Lecture Notes on :

FUNDAMENTALS OF PHYSICS

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Chapters

Physics is the most interesting subject in the world

because

it is about how the world works

Chapter 1 : Measurements

- There are two kinds of physical quantities
- **Basic quantities:** length, time, mass, temperature, pressure, and electric current.
- **Derived quantities:** all other physical quantities
- For example, speed is defined in terms of length and time .
- The unit of each quantity is a unique name. **Example:** meter (m) for the quantity length.

Three Systems of Units (SI)

French (international) system [SI]: MKS: meter, Kg, second

French system: CGS: centimeter, gram, second

British system: FPS: feet, pound, second

The International System of Units (SI)

- Seven quantities are base quantities

Table 1-1

Units for Three SI Base Quantities

Quantity	Unit Name	Unit Symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

Temperature : *Kelvin*

Electric current : *ampere*

Luminous intensity : *candela*

Amount of substance: *mole*

The International System of Units (SI)

- Example of derived quantities: SI unit for power (watt W) is

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^3$$

(kilogram-meter squared per second cubic)

- Speed (velocity) v = distance / time = m/s
- Acceleration a = distance / time^2 = m/s^2

Length

The standard of meter was defined

1. one ten-millionth of the distance from the north pole to the equator
2. the distance between two fine lines near the ends of a platinum-iridium bar (the standard meter bar)
3. 1 650 763.73 wavelengths of an orange-red light emitted by atoms of krypton-86
4. **Length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second**

Standard of Time

1. Earth's rotation has been used
2. Atomic clocks were then developed

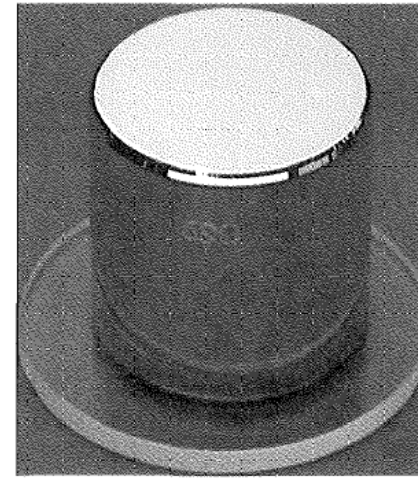
➢ A standard second based on the cesium clock is:

One second is the time taken by 9 192 631 770 oscillations of the light (of a specified wavelength) emitted by a cesium-133 atom.

Why are atomic clocks used to measure the most precise standard second?

Because these atoms are very consistent

Standard of Mass



- The SI standard of mass (kilogram) is a platinum-iridium cylinder

Second Mass Standard

- The masses of atoms are measured by the mass of the carbon-12 atom
- This atom has a mass of 12 **atomic mass units** (u)
- The relation between the two units is

$$1\ \text{u} = 1.660\,538\,86 \times 10^{-27}\ \text{kg},$$

Density

Density ρ is the mass per unit volume

$$\rho = \frac{m}{V}.$$

- The units are kg/m^3 or gm/cm^3
- The density of water ($1.0\ gm/cm^3$) is often used as a comparison

Dimensional Analysis

- *Dimension* [ ] denotes the physical nature of a quantity.
- Whether a distance is measured in units of feet or meters, it is still a distance.
- We say its dimension is *length.*
- The symbols of the dimensions of:

  length, mass, and time are **L, M, and T**
- The dimensions of speed v are written

$$[\mathrm{v}] = \frac{\text{distance}}{\text{time}} = \frac{L}{T} = [LT^{-1}]$$

Examples **The dimensions of**

- Area *A* : $[A]=L^2$
- Acceleration : $[a]=\frac{[\text{Velocity}]}{[\text{Time}]}=\frac{LT^{-1}}{T}=LT^{-2}$
- Force F : [F] = acceleration x mass= [a] x [mass]

$= MLT^{-2} = MLT^{-2}$

- Pressure = (Force / area) = $MLT^{-2} / L^2 = ML^{-1}T^{-2}$
- Work = Force x displacement = $LMLT^{-2} = ML^2T^{-2}$
- Viscosity parameter = (F x r / A x V)

= (Force x displacement / area x velocity)

$= MLT^{-2}L / L^2LT^{-1} = ML^{-1}T^{-1}$.

Units and Dimensions of Famous Quantities

Quantity	Unit		Dimension
	MKS	**CGS**	
Length	m	cm	L
Mass	Kg	Gm	M
Time	s	s	T
Area	m^2	cm^2	L^2
Volume	m^3	cm^3	L^3
Speed (velocity)	$\frac{\text{distance}}{\text{time}} = m/s$	cm/s	LT^{-1}
Acceleration	$\frac{\text{speed}}{\text{time}} = m/s^2$	cm/s^2	LT^{-2}
Force	$\text{mass X acceleration} = mKg/s^2$	$cm\,gm/s^2$	MLT^{-2}
Pressure	$\frac{\text{force}}{\text{area}} = \frac{mKg/s^2}{m^2} = Kg/ms^2$	gm/cms^2	$ML^{-1}T^{-2}$
Work	$\text{force X displacement} = m^2Kg/s^2$	cm^2gm/s^2	ML^2T^{-2}

Changing Units

- Units are changed by a method called ***chain-link conversion***
- We multiply the original measurement by a ***conversion factor*** (ratio of units equal to unity)

Example

1 min and 60 s are identical time intervals

$$\frac{1\ \text{min}}{60\ \text{s}} = 1 \quad \text{and} \quad \frac{60\ \text{s}}{1\ \text{min}} = 1$$

Thus, the ratios (1 min)/(60 s) and (60 s)/(1 min) can be used as conversion factors

Examples

to convert 2 min to seconds

$$2\ \text{min} = (2\ \text{min})(1) = (2\ \cancel{\text{min}})\left(\frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right) = 120\ \text{s}$$

to convert 15 inch to centimeters

$$15.0\ \text{in.} = (15.0\ \cancel{\text{in.}})\left(\frac{2.54\ \text{cm}}{1\ \cancel{\text{in.}}}\right) = 38.1\ \text{cm}$$

to convert 15 h to seconds

$$15\ \text{h} = 15\ \text{h}\ \text{X}\ 1 = 15\ \text{h}\ \ \text{X}\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 900\ \text{min}$$

$$= 900\ \text{min}\ \text{X}\ 1 = 900\ \text{min}\ \text{X}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 54000\ s$$

to convert 10 km/h to m/s`

$$10\ \text{km/h} = 10\ \text{km/h}\left(\frac{1000\ \text{m/h}}{1\ \text{km/h}}\right) = 10000\ \text{m/h}$$

$$= 10000\ \text{m/h}\left(\frac{1\ \text{m/s}}{3600\ \text{m/h}}\right) = \frac{100}{36}\ \text{m/s} = \frac{100}{36}\ \text{m/s} = 2.78\ \text{m/s}$$

to convert 15 m/s to km/h

$$15\ \text{m/s} = 15\ \text{m/s}\left(\frac{1\ \text{km/s}}{1000\ \text{m/s}}\right) = 0.015\ \text{km/s}$$

$$= 0.015\ \text{km/s}\left(\frac{3600\ \text{km/h}}{1\ \text{km/s}}\right) = 54\ \text{km/h}$$

Samples of Exam Questions

Q.1 10^4 milliseconds is equal to:

(A) 10^3 s (B) 10^2 s (C) 1 s (D) 10 s (E) 10^{-1} s

Q.2 A cubic box with an edge of exactly 3 cm has a volume of: (volume = edge^3)

(A) 10^{-6} m^3 (B) 8×10^{-6} m^3 (C) 2.7×10^{-5} m^3 (D) 6.4×10^{-5} m^3 (E) 4×10^{-6} m^3

Q.3 The speed υ in m/s of a car is given by $\upsilon = bt^3$ where the time t is in seconds. The unit of b is:

(A) m/s^4 (B) ms (C) m/s (D) m/s^3 (E) m/s^2

(1)

$$10^4 \text{ ms} = 10^4 \text{ ms}\left(\frac{1\,\text{s}}{1000\,\text{ms}}\right) = 10\,\text{s}$$

(2)

$$V = 3\text{x}3\text{x}3 = 27\,\text{cm}^3 = 27\,\text{cm}^3\left(\frac{1\,\text{m}^3}{10^6\,\text{cm}^3}\right)$$

$$= 27\text{x}10^{-6}\ \text{m}^3 = 2.7\text{x}10^{-5}\ \text{m}^3$$

(3)

m/s = unit(b) s^3 unit(b) = m/s^4

Using the dimensional analysis:

$$[v]=[b][t^3] \Rightarrow \frac{L}{T} = [b]\,T^3 \Rightarrow [b] = \frac{L}{T^4}$$

Then the unit of b is m/s^4

Q.1 The **SI** unit of acceleration is:

(A) m/s^2 (B) s/m (C) kg m/s (D) m/s (E) kg

Q.2 A car is traveling at 15 m/s. The speed of this car is equivalent to:

(A) 45 km/h (B) 20 km/h (C) 54 km/h (D) 11 km/h (E) 72 km/h

Q.3 A cube of edge 30.5 mm, its volume is:

(A) 2.84×10^{-5} m^3 (B) 2.84×10^{-6} m^3 (C) 2.84×10^{-4} m^3 (D) 28.4 m^3 (E) 2.84 m^3

Q.11 A cube of edge 30.5 mm, its volume is:

(A) 2.84×10^{-5} m^3 (B) 2.84×10^{-6} m^3 (C) 2.84×10^{-4} m^3 (D) 28.4 m^3 (E) 2.84 m^3

Q.1 A man of mass 50 kg. His weight is:

(A) <u>490 N</u> (B) 50 N (C) zero (D) 98 N (E) 980 N

Q.2 1Newton is equivalent to:

(A) 9.8 $kg.m/s^2$ (B) <u>1 kg. m/s^2</u> (C) 1 kg of mass (D) 1 kg of force (E) none of these

Q.1 The **SI** unit of velocity is:

(A) m/s^2 (B) s/m (C) kg m/s (D) m/s (E) kg

Q.2 A car is traveling at 20 m/s. The speed of this car is equivalent to:

(A) 40 km/h (B) 20 km/h (C) 10 km/h (D) 11 km/h (E) 72 km/h

Q.3 A cube of edge 47.5 mm, its volume is:

(A) 43 m^3 (B) 0.473 m^3 (C) 1.072×10^{-4} m^3 (D) 47.3 m^3 (E) 475 m^3

Scientific notation (powers of 10) is used to express the very large and very small quantities.

$$3\,560\,000\,000 \text{ m} = 3.56 \times 10^{9} \text{ m}$$

$$0.000\,000\,492 \text{ s} = 4.92 \times 10^{-7} \text{ s.}$$

In computers

3.56 E9

4.92 E-7

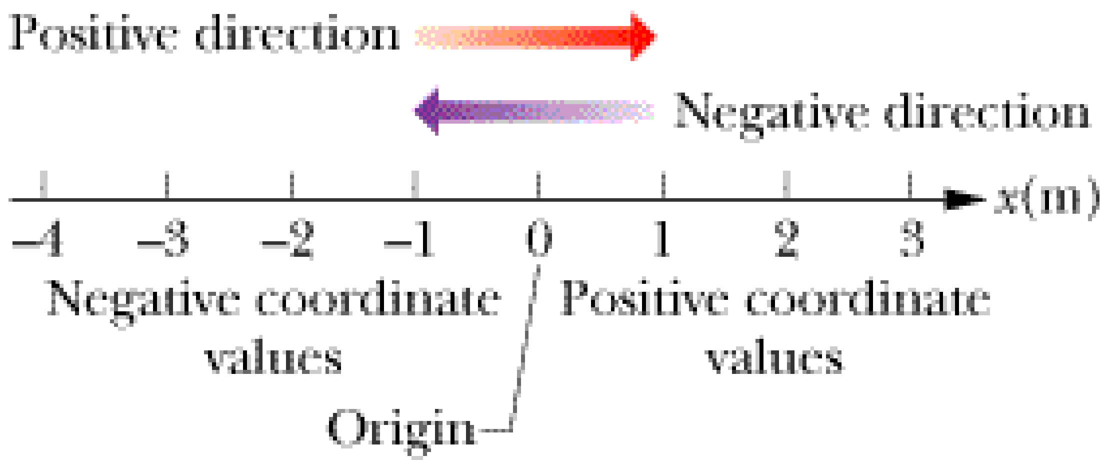
Prefixes for SI Units

Factor	Prefix[a]	Symbol	Factor	Prefix[a]	Symbol
10^{24}	yotta-	Y	10^{-1}	deci-	d
10^{21}	zetta-	Z	$\mathbf{10^{-2}}$	**centi-**	**c**
10^{18}	exa-	E	$\mathbf{10^{-3}}$	**milli-**	**m**
10^{15}	peta-	P	$\mathbf{10^{-6}}$	**micro-**	$\boldsymbol{\mu}$
10^{12}	tera-	T	$\mathbf{10^{-9}}$	**nano-**	**n**
$\mathbf{10^{9}}$	**giga-**	**G**	$\mathbf{10^{-12}}$	**pico-**	**p**
$\mathbf{10^{6}}$	**mega-**	**M**	10^{-15}	femto-	f
$\mathbf{10^{3}}$	**kilo-**	**k**	10^{-18}	atto-	a
10^{2}	hecto-	h	10^{-21}	zepto-	z
10^{1}	deka-	da	10^{-24}	yocto-	y

Chapter 2 Motion Along a Straight Line

2.2. Motion
2.3. Position and Displacement
2.4. Average Velocity and Average Speed
2.5. Instantaneous Velocity and Speed
2.6. Acceleration
2.7. Constant Acceleration: A Special Case
2.8. Another Look at Constant Acceleration
2.9. Free-Fall Acceleration

One-dimensional Coordinate System



It consists of:

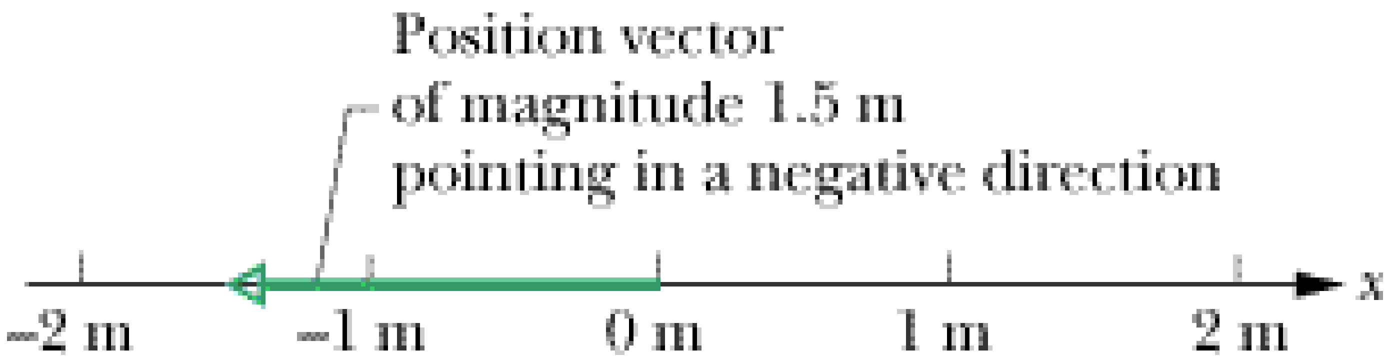
- *the origin (or zero point)*,
- a *coordinate axis:* the direction along it is positive. The other direction is negative

Scalars and Vectors

- A ***scalar quantity*** can be described by its magnitude only
- *A* ***Vector*** is described *with* both its magnitude and direction.

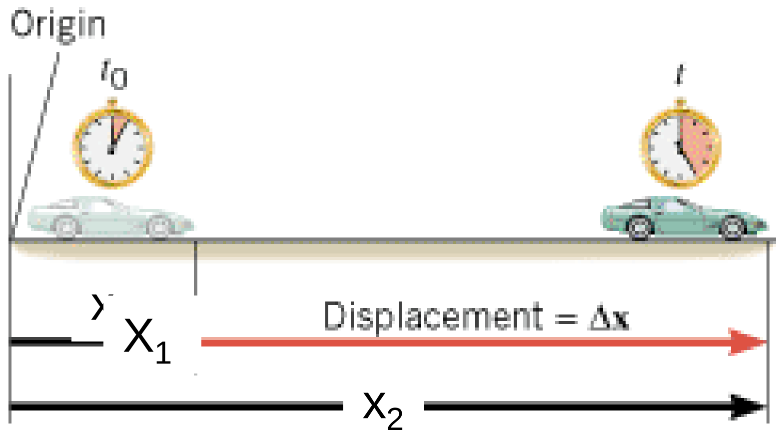
A vector can be represented by an arrow:

Position vector



- Its **magnitude** is the distance between the object and the origin.
- Its **direction** is positive when the object is in the positive side of axis, and negative when the object is in the negative side.

Displacement Vector



$$\vec{\Delta x} = \vec{x}_2 - \vec{x}_1$$

- **It** is the change of the object's position
- It points from the initial position to the final position of the object
- Its magnitude equals the distance between the two positions.
- ***SI Unit of Displacement:*** meter (m)

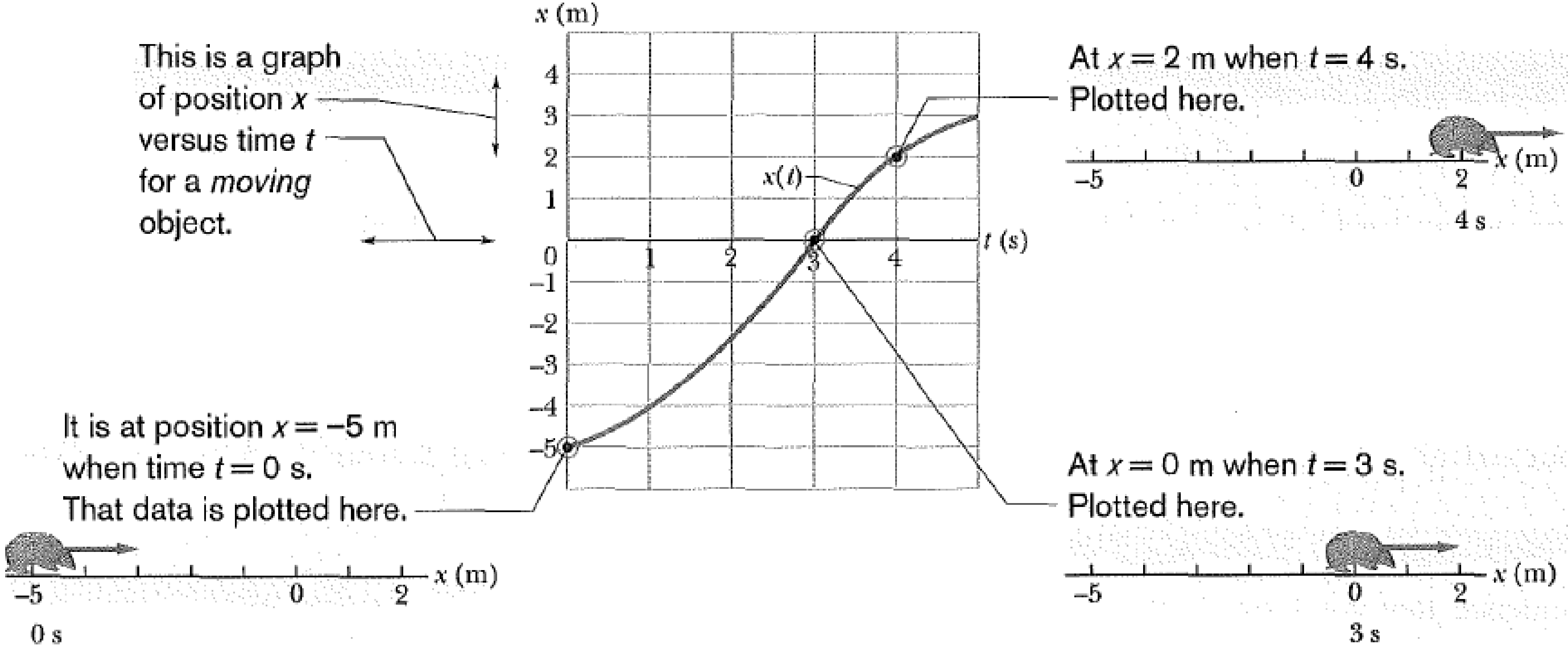
Average Velocity

$$\text{Average velocity} = \frac{\text{displacement}}{\text{elapsed time}}$$

$$\langle v \rangle = \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}\hat{\imath}$$

- x_2 and x_1 are the position vectors at the final and initial times
- Angle brackets denotes the average of a quantity.
- ***SI Unit of Average Velocity:*** meter per second (m/s)

Example



$$\vec{v}_1 = (\vec{x}_2 - \vec{x}_1)/(t_2 - t_1)$$
$$= (0 – (-5)) / (3 - 0) = 5/3 \text{ m/s}$$

$$\vec{v}_2 = (\vec{x}_3 – \vec{x}_2)/(t_3 – t_2)$$
$$= (2 – 0)/(4 - 3) = 2 \text{ m/s}$$

$$\langle \vec{V} \rangle = (\vec{v}_1 + \vec{v}_2)/2 = (5/3 + 2)/2 \text{ m/s}$$

Average Speed

$$\langle s \rangle \equiv \frac{\text{total distance}}{\Delta t} \quad \text{(definition of average speed).}$$

speed: the magnitude of velocity

Average speed is always positive

Average velocity could be negative, positive or zero

depending on the direction of the partial velocities

SAMPLE PROBLEM 2-1

One drives a beat-up pickup truck along a straight road for *8.4* km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, he walks another 2.0 km farther along the road to a gasoline station.

