{h\to 0} \frac{6ht^2 + 6th^2 + 2h^3 + h}{\mathrm{h}}$$

$$= \lim_{h\to 0} \frac{h(6\mathrm{t}^2 + 6\mathrm{th} + 2\mathrm{h}^2 + 1)}{h} = \lim_{h\to 0} 6\mathrm{t}^2 + 6\mathrm{th} + 2\mathrm{h}^2 + 1$$

$$= 6\mathrm{t}^2 + 0 + 0 + 1 = 6\mathrm{t}^2 + 1$$

so $v(t) = s'(t) = 6\mathrm{t}^2 + 1$

the instantaneous velocity of the particle when $t = 2$ *is*

$$v(2) = s'(2) = 6(2^2) + 1 = 6(4) + 1 = 24 + 1 = 25\ m/s$$

34. *An object moves along a coordinate line so that its* directed distance from the origin after t seconds is $t^2 + 1$ ft

$a.$ *Find its instantaneous velocity at* $t = a\ , a > 0$

$b.$ *When will it reach a velocity of* $6\ ft/sec$

يتحرك جسم على طول خط الأعداد بحيث أن بعده عن نقطة الأصل بعد t ثانية هي $t^2 + 1$ قدم (ft)

a) أوجد سرعته اللحظية عندما t=a ,a>0

b) متى تكون سرعته 6 أقدام في الثانية

$$v(t) = s'(t) = \lim_{h\to 0} \frac{s(t+h) - s(t)}{h}$$

$$= \lim_{h\to 0} \frac{(t+h)^2 + 1 - t^2 - 1}{h}$$

$$= \lim_{h\to 0} \frac{t^2 + 2th + h^2 + 1 - t^2 - 1}{\mathrm{h}} = \lim_{h\to 0} \frac{2th + h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2t+h)}{h} = \lim_{h\to 0} 2t + h = 2t + 0 = 2t$$

$v(t) = 2t$

$a.$ *its instantaneous velocity at* $t = a$ *is* $v(a) = 2a$

$b.$ *it will reach a velocity of* $6\ ft/\sec$

when $2t = 6$, *when* $t = 3$ *seconds*

35. *An object moves along a coordinate line so that its* directed distance from the origin after t seconds is $\sqrt{2t+1}$ ft

a. *Find its instantaneous velocity at* $t = a\,, a > 0$

b. *When will it reach a velocity of* $\frac{1}{3}\ ft/sec$

يتحرك جسم على طول خط الأعداد بحيث أن بعده عن نقطة الأصل بعد t ثانية هي t^2+1 قدم (ft)

a) أوجد سرعته اللحظية عندما t=a ,a>0

b) متى تكون سرعته $\frac{1}{3}$ قدم في الثانية

$$v(t) = s'(t) = \lim_{h\to 0} \frac{s(t+h)-s(t)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(t+h)+1}-\sqrt{2t+1}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{2(t+h)+1}-\sqrt{2t+1}\right)\left(\sqrt{2(t+h)+1}+\sqrt{2t+1}\right)}{h\left(\sqrt{2(t+h)+1}+\sqrt{2t+1}\right)}$$

$$\lim_{h\to 0} \frac{2(t+h)+1-2t-1}{h\left(\sqrt{2(t+h)+1}+\sqrt{2t+1}\right)}$$

$$= \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2(t+h)+1} + \sqrt{2t+1}\right)}$$

$$= \lim_{h\to 0} \frac{2}{\left(\sqrt{2(t+h)+1} + \sqrt{2t+1}\right)}$$

$$= \frac{2}{\left(\sqrt{2(t+0)+1} + \sqrt{2t+1}\right)} = \frac{2}{2\sqrt{2t+1}} = \frac{1}{\sqrt{2t+1}}$$

$$v(t) = \frac{1}{\sqrt{2t+1}}$$

$a.\, its\ instantaneous\ velocity\ at\ t = a\ \ is\ v(a) = \dfrac{1}{\sqrt{2a+1}}$

$b.\, it\ will\ reach\ a\ velocity\ of\ \ \dfrac{1}{3}\ ft/\sec$

$when\ \dfrac{1}{\sqrt{2t+1}} = \dfrac{1}{3}\ \ , \sqrt{2t+1} = 3,\ \ 2t+1 = 9$

$2t = 8\,, when\ \ t = 4\ seconds$

Section 2.2

حل تمارين (2.2) Exercises صفحة 174 و 175 و 176 في الكتاب

In Exercises 1-18 find the derivative of the given function

في التمارين 1-18 أوجد مشتقة الدالة المعطاة

إن إيجاد المشتقة بواسطة تعريف النهاية عمل ممل وطويل لذلك سنورد قوانين ونظريات تسهل إيجاد المشتقة

1) $if\ \ f(x) = mx + c\ \ \ , m, c\ are\ real\ numbers\ then$

$$f'(x) = \frac{d}{dx}(mx + c) = m$$

2) $if\ \ f(x) = c\ \ , c\ is\ real\ number\ then$

$$f'(x) = \frac{d}{dx}(c) = 0$$

3) $if\ \ f(x) = x\ \ \ then\ \ \ f'(x) = \frac{d}{dx}(x) = 1$

4) $if\ f(x) = x^r\ \ then\ \ \ f'(x) = \frac{d}{dx}(x^r) = rx^{r-1}$

5) $$(cf)'(x) = cf'(x)$$

6) $$(f + g)'(x) = f'(x) + g'(x)$$

7) $$(f - g)'(x) = f'(x) - g'(x)$$

8) $$(fg)'(x) = f(x)g'(x) + g(x)f'(x)$$

مشتقة حاصل ضرب حدين= الأول في مشتقة الثاني + الثاني في مشتقة الأول

9) $$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

مشتقة القسمة = (المقام في مشتقة البسط – البسط في مشتقة المقام) / (مربع المقام)

إنتبه: الترتيب هنا مهم تذكر المقام السامي هو الأول

1. $f(x) = 3x^3 - 4x^2 + 5x - 2$

$$f'(x) = (3)(3x^{3-1}) - (4)(2x^{2-1}) + 5 - 0$$

$$= 9x^2 - 8x + 5$$

2. $f(x) = 3x^7 - \frac{1}{5}x^5$

$$f'(x) = 3(7x^{7-1}) - \frac{1}{5}(5x^{5-1}) = 21x^6 - x^4$$

3. $f(x) = 3x^7 - \frac{1}{5}x^5 + 2x^{\frac{2}{3}}$

$$f'(x) = 21x^6 - x^4 + 2\left(\frac{2}{3}x^{\frac{2}{3}-1}\right) = 21x^6 - x^4 + \frac{4}{3}x^{\frac{-1}{3}}$$

$$= 21x^6 - x^4 + \frac{4}{3x^{\frac{1}{3}}} = 21x^6 - x^4 + \frac{4}{3\sqrt[3]{x}}$$

4. $f(x) = x^{-4} - \dfrac{1}{3x^3} + \dfrac{x^2}{2} - x$

$$f(x) = x^{-4} - \frac{1}{3}x^{-3} + \frac{x^2}{2} - x$$

$$f'(x) = -4x^{-5} - \frac{-3}{3}x^{-4} + \frac{2x}{2} - 1 = -\frac{4}{x^5} + \frac{1}{x^4} + x - 1$$

5. $g(x) = \dfrac{1}{x^4 + x^2 + 1}$

$$g'(x) = \frac{(x^4 + x^2 + 1)(0) - (1)(4x^3 + 2x)}{(x^4 + x^2 + 1)^2}$$

$$= -\frac{(4x^3 + 2x)}{(x^4 + x^2 + 1)^2}$$

6. $R(x) = \dfrac{\sqrt{7}}{x^5} = \sqrt{7}x^{-5}$

$$R'(x) = (\sqrt{7})\,(-5x^{-6}) = -\frac{5\sqrt{7}}{x^6}$$

7. $g(t) = (t^2 + t + 1)(t^2 + 2)$

$$g'(t) = (t^2 + t + 1)(2t) + (t^2 + 2)(2t + 1)$$

$$= 2t^3 + 2t^2 + 2t + 2t^3 + t^2 + 4t + 2$$

$$= 4t^3 + 3t^2 + 6t + 2$$

8. $h(x) = x^3(x+1)(x^2+2x)$

$= (x^4 + x^3)(x^2 + 2x)$

$h'(x) = (x^4 + x^3)(2x + 2) + (x^2 + 2x)(4x^3 + 3x^2)$

$= 2x^5 + 2x^4 + 2x^4 + 2x^3 + 4x^5 + 3x^4 + 8x^4 + 6x^3$

$= 6x^5 + 15x^4 + 8x^3$

9. $g(x) = (5x^2 - 7)(3x^2 - 2x + 1)$

$g'(x) = (5x^2 - 7)(6x - 2) + (3x^2 - 2x + 1)(10x)$

$= 30x^3 - 10x^2 - 42x + 14 + 30x^3 - 20x^2 + 10x$

$= 60x^3 - 30x^2 - 32x + 14$

10. $h(x) = (3x^2 + 2x)(x^4 - 3x + 1)$

$h'(x) = (3x^2 + 2x)(4x^3 - 3) + (x^4 - 3x + 1)(6x + 2)$

$= 12x^5 - 9x^2 + 8x^4 - 6x + 6x^5 - 18x^2 + 6x + 2x^4 - 6x$
$+ 2$

$= 18x^5 + 10x^4 - 27x^2 - 6x + 2$

11. $g(t) = \dfrac{t+2}{t-2}$

$$g'(t)=\frac{(t-2)(1)-(t+2)(1)}{(t-2)^2}=\frac{t-2-t-2}{(t-2)^2}$$

$$=-\frac{4}{(t-2)^2}$$

$$-\frac{4}{(t-2)^2}=\frac{-4}{(t-2)^2}=\frac{4}{-(t-2)^2}$$ لا تنسى

12. $g(x)=\dfrac{x^2-3}{x+4}$

$$g'(x)=\frac{(x+4)(2x)-(x^2-3)(1)}{(x+4)^2}=\frac{2x^2-x^2+3}{(x+4)^2}$$

$$=\frac{x^2+3}{(x+4)^2}$$

13. $f(x)=\dfrac{x}{3-2x}$

$$f'(x)=\frac{(3-2x)(1)-x(-2)}{(3-2x)^2}=\frac{3-2x+2x}{(3-2x)^2}=\frac{3}{(3-2x)^2}$$

14. $f(x)=|x+1|=\sqrt{(x+1)^2}=\;((x+1)^2)^{\frac{1}{2}}$

$$f'(x)=\frac{1}{2}\,((x+1)^2)^{\frac{-1}{2}}\,(2(x+1))(1)$$

$$=\frac{x+1}{((x+1)^2)^{\frac{1}{2}}}=\frac{x+1}{\sqrt{(x+1)^2}}=\frac{x+1}{|x+1|}=\frac{(x+1)(|x+1|)}{|x+1||x+1|}$$

$$= \frac{(x+1)(|x+1|)}{|(x+1)^2|} = \frac{(x+1)(|x+1|)}{(x+1)^2} = \frac{|x+1|}{x+1}$$

$$|x| = \sqrt{x^2} \quad , \quad \frac{x}{|x|} = \frac{|x|}{x} \quad , |x^2| = x^2$$

15. $f(z) = \dfrac{z^2+2}{z^3}$

$$f'(z) = \frac{z^3(2z) - (z^2+2)(3z^2)}{z^6} = \frac{2z^4 - 3z^4 - 6z^2}{z^6}$$

$$= \frac{-z^4 - 6z^2}{z^6} = \frac{z^2(-z^2-6)}{z^6} = -\frac{z^2+6}{z^4}$$

16. $\left(1+\dfrac{1}{x}\right)\left(1+\dfrac{1}{x^2}\right) = \left(\dfrac{x+1}{x}\right)\left(\dfrac{x^2+1}{x^2}\right)$

$$= \frac{x^3+x+x^2+1}{x^3}$$

$$f'(x) = \frac{x^3(3x^2+1+2x) - (x^3+x+x^2+1)(3x^2)}{x^6}$$

$$= \frac{3x^5+3x^4+x^3-3x^5-3x^4-3x^3-3x^2}{x^6}$$

$$= \frac{-2x^3-3x^2}{x^6} = \frac{x^2(-2x-3)}{x^6} = -\frac{2x+3}{x^4}$$

17. $f(t) = \dfrac{2t^2 - 3t + 1}{2t + 1}$

$$f'(t) = \frac{(2t+1)(4t-3) - (2t^2 - 3t + 1)(2)}{(2t+1)^2}$$

$$= \frac{8t^2 - 6t + 4t - 3 - 4t^2 + 6t - 2}{(2t+1)^2}$$

$$= \frac{4t^2 + 4t - 5}{(2t+1)^2}$$

18. $g(x) = \dfrac{x^2 - x + 1}{x^2 + 1}$

$$f'(x) = \frac{(x^2+1)(2x-1) - (x^2 - x + 1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^3 - x^2 + 2x - 1 - 2x^3 + 2x^2 - 2x}{(x^2+1)^2}$$

$$= \frac{x^2 - 1}{(x^2+1)^2}$$

In Exercises 19-27 find the equation of the tangent line to the graph of the function at the indicated point

في التمارين 19-27 أوجد معادلة الخط المماس (tangent line) لرسم الدالة عند النقطة المعينة

19. $f(x) = 2x^2 - 3x + 4 \quad ,(2,6)$

1) معادلة الخط المماس عند النقطة $(2,6)$ هي

$$y = m(x-2) + 6$$

2) m هي ميل المماس عند x=2 وتساوي مشتقة الدالة عند x=2

3) نوجد مشتقة الدالة $f'(x)$

4) $m = f'(2)$

$$f'(x) = 4x - 3 \quad , \qquad m = f'(2) = 4(2) - 3 = 8 - 3 = 5$$

the equation of the tangent line is

$$y = 5(x-2) + 6 = 5x - 10 + 6 = 5x - 4$$

20. $f(x) = -\frac{5}{3}x^2 + 2x + 2 \quad ,\left(-1, -\frac{5}{3}\right)$

1) معادلة الخط المماس عند النقطة $\left(-1, -\frac{5}{3}\right)$ هي

$$y = m(x - (-1)) - \frac{5}{3} = m(x+1) - \frac{5}{3}$$

2) m هي ميل المماس عند x=-1 وتساوي مشتقة الدالة عند x=-1

3) نوجد مشتقة الدالة $f'(x)$

$m = f'(-1)$ (4

$$f'(x) = -\frac{10}{3}x + 2 \;, \qquad m = f'(-1) = \left(-\frac{10}{3}\right)(-1) + 2$$

$$m = \frac{10}{3} + 2 = \frac{10+6}{3} = \frac{16}{3}$$

the equation of the tangent line is

$$y = \frac{16}{3}(x+1) - \frac{5}{3} = \frac{16}{3}x + \frac{16}{3} - \frac{5}{3} = \frac{16}{3}x + \frac{11}{3}$$

21. $f(x) = x^2 - 2x + 2 \;, \qquad (1,1)$

1) معادلة الخط المماس عند النقطة $(1,1)$ هي

$$y = m(x-1) + 1$$

2) m هي ميل المماس عند x=1 وتساوي مشتقة الدالة عند x=1

3) نوجد مشتقة الدالة $f'(x)$

$$f'(x) = 2x - 2 \;, \qquad m = f'(1) = 2(1) - 2 = 0$$

the equation of the tangent line is

$$y = (0)(x-1) + 1 = 0 - 0 + 1 = 1$$

22. $f(x) = \frac{x}{x-3}$, $(6,2)$

1) معادلة الخط المماس عند النقطة $(6,2)$ هي

$$y = m(x-6)+2$$

2) m هي ميل المماس عند x=6 وتساوي مشتقة الدالة عند x=6

3) نوجد مشتقة الدالة $f'(x)$

$$f'(x) = \frac{(x-3)(1)-x(1)}{(x-3)^2} = \frac{x-3-x}{(x-3)^2} = -\frac{3}{(x-3)^2}$$

$$m = f'(6) = -\frac{3}{(6-3)^2} = -\frac{3}{9} = -\frac{1}{3}$$

the equation of the tangent line is

$$y = -\frac{1}{3}(x-6)+2 = -\frac{1}{3}x+2+2 == -\frac{1}{3}x+4$$

23. $f(x) = \frac{1}{x^2+1}$, $\left(-1,\frac{1}{2}\right)$

1) معادلة الخط المماس عند النقطة $\left(-1,\frac{1}{2}\right)$ هي

$$y = m(x+1)+\frac{1}{2}$$

2) m هي ميل المماس عند x=-1 وتساوي مشتقة الدالة عند x=-1

3) نوجد مشتقة الدالة $f'(x)$

$$f'(x) = \frac{(x^2+1)(0)-(1)(2x)}{(x^2+1)^2} = -\frac{2x}{(x^2+1)^2}$$

$$m = f'(-1) = -\frac{2(-1)}{((-1)^2+1)^2} = \frac{2}{2^2} = \frac{2}{4} = \frac{1}{2}$$

the equation of the tangent line is

$$y = \frac{1}{2}(x+1) + \frac{1}{2} = \frac{1}{x}x + \frac{1}{2} + \frac{1}{2} = \frac{1}{x}x + 1$$

24. $f(x) = x^{-2}$, $(1,1)$

1) معادلة الخط المماس عند النقطة $(1,1)$ هي

$$y = m(x-1) + 1$$

2) m هي ميل المماس عند x=1 وتساوي مشتقة الدالة عند x=1

3) نوجد مشتقة الدالة $f'(x)$

$$f'(x) = -2x^{-3} = -\frac{2}{x^3}$$

$$m = f'(1) = -\frac{2}{1^3} = -\frac{2}{1} = -2$$

the equation of the tangent line is

$$y = m(x-1) + 1 = -2(x-1) + 1 = -2x + 2 + 1$$
$$= -2x + 3$$

25. $f(x) = \frac{x}{x+1} \quad , \qquad \left(1, \frac{1}{2}\right)$

1) معادلة الخط المماس عند النقطة $\left(1, \frac{1}{2}\right)$ هي

$$y = m(x-1) + \frac{1}{2}$$

2) m هي ميل المماس عند x=1 وتساوي مشتقة الدالة عند x=1

3) نوجد مشتقة الدالة $f'(x)$

$$f'(x) = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$$

$$m = f'(1) = \frac{1}{(1+1)^2} = \frac{1}{4}$$

the equation of the tangent line is

$$y = \frac{1}{4}(x-1) + \frac{1}{2} = \frac{1}{4}x - \frac{1}{4} + \frac{1}{2} = \frac{1}{4}x + \frac{1}{4}$$

$$-\frac{1}{4} + \frac{1}{2} = \frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}$$

26. $f(x) = x^2 - \frac{10}{x} = x^2 - 10x^{-1}$, $(-2,9)$

1) معادلة الخط المماس عند النقطة $(-2,9)$ هي

$$y = m(x+2) + 9$$

2) m هي ميل المماس عند x=-2 وتساوي مشتقة الدالة عند x=-2

3) نوجد مشتقة الدالة $f'(x)$

$$f'(x) = 2x - (-1)x^{-2} = 2x + \frac{1}{x^2}$$

$$m = f'(-2) = 2(-2) + \frac{1}{(-2)^2} = -4 + \frac{1}{4} = \frac{-16+1}{4} = -\frac{15}{4}$$

the equation of the tangent line is

$$y = -\frac{15}{4}(x+2) + 9 = -\frac{15}{4}x - \frac{15}{2} + 9$$

$$= -\frac{15}{4}x - \frac{15}{2} + \frac{18}{2} = -\frac{15}{4}x + \frac{3}{2}$$

27. $f(x) = x^3 - x$, $(-1,0)$

1) معادلة الخط المماس عند النقطة $(-1,0)$ هي

$$y = m(x+1)$$

2) m هي ميل المماس عند x=-1 وتساوي مشتقة الدالة عند x=-1

3) نوجد مشتقة الدالة $f'(x)$

$f'(x) = 3x^2 - 1$

$m = f'(-1) = 3(-1)^2 - 1 = 3 - 1 = 2$

the equation of the tangent line is

$y = 2(x + 1) = 2x + 2$

$28. Suppose\ f\ and\ g\ are\ functions\ of\ x\ that\ are$

$differentiable\ at\ x = 0\ and\ that$

$f(0) = 5\,, f'(0) = -3\ , \qquad g(0) = -1\ , and \quad g'(0) = 2$

Find the values of the following derivatives at $x = 0$

28. افرض أن *f* و *g* دوال للمتغير *x* وأنها قابلة للاشتقاق عند *x=0* وأن

$f(0) = 5\,, f'(0) = -3\ , \qquad g(0) = -1\ , and \quad g'(0) = 2$

أوجد قيم المشتقات الآتية عند *x=0*

$a. (fg)'(0) = f'(0)g'(0) = (-3)(2) = -6$

$$b. \left(\frac{f}{g}\right)'(0) = \frac{g(0)(f'(0)) - f(0)(g'(0))}{(g(0))^2} = \frac{-1(-3) - 5(2)}{(-1)^2}$$

$$= \frac{3 - 10}{1} = -7$$

$$c. \left(\frac{g}{f}\right)'(0) = \frac{f(0)g'(0) - g(0)f'(0)}{(f(0))^2} = \frac{5(2) - (-1)(-3)}{5^2}$$

$$= \frac{10 - 3}{25} = \frac{7}{25}$$

$d. (7g - 2f)'(0) = 7g'(0) - 2f'(0) = 7(2) - 2(-3) = 2$

$29. Suppose\ f\ and\ g\ are\ functions\ of\ x\ that\ are\ differentiable\ at\ x = 3\ and\ that$

