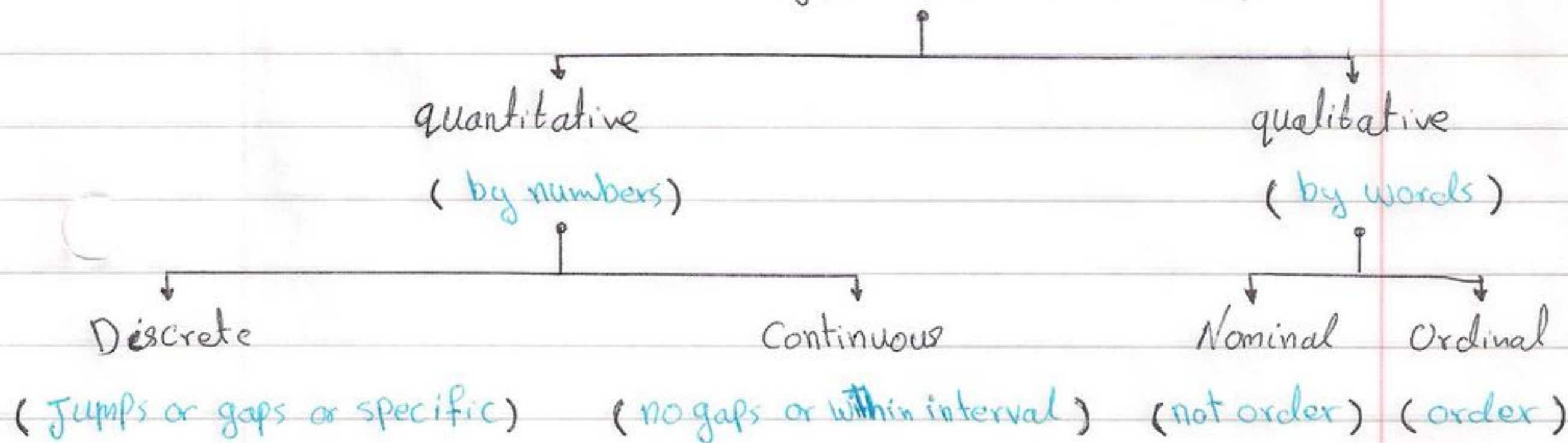


"Basic Concepts"

- * population: is the largest collection of entities (elements or individuals) in which we are interested at particular time and about which we want to draw some conclusions ($N = \text{population size}$).
- * Sample: is a part of population ($n = \text{sample size}$).
- * Variable: characteristic to be measured (number or words) on the elements (in population or in sample).

* types of Variables



- * data: raw material of statistics.
- * statistics: field of study concerned with.
- * descriptive statistics: collection, organization, summarization, describe, and analysis of data.
- * inferential statistics: reach decisions, inference, and conclusions about a large body of data (population) by only part of the data (sample) is observed.

"Frequency Distribution"

* Ordered array or set: listing of values in order from the smallest to the largest values with frequency.

class interval	true class interval	mid point or mid interval
$[lower(1) - upper(1)]$	$[true\ lower(1) - true\ upper(1)]$	$mid(1) = \frac{lower(1) + upper(1)}{2} = \frac{true\ lower(1) + true\ upper(1)}{2}$
$[lower(2) - upper(2)]$	$[true\ lower(2) - true\ upper(2)]$	$mid(2) = \frac{lower(2) + upper(2)}{2} = \frac{true\ lower(2) + true\ upper(2)}{2}$
$[lower(3) - upper(3)]$	$[true\ lower(3) - true\ upper(3)]$	$mid(3) = \frac{lower(3) + upper(3)}{2} = \frac{true\ lower(3) + true\ upper(3)}{2}$

The method to find the following:

* class interval \Rightarrow true class interval:

$$d = lower(2) - upper(1) = lower(3) - upper(2) \quad (\text{gaps})$$

$$\begin{cases} true\ lower(1) = lower(1) - \frac{d}{2} \\ true\ upper(1) = upper(1) + \frac{d}{2} \\ true\ lower(2) = lower(2) - \frac{d}{2} \\ true\ upper(2) = upper(2) + \frac{d}{2} \\ true\ lower(3) = lower(3) - \frac{d}{2} \\ true\ upper(3) = upper(3) + \frac{d}{2} \end{cases}$$

