KING SAUD UNIVERSITY

COMMON FIRST YEAR

BASIC SCIENCES DEPARTMENT

Math 101 First Midterm Exam 1441 H.

First Semester

Time Allowed – 90 min

جامعة الملك سعود King Saud University

السنة الأولى المشتركة

St. Name: ____________ St. ID: ____________ Section: ____________

ملاحظات :

1- اكتب خطوات الحل بالتفصيل لجميع الأسئلة داخل دفتر الإجابة (الإجابة على ورقة الأسئلة غير معتمدة).
علما بأن عدد الأسئلة (٤)، وعدد الصفحات (٢).

2- لا يسمح بالكتابة إلا بالقلم الأزرق فقط.

3- لا يسمح بتدوال الآلة الحاسبة بين الطلاب.

4- لا تستخدم آلة حاسبة قابلة للبرمجة أو آلة حاسبة ترسم دوال.

Question 1: **(6 Marks)**

A) Classify each of the following numbers into rational or irrational

$$\left\{\sqrt[3]{27}\,,\ \frac{\sqrt{8}}{\sqrt{2}},\ 3\pi,\ \sqrt{\sqrt{25}+\sqrt{16}}\,,\ 7.\overline{5},\ \sqrt[3]{2},\ 4.952+\frac{1}{3},\ 2.45971\ldots\right\}$$

B) Solve the following inequalities, and write your answer in an interval notation:

i) $6x-2 \geq x+8$

ii) $\dfrac{1}{|\,x-1\,|} < \dfrac{1}{|\,x-2\,|}$

Question 2: **(6 Marks)**

A) Find the domain of each of the following:

i) $h(x) = x^2 - 3x - 4$

ii) $g(x) = \dfrac{5x}{\sin x}$

B) Let $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{2}{x-1}$. Find:

i) The rule $(f+g)(x)$.

ii) D_{f+g}.

iii) The rule $(f \circ g)(x)$.

Question 3: **(6 Marks)**

A) Show that all linear functions are one-to-one.

B) Given that the function $f(x) = 5x + 7$ is one-to-one function

i) Find the inverse of f.

ii) Find the range of f.

Question 4: **(7 Marks)**

A) Let ϕ be an angle in standard position, where its rotation is clockwise, with arc length 110 cm, and the diameter of the circle is 40 cm.
Determine the angle ϕ in degree.

B) Find the reference angle of $\frac{3\pi}{4}$.

C) Find the exact value of $\cos(\sin^{-1}(\frac{2}{3}) + \tan^{-1}(\frac{-1}{3}))$, without using calculator.

D) Verify the identity:

$$\frac{\tan^2 x}{\sec^2 x} = 1 - \cos^2 x$$

Good Luck

حل إختبار الميد الأول Math 101 الترم الأول ١٤٤١هـ

Q1

(A)

$\sqrt[3]{27} = 3$ rational $\in Q$

$\frac{\sqrt{8}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{16}}{2} = \frac{4}{2} = 2$ rational $\in Q$

3π irrational $\in I$

$\sqrt{\sqrt{25}+\sqrt{16}} = \sqrt{5+4} = \sqrt{9} = 3$ rational $\in Q$

$7.\bar{5}$ rational $\in Q$

$\sqrt[3]{2}$ Irrational $\in I$

$4.952 + \frac{1}{3} = \frac{1982}{375}$ rational $\in Q$

$2,45971\ldots$ Irrational

(B) 1) $6x - 2 \geq x + 8$

$6x - x \geq 8 + 2$

$\frac{\cancel{5}x}{\cancel{5}} \geq \frac{10}{5}$

$x \geq 2$

$S.S = [2, \infty)$

Q1 (B) ii)