$f(3) = 4\ , f'(3) = -6\ ,\qquad g(3) = 2\ \ , and\ \ g'(3) = 5$

Find the following

29. افرض أن f و g دوال للمتغير x وأنها قابلة للاشتقاق عند $x=3$ وأن

$f(3) = 4\ , f'(3) = -6\ ,\qquad g(3) = 2\ \ , and\ \ g'(3) = 5$

أوجد ما يلي

$a. (f+g)'(3) = f'(3) + g'(3) = -6 + 5 = -1$

$b. (fg)'(3) = f(3)g'(3) + g(3)\, f'(3) = 4(5) + 2(-6)$

$= 20 - 12 = 8$

$$c. \left(\frac{f}{g}\right)'(3) = \frac{g(3)(f'(3)) - f(3)(g'(3))}{(g(3))^2}$$

$$= \frac{2(-6) - 4(5)}{2^2} = \frac{-32}{4} = -8$$

$$d. \left(\frac{f}{f-g}\right)'(3) = \frac{(f-g)(3)f'(3) - f(3)(f-g)'(3)}{(\,f(3) - g(3)^2}$$

$$= \frac{\big(f(3) - g(3)\big)(-6) - 4(f'(3) - g'(3))}{(4-2)^2}$$

$$= \frac{(4-2)(-6) - 4(-6-5)}{2^2} = \frac{-12 + 44}{4} = \frac{32}{4} = 8$$

In Exercises 30 – 33 find x-coordinates points where the tangent line to the graph of the given function is horizontal

في التمارين 30 – 33 أوجد نقاط الإحداثي السيني والتي عندها يكون الخط المماس لرسم الدالة المعطاة أفقياً

يكون المماس أفقي أي أنه مواز لمحور *x* أي أن ميله يساوي 0

أي ما هو الإحداثي السيني للنقاط التي عندها تكون المشتقة تساوي 0

$$30. f(x) = \frac{5x}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)(5) - 5x(2x)}{(x^2+1)^2} = 0$$

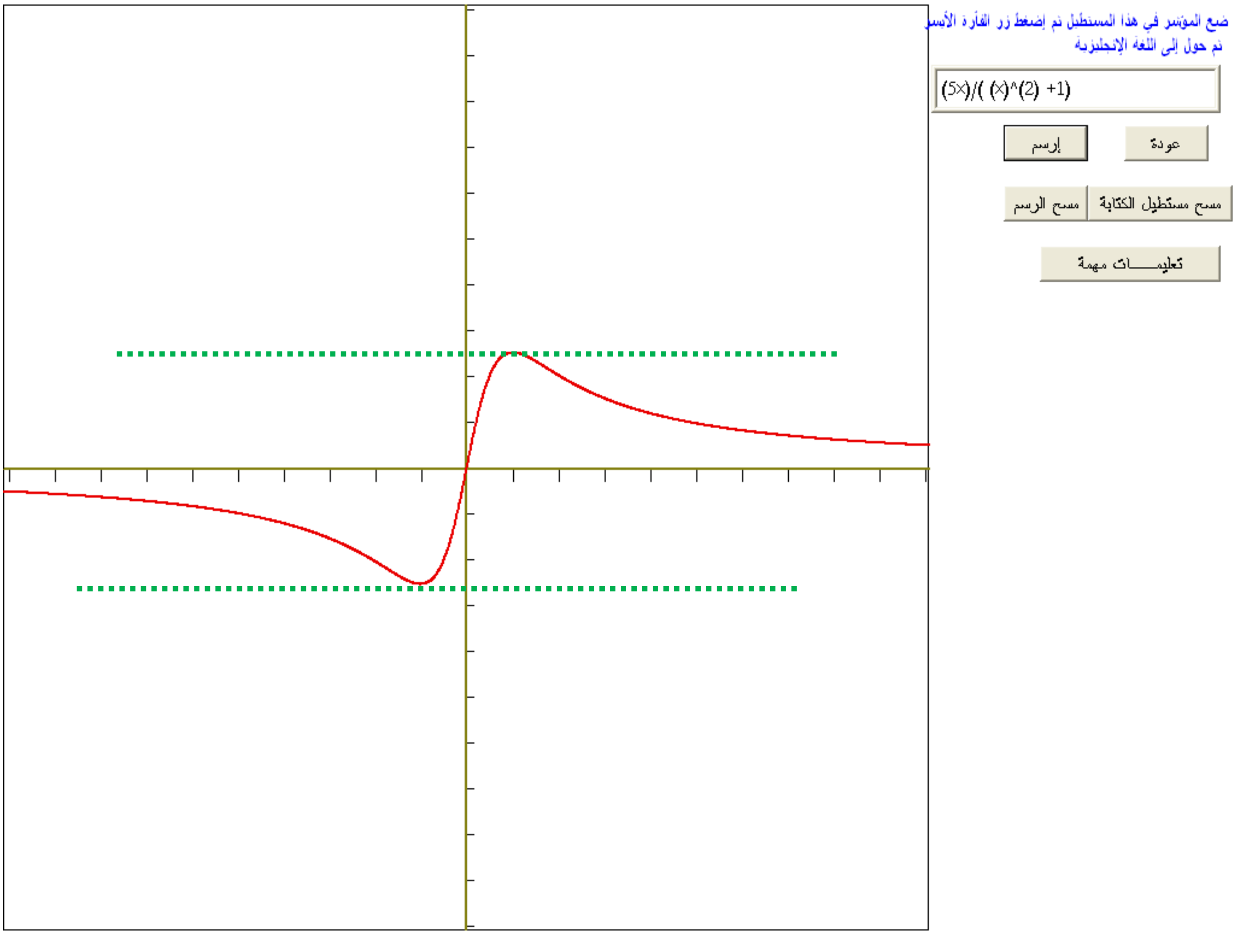
$$(x^2+1)(5) - 5x(2x) = 0\,, 5x^2 + 5 - 10x^2 = 0$$

$$, -5x^2 + 5 = 0\,, 5(-x^2+1) = 0\,, -x^2 + 1, x^2 = 1, x = \pm 1$$

The x-coordinates points where the tangent line to the graph of the given function is horizontal are

X=1 ,x=-1

أنظر الرسم ولاحظ أن المماس يكون خط أفقي عند *x=1* وعند *x=-1*



31. $f(x) = x^2 - x - 11$

$$f'(x) = 2x - 1 = 0\,, 2x = 1, x = \frac{1}{2}$$

The x-coordinates points where the tangent line to the graph of the given function is horizontal is

$$x = \frac{1}{2}$$

$32. f(x) = (x+2)(x^2 - 2x - 8)$

$f'(x) = (x+2)(2x-2) + (x^2 - 2x - 8)(1) = 0$

$, 2x^2 - 2x + 4x - 4 + x^2 - 2x - 8 = 0$

$, 3x^2 - 12 = 0 , 3x^2 = 12 , x^2 = 4, x = \pm 2$

The x-coordinates points where the tangent line to the graph of the given function is horizontal are

X=2 , x=-2

$33. f(x) = x^3 - x^2$

$f'(x) = 3x^2 - 2x = 0 , x(3x-2) = 0 , x = 0 \ , 3x - 2 = 0$

$, 3x = 2 , x = \frac{2}{3}$

The x-coordinates points where the tangent line to the graph of the given function is horizontal are

$x = 0 \ , \qquad x = \frac{2}{3}$

$34. Find\ all\ points\ (x, y) on\ the\ graph\ of\ \ f(x) = x^2\ with$

$tangent\ lines\ passing\ through\ the\ point\ (3,8)$

أوجد جميع النقاط (x,y) التي على رسم الدالة $f(x) = x^2$ والتي من أجلها خطوط التماس تمر بالنقطة (3,8)

$f'(x) = 2x, \qquad m = f'(3) = 6$

$y = 6(x - 3) + 8 = 6x - 18 + 8 = 6x - 10$

$35. Find\ equations\ for\ the\ tangent\ to\ the\ curve$

$y = x^3 - 4x + 1\ \ at\ the\ points\ where\ the\ slope\ of$

$the\ tangent\ is\ 8$

أوجد معادلات المماس للمنحني $y = x^3 - 4x + 1$ *عند النقاط* حيث *يكون ميل المماس 8*

$f'(x) = 3x^2 - 4\ , \qquad 3x^2 - 4\ = 8\, , 3x^2 - 4\ - 8 = 0$

$= 3x^2 -\ 12 = 0\ \, , 3x^2 = 12\ \ , \qquad x^2 = 4\, , x = \pm 2$

$y = 8 - 8 + 1 = 1\, , \qquad y = -8 + 8 + 1 = 1$

$y = 8(x - 2) + 1\ \ , \qquad y = 8(x + 2) + 1$

$y = 8x - 16 + 1 = 8x - 15$

$y = 8x + 16 + 1 = 8x + 17$

$36. Let\ f(x) = \frac{2}{3}x^3 + x^2 - 12x + 6\ .Find\ the\ values\ of\ x$

$for\ which$

$a.\ f'(x) = -12 \qquad b. f'(x) = 0 \quad c.\ f'(x) = 12$

لتكن $f(x) = \frac{2}{3}x^3 + x^2 - 12x + 6$ أوجد قيم x التي من أجلها

$a.\ f'(x) = -12 \qquad b. f'(x) = 0 \quad c.\ f'(x) = 12$

$f'(x) = 2x^2 + 2x - 12$

$a.\ 2x^2 + 2x - 12 = -12\ , \qquad 2x^2 + 2x = 0\ , x(2x+2) = 0$

$, \qquad x = 0\ \ or\ \ 2x + 2 = 0\ \ , \qquad x = \frac{-2}{2} = -1$

$b. 2x^2 + 2x - 12 = 0\ , x^2 + x - 6 = 0\ , (x-2)(x+3) = 0$

$x = 2\ \ or\ \ x = -3$

$c.\ 2x^2 + 2x - 12 = 12\ , \qquad 2x^2 + 2x - 24 = 0$

$x^2 + x - 12 = 0\ \ , \qquad (x-3)(x+4) = 0$

$x = 3\ \ or\ \ x = -4$

$37.$ *Find the point(s) on the graph of the function*

$f(x) = 2x + \frac{1}{x}$ *at which the slope of tangent line is* -2

أوجد النقاط التي على منحني (رسم) الدالة $f(x) = 2x + \frac{1}{x}$ والتي عندها بكون ميل الخط المماس يساوي -2

$$f'(x) = 2 + \frac{x(0) - 1(1)}{x^2} = 2 - \frac{1}{x^2}$$

$$2 - \frac{1}{x^2} = -2 , - \frac{1}{x^2} = -4 , x^2 = \frac{1}{4}, x = \pm\frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + \frac{1}{\frac{1}{2}} = 1 + 2 = 3$$

$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) + \frac{1}{-\frac{1}{2}} = -1 - 2 = -3$$

the points are $\left(\frac{1}{2}\right), 3)$, $(-\frac{1}{2}, -3)$

$38.$ *Solve the equation* $\frac{1}{x} + \frac{1}{y} = 6$ *for y and find* $\frac{dy}{dx}$

حل المعادلة $\frac{1}{x} + \frac{1}{y} = 6$ من أجل y وأوجد $\frac{dy}{dx}$

$$\frac{1}{x} + \frac{1}{y} = 6, \frac{1}{y} = 6 - \frac{1}{x} = \frac{6x - 1}{x}, y = \frac{x}{6x - 1}$$

$$\frac{dy}{dx} = \frac{(6x - 1)(1) - x(6)}{(6x - 1)^2} = \frac{6x - 1 - 6x}{(6x - 1)^2} = \frac{-1}{(6x - 1)^2}$$

39. $\textit{Find the values of } a \textit{ and } b \textit{ if the tangent to}$

$y = ax^2 + bx \;\; at \; (1,5) has \; slope \; 8$

أوجد قيم a و b إذا كان ميل المماس للدالة $y = ax^2 + bx$ عند (1,5)هو 8

$5 = a(1^2) + b(1) \, , a + b = 5$

$\frac{dy}{dx} = 2ax + b \, , 2ax + b = 8 \;\; , 2a(1) + b = 8 \, , 2a + b = 8$

$a + b = 5 \;\; , 2a + b = 8$

$a = 5 - b \, , 2(5 - b) + b = 8 \, , 10 - 2b + b = 8$

$$-b = -2 \;\; , b = 2 \;\; , \;\; a + 2 = 5, a = 3$$

40. $\textit{Determine whether } f(x) = \begin{cases} x^3 & , x \leq 1 \\ 4x - 3 & , x < 1 \end{cases}$

$is \; defferentiable \; at \; x = 1 \; or \; not$

$f'(x) = 3x^2 \, , \qquad f'(1) = 3(1^2) = 3 \quad , x \leq 1$

$f'(x) = 4 \, , \qquad f'(1) = 4 \qquad\qquad , x < 1$

$3 \neq 4 \; so \; f(x) is \; not \; defferentiable \; at \; x = 1$

$$41. \textit{Find a and b given that } f(x) = \begin{cases} ax^2 + b & , x \leq -1 \\ bx^5 + ax + 4 & , x > -1 \end{cases}$$

is everywhere continuous

$$\lim_{x \to -1} ax^2 + b = a(-1)^2 + b = a + b$$

$$\lim_{x \to -1} bx^5 + ax + 4 = b(-1)^5 - a + 4 = -b - a + 4$$

$$a + b = -b - a + 4 \ , \ \ 2a = -2b + 4 \, , \qquad a = -b + 2$$

$$f'(x) = 2ax \qquad , f'(-1) = 2a(-1) = -2a \ \ , x \leq -1$$

$$f'(x) = 5bx^4 + a \, , f'(-1) = \ 5b(-1)^4 + a = 5b + a \, , x > -1$$

$$-2a = 5b + a \, , -3a = 5b \, , a = -\frac{5}{3}b$$

$$a = -b + 2 \, , a = -\frac{5}{3}b \ so \ -\frac{5}{3}b = -b + 2$$

$$-\frac{5}{3}b + b = 2 \, , \frac{-5b + 3b}{3} = 2 \, , \frac{-2b}{3} = 2 \, , -2b = 6$$

$$, b = \frac{6}{-2} = -3$$

$$b = -3 \ \ , \qquad a = -b + 2 = -(-3) + 2 = 3 + 2 = 5$$

$42. Use\ product\ rule\ to\ show\ that$

$$\frac{d}{dx}\left[\left(f(x)\right)^2\right] = 2f(x)f'(x)$$

$$\left(f(x)\right)^2 = \left(f(x)\right)\left(f(x)\right)$$

$$\frac{d}{dx}\left[\left(f(x)\right)\left(f(x)\right)\right] = f(x)f'(x) + f(x)f'(x) = 2f(x)f'(x)$$

$43. The\ position\ of\ a\ particle\ is\ given\ by$

$s(t) = t^3 - 4.5t^2 - 7t\ \ , t \geq 0$

$where\ t\ is\ measured\ in\ seconds\ and\ s\ in\ meter$

$When\ the\ particle\ reaches\ a\ velocity\ of\ 5m/s$

موقع جزئ معطى بالمعادلة

$s(t) = t^3 - 4.5t^2 - 7t\ \ , t \geq 0$

حيث t مقاسة بالثانية و s بالمتر

متى تصل سرعة الجزئ إلى 5m/s (5 متر في الثانية)

$v(t) = s'(t) = 3t^2 - 9t - 7$

$3t^2 - 9t - 7 = 5\ , 3t^2 - 9t - 12 = 0\ , t^2 - 3t - 4 = 0$

$, (t-4)(t+1) = 0\ \ , \qquad t = 4\ \ or\ t = -1$

$so\ only\ t = 4\ \ because\ t \geq 0$

S(t) تعني المسافة و v(t) تعني السرعة والسرعة = مشتقة المسافة

$44. A\ particle\ moves\ according\ to$

$s(t) = t^2 - 6t + 9\ \ , t \geq 0$

$where\ t\ is\ measured\ in\ seconds\ and\ s\ in\ feet$

$a. Find\ the\ velocity\ of\ the\ particle\ at\ time\ t$

$b. What\ is\ the\ velocity\ of\ the\ particle\ after\ 2\ seconds?$

يتحرك جزئ بحسب المعادلة

$s(t) = t^2 - 6t + 9\ \ , t \geq 0$

حيث t مقاسة بالثانية و s بالقدم

a) أوجد سرعة الجزئ عند الزمن t

a) ماهي سرعة الجزئ بعد مرور 2 ثانية

$a.\ v(t) = s'(t) = 2t - 6$

$b.\ v(2) = 2(2) - 6 = -2$?????

Section 2.3

حل تمارين (Exercises (2.3 صفحة 184 و 185 في الكتاب

1. $\frac{d}{dx}(\sin x) = \cos x$, 2. $\frac{d}{dx}(\cos x) = -\sin x$ تحفظ

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x cos\ x - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{cos^2 x + sin^2 x}{(\cos x)^2} = \frac{1}{(\cos x)^2} = sec^2 x$$

3. $\frac{d}{dx}(\tan x) = sec^2 x$

$$\frac{d}{dx}(\cot x) = \frac{\cos x}{\sin x} = \frac{\sin x(-\sin x) - \cos x(\cos x)}{(\sin x)^2}$$

$$= \frac{-(sin^2 x + cos^2 x)}{(\sin x)^2} = \frac{-1}{(\sin x)^2} = -csc^2 x$$

4. $\frac{d}{dx}(\cot x) = -csc^2 x$

$$\frac{d}{dx}(sec\ x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{\cos x(0) - (1)(-\sin x)}{(cos^2 x)}$$

$$= \frac{\sin x}{(cos^2 x)} = \frac{\sin x}{\cos x}\frac{1}{\cos x} = \tan x \sec x$$

5. $\frac{d}{dx}(sec\ x) = \tan x \sec x$

$$\frac{d}{dx}(csc\ x) = \frac{d}{dx}\left(\frac{1}{sin\ x}\right) = \frac{\sin x(0) - (1)\cos x}{\sin x \sin x}$$

$$= \frac{-\cos x}{\sin x \sin x} = \frac{-\cos x}{\sin x}\frac{1}{\sin x} = -\cot x \csc x$$

6. $\frac{d}{dx}(csc\ x) = -\cot x \csc x$

$sin^2 x = \sin x\ sinx = (\sin x)^2$ لا تنسى مثلاً

$In\ Exercises\ 1-20\ \ find\ \ \frac{dy}{dx}$

$1. y = x\ csc\ x$

$$\frac{dy}{dx} = x(-\cot x csc\ x) + \csc x(1) = \csc x(1 - xcotx)$$

$2. y = \cot x\ cscx$

$$\frac{dy}{dx} = \cot x(\,-\cot x \csc x) + \csc x(\,-csc^2 x)$$

$$= -\csc x\ cot^2 x - csc^3 = -\csc x(\,cot^2 x + csc^2 x)$$

$$= -\ \frac{1}{\sin x}\left(\frac{cos^2 x}{sin^2\, x} + \frac{1}{sin^2\, x}\right) = \ -\frac{cos^2 x + 1}{sin^3 x}$$