* true class interval \Rightarrow class interval (where d or $\frac{d}{2}$ is known):

$$\left[\begin{array}{l} \text{lower } ① = \text{true lower } ① + \frac{d}{2} \\ \text{upper } ① = \text{true upper } ① - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ② = \text{true lower } ② + \frac{d}{2} \\ \text{upper } ② = \text{true upper } ② - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ③ = \text{true lower } ③ + \frac{d}{2} \\ \text{upper } ③ = \text{true upper } ③ - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ③ = \text{true lower } ③ + \frac{d}{2} \\ \text{upper } ③ = \text{true upper } ③ - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ③ = \text{true lower } ③ + \frac{d}{2} \\ \text{upper } ③ = \text{true upper } ③ - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ③ = \text{true lower } ③ + \frac{d}{2} \\ \text{upper } ③ = \text{true upper } ③ - \frac{d}{2} \end{array} \right.$$

* midpoint \Rightarrow true class interval:

$$w = \text{width} = \text{lower } ② - \text{lower } ① = \text{lower } ③ - \text{lower } ② \quad \left. \begin{array}{l} \text{by using the class} \\ \text{interval} \end{array} \right\}$$

$$= \text{upper } ② - \text{upper } ① = \text{upper } ③ - \text{upper } ②$$

$$= \text{true upper } ① - \text{true lower } ①$$

$$= \text{true upper } ② - \text{true lower } ② \quad \left. \begin{array}{l} \text{by using the true class interval} \end{array} \right\}$$

$$= \text{true upper } ③ - \text{true lower } ③$$

$$= \text{mid } ② - \text{mid } ① = \text{mid } ③ - \text{mid } ② \quad \left. \begin{array}{l} \text{by using the mid point} \end{array} \right\}$$

$$\left[\begin{array}{l} \text{true lower } ① = \text{mid } ① - \frac{w}{2} \\ \text{true upper } ① = \text{mid } ① + \frac{w}{2} \end{array} \right. =$$

$$\left[\begin{array}{l} \text{true lower } ② = \text{mid } ② - \frac{w}{2} \\ \text{true upper } ② = \text{mid } ② + \frac{w}{2} \end{array} \right. =$$

$$\left[\begin{array}{l} \text{true lower } ③ = \text{mid } ③ - \frac{w}{2} \\ \text{true upper } ③ = \text{mid } ③ + \frac{w}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{true lower } ③ = \text{mid } ③ - \frac{w}{2} \\ \text{true upper } ③ = \text{mid } ③ + \frac{w}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{true lower } ③ = \text{mid } ③ - \frac{w}{2} \\ \text{true upper } ③ = \text{mid } ③ + \frac{w}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{true lower } ③ = \text{mid } ③ - \frac{w}{2} \\ \text{true upper } ③ = \text{mid } ③ + \frac{w}{2} \end{array} \right.$$

جزء الثاني

* Frequency \cong Freq.

* Relative \cong r. (relative \cong proportion)

* Percentage \cong per. (percentage \cong percent)

* Cumulative \cong Cum.

	Freq.	Cum. Freq.	r. Freq.	Cum. r. Freq.	Per. Freq.	Cum. Per. Freq.
Total = n		n	Total = 1	1	Total = 100%	100%

Diagram illustrating the relationships between different frequency measures:

- From Freq. to Cum. Freq.: $\div n$
- From Cum. Freq. to r. Freq.: $\div n$
- From r. Freq. to Cum. r. Freq.: $\times 100\%$
- From Cum. r. Freq. to Per. Freq.: $\times 100\%$
- From Per. Freq. to Cum. Per. Freq.: $\times 100\%$

We have another Method to Find the Following :

* Cumulative Frequency :

Cum. Freq. of 1st class interval = Freq.

Cum. Freq. of any class interval = Freq. + Cum. Freq. of preceding class interval

* Cumulative relative Frequency :

Cum. r. Freq. of 1st class interval = r. Freq.