$$\frac{1}{|x-1|} < \frac{1}{|x-2|} \qquad x \neq 1, 2$$

$|a| > |b|$
$a^2 > b^2$

$$|x-1| > |x-2|$$ بالتربيع

$$(x-1)^2 > (x-2)^2$$

فرق بين مربعين

$$(x-1)^2 - (x-2)^2 > 0$$

$a^2 - b^2$

$(a+b)(a-b)$

$$[(x-1)+(x-2)][(x-1)-(x-2)] > 0$$

$$\cancel{x} - 1 - \cancel{x} + 2$$

$$(2x-3)(1) > 0$$

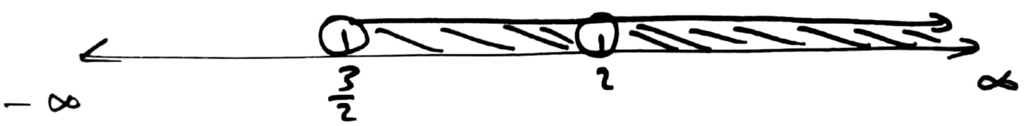
$$2x - 3 > 0$$

$$\frac{\cancel{2}x}{\cancel{2}} > \frac{3}{2}$$

$$x > \frac{3}{2}$$

$$S.S = \left(\frac{3}{2}, \infty\right) - \{2\}$$

or $S.S = \left(\frac{3}{2}, 2\right) \cup (2, \infty)$



Q2 (A) ① $h(x) = x^2 - 3x - 4$

h is polynomail $\Rightarrow$ $D_h = R$

ii) $g(x) = \dfrac{5x}{\sin x}$

$\sin x = 0$ $x = 0, \frac{180}{\pi}, 2\pi, 3\pi, ..$

$D_g = R - \{n\pi\}, n \in Z$

(B) $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{2}{x-1}$

i) $f+g(x) = \dfrac{1}{x} + \dfrac{2}{x-1}$

$= \dfrac{x-1+2x}{x(x-1)} = \dfrac{3x-1}{x(x-1)}$

ii) $D_{f+g} = D_f \cap D_g$

$D_f \Rightarrow R - \{0\}$

$D_g \Rightarrow R - \{1\}$

$D_{f+g} = R - \{0, 1\}$

Q3 (A) show that all Linear functions are (one-to-one)

Linear $\Rightarrow$ $f(x) = ax + b$ $a, b \in R$, $a \neq 0$

let $x_1, x_2 \in D_f$ such that

$$f(x_1) = f(x_2)$$

$$ax_1 \cancel{+b} = ax_2 \cancel{+b}$$
$$\cancel{-b} \qquad \cancel{-b}$$

$$\frac{\cancel{a}x_1}{\cancel{a}} = \frac{\cancel{a}x_2}{\cancel{a}}$$

$$x_1 = x_2$$

$\therefore f(x)$ is (one-to-one) function

(B) (i) $f(x) = 5x + 7$ is (1-1)

$$y = 5x + 7$$

$$x = 5y + 7$$

$$\frac{x-7}{5} = \frac{\cancel{5}y}{\cancel{5}}$$

$$y = \frac{x-7}{5}$$

$$f^{-1}(x) = \frac{x-7}{5}$$

(ii) $R_f = D_{f^{-1}}$

$D_{f^{-1}}$ Polynomail

$\therefore D_{f^{-1}} = R$

$\therefore$ Range $f = D_{f^{-1}} = R$

Q4

(A) arc length $= \boxed{S = 110}$ km

diameter $= 2r = 40 \Rightarrow \boxed{r = 20}$ cm

$\phi = ??$ in degree

$$\phi = \frac{S}{r} = \frac{110}{20} = 5.5 \text{ Radian}$$

$$\therefore \phi = 5.5 \times \frac{180}{\pi} = 315.13^\circ$$

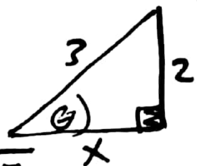
(B) $\theta = \frac{3\pi}{4} = 135^\circ$ in quarter Q_2

$$\theta' = (180 - \theta) = (180 - 135^\circ) = 45^\circ = \frac{\pi}{4}$$

(C) $$\cos\Big(\underbrace{\sin^{-1}\left(\tfrac{2}{3}\right)}_{\theta} + \underbrace{\tan^{-1}\left(\tfrac{-1}{3}\right)}_{\alpha}\Big)$$

let $\theta = \sin^{-1}\left(\frac{2}{3}\right)^{+}$

$\boxed{\sin\theta = \frac{2}{3}}$



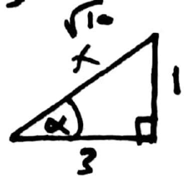
$x = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

θ in Q_1

$\boxed{\cos\theta = \frac{\sqrt{5}}{3}}$

let $\alpha = \tan^{-1}\left(\frac{-1}{3}\right)$

$\tan\alpha = -\frac{1}{3}$



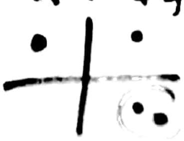
$\boxed{\sin\alpha = \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10}}$

$x = \sqrt{1^2 + 3^2}$

$x = \sqrt{10}$

$\boxed{\cos\alpha = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}}$

α in Q_4



$$\cos(\theta + \alpha) = \cos\theta\cos\alpha - \sin\theta\sin\alpha$$

$$= \frac{\sqrt{5}}{3} \cdot \frac{3\sqrt{10}}{10} - \frac{2}{3} \cdot \frac{-\sqrt{10}}{10}$$

$$= \frac{3\sqrt{50}}{30} + \frac{2\sqrt{10}}{30} = \frac{2\sqrt{10} + 15\sqrt{2}}{30}$$

Q 4 D

$$\frac{\tan^2 x}{\sec^2 x} = 1 - \cos^2 x$$

$$L.H.S = \tan^2 x \cdot \cos^2 x$$

$$= \frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x}$$

$$= \sin^2 x = 1 - \cos^2 x = R.H.S$$

مع تمنياتي بالتوفيق والنجاح   أ. أيمن رفاعي