$3. y = x(1 + \sec x)$

$$\frac{dy}{dx} = x(\tan x \sec x) + (1 + \sec x)\,(1)$$

$$= xtan\ x \sec x + \sec x + 1$$

$$4.\ y = \frac{\tan x}{x}$$

$$\frac{dy}{dx} = \frac{x(sec^2x) - \tan x(1)}{x^2} = \frac{xsec^2x - \tan x}{x^2}$$

$$5.\ y = \frac{\sec x}{x - 1}$$

$$\frac{dy}{dx} = \frac{(x - 1)(\tan x \sec x) - \sec x(1)}{(x - 1)^2}$$

$$= \frac{\sec x[(x - 1)\tan x) - 1]}{(x - 1)^2}$$

$$6.\ y = \frac{\csc x}{\tan x}$$

$$\frac{dy}{dx} = \frac{\tan x(-\cot x \csc x) - \csc x(sec^2x)}{tan^2x}$$

$$= \frac{-\tan x \cot x \csc x - \csc x sec^2x}{tan^2x}$$

$$= \frac{-(1)\csc x - \csc x sec^2x}{tan^2x} = -\frac{\csc x(1 + sec^2x)}{tan^2x}$$

$$7.\ y = \frac{\cot x}{1+\csc x}$$

$$\frac{dy}{dx} = \frac{(1+\csc x)(-csc^2 x) - \cot x(-\cot x \csc x)}{(1+\csc x)^2}$$

$$= \frac{-csc^2 x\,(1+\csc x) + cot^2 x \csc x}{(1+\csc x)^2}$$

$$= \frac{-csc^2 x\,(1+\csc x) + (csc^2 x - 1)\csc x}{(1+\csc x)^2}$$

$$= \frac{-csc^2 x(1+\csc x) + (\csc x - 1)(\csc x + 1)\csc x}{(1+\csc x)^2}$$

$$= \frac{-csc^2 x + (\csc x - 1)\csc x}{1+\csc x}$$

$$= \frac{-csc^2 x + csc^2 x - \csc x}{1+\csc x} = -\frac{\csc x}{1+\csc x}$$

انظر الكتاب صفحة 58 لتجد أن

$$csc^2 x = 1 + cot^2 x$$

8. $y = x^2 \cos x - 4\sin x$

$$\frac{dy}{dx} = x^2(-\sin x) + \cos x(2x) - 4\cos x$$

$$= -x^2 \sin x + 2xcos\ x - 4\cos x$$

$$-x^2 \sin x + \cos x(2x - 4)$$

9. $y = sin^2 x + cos^2 x = 1\,, \dfrac{dy}{dx} = 0$

$$10.\ y = \frac{(x^2+1)\cot x}{3-\cos x\csc x} = \frac{(x^2+1)\cot x}{3-\cos x\,\frac{1}{\sin x}}$$

$$= \frac{(x^2+1)\cot x}{3-\cot x}$$

$$\frac{dy}{dx}\left[\frac{(x^2+1)\cot x}{3-\cot x}\right] = \frac{dy}{dx}\left[\frac{x^2+1}{3-\cot x}\right]\ \frac{dy}{dx}\left[\frac{\cot x}{3-\cot x}\right]$$

$$\frac{dy}{dx}\left[\frac{x^2+1}{3-\cot x}\right] = \frac{(3-\cot x)(2x)-(x^2+1)(csc^2x)}{(3-\cot x)^2}$$

$$\frac{dy}{dx}\left[\frac{\cot x}{3-\cot x}\right] = \frac{(3-\cot x)(-csc^2x)-\cot x(csc^2x)}{(3-\cot x)^2}$$

$$let\ a = \frac{(3-\cot x)(2x)-(x^2+1)(csc^2x)}{(3-\cot x)^2}$$

$$,\qquad b = \frac{(3-\cot x)(-csc^2x)-\cot x(csc^2x)}{(3-\cot x)^2}$$

$$\frac{dy}{dx} = (a)(b)$$

$$11.\ y = \frac{\sin x\sec x}{1+xtan\ x} = \frac{\sin x(\frac{1}{\cos x})}{1+xtan\ x} = \frac{\tan x}{1+xtan\ x}$$

$$\frac{dy}{dx} = \frac{(1+xtan\ x)(sec^2x)-[\tan x\{(xsec^2x)+\tan x\}]}{(1+xtan\ x)^2}$$

$$= \frac{(1 + xtan\, x)(sec^2x)}{(1 + xtan\, x)^2} - \frac{xtanxsec^2x + tan^2x}{(1 + xtan\, x)^2}$$

$$= \frac{sec^2x + xtan\, sec^2x - xtanxsec^2x - tan^2x}{(1 + xtan\, x)^2}$$

$$= \frac{sec^2x - tan^2x}{(1 + xtan\, x)^2} = \frac{\frac{1}{cos^2x} - \frac{sin^2x}{cos^2x}}{(1 + xtan\, x)^2} = \frac{\frac{1 - sin^2x}{cos^2x}}{(1 + xtan\, x)^2}$$

$$= \frac{\frac{cos^2x}{cos^2x}}{(1 + xtan\, x)^2} = \frac{1}{(1 + xtan\, x)^2}$$

$$12.\, y = \frac{tanx - 1}{secx} = \frac{\frac{sinx}{cosx} - 1}{\frac{1}{cosx}} = \frac{\frac{sinx - cosx}{cosx}}{\frac{1}{cosx}}$$

$$= \frac{sinx - cosx}{cosx}\ \frac{cosx}{1} = sinx - cosx$$

$$\frac{dy}{dx} = cosx - (-sinx) = cosx + sinx$$

$$13.\, y = x^{-3}sinxtanx = \frac{sinxtanx}{x^3}$$

$$\frac{dy}{dx} = x^{-3}[sinx(sec^2x) + tanxcosx] + sinxtanx(-3x^{-4})$$

$$\frac{dy}{dx} = \frac{x^3[(sinx(sec^2x) + tanxcosx)] - 3x^2sinxtanx}{x^6}$$

$$= \frac{x(sinxsec^2x + tanxcosx) - 3sinxtanx}{x^4}$$

$$= \frac{x\left(sinxsec^2x + \frac{sinx}{cosx}cosx\right) - 3sinxtanx}{x^4}$$

$$= \frac{xsinxsec^2x + xsinx - 3sinxtanx}{x^4}$$

$$= \frac{sinx(xsec^2x + x - 3tanx)}{x^4}$$

14. $y = xsinxcosx$

$$\frac{dy}{dx} = x(-sin^2x + cos^2x) + sinxcosx(1)$$

$$= x(-sin^2x + 1 - sin^2x) + sinxcosx$$

$$= x(1 - 2sin^2x) + sinxcosx = x - 2xsin^2x + sinxcosx$$

15. $y = tan^2x$

$$\frac{dy}{dx} = 2tanx(sec^2x)$$

16. $y = sec^3x$

$$\frac{dy}{dx} = 3sec^2x(\tan x \sec x) = 3tanxsec^3x$$

17. $y = x^2 cosx$

$$\frac{dy}{dx} = x^2(-sinx) + cosx(2x) = -x^2 sinx + 2xcosx$$

18. $y = \frac{1 - cosx}{x}$

$$\frac{dy}{dx} = \frac{x(sinx) - (1 - cosx)(1)}{x^2} = \frac{xsinx - 1 + cosx}{x^2}$$

19. $y = \frac{sinx + cosx}{cosx} = \frac{sinx}{cosx} + \frac{cosx}{cosx} = tanx + 1$

$$\frac{dy}{dx} = sec^2 x$$

20. $y = \frac{xcosx + sinx}{x^2 + 1} = \frac{xcosx}{x^2 + 1} + \frac{sinx}{x^2 + 1}$

$$\frac{dy}{dx} = \frac{(x^2 + 1)(-xsinx + cosx) - (xcosx)(2x)}{(x^2 + 1)^2}$$

$$+ \frac{(x^2 + 1)cosx - sinx(2x)}{(x^2 + 1)^2}$$

$$= \frac{-xsinx + cosx}{x^2 + 1} - \frac{2x^2 cosx}{(x^2 + 1)^2}$$

$$+ \frac{cosx}{x^2 + 1} - \frac{2x^2 cosx}{(x^2 + 1)^2}$$

$$= \frac{-xsinx + cosx}{x^2+1} + \frac{cosx}{x^2+1} - \frac{4x^2cosx}{(x^2+1)^2}$$

$$= \frac{-xsinx}{x^2+1} + \frac{cosx}{x^2+1} + \frac{cosx}{x^2+1} - \frac{4x^2cosx}{(x^2+1)^2}$$

$$= \frac{-xsinx}{x^2+1} + \frac{2cosx}{x^2+1} - \frac{4x^2cosx}{(x^2+1)^2}$$

In Exercises 21-24 find the equation of the tangent line to the graph of the function at the indicated point

في التمارين 21-24 أوجد معادلة الخط المماس لرسم الدالة عند النقطة المشار إليها

$21. f(x) = sinx \quad , \left(\frac{\pi}{6}, \frac{1}{2}\right)$

$$f'(x) = cosx \, , m = f'\left(\frac{\pi}{6}\right) = cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

the equation of the tangent line is

$$y = m(x-a) + f(a) = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}x - \frac{\sqrt{3}\pi}{12} + \frac{1}{2} = \frac{\sqrt{3}}{2}x + \frac{-\sqrt{3}\pi + 6}{12}$$

$$= \frac{\sqrt{3}}{2}x + \frac{6 - \sqrt{3}\pi}{12}$$

$22. f(x) = tanx , \left(\frac{\pi}{4}, 1\right)$

$f'(x) = sec^2 x , \qquad m = f'\left(\frac{\pi}{4}\right) = (\sec\left(\frac{\pi}{4}\right))^2 = (\sqrt{2})^2 = 2$

the equation of the tangent line is

$y = m(x-a) + f(a) = 2\left(x - \frac{\pi}{4}\right) + \ 1 = 2x - \frac{\pi}{2} + 1$

$= 2x + \frac{-\pi + 2}{2} = 2x + \frac{2-\pi}{2}$

$23. f(x) = secx , \left(\frac{\pi}{3}, 2\right)$

$f'(x) = \tan x \sec x \ , \ m = \ f'\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) \sec\left(\frac{\pi}{3}\right)$

$m = \ \sqrt{3}\,(2) = 2\sqrt{3}$

the equation of the tangent line is

$y = m(x-a) + f(a) = 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + \ 2$

$= 2\sqrt{3}x - \frac{2\sqrt{3}\pi}{3} + 2 = 2\sqrt{3}x + \frac{6 - 2\sqrt{3}\pi}{3}$

$24. f(x) = secx - 2cosx , \left(\frac{\pi}{3}, 1\right)$

$f'(x) = \tan x \sec x + 2sinx$

$$m = \ f'\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right)\sec\left(\frac{\pi}{3}\right) + 2\sin\ \left(\frac{\pi}{3}\right)$$

$$= \sqrt{3}\ (2) + 2\frac{\sqrt{3}}{2} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$$

the equation of the tangent line is

$$y = m(x-a) + f(a) = 3\sqrt{3}\left(x - \frac{\pi}{3}\right) + \ 1$$

$$= 3\sqrt{3}x - \frac{3\sqrt{3}\pi}{3} + 1 = 3\sqrt{3}x - \sqrt{3}\,\pi + 1$$

$In\ Exersises\ 25 - 26\ prove\ each\ formula$

في التمارين 25-26 أثبت كل صيغة

$$25. \frac{d}{dx}(cotx) = -sec^2x$$

$$\frac{d}{dx}(\cot x) = \frac{\cos x}{\sin x} = \frac{\sin x(-\sin x) - \cos x(\cos x)}{(\sin x)^2}$$

$$= \frac{-(sin^2x + cos^2x)}{(\sin x)^2} = \frac{-1}{(\sin x)^2} = -csc^2x$$

$$26. \frac{d}{dx}(\csc x) = -cscxcotx$$

$$\frac{d}{dx}(csc\ x) = \frac{d}{dx}\left(\frac{1}{sin\ x}\right) = \frac{\sin x(0) - (1)\cos x}{\sin x \sin x}$$

$$= \frac{-\cos x}{\sin x \sin x} = \frac{-\cos x}{\sin x}\frac{1}{\sin x} = -\cot x \csc x$$

$27. A\ particle\ moves\ according\ to\ the\ position\ \ function$

$$s(t) = \frac{cost}{t}$$

Where t is measured in seconds and s in feet , Find the

$instantaneous\ velocity\ at\ t = \pi$

يتحرك جزئ بحسب دالة الموقع

$$s(t) = \frac{cost}{t}$$

حيث t مقاسة بالثواني و s مقاسة بالأقدام . أوجد السرعة اللحظية عند $t= \pi$

$$v(t) = s'(t) = \frac{t(-sint) - cost}{t^2} = -\frac{tsint + cost}{t^2}$$

$$v(\pi) = s'(\pi) = -\frac{tsin(\pi) + \cos{(\pi)}}{\pi^2} = -\frac{0 + (-1)}{\pi^2} = \frac{1}{\pi^2}\ f/s$$

$In\ Exercises\ 28 - 31\ , find\ the\ x - coordinate(s)$

$of\ the\ point(s)\ on\ the\ graph\ of\ the\ function\ at\ which$

$the\ tangent\ line\ has\ the\ indicated\ slope$

في التمارين 28-31 أوجد الاحداثيات السينية للنقاط التي على رسم الدالة والتي عندها الخط المماس له الميل المشار إليه

$28. g(x) = x + sinx\ \ , \qquad m = 1$

$g'(x) = 1 + cosx\ , \qquad 1 + cosx = 1\ , cosx = 0$

$$x = \frac{\pi}{2}\ , \frac{3\pi}{2}\ , \frac{5\pi}{2}\ , \dots\ \ , -\frac{\pi}{2}\ \ , -\ \frac{3\pi}{2}\ , -\frac{5\pi}{2}\ , \dots = \frac{\pi}{2} \pm n\pi$$

29. $f(x) = sinx \;, \qquad m = 1$

$f'(x) = cosx \;, \qquad cosx = 1$

$, x = 0,2\pi, 4\pi, 6\pi \ldots., -2\pi, -4\pi, -6\pi = \pm 2n\pi$

30. $g(x) = cotx \;, \qquad m = -2$

$g'(x) = -csc^2x \;,$

$$-csc^2x = -2 \, , csc^2x = 2 \, , cscx = \pm\sqrt{2}$$

$x = \frac{\pi}{4} \pm \frac{n\pi}{2} \; , n \; is \; integer$

31. $f(x) = cscx \;, \qquad m = 0$

$f'(x) = -\cot x \csc x \;, \; -\cot x \csc x = 0$

$\cot x \csc x = 0 \, , \quad \frac{cosx}{sinx} \, \frac{1}{sinx} = 0 \, , cosx = 0$

$x = \frac{\pi}{2} \;, \frac{3\pi}{2} \;, \frac{5\pi}{2} \;, \ldots \;\;, -\;\frac{\pi}{2} \;\;, -\;\frac{3\pi}{2} \;, -\frac{5\pi}{2} \;, \ldots = \frac{\pi}{2} \pm n\pi$

In $Exercises\ 32 - 34\ find\ all\ points\ on\ the\ graph\ of\ f(x)$ $where\ the\ tangent\ line\ is\ horizonal$

في التمارين 32-34 جد كل النقاط التي على رسم الدالة $f(x)$ والتي عندها يكون خط التماس أفقياً

ملاحظة : إذا كان المماس أفقي فهذا يعني أن ميله يساوي 0

$32. f(x) = tan^2x$

$f'(x) = 2tanx(sec^2x), \qquad 2tanx(sec^2x) = 0$

$, tanx(sec^2x) = 0\,, \qquad \frac{sinx}{cosx}\,\frac{1}{cos^2x} = 0\ , sinx = 0$

$x = 0, \pi, 2\pi, 3\pi, \dots, -\pi, -2\pi, -3\pi = \pm n\pi$

$33. f(x) = 9sinxcosx$

$f'(x) = 9[-sin^2x + cos^2x]$

$-sin^2x + cos^2x = 0\,, -sin^2x + 1 - sin^2x = 0$

$2sin^2x = 1\,, sin^2x = \frac{1}{2}\ , \qquad sinx = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4} + \frac{n\pi}{2}\ , \qquad n\ is\ integer$

$34. f(x) = \frac{cosx}{2 + sinx}$

$f'(x) = \frac{(2 + sinx)(-sinx) - cosx(cosx)}{(2 + sinx)^2}$

$= \frac{-2sinx - sin^2(x) - cos^2(x)}{(2 + sinx)^2} = \frac{-2sinx - 1}{(2 + sinx)^2} = 0$

$-2sinx - 1 = 0\,, sinx = -\frac{1}{2}$

$x = -\frac{\pi}{6} + 2n\pi\ , n\ is\ integer$

Section 2.4

حل تمارين (2.4) EXERCISES صفحة 194 و 195 و 196 في الكتاب

قانون السلسلة the chain rule

$$if\ \ y = f(u)\ , \qquad u = g(x)\ , \qquad then\ \frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

$$or \quad \frac{dy}{dx} = f'(\,g(x))g'(x)$$

قانون القوة العامة the general power rule

$$\frac{d}{dx}\,(\,g(x)\,)^r = r(\,g(x)\,)^{r-1}\frac{d}{dx}(\,g(x))$$

$$or \quad \frac{d}{dx}\,(\,g(x)\,)^r = r(\,g(x)\,)^{r-1}(\,g'(x))$$

In exercises 1-30 find the derivative of the following functions

في التمارين 1-30 جد مشتقة الدوال الآتية

1. $f(x) = (2x+3)^3$

$f'(x) = 3(2x+3)^2(2) = 6(2x+3)^2$

2. $f(x) = (x^2+2x)^5$

$f'(x) = 5(x^2+2x)^4(2x+2) = (10x+10)(x^2+2x)^4$

3. $g(x) = (x^5 - 10)^{30}$

$g'(x) = 30(x^5 - 10)^{29}(5x^4) = 150x^4(x^5 - 10)^{29}$

4. $h(x) = (x^2 + 3x + 1)^4$

$h'(x) = 4(x^2 + 3x + 1)^3(2x + 3)$

$= (8x + 12)(x^2 + 3x + 1)^3$

5. $f(t) = \dfrac{1}{(1 + t^{2)})^2} = (1 + t^{2)})^{-2}$

$f'(t) = -2(1 + t^{2)})^{-3}(2t) = -\dfrac{4t}{(1 + t^{2)})^3}$

6. $g(x) = \sqrt{\dfrac{x}{x+1}}$, $g(x) = \sqrt{u} = u^{\frac{1}{2}}$, $u(x) = \dfrac{x}{x+1}$

$u'(x) = \dfrac{(x+1)(1) - x(1)}{(x+1)^2} = \dfrac{1}{(x+1)^2}$

$g'(x) = \dfrac{1}{2}u^{-\frac{1}{2}}(u'(x)\ ) = \dfrac{1}{2\sqrt{u}}\left(\dfrac{1}{(x+1)^2}\right)$

$= \dfrac{1}{2\sqrt{\dfrac{x}{x+1}}}\left(\dfrac{1}{(x+1)^2}\right) = \dfrac{1}{\dfrac{2\sqrt{x}}{\sqrt{x+1}}}\left(\dfrac{1}{(x+1)^2}\right)$

$$= \frac{\sqrt{x+1}}{2\sqrt{x}}\left(\frac{1}{(x+1)^2}\right) = \frac{1}{2\sqrt{x}(x+1)^2(x+1)^{-\frac{1}{2}}}$$

$$= \frac{1}{2\sqrt{x}\,(x+1)^{\frac{3}{2}}} = \frac{1}{2\sqrt{x}\,\sqrt{(x+1)^3}} = \frac{1}{2\sqrt{x(x+1)^3}}$$

or

$$6.\ g(x) = \sqrt{\frac{x}{x+1}} = (x^{\frac{1}{2}})(x+1)^{-\frac{1}{2}}$$

$$g'(x) = \left(x^{\frac{1}{2}}\right)\left(-\frac{1}{2}(x+1)^{-\frac{3}{2}}\right) + (x+1)^{-\frac{1}{2}}\left(\frac{1}{2}(x^{-\frac{1}{2}}\right)$$

$$= -\frac{\left(x^{\frac{1}{2}}\right)}{2(x+1)^{\frac{3}{2}}} + \frac{1}{2(x+1)^{\frac{1}{2}}\left(x^{\frac{1}{2}}\right)}$$

$$= \frac{1}{2\sqrt{x+1}}\left[-\frac{\left(x^{\frac{1}{2}}\right)}{x+1} + \frac{1}{\left(x^{\frac{1}{2}}\right)}\right] = \frac{1}{2\sqrt{x+1}}\left[\frac{-x+x+1}{(x+1)\left(x^{\frac{1}{2}}\right)}\right]$$

$$= \frac{1}{2\sqrt{x}\,\sqrt{(x+1)^3}} = \frac{1}{2\sqrt{x(x+1)^3}}$$

$$7.\ f(t) = \left(t - \frac{2}{t}\right)^3 = \left(\frac{t^2-2}{t}\right)^3$$

$$f'(t) = 3\left(\frac{t^2-2}{t}\right)^2\left(\frac{t(2t)-(t^2-2)(1)}{t^2}\right)$$

$$= 3\left(\frac{t^2-2}{t}\right)^2\left(\frac{2t^2-t^2+2}{t^2}\right) = 3\left(\frac{t^2-2}{t}\right)^2\left(\frac{t^2+2)}{t^2}\right)$$

$$= 3\left(t-\frac{2}{t}\right)^2\left(1+\frac{2}{t^2}\right)$$

8. $f(x) = (x^2+1)^2\sqrt[3]{2x+1} = (x^2+1)^2\,(2x+1)^{\frac{1}{3}}$

$$f'(x) = (x^2+1)^2\left(\frac{1}{3}\,(2x+1)^{-\frac{2}{3}}(2)\right)$$

$$+(2x+1)^{\frac{1}{3}}(2(x^2+1)(2x)$$

$$= \frac{2}{3}\,\frac{(x^2+1)^2}{(2x+1)^{\frac{2}{3}}} + \ 4x(x^2+1)\sqrt[3]{2x+1}$$

9. $g(x) = (3x^2+1)(2x-1)^3$

$$g'(x) = (3x^2+1)(3(2x-1)^2(2)) + (2x-1)^3(6x)$$

$$= (2x-1)^2[3(3x^2+1)(2) + (2x-1)(6x)]$$

$$= (2x-1)^2[18x^2+6+12x^2-6x]$$

$$= (2x-1)^2[30x^2-6x+6] = 6(2x-1)^2[5x^2-x+1]$$

10. $g(x) = \cos(x^3)$

$g'(x) = -\sin(x^3)(3x^2) = -(3x^2)\sin(x^3)$

11. $f(x) = (1 + cos^2x)^6$

$f'(x) = 6(1 + cos^2x)^5(2cosx)(-sinx)$

$= -12sinxcosx(1 + cos^2x)^5$

12. $h(x) = \sqrt[4]{\tan(x^3)} = (\tan(x^3))^{\frac{1}{4}}$

$$h'(x) = \frac{1}{4}(\tan(x^3))^{\frac{-3}{4}}(3x^2) = \frac{(3x^2)}{4(\tan(x^3))^{\frac{3}{4}}}$$

$$= \frac{3}{4}\frac{x^2}{\sqrt[4]{(\tan(x^3))^2}}$$

13. $y = sec^2(2x) - tan^2(2x)$

$$\frac{dy}{dx} = 2\sec(2x)(\tan(2x)\sec(2x)(2) - 2\tan(2x)(sec^2(2x)(2)$$

$= 16x^2tan(2x)sec^2(2x) - 16x^2tan(2x)sec^2(2x) = 0$

14. $h(\theta) = \csc(\frac{\theta}{3})$

$$h'(\theta) = -\cot\left(\frac{\theta}{3}\right)\csc\left(\frac{\theta}{3}\right)\left(\frac{1}{3}\right) = -\frac{1}{3}\cot\left(\frac{\theta}{3}\right)\csc\left(\frac{\theta}{3}\right)$$