Cum. r. Freq. of any class interval = r. Freq. + Cum. r. Freq. of preceding class interval

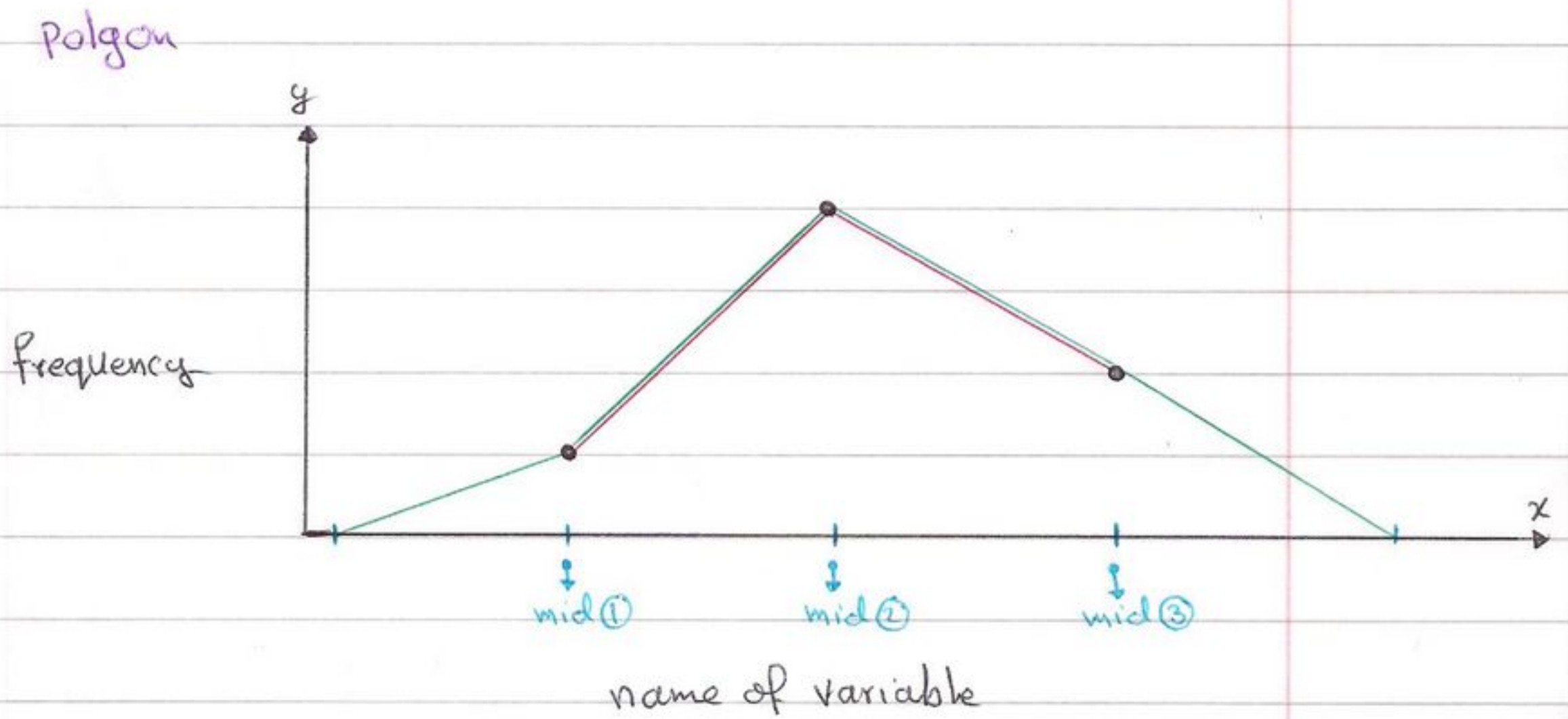
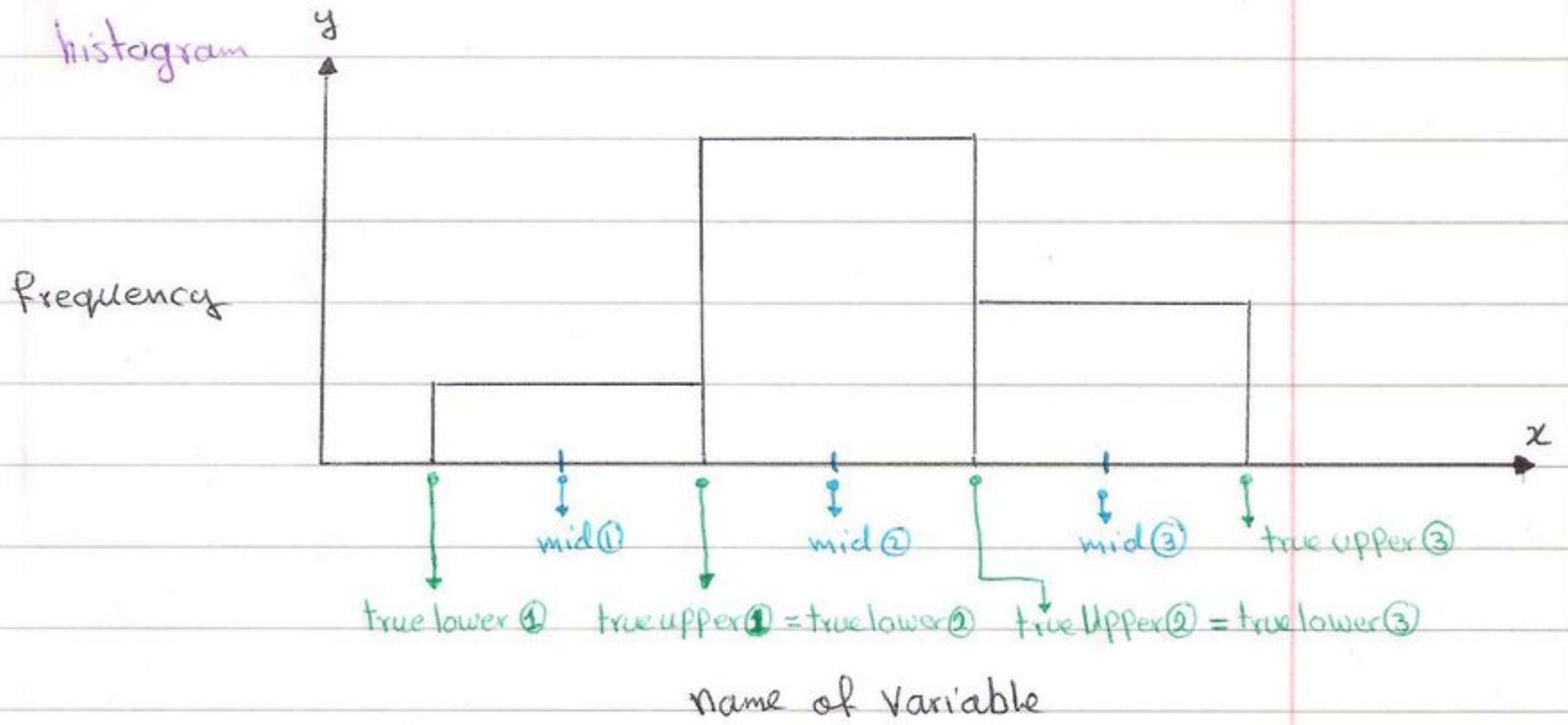
* Cumulative Percentage Frequency :