(a) What is the overall displacement from the beginning of his drive to his arrival at the station?

Calculation:

$$\Delta x = x_2 - x_1 = 10.4 \text{ km} - 0 = 10.4 \text{ km}.$$

Thus, the overall displacement is *10.4* km in the positive direction of the *X* axis.

(b) What is the time interval Δt from the beginning of his drive to his arrival at the station?

Calculation: We first write $$v_{avg,dr} = \frac{\Delta x_{dr}}{\Delta t_{dr}}$$

Rearranging and substituting data then give us

$$\Delta t_{dr} = \frac{\Delta x_{dr}}{v_{avg,dr}} = \frac{8.4\text{ km}}{70\text{ km/h}} = 0.12\text{ h}$$

So,

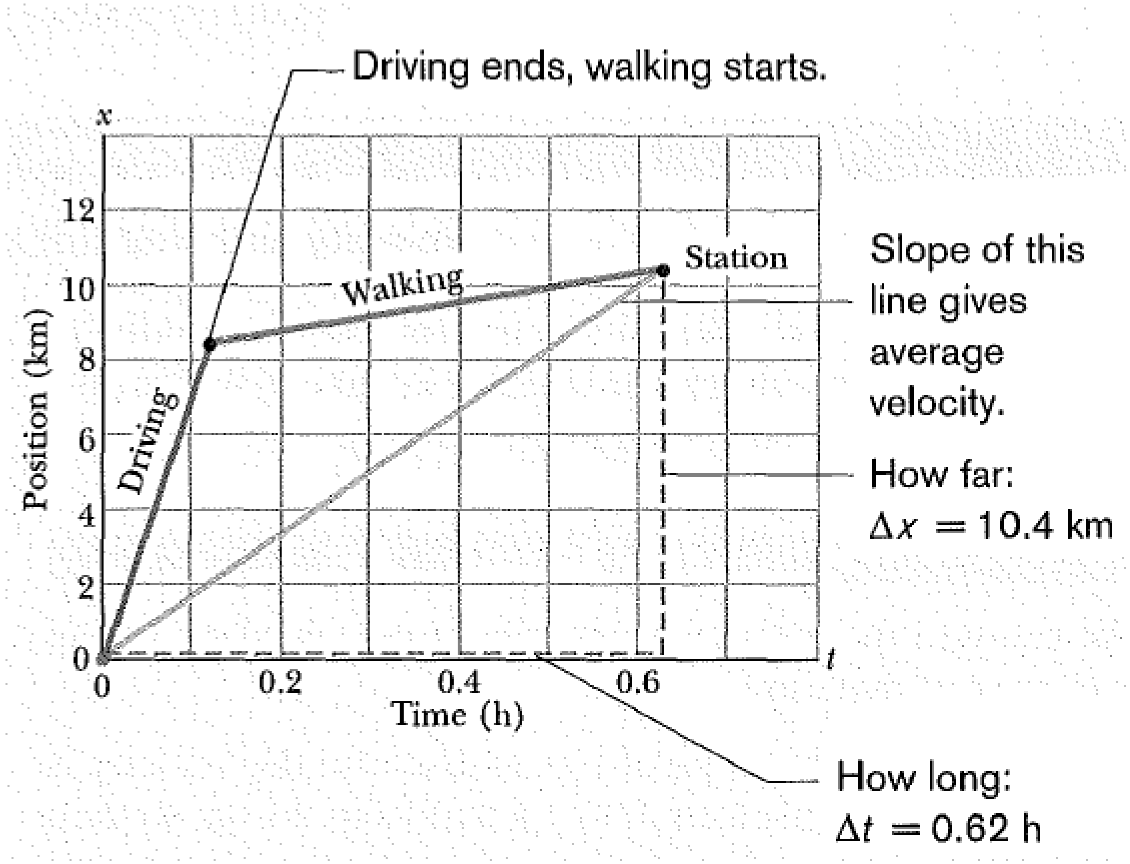
$$\Delta t = \Delta t_{dr} + \Delta t_{wlk}$$
$$= 0.12\text{ h} + 0.50\text{ h} = 0.62\text{ h}.$$

(d) What is the average speed v_{avg} from the beginning of his drive to his arrival at the station? Find it both numerically and graphically.

Calculation: Here we find

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10.4\text{ km}}{0.62\text{ h}}$$
$$= 16.8\text{ km/h} \approx 17\text{ km/h}.$$

SAMPLE PROBLEM 2-1



The average velocity is the slope of the straight line connecting the origin to the final position

Instantaneous Velocity and Speed

$$\lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{\overrightarrow{dx}}{dt} = \frac{dx}{dt}\,\hat{\imath}$$

- It is the time derivative of the object's position.
- It is obtained at any instant from the average velocity by shrinking the time interval Δt closer and closer to zero
- **Instantaneous speed (speed)** is the magnitude of the instantaneous velocity vector

Sample Problem 2-3

The position of a particle moving on an x axis is given by

$$x = 7.8 + 9.2t - 2.1t^3, \qquad (2\text{-}5)$$

with x in meters and t in seconds. What is its velocity at $t = 3.5$ s? Is the velocity constant, or is it continuously changing?

$$v = \frac{dx}{dt} = \frac{d}{dt}(7.8 + 9.2t - 2.1t^3),$$

which becomes

$$v = 0 + 9.2 - (3)(2.1)t^2 = 9.2 - 6.3t^2. \qquad (2\text{-}6)$$

At $t = 3.5$ s,

$$v = 9.2 - (6.3)(3.5)^2 = -68 \text{ m/s}. \qquad \text{(Answer)}$$

Definition of Acceleration

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Elapsed time}}$$

$$\langle \vec{a} \rangle = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

SI Unit : meter per second squared (m/s^2)

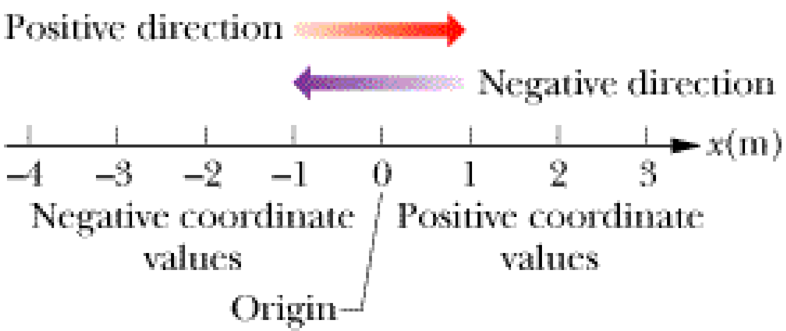
If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

Instantaneous Acceleration

$$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \quad \text{(definition of 1D instantaneous acceleration)}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}$$

Sample Problem 2-4 Build your skill



A particle's position on the x axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

Calculations: Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with v in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with a in meters per second squared.

(b) Is there ever a time when $v = 0$?

Calculation: Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s}. \qquad \text{(Answer)}$$

Constant Acceleration

Typical example, acceleration of a car at a constant rate when a traffic light turns from red to green

$$a = a_{avg} = \frac{\mathrm{v} - v_0}{\mathrm{t} - 0}$$

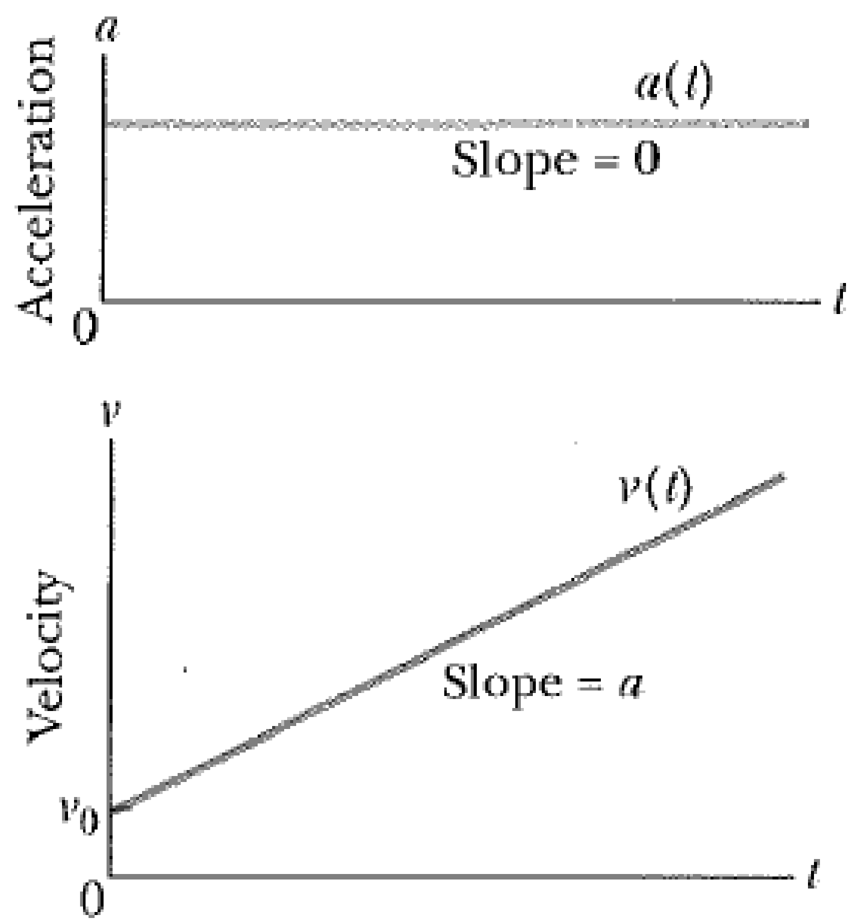
$$at = \mathrm{v} - v_0$$

$$\mathrm{v} = v_0 + at \qquad (1)$$

$$\mathrm{x} = x_0 + v_{avg}t \qquad (2)$$

$$v_{avg} = \frac{1}{2}(v_0 + \mathrm{v}) \qquad (3)$$

$$v_{avg} = v_0 + \frac{1}{2}at \qquad (4)$$

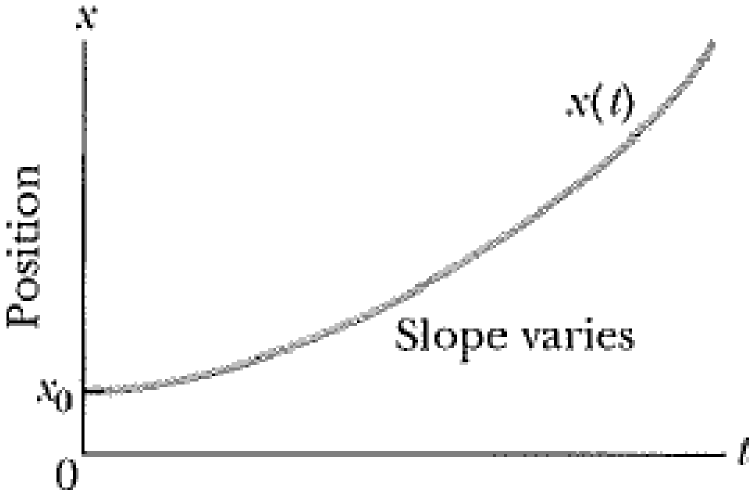


Constant Acceleration

$$x = x_0 + v_{avg}t \qquad (2)$$

$$v_{avg} = v_0 + \frac{1}{2}at \qquad (4)$$

$$x = x_0 + v_0t + \frac{1}{2}at^2 \qquad (5)$$



Equation	Missing Quantity
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0t + \frac{1}{2}at^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Equations for motion with constant acceleration

CHECKPOINT 4

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the above equations apply?

$$(1)\ v = \frac{dx}{dt} = 3 \qquad a = \frac{d^2x}{dt^2} = 0 \ldots.\ constant$$

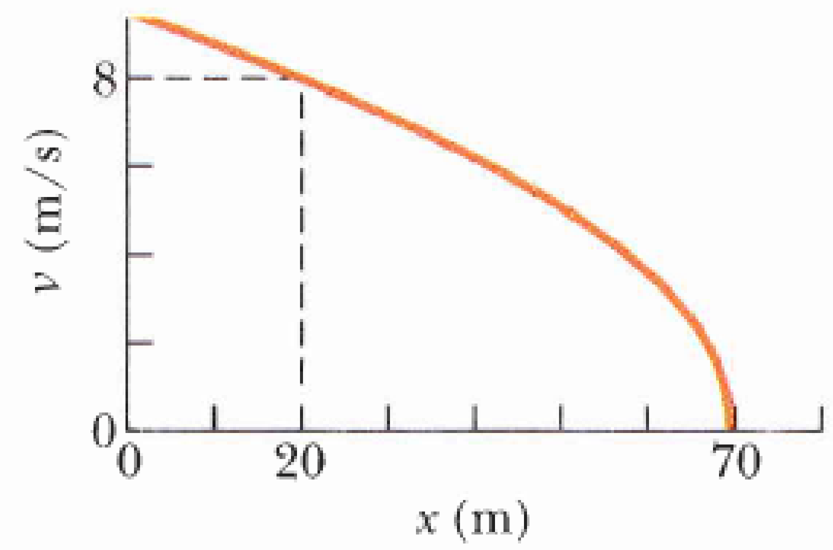
$$(2)\ v = \frac{dx}{dt} = -15t^2 + 8t \qquad a = \frac{d^2x}{dt^2} = -30t + 8 \ldots.\ not\ constant$$

$$(3)\ a = \frac{d^2x}{dt^2} = not\ constant$$

$$(4)\ v = \frac{dx}{dt} = 10t \qquad a = \frac{d^2x}{dt^2} = 10 \ldots.\ constant$$

Sample Problem

The figure gives a particle's velocity v versus its position as it moves along an x axis with constant acceleration. What is its velocity at position x = O?



From the graph, We have: *v = 0 and x = 70 m*. then using

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$(0\,\text{m/s})^2 = (8\,\text{m/s})^2 + 2a(70\,\text{m} - 20\,\text{m})$$

which gives us $a = -0.64\ m/s^2$

Also we have: *v = 8 m/s and x = 20 m,*

$$(8\,\text{m/s})^2 = v_0^2 + 2a(20\,\text{m} - 0)$$

Then substituting for *a* and solving for v0 results in $v_0 = 9.5\ \text{m/s}.$

Free-Fall Acceleration

- its magnitude is g; it is independent of the object's characteristics, such as mass, density, or shape
- g varies slightly with latitude and with elevation; at the sea level g=9.8 m/s^2 (or 32 ft/s^2)
- The equations of motion for constant acceleration also apply to free fall near Earth's surface either up or down
- The directions of motion are now along a vertical *y* axis: it is +ve for upward motion and –ve for downward motion (*a* = - *g*)

The free-fall acceleration near Earth's surface is $a = -g = -9.8\ m/s^2$, and the *magnitude* of the acceleration is $g = 9.8\ m/s^2$. Do not substitute $-9.8\ m/s^2$ for g.

CHECKPOINT 5

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

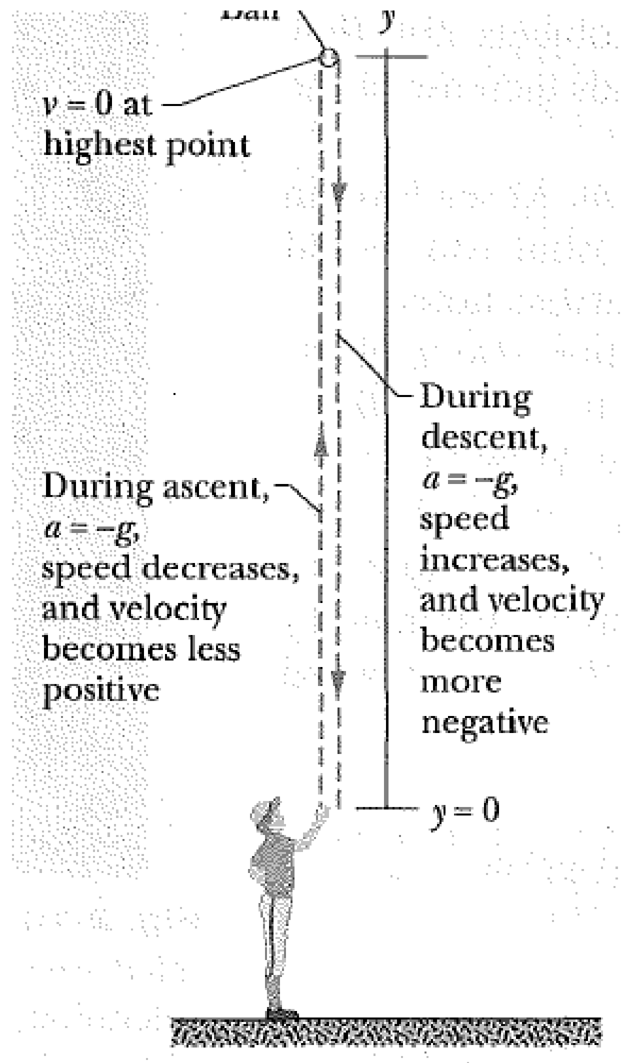
Sample Problem

A pitcher tosses a baseball up along a *y axis, with* an initial speed of 12 *m/s.*
(a) How long does the ball take to reach its maximum height?

***Calculation:* Knowing** *v, a*, and the initial velocity Vo = 12 m/s, and seeking t, we solve the equation

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{0 - 12\text{ m/s}}{-9.8\text{ m/s}^2} = 1.2\text{ s}$$



Sample Problem

(b) What is the ball's maximum height above its release point?

Calculation: We can take the ball's release point as $y_0 = 0$. Set *$y - y_0 = y$ and $v = 0$ (at the maximum height), and solve the equation*

$$v^2 = v_0^2 + 2ay \qquad y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 7.3\ \text{m}$$

(C) How long does the ball take to reach a point 5.0 m above its release point?