15. $f(x) = cos^2(cosx) + sin^2(cosx) = 1$

$f'(x) = 0$

$cos^2(x) + sin^2(x) = 1$ لا تنسى

16. $h(x) = \sin(\sin(\sin x))$

$h(x) = \sin(u) \quad , \qquad u = \sin(\sin x)$

$h'(x) = \cos(u)\,(u'(x)) \;, u'(x) = \cos(sinx)\, cosx$

$h'(x) = \cos(\sin(\sin x))\,(\cos(sinx)\, cosx)$

17. $f(x) = \sqrt[3]{x^3 + (2x-1)^3}$

$f(x) = \sqrt[3]{u} = u^{\frac{1}{3}} \;, \qquad u = x^3 + (2x-1)^3$

$f'(x) = \frac{1}{3}u^{-\frac{2}{3}}\left(u'(x)\right), \qquad u'(x) = 3x^2 + 3\,(2x-1)^2\,(2)$

$$f'(x) = \frac{u'(x)}{3u^{\frac{2}{3}}} = \frac{3x^2 + 6\,(2x-1)^2}{3\,\sqrt[3]{u^2}}$$

$$= \frac{3x^2 + 6\,(2x-1)^2}{3\sqrt[3]{(x^3 + (2x-1)^3\;)^2}}$$

18. $g(x) = \left(\frac{1-8x}{1+8x}\right)^4$

$$g'(x) = 4\left(\frac{1-8x}{1+8x}\right)^3\left[\frac{(1+8x)(-8) - (1-8x)(8)}{(1+8x)^2}\right]$$

$$= 4\left(\frac{1-8x}{1+8x}\right)^3\left[\frac{-8-64x-8+64x}{(1+8x)^2}\right]$$

$$= 4\left(\frac{1-8x}{1+8x}\right)^3\left[\frac{-16}{(1+8x)^2}\right] = -\frac{64}{(1+8x)^2}\left(\frac{1-8x}{1+8x}\right)^3$$

19. $f(u) = \left[\frac{1-2u}{1+u}\right]^3$

$$f'(u) = 3\left[\frac{1-2u}{1+u}\right]^2\left[\frac{(1+u)(-2)-(1-2u)(1)}{(1+u)^2}\right]$$

$$= 3\left[\frac{1-2u}{1+u}\right]^2\left[\frac{-2-2u-1+2u}{(1+u)^2}\right] = 3\left[\frac{1-2u}{1+u}\right]^2\left[\frac{-3}{(1+u)^2}\right]$$

$$= -9\frac{(1-2u)^2}{(1+u)^4}$$

20. $g(x) = \left(\frac{x+5}{x^2+2}\right)^2$

$$g'(x) = 2\left(\frac{x+5}{x^2+2}\right)\left(\frac{(x^2+2)(1)-(x+5)(2x)}{(x^2+2)^2}\right)$$

$$= 2\left(\frac{x+5}{x^2+2}\right)\left(\frac{x^2+2-2x^2-10x}{(x^2+2)^2}\right)$$

$$= 2\left(\frac{x+5}{x^2+2}\right)\left(\frac{-x^2-10x+2}{(x^2+2)^2}\right)$$

$$= -2\left(\frac{x+5}{x^2+2}\right)\left(\frac{x^2+10x-2}{(x^2+2)^2}\right)$$

21. $g(x) = (2 + (x^2 + 1)^4)^3$

$g'(x) = 3(2 + (x^2 + 1)^4)^2(4(x^2 + 1)^3)(2x))$

$= 3(2 + (x^2 + 1)^4)^2(8x(x^2 + 1)^3$

$= 24x(x^2 + 1)^3(2 + (x^2 + 1)^4)^2$

22. $f(t) = 3sec^2(\pi t - 1)$

$f'(t) = 6\sec(\pi t - 1)\,((\tan(\pi t - 1)\sec(\pi t - 1))(\pi)$

$= 6\pi\ \sec(\pi t - 1)\,(\tan(\pi t - 1)\sec(\pi t - 1)$

23. $f(x) = cos(\,(1 - 2x)^2\,)$

$= -sin(\,(1 - 2x)^2\,)(2(1 - 2x)(-2)$

$= -sin(\,(1 - 2x)^2\,)(-4(1 - 2x))$

$= 4(1 - 2x)sin(\,(1 - 2x)^2\,)$

24. $g(t) = 2cot^2(\pi t + 2)$

$g'(t) = 4\cot(\pi t + 2)\,(-csc^2(\pi t + 2)(\pi))$

$= -4\pi\cot(\pi t + 2)csc^2(\pi t + 2)$

$$25.\ g(x) = \sin(\tan(2x))$$

$$g'(x) = co\,s(\tan(2x))\,(sec^2(2x)(2))$$

$$= 2co\,s(\tan(2x))\,sec^2(2x)$$

$$26.\ g(t) = \sec\left(\frac{1}{2}t\right)\tan\left(\frac{1}{2}t\right)$$

$$g'(t) = \sec\left(\frac{1}{2}t\right)\left(sec^2\left(\frac{1}{2}t\right)\right)\left(\frac{1}{2}\right) + \tan\left(\frac{1}{2}t\right)(\tan\left(\frac{1}{2}t\right)\sec\left(\frac{1}{2}t\right)\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)sec^3\left(\frac{1}{2}t\right) + \left(\frac{1}{2}\right)\sec\left(\frac{1}{2}t\right)tan^2\left(\frac{1}{2}t\right)$$

$$= \left(\frac{1}{2}\right)\sec\left(\frac{1}{2}t\right)\left(sec^2\left(\frac{1}{2}t\right) + tan^2\left(\frac{1}{2}t\right)\right)$$

$$27.\ f(x) = \left(\frac{sinx}{\cos(2x)}\right)^3$$

$$f'(x) = 3\left(\frac{sinx}{\cos(2x)}\right)^2\left(\frac{\cos(2x)\,cosx - sinx\big(-\sin(2x)\,(2)\big)}{cos^2(2x)}\right)$$

$$= 3\left(\frac{sinx}{\cos(2x)}\right)^2\left(\frac{cosxcos(2x) + 2sinxsin(2x)}{cos^2(2x)}\right)$$

$$= 3\frac{sin^2x}{cos^2(2x)}\left(\frac{cosxcos(2x) + 2sinxsin(2x)}{cos^2(2x)}\right)$$

$$= 3sin^2x\left(\frac{cosxcos(2x) + 2sinxsin(2x)}{cos^4(2x)}\right)$$

28. $g(t) = cos^5(4t - 19)$

$g'(t) = 5cos^4(4t - 19)(-\sin(4t - 19)\,(4))$

$= -20\sin(4t - 19)\,cos^4(4t - 19)$

29. $g(t) = cos^4(sin\ t^2)$

$g(t) = cos^4(u)\ ,\qquad u(t) = sin\ t^2, u'(t) = cos\ t^2(2t)$

$g'(t) = 4cos^3(u)(-sinu)u'(t)$

$= -4\sin(sin\ t^2)(\ cos^3(sin\ t^2))(cos\ t^2(2t))$

$= -8tcos\ t^2\sin(sin\ t^2)cos^3(sin\ t^2)$

30. $h(x) = sin^3(cosx)$

$h'(x) = 3sin^2(cosx)\cos(cosx)\,(-sinx)$

$= -3sinx\cos(cosx)\,sin^2(cosx)$

In Exercises 31-40 find the equation of the tangent line to the given curve at the given point

في التمارين 31-40 أوجد معادلة الخط المماس للمنحني(للرسم) المعطى عند النقطة المعطاة

31. $y = (2x+1)^5$, $\qquad (0,1) = (a, f(a))$

$$\frac{dy}{dx} = 5(2x+1)^4(2) = 10(2x+1)^4$$

$$m = \frac{dy}{dx}|_0 = 10(2(0)+1)^4 = 10(1)^4 = 10$$

الرمز $\frac{dy}{dx}|_0$ يعني المشتقة عند x=0

the equation of the tangent line is

$$y = m(x-a) + f(a) = 10(x-0) + 1 = 10x + 1$$

32 $y = \dfrac{x}{(3-x^2)^5}$, $\qquad (2,-2)$

$$\frac{dy}{dx} = \frac{((3-x^2)^5)(1) - x(\,5(3-x^2)^4\,)(-2x)}{(3-x^2)^{10}}$$

$$= \frac{(3-x^2)^4[3-x^2+10x^2]}{(3-x^2)^{10}} = \frac{9x^2+3}{(3-x^2)^6}$$

$$m = \frac{dy}{dx}|_2 = \frac{9(2)^2 + 3}{(3 - (2^{\,2}))^6} = \frac{39}{(-1)^6} = 39$$

the equation of the tangent line is

$y = m(x - a) + f(a) = 39(x - 2) - 2 = 39x - 76 - 2$

$= 39x - 78$

33. $y = (x^3 - x^2 + x - 1)^{10}$, $(1,0)$

$$\frac{dy}{dx} = 10(x^3 - x^2 + x - 1)^9(3x^2 - 2x + 1)$$

$$m = \frac{dy}{dx}|_1 = 10\,(1 - 1 + 1 - 1)^9(3 - 2 + 1) = 10(0)(2)$$

$$= 0$$

the equation of the tangent line is

$y = m(x - a) + f(a) = 0(x - 1) + 0 = 0 + 0 = 0$

34. $y = (1 + 2x^2)^3$, $(1,27)$

$$\frac{dy}{dx} = 3(1 + 2x^2)^2(4x) = 12x(1 + 2x^2)^2$$

$$m = \frac{dy}{dx}|_1 = 12(1)(1 + 2\,1^2)^2 = 12(1 + 2)^2 = 12(9) = 108$$

the equation of the tangent line is

$$y = m(x-a) + f(a) = 108(x-1) + 27$$
$$= 108x - 108 + 27 = 108x - 81$$

35. $y = cot^2x \ , \left(\frac{\pi}{4}, 1\right)$

$$\frac{dy}{dx} = 2cotx\,(-csc^2x) = -2csc^2xcotx$$

$$m = \frac{dy}{dx}|_{\frac{\pi}{4}} = -2\csc^2\left(\frac{\pi}{4}\right) cot\left(\frac{\pi}{4}\right) = -2(\sqrt{2})^2(1) = -4$$

the equation of the tangent line is

$$y = m(x-a) + f(a) = -4\left(x - \frac{\pi}{4}\right) + 1 = -4x + \ \pi + 1$$

36. $y = \cos(3x) \ , \ \left(\frac{\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

$$\frac{dy}{dx} = -\sin(3x)\,(3) = -3\sin\,(3x)$$

$$m = \frac{dy}{dx}|_{\frac{\pi}{4}} = -3\sin\left(3\,\frac{\pi}{4}\right) = -\sin\left(\frac{3\pi}{4}\right) = -3\left(\frac{\sqrt{2}}{2}\right)$$

the equation of the tangent line is

$$y = m(x-a) + f(a) = -3\left(\frac{\sqrt{2}}{2}\right)\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2}$$

$$= -\frac{3\sqrt{2}}{2}x + \frac{3\sqrt{2}\,\pi}{8} - \frac{\sqrt{2}}{2}$$

37. $y = tan^2x$, $\left(\frac{\pi}{4}, 1\right)$

$$\frac{dy}{dx} = 2tanx(sec^2x)$$

$$m = \frac{dy}{dx}|_{\frac{\pi}{4}} = 2\tan\left(\frac{\pi}{4}\right)\left(sec^2\left(\frac{\pi}{4}\right)\right) = 2(1)(\sqrt{2})^2 = 4$$

the equation of the tangent line is

$$y = m(x-a) + f(a) = 4\left(x - \frac{\pi}{4}\right) + 1 = 4x - \pi + 1$$

38. $y = 2tan^3x$, $\left(\frac{\pi}{4}, 2\right)$

$$\frac{dy}{dx} = 6tan^2x\ (sec^2x)$$

$$m = \frac{dy}{dx}|_{\frac{\pi}{4}} = 6tan^2\left(\frac{\pi}{4}\right) sec^2\left(\frac{\pi}{4}\right) = 6(1)^2(\sqrt{2})^2 = 24$$

the equation of the tangent line is

$$y = m(x-a) + f(a) = 24\left(x - \frac{\pi}{4}\right) + 2 = 24x - 6\pi + 2$$

39. $y = (x^2+1)^{-2}$, $\left(1, \frac{1}{4}\right)$

$$\frac{dy}{dx} = -2(x^2+1)^{-3}(2x) = -4x(x^2+1)^{-3}$$

$$m = \frac{dy}{dx}|_1 = -4(1)(\,1^2+1)^{-3} = -4(2^{-3}) = \frac{-4}{2^3} = -\frac{1}{2}$$

the equation of the tangent line is

$$y = m(x-a) + f(a) = -\frac{1}{2}(x-1) + \frac{1}{4} = -\frac{1}{2}x + \frac{1}{2} + \frac{1}{4}$$

$$= -\frac{1}{2}x + \frac{3}{4}$$

40. $y = 1 + xsin(3x)$, $\left(\frac{\pi}{3}, 1\right)$

$$\frac{dy}{dx} = x(-\cos(3x)\,(3) + sin(3x)(1)$$

$$= -3xcos(3x) + \sin\,(3x)$$

$$m = \frac{dy}{dx}|_{\frac{\pi}{3}} = -3\cos\left(3\frac{\pi}{3}\right) + sin\left(3\frac{\pi}{3}\right) = -3cos\pi + sin\pi$$

$= -3(-1) + 0 = 3$

equation of the tangent line is

$$y = m(x-a) + f(a) = 3\left(x - \frac{\pi}{3}\right) + 1 = 3x - \pi + 1$$

In Exercises 41-44 find the x-coordinate of the point(s) at which the graph of the equation has horizontal tangent line

في التمارين 41-44 أوجد الاحداثي السيني للنقاط التي عندها يكون الخط المماس لمنحني المعادلة أفقياً

الخط المماس لمنحني المعادلة هو خط أفقي تعني أن ميله يساوي صفراً أي أن المشتقة تساوي صفراً

$41. y = x^2(x-1)^3$

$$\frac{dy}{dx} = x^2[\,3(x-1)^2(1) + (x-1)^3(2x) = 0$$

$$x(x-1)^2\big(3x + 2(x-1)\big) = 0$$

$$x(x-1)^2(5x-2) = 0$$

$$x = 0 \quad or\ (x-1)^2 = 0 \ \ or\ \ (5x-2)$$

$$x = 0\ \ or\ \ x = 1\ \ or\ \ x = \frac{2}{5}$$

the x-coordinate of the points at which the graph of the equation has horizontal tangent line are

$$x = 0\ , \qquad x = 1\ , \qquad x = \frac{2}{5}$$

$42.\ y = (2x+1)^2(x-3)^3$

$$\frac{dy}{dx} = (2x+1)^2(\ 3(x-3)^2\ )(1) + (x-3)^3(\ 2(2x+1)\ )(2)$$

$$= (2x+1)(x-3)^2[\ 3(2x+1) + (x-3)(4)]$$

$$= (2x+1)(x-3)^2[\ 6x + 3 + 4x - 12]$$

$$(2x+1)(x-3)^2(10x-9) = 0$$

$$(2x+1) = 0 \quad or\ (x-3)^2 = 0 \quad or\ (10x-9) = 0$$

$$x = -\frac{1}{2} \quad or\ \ x = 3 \quad or \quad x = \frac{9}{10}$$

the x-coordinate of the points at which the graph of the equation has horizontal tangent line are

$$x = -\frac{1}{2}\ , \qquad x = 3\ , \qquad x = \frac{9}{10}$$

$43.\ y = \dfrac{(x+3)^2}{x}$

$$\frac{dy}{dx} = \frac{x\big(\ 2(x+3)\big) - (x+3)^2(1)}{x^2} = \frac{(x+3)(2x-(x+3))}{x^2}$$

$$= (x+3)(2x-x-3) = \ (x+3)(x-3) = 0$$

$$x = -3 \ \ or\ \ x = 3$$

the x-coordinate of the points at which the graph of the equation has horizontal tangent line are

$x = -3$, $\quad x = 3$

$44.\ y = \left(\frac{1-8x}{1+8x}\right)^4$

$$\frac{dy}{dx} = 4\left(\frac{1-8x}{1+8x}\right)^3 \left[\ \frac{(1+8x)(-8)-(1-8x)(8)}{(1+8x)^2}\right]$$

$$= 4\left(\frac{1-8x}{1+8x}\right)^3 \left[\frac{-8-64x-8+64x}{(1+8x)^2}\right]$$

$$= 4\left(\frac{1-8x}{1+8x}\right)^3 \left[\frac{-16}{(1+8x)^2}\right] = \frac{-16(4)(1-8x)^{\,3}}{(1+8x)^5}$$

$$= -16(4)(1-8x)^{\,3} = 0\ ,(1-8x)^{\,3} = \ 0\ \ ,$$

$$1-8x = 0\,, x = \frac{1}{8}$$

the x-coordinate of the point at which the graph of the equation has horizontal tangent line is

$$x = \frac{1}{8}$$

$45.\ Determine\ the\ point(s) in\ the\ interval\ (0{,}2\pi) at\ which$

$the\ graph\ of\ f(x) = 2cosx + \sin(2x)\ \ has\ a\ horizontal$

$tangent$

حدد النقاط التي في الفترة $(0,2\pi)$ والتي عندها منحني الدالة

$f(x) = 2cosx + \sin(2x)$

له مماس أفقي

$f'(x) = 2(-sinx) + \cos(2x)\,(2) = -2(sinx - \cos(2x))$

$-2(sinx - \cos(2x)) = 0 \;, \qquad sinx - \cos(2x) = 0$

$sinx - (cos^2x - sin^2x) = 0$

$sinx - (1 - sin^2x - sin^2x)$

$sinx - 1 + 2sin^2x = 0\,, \qquad sin^2x + \frac{1}{2}sinx - \frac{1}{2} = 0$

$(sinx + 1)\left(sinx - \frac{1}{2}\right) = 0 \;, sinx + 1 = 0 \;\; or \; sinx - \frac{1}{2} = 0$

$sinx = -1 \;\; or \;\; sinx = \frac{1}{2}$

$x = \frac{3\pi}{2} \quad or \;\; x = \frac{\pi}{6} \quad or \; x = \left(\pi - \frac{\pi}{6}\right) = \frac{5\pi}{6}$

$f\left(\frac{3\pi}{2}\right) = 2cos\left(\frac{3\pi}{2}\right) + \sin\left(2\left(\frac{3\pi}{2}\right)\right)$

$= 2(0) + \sin(3\pi) = 0 + sin\pi = 0 + 0 = 0$

$\left(\frac{3\pi}{2}, 0\right)$

$$f\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right) + \sin\left(2\frac{\pi}{6}\right) = 2\frac{\sqrt{3}}{2} + \sin\left(\frac{\pi}{3}\right)$$

$$= \sqrt{3} + \frac{\sqrt{3}}{2} = \frac{2\sqrt{3} + \sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

$$\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right)$$

$$f\left(\frac{5\pi}{6}\right) = 2\cos\left(\frac{5\pi}{6}\right) + \sin\left(2\frac{5\pi}{6}\right) = 2\frac{-\sqrt{3}}{2} + \sin\left(\frac{5\pi}{3}\right)$$

$$= -\sqrt{3} + \frac{-\sqrt{3}}{2} = \frac{-2\sqrt{3} - \sqrt{3}}{2} = -\frac{3\sqrt{3}}{2}$$

$$\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)$$

46. *Find all point(s) on the graph of* $y = sin^2x$ *where the* Tangent line has slope 1.