Cum. per. Freq. of 1st class interval = per. Freq.

Cum. per. Freq. of any class interval = per. Freq. + Cum. per. Freq. of preceding class interval

جزء الثاني

* displaying the frequency distribution:



≡ polygon closed

≡ polygon open

"Descriptive Statistics"

measures of

Central Tendency (Location)

- * mean (unit)
- * median (unit)
- * mode (unit)

Dispersion (Variation)

- * range (unit)
- * Variance = (standard deviation)² (unit)²
- * standard deviation = $\sqrt{\text{variance}}$ (unit)
- * Coefficient of variation (unit-less)
C.V.

population

* X_1, X_2, \dots, X_N
* any measure here
it called "parameter"

sample

* X_1, X_2, \dots, X_n
* any measure here
it called "statistic"

	Population	sample
mean	$M = \frac{\sum_{i=1}^N X_i}{N}$	$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$
Variance	$\sigma^2 = \frac{\sum_{i=1}^N (X_i - M)^2}{N}; \sigma^2 \geq 0$	$S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}; S^2 \geq 0$
Standard deviation	$\sigma = \sqrt{\sigma^2}; \sigma \geq 0$	$S = \sqrt{S^2}; S \geq 0$
size	N	n

مجرب الثاني

«الاستخدام الآلة الحاسبة لإيجاد المقاييس الإحصائية»

① تنظيف الجهاز: نعمل هذه العملية كلما كانت لدينا أي بيانات جديدة أو إرجاع الجهاز لوضع D....