Calculation: We know y_0, $a=-g$, and displacement $y - y_0 = 5.0$ m, and we want t, so we set $y_0 = 0$ and use the equation

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \longrightarrow y = v_0 t - \tfrac{1}{2}gt^2$$

$$5.0\ \text{m} = (12\ \text{m/s})t - (\tfrac{1}{2})(9.8\ \text{m/s}^2)t^2$$

$$\longrightarrow 4.9t^2 - 12t + 5.0 = 0 \longrightarrow t = 0.53\ \text{s} \quad \text{and} \quad t = 1.9\ \text{s}$$

Sample Problem

A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

SOLUTION

(a) $$x - x_0 = \frac{1}{2}(v_0 + v)t$$

$$40\ \text{m} = \frac{1}{2}\left(v_0 + 2.8\,\frac{\text{m}}{\text{s}}\right)(8.5\ s) \qquad v_0 = 6.61\ \text{m/s}$$

(b) $$a = \frac{v - v_0}{t} = \frac{2.8 - 6.61\ \text{m/s}}{8.5\ \text{s}} = -0.448\ \text{m/s}^2$$

Samples of Exam Questions

Displacement

Q.14 The position of a ball thrown vertically upward is given by the equation y = 10.0+12.0t- 5.00t^2 (SI units), the height at t=0 is:

(A) 15 m (B) 1 m (C) 5 m (D) Zero (E) 10 m

$$y = 10 + 12t - 5t^2 \quad \Rightarrow \quad y(t=0) = 10\ \text{m}$$

Average & instantaneous Velocity

Q.8 A bicycle travels 12 km in 90 min. Its average speed is:

(A) 48 km/h (B) 18 km/h (C) 8 km/h (D) 0.3 km/h (E) 36 km/h

$$t = 90\,\text{min} = \frac{90}{60} = 1.5\,\text{h}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{12}{1.5} = 8\,\text{km/h}$$

Q.7 A bicycle travels 15 km in 30 min. Its average speed is:

(A) 48 km/h (B) 18 km/h (C) 8 km/h (D) 0.3 km/h (E) 30 km/h

$$t = 30\,\text{min} = \frac{30}{60} = 0.5\,\text{h}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{15}{0.5} = 30\,\text{km/h}$$

Average & instantaneous Velocity

Q.6 A car moves along a straight line with velocity in m/s given by $v = t^2 + 3$. The velocity at t=0 is:

(A) zero (B) 4 m/s (C) 3 m/s (D) 2 m/s (E) 6 m/s

$$v = t^2 + 3 \quad \Rightarrow \quad v(t=0) = 3 \text{ m/s}$$

Q.13 A car moves along the x-axis with constant deceleration, the speed of the car is:

(A) Decreasing (B) Increasing (C) 9.8 m/s^2 (D) Zero (E) none of these

Q.12 An object falling toward the earth's surface will have velocity that its magnitude is: (Ignore air resistance)

(A) Decreasing (B) Zero (C) 9.8 m/s^2 (D) Increasing (E) none of these

Average & instantaneous Velocity

Q.8 The position of an object is given by $x = 4t + 2t^2$. Its average velocity over the time interval from t = 0 to t = 4 s is:

(A) 8 m/s (B) 10 m/s (C) 12 m/s (D) 14 m/s (E) 16 m/s

$$x(t=4) = 4x4 + 2x4^2 = 48m \qquad x(t=0) = 0$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(t=4) - x(t=0)}{4-0} = \frac{48-0}{4-0} = 12 \text{ m/s}$$

Average & instantaneous Acceleration

Q.4 The instantaneous acceleration $\vec{a}$ is given as:

(A) $\frac{dx}{dt}$ (B) $\frac{d}{dt}\left(\frac{d^2x}{dt^2}\right)$ (C) $\frac{d^2}{dt^2}\left(\frac{dx}{dt}\right)$ (D) $\frac{d^2}{dt^2}\left(\frac{dv}{dt}\right)$ (E) $\frac{d}{dt}\left(\frac{dx}{dt}\right)$

Q.5 A particle is moving along the negative x-axis with constant velocity. The magnitude of its acceleration is:

(A) -9.8 m/s^2 (B) zero (C) constant (D) 9.8 m/s^2 (E) 980 cm/s^2

Since the particle moves with constant velocity, its acceleration is zero

Average & instantaneous Acceleration

Q.6 A car moves along a straight line with velocity in m/s given by $\upsilon = t^2 + 3$. The velocity at t=0 is:

(A) zero (B) 4 m/s (C) 3 m/s (D) 2 m/s (E) 6 m/s

Q.7 Referring to question 6, the acceleration of the car at t = 4s is:

(A) 6 m/s^2 (B) 8 m/s^2 (C) 10 m/s^2 (D) 12m/s^2 (E) 4m/s^2

$$a = \frac{dv}{dt} = 2t \quad \Rightarrow \quad a(t=4) = 2\text{x}4 = 8\ \text{m/s}^2$$

Average & instantaneous Acceleration

Q.9 A particle is moving along a straight line. At t=3s its velocity is 20 m/s and at t=8s its velocity is zero. The average acceleration is:

(A) -6 m/s^2 (B) -2 m/s^2 (C) -3 m/s^2 (D) -4m/s^2 (E) -5 m/s^2

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(t=8) - v(t=3)}{8-3} = \frac{0-20}{8-3} = \frac{-20}{5} = -4 \text{ m/s}^2$$

Constant Acceleration

Q.10 A car travels in a straight line with an initial velocity of 4 m/s and an acceleration of 2 m/s^2. The distance traveled in 4s is:

((A) 36 m (B) 40 m (C) 24 m (D) 28 m (E) 32 m

$v_0 = 4\text{ m/s}$ $a = 2\text{ m/s}^2$ $t = 4\text{ s}$ $x\text{-}x_0$ (?) $\Rightarrow$ v (missed)

$$x - x_0 = v_0 t + \frac{1}{2}at^2 = 4\text{x}4 + \frac{1}{2}\text{x}2\text{x}4^2 = 32\text{ m}$$

Constant Acceleration

Q.11 A car, initially at rest, travels 32 m in 4 s along a straight line with constant acceleration. The acceleration of the car is:

(A) 4 m/s^2 (B) 5 m/s^2 (C) 6 m/s^2 (D) 2 m/s^2 (E) 3 m/s^2

$v_0 = 0 \text{ m/s} \qquad a = ? \qquad t = 4 \text{ s} \qquad x\text{-}x_0 = 32 \text{ m} \quad \Rightarrow \quad v \text{ (missed)}$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2 = \frac{1}{2}at^2 \quad \Rightarrow \quad a = \frac{2(x\text{-}x_0)}{t^2} = \frac{2\text{x}32}{4\text{x}4} = 4 \text{ m/s}^2$$

Constant Acceleration

Q.12 What is the initial speed of a car moving a distance of 60 m in 6 s if the final speed was 15 m/s?
(A) 15 m/s (B) 10 m/s (C) 5 m/s (D) zero (E) 20 m/s

$v_0 = ?$ $v = 15\,\text{m/s}$ $t = 6\,\text{s}$ $x\text{-}x_0 = 60\,\text{m}$ $\Rightarrow$ a (missed)

$$x - x_0 = \frac{1}{2}(v + v_0)t \Rightarrow v + v_0 = \frac{2(x - x_0)}{t}$$

$$\Rightarrow v_0 = \frac{2(x - x_0)}{t} - v = \frac{2\text{x}60}{6} - 15 = 5\,\text{m/s}$$

Constant Acceleration

Q.30 A car moving with constant acceleration covers the distance between two points 60 m apart in 4 seconds. If its speed as it passes the second point is 20 m/s, its speed at the first point is:

(A) 20 m/s (B) 10 m/s (C) 5 m/s (D) 45 m/s (E) 30 m/s

$v_0 = ?$ $\quad v = 20\text{ m/s}$ $\quad t = 4\text{ s}$ $\quad x\text{-}x_0 = 60\text{ m}$ $\quad \Rightarrow \quad a$ (missed)

$$x - x_0 = \frac{1}{2}(v + v_0)t \quad \Rightarrow \quad v + v_0 = \frac{2(x - x_0)}{t}$$

$$\Rightarrow \quad v_0 = \frac{2(x - x_0)}{t} - v = \frac{2\text{x}60}{4} - 20 = 10\text{ m/s}$$

Constant Acceleration

Q.7 A car uniformly changes its speed from 20 m/s to 5 m/s in 5 s. The distance moved in the third second is:
(A) 56 m (B) 46.5 m (C) 34 m (D) 12.5 m (E) 9.5 m

(1) $v_0 = 20\text{ m/s} \qquad v = 5\text{ m/s} \qquad t = 5\text{ s} \qquad x\text{-}x_0(t = 5\text{ s}) = \text{missed} \quad \& \; a = ?$

$$v = v_0 + at \;\Rightarrow\; v - v_0 = at \;\Rightarrow\; a = \frac{v - v_0}{t} = \frac{5 - 20}{5} = \text{-}3\text{ m/s}^2$$

(2) $v_0 = 20\text{ m/s} \qquad t = 3\text{ s} \qquad x\text{-}x_0(t = 3\text{ s}) = ? \qquad a = \text{-}3\text{ m/s}^2 \qquad v\text{ (missed)}$

$$x\text{-}x_0 = v_0 t + \frac{1}{2}at^2 = 20\text{x}3 + \frac{1}{2}\text{x(-3)x}3^2 = 60 - 13.5 = 46.5\text{ m}$$

Q.6 A car uniformly changes its speed from 20 m/s to 5 m/s in 5 s. The distance moved in the fourth second is:
(A) 56 m (B) 9.5 m (C) 62.5 m (D) 3 m (E) 46.5 m

(1) $v_0 = 20\text{ m/s} \qquad v = 5\text{ m/s} \qquad t = 5\text{ s} \qquad x\text{-}x_0(t = 5\text{ s}) = \text{missed} \quad \& \; a = ?$

$$a = \frac{v - v_0}{t} = \frac{5 - 20}{5} = \text{-}3\text{ m/s}^2$$

(2) $v_0 = 20\text{ m/s} \qquad t = 4\text{ s} \qquad x\text{-}x_0(t = 4\text{ s}) = ? \qquad a = \text{-}3\text{ m/s}^2 \qquad v\text{ (missed)}$

$$x\text{-}x_0 = v_0 t + \frac{1}{2}at^2 = 20\text{x}4 + \frac{1}{2}\text{x(-3)x}4^2 = 80 - 24 = 56\text{ m}$$

Free fall acceleration

Q.6 An object thrown vertically upwards will have velocity that its magnitude is: (Ignore air resistance)
(A) Zero (B) Increasing (C) Constant (D) Decreasing (E) none of these

Q.10 At the earth's surface, a ball thrown straight up from a bridge would have an acceleration of magnitude:
(A) less than 9.8 m/s^2 (B) 9.8 m/s^2 (C) more than 9.8 m/s^2 (D) Zero (E) none of these

Q.13 A baseball is thrown vertically up into the air. The acceleration of the ball at its highest point is:
(A) -19.6 m/s^2 (B) 19.6 m/s^2 (C) + 9.8 m/s^2 (D) - 9.8 m/s^2 (E) zero

The acceleration is a vector, then it is equal to -9.8 m/s^2

Free fall acceleration

Q.14 An object is thrown straight up from ground level with a speed of 30 m/s. Its height after 1.0 s is:

(A) 15.1 m (B) 5.1 m (C) 45.1 m (D) 35.1 m (E) 25.1 m

$$v_0 = 30\,\text{m/s} \qquad g = 9.8\,\text{m/s}^2 \qquad t = 1\,\text{s} \qquad y\text{-}y_0 = ? \quad \Rightarrow \quad v\,\text{(missed)}$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 = 30\text{x}1 - \frac{1}{2}\text{x}9.8\text{x}1^2 = 30 - 4.9 = 25.1\,\text{m}$$

Free fall acceleration

Q.16 A stone dropped off a 75 m high building reaches the ground in:

(A) 3.91 s (B) 2.86 s (C) 1.35 s (D) 5.53 s (E) 4.95 s

$v_0 = 0\text{ m/s (free drop)} \quad g = 9.8\text{ m/s}^2 \quad t = ? \quad y\text{-}y_0 = -75\text{ m} \quad \Rightarrow \quad v\text{ (missed)}$

$$y - y_0 = v_0 t - \frac{1}{2}gt^2 = 0 - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2 \quad \Rightarrow \quad t^2 = -\frac{2(y\text{-}y_0)}{g}$$

$$t = \sqrt{-\frac{2(y\text{-}y_0)}{g}} = \sqrt{-\frac{2\text{x}(-75)}{9.8}} = 3.91\text{ s}$$

Q.17 Referring to question 16,the speed of the stone just before reaching the ground is: Morouj Q

(A) 54.2 m/s (B) 48.5 m/s (C) 38.3 m/s (D) 28 m/s (E) zero

$v_0 = 0\text{ m/s} \quad g = 9.8\text{ m/s}^2 \quad t = 3.91\text{ s} \quad y\text{-}y_0 = -75\text{ m} \quad v = ?$

$$v = v_0 + at = 0 - gt = -9.8\text{x}3.91 = -38.8\text{ m/s} \Rightarrow \text{speed} = 38.8\text{ m/s}$$

Free fall acceleration

Q.9 A ball is thrown vertically upward at a speed of 21 m/s. It will reach its maximum height in:

(A) 1.8 s (B) 2.1 s (C) 0.60 s (D) 0.33 s (E) 1.2 s

$$v_0 = 21\,\text{m/s} \qquad v = 0\,\text{m/s} \qquad g = 9.8\,\text{m/s}^2 \qquad t = ? \qquad y\text{-}y_0\,(\text{missed})$$

$$v = v_0 - gt \quad \Rightarrow \quad gt = v_0 - v \quad \Rightarrow \quad \text{t} = \frac{v_0 - v}{g} = \frac{21-0}{9.8} = 2.1\,\text{s}$$

Q.12 A ball is thrown vertically upward from ground level to reach a maximum height of 98 m. The initial speed is:

(A) 43.8 m/s (B) 100 m/s (C) 25 m/s (D) 31.3 m/s (E) 49 m/s

$$v = 0\,\text{m/s} \qquad g = 9.8\,\text{m/s}^2 \qquad v_0 = ? \qquad y\text{-}y_0 = 98\,\text{m} \qquad t\,(\text{missed})$$

$$v^2 = v_0^2 - 2g\left(y - y_0\right) \quad \Rightarrow \quad v_0^2 = v^2 + 2g\left(y - y_0\right) = 0 + 2x9.8x98 = 1920.8$$

$$v = 43.8s\ \text{m/s}$$

Free fall acceleration

Q.29 A boy shot a football vertically up with an initial speed v_o. When the ball was 2 m above the ground, the speed was 0.4 of the initial speed. The initial speed is :

(A) 6.8 m/s (B) 3.4 m/s (C) 11.8 m/s (D) 4.8 m/s (E) 19.6 m/s

$v = 0.4v_0$ $g = 9.8\,\text{m/s}^2$ $v_0 = ?$ $y\text{-}y_0 = 2\,\text{m}$ t (missed)

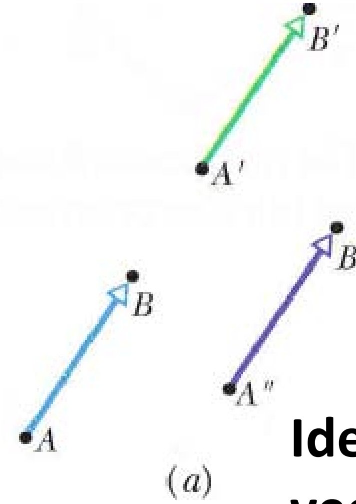
$$v^2 = v_0^2 - 2g(y - y_0) \Rightarrow v_0^2 = v^2 + 2g(y - y_0) = 0.4\text{x}0.4v_0^2 + 2\text{x}9.8\text{x}2 = 0.16v_0^2 + 39.2$$

$$v_0^2 = 0.16v_0^2 + 39.2 \Rightarrow v_0^2 - 0.16v_0^2 = 39.2 \Rightarrow 0.84v_0^2 = 39.2$$

$$v_0^2 = \frac{39.2}{0.84} \Rightarrow v = 6.8\,\text{m/s}$$

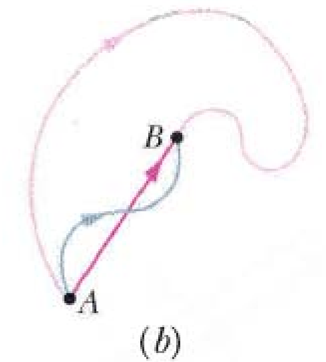
3-2 | Vectors and Scalars

A vector quantity has both a magnitude and a direction

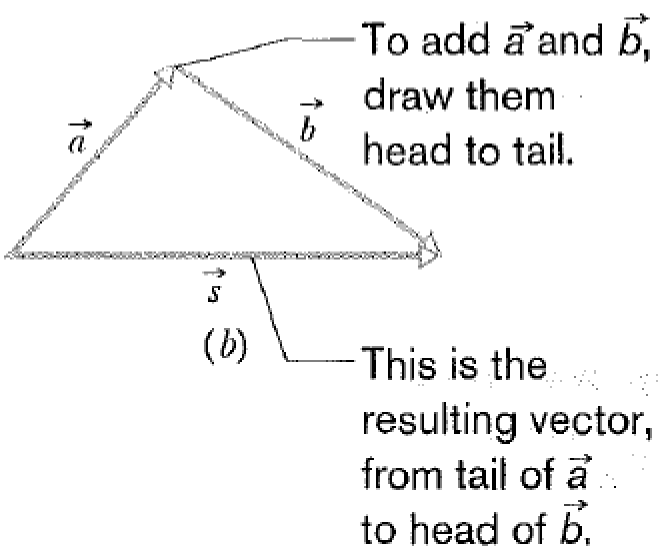


Identical vectors

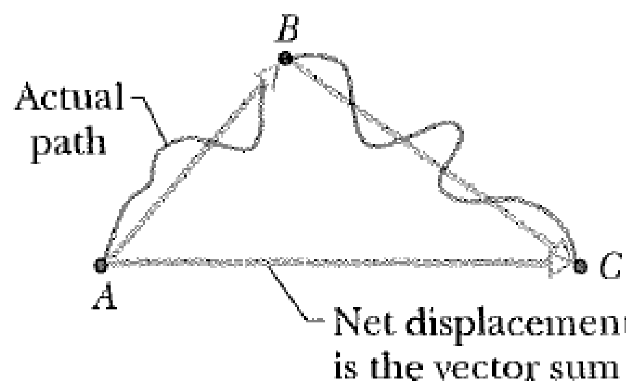
All paths correspond to the same displacement vector

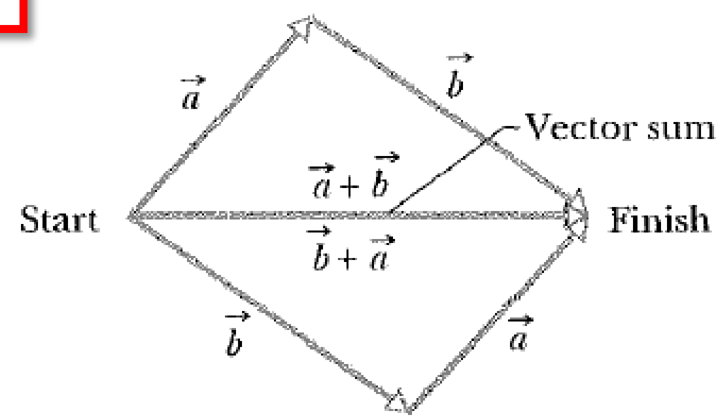


3-3 | Adding Vectors Geometrically



$$\vec{s} = \vec{a} + \vec{b}$$

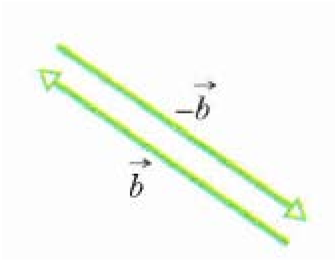




The vector sum $\vec{s}$ is the vector that extends from the tail of $\vec{a}$ to the head of $\vec{b}$

PROPERTIES OF VECTOR ADDITION

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad \text{(commutative law)}$$

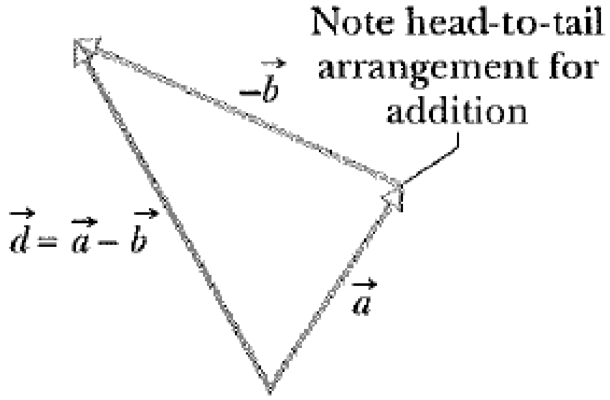


$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \quad \text{(associative law)}$$

$$\vec{b} + (-\vec{b}) = 0.$$

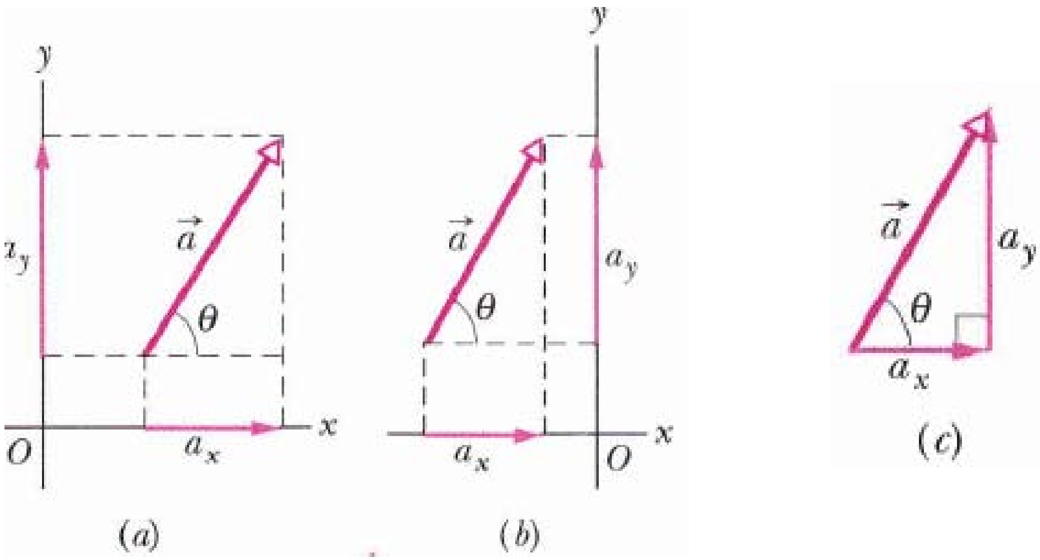
$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \quad \text{(vector subtraction)}$$

$$\vec{d} + \vec{b} = \vec{a} \quad \text{or} \quad \vec{a} = \vec{d} + \vec{b}$$



3-4 | Components of Vectors

- A **component of a vector is the projection of the vector on an axis.**
- The process of finding the components of a a vector is called resolving **the vector**
- A component of a vector has the same direction (along an axis) as the vector