أوجد كل النقاط التي على منحني $y = sin^2x$ والتي عندها ميل الخط المماس يساوي 1

$$\frac{dy}{dx} = 2sinxcosx \ \ , 2sinxcosx = 1\ , sinxcosx = \ \frac{1}{2}$$

$$\sin\left(\frac{\pi}{4}\right) cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$x = \frac{\pi}{4} + 2n\pi = \frac{\pi + 8n\pi}{4} = \frac{9n\pi}{4}$$

$$\sin\left(\frac{5\pi}{4}\right) cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}\ (-\ \frac{1}{\sqrt{2}}) = \frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$x = \frac{5\pi}{4} + 2n\pi = \frac{5\pi + 8n\pi}{4} = \frac{13n\pi}{4}$$

$$\frac{dy}{dx}|_{\frac{\pi}{4}} = sin^2\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\left(\frac{9n\pi}{4}, \frac{1}{2}\right) \ , \qquad n\ is\ integer$$

$$\frac{dy}{dx}|_{\frac{5\pi}{4}} = sin^2\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}(-\ \frac{1}{\sqrt{2}}) = \frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\left(\frac{13n\pi}{4}, \frac{1}{2}\right) \ , \qquad n\ is\ integer$$

$47. In\ Exercises\ 47 - 48\ use\ the\ position\ function\ to\ find$

$the\ velocity\ at\ t = 2$

$(Assume\ units\ of\ meters\ and\ seconds)$

في التمارين 47-48 استخدم دالة المكان لتجد السرعة عند t=2

(افرض وحدات الامتار والثواني)

$$47. s(t) = \sqrt{t^2+8} = (t^2+8)^{\frac{1}{2}}$$

$$v(t) = s'(t) = \frac{1}{2}(t^2+8)^{-\frac{1}{2}}(2t) = \frac{t}{(t^2+8)^{\frac{1}{2}}} = \frac{t}{\sqrt{t^2+8}}$$

$$v(2) = \frac{2}{\sqrt{4+8}} = \frac{2}{\sqrt{12}} = \frac{2}{\sqrt{3(4)}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}\ \ m/s$$

$$48. s(t) = \frac{60t}{\sqrt{t^2+1}} = 60t(t^2+1)^{\frac{1}{2}}$$

$$v(t) = s'(t) = 60t\left(\frac{1}{2}(t^2+1)^{-\frac{1}{2}}(2t)\right) + (t^2+1)^{\frac{1}{2}}(60)$$

$$= 60t^2\left((t^2+1)^{-\frac{1}{2}}\right) + 60(t^2+1)^{\frac{1}{2}}$$

$$v(2) = 60(4)\,5^{-\frac{1}{2}} + 60\,5^{\frac{1}{2}} = 60\left(\frac{4}{\sqrt{5}}\right) + 60\sqrt{5}$$

$$=60\left(\frac{4}{\sqrt{5}} + \sqrt{5}\right) = 60\left(\frac{4(5)}{\sqrt{5}}\right) = 240\sqrt{5}$$

Section 2.5

تمارين (2.5) **Exercises** صفحة 205 و 206 في الكتاب

$In\ Exercises\ \ 1-21\,.\,Find\ \ \frac{dy}{dx}$

$using\ implicit\ differentiation$

في التمارين 1-21 أوجد $\frac{dy}{dx}$ مستخدماً الاشتقاق الضمني

في الاشتقاق الضمني$(implicit\ differentiation)$ نوجد مشتقة x
ومشتقة y كالعادة فقط نضرب مشتقة y بالحد $\frac{dy}{dx}$

$1.\,x^2 + y = 2xy$

$$2x + (1)\frac{dy}{dx} = 2\left(x\left((1)\frac{dy}{dx}\right) + y(1)\right)$$

$$2x + \frac{dy}{dx} = 2x\frac{dy}{dx} + 2y$$

$$\frac{dy}{dx} - 2x\frac{dy}{dx} = 2y - 2x$$

$$\frac{dy}{dx}(1-2x) = 2(y-x) \ , \qquad \frac{dy}{dx} = \frac{2(y-x)}{1-2x} = \frac{-2(x-y)}{-(2x-1)}$$

$$= \frac{2(x-y)}{2x-1}$$

2. $x^2 = y^2 + 1$

$$2x = 2y\frac{dy}{dx} \ , \quad \frac{dy}{dx} = \frac{2x}{2y} = \frac{x}{y}$$

3. $x^2y + xy^2 = 3x$

$$x^2\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^2(1) = 3$$

$$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = 3 - 2xy - y^2$$

$$\frac{dy}{dx}(x^2 + 2xy) = 3 - 2xy - y^2$$

$$\frac{dy}{dx} = \frac{3 - 2xy - y^2}{x^2 + 2xy} = \frac{3 - 2xy - y^2}{x(x + 2y)}$$

4. $2x^2 - y^2 = 1$

$$4x - 2y\frac{dy}{dx} = 0 \ , \qquad \frac{dy}{dx} = \frac{-4x}{-2y} = \frac{2x}{y}$$

5. $(2x + y)^2 = y$

$$2(2x + y)\left(2 + \frac{dy}{dx}\right) = \frac{dy}{dx}$$

$$4(2x + y) + 2(2x + y)\frac{dy}{dx} = \frac{dy}{dx}$$

$$2(2x + y)\frac{dy}{dx} - \frac{dy}{dx} = -4(2x + y)$$

$$\frac{dy}{dx}(2(2x + y) - 1) = -4(2x + y)$$

$$\frac{dy}{dx} = \frac{-4(2x + y)}{2(2x + y) - 1} = -\frac{4(2x + y)}{4x + 2y - 1}$$

6. $x^2 - y^2x = 1$

$$2x - \left(y^2(1) + 2xy\frac{dy}{dx}\right) = 0$$

$$2x - y^2 - 2xy\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x + y^2}{-2xy} = \frac{-(2x - y^2)}{-2xy} = \frac{2x - y^2}{2xy}$$

7. $x^3 - y^3 = 6xy$

$$3x^2 - 3y^2\frac{dy}{dx} = 6(y + x\frac{dy}{dx})$$

$$3x^2 - 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$$

$$3x^2 - 6y = (3y^2 + 6x)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - 6y}{3y^2 + 6x} = \frac{x^2 - 2y}{y^2 + 2x}$$

8. $x^2y + 3xy^3 - x = 3$

$$2xy + x^2\frac{dy}{dx} + 3(3xy^2 + y^3) - 1 = 0$$

$$x^2\frac{dy}{dx} = -3(3xy^2 + y^3) + 1 - 2xy$$

$$x^2\frac{dy}{dx} = -9xy^2 - 3y^3 - 2xy + 1$$

$$\frac{dy}{dx} = \frac{-9xy^2 - 3y^3 - 2xy + 1}{x^2}$$

9. $\sqrt{1 + xy} = y$

$$1 + xy = y^2$$

$$y + x\frac{dy}{dx} = 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = -y$$

$$\frac{dy}{dx}(x - 2y) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x - 2y} = \frac{-y}{-(-x + 2y)} = \frac{y}{-x + 2\sqrt{1 + xy}}$$

10. $tany = 3x^2$

$$sec^2y\frac{dy}{dx} = 6x$$

$$\frac{dy}{dx} = \frac{6x}{sec^2y}$$

11. $\cos(xy) = x$

$$-\sin(xy)\left(x\frac{dy}{dx} + y\right) = 1$$

$$-xsin(xy)\frac{dy}{dx} - ysin(xy) = 1$$

$$\frac{dy}{dx} = \frac{1 + ysin(xy)}{-xsin(xy)} = -\frac{1}{xsin(xy)} + \frac{ysin(xy)}{xsin(xy)}$$

$$= -\frac{\csc(xy)}{x} + \frac{y}{x} = -\frac{y + \csc(xy)}{x}$$

12. $2siny = (x + 1)^2$

$$2cosy\frac{dy}{dx} = 2(x+1)$$

$$\frac{dy}{dx} = \frac{2(x+1)}{2cosy} = \frac{x+1}{cosy} = \frac{x}{cosy} + \frac{1}{cosy} = xsecy + secy$$

$$= secy(x+1)$$

13. $xsiny + ysinx = 1$

$$xcosy\frac{dy}{dx} + siny\ + ycosx + sinx\frac{dy}{dx} = 0$$

$$xcosy\frac{dy}{dx} + sinx\frac{dy}{dx} = -siny - ycosx$$

$$\frac{dy}{dx}(xcosy + sinx) = -(siny + ycosx)$$

$$\frac{dy}{dx} = -\frac{siny + ycosx}{xcosy + sinx}$$

14. $(x+1)^2 + (y-2)^2 = 9$

$$2(x+1) + 2(y-2)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2(x+1)}{2(y-2)} = -\frac{x+1}{y-2}$$

15. $\sqrt{x} + \sqrt{y} = 1$, $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 1$

$$\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}}=-\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}=-\frac{\sqrt{y}}{\sqrt{x}}=-\sqrt{\frac{y}{x}}$$

16. $x=\sec(2y)$, $x=\dfrac{1}{\cos(2y)}$, $x\cos(2y)=1$

$$x\left(-\sin(2y)\,(2)\frac{dy}{dx}\right)+\cos(2y)=0$$

$$-2x\sin(2y)\frac{dy}{dx}=-\cos(2y)$$

$$\frac{dy}{dx}=\frac{\cos(2y)}{2x\sin(2y)}=\frac{\cot(2y)}{2x}$$

17. $\sqrt{1+\cos^2y}=xy$, $(1+\cos^2y)^{\frac{1}{2}}=xy$

$$\frac{1}{2}(1+\cos^2y)^{-\frac{1}{2}}\left(2\cos y(-\sin y)\frac{dy}{dx}\right)=x\frac{dy}{dx}+y$$

$$-\sin y\cos y\,(1+\cos^2y)^{-\frac{1}{2}}\frac{dy}{dx}-x\frac{dy}{dx}=y$$

$$\frac{dy}{dx}\left(-\sin y\cos y\,(1+\cos^2y)^{-\frac{1}{2}})-x\right)=y$$

$$\frac{dy}{dx} = -\frac{y}{\left(sinycosy\,(1+cos^2y)^{-\frac{1}{2}} + x\right)}$$

$$= -\frac{y}{\frac{sinycosy}{(1+cos^2y)^{\frac{1}{2}}} + x} = -\frac{y}{\frac{sinycosy + x(1+cos^2y)^{\frac{1}{2}}}{(1+cos^2y)^{\frac{1}{2}}}}$$

$$= -\frac{y(1+cos^2y)^{\frac{1}{2}}}{sinycosy + x(1+cos^2y)^{\frac{1}{2}}}$$

$$= -\frac{y\sqrt{1+cos^2y}}{sinycosy + x\sqrt{1+cos^2y}}$$

18. $x + y^2 = \cot(xy)$

$$1 + 2y\frac{dy}{dx} = -csc^2(xy)(\,y + x\frac{dy}{dx})$$

$$1 + 2y\frac{dy}{dx} = -ycsc^2(xy) - xcsc^2(xy)\frac{dy}{dx}$$

$$2y\frac{dy}{dx} + xcsc^2(xy)\frac{dy}{dx} = -ycsc^2(xy) - 1$$

$$\frac{dy}{dx}(2y + xcsc^2(xy)\,) = -ycsc^2(xy) - 1$$

$$\frac{dy}{dx} = -\frac{ycsc^2(xy) + 1}{2y + xcsc^2(xy)}$$

$$19. \frac{xy}{x^2+y^2} = x+1$$

$$\frac{(x^2+y^2)\left(x\frac{dy}{dx}+y\right) - xy(2x+2y\frac{dy}{dx})}{(x^2+y^2)^2} = 1$$

$$\frac{x(x^2+y^2)\frac{dy}{dx}+y(x^2+y^2)-2yx^2-2xy^2\frac{dy}{dx} = 1}{(x^2+y^2)^2}$$

$$\frac{dy}{dx}(\,x(x^2+y^2)-2xy^2)$$
$$= 2yx^2 - y(x^2+y^2) + (x^2+y^2)^2$$

$$\frac{dy}{dx} = \frac{2yx^2 - y(x^2+y^2) + (x^2+y^2)^2}{x(x^2+y^2)-2xy^2}$$

$$= \frac{2yx^2 - yx^2 - y^3 + (x^2+y^2)^2}{x(x^2+y^2)-2xy^2}$$

$$= \frac{yx^2 - y^3 + (x^2+y^2)^2}{x(x^2+y^2-2y^2)}$$

$$= \frac{y(x^2-y^2)+(x^2+y^2)^2}{x(x^2-y^2)}$$

$$= \frac{y}{x} + \frac{(x^2+y^2)^2}{x(x^2-y^2)}$$

20. $(2x^2+3y^2)^{\frac{5}{2}}=x$

$$\frac{5}{2}(2x^2+3y^2)^{\frac{3}{2}}\left(4x+6y\frac{dy}{dx}\right)=1$$

$$10x(2x^2+3y^2)^{\frac{3}{2}}+15(2x^2+3y^2)^{\frac{3}{2}}\frac{dy}{dx}=1$$

$$15(2x^2+3y^2)^{\frac{3}{2}}\frac{dy}{dx}=1-10x(2x^2+3y^2)^{\frac{3}{2}}$$

$$\frac{dy}{dx}=\frac{1-10x(2x^2+3y^2)^{\frac{3}{2}}}{15(2x^2+3y^2)^{\frac{3}{2}}}=\frac{1}{15(2x^2+3y^2)^{\frac{3}{2}}}+\frac{2}{3}$$

$$\frac{5}{2}-1=\frac{5-2}{2}=\frac{3}{2}$$

21. $\frac{1}{x}+\frac{1}{y}=1$

$$\frac{x(0)-(1)(1)}{x^2}+\frac{y(0)-(1)\frac{dy}{dx}}{y^2}=0$$

$$\frac{-1}{x^2}-\frac{\frac{dy}{dx}}{y^2}=0\ ,-\frac{\frac{dy}{dx}}{y^2}=\frac{1}{x^2}$$

$$\frac{dy}{dx}=-\frac{y^2}{x^2}$$

In Exercises 22-30 , find an equation of the tangent line at the given point.

في التمارين 22-30 أوجد معادلة الخط المماس عند النقطة المعطاة

$22.\ x^3y^2 = -3xy \quad ,(-1,-3)$

$$x^3 2y\frac{dy}{dx} + y^2 3x^2 = -3(x\frac{dy}{dx} + y)$$

$$2yx^3\frac{dy}{dx} + 3y^2x^2 = -3x\frac{dy}{dx} - 3y$$

$$yx^3\frac{dy}{dx} + 3x\frac{dy}{dx} = -3y - 3y^2x^2$$

$$\frac{dy}{dx}(yx^3 + 3x) == -3y - 3y^2x^2$$

$$\frac{dy}{dx} = -\frac{3y + 3y^2x^2}{yx^3 + 3x}$$

$$m = \frac{dy}{dx}|_{(-1,-3)} = -\frac{3(-3) + 3(3^2)(-1^{2)}}{3(-1)^3 + 3(-1)}$$

$$m = -\frac{-9 + 27(1)}{-3 - 3} = -\frac{18}{-6} = 3$$

ميل الخط المماس عند النقطة (-1,-3) هو 3

معادلة الخط المماس عند النقطة (-1,-3) والذي ميله هو العدد 3 هي

$$y = m(x-a) + f(a) = 3\big(x - (-1)\big) + (-3)$$

$$= 3x + 3 - 3 = 3x$$

$23.\ x^2 - 4y^3 = 0 \ , \qquad (2,1)$

$$2x-12y^2\frac{dy}{dx}=0\ ,\ -12y^2\frac{dy}{dx}=-2x\ ,\frac{dy}{dx}=\frac{x}{6y^2}$$

$$m=\frac{dy}{dx}|_{(2,1)}=\frac{2}{6(1^2)}=\frac{1}{3}$$

ميل الخط المماس عند النقطة (2,1) هو $\frac{1}{3}$

معادلة الخط المماس عند النقطة (2,1) والذي ميله هو العدد $\frac{1}{3}$ هي

$$y=m(x-a)+f(a)=\frac{1}{3}(x-2)+1=\frac{1}{3}x-\frac{2}{3}+1$$

$$=\frac{1}{3}x+\frac{1}{3}$$

24. $x^4=8(x^2-y^2)\ ,\qquad (2,-\sqrt{2})$

$$4x^3=8(2x-2y\frac{dy}{dx})$$

$$4x^3=16x-16y\frac{dy}{dx}$$

$$16y\frac{dy}{dx}=16x-4x^3$$

$$\frac{dy}{dx}=\frac{16x-4x^3}{16y}=\frac{x}{y}-\frac{x^3}{4y}$$

$$m = \frac{dy}{dx}|_{(2,-\sqrt{2})} = \frac{2}{-\sqrt{2}} - \frac{(2)^3}{4(-\sqrt{2})} = \frac{2}{-\sqrt{2}} - \frac{2}{-\sqrt{2}}$$
$$= -\frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 0$$

ميل الخط المماس عند النقطة $(2,-\sqrt{2})$ هو 0

معادلة الخط المماس عند النقطة $(2,-\sqrt{2})$ والذي ميله هو العدد 0 هي

$$y = m(x-a) + f(a) = 0(x-2) - \sqrt{2} = -\sqrt{2}$$

25. $x^4 = 4(x^2 - y^2)\ ,(1,\frac{\sqrt{3}}{2})$

$$4x^3 = 4(2x - 2y\frac{dy}{dx})$$

$$4x^3 = 8x - 8y\frac{dy}{dx}$$

$$8y\frac{dy}{dx} = 8x - 4x^3$$

$$\frac{dy}{dx} = \frac{8x - 4x^3}{8y} = \frac{x}{y} - \frac{x^3}{2y}$$

$$m = \frac{dy}{dx}|_{(1,\frac{\sqrt{3}}{2})} = \frac{1}{\frac{\sqrt{3}}{2}} - \frac{1}{2\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

ميل الخط المماس عند النقطة $(1,\frac{\sqrt{3}}{2})$ هو $\frac{1}{\sqrt{3}}$

معادلة الخط المماس عند النقطة $(1,\frac{\sqrt{3}}{2})$ والذي ميله هو العدد $\frac{1}{\sqrt{3}}$ هي

$$y = m(x-a) + f(a) = \frac{1}{\sqrt{3}}(x-1) + \frac{\sqrt{3}}{2}$$

$$= \frac{x}{\sqrt{3}} - \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2} = \frac{x}{\sqrt{3}} + \frac{-2+3}{2\sqrt{3}} = \frac{x}{\sqrt{3}} + \frac{1}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}\left(x + \frac{1}{2}\right) = \frac{\sqrt{3}}{3}\left(x + \frac{1}{2}\right)$$

26. $\frac{x^2}{16} - \frac{y^2}{9} = 1 \quad , \quad \left(-5, \frac{9}{4}\right)$

$$\frac{2x}{16} - \frac{2y}{9}\frac{dy}{dx} = 0 \quad , \frac{2y}{9}\frac{dy}{dx} = \frac{x}{8}$$

$$\frac{dy}{dx} = \frac{x}{8}\,\frac{9}{2y} = \frac{9x}{16y}$$

$$m = \frac{dy}{dx}\Big|_{(-5,\frac{9}{4})} = \frac{9(-5)}{16(\frac{9}{4})} = \frac{9(-5)}{4(9)} = -\frac{5}{4}$$

ميل الخط المماس عند النقطة $(-5,\frac{9}{4})$ هو $\frac{-5}{4}$

معادلة الخط المماس عند النقطة $(-5,\frac{9}{4})$ والذي ميله هو العدد $\frac{-5}{4}$ هي

$$y = m(x-a) + f(a) = \frac{-5}{4}(x+5) + \frac{9}{4}$$

$$= -\frac{5x}{4} - \frac{25}{4} + \frac{9}{4} = -\frac{5x}{4} - \frac{16}{4} = -\frac{5x}{4} - 4$$

27. $\frac{x^2}{2} + \frac{y^2}{8} = 1 \quad , (1,2)$

$$x + \frac{y}{4}\frac{dy}{dx} = 0 \ , \qquad \frac{dy}{dx} = -x\frac{4}{y} = -\frac{4x}{y}$$

$$m = \frac{dy}{dx}|_{(1,2)} = -\frac{4}{2} = -2$$

ميل الخط المماس عند النقطة $(1,2)$ هو -2

معادلة الخط المماس عند النقطة $(1,2)$ والذي ميله هو العدد -2 هي

$$y = m(x-a) + f(a) = -2(x-1) + 2 = -2x + 2 + 2$$
$$= -2x + 48.$$

28. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4 \ , \qquad \left(-3\sqrt{3}, 1\right) = (-\sqrt{27}, 1)$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0 \ , \frac{dy}{dx} = -\frac{\frac{2}{3}x^{-\frac{1}{3}}}{\frac{2}{3}y^{-\frac{1}{3}}} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

$$m = \frac{dy}{dx}|_{(-3\sqrt{3},1)} = -\frac{1}{\sqrt[3]{-27^{\frac{1}{2}}}} = -\frac{1}{(27)^{\frac{1}{6}}}$$