SHIFT + CLR (9) → 3:ALL → = → =

② تحويل الآلة إلى آلة إحصائية وتهيئة البيانات:....

③ جدول لا يحتوي على عمود التكرار:....

MODE → 3:STAT → 1:VAR →

	X
1	a
2	b
3	c

→ AC

④ جدول يحتوي على عمود التكرار:....

SHIFT + MODE → 4:STAT → 1:ON

MODE → 3:STAT → 1:VAR →

	X	FREQ
1	a	d
2	b	e
3	c	f

→ AC

③ إيجاد المقاييس الإحصائية:....

SHIFT + STAT (1) → 5:Var → 1:n

حساب \bar{x} أو M (متوسط أو وسط حسابي) → 2: \bar{x}

حساب s (الانحراف المعياري) → 3: $\sigma x - n$ (σx)

حساب S (الانحراف المعياري) → 4: $\sigma x - n - 1$ (Sx)

→ نختار الرقم حسب المقياس المطلوب → =

ملاحظة: لحساب التباين:....

4 أو 3 → = → x^2 → =

"Probability"

- * **Experiment**: some procedure or process that we do.
- * **Sample space (Ω)**: set of all possible outcomes of experiment; where $n(\Omega)$ is the number of outcomes (elements) in Ω .
- * **Event (E)**: any subset of Ω ; where $n(E)$ is the number of outcomes in E .
 - $E \subseteq \Omega$
 - $\emptyset \subseteq \Omega$ (impossible event)
 - $\Omega \subseteq \Omega$ (sure event)
- * **Equally likely outcomes of experiment**: if the outcomes have the same chance of occurrence.
- * **Probability**: measure used to measure the chance of occurrence of event; which is between 0 and 1.
- * **Probability of event E** : $0 \leq P(E) = \frac{n(E)}{n(\Omega)} \leq 1$
 - $P(\emptyset) = 0$
 - $P(\Omega) = 1$

* Operations on events:

① Union:

- $A \cup B \cong A \text{ or } B \cong A + B$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B) \Leftrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $A \cup B = \Omega$ or $P(A \cup B) = 1 \Leftrightarrow A$ and B are exhaustive
- $A \cup A^c = \Omega$ (exhaustive)
- $P(A \cup B) = \frac{n(A \cup B)}{n(\Omega)}$

② intersection:

- $A \cap B \cong A$ and B
- $P(A \cap B) = \frac{n(A \cap B)}{n(\Omega)}$
- $A \cap B = \emptyset$ or $P(A \cap B) = 0 \Leftrightarrow A$ and B are disjoint (mutually exclusive)
- $A \cap A' = \emptyset$ (disjoint or mutually exclusive)

③ Complement:

- $\bar{A} \cong A^c \cong A' \cong \text{not } A$
- $n(A) + n(A') = n(\Omega) \Leftrightarrow P(A) + P(A') = 1$

* Marginal probability:

① table 2x2:

	B	B ^c	
A	$n(A \cap B)$	$n(A \cap B^c)$	$= n(A)$
	+	+	+
A ^c	$n(A^c \cap B)$	$n(A^c \cap B^c)$	$= n(A^c)$
	$n(B)$	$n(B^c)$	$= n(\Omega)$

⇔

	B	B ^c	
A	$P(A \cap B)$	$P(A \cap B^c)$	$= P(A)$
	+	+	+
A ^c	$P(A^c \cap B)$	$P(A^c \cap B^c)$	$= P(A^c)$
	$P(B)$	$P(B^c)$	$= 1$

الدمج عامودياً أو عرضياً للتقاطع يساوي الأجزاء عامودياً أو عرضياً على التوالي...