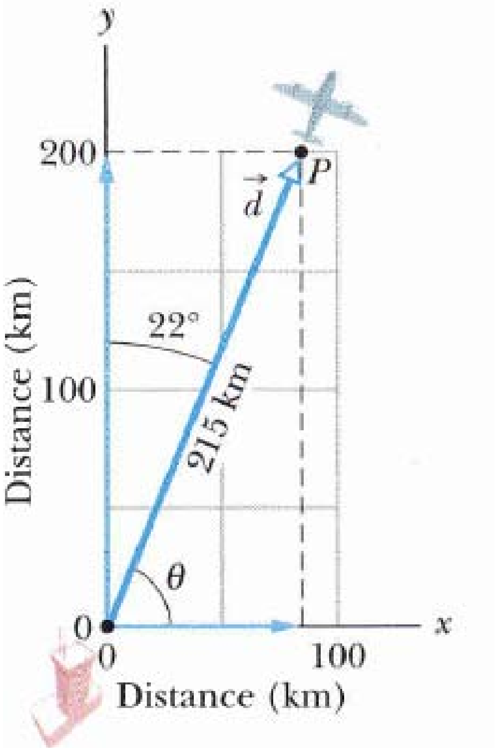
The vector components can be found from the right triangle as

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta$$

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}$$

Sample Problem 3-2

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?



$$d_x = d\cos\theta = (215\text{ km})(\cos 68°)$$
$$= 81\text{ km} \qquad \text{(Answer)}$$
$$d_y = d\sin\theta = (215\text{ km})(\sin 68°)$$
$$= 199\text{ km} \approx 2.0 \times 10^2\text{ km.} \qquad \text{(Answer)}$$

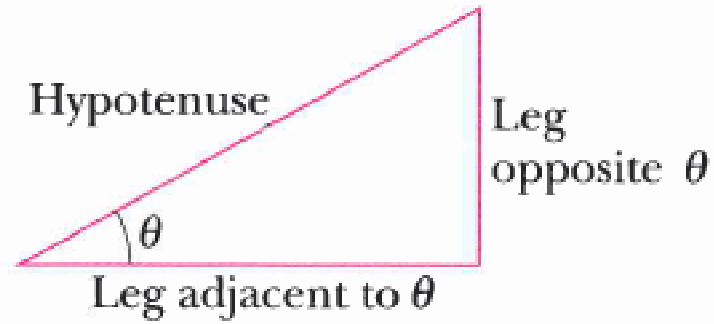
Thus, the airplane is 81 km east and 2.0×10^2 km north of the airport.

common trigonometric functions

$$\sin\theta = \frac{\text{leg opposite }\theta}{\text{hypotenuse}}$$

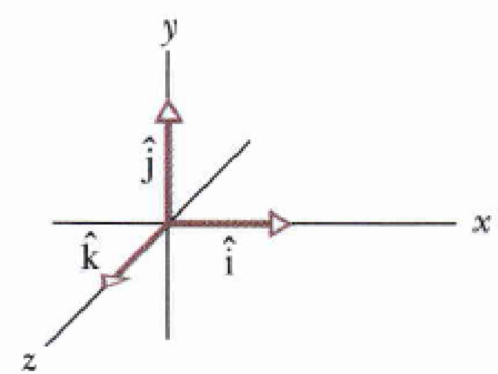
$$\cos\theta = \frac{\text{leg adjacent to }\theta}{\text{hypotenuse}}$$

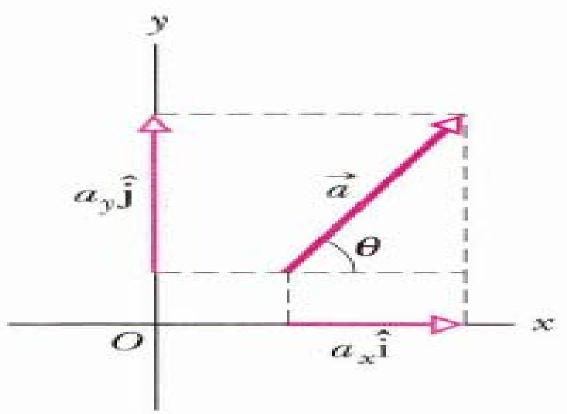
$$\tan\theta = \frac{\text{leg opposite }\theta}{\text{leg adjacent to }\theta}$$



3-5 | Unit Vectors

- They are equal 1 and points in a particular direction
- The unit vectors in the positive directions of the *x, y, and z axes are labeled i, J, and k*
- They are very useful for expressing other vectors





$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j}.$$

3-6 | Adding Vectors by Components

Consider the statement

$$\vec{r} = \vec{a} + \vec{b}$$

$$r_x = a_x + b_x$$
$$r_y = a_y + b_y$$
$$r_z = a_z + b_z$$

This procedure applies also to vector subtractions

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$
$$\vec{d} = d_x\hat{\mathrm{i}} + d_y\hat{\mathrm{j}} + d_z\hat{\mathrm{k}}$$

$$d_x = a_x - b_x, \quad d_y = a_y - b_y, \quad \text{and} \quad d_z = a_z - b_z$$

Sample Problem 3-4

Figure 3-16*a* shows the following three vectors:

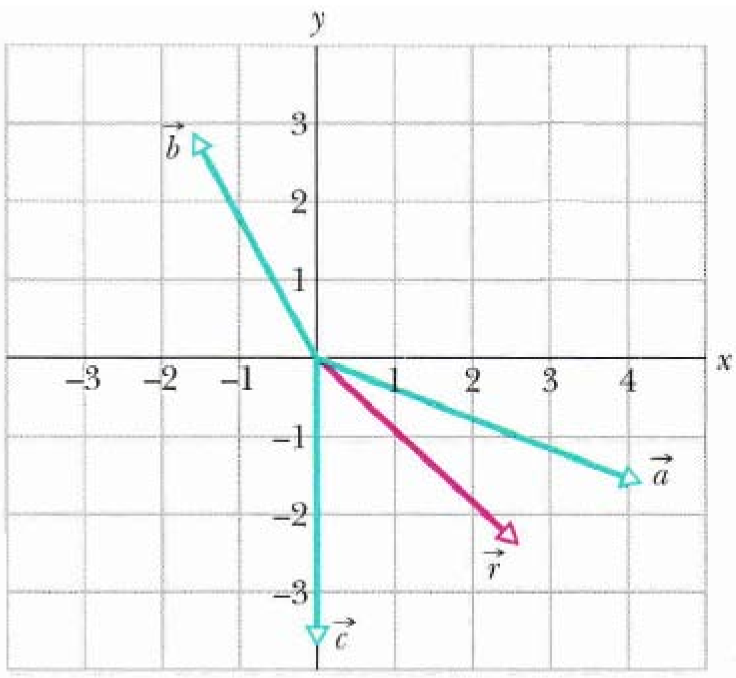
$$\vec{a} = (4.2 \text{ m})\hat{\text{i}} - (1.5 \text{ m})\hat{\text{j}},$$

$$\vec{b} = (-1.6 \text{ m})\hat{\text{i}} + (2.9 \text{ m})\hat{\text{j}},$$

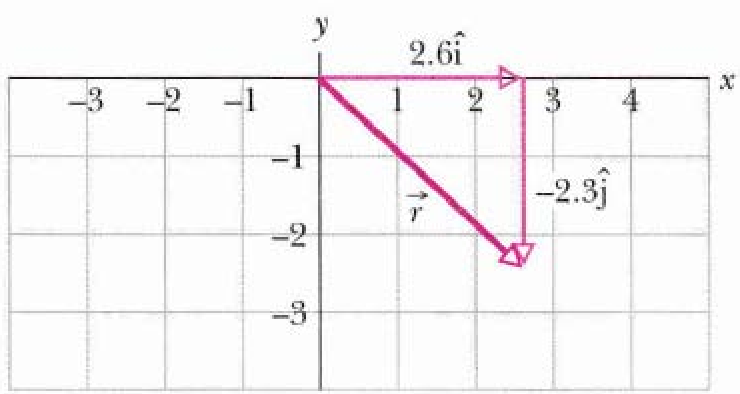
and

$$\vec{c} = (-3.7 \text{ m})\hat{\text{j}}.$$

What is their vector sum $\vec{r}$ which is also shown?



(*a*)



(*b*)

Calculations: For the x axis, we add the x components of $\vec{a}$, $\vec{b}$, and $\vec{c}$, to get the x component of the vector sum $\vec{r}$:

$$r_x = a_x + b_x + c_x$$

$$= 4.2 \text{ m} - 1.6 \text{ m} + 0 = 2.6 \text{ m}.$$

Similarly, for the y axis,

$$r_y = a_y + b_y + c_y$$

$$= -1.5 \text{ m} + 2.9 \text{ m} - 3.7 \text{ m} = -2.3 \text{ m}.$$

$$\vec{r} = (2.6 \text{ m})\hat{\text{i}} - (2.3 \text{ m})\hat{\text{j}}, \qquad \text{(Answer)}$$

$$r = \sqrt{(2.6 \text{ m})^2 + (-2.3 \text{ m})^2} \approx 3.5 \text{ m} \qquad \text{(Answer)}$$

$$\theta = \tan^{-1}\left(\frac{-2.3 \text{ m}}{2.6 \text{ m}}\right) = -41°, \qquad \text{(Answer)}$$

Multiplying Vectors

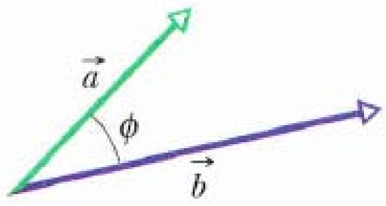
Multiplying a Vector by a Scalar

$$s\vec{a} = \vec{b} \qquad |\vec{b}| = |s||\vec{a}|$$

The Scalar Product : dot product

$$\vec{a}\cdot\vec{b} = ab\cos\phi$$

Scalar quantity



where a is the magnitude of $\vec{a}$, b is the magnitude of $\vec{b}$

ϕ is the angle between $\vec{a}$ and $\vec{b}$

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

commutative law

When two vectors are in unit-vector notation, we write their dot product as

$$\vec{a}\cdot\vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}})\cdot(b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}}),$$

$$= a_xb_x + a_yb_y + a_zb_z$$

Sample Problem

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$?

Calculations

$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

$$a = \sqrt{3.0^2 + (-4.0)^2} = 5.00 \qquad b = \sqrt{(-2.0)^2 + 3.0^2} = 3.61$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (3.0\hat{i} - 4.0\hat{j}) \cdot (-2.0\hat{i} + 3.0\hat{k}) \\ &= (3.0\hat{i}) \cdot (-2.0\hat{i}) + (3.0\hat{i}) \cdot (3.0\hat{k}) \\ &\quad + (-4.0\hat{j}) \cdot (-2.0\hat{i}) + (-4.0\hat{j}) \cdot (3.0\hat{k}) \end{aligned}$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= -(6.0)(1) + (9.0)(0) + (8.0)(0) - (12)(0) \\ &= -6.0. \end{aligned}$$

$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

$$-6.0 = (5.00)(3.61) \cos \phi,$$

$$\phi = \cos^{-1} \frac{-6.0}{(5.00)(3.61)} = 109° \approx 110°$$

The Vector Product **cross product**

The **vector product** of $\vec{a}$ and $\vec{b}$, written $\vec{a} \times \vec{b}$, produces a third vector $\vec{c}$ whose magnitude is

$$c = ab \sin \phi$$

> If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$$ **Not cumulative**

$$\vec{a} \times \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \times (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$$

$$\vec{a} \times \vec{b} = (a_yb_z - b_ya_z)\hat{\mathrm{i}} + (a_zb_x - b_za_x)\hat{\mathrm{j}} + (a_xb_y - b_xa_y)\hat{\mathrm{k}} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$$

Product of unit vectors

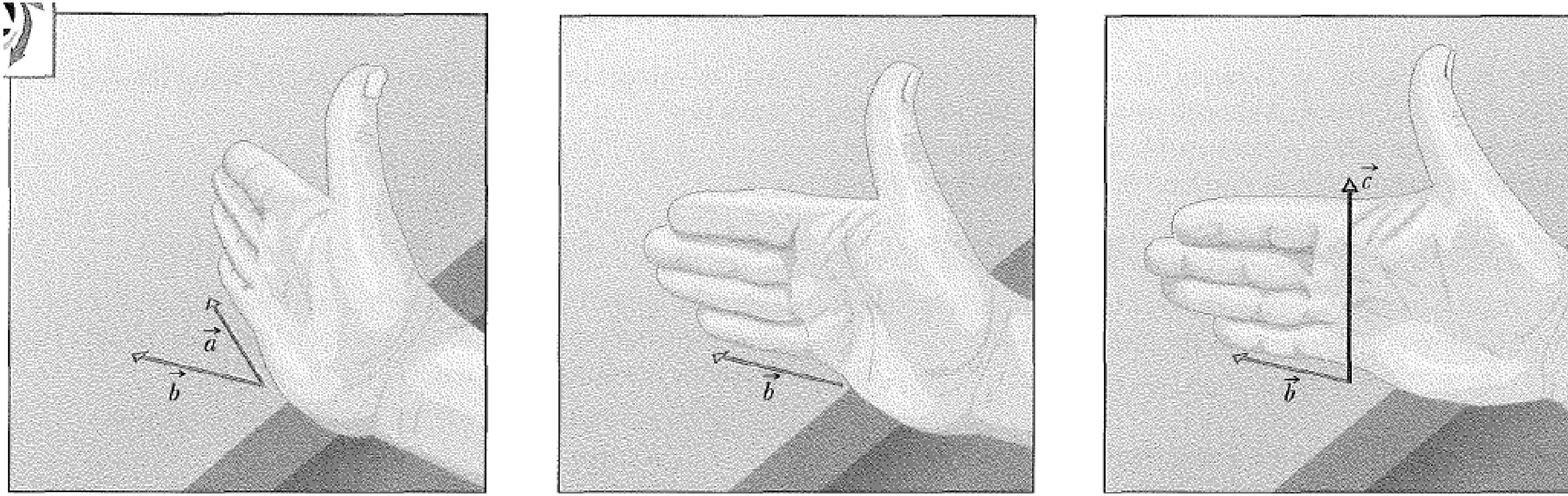
$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{k} = \hat{i} \qquad \hat{k} \times \hat{i} = \hat{j}$$

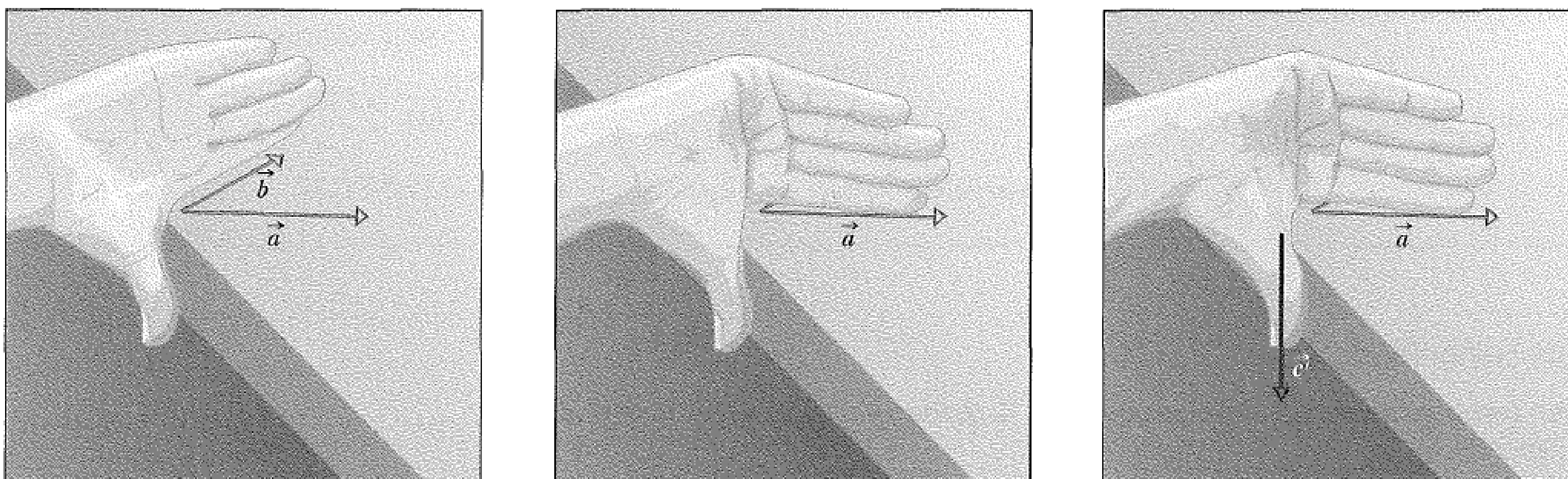
$$\hat{j} \times \hat{i} = -\hat{k} \qquad \hat{k} \times \hat{j} = -\hat{i} \qquad \hat{i} \times \hat{k} = -\hat{j}$$

$$a_x\hat{\mathrm{i}} \times b_x\hat{\mathrm{i}} = a_xb_x(\hat{\mathrm{i}} \times \hat{\mathrm{i}}) = 0 \qquad a_x\hat{\mathrm{i}} \times b_y\hat{\mathrm{j}} = a_xb_y(\hat{\mathrm{i}} \times \hat{\mathrm{j}}) = a_xb_y\hat{\mathrm{k}}$$

right-hand rule The direction of $\vec{c}$ is perpendicular to the plane that contains $\vec{a}$ and $\vec{b}$



(a)



Sample Problem 3-9

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

Calculations: Here we write

$$\begin{aligned}\vec{c} &= (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k}) \\ &= 3\hat{i} \times (-2\hat{i}) + 3\hat{i} \times 3\hat{k} + (-4\hat{j}) \times (-2\hat{i}) \\ &\quad + (-4\hat{j}) \times 3\hat{k}.\end{aligned}$$

$$\begin{aligned}\vec{c} &= -6(0) + 9(-\hat{j}) + 8(-\hat{k}) - 12\hat{i} \\ &= -12\hat{i} - 9\hat{j} - 8\hat{k}. \qquad \text{(Answer)}\end{aligned}$$

Samples of Exam Questions

Logic Questions

Q.21 The scalar product $\hat{i} \cdot \hat{j}$ is equal to:

(A) $\hat{k}$ (B) $2\hat{i}$ (C) $2\hat{j}$ (D) zero (E) $\hat{i}\hat{j}$

$$\hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1 \quad \hat{i} \bullet \hat{j} = \hat{j} \bullet \hat{k} = \hat{i} \bullet \hat{k} = 0$$

Q.13 The result of $\hat{j} \bullet \hat{j}$ is:

(A) $\hat{i}$ (B) $\hat{k}$ (C) $\hat{j}$ (D) Zero (E) 1

Logic Questions

Q.27 The vector product $\hat{j} \times \hat{i}$ is equal to:

(A) $\hat{j}$ (B) $-\hat{i}$ (C) $\hat{k}$ (D) 1 (E) $-\hat{k}$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k} \quad \hat{k} \times \hat{j} = -\hat{i} \quad \hat{i} \times \hat{k} = -\hat{j}$$

Logic Questions

Q.28 The value of $\hat{i} \cdot (\hat{k} \times \hat{j})$ is:

(A) $\hat{j}$ (B) zero (C) $\hat{k}$ (D) -1 (E) 1

$$\hat{i} \bullet (\hat{k} \times \hat{j}) = \hat{i} \bullet (-\hat{i}) = -\hat{i} \bullet \hat{i} = -1$$

Q.15 The result of $(\hat{i} \times \hat{k}) \bullet \hat{j}$ is:

(A) $\hat{i}$ (B) 1 (C) $\hat{j}$ (D) -1 (E) Zero

$$(\hat{i} \times \hat{k}) \bullet \hat{j} = -\hat{j} \bullet \hat{j} = -1$$

Q.20 The result of $(\hat{k} \times \hat{i}) \bullet \hat{j}$ is:

(A) $\hat{i}$ (B) 1 (C) $\hat{j}$ (D) $\hat{k}$ (E) Zero

$$(\hat{k} \times \hat{i}) \bullet \hat{j} = \hat{j} \bullet \hat{j} = 1$$

Q.15 The result of $(\hat{k} \times \hat{j}) \times \hat{i}$ is:

(A) $\hat{i}$ (B) 1 (C) Zero (D) $\hat{k}$ (E) $\hat{j}$

Logic Questions

Q.15 The result of $(\hat{i}\times\hat{j})\times\hat{i}$ is:

(A) $\hat{i}$ (B) 1 (C) Zero (D) $\hat{k}$ (E) $\hat{j}$

$$(\hat{i}\times\hat{j})\times\hat{i} = \hat{k}\times\hat{i} = \hat{j}$$

Q.26 If $\bar{A}\bullet\bar{B}=0$, the angle between the vectors $\bar{A}$ and $\bar{B}$ is: (Hint: $\bar{A}$ and $\bar{B}$ are non-zero vectors)

(A) 180° (B) Zero (C) 90° (D) 315° (E) 45°

If scalar product is zero, the vectors are perpendicular (متعامدين) and the angle between them is 90^0

(A) 45.6°

Q.26 If $\bar{A}\times\bar{B}=0$, the angle between the vectors $\bar{A}$ and $\bar{B}$ should be: (Hint: $\bar{A}$ and $\bar{B}$ are no