ميل الخط المماس عند النقطة $(-\sqrt{27}, 1)$ هو $-\frac{1}{(27)^{\frac{1}{6}}}$

معادلة الخط المماس عند النقطة $(-\sqrt{27}, 1)$ والذي ميله هو $-\frac{1}{(27)^{\frac{1}{6}}}$ هي

$$y = m(x-a) + f(a) = -\frac{1}{(\,27)^{\frac{1}{6}}}(x + \sqrt{27}) + 1$$

$$= -\frac{x}{\sqrt[6]{27}} - 27^{\frac{1}{2}-\frac{1}{6}} + 1 = -\frac{x}{\sqrt[6]{27}} - 27^{\frac{1}{3}} + 1$$

$$= -\frac{x}{\sqrt[6]{27}} - 3 + 1 = -\frac{x}{\sqrt[6]{27}} - 2$$

29. $y^2 = x^3(2-x)\,, \qquad (1{,}1)$

$y^2 = 2x^3 - x^4$

$$2y\frac{dy}{dx} = 6x^2 - 4x^3\,, \frac{dy}{dx} = \frac{6x^2 - 4x^3}{2y} = \frac{3x^2 - 2x^3}{y}$$

$$m = \frac{dy}{dx}|_{(1,1)} = \frac{3-2}{1} = 1$$

ميل الخط المماس عند النقطة $(1,1)$ هو 1

معادلة الخط المماس عند النقطة $(1,1)$ والذي ميله هو العدد 1 هي

$$y = m(x-a) + f(a) = 1(x-1) + 1 = x - 1 + 1 = x$$

30. $2(x^2 + y^2)^2 = 25(x^2 - y^2)\,, \qquad (3{,}1)$

$$4(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right) = 50x - 50y\frac{dy}{dx}$$

$$8x^3 + 8x^2y\frac{dy}{dx} + 8y^2x + 8y^3\frac{dy}{dx} = 50x - 50y\frac{dy}{dx}$$

$$\frac{dy}{dx}(8x^2y + 8y^3 + 50y) = 50x - 8x^3 - 8y^2x$$

$$\frac{dy}{dx} = \frac{50x - 8x^3 - 8y^2x}{8x^2y + 8y^3 + 50y}$$

$$= \frac{dy}{dx}|_{(3,1)} = \frac{50(3) - 8(27) - 8(1)(3)}{8(9)(1) + 8 + 50}$$

$$= \frac{150 - 216 - 24}{72 + 58} = \frac{-80}{130} = -\frac{8}{13}$$

ميل الخط المماس عند النقطة $(3,1)$ هو $-\frac{8}{13}$

معادلة الخط المماس عند النقطة $(3,1)$ والذي ميله هو $-\frac{8}{13}$ هي

$$y = m(x-a) + f(a) = -\frac{8}{13}(x-3) + 1 = -\frac{8x}{13} + \frac{37}{13}$$

In exercises 31-32 find the points at which the graph of the equation has a horizontal tangent line

في التمارين 31-32 أوجد النقاط التي عندها منحنى المعادلة له خط تماس أفقي

خط تماس أفقي تعني أن ميل المماس =0 أي أن المشتقة =0

31 $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

$$50x + 32y\frac{dy}{dx} + 200 - 160\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(32y - 160) = -200 - 50x$$

$$\frac{dy}{dx} = \frac{-200 - 50x}{32y - 160} = 0$$

$$-200 - 50x \ , \qquad x = -\frac{200}{50} = -4$$

الآن نعوض في المعادلة عن x بالعدد 4- لنجد قيمة y المناظرة

$$25(-4)^2 + 16y^2 + 200(-4) - 160y + 400 = 0$$

$$400 + 16y^2 - 800 - 160y + 400 = 0$$

$$16y^2 - 160y = 0 \ , \qquad y(16y - 160) = 0$$

$$y = 0 \ \ or \ \ 16y - 160 = 0 \ \ , y = \frac{160}{16} = 10$$

إذاً النقاط هي

$$(-4{,}0) \ \ , \qquad (-4{,}10)$$

32. $4x^2 + y^2 - 8x + 4y + 4 = 0$

$$8x + 2y\frac{dy}{dx} - 8 + 4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y + 4) = 8 - 8x$$

$$\frac{dy}{dx} = \frac{8 - 8x}{8 - 8x} = 0 \ , \qquad 8 - 8x = 0 \ , \qquad x = 1$$

الآن نعوض في المعادلة عن x بالعدد 1 لنجد قيمة y المناظرة

$$4 + y^2 - 8 + 4y + 4 = 0 \ , \qquad y^2 + 4y = 0$$

$$y(y + 4) = 0 \ \ , \ \ y = 0 \ \ or \ \ y = -4$$

إذاً النقاط هي

$(1,0)\ , \qquad (1,-4)$

$33.\ Find\ all\ point(s) on\ the\ curve$

$x^2y^2+xy=2$

$where\ the\ slope\ of\ the\ tangent\ line\ is -1$

أوجد جميع النقاط على المنحني

$x^2y^2+xy=2$

حيث ميل الخط المماس هو 1-

$$x^2\left(2y\frac{dy}{dx}\right)+2xy^2+x\frac{dy}{dx}+y=0$$

$$\frac{dy}{dx}(2yx^2+x)=-2xy^2-y$$

$$\frac{dy}{dx}=\frac{-2xy^2-y}{2yx^2+x}=-1$$

$$-2xy^2-y=-2yx^2-x$$

$$2yx^2-2xy^2+x-y=0$$

$$2yx(x-y)+x-y=0$$

$$(x-y)(2yx+1)=0$$

$$x-y=0\ \ or\ \ 2yx+1=0$$

$$x=y\quad or\quad x=-\frac{1}{2y}$$

$when\ \ x = y \qquad x^2x^2 + xx = 2\ \ , \qquad x^4 + x^2 - 2 = 0$

$(x^2 + 2)(x^2 - 1) = 0$

$x^2 + 2 = 0\ \ or\ \ x^2 - 1$

$x^2 + 2 \geq 2\ \ so\ \ x^2 = 1\ \ , \qquad x = \pm 1$

$when\ \ x = 1\ \ \ , x = y\ \ so\ \ y = 1\ so\ the\ point\ is\ (1,1)$

$when\ \ x = -1\ \ \ , x = y\ \ so\ \ y = -1\ so\ the\ point\ is\ (-1,-1)$

$when\ \ \ x = -\frac{1}{2y}\ \ , \qquad (-\frac{1}{2y})^2 y^2 + \left(-\frac{1}{2y}\right) y = 2$

$\frac{1}{4y^2}\ y^2 - \frac{1}{2} = 2\ \ , \qquad \frac{1}{4} - \frac{1}{2} \neq 2\ \ \ no\ \ solution$

$so\ \ \ points\ \ are\ \ \ (1,1)\ \ , \qquad (-1,-1)$

34. $Show\ that\ the\ equation\ of\ the\ tangent\ line\ tothe\ parabola$

$y^2 = 4px$

$at\ the\ point\ (a,b)\ is\ \ \ by = 2p(a + x)$

وضح أن معادلة الخط المماس للقطع

$y^2 = 4px$

عند النقطة (a,b) هي $by = 2p(a + x)$

$$2y\frac{dy}{dx} = 4p \quad , \quad m = \frac{dy}{dx} = \frac{4p}{2y} = \frac{2p}{y}$$

$$y = m(x-a) + b = \frac{2p}{y}(x-a) + b$$

$$y^2 = 2p(x-a) + by$$

$$4px = 2p(x-a) + by \quad , because\ y^2 = 4px$$

$$by = 4px - 2p(x-a) = 4px - 2px + 2pa$$

$$by = 2px + 2pa = 2p(x+a)$$

Section 2.6

تمارين (2.6) EXERCISES صفحة *213* و *214* في الكتاب

In exercises 1-16 find the second derivative of the given function.

في التمارين 1-16 أوجد المشتقة الثانية للدالة المعطاة

ملاحظة : المشتقة الثانية هي مشتقة المشتقة ونرمز لها بالرمز

$$f''(x) \quad or \quad y'' \quad or \quad \frac{d^2y}{dx^2}$$

1. $f(x) = x^7 - 2x^4 + x^3 + 6x + 2$

$$f'(x) = 7x^6 - 8x^3 + 3x^2 + 6$$

$$f''(x) = 42x^5 - 24x^2 + 6x$$

2. $g(x) = x^8 - 2x^5 - 3x^2 + 1$

$$g'(x) = 8x^7 - 10x^4 - 6x$$

$$g''(x) = 56x^6 - 40x^3 - 6$$

3. $h(t) = \sqrt{t^2 + 1} = (t^2 + 1)^{\frac{1}{2}}$

$$h'(t) = \frac{1}{2}(t^2 + 1)^{-\frac{1}{2}}(2t) = t(t^2 + 1)^{-\frac{1}{2}} = \frac{t}{(t^2 + 1)^{\frac{1}{2}}}$$

$$h''(t) = \frac{(t^2 + 1)^{\frac{1}{2}} - t(\frac{1}{2}(t^2 + 1)^{-\frac{1}{2}}(2t))}{t^2 + 1}$$

$$= \frac{(t^2 + 1)^{\frac{1}{2}} - t^2(t^2 + 1)^{-\frac{1}{2}}}{t^2 + 1}$$

$$= \frac{(t^2 + 1)^{-\frac{1}{2}}((t^2 + 1) - t^2}{t^2 + 1} = \frac{(t^2 + 1)^{-\frac{1}{2}}(1)}{t^2 + 1}$$

$$= \frac{1}{(t^2 + 1)(t^2 + 1)^{\frac{1}{2}}} = \frac{1}{(t^2 + 1)^{\frac{3}{2}}}$$

لاحظ أن

$$(t^2 + 1)^{\frac{1}{2}} = (t^2 + 1)^{\frac{1}{2}}(t^2 + 1)^{\frac{1}{2}}(t^2 + 1)^{-\frac{1}{2}}$$

$$= (t^2+1)^1(t^2+1)^{-\frac{1}{2}}$$

4. $f(x) = \sqrt[3]{x^2+2x} = (x^2+2x)^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}(x^2+2x)^{\frac{-2}{3}}(2x+2) = \frac{2x+2}{3(x^2+2x)^{\frac{2}{3}}}$$

$f''(x)$

$$= \frac{3(x^2+2x)^{\frac{2}{3}}(2) - (2x+2)(\,3\,\frac{2}{3}(x^2+2x)^{\frac{-1}{3}}(2x+2))}{9(x^2+2x)^{\frac{4}{3}}}$$

$$= \frac{(x^2+2x)^{\frac{-1}{3}}(\;6(x^2+2x) - 2\,(2x+2)(2x+2)}{9(x^2+2x)^{\frac{4}{3}}}$$

$$= \frac{(x^2+2x)^{\frac{-1}{3}}(6x^2+12x-8x^2-16x-8)}{9(x^2+2x)^{\frac{4}{3}}}$$

$$= \frac{(x^2+2x)^{\frac{-1}{3}}(-2x^2-4x-8)}{9(x^2+2x)^{\frac{4}{3}}}$$

$$= \frac{(x^2+2x)^{\frac{-1}{3}}(\,-2(x^2-2x-4)\,)}{9(x^2+2x)^{\frac{4}{3}}}$$

$$= -\frac{2(x^2-2x-4)}{9(x^2+2x)^{\frac{5}{3}}}$$

5. $g(x) = -\frac{1}{2}x^3 + \sqrt{2}x^2 - 18$

$g'(x) = -\frac{3}{2}x^2 + 2\sqrt{2}x$

$g''(x) = -\frac{6}{2}x + 2\sqrt{2} = -3x + 2\sqrt{2}$

6. $h(x) = \sqrt{x^3} - \sqrt[3]{x} - 2x = (x^3)^{\frac{1}{2}} - x^{\frac{1}{3}} - 2x$

$h'(x) = \frac{1}{2}(x^3)^{-\frac{1}{2}}(3x^2) - \frac{1}{3}x^{-\frac{2}{3}} - 2$

$= \frac{3}{2}\ \frac{x^2}{(x^3)^{\frac{1}{2}}} - \frac{1}{3}\ \frac{1}{x^{\frac{2}{3}}} - 2 = \frac{3}{2}\frac{x}{x^{\frac{1}{2}}} - \frac{1}{3}\ \frac{1}{x^{\frac{2}{3}}} - 2$

$h''(x) = \frac{3}{2}\left[\frac{x^{\frac{1}{2}} - x(\frac{1}{2}x^{-\frac{1}{2}})}{x}\right] - \frac{1}{3}\left[\frac{-\frac{2}{3}x^{-\frac{1}{3}}}{x^{\frac{4}{3}}}\right]$

$= \frac{3}{2}\left[\frac{x^{-\frac{1}{2}}(x - \frac{1}{2}x)}{x}\right] + \frac{\frac{2}{3}}{3}\left[\frac{1}{x^{\frac{5}{3}}}\right]$

$= \frac{3}{2}\left[\frac{x^{-\frac{1}{2}}(\frac{1}{2}x)}{x}\right] + \frac{2}{9}\left[\frac{1}{x^{\frac{5}{3}}}\right] = \frac{3}{4x^{\frac{1}{2}}} + \frac{2}{9x^{\frac{5}{3}}}$

$$7.\ g(x) = \frac{x}{x-1}$$

$$g'(x) = \frac{(x-1)(1) - x(1)}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$$g''(x) = \frac{(x-1)^2(0) - (\,(-1)(2(x-1))}{(x-1)^4}$$

$$= \frac{2x-2}{(x-1)^4} = \frac{2(x-1)}{(x-1)^4} = \frac{2}{(x-1)^3}$$

$$8.\ f(x) = \frac{1-x}{1+x}$$

$$f'(x) = \frac{(1+x)(-1) - \ (1-x)(1)}{(1+x)^2}$$

$$= \frac{-1-x-1+x}{(1+x)^2} = \ \frac{-2}{(1+x)^2}$$

$$f''(x) = \frac{(1+x)^2(0) - (-2)(\ 2(1+x))}{(1+x)^4}$$

$$= \frac{4+4x}{(1+x)^4} = \frac{4(1+x)}{(1+x)^4} = \frac{4}{(1+x)^3}$$

$$9.\ g(x) = (1 - x^{2)^{\frac{3}{4}}}$$

$$g'(x) = \frac{3}{4}(1 - x^{2)^{\frac{-1}{4}}}(-2x) = -\frac{3}{2}\left(\frac{x}{(1-x^2)^{\frac{1}{4}}}\right)$$

$$g''(x) = -\frac{3}{2}\left(\frac{(1-x^2)^{\frac{1}{4}}) - x\left(\frac{1}{4}(1-x^2)^{\frac{-3}{4}}\right)(-2x)}{(1-x^2)^{\frac{1}{2}}} \right)$$

$$= -\frac{3}{2}\left(\frac{(1-x^2)^{\frac{1}{4}}) + \frac{x^2}{2}(1-x^2)^{\frac{-3}{4}}}{(1-x^2)^{\frac{1}{2}}} \right)$$

$$= -\frac{3}{2}(\frac{(1-x^2)^{\frac{-3}{4}}(1-x^2+\frac{x^2}{2})}{(1-x^2)^{\frac{1}{2}}}$$

$$= -\frac{3}{2}(\frac{1-\frac{x^2}{2}}{(1-x^2)^{\frac{5}{4}}} = -\frac{3}{4}\left(\frac{2-x^2}{(1-x^2)^{\frac{5}{4}}} \right) = \frac{3}{4}\left(\frac{x^2-2)}{(1-x^2)^{\frac{5}{4}}} \right)$$

10. $h(x) = (x^3 - 2x^2)^{\frac{6}{5}}$

$$h'(x) = \frac{6}{5}(x^3 - 2x^2)^{\frac{1}{5}}(3x^2 - 4x)$$

$$h''(x) = \frac{6}{5}\Big[(x^3 - 2x^2)^{\frac{1}{5}}(6x - 4)$$
$$+ (3x^2 - 4x)\frac{1}{5}(x^3 - 2x^2)^{\frac{-4}{5}}(3x^2 - 4x)\Big]$$

$$= \frac{6}{5}\left[(x^3 - 2x^2)^{\frac{-4}{5}}\left\{(x^3 - 2x^2)(6x - 4) + \frac{1}{5}(3x^2 - 4x)^2\right\}\right]$$

$$= \frac{6}{5}\left[\frac{(x^3-2x^2)(6x-4)+\frac{1}{5}\ \ (3x^2-4x)^2}{(x^3-2x^2)^{\frac{4}{5}}}\right]$$

$$\frac{6}{25}\left[\frac{5(x^3-2x^2)(6x-4)+\ \ (3x^2-4x)^2}{(x^3-2x^2)^{\frac{4}{5}}}\right]$$

$$= \frac{30(x^3-2x^2)(6x-4)+\ 6\ (3x^2-4x)^2}{25(x^3-2x^2)^{\frac{4}{5}}}$$

11. $g(t) = sin^3(t)$

$g'(t) = 3sin^2(t)\cos\ t$

$g''(t) = 3[sin^2(t)(-sint) + cost(2sintcost)]$

$= 3[\,-sin^3(t) + 2sintcos^2t]$

$= 3sint[\,-sin^2(t) + 2cos^2t]$

12. $f(x) = x^2\cos\,(2x)$

$f'(x) = x^2(-\sin(2x)\,(2)) + 2x\cos\,(2x)$

$= -2x^2\sin(2x) + 2x\cos\,(2x)$

$f''(x) = -2x^2\cos(2x)\,(2) + \sin(2x)\,(-4x)$

$$+2x\left(-\sin(2x)\,(2)\right)+2\cos\,(2x)$$

$$=-4x^2\cos(2x)-4xsin(2x)$$

$$-4xsin(2x)\ +2\cos\,(2x)$$

$$=-4x^2\cos(2x)+2\cos(2x)-8xsin(2x)$$

$$=2\cos(2x)\,(-2x^2+1)-8xsin(2x)$$

13. $g(x)=\dfrac{1}{1-\cos\,(3x)}$

$$g'(x)=\frac{-\left(-\left(-\sin(3x)\,(3)\right)\right)}{(1-\cos(3x))^2}=\frac{-3\sin\,(3x)}{(1-\cos(3x))^2}$$

$$g''(x)=\frac{(1-\cos(3x))^2(\,-9\cos(3x))}{(1-\cos(3x))^4}$$

$$-\ \frac{(-3\sin\,(3x)(\ \ 2(1-\cos(3x)\,(-\left(-\sin(3x)\,(3)\right)))}{(1-\cos(3x))^4}$$

$$=\frac{-9\cos\,(3x)(1-\cos(3x))^2}{(1-\cos(3x))^4}+\frac{18sin^2(3x)(1-\cos(3x)}{(1-\cos(3x))^4}$$

$$=\frac{-9\cos\,(3x)}{(1-\cos(3x))^2}+\frac{18sin^2(3x)}{(1-\cos(3x))^3}$$

$$= \frac{-9\cos(3x)\,(1-\cos(3x)) + 18sin^2(3x)}{(1-\cos(3x))^3}$$

$$= \frac{-9\cos(3x) + 9cos^2(3x) + 18sin^2(3x)}{(1-\cos(3x))^3}$$

$$= \frac{9(\,2sin^2(3x) + cos^2(3x) - \cos\,(3x)}{(1-\cos(3x))^3}$$

14. $r(t) = \tan\,(t^2)$

$r'(t) = sec^2(t^2)(2t)$

$r''(t) = sec^2(t^2)(2)\ + 2t(2\sec\left(\mathrm{t}^{2)}\right)\tan(t^2)\ sec\ (t^2)\ \ (2t)$

$= 2sec^2(t^2) + 8t^2\tan(t^2)\ \ sec^2(t^2)$

$= 2sec^2(t^2)(1 + 4t^2\tan(t^2)\ )$

15. $h(t) = (3t+1)^4$

$h'(t) = 4(3t+1)^3(3) = 12(3t+1)^3$

$h''(t) = 36(3t+1)^2(3) = 108(3t+1)^2$

$16.\ f(x) = \sqrt[3]{2-9x} = (2-9x)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}(2-9x)^{\frac{-2}{3}}(-9) = -3(2-9x)^{\frac{-2}{3}}$

$f''(x) = (-3)\left(-\frac{2}{3}\right)(2-9x)^{\frac{-5}{3}}(-9)$

$= -\frac{18}{(2-9x)^{\frac{5}{3}}}$

In Exercises $17-22$, *find* y'''

في التمارين 17-22 أوجد y''' (أوجد المشتقة الثالثة)

$17.\ y = \sqrt{5t-1} = (5t-1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(5t-1)^{\frac{-1}{2}}(5) = \frac{5}{2}(5t-1)^{\frac{-1}{2}}$

$y'' = -\frac{5}{4}(5t-1)^{\frac{-3}{2}}(5) = -\frac{25}{4}(5t-1)^{\frac{-3}{2}}$

$y''' = \frac{75}{8}(5t-1)^{\frac{-5}{2}}(5) = \frac{375}{8}(5t-1)^{\frac{-5}{2}}$