② tables of 2×3 ; 3×3 ; 2×4 ; 3×4 , 4×4 , ... : for example if we take

	B_1	B_2	B_3	
A_1	$n(A_1 \cap B_1)$	$+ n(A_1 \cap B_2)$	$+ n(A_1 \cap B_3)$	$= n(A_1)$
	+	+	+	+
A_2	$n(A_2 \cap B_1)$	$+ n(A_2 \cap B_2)$	$+ n(A_2 \cap B_3)$	$= n(A_2)$
	+	+	+	+
A_3	$n(A_3 \cap B_1)$	$+ n(A_3 \cap B_2)$	$+ n(A_3 \cap B_3)$	$= n(A_3)$
	$n(B_1)$	$+ n(B_2)$	$+ n(B_3)$	$= n(\mathcal{U})$

	B_1	B_2	B_3	
A_1	$P(A_1 \cap B_1)$	$+ P(A_1 \cap B_2)$	$+ P(A_1 \cap B_3)$	$= P(A_1)$
	+	+	+	+
A_2	$P(A_2 \cap B_1)$	$+ P(A_2 \cap B_2)$	$+ P(A_2 \cap B_3)$	$= P(A_2)$
	+	+	+	+
A_3	$P(A_3 \cap B_1)$	$+ P(A_3 \cap B_2)$	$+ P(A_3 \cap B_3)$	$= P(A_3)$
	$P(B_1)$	$+ P(B_2)$	$+ P(B_3)$	$= 1$

الجمع عامودياً أو عرضياً للتقاطع يساوي الأجزاء عامودياً أو عرضياً على التوالي

• $A_1 \cap A_2 = A_1 \cap A_3 = A_2 \cap A_3 = A_1 \cap A_2 \cap A_3 = \emptyset$;

$B_1 \cap B_2 = B_1 \cap B_3 = B_2 \cap B_3 = B_1 \cap B_2 \cap B_3 = \emptyset$.

• $A_1' = A_2 \cup A_3$; $A_2' = A_1 \cup A_3$; $A_3' = A_1 \cup A_2$; $B_1' = B_2 \cup B_3$; $B_2' = B_1 \cup B_3$; $B_3' = B_1 \cup B_2$.

• $A_1' \cap B_1 = (A_2 \cup A_3) \cap B_1 = (A_2 \cap B_1) \cup (A_3 \cap B_1) \Leftrightarrow P(A_1' \cap B_1) = P(A_2 \cap B_1) + P(A_3 \cap B_1)$

$A_1 \cap B_1' = A_1 \cap (B_2 \cup B_3) = (A_1 \cap B_2) \cup (A_1 \cap B_3) \Leftrightarrow P(A_1 \cap B_1') = P(A_1 \cap B_2) + P(A_1 \cap B_3)$

وعلى هذا يقاس الأمر لأي حادثتين

* conditional probability:

• $A|B \cong A$ given $B \cong A$ knowing B

$$\bullet P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)}$$

• A and B independent $\Leftrightarrow P(A \cap B) = P(A)P(B)$

$$\Leftrightarrow P(A|B) = P(A)$$

$$\Leftrightarrow P(B|A) = P(B)$$

"Bayes' Theorem"

		Disease		
		Present (D)	absent (D')	
test result	+ve (T)	$n(T \cap D)$	$n(T \cap D')$	$n(T)$
	-ve (T')	$n(T' \cap D)$	$n(T' \cap D')$	$n(T')$
		$n(D)$	$n(D')$	$n(\mathcal{U})$

From the table, we can find:

- ① False +ve result $P(T|D') = \frac{n(T \cap D')}{n(D')}$
- ② False -ve result $P(T'|D) = \frac{n(T' \cap D)}{n(D)}$
- ③ sensitivity of the test $P(T|D) = \frac{n(T \cap D)}{n(D)}$
- ④ specificity of the test $P(T'|D') = \frac{n(T' \cap D')}{n(D')}$

Bayes' theorem: if we have $P(D)$ is the relevant disease in general population which obtained from another independent study ($P(\bar{D}) = 1 - P(D)$), then:

- ⑤ Predictive value +ve of the test

$$P(D|T) = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D')P(D')}$$

- ⑥ Predictive value -ve of the test

$$P(D'|T') = \frac{P(T'|D')P(D')}{P(T'|D')P(D') + P(T'|D)P(D)}$$