(A) 315° (B) 30 (C) 90° (D) 180° (E) 4

$\vec{b}\times\vec{a}$ is:

If cross product is zero, the vectors are parallel (متوازيين) and the angle between them is zero

Vector Components

Q.18 A vector $\vec{A}$ has x-component of 10 m and y-component of 15 m. The magnitude of this vector is:
(A) 14.14 m (B) 18 m (C) 22.36 m (D) 35.12 m (E) 11.18 m

$$A_x = 10\,\text{m} \qquad A_y = 15\,\text{m}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{100 + 225} = \sqrt{325} = 18.02 \approx 18$$

Q.28 The components of vector $\bar{A}$ are given as A_x=5.5 m and A_y=-5.3 m. The magnitude of vector $\bar{A}$ is:
(A) 9.2 m (B) 8.4 m (C) 6.9 m (D) 6.1 m (E) 7.6 m

$$A_x = 5.5\,\text{m} \qquad A_y = -5.3\,\text{m}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(5.5)^2 + (-5.3)^2} = \sqrt{30.25 + 28.09} = \sqrt{58.34} = 7.64 \approx 7.6$$

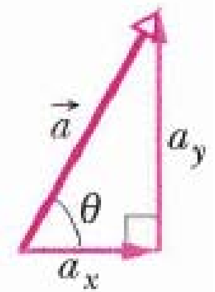
Vector Components

Q.19 A vector has a magnitude of 14 units makes an angle of 30° with the x axis. Its y component is:

A) 8 units (B) 9 units (C) 5 units (D) 6 units (E) 7 units

$$A = 14 \text{ units} \qquad \theta = 30^0$$

$$A_y = A\sin\theta = 14\text{x}\sin 30^0 = 14\text{x}\frac{1}{2} = 7 \text{ units}$$



(c)

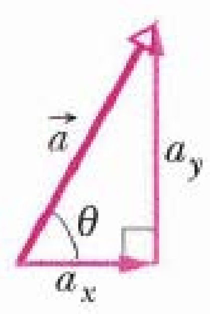
Vector Components

Q.24 If the magnitude of a vector is 18m and its x-component of 10m. The angle it makes with the positive x-axis is:

(A) 48.2° (B) 63.4° (C) 66.4° (D) 60° (E) 56.25°

$A = 18\,\text{m} \qquad A_x = 10\,\text{m}$

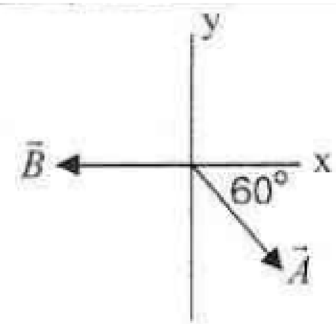
$$A_x = A\cos\theta \;\Rightarrow\; \cos\theta = \frac{A_x}{A} \Rightarrow\; \theta = \cos^{-1}\left(\frac{A_x}{A}\right) = \cos^{-1}\left(\frac{10}{18}\right) = \cos^{-1}(0.555) = 56.25^0$$



(c)

Vector Addition

Q.20 As shown in the figure, if the magnitudes of $\vec{A}$ *and* $\vec{B}$ are 10 units and 15 units respectively then the x-component of the resultant of $\vec{A}$ *and* $\vec{B}$ is:



(A) -10 units (B) -15 units (C) -20units (D) zero (E) -5 units

$$A = 10 \text{ units} \qquad B = 15 \text{ units}$$

$$\vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j} + \left(A_z + B_z\right)\hat{k}$$

$$A_x = A\cos\theta = 10\text{x}\cos(60) = 10\text{x}\frac{1}{2} = 5 \text{ units}$$

$$B_x = -15 \text{ units} \qquad \Rightarrow A_x + B_x = 5 - 15 = -10 \text{ units}$$

Vector Addition

Q.22 if $\vec{A} = 4\hat{i} - 6\hat{j}$ then the vector $\frac{1}{2}\vec{A}$ is:

A) $2\hat{i} - \hat{j}$ (B) $2\hat{i} - 5\hat{j}$ (C $2\hat{i} - 4\hat{j}$ (D) $2\hat{i} - 3\hat{j}$ (E) $2\hat{i} - 2\hat{j}$

A = 4i -6j =====> ½ A = 2i -3j

Vector Addition

Q.23 Two vectors are given as $\vec{A} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = -\hat{i} + \hat{j} + 4\hat{k}$. The result of $\vec{A} - \vec{B}$ is:

(A) $5\hat{i} - 3\hat{j}$ (B) $4\hat{i} - 3\hat{j}$ (C) $3\hat{i} - 3\hat{j}$ (D) $2\hat{i} - 3\hat{j}$ (E) $\hat{i} - 3\hat{j}$

$$A_x = 2 \quad A_y = -2 \quad A_z = 4$$

$$B_x = -1 \quad B_y = 1 \quad B_z = 4$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k}$$

$$= (2 - (-1))\hat{i} + (-2 - 1)\hat{j} + (4 - 4)\hat{k}$$

$$= 3\hat{i} - 3\hat{j}$$

Vector Addition

Q.22 Given $\bar{A} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{B} = 2\hat{i} - 6\hat{j} + 7\hat{k}$, $\bar{C} = 2\hat{i} - \hat{j} + 4\hat{k}$. Then the vector $\bar{D} = 2\bar{A} + \bar{B} - \bar{C}$ is:

(A) $-\hat{i} - 2\hat{j} + 3\hat{k}$ (B) $3\hat{i} + 2\hat{j} - 5\hat{k}$ (C) $3.5\hat{i}$ (D) $4\hat{i} - 3\hat{j} + 9\hat{k}$ (E) $\hat{i} + 2\hat{j} - 5k$

$$A_x = 2 \quad A_y = 1 \quad A_z = 3$$

$$B_x = 2 \quad B_y = -6 \quad B_z = 7$$

$$C_x = 2 \quad C_y = -1 \quad C_z = 4$$

$$\vec{D} = 2\vec{A} + \vec{B} - \vec{C} = (2A_x + B_x - C_x)\hat{i} + (2A_y + B_y - C_y)\hat{j} + (2A_z + B_z - C_z)\hat{k}$$

$$= (2x2 + 2 - 2)\hat{i} + (2x1 - 6 - (-1))\hat{j} + (2x3 + 7 - 4)\hat{k}$$

$$= (4 + 2 - 2)\hat{i} + (2 - 6 + 1)\hat{j} + (6 + 7 - 4)\hat{k}$$

$$= 4\hat{i} - 3\hat{j} + 9\hat{k}$$

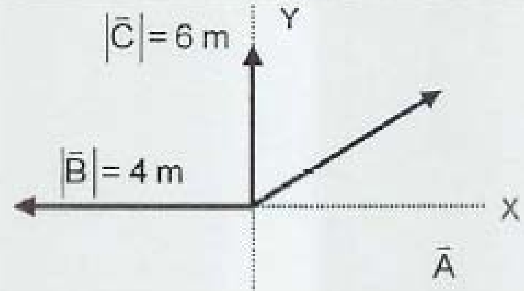
Q.19 Refer to question (18) the angle between the vector $\bar{A}$ and the positive z-axis is:

(A) 36.7° (B) Zero (C) 180° (D) 315° (E) 90°

Vector Addition

Q.19 In figure, if $\vec{A} + \vec{B} - \vec{C} = 4\hat{i}$ then the vector $\vec{A}$ in unit vector notation is:

(A) $4\hat{i} + 2\hat{j}$ (B) $9\hat{i} + 4\hat{j}$ (C) $8\hat{i} + 6\hat{j}$ (D) $5\hat{i} - 4\hat{j}$ (E) $4\hat{i}$



$B_x = -4\text{ m} \qquad B_y = 0$

$C_x = 0 \qquad C_y = 6\text{ m}$

$\vec{A} + \vec{B} - \vec{C} = 4\hat{i} \qquad \Rightarrow \vec{A} = 4\hat{i} - \left(\vec{B} - \vec{C}\right)$

$\vec{B} - \vec{C} = \left(B_x - C_x\right)\hat{i} + \left(B_y - C_y\right)\hat{j} = \left(-4 - 0\right)\hat{i} + \left(0 - 6\right)\hat{j} = -4\hat{i} - 6\hat{j}$

$\vec{A} = 4\hat{i} - \left(\vec{B} - \vec{C}\right) = 4\hat{i} - \left(-4\hat{i} - 6\hat{j}\right) = 4\hat{i} + 4\hat{i} + 6\hat{j} = 8\hat{i} + 6\hat{j}$

Scalar Product

Q.25 If the magnitude of two vectors are 10 units and 20 units and the angle between them is 60° then their scalar product is:

(a) 100 (B) 125 (C) zero (D) 25 (E) 75

$$A = 10 \text{ units} \qquad B = 20 \text{ units} \qquad \varphi = 60^0$$

$$\vec{A} \bullet \vec{B} = AB\cos\varphi = 10x20x\cos 60^0 = 200x\frac{1}{2} = 100$$

Q.26 Two vectors are given as $\vec{A} = 5\hat{j} + 4\hat{k}$ and $\vec{B} = -\hat{i} + \hat{j}$, their scalar product $\vec{A} \cdot \vec{B}$ is:

(A) 4 (B) 5 (C) 6 (D) 7 (E) 3

$$A.B = A_x B_x + A_y B_y + A_z B_z = 0\times(-1) + 5\times1 + 4\times0 = 5 \text{ units}$$

Scalar Product

Q.26 Two vectors are given as $\vec{A} = 5\hat{j} + 4\hat{k}$ and $\vec{B} = -\hat{i} + \hat{j}$, their scalar product $\vec{A} \cdot \vec{B}$ is:

(A) 4 (B) 5 (C) 6 (D) 7 (E) 3

$$A_x = 0 \qquad A_y = 5 \qquad A_z = 4$$

$$B_x = -1 \qquad B_y = 1 \qquad B_z = 0$$

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z = 0x(-1) + 5x1 + 4x0 = 0 + 5 + 0 = 5$$

Scalar Product

Q.24 Given $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, Then $(\bar{a} \cdot \bar{b})$ is:

(A) $3\hat{i} + 4\hat{j} - 5\hat{k}$ (B) 40 (C) 8 (D) $\hat{i} + \hat{j} - 5\hat{k}$ (E) $\hat{i} + 2\hat{j}$

$$A_x = 1 \qquad A_y = 2 \qquad A_z = 3$$

$$B_x = 2 \qquad B_y = -3 \qquad B_z = 4$$

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z = 1x2 + 2x(-3) + 3x4 = 2 - 6 + 12 = 8$$

Scalar Product

(A) 74.5°

Q.24 Given $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Then $\left(5\vec{a} \bullet \vec{b}\right)$ is:

(A) $3\hat{i} + 4\hat{j} - 5\hat{k}$ (B) 40 (C) 8 (D) $\hat{i} + 2\hat{j} - 5\hat{k}$ (E) 60

... then the angle between vector $\vec{c}$ and $\vec{d}$ is:

$$A_x = 1 \qquad A_y = 2 \qquad A_z = 3$$

$$B_x = 2 \qquad B_y = -3 \qquad B_z = 4$$

$$5\vec{A} \bullet \vec{B} = 5A_xB_x + 5A_yB_y + 5A_zB_z = 5x1x2 + 5x2x(-3) + 5x3x4 = 10 - 30 + 60 = 40$$

Scalar Product

Q.25 Given $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{d} = 2\hat{i} - \hat{j} + 4\hat{k}$,then the angle between vector $\vec{c}$ and $\vec{d}$ is:

(A) 45.6° (B) 15° (C) 120° (D) 90° (E)Zero

$$\vec{c} \bullet \vec{d} = c\,d\cos\varphi \quad \Rightarrow \quad \cos\varphi = \frac{\vec{c} \bullet \vec{d}}{cd} \Rightarrow \quad \varphi = \cos^{-1}\left(\frac{\vec{c} \bullet \vec{d}}{cd}\right)$$

$$c_x = 1 \qquad c_y = 2 \qquad c_z = 3$$

$$d_x = 2 \qquad d_y = -1 \qquad d_z = 4$$

$$c = \sqrt{c_x^2 + c_y^2 + c_z^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{4+1+16} = \sqrt{21}$$

$$\vec{c} \bullet \vec{d} = c_x d_x + c_y d_y + c_z d_z = 1x2 + 2x(-1) + 3x4 = 2 - 2 + 12 = 12$$

$$\varphi = \cos^{-1}\left(\frac{12}{\sqrt{14}\sqrt{21}}\right) = \cos^{-1}\left(\frac{12}{\sqrt{294}}\right) = \cos^{-1}\left(\frac{12}{17.15}\right) = 45.6^0$$

Vector Product

Q.30 If the angle between $\vec{A}$ *and* $\vec{B}$ is 30°, and A = 5 units, B = 10 units, then the magnitude of the vector product $\vec{A} \times \vec{B}$ is:

(A) 25 (B) 20 (C) 15 (D) 30 (E) 35

$$A = 5\,\text{units} \qquad B = 10\,\text{units} \qquad \varphi = 30^0$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\varphi = 5x10x\sin 30^0 = 50x\frac{1}{2} = 25 \quad \text{unit}^2$$

Vector Product

Q.21 If $\bar{A}$ and $\bar{B}$ are vectors with magnitudes 5 and 4, respectively, and the magnitude of their cross product is 17.32, then the angle between $\bar{A}$ and $\bar{B}$ is:

(A) 90° (B) 60° (C) 45° (D) 180° (E) 30°

$$A = 5\text{ units} \qquad B = 4\text{ units} \qquad \left|\vec{A}\times\vec{B}\right| = 17.32$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\varphi \quad \Rightarrow \quad \sin\varphi = \frac{\left|\vec{A}\times\vec{B}\right|}{AB} = \frac{17.32}{5x4} = \frac{17.32}{20} \quad \Rightarrow \quad \varphi = \sin^{-1}\frac{17.32}{20} = 60^0$$

Q.21 If $\bar{A}$ and $\bar{B}$ are vectors with magnitudes 5 and 4, respectively, and the magnitude of their cross product is 10, then the angle between $\bar{A}$ and $\bar{B}$ is:

(A) 90° (B) 60° (C) 45° (D) 180° (E) 30°

Vector Product

Q.29 Two vectors $\vec{A} = 8\hat{i} + 6\hat{j}$ and $\vec{B} = -6\hat{i}$, their vector product $\vec{A} \times \vec{B}$ is:

(A) $48\hat{k}$ (B) $30\hat{k}$ (C) $36\hat{k}$ (D) $42\hat{k}$ (E) $48\hat{k}$

$$\mathrm{A}\times\mathrm{B} = A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 6 & 0 \\ -6 & 0 & 0 \end{vmatrix} = 0\hat{i} + 0\hat{j} + (8 \times 0 - 6 \times (-6))\hat{k} = 36\hat{k}$$

Vector Product

Q.27 Given that $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\bar{b} = 2\hat{i} - \hat{j} + 4\hat{k}$,then $\bar{a} \times \bar{b}$ is:

(A) $11\hat{i} + 2\hat{j} - 5\hat{k}$ (B) $-\hat{i} - 2\hat{j} + 3\hat{k}$ (C) $3.5\hat{i}$ (D) 4 (E) $\hat{i} + 2\hat{j} - 5k$

$$a_x = 1 \qquad a_y = 2 \qquad a_z = 3$$

$$b_x = 2 \qquad b_y = -1 \qquad b_z = 4$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{vmatrix} = \hat{i}\left(2x4 - 3x(-1)\right) - \hat{j}\left(1x4 - 3x2\right) + \hat{k}\left(1x(-1) - 2x2\right)$$

$$= \hat{i}\left(8 + 3\right) - \hat{j}\left(4 - 6\right) + \hat{k}\left(-1 - 4\right) = 11\hat{i} + 2\hat{j} - 5\hat{k}$$

Motion in two and three dimensions

Position vector

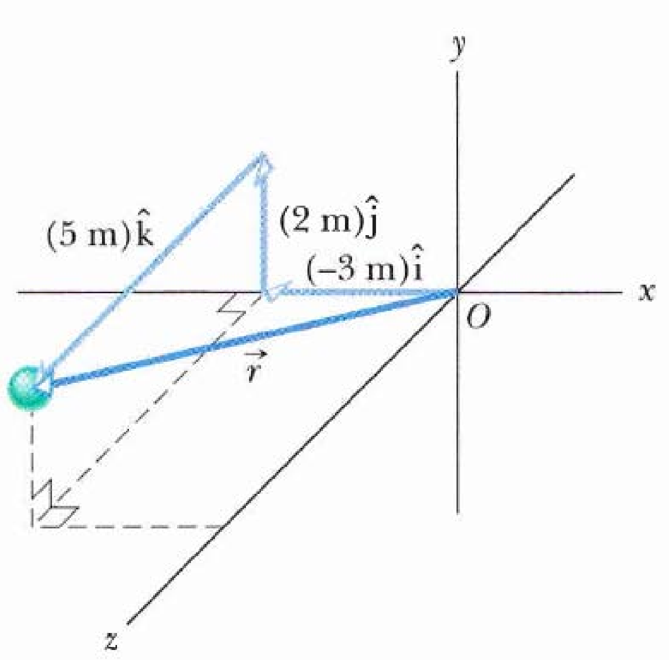
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

If $y = z = 0 \Rightarrow \vec{r} = x\hat{i}$

position in one dimension

Example:

$$\vec{r} = (-3\ \text{m})\hat{i} + (2\ \text{m})\hat{j} + (5\ \text{m})\hat{k}$$



Displacement vector

If the object is displaced from position r_1 to r_2

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

Sample Problem 4-1

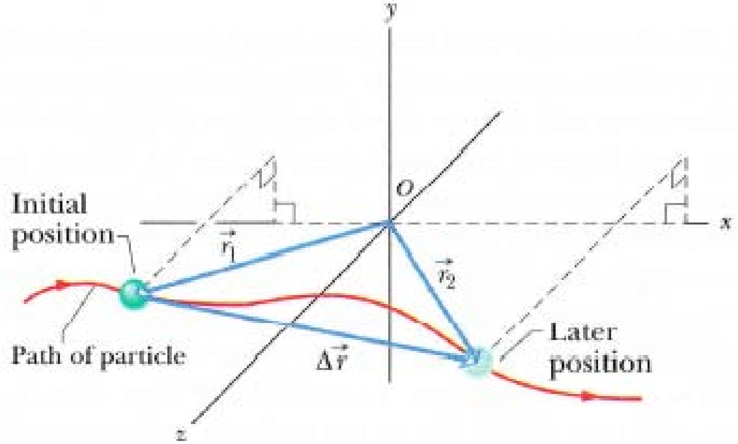
In Fig. 4-2, the position vector for a particle initially is

$$\vec{r}_1 = (-3.0\text{ m})\hat{i} + (2.0\text{ m})\hat{j} + (5.0\text{ m})\hat{k}$$

and then later is

$$\vec{r}_2 = (9.0\text{ m})\hat{i} + (2.0\text{ m})\hat{j} + (8.0\text{ m})\hat{k}.$$

What is the particle's displacement $\Delta\vec{r}$ from $\vec{r}_1$ to $\vec{r}_2$?