$18.\ y = (x+1)^{\frac{2}{3}}$

$y' = \frac{2}{3}(x+1)^{\frac{-1}{3}}$

$$y'' = -\frac{2}{9}\,(x+1)^{\frac{-4}{3}}$$

$$y''' = \frac{8}{27}(x+1)^{\frac{-7}{3}}$$

19. $y = \dfrac{1}{x-1}$

$$y' = \frac{(x-1)(0) - 1(1)}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

$$y'' = -\frac{(x-1)^2(0) - 1(2(x-1))}{(x-1)^4} = \frac{2(x-1)}{(x-1)^4} = \frac{2}{(x-1)^3}$$

$$y''' = \frac{(x-1)^3(0) - 2(\,3(x-1)^2}{(x-1)^6} = -\frac{6}{(x-1)^4}$$

20. $y = \dfrac{3x}{1-x}$

$$y' = \frac{(1-x)(3) - 3x(-1)}{(1-x)^2} = \frac{3-3x+3x}{(1-x)^2} = \frac{3}{(1-x)^2}$$

$$y'' = \frac{(1-x)^2(0) - 3(\,2(1-x)(-1))}{(1-x)^4} = \frac{6(1-x)}{(1-x)^4}$$

$$= \frac{6}{(1-x)^3}$$

$$y''' = \frac{(1-x)^3(0) - 6(\,3(1-x)^2(-1)}{(1-x)^6} = \frac{18}{(1-x)^4}$$

21. $y = \sin(7x)$

$y' = 7\cos(7x)$

$y'' = 49(-\sin(7x)$

$y''' = -343\cos(7x)$

22. $y = \sin(x^3)$

$y' = cos(x^3)\,(3x^2) = 3x^2\ \ cos(x^3)$

$y'' = 3x^2\big(-sin(x^3)(3x^2)\big) + cos(x^3)\,(6x)$

$= \ -9x^4 sin(x^3) + 6x\,cos(x^3)$

$y''' = -9x^4\big(cos(x^3)(3x^2)\big) + sin(x^3)(\,-36x^3)$

$+6x(-sin(x^3)(3x^2)\,) + cos(x^3)\,(6)$

$= -27x^6 cos(x^3) - 36x^3 sin(x^3)$

$-18x^3 sin(x^3) + 6\,cos(x^3)$

$= -54x^3 sin(x^3) + 3\,cos(x^3)\,(2 - 9x^6)$

In exercises $23 - 28$, find $\mathrm{g}''(2)$

23. $g(x) = \frac{2}{x}$

$$g'(x) = \frac{x(0) - 2(1)}{x^2} = \frac{-2}{x^2}$$

$$g''(x) = \frac{x^2(0) - (-2(2x))}{x^4} = \frac{4x}{x^4} = \frac{4}{x^3}$$

$$g''(2) = \frac{4}{(2)^3} = \frac{4}{8} = \frac{1}{2}$$

24. $g(x) = \frac{2x^2}{5 - x}$

$$g'(x) = \frac{(5 - x)(4x) - 2x^2(-1)}{(5 - x)^2} = \frac{20x - 4x^2 + 2x^2}{(5 - x)^2}$$

$$= \frac{20x - 2x^2}{(5 - x)^2}$$

$$g''(x) = \frac{(5 - x)^2(20 - 4x) - (20x - 2x^2)(\ 2(5 - x)(-1))}{(5 - x)^4}$$

$$= \frac{4(5 - x)^2(5 - x) + 2(20x - 2x^2)(5 - x)}{(5 - x)^4}$$

$$= \frac{4}{5 - x} + \frac{40x - 4\,x^2}{(5 - x)^3}$$

$$g''(2) = \frac{4}{5 - 2} + \frac{40(2) - 4(4)}{(5 - 2)^2} = \frac{4}{3} + \frac{64}{9} = \frac{12 + 64}{9} = \frac{52}{9}$$

25. $g(t) = (\cos(\pi t))^{-2}$

$g'(t) = -2(\cos(\pi t))^{-3}\,(-\sin(\pi t)\,(\pi)$

$= 2\pi\sin(\pi t)\,(\cos(\pi t))^{-3}$

$g''(x) = 2\pi[\,\sin(\pi t)\,(\,-3(\cos(\pi t))^{-4}\,(-\sin(\pi t)(\pi)\;)$

$+(\cos(\pi t))^{-3}\,(cos(\pi t)(\pi)]$

$= 6\pi^2 sin^2(\pi t)(\cos(\pi t))^{-4} + 2\pi^2(\cos(\pi t))^{-2}$

$g''(2) = 6\pi^2 sin^2(2\pi)(\cos(2\pi))^{-4} + 2\pi^2(\cos(2\pi))^{-2}$

$= 6\pi^2(0)(1) + 2\pi^2(1) = 2\pi^2$

$\sin(2\pi) = 0$, $cos(2\pi) = 1$ لا تنسى

26. $g(t) = xsin\left(\frac{\pi}{x}\right)$

$g'(t) = xcos\left(\frac{\pi}{x}\right)\left(\frac{0-\pi}{x^2}\right) + sin\left(\frac{\pi}{x}\right) = \frac{-\pi}{x}cos\left(\frac{\pi}{x}\right) + sin\left(\frac{\pi}{x}\right)$

$g''(x) = \frac{-\pi}{x}\left(-sin\left(\frac{\pi}{x}\right)\left(\frac{-\pi}{x}\right)\right) + cos\left(\frac{\pi}{x}\right)(\frac{\pi}{x^2})$

$+cos\left(\frac{\pi}{x}\right)(\frac{-\pi}{x^2})$

$= \frac{-\pi^2}{x^2}sin\left(\frac{\pi}{x}\right) + \left(\frac{\pi}{x^2}\right)cos\left(\frac{\pi}{x}\right) + \frac{-\pi}{x^2}\;cos\left(\frac{\pi}{x}\right)$

$$g''(2) = \frac{-\pi^2}{4}\sin\left(\frac{\pi}{2}\right) + \left(\frac{\pi}{x^2}\right)\cos\left(\frac{\pi}{2}\right) + \frac{-\pi}{x^2}\ cos\left(\frac{\pi}{2}\right)$$

$$= \frac{-\pi^2}{4}(1) + \left(\frac{\pi}{x^2}\right)(0) + \frac{-\pi}{x^2}(0) = \frac{-\pi^2}{4}$$

$\sin\left(\frac{\pi}{2}\right) = 1\ , cos\left(\frac{\pi}{2}\right) = 0$ لا تنسى

27. $g(x) = x(1 - x^2)^3$

$g'(x) = x\big(\ 3(1 - x^2)^2(-2x)\big) + (1 - x^2)^3$

$= -6x^2(1 - x^2)^2 + (1 - x^2)^3$

$g''(x) = -6x^2\big(\ 2(1 - x^2)(-2x)\big) + (1 - x^2)^2(-12x)$

$+3(1 - x^2)^2(-2x)$

$= 24x^3(1 - x^2) - 12x(1 - x^2)^2 - 6x(1 - x^2)^2$

$g''(2) = 24(8)(1 - 4) - 12(2)(1 - 4)^2 - 6(2)(9)$

$= 24(8)(-3) - 12(2)(9) = -576 - 216 - 108 = 900$

28. $g(s) = \dfrac{(s + 1)^2}{s - 1}$

$$g'(s) = \frac{(s-1)\big(\,2(s+1)\big) - (s+1)^2\,(1)}{(s-1)^2}$$

$$= \frac{(s+1)(\,2(s-1) - (s+1))}{(s-1)^2} = \frac{(s+1)(s-3)}{(s-1)^2}$$

$$= \frac{s^2 - 2s - 3}{(s-1)^2}$$

$$g''(s) = \frac{(s-1)^2(2s-2) - (s^2 - 2s - 3)(\,2(s-1))}{(s-1)^4}$$

$$= \frac{2(s-1)^3}{(s-1)^4} - \frac{(s^2 - 2s - 3)(\,2(s-1))}{(s-1)^4}$$

$$= \frac{2}{s-1} - \frac{2(s^2 - 2s - 3)}{(s-1)^3}$$

$$g''(2) = \frac{2}{2-1} - \frac{2(\,4 - 4 - 3)}{(2-1)^4}$$

$$= 2 + \frac{6}{1} = 8$$

In exercises $29 - 36$, find y$''$ by implicit differentiation

في التمارين 29-36 أوجد y'' عن طريق الاشتقاق الضمني

29. $x^2 + y^2 = 1$

$$2x + 2yy' = 0 \quad , y' = -\frac{x}{y}$$

$$y'' = -\frac{y - xy'}{y^2} = -\frac{y - x(-\frac{x}{y})}{y^2} = -\frac{\frac{y^2 + \ x^2}{y}}{y^2} = -\frac{y^2 + \ x^2}{y^3}$$

30. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$

$$\frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}y' = 0$$

$$x^{\frac{-1}{3}} + y^{\frac{-1}{3}}y' \ = 0$$

$$y' = -\frac{x^{\frac{-1}{3}}}{y^{\frac{-1}{3}}} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$y'' = -\frac{x^{\frac{1}{3}}\left(\frac{1}{3}y^{\frac{-2}{3}}y'\right) - y^{\frac{1}{3}}(\frac{1}{3}x^{\frac{-2}{3}})}{x^{\frac{2}{3}}}$$

$$= -\frac{1}{3}\frac{x^{\frac{-2}{3}}(y^{\frac{-2}{3}}y' - y^{\frac{1}{3}})}{x^{\frac{2}{3}}}$$

$$= -\frac{1}{3}\frac{y^{\frac{-2}{3}}y' - y^{\frac{1}{3}}}{x^{\frac{2}{3}}x^{\frac{2}{3}}} = -\frac{1}{3}\frac{y^{\frac{-2}{3}}y' - y^{\frac{1}{3}}}{x^{\frac{4}{3}}}$$

$$= -\frac{1}{3}\;\frac{y^{\frac{-2}{3}}\left(-\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right) - y^{\frac{1}{3}}}{x^{\frac{4}{3}}} = \frac{1}{3}\;\frac{y^{\frac{-2}{3}}\left(\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right) + y^{\frac{1}{3}}}{x^{\frac{4}{3}}}$$

$$= \frac{1}{3}\;\frac{\frac{y^{\frac{-2}{3}}y^{\frac{1}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}}}{x^{\frac{1}{3}}}}{x^{\frac{4}{3}}} = \;\frac{1}{3}\;\frac{y^{\frac{-2}{3}}y^{\frac{1}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}}}{x^{\frac{5}{3}}}$$

$$= \frac{1}{3}(\;\frac{y^{\frac{1}{3}}}{y^{\frac{2}{3}}x^{\frac{5}{3}}} + \frac{y^{\frac{1}{3}}}{x^{\frac{4}{3}}})$$

31 $x^2 + 6xy + y^2 = 8$

$$2x + 6xy' + 6y + 2yy' = 0$$

$$y' = -\frac{2x + 6y}{6x + 2y} = -\frac{x + 3y}{3x + y}$$

$$y'' = -\frac{(3x + y)(1 + 3y') - (x + 3y)(3 + y')}{(3x + y)^2}$$

$$= -\frac{3x + y + 9xy' + 3yy' - 3x - 9y - xy' - 3yy'}{(3x + y)^2}$$

$$= -\frac{8xy' - 8y}{(3x + y)^2}$$

$$= -\frac{8x\left(-\frac{x + 3y}{3x + y}\right) - 8y}{(3x + y)^2}$$

$$= -\frac{\frac{8x(-x - 3y) - 8y(3x + y)}{3x + y}}{(3x + y)^2}$$

$$= -\frac{8x(-x - 3y) - 8y(3x + y)}{(3x + y)^3}$$

$$= -\frac{-8x^2 - 24xy - 24xy - 8y^2}{(3x + y)^3}$$

$$= \frac{8x^2 + 24xy + 24xy + 8y^2}{(3x + y)^3}$$

$$= \frac{8(x^2 + 6xy + y^2)}{(3x + y)^3} = \frac{8(8)}{(3x + y)^3} = \frac{64}{(3x + y)^3}$$

لأن أصل المسألة هو

$$x^2 + 6xy + y^2 = 8$$

32 $\sqrt{x} + \sqrt{y} = 1$, $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 1$

$$\frac{1}{3}x^{\frac{-2}{3}} + \frac{1}{3}y^{\frac{-2}{3}}y' = 0$$

$$y' = -\frac{x^{\frac{-2}{3}}}{y^{\frac{-2}{3}}} = -\frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}}$$

$$y'' = -\frac{x^{\frac{2}{3}}\left(\frac{2}{3}y^{\frac{-1}{3}}y'\right) - y^{\frac{2}{3}}(\frac{2}{3}x^{\frac{-1}{3}})}{x^{\frac{4}{3}}}$$

$$= -\frac{2}{3}\frac{x^{\frac{2}{3}}y^{\frac{-1}{3}}\left(-\frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}}\right) - y^{\frac{2}{3}}x^{\frac{-1}{3}}}{x^{\frac{4}{3}}}$$

$$= \frac{2}{3}\frac{x^{\frac{2}{3}}y^{\frac{-1}{3}}y^{\frac{2}{3}} + x^{\frac{2}{3}}y^{\frac{2}{3}}x^{\frac{-1}{3}}}{x^{\frac{6}{3}}}$$

$$= \frac{2}{3}\frac{x^{\frac{2}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}}x^{\frac{1}{3}}}{x^{\frac{6}{3}}}$$

33. $x + cosy = y$

$$1 - siny\, y' = y'$$

$$-siny\, y' - y' = -1$$

$$siny\, y' + y' = 1$$

$$y' = \frac{1}{siny + 1}$$

$$y'' = \frac{(siny + 1)(0) - cosyy'}{(siny + 1)^2}$$

$$= -\frac{cosy\frac{1}{siny + 1}}{(siny + 1)^2}$$

$$= -\frac{cosy}{(siny + 1)^3}$$

34. $2\sin(xy) = 1$

$$2\cos(xy)\,(xy' + y) = 0$$

$$2\cos(xy)\,xy' + 2\cos(xy)\,y = 0$$

$$y' = -\ \frac{ycos(xy)}{xcos(xy)} = -\frac{y}{x}$$

$$y'' = -\ \frac{xy' - y}{x^2}$$

$$= -\ \frac{x\left(-\frac{y}{x}\right) - y}{x^2}$$

$$= -\frac{-xy - xy}{x^3} = \frac{2xy}{x^3} = \frac{2y}{x^2}$$

35. $2tany = x$

$2sec^2y\, y' = 1$

$$y' = \frac{1}{2sec^2y}$$

$$y'' = \frac{(2sec^2y)(0) - 4secy(secytany)y'}{4sec^4y}$$

$$= -\frac{tanyy'}{sec^2y} = -\frac{tany(\frac{1}{2sec^2y})}{sec^2y}$$

$$= -\frac{tany}{2sec^4y} = -\frac{1}{2}\ tanycos^4y = -\frac{1}{2}\frac{sinycos^4y}{cosy}$$

$$= -\frac{1}{2}\ siny\ cos^3y$$

36. $2cosy = x$

$-2sinyy' = 1$

$$y' = \frac{1}{2siny}$$

$$y'' = \frac{2siny(0) - 2cosyy'}{(2siny)^2}$$

$$= -\frac{2cosy(\frac{1}{2siny})}{(2siny)^2}$$

$$= -\frac{2cosy}{(2siny)^3} = -\frac{2cosy}{8sin^3y} = -\frac{cosy}{4sin^3y}$$

$37. If\ \ g\ is\ twice\ differentiable\ function\ find$

$\frac{d^2}{dx^2}\left(\frac{g(x)}{x}\right)\ \ in\ terms\ g\ , g', g''$

إذا كانت g دالة قابلة للاشتقاق مرتين فأوجد

$$\frac{d^2}{dx^2}\left(\frac{g(x)}{x}\right)$$

بدلالة $g\ , g', g''$

$$\frac{d}{dx}\left(\frac{g(x)}{x}\right) = \frac{xg'(x) - g(x)}{x^2}$$

$$\frac{d^2}{dx^2}\left(\frac{g(x)}{x}\right)$$

$$= \frac{x^2\big(xg''(x) + g'(x) - g'(x)\big) - \big(xg'(x) - g(x)\big)(2x)}{x^4}$$

$$= \frac{x^3g''(x)\ - 2x^2g'(x) + 2xg(x)}{x^4}$$

$$= \frac{x^2g''(x)\ - 2x\ g'(x) + 2g(x)}{x^3}$$

38. If $f(\mathrm{x}) = \mathrm{x}^3 - x^2 + 6x - 5$, $find\ an\ equation\ of\ the$ $tangent$ line to the graph of f' $at\ the\ point\ p(1,7)$

إذا كانت $f(\mathrm{x}) = \mathrm{x}^3 - x^2 + 6x - 5$

فأوجد معادلة الخط المماس لرسم f' عند النقطة p(1,7)

$f'(x) = 3x^2 - 2x + 6$

$f''(x) = 6x - 2$

$m = f''(1) = 6(1) - 2 = 4$

$$y = m(x - a) + f(a) = 4(x - 4) + 7 = 4x - 16 + 7$$
$$= 4x - 9$$

39. $If\ \ y = x^4\ show\ that\ x^2 \dfrac{d^2y}{dx^2} - 12y = 0$

إذا كانت $y = x^4$ فأثبت أن $x^2 \frac{d^2y}{dx^2} - 12y = 0$

$\dfrac{dy}{dx} = 4x^3 \quad , \quad \dfrac{d^2y}{dx^2} = 12x^2$

بضرب الطرفين ب x^2

$$x^2 \frac{d^2y}{dx^2} = 12x^2(x^2) = 12x^4$$

$$x^2 \frac{d^2y}{dx^2} - 12x^4 = 0$$

$$x^2 \frac{d^2y}{dx^2} - 12y = 0\ \ because\ y = x^4$$

40. $If\ y = \frac{1}{x}\ show\ that\ x^2y'' + 3xy' + y = 0$

إذا كانت $y = \frac{1}{x}$ فأثبت أن $x^2y'' + 3xy' + y = 0$

$$y' = \frac{x(0) - 1}{x^2} = -\frac{1}{x^2}$$

$$y'' = -\frac{-2x}{x^4} = \frac{2x}{x^4}$$

$$x^2y'' = \frac{2x}{x^2} = 2x\left(\frac{1}{x^2}\right) = 2x(-y') = -2xy'$$

$$x^2y'' + 2xy' = 0$$

$$x^2y'' + 2xy' + xy' = xy'$$

$$x^2y'' + 3xy' = x\left(-\frac{1}{x^2}\right) = -\frac{1}{x} = -\left(\frac{1}{x}\right) = -y$$

$$x^2y'' + 3xy' + y = 0$$

41. $Evaluate\quad \frac{d^2}{dx^2}\left[(x^2+1)\frac{d}{dx}\left(\frac{1}{x}\right)\right]$

إحسب $\frac{d^2}{dx^2}\left[(x^2+1)\frac{d}{dx}\left(\frac{1}{x}\right)\right]$

$$\frac{d}{dx}(x^2 + 1) = 2x$$

$$\frac{d^2(2x)}{dx^2} = 2$$

$$\frac{d^2}{dx^2}\left[\frac{d}{dx}\left(\frac{1}{x}\right)\right] = \frac{d^3}{dx^3}\left(\frac{1}{x}\right)$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$$

$$\frac{d^2}{dx^2}\left(\frac{1}{x}\right) = \frac{2x}{x^4} = \frac{2}{x^3}$$

$$\frac{d^3}{dx^3}\left(\frac{1}{x}\right) = \frac{-2(3x^2)}{x^6} = \frac{-6}{x^4}$$

$$\frac{d^2}{dx^2}\left[(x^2+1)\frac{d}{dx}\left(\frac{1}{x}\right)\right] = (2)\left(\frac{-6}{x^4}\right) = -\frac{12}{x^4}$$

$42. \, Evaluate \quad \frac{d^2}{dx^2}\left[(1+2x)\frac{d^2}{dx^2}(5-x^3)\right]$

$$\frac{d}{dx}(1+2x) = 2$$

$$\frac{d^2}{dx^2}(1+2x) = 0$$

$$\frac{d^2}{dx^2}\left[(1+2x)\frac{d^2}{dx^2}(5-x^3)\right] = (0)\frac{d^4}{dx^4}(5-x^3) = 0$$

$43. The\ position\ of\ a\ particle\ is\ givem\ by\ the\ equation$

$s(t) = t^4 - 4t - 1$

$where\ s\ is\ measured\ in\ meters\ and\ t\ in\ seconds$

$a)\ What\ are\ v(t) and\ a(t), the\ velocity\ and\ acceleration$

$of\ the\ particle\ at\ time\ t\ ?$

$b)\ What\ is\ the\ velocity\ of\ the\ particle\ after\ 2\ seconds\ ?$

$c)\ What\ is\ the\ acceleration\ of\ the\ particle\ after\ \frac{1}{2}\ second\ ?$

$d)\ When\ is\ the\ acceleration\ of\ the\ particle\ positive\ ?$

موقع أو مكان جزيء معطى بالمعادلة

$s(t) = t^4 - 4t - 1$

حيث s مقاسة بالامتار و t بالثواني

a) ما ذا تكون v(t) و a(t) , أي سرعة وعجلة الجزيء عند الزمن t ؟

b) ماهي سرعة الجزيء بعد مرور 2 ثانية ؟

c) ماهي عجلة الجزيء بعد مرور $\frac{1}{2}$ ثانية ؟

d) متى تكون عجلة الجزيء موجبة ؟

$a)\ v(t) = s'(t) = 4t^3 - 4$

$a(t) = v'(t) = 12t^2$

$b)\ the\ velocity\ of\ the\ particle\ after\ 2\ seconds\ is$

$v(2) = 4(2)^3 - 4 = 32 - 4 = 28\ m/s$

$c)\ the\ acceleration\ \ of\ the\ particle\ after\ \frac{1}{2}\ second\ is$

$$a\left(\frac{1}{2}\right) = 12(\frac{1}{2})^2 = 12\left(\frac{1}{4}\right) = 3\ m/sec^2$$