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$
$$= [9.0 - (-3.0)]\hat{i} + [2.0 - 2.0]\hat{j} + [8.0 - 5.0]\hat{k}$$
$$= (12\text{ m})\hat{i} + (3.0\text{ m})\hat{k}. \qquad \text{(Answer)}$$

Displacement

1. A particle goes from $x = -2$ m, $y = 3$ m, $z = 1$ m to $x = 3$ m, $y = -1$ m, $z = 4$ m. Its displacement is:

a) $(1\text{ m})\hat{i} + (2\text{ m})\hat{j} + (5\text{ m})\hat{k}$

b) $(5\text{ m})\hat{i} - (4\text{ m})\hat{j} + (3\text{ m})\hat{k}$

c) $-(5\text{ m})\hat{i} + (4\text{ m})\hat{j} - (3\text{ m})\hat{k}$

d) $-(5\text{ m})\hat{i} - (2\text{ m})\hat{j} = (3\text{ m})\hat{k}$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (3\text{m} - (-2\text{m}))\hat{\text{i}} + (-1\text{m} - 3\text{m})\hat{\text{j}} + (4\text{m} - 1\text{m})\hat{\text{k}} = 5\text{ m }\hat{\text{i}} - 4\text{ m }\hat{\text{j}} + 3\text{ m }\hat{\text{k}}$$

Q.3 A particle moving from $\bar{r}_1 = 2\hat{i} + 5\hat{j} + 8\hat{k}$ *to* $\bar{r}_2 = 12\hat{i} + 10\hat{j} + 8\hat{k}$ then the displacement is:

(A) $10\hat{i} - 3\hat{j}$ (B) $4\bar{j} + 6\bar{k}$ (C) $\underline{10\hat{i} + 5\hat{j}}$ (D) $5\hat{j}$ (E) 8

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= (12 - 2)\hat{i} + (10 - 5)\hat{j} + (8 - 8)\hat{k} = 10\hat{i} + 5\hat{j} + 0\hat{k} = 10\hat{i} + 5\hat{j}$$

Q.3 A particle moving from $\bar{r}_1 = 2\hat{i} + 5\hat{j} - 12\hat{k}$ to $\bar{r}_2 = 2\hat{i} + 5\hat{j} - 8\hat{k}$ then the displacement is:

(A) $10\hat{i} - 3\hat{j}$ (B) $4\bar{j} + 6\bar{k}$ (C) $10\hat{i} + 5\hat{j}$ (D) $5\hat{j}$ (E) $\underline{4\hat{k}}$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= (2 - 2)\hat{i} + (5 - 5)\hat{j} + (-8 - (-12))\hat{k} = 0\hat{i} + 0\hat{j} + 4\hat{k} = 4\hat{k}$$

Average Velocity

$$\vec{v}_{avg} = \frac{\text{displacement}}{\text{interval time}} = \frac{\Delta\vec{r}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$= v_{avg(x)}\hat{i} + v_{avg(y)}\hat{j} + v_{avg(z)}\hat{k}$$

Instantaneous Velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

Average acceleration

$$\vec{a}_{avg} = \frac{\text{change in veleocity}}{\text{interval time}} = \frac{\Delta\vec{v}}{\Delta t}$$

$$= \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \frac{\Delta v_z}{\Delta t}\hat{k}$$

$$= a_{avg(x)}\hat{i} + a_{avg(y)}\hat{j} + a_{avg(z)}\hat{k}$$

Instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

If

Sample Problem 4-2

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \qquad (4\text{-}5)$$

and

$$y = 0.22t^2 - 9.1t + 30. \qquad (4\text{-}6)$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

$$\vec{r} = x\hat{i} + y\hat{j}$$
$$= \left(-0.31t^2 + 7.2t - 28\right)\hat{i} + \left(0.22t^2 - 9.1t + 30\right)\hat{j}$$

$$\vec{r}(t = 15\text{s}) = \left(-0.31\text{x}15^2 + 7.2\text{x}15 - 28\right)\hat{i} + \left(0.22\text{x}15^2 - 9.1\text{x}15 + 30\right)\hat{j}$$
$$= (66\text{ m})\hat{i} + (-57\text{ m})\hat{j}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{66^2 + (-57)^2} = 87\text{ m}$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-57}{66} = -41^o$$

(b) Find the velocity of the rabbit at the instant t=15 s?

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(-0.31t^2 + 7.2t + 28\right) = -0.62t + 7.2$$

$$v_x(t = 15\,\text{s}) = -0.62\text{x}15 + 7.2 = -2.1\,\text{m/s}$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(0.22t^2 - 9.1t + 30\right) = 0.44t - 9.1$$

$$v_y(t = 15\,\text{s}) = 0.44\text{x}15 - 9.1 = -2.5\,\text{m/s}$$

$$\vec{v} = (-2.1\text{ m/s})\hat{\text{i}} + (-2.5\text{ m/s})\hat{\text{j}}, \quad \text{(Answer)}$$

$$\vec{v} = \left(-2.1\,\text{m}\right)\hat{i} + \left(-2.1\,\text{m}\right)\hat{j}$$

Sample Problem 4-4

For the rabbit in Sample Problems 4-2 and 4-3, find the acceleration $\vec{a}$ at time $t = 15$ s.

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j}$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 \text{ m/s}^2.$$

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2.$$

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{i} + (0.44 \text{ m/s}^2)\hat{j}, \text{ (Answer)}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.62 \text{ m/s}^2)^2 + (0.44 \text{ m/s}^2)^2}$$
$$= 0.76 \text{ m/s}^2. \quad \text{(Answer)}$$

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\left(\frac{0.44 \text{ m/s}^2}{-0.62 \text{ m/s}^2}\right) = -35°.$$

$$-35° + 180° = 145°. \quad \text{(Answer)}$$

•1 A positron undergoes a displacement $\Delta\vec{r} = 2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k}$, ending with the position vector $\vec{r} = 3.0\hat{j} - 4.0\hat{k}$, in meters. What was the positron's initial position vector?

1. The initial position vector $\vec{r}_o$ satisfies $\vec{r} - \vec{r}_o = \Delta\vec{r}$, which results in

$$\vec{r}_o = \vec{r} - \Delta\vec{r} = (3.0\hat{j} - 4.0\hat{k})\text{m} - (2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k})\text{m} = (-2.0\text{ m})\hat{i} + (6.0\text{ m})\hat{j} + (-10\text{ m})\hat{k}.$$

•5 An ion's position vector is initially $\vec{r} = 5.0\hat{i} - 6.0\hat{j} + 2.0\hat{k}$, and 10 s later it is $\vec{r} = -2.0\hat{i} + 8.0\hat{j} - 2.0\hat{k}$, all in meters. In unit-vector notation, what is its $\vec{v}_{avg}$ during the 10 s?

$$\vec{v}_{avg} = \frac{(-2.0\hat{i} + 8.0\hat{j} - 2.0\hat{k})\ \text{m} - (5.0\hat{i} - 6.0\hat{j} + 2.0\hat{k})\ \text{m}}{10\ \text{s}} = (-0.70\hat{i} + 1.40\hat{j} - 0.40\hat{k})\ \text{m/s}.$$

•11 A particle moves so that its position (in meters) as a function of time (in seconds) is $\vec{r} = \hat{\mathrm{i}} + 4t^2\hat{\mathrm{j}} + t\hat{\mathrm{k}}$. Write expressions for (a) its velocity and (b) its acceleration as functions of time. SSM

(a) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\mathrm{i}} + 4t^2\,\hat{\mathrm{j}} + t\,\hat{\mathrm{k}}) = 8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}\,.$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}\,(8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}) = 8\,\hat{\mathrm{j}}\,.$$

Velocity & Acceleration

Q.10 A particle moves in xy plane as x(t)= 2t (m) and y(t)=t^2-1 (m). The velocity of the particle at t=1 s is:

(A) $\hat{i}+\hat{j}$ (m/s) (B) $2\hat{i}+\hat{j}$ (m/s) (C) $2\hat{i}+2\hat{j}$ (m/s) (D) $2\hat{i}-\hat{j}$ (m/s) (E) 10 (m/s)

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$v_x = \frac{dx}{dt} = 2\text{ m/s} \qquad v_x = \frac{dy}{dt} = 2t\ \text{ m/s}$$

$$v_x(t=1s) = 2\text{ m/s} \qquad v_y(t=1s) = 2\text{ m/s}$$

$$\vec{v} = 2\hat{i} + 2\hat{j}\ \text{ (m/s)}$$

Q.10 A particle moves in xy plane as x(t)= 2t (m) and y(t)=t^2-1 (m). The velocity of the particle at t=2 s is:

(A) $2\hat{i}+4\hat{j}$ (m/s) (B) $2\hat{i}+\hat{j}$ (m/s) (C) $2\hat{i}+2\hat{j}$ (m/s) (D) $2\hat{i}-\hat{j}$ (m/s) (E) 10 (m/s)

$$v_x = \frac{dx}{dt} = 2\text{ m/s} \qquad v_x = \frac{dy}{dt} = 2t\ \text{ m/s}$$

$$v_x(t=2s) = 2\text{ m/s} \qquad v_y(t=2s) = 4\text{ m/s}$$

$$\vec{v} = 2\hat{i} + 4\hat{j}\ \text{ (m/s)}$$

Sample Problem 4-5

A particle with velocity $\vec{v}_0 = -2.0\hat{i} + 4.0\hat{j}$ (in meters per second) at $t = 0$ undergoes a constant acceleration $\vec{a}$ of magnitude $a = 3.0$ m/s^2 at an angle $\theta = 130°$ from the positive direction of the x axis. What is the particle's velocity $\vec{v}$ at $t = 5.0$ s?

In these equations, v_{0x} ($= -2.0$ m/s) and v_{0y} ($= 4.0$ m/s)

$$a_x = a\cos\theta = (3.0\ \text{m/s}^2)(\cos 130°) = -1.93\ \text{m/s}^2,$$

$$a_y = a\sin\theta = (3.0\ \text{m/s}^2)(\sin 130°) = +2.30\ \text{m/s}^2.$$

$$v = v_0 + at$$

$$v_x = -2.0\ \text{m/s} + (-1.93\ \text{m/s}^2)(5.0\ \text{s}) = -11.65\ \text{m/s},$$

$$v_y = 4.0\ \text{m/s} + (2.30\ \text{m/s}^2)(5.0\ \text{s}) = 15.50\ \text{m/s}.$$

$$\vec{v} = (-12\ \text{m/s})\hat{i} + (16\ \text{m/s})\hat{j}. \qquad \text{(Answer)}$$

$$v = \sqrt{v_x^2 + v_y^2} = 19.4 \approx 19\ \text{m/s} \qquad \text{(Answer)}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = 127° \approx 130°. \qquad \text{(Answer)}$$

Constant Acceleration

Q.9 At t=0, a car moves with velocity $\vec{v}_0 = 2\hat{i} + \hat{j}$ (m/s) and acceleration $\vec{a} = 2\hat{j}$ (m/s^2). The velocity of the car at t=2 s is:

(A) $6\hat{i} + \hat{j}$ (B) $2\hat{i} + 5\hat{j}$ (C) $2\hat{i} + \hat{j}$ (D) $\hat{i} + 5\hat{j}$ (E) 1

$$v_0 = 2\hat{i} + \hat{j} \qquad a = 2\hat{j} \text{ m/s} \qquad t = 2\text{ s} \qquad \vec{v} = ? \qquad \vec{r} = missed$$

$$v_{0x} = 2 \quad v_{0y} = 1 \qquad a_x = 0 \quad a_y = 2$$

$$v_x = v_{0x} + a_x t = 2 + 0\text{x}2 = 2 \qquad v_y = v_{0y} + a_y t = 1 + 2\text{x}2 = 5$$

$$\vec{v} = v_{0x}\hat{i} + v_{0x}\hat{j} = 2\hat{i} + 5\hat{j}$$

Projectile Motion

The projectile is launched at initial velocity

$$\vec{v}_0 = v_{0x}\hat{i} + \vec{v}_{0y}\hat{j}$$

$$v_{0x} = v_0 \cos\theta_0 \qquad v_{0y} = v_0 \sin\theta_0$$

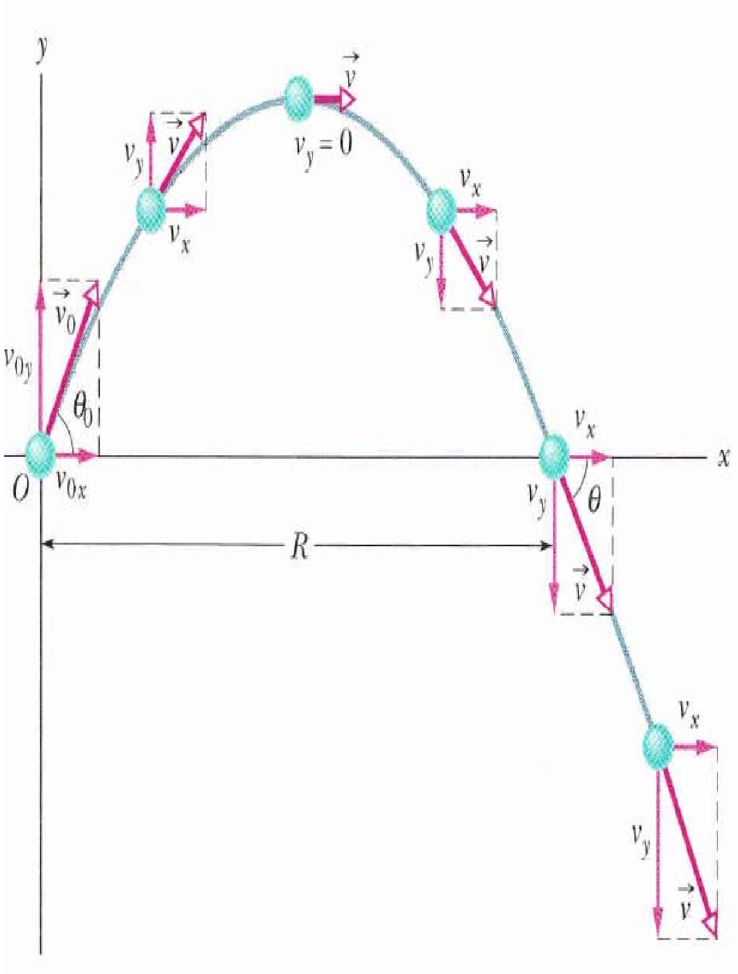
The Horizontal Motion

$$x - x_0 = v_{0x}t.$$

Because there is *no acceleration* in the horizontal direction

$$x\text{-}x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x\text{-}x_0 = v_{0x}t = (v_0 \cos\theta_0)t$$



Projectile Motion

The Vertical Motion

$$y\text{-}y_0 = v_{0y}t - \frac{1}{2}gt^2 = v_0 \sin\theta_0 t - \frac{1}{2}gt^2$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2gt(v_0 \sin\theta_0) + g^2t^2 = (v_0 \sin\theta_0)^2 - 2g\left(t(v_0 \sin\theta_0) - \frac{1}{2}gt^2\right)$$
$$= (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

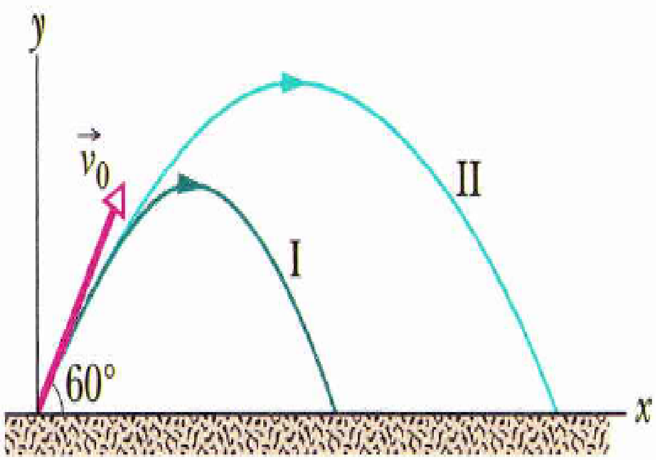
Equation of path (trajectory)

$$y = (\tan\theta_x)x - \frac{gx^2}{2(v_0 \cos\theta_0)^2}$$ **Converted parabola**

The Horizontal Range

$$x\text{-}x_0 = (v_0 \cos\theta_0)t \qquad y\text{-}y_0 = v_0 \sin\theta_0 t - \frac{1}{2}gt^2$$

put $x\text{-}x_0 = R$ put $y\text{-}y_0 = R$



$$R = (v_0 \cos\theta_0)t$$

and

$$0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2.$$

Eliminating t between these two equations yields

$$R = \frac{2v_0^2}{g}\sin\theta_0\cos\theta_0.$$

Using the identity $\sin 2\theta_0 = 2\sin\theta_0\cos\theta_0$ (see Appendix E), we obtain

$$R = \frac{v_0^2}{g}\sin 2\theta_0.$$

The horizontal range R is maximum for a launch angle of 45°.

Sample Problem 4-7

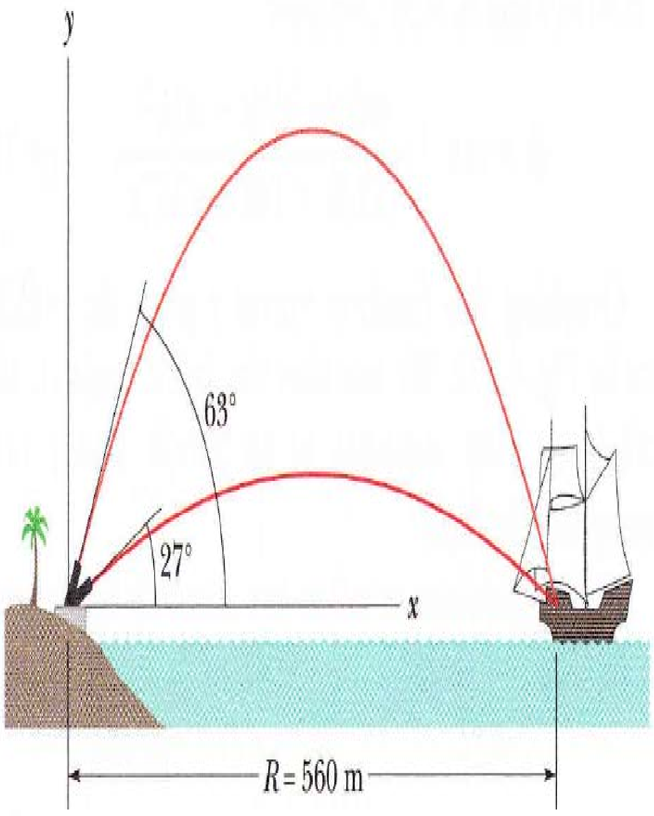
Figure 4-16 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?

$$(R = (v_0^2/g)\sin 2\theta_0) \qquad \sin 2\theta_0 = \frac{gR}{v_0^2}$$

$$\theta_0 = \frac{1}{2}\sin^{-1}\frac{gR}{v_0^2} = \frac{1}{2}\sin^{-1}\frac{(9.8\text{ m/s}^2)(560\text{ m})}{(82\text{ m/s})^2}$$

$$= \frac{1}{2}\sin^{-1} 0.816. \qquad (4\text{-}31)$$



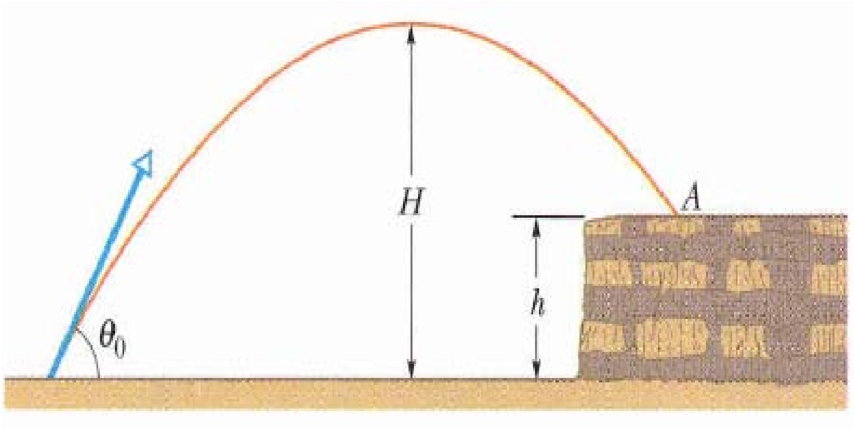
(b) What is the maximum range of the cannonballs?

$$R = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{(82\text{ m/s})^2}{9.8\text{ m/s}^2}\sin(2 \times 45°)$$

$$= 686\text{ m} \approx 690\text{ m}. \qquad \text{(Answer)}$$

Exercise

•26 In Fig. 4-36, a stone is projected at a cliff of height h with an initial speed of 42.0 m/s directed at angle $\theta_0 = 60.0°$ above the horizontal. The stone strikes at A, 5.50 s after launching. Find (a) the height h of the cliff, (b) the speed of the stone just before impact at A, and (c) the maximum height H reached above the ground.



$$(a)\quad h = y\text{-}y_0 = v_0 \sin\theta_0 t - \frac{1}{2}gt^2 = 42\text{x}\sin 60\text{x}5.5 - \frac{1}{2}\text{x}9.8\text{x}(5.5)^2 = **\text{ m}$$

$$(b)\quad v_x = v_0 \cos\theta_0 \qquad v_y = v_0 \sin\theta_0 - gt$$

$$v = \sqrt{(v_0\cos\theta_0)^2 + (v_0\sin\theta_0 - gt)^2} = \sqrt{(42\cos 60)^2 + (42\sin 60 - 9.8\text{x}5.5)^2} = 27\text{ m/s}$$

$$(c)\quad v_y = 0 \qquad \text{y - y}_0 = \text{H}$$

$$v_y^2 = (v_0\sin\theta_0)^2 - 2g(y - y_0) \;\Rightarrow\; 0 = (v_0\sin\theta_0)^2 - 2gH$$

$$H = \frac{(v_0\sin\theta_0)^2}{2g} = \frac{(42\text{x}\sin 60)^2}{2\text{x}9.8} = 67.5\text{ m}$$

Exercise

•21 A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air? (b) At what horizontal distance from the firing point does it strike the ground? (c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

$\theta_0 = 0$ motion is downward $[-(\mathrm{y} - \mathrm{y}_0) = -\mathrm{h}]$

(a)

$$y\text{-}y_0 = v_0 \sin\theta_0 t - \frac{1}{2} g t^2 \quad \Rightarrow \quad -h = -\frac{1}{2} g t^2 \quad \Rightarrow \quad \mathrm{t} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\mathrm{x}45}{9.8}} = 3.03\,\mathrm{s}$$

(b)

$$x\text{-}x_0 = v_0 t = 250\mathrm{x}3.03 = 758\,\mathrm{m}$$

(c)

$$v_y = v_0 \sin\theta_0 - gt = -gt \quad \Rightarrow \quad \left|v_y\right| = gt = 9.8\mathrm{x}3.03 = 29.7\,\mathrm{m/s}$$

•27 A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0°$ below the horizontal when the pilot releases a radar decoy (Fig. 4-37). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700$ m. (a) How long is the decoy in the air? (b) How high was the release point? ILW

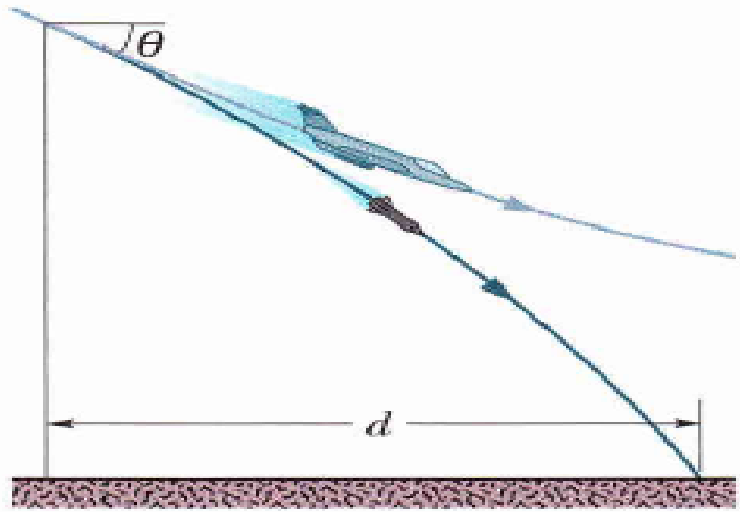


FIG. 4-37 Problem 27.

$$(a)\quad v0 = 290\ \text{km/h} = 290\ \text{km/h}\ \frac{5\ \text{m/s}}{18\ \text{km/h}} = 80.6\ \text{m/s}$$

$$\Delta x = x\text{-}x_0 = \left(v_0 \cos\theta_0\right)t \quad \Rightarrow \quad \text{t} = \frac{\Delta x}{v_0 \cos\theta_0} = \frac{700}{80.6\cos(-30)} = 10\ \text{s}$$

$$(b)\quad y\text{-}y_0 = v_0 \sin\theta_0 t - \frac{1}{2} g t^2 \quad \Rightarrow \quad -h = v_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

$$-h = -80.6\text{x}\sin(-30)\text{x}10 - \frac{1}{2}\text{x}9.8\text{x}10^2 \quad \Rightarrow \quad \text{h} = 897\ \text{m}$$

Projectile

Q.5 A boy kicks a ball at an angle of 40º to the horizontal with a speed of 14.0 m/s. The time it takes to reach the highest point is:

(A) 0.92 s (B) 0.77 s (C) 0.15 s (D) 1.12 s (E) 0.38 s

$$\theta_0 = 40^o \quad \mathrm{v}_0 = 14\,\mathrm{m/s} \quad \mathrm{v}_\mathrm{y} = 0$$

$$v_\mathrm{y} = v_0 \sin\theta_0 - gt \quad \Rightarrow \quad 0 = v_0 \sin\theta_0 - gt$$

$$\Rightarrow \quad \mathrm{t} = \frac{v_0 \sin\theta_0}{g} = \frac{14\sin 40}{9.8} = 0.92\,\mathrm{s}$$

Q.6 Referring to question 5, the maximum height that the ball can reach is:

(A) 9.87 m (B) 4.13 m (C) 15.33 m (D) 12.68 m (E) 14.0 m

$$v_y = 0 \qquad \mathrm{y - y_0 = H}$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0) \Rightarrow 0 = (v_0 \sin\theta_0)^2 - 2gH$$

$$H = \frac{(v_0 \sin\theta_0)^2}{2g} = \frac{(14\mathrm{x}\sin 40)^2}{2\mathrm{x}9.8} = \frac{(14\mathrm{x}0.643)^2}{2\mathrm{x}9.8} = 4.13\,\mathrm{m}$$

Q.7 Referring to question 5, the horizontal range that the ball can reach is:

(A) 9.87 m (B) 14.7 m (C) 15.33 m (D) 12.68 m (E) 19.7 m

$$\mathrm{R} = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{14\mathrm{x}14}{9.8}\sin 2\mathrm{x}40 = \frac{196}{9.8}\sin 80 = 19.7\,\mathrm{m}$$

Q.5 A boy kicks a ball at an angle of 30° to the horizontal with a speed of 14.0 m/s. The time it takes to reach the highest point is:

(A) 0.92 s (B) 0.71 s (C) 0.15 s (D) 1.12 s (E) 0.38 s

Q.6 Referring to question 5, the maximum height that the ball can reach is:

(A) 9.87 m (B) 4.13 m (C) 15.33 m (D) 12.68 m (E) 2.5 m

Q.7 Referring to question 5, the horizontal range that the ball can reach is:

(A) 17.32 m (B) 19.7 m (C) 15.33 m (D) 12.68 m (E) 14.0 m

(1) $\theta_0 = 30^o \quad \mathrm{v}_0 = 14\ \mathrm{m/s} \quad \mathrm{v}_y = 0$

$$v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \quad 0 = v_0 \sin\theta_0 - gt \Rightarrow \quad \mathrm{t} = \frac{v_0 \sin\theta_0}{g} = \frac{14 \sin 30}{9.8} = 0.71\,\mathrm{s}$$

(2) $$H = \frac{(v_0 \sin\theta_0)^2}{2g} = \frac{(14\mathrm{x} \sin 30)^2}{2\mathrm{x}9.8} = 2.5\,\mathrm{m}$$

(3) $$\mathrm{R} = \frac{v_0^2}{g} \sin 2\theta_0 = \frac{14\mathrm{x}14}{9.8} \sin 2\mathrm{x}30 = \frac{196}{9.8} \sin 60 = 17.32\,\mathrm{m}$$

Q.5 A boy kicks a ball at an angle of 35° to the horizontal with a speed of 14.0 m/s. The time it takes to reach the highest point is:

(A) 0.92 s (B) 0.71 s (C) 0.15 s (D) 0.82 s (E) 0.38 s

Q.6 Referring to question 5, the maximum height that the ball can reach is:

(A) 9.87 m (B) 4.13 m (C) 3.29 m (D) 12.68 m (E) 2.5 m

Q.7 Referring to question 5, the horizontal range that the ball can reach is:

(A) 17.32 m (B) 18.79 m (C) 15.33 m (D) 12.68 m (E) 14.0 m

(1) $\theta_0 = 35^o \quad v_0 = 14\text{ m/s} \quad v_y = 0$

$$v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \quad 0 = v_0 \sin\theta_0 - gt \Rightarrow \quad t = \frac{v_0 \sin\theta_0}{g} = \frac{14 \sin 35}{9.8} = 0.82\text{ s}$$

(2) $$H = \frac{(v_0 \sin\theta_0)^2}{2g} = \frac{(14\text{x} \sin 35)^2}{2\text{x}9.8} = 3.29\text{ m}$$

(3) $$R = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{14\text{x}14}{9.8}\sin 2\text{x}35 = \frac{196}{9.8}\sin 70 = 18.79\text{ m}$$

Q.11 A projectile is launched to achieve a maximum range of 140 m, the speed of the projectile must be:

(A) 17 m/s (B) 27 m/s (C) 45 m/s (D) 37 m/s (E) 10 m/s

$$R = \frac{v_0^2}{g}\sin 2\theta_0 \quad \Rightarrow \quad R_{max} = \frac{v_0^2}{g} \quad \Rightarrow v_0 = \sqrt{gR_{max}} = \sqrt{9.8\text{x}140} = 37 \text{ m/s}$$

Q.11 A projectile is launched to achieve a maximum range of 100 m, the speed of the projectile must be:

(A) 17 m/s (B) 31.3 m/s (C) 37 m/s (D) 45 m/s (E) 10 m/s

$$R = \frac{v_0^2}{g}\sin 2\theta_0 \quad \Rightarrow \quad R_{max} = \frac{v_0^2}{g} \quad \Rightarrow v_0 = \sqrt{gR_{max}} = \sqrt{9.8\text{x}100} = 31.3 \text{ m/s}$$

Q.11 A projectile is launched to achieve a maximum range of 150 m, the speed of the projectile must be:

(A) 17 m/s (B) 27 m/s (C) 37 m/s (D) 38.34 m/s (E) 10 m/s

$$R = \frac{v_0^2}{g}\sin 2\theta_0 \quad \Rightarrow \quad R_{\max} = \frac{v_0^2}{g} \quad \Rightarrow v_0 = \sqrt{gR_{\max}} = \sqrt{9.8\text{x}150} = 38.34\text{ m/s}$$

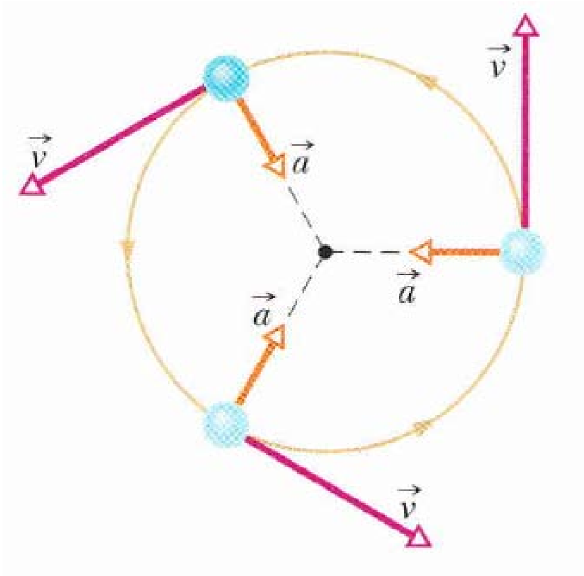
Q.11 A projectile is launched to achieve a maximum range of 120 m, the speed of the projectile must be:

(A) 17 m/s (B) 31.3 m/s (C) 37 m/s (D) 45 m/s (E) 34.29 m/s

$$R = \frac{v_0^2}{g}\sin 2\theta_0 \quad \Rightarrow \quad R_{\max} = \frac{v_0^2}{g} \quad \Rightarrow v_0 = \sqrt{gR_{\max}} = \sqrt{9.8\text{x}120} = 34.29\text{ m/s}$$

Uniform Circular Motion

$$ma = m\frac{v^2}{r}$$ **Centripetal force**



$$a = \frac{v^2}{r} \text{ : centripetal acceleration}$$

force & acceleration are perpendicular to velocity

to the center **tangent**

$$\text{Rotation period } T = \frac{2\pi r}{v}$$

$$\text{Angular velocity } \omega = \frac{2\pi}{T} = \frac{v}{r}$$

Circular Motion

Q.15 The velocity and acceleration of a body in a uniform circular motion are:
(A) differed by 45^0 (B) perpendicular (C) differed by 135^0 (D) parallel (E) none of these

Q.24 A car rounds a 20 m radius curve at 10 m/s. The magnitude of its acceleration is:
(A) Zero (B) $5\ m/s^2$ (C) $2\ m/s^2$ (D) $4\ m/s^2$ (E) $6\ m/s^2$

$$a = \frac{v^2}{r} = \frac{10^2}{20} = 5\ \mathrm{m/s^2}$$

2. A projectile is fired over level ground with an initial velocity that has a vertical component of 20 m/s and a horizontal component of 30 m/s. The distance from launching to landing points is:

a) 40 m
b) 60 m
c) 80 m
d) 122.5 m

•6 An electron's position is given by $\vec{r} = 3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}$, with t in seconds and $\vec{r}$ in meters. (a) In unit-vector notation, what is the electron's velocity $\vec{v}(t)$? At $t = 2.00$ s, what is $\vec{v}$ (b) in unit-vector notation and as (c) a magnitude and (d) an angle relative to the positive direction of the x axis?

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for dx/dt.

(a) Eq. 4-10 leads to

$$v(t) = \frac{d}{dt}\left(3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}\right) = (3.00 \text{ m/s})\hat{i} - (8.00t \text{ m/s})\hat{j}$$

(b) Evaluating this result at $t = 2.00$ s produces $\vec{v} = (3.00\hat{i} - 16.0\hat{j})$ m/s.

(c) The speed at $t = 2.00$ s is $v = |\vec{v}| = \sqrt{(3.00 \text{ m/s})^2 + (-16.0 \text{ m/s})^2} = 16.3$ m/s.

(d) The angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-16.0 \text{ m/s}}{3.00 \text{ m/s}}\right) = -79.4° \text{ or } 101°$$

••17 A particle leaves the origin with an initial velocity $\vec{v} = (3.00\hat{i})$ m/s and a constant acceleration $\vec{a} = (-1.00\hat{i} - 0.500\hat{j})$ m/s^2. When it reaches its maximum x coordinate, what are its (a) velocity and (b) position vector? SSM ILW

(a) The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. The x component is $v_x = v_{0x} + a_x t = 3.00 - 1.00t$, and the y component is

$$v_y = v_{0y} + a_y t = -0.500t$$

since $v_{0y} = 0$. When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.00 - 1.00t_m = 0$ or $t_m = 3.00$ s. The y component of the velocity at this time is

$$v_y = 0 - 0.500(3.00) = -1.50 \text{ m/s};$$

this is the only nonzero component of $\vec{v}$ at t_m.

(b) Since it started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. At $t = t_m$ this becomes

$$\vec{r} = \left(3.00\hat{i}\right)(3.00) + \frac{1}{2}\left(-1.00\hat{i} - 0.50\hat{j}\right)(3.00)^2 = (4.50\hat{i} - 2.25\hat{j}) \text{ m}.$$

Newton's First Law

Newton's First Law: If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.

In other words, if the body is at rest, it stays at rest. If it is moving, it continues to move with the same velocity (same magnitude *and* same direction).

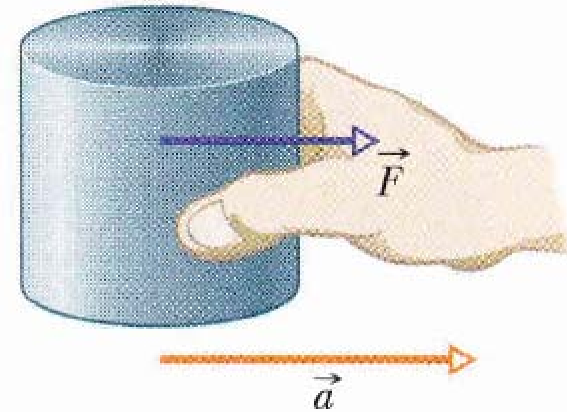


FIG. 5-1 A force $\vec{F}$ on the standard kilogram gives that body an acceleration $\vec{a}$.

Newton's First Law: If no *net* force acts on a body ($\vec{F}_{net} = 0$), the body's velocity cannot change; that is, the body cannot accelerate.

Newton's Second Law

Newton's Second Law: The net force on a body is equal to the product of the body's mass and its acceleration.

In equation form,

$$\vec{F}_{net} = m\vec{a} \quad \text{(Newton's second law).} \tag{5-1}$$

Like other vector equations, Eq. 5-1 is equivalent to three component equations, one for each axis of an *xyz* coordinate system:

$$F_{net,x} = ma_x, \quad F_{net,y} = ma_y, \quad \text{and} \quad F_{net,z} = ma_z. \tag{5-2}$$

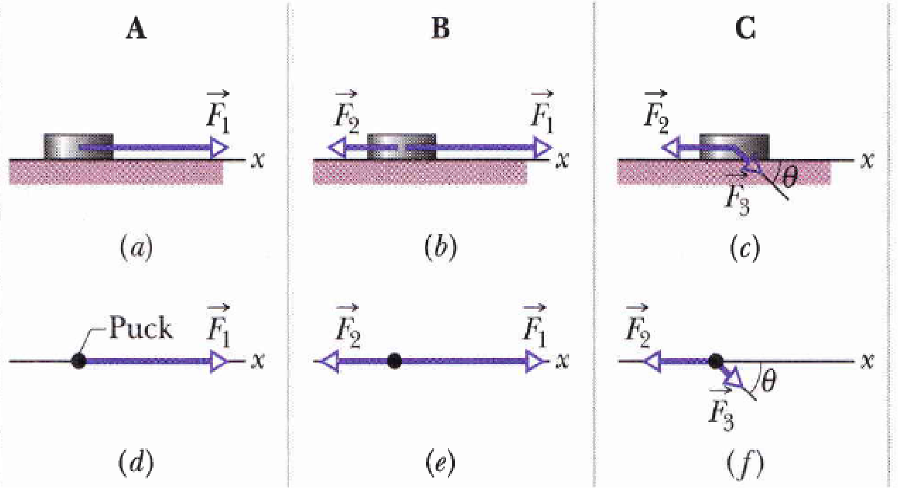
The acceleration component along a given axis is caused *only by* the sum of the force components along that *same* axis, and not by force components along any other axis.

For SI units, Eq. 5-1 tells us that

$$1\text{ N} = (1\text{ kg})(1\text{ m/s}^2) = 1\text{ kg}\cdot\text{m/s}^2. \tag{5-3}$$

Sample Problem 5-1

Figures 5-3*a* to *c* show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is $m = 0.20$ kg. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\vec{F}_3$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?



Situation A: For Fig. 5-3*d*, where only one horizontal force acts, Eq. 5-4 gives us

$$F_1 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1}{m} = \frac{4.0\text{ N}}{0.20\text{ kg}} = 20\text{ m/s}^2. \quad \text{(Answer)}$$

The positive answer indicates that the acceleration is in the positive direction of the x axis.

Situation B: In Fig. 5-3*e*, two horizontal forces act on the puck, $\vec{F}_1$ in the positive direction of x and $\vec{F}_2$ in the negative direction. Now Eq. 5-4 gives us

which, with given data, yields

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0\text{ N} - 2.0\text{ N}}{0.20\text{ kg}} = 10\text{ m/s}^2.$$

(Answer)

Situation C: In Fig. 5-3*f*, force $\vec{F}_3$ is not directed along the direction of the puck's acceleration; only *x* component $F_{3,x}$ is. (Force $\vec{F}_3$ is two-dimensional but the motion is only one-dimensional.) Thus, we write Eq. 5-4 as

$$F_{3,x} - F_2 = ma_x. \tag{5-5}$$

From the figure, we see that $F_{3,x} = F_3 \cos \theta$. Solving for the acceleration and substituting for $F_{3,x}$ yield

$$a_x = \frac{F_{3,x} - F_2}{m} = \frac{F_3 \cos \theta - F_2}{m}$$

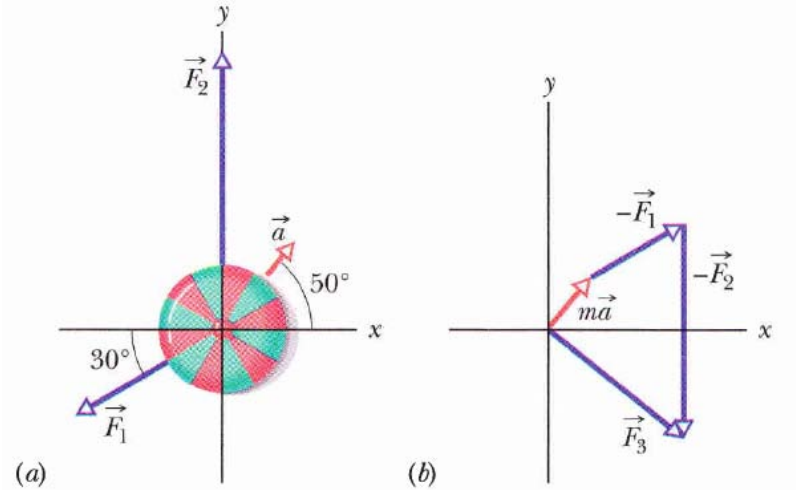
$$= \frac{(1.0\text{ N})(\cos 30^\circ) - 2.0\text{ N}}{0.20\text{ kg}} = -5.7\text{ m/s}^2.$$

(Answer)

Thus, the net force accelerates the puck in the negative direction of the *x* axis.