$d)\ the\ acceleration\ \ of\ the\ particle\ is\ \ positive\ \ when$

$$a(t) = 12t^2 > 0\ , t^2 > 0\ \ , t > 0$$

$44. If\ \ s(t) = \frac{1}{10}(t^4 - 14t^3 + 60t^2)$

$, find\ the\ velocity\ of\ the\ moving\ object\ when\ its$

$acceleration\ is\ zero.$

إذا كانت $s(t) = \frac{1}{10}(t^4 - 14t^3 + 60t^2)$ فأوجد سرعة الجسم المتحرك
عندما تكون عجلته تساوي صفر

$$v(t) = \ s'(t) = \frac{1}{10}(4t^3 - 14(3)t^2 + 120t)$$

$$a(t) = v'(t) = \frac{1}{10}(12t^2 - 14(3)(2)t + 120)$$

$its\ acceleration\ is\ zero\ \ when$

$$\frac{1}{10}(12t^2 - 14(3)(2)t + 120) = 0$$

$$12t^2 - 14(3)(2)t + 120 = 0$$

$$12t^2 - 7(12)t + 12(10) = 0$$

$$t^2 - 7t + 10 = 0$$

$$(t-2)(t-5) = 0$$

$$t = 2 \quad or \quad t = 5$$

$$its\ acceleration\ is\ zero\ \ when\quad t = 2\ \ or\ \ t = 5$$

the velocity of the moving object when its acceleration is zero are

$$v(2) = \frac{1}{10}(4(2)^3 - 14(3)2^2 + 120(2))$$

$$= \frac{1}{10}(\ 32 - 168 + 240) = \frac{1}{10}(104) = \frac{104}{10}$$

$$or\ \ v(5) = \frac{1}{10}(4(5)^3 - 14(3)5^2 + 120(5))$$

$$= \frac{1}{10}(500 - 1050 + 600) = \frac{1}{10}(50) = 5$$

Section 2.7

تمارين Exercises (2.7) صفحة 223 في الكتاب

مشتقة الدوال العكسية the derivative of inverse functions

الطريقة الأولى

بما أن $f\big(f^{-1}(x)\big) = x$

إذاً باستخدام قانون السلسلة (the chain rule) في الاشتقاق ينتج أن

$$\frac{d}{dx}[\, f\big(f^{-1}(x)\big) = \frac{d}{dx}(x)$$

$$f'(\, f^{-1}(x)\,)(f^{-1})'(x) = 1$$

$$(f^{-1})'(x) = \frac{1}{f'(\, f^{-1}(x)\,)} \qquad (1)$$

الطريقة الثانية

لنفرض أن $y = f^{-1}(x)$ ومنه تكون $y = f(x)$ إذاً

$$\frac{dy}{dx} = (f^{-1})'(x) \quad and \quad \frac{dx}{dy} = f'(y)$$

وبتعويض هاتين القيمتين في المعادلة (1) تنتج الصيغة الثانية للمعادلة (1)

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} \qquad (2)$$

$In\ Exercises\ 1-3\ use\ formula\ (2)$

$to\ find\ the\ derivative\ of\ f^{-1}$

في التمارين 1-3 استخدم الصيغة (2) لتجد مشتقة f^{-1}

$1.\ f(x) = 2x^3 + x - 3$

$let\ \ y = f^{-1}(x)\ then\ \ x = f(y) = 2y^3 + y - 3$

$$\frac{dx}{dy} = 6y^2 + 1$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{6y^2 + 1}$$

$2.\ f(x) = 2x^5 + x^3 + x$

$let\ \ y = f^{-1}(x)\ then\ \ x = f(y) = 2y^5 + y^3 + y$

$$\frac{dx}{dy} = 10y^4 + 3y^2 + 1$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{10y^4 + 3y^2 + 1}$$

$3.\ f(x) = x^7 + x^5 + x$

$let\ \ y = f^{-1}(x)\ then\ \ x = f(y) = y^7 + y^5 + y$

$$\frac{dx}{dy} = 7y^6 + 5y^4 + 1$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{7y^6 + 5y^4 + 1}$$

Derivatives of inverse trigonometric functions

مشتقات معكوسات الدوال المثلثية (انظر الكتاب صفحة 217)

$$a.\quad \frac{d}{dx}(\,sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\quad ,-1 < x < 1$$

$$b.\quad \frac{d}{dx}(\,cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}\quad ,-1 < x < 1$$

$$c.\quad \frac{d}{dx}(\,tan^{-1}x) = \frac{1}{1+x^2}$$

$$d.\quad \frac{d}{dx}(\,cot^{-1}x) = -\frac{1}{1+x^2}$$

$$e.\quad \frac{d}{dx}(\,sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}\quad ,\qquad |x| > 1$$

$$f.\quad \frac{d}{dx}(\,csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}\quad ,\qquad |x| > 1$$

In Exercises 4-17 find the derivative of the function

في التمارين 4-17 أوجد مشتقة الدالة

$4. f(x) = sec^{-1}(3x)$

$$\frac{d}{dx}(sec^{-1}(3x) = \frac{1}{3x\sqrt{(3x)^2-1}}(3) = \frac{1}{x\sqrt{9x^2-1}}$$

$$5. g(x) = \frac{cos^{-1}x}{x+1}$$

$$\frac{d}{dx}\left(\frac{cos^{-1}x}{x+1}\right)=\frac{(x+1)\left(-\frac{1}{\sqrt{1-x^2}}\right)-cos^{-1}x(1)}{(x+1)^2}$$

$$=\frac{-(x+1)-cos^{-1}x(\sqrt{1-x^2})}{\sqrt{1-x^2}(x+1)^2}$$

$$=\frac{-(x+1)}{\sqrt{1-x^2}(x+1)^2}-\frac{cos^{-1}x(\sqrt{1-x^2})}{\sqrt{1-x^2}(x+1)^2}$$

$$=-\frac{1}{\sqrt{1-x^2}(x+1)}-\frac{cos^{-1}x}{(x+1)^2}$$

6. $f(x)=cot^{-1}(\sqrt{x})=cot^{-1}(x^{\frac{1}{2}})$

$$\frac{d}{dx}\left(cot^{-1}\left(x^{\frac{1}{2}}\right)\right)=-\frac{1}{1+(x^{\frac{1}{2}})^2}(\frac{1}{2}x^{\frac{-1}{2}})$$

$$=-\frac{1}{2}\frac{1}{(1+x)(x^{\frac{1}{2}})}=-\frac{1}{2(1+x)\sqrt{x}}$$

7. $f(x)=xcos^{-1}x-\sqrt{1-x^2}=xcos^{-1}x-(1-x^2)^{\frac{1}{2}}$

$$\frac{df}{dx}=x\left(-\frac{1}{\sqrt{1-x^2}}\right)+cos^{-1}x-\frac{1}{2}(1-x^2)^{\frac{-1}{2}}(-2x)$$

$$=-\frac{x}{\sqrt{1-x^2}}+cos^{-1}x+\frac{x}{\sqrt{1-x^2}}=cos^{-1}x$$

8. $f(t)=\frac{1}{2}t\sqrt{4-t^2}+2sin^{-1}\left(\frac{t}{2}\right)$

$$=\frac{1}{2}t(4-t^2)^{\frac{1}{2}}+2sin^{-1}\left(\frac{t}{2}\right)$$

$$\frac{df}{dt}=\frac{1}{2}t(\frac{1}{2}(4-t^2)^{\frac{-1}{2}}(-2t)+(4-t^2)^{\frac{1}{2}}(\frac{1}{2})$$

$$+2\frac{1}{\sqrt{1-\left(\frac{t}{2}\right)^2}}(\frac{1}{2})$$

$$=-\frac{1}{2}t^2(4-t^2)^{\frac{-1}{2}}+\frac{1}{2}(4-t^2)^{\frac{1}{2}}+\frac{1}{\sqrt{1-\left(\frac{t}{2}\right)^2}}$$

$$=(4-t^2)^{\frac{-1}{2}}(-\frac{1}{2}t^2+\frac{1}{2}(4-t^2)+\frac{1}{\sqrt{1-\left(\frac{t}{2}\right)^2}}$$

$$=(4-t^2)^{\frac{-1}{2}}(-\frac{1}{2}t^2+2-\frac{1}{2}t^2+\frac{1}{\sqrt{1-\left(\frac{t}{2}\right)^2}}$$

$$=(4-t^2)^{\frac{-1}{2}}(2-t^2)+\frac{1}{\sqrt{1-\left(\frac{t}{2}\right)^2}}$$

$$=\frac{2-t^2}{(4-t^2)^{\frac{1}{2}}}+\frac{1}{\sqrt{1-\left(\frac{t}{2}\right)^2}}=\frac{2-t^2}{(4-t^2)^{\frac{1}{2}}}+\frac{1}{\sqrt{1-\frac{t^2}{4}}}$$

$$= \frac{2-t^2}{(4-t^2)^{\frac{1}{2}}} + \frac{1}{\sqrt{\frac{4-t^2}{4}}}$$

$$= \frac{2-t^2}{(4-t^2)^{\frac{1}{2}}} + \frac{1}{\frac{\sqrt{4-t^2}}{2}} = \frac{2-t^2}{(4-t^2)^{\frac{1}{2}}} + \frac{2}{\sqrt{4-t^2}}$$

$$= \frac{2-t^2}{\sqrt{4-t^2}} + \frac{2}{\sqrt{4-t^2}} = \frac{2-t^2+2}{\sqrt{4-t^2}} = \frac{4-t^2}{(4-t^2)^{\frac{1}{2}}}$$

$$= (4-t^2)(4-t^2)^{\frac{-1}{2}} = (4-t^2)^{\frac{1}{2}} = \sqrt{4-t^2}$$

9. $f(t) = \tan(sin^{-1}t)$

$$\frac{df}{dt} = \ sec^2(sin^{-1}t)\frac{1}{\sqrt{1-t^2}}$$

$$= \frac{1}{cos^2(sin^{-1}t)}\frac{1}{\sqrt{1-t^2}}$$

$$= \frac{1}{1-sin^2(sin^{-1}t)}\frac{1}{\sqrt{1-t^2}}$$

$$= \frac{1}{1-sin\,(sin^{-1}t)sin\,(sin^{-1}t)}\frac{1}{\sqrt{1-t^2}}$$

$$= \frac{1}{1-(t)(t)}\frac{1}{\sqrt{1-t^2}} = \frac{1}{(1-t^2)(1-t^2)^{\frac{1}{2}}}$$

$$= \frac{1}{(1-t^2)^{\frac{3}{2}}}$$

$sin^2x + cos^2x = 1 \ , cos^2x = 1 - sin^2x$ لا تنسى

ماهو جيب الزاوية التي جيبها $sin\,(sin^{-1}t)=$ t والجواب هو t أي أن

$$sin\,(sin^{-1}t) = t$$

10. $f(x) = (\,\csc(2x) + \cot(2x)\,)^4$

$$\frac{df}{dx} = \frac{1}{4}(\,\csc(2x) + \cot(2x)\,)^3(-\cot(2x)\csc(2x)(2) +$$

$$-csc^2(2x)(2)\,)$$

$$= \frac{1}{4}(\,\csc(2x) + \cot(2x)\,)^3(-2\cot(2x)\csc(2x) - 2csc^2(2x))$$

$$= -\frac{1}{2}(\,\csc(2x) + \cot(2x)\,)^3(\cot(2x)\csc(2x) + csc^2(2x)\,)$$

$$= -\frac{\csc(2x)}{2}(\,\csc(2x) + \cot(2x)\,)^3(\cot(2x) + \csc(2x))$$

$$= -\frac{\csc(2x)}{2}(\,\csc(2x) + \cot(2x)\,)^3(\csc(2x) + \cot(2x))$$

$$= -\frac{\csc(2x)}{2}(\,\csc(2x) + \cot(2x)\,)^4$$

11. $f(x) = sec^{-1}\left(\sqrt{x^2-1}\,\right) = sec^{-1}(x^2-1)^{\frac{1}{2}}$

$$\frac{df}{dx} = \frac{1}{(x^2-1)^{\frac{1}{2}}\sqrt{x^2-1-1}}\left(\frac{1}{2}(x^2-1)^{\frac{-1}{2}}\right)(2x)$$

$$= \frac{x}{(x^2-1)(x^2-2)^{\frac{1}{2}}}$$

12. $f(x) = x^2 tan^{-1}(x^2)$

$$\frac{df}{dx} = x^2\frac{1}{1+x^4}(2x) + tan^{-1}(x^2)(2x)$$

$$= \frac{2x^3}{1+x^4} + 2x tan^{-1}(x^2)$$

13. $f(x) = \cos(x^{-1}) + (cosx)^{-1} + cos^{-1}x$

$$= \cos(x^{-1}) + secx + cos^{-1}x$$

$$\frac{df}{dx} = -\sin(x^{-1})(-1x^{-2}) + \tan x \sec x + \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$= \frac{sin(x^{-1})}{x^2} + \tan x \sec x - \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{\sin\left(\frac{1}{x}\right)}{x^2} + \tan x \sec x - \frac{1}{\sqrt{1-x^2}}$$

$$14.\ f(x) = \frac{sin^{-1}(x)}{x^2+1}$$

$$\frac{df}{dx} = \frac{(x^2+1)\frac{1}{\sqrt{1-x^2}} - sin^{-1}(x)(2x)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\frac{1}{\sqrt{1-x^2}}}{(x^2+1)^2} - \frac{2xsin^{-1}(x)}{(x^2+1)^2}$$

$$= \frac{1}{(x^2+1)\sqrt{1-x^2}} - \frac{2xsin^{-1}(x)}{(x^2+1)^2}$$

$$15.\ f(x) = \sqrt{x}\ sec^{-1}(\sqrt{x})$$

$$\frac{df}{dx} = \sqrt{x}\frac{1}{\sqrt{x}\sqrt{x\ -1}}(\frac{1}{2}x^{-\frac{1}{2}}) + sec^{-1}\left(\sqrt{x}\right)(\frac{1}{2}x^{-\frac{1}{2}})$$

$$= \ \frac{1}{2}\frac{1}{\sqrt{x}\sqrt{x\ -1}} + \frac{1}{2}\frac{sec^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$= \left(\frac{1}{2\sqrt{x}}\right)\left(\frac{1}{\sqrt{x\ -1}} + sec^{-1}(\sqrt{x})\right)$$

$$16.\ f(t) = tan^{-1}\left(\ t - \sqrt{t^2+1}\ \right) = tan^{-1}(\ t - (t^2+1)^{\frac{1}{2}}\ )$$

$$\frac{df}{dt} = \frac{1}{1+\left(\ t-(t^2+1)^{\frac{1}{2}}\ \right)^2}(\ 1 - \frac{1}{2}(t^2+1)^{\frac{-1}{2}}(2t)\ )$$

$$= \frac{1 - t(t^2+1)^{\frac{-1}{2}}}{1 + \left(t - (t^2+1)^{\frac{1}{2}} \right)^2}$$

$17. f(t) = cot^{-1}t + cot^{-1}(\frac{1}{t})$

$$\frac{df}{dt} = - \frac{1}{1+t^2} + (- \frac{1}{1 + (\frac{1}{t})^2} \left(\frac{-1}{t^2}\right))$$

$$= - \frac{1}{1+t^2} + \frac{1}{t^2(1 + (\frac{1}{t})^2)}$$

$$= - \frac{1}{1+t^2} + \frac{1}{t^2(1 + \frac{1}{t^2})}$$

$$= - \frac{1}{1+t^2} + \frac{1}{t^2(\frac{t^2+1)}{t^2})}$$

$$= - \frac{1}{1+t^2} + \frac{1}{t^2+1}$$

$$= - \frac{1}{1+t^2} + \frac{1}{1+t^2} = 0$$

In Exercises 18-22 find an equation of the tangent line to the graph of the function at the indicated point

في التمارين 18-22 أوجد معادلة الخط المماس لرسم الدالة عند النقطة المحددة

$18. y = 3cos^{-1}\left(\frac{x}{2}\right) \quad , \qquad (1, \pi)$

$$\frac{dy}{dx} = 3(-\frac{1}{\sqrt{1-x^2}}\left(\frac{1}{2}\right)) = -\frac{3}{2}\frac{1}{\sqrt{1-x^2}}$$

$$m = \frac{dy}{dx}|_{1,\pi} = -\frac{3}{2}\frac{1}{\sqrt{1-1}} = -\frac{3}{0} = ???$$

No tangent line at this point

$19. y = xsin^{-1}x \ , \ (\frac{1}{2}, \frac{\pi}{12})$

$$\frac{dy}{dx} = x\left(\frac{1}{\sqrt{1-x^2}}\right) + sin^{-1}x$$

$$m = \frac{dy}{dx}|_{\frac{1}{2},\frac{\pi}{12}} = \frac{1}{2}(\frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}) + sin^{-1}(\frac{1}{2})$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{1-\frac{1}{4}}}\right) + \frac{\pi}{6} \qquad , because \ \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{\frac{3}{4}}}\right) + \frac{\pi}{6}$$

$$= \frac{1}{2\sqrt{\frac{3}{4}}} + \frac{\pi}{6} = \frac{1}{\sqrt{4(\frac{3}{4})}} + \frac{\pi}{6} = \frac{1}{\sqrt{3}} + \frac{\pi}{6} = \frac{2\sqrt{3} + \pi}{6}$$

$$y = m(x - a) + f(a) = \frac{2\sqrt{3} + \pi}{6}\left(x - \frac{1}{2}\right) + \frac{\pi}{12}$$

$$= \frac{2\sqrt{3} + \pi}{6}x - \frac{2\sqrt{3} + \pi}{6}\left(\frac{1}{2}\right) + \frac{\pi}{12}$$

$$= \frac{2\sqrt{3} + \pi}{6}x - \frac{2\sqrt{3} + \pi}{12} + \frac{\pi}{12}$$

$$= \frac{2\sqrt{3} + \pi}{6}x + \frac{-(2\sqrt{3} + \pi) + \pi}{12}$$

$$= \frac{2\sqrt{3} + \pi}{6}x + \frac{-2\sqrt{3} - \pi + \pi}{12}$$

$$= \frac{2\sqrt{3} + \pi}{6}x + \frac{-2\sqrt{3}}{12}$$

$$= \frac{2\sqrt{3} + \pi}{6}x - \frac{\sqrt{3}}{6}$$

$$20.\ y = sec^{-1}(2x)\ ,\quad (\frac{\sqrt{2}}{2},\frac{\pi}{4})$$

$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}(2) = \frac{2}{x\sqrt{x^2-1}}$$

$$\frac{dy}{dx}|_{\frac{\sqrt{2}}{2},\frac{\pi}{4}} = \frac{2}{\frac{\sqrt{2}}{2}\sqrt{\frac{2}{4}-1}} = \frac{2}{\frac{\sqrt{2}}{2}\sqrt{-\frac{3}{4}}}$$

$\sqrt{-\frac{3}{4}}$ *not real number so* No tangent line at this point

$$21.\ y = cos^{-1}(x^2)\ ,\ (0,\frac{\pi}{2})$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^4}}(2x) = -\frac{2x}{\sqrt{1-x^4}}$$

$$m = \frac{dy}{dx}|_{0,\frac{\pi}{2}} = -\frac{0}{1} = 0$$

$$y = m(x-a) + f(a) = 0(x-0) + \frac{\pi}{2} = \frac{\pi}{2}$$

$$22.\ y = tan^{-1}(x)\ ,\qquad (-1,-\frac{\pi}{4})$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}\ ,\qquad m = \frac{dy}{dx}|_{-1,-\frac{\pi}{4}} = \frac{1}{1+1} = \frac{1}{2}$$

$$y = m(x-a) + f(a) = \frac{1}{2}(x+1) - \frac{\pi}{4} = \frac{1}{2}x + \frac{1}{2} - \frac{\pi}{4}$$

$$= \frac{1}{2}x + \frac{2-\pi}{4}$$

23. $If\ \ h(x) = xsin^{-1}\left(\frac{x}{4}\right) + \sqrt{16-x^2}\ \ , find\ h'(2)$

$$h'(x) = x\frac{1}{\sqrt{1-\frac{x^2}{16}}}(\frac{1}{4}) + sin^{-1}\left(\frac{x}{4}\right) + \frac{1}{2}(16-x^2)^{-\frac{1}{2}}(-2x)$$

$$h'(2) = 2\frac{1}{\sqrt{1-\frac{4}{16}}}\left(\frac{1}{4}\right) + sin^{-1}\left(\frac{2}{4}\right) - 2(16-4)^{-\frac{1}{2}}$$

$$= \frac{2}{\sqrt{\frac{3}{4}}}(\frac{1}{4}) + sin^{-1}\left(\frac{1}{2}\right) - \frac{2}{\sqrt{12}} = \frac{2}{\frac{\sqrt{3}}{2}}(\frac{1}{4}) + \frac{\pi}{6} - \frac{2}{2\sqrt{3}}$$

$$= \frac{4}{4\sqrt{3}} + \frac{\pi}{6} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} + \frac{\pi}{6} - \frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$24. Prove\ that\ \frac{d}{dx}(cot^{-1}u) = -\frac{1}{1+u^2}\frac{du}{dx}$$

$let\ \ y = cot^{-1}u \quad so\ \cot y = u$

$$tany = \frac{1}{u} = \frac{\text{المقابل}}{\text{المجاور}}$$

المقابل =1 والمجاور=u إذن الوتر = $\sqrt{1+u^2}$

$$coty = u\ , \qquad \frac{d}{dx}(coty) = \frac{d}{dx}\ u$$

$$-csc^2y\ \frac{dy}{dx} = 1\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{-csc^2y}\ \frac{du}{dx}$$

$$\frac{dy}{dx} = -sin^2y\frac{du}{dx}$$

$$siny = \frac{\text{المقابل}}{\text{الوتر}} = \frac{1}{\sqrt{1+u^2}} \quad , \qquad sin^2y = \frac{1}{1+u^2}$$

$$\frac{dy}{dx} = -sin^2y\frac{du}{dx} = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(y) = \frac{d}{dx}(cot^{-1}u\ ) = -\frac{1}{1+u^2}\frac{du}{dx}$$

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