Sample Problem 5-2

In the overhead view of Fig. 5-4*a*, a 2.0 kg cookie tin is accelerated at 3.0 m/s² in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. What is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation?



x components: Along the x axis we have

$$F_{3,x} = ma_x - F_{1,x} - F_{2,x}$$
$$= m(a\cos 50°) - F_1\cos(-150°) - F_2\cos 90°.$$

Then, substituting known data, we find

$$F_{3,x} = (2.0\text{ kg})(3.0\text{ m/s}^2)\cos 50° - (10\text{ N})\cos(-150°)$$
$$- (20\text{ N})\cos 90°$$
$$= 12.5\text{ N}.$$

y components: Similarly, along the y axis we find

$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$
$$= m(a\sin 50°) - F_1\sin(-150°) - F_2\sin 90°$$
$$= (2.0\text{ kg})(3.0\text{ m/s}^2)\sin 50° - (10\text{ N})\sin(-150°)$$
$$- (20\text{ N})\sin 90°$$
$$= -10.4\text{ N}.$$

Vector: In unit-vector notation, we can write

$$\vec{F}_3 = F_{3,x}\hat{\imath} + F_{3,y}\hat{\jmath} = (12.5\text{ N})\hat{\imath} - (10.4\text{ N})\hat{\jmath}$$
$$\approx (13\text{ N})\hat{\imath} - (10\text{ N})\hat{\jmath}. \qquad \text{(Answer)}$$

We can now use a vector-capable calculator to get the magnitude and the angle of $\vec{F}_3$. We can also use Eq. 3-6 to obtain the magnitude and the angle (from the positive direction of the x axis) as

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = 16\text{ N}$$

and

$$\theta = \tan^{-1}\frac{F_{3,y}}{F_{3,x}} = -40°. \qquad \text{(Answer)}$$

5-7 | Some Particular Forces

The Gravitational Force

Newton's second law can be written in the form $F_{net,y} = ma_y$, which, in our situation, becomes

$$-F_g = m(-g)$$

or

$$F_g = mg. \tag{5-8}$$

Weight

$$F_{net,y} = ma_y.$$

In our situation, this becomes

$$W - F_g = m(0) \tag{5-10}$$

or

$$W = F_g \quad \text{(weight, with ground as inertial frame).} \tag{5-11}$$

$W=mg$ $m=W/g$ $F=(W/g)a$

The Normal Force

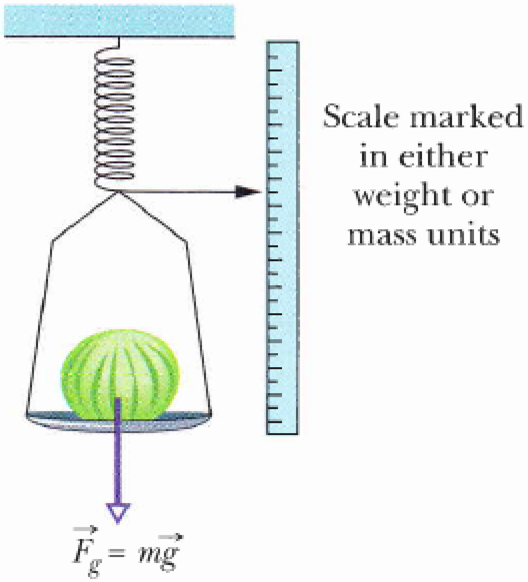
$$F_N - F_g = ma_y.$$

From Eq. 5-8, we substitute mg for F_g, finding

$$F_N - mg = ma_y.$$

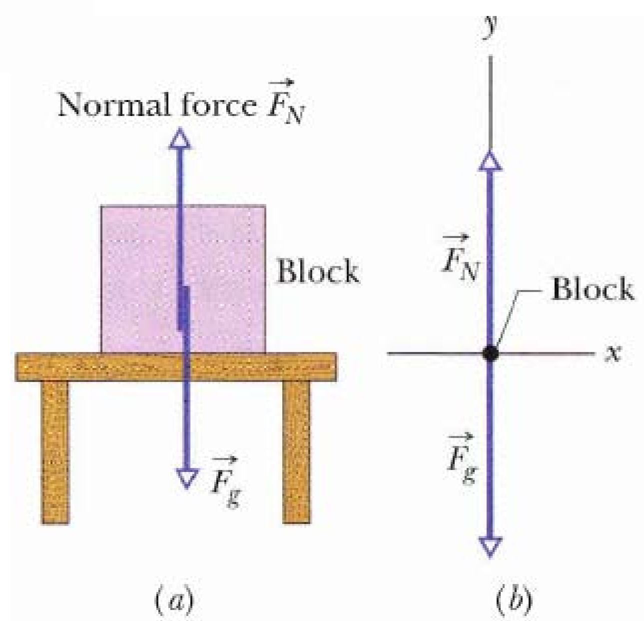
Then the magnitude of the normal force is

$$F_N = mg + ma_y = m(g + a_y) \quad (5\text{-}13)$$



$a_y = 0$ and Eq. 5-13 yields

$$F_N = mg. \quad (5\text{-}14)$$



Friction

If we either slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface. (We discuss this bonding more in the next chapter.) The resistance is considered to be a single force $\vec{f}$, called either the **frictional force** or simply **friction.** This force is directed along the surface, opposite the direction of the intended motion (Fig. 5-8). Sometimes, to simplify a situation, friction is assumed to be negligible (the surface is *frictionless*).

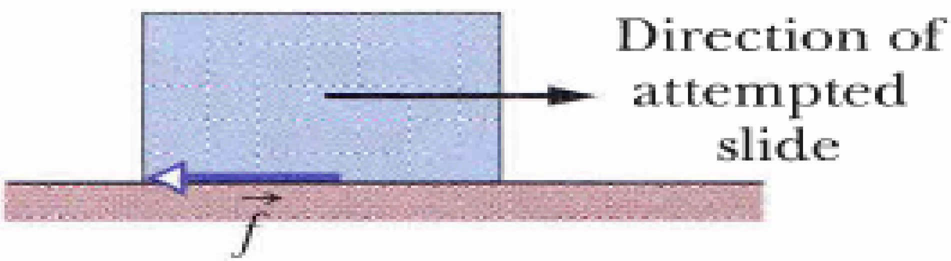
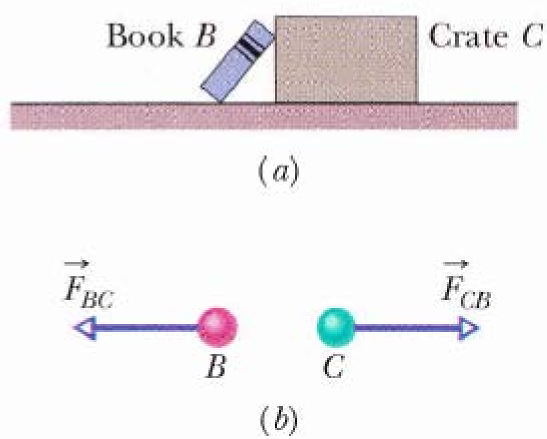


FIG. 5-8 A frictional force $\vec{f}$ opposes the attempted slide of a body over a surface.

5-8 | Newton's Third Law

Newton's Third Law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.



For the book and crate, we can write this law as the scalar relation

$$F_{BC} = F_{CB} \qquad \text{(equal magnitudes)}$$

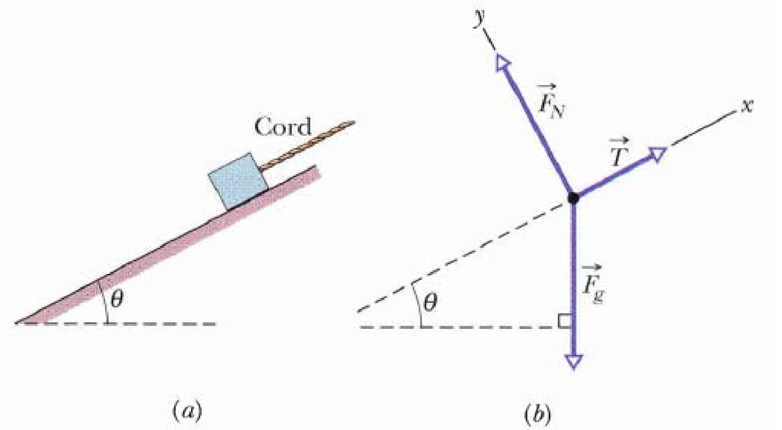
or as the vector relation

$$\vec{F}_{BC} = -\vec{F}_{CB} \qquad \text{(equal magnitudes and opposite directions),}$$

The reaction of force on a body is always an equal and opposite direction of the force

Sample Problem 5-5

In Fig. 5-16*a*, a cord pulls on a box of sea biscuits up along a frictionless plane inclined at $\theta = 30°$. The box has mass $m = 5.00$ kg, and the force from the cord has magnitude $T = 25.0$ N. What is the box's acceleration component a along the inclined plane?



(*a*) (*b*)

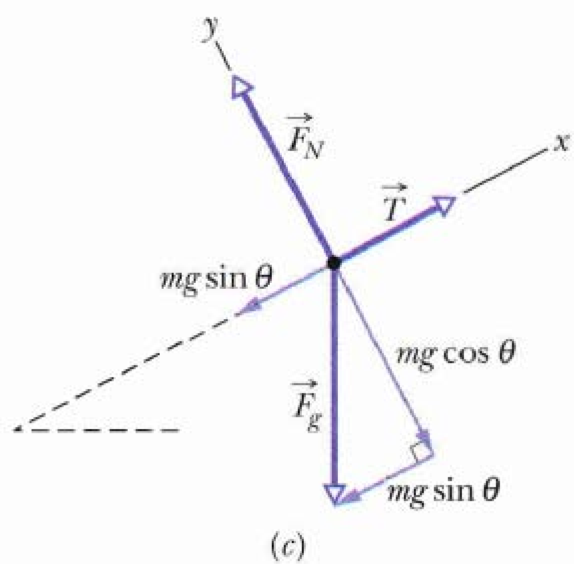
We write Newton's second law ($\vec{F}_{net} = m\vec{a}$) for motion along the x axis as

$$T - mg\sin\theta = ma. \tag{5-22}$$

Substituting data and solving for a, we find

$$a = 0.100 \text{ m/s}^2, \quad \text{(Answer)}$$

where the positive result indicates that the box accelerates up the plane.



(*c*)

Sample Problem 5-8 Build your skill

In Fig. 5-19*a*, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

(a) Find a general solution for the scale reading, whatever the vertical motion of the cab

$$F_N - F_g = ma$$

or

$$F_N = F_g + ma. \qquad (5\text{-}27)$$

This tells us that the scale reading, which is equal to F_N, depends on the vertical acceleration. Substituting mg for F_g gives us

$$F_N = m(g + a) \qquad \text{(Answer)} \qquad (5\text{-}28)$$

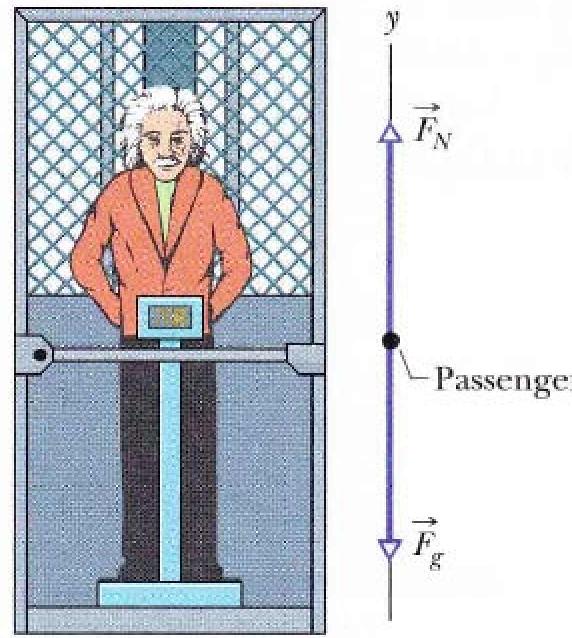
(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

KEY IDEA For any constant velocity (zero or otherwise), the acceleration a of the passenger is zero.

Calculation: Substituting this and other known values into Eq. 5-28, we find

$$F_N = (72.2\ \text{kg})(9.8\ \text{m/s}^2 + 0) = 708\ \text{N}.$$

(Answer)



(*a*) (*b*)

(c) What does the scale read if the cab accelerates upward at 3.20 m/s² and downward at 3.20 m/s²?

Calculations: For $a = 3.20\ \mathrm{m/s^2}$, Eq. 5-28 gives

$$F_N = (72.2\ \mathrm{kg})(9.8\ \mathrm{m/s^2} + 3.20\ \mathrm{m/s^2})$$
$$= 939\ \mathrm{N}, \qquad \text{(Answer)}$$

and for $a = -3.20\ \mathrm{m/s^2}$, it gives

$$F_N = (72.2\ \mathrm{kg})(9.8\ \mathrm{m/s^2} - 3.20\ \mathrm{m/s^2})$$
$$= 477\ \mathrm{N}. \qquad \text{(Answer)}$$

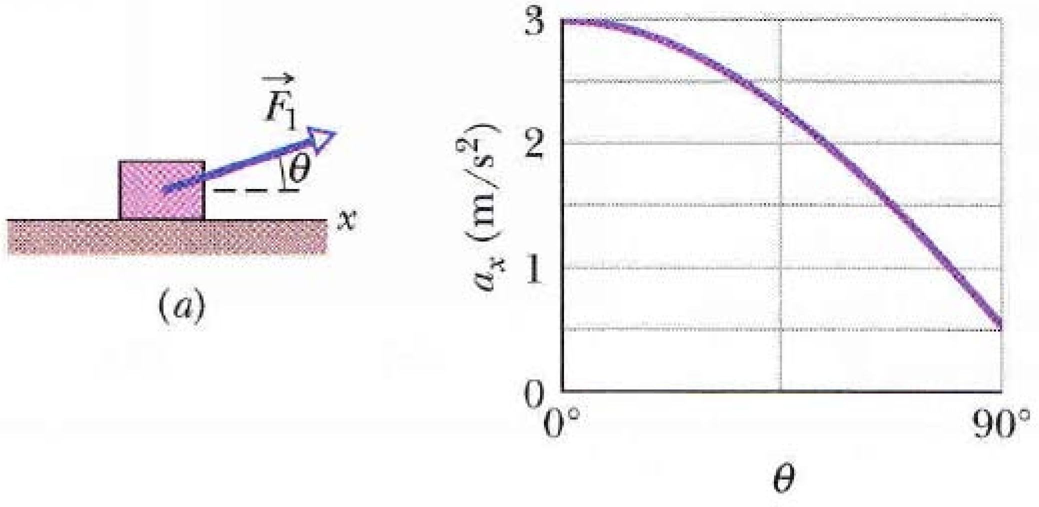
(d) During the upward acceleration in part (c), what is the magnitude F_{net} of the net force on the passenger, and what is the magnitude $a_{p,cab}$ of his acceleration as measured in the frame of the cab? Does $\vec{F}_{net} = m\vec{a}_{p,cab}$?

Calculation: The magnitude F_g of the gravitational force on the passenger does not depend on the motion of the passenger or the cab; so, from part (b), F_g is 708 N. From part (c), the magnitude F_N of the normal force on the passenger during the upward acceleration is the 939 N reading on the scale. Thus, the net force on the passenger is

$$F_{net} = F_N - F_g = 939\ \mathrm{N} - 708\ \mathrm{N} = 231\ \mathrm{N}, \qquad \text{(Answer)}$$

Sample Problem 5-7 Build your skill

Figure 5-18*a* shows the general arrangement in which two forces are applied to a 4.00 kg block on a frictionless floor, but only force $\vec{F}_1$ is indicated. That force has a fixed magnitude but can be applied at angle θ to the positive direction of the x axis. Force $\vec{F}_2$ is horizontal and fixed in both magnitude and angle. Figure 5-18*b* gives the horizontal acceleration a_x of the block for any given value of θ from 0° to 90°. What is the value of a_x for $\theta = 180°$?



Calculations: The x component of $\vec{F}_2$ is F_2 because the vector is horizontal. The x component of $\vec{F}_1$ is $F_1 \cos\theta$. Using these expressions and a mass m of 4.00 kg, we can write Newton's second law ($\vec{F}_{net} = m\vec{a}$) for motion along the x axis as

$$F_1 \cos\theta + F_2 = 4.00a_x. \tag{5-25}$$

From this equation we see that when $\theta = 90°$, $F_1 \cos\theta$ is zero and $F_2 = 4.00a_x$. From the graph we see that the corresponding acceleration is $0.50\ \mathrm{m/s^2}$. Thus,

$F_2 = 2.00$ N and $\vec{F}_2$ must be in the positive direction of the x axis.

From Eq. 5-25, we find that when $\theta = 0°$,

$$F_1 \cos 0° + 2.00 = 4.00a_x. \tag{5-26}$$

From the graph we see that the corresponding acceleration is $3.0\ \mathrm{m/s^2}$. From Eq. 5-26, we then find that $F_1 = 10$ N.

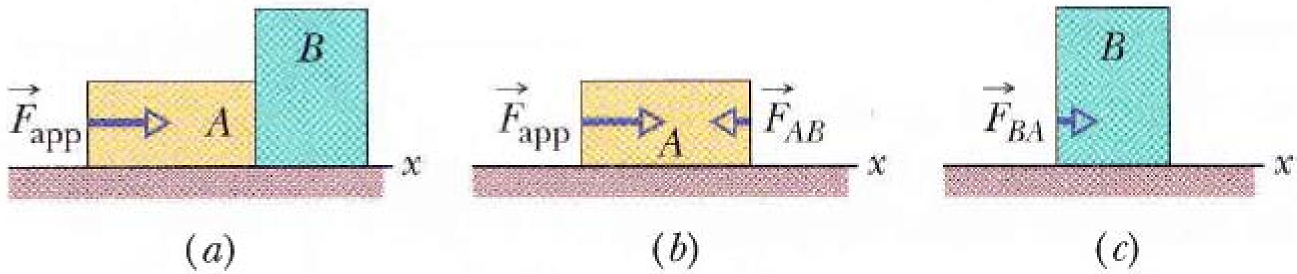
Substituting $F_1 = 10$ N, $F_2 = 2.00$ N, and $\theta = 180°$ into Eq. 5-25 leads to

$$a_x = -2.00\ \mathrm{m/s^2}. \qquad \text{(Answer)}$$

Sample Problem 5-9 Build your skill

In Fig. 5-20*a*, a constant horizontal force $\vec{F}_{app}$ of magnitude 20 N is applied to block A of mass $m_A = 4.0$ kg, which pushes against block B of mass $m_B = 6.0$ kg. The blocks slide over a frictionless surface, along an x axis.

(a) What is the acceleration of the blocks?



(b) What is the (horizontal) force $\vec{F}_{BA}$ on block B from block A (Fig. 5-20*c*)?

Here, once again for the x axis, we can write that law as

$$F_{app} = (m_A + m_B)a,$$

where now we properly apply $\vec{F}_{app}$ to the system with total mass $m_A + m_B$. Solving for a and substituting known values, we find

$$a = \frac{F_{app}}{m_A + m_B} = \frac{20\ \text{N}}{4.0\ \text{kg} + 6.0\ \text{kg}} = 2.0\ \text{m/s}^2.$$

(Answer)

Calculation: Here we can write that law, still for components along the x axis, as

$$F_{BA} = m_B a,$$

which, with known values, gives

$$F_{BA} = (6.0\ \text{kg})(2.0\ \text{m/s}^2) = 12\ \text{N}. \quad \text{(Answer)}$$

Thus, force $\vec{F}_{BA}$ is in the positive direction of the x axis and has a magnitude of 12 N.

••5 There are two forces on the 2.00 kg box in the overhead view of Fig. 5-31, but only one is shown. For $F_1 = 20.0$ N, $a = 12.0$ m/s^2, and $\theta = 30.0°$, find the second force (a) in unit-vector notation and as (b) a magnitude and (c) an angle relative to the positive direction of the x axis. SSM

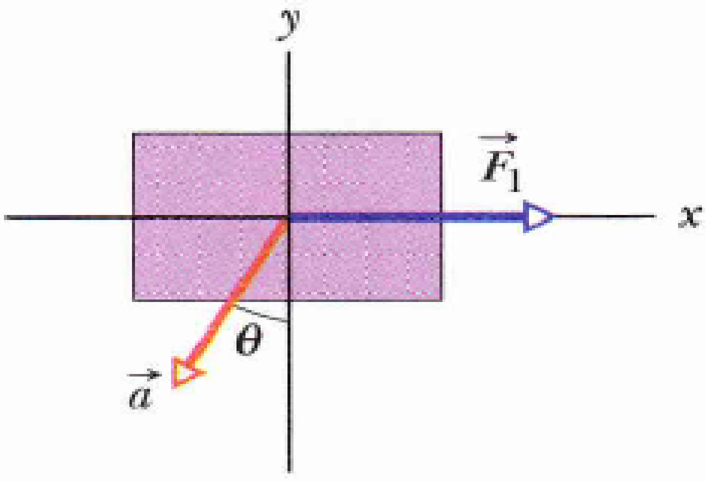


FIG. 5-31 Problem 5.

5. We denote the two forces $\vec{F}_1$ and $\vec{F}_2$. According to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$, so $\vec{F}_2 = m\vec{a} - \vec{F}_1$.

(a) In unit vector notation $\vec{F}_1 = (20.0 \text{ N})\hat{\text{i}}$ and

$$\vec{a} = -\left(12.0 \sin 30.0° \text{m/s}^2\right)\hat{\text{i}} - \left(12.0 \cos 30.0° \text{m/s}^2\right)\hat{\text{j}} = -\left(6.00 \text{ m/s}^2\right)\hat{\text{i}} - \left(10.4 \text{m/s}^2\right)\hat{\text{j}}.$$

Therefore,

$$\begin{aligned}\vec{F}_2 &= (2.00 \text{kg})\left(-6.00 \text{m/s}^2\right)\hat{\text{i}} + (2.00 \text{ kg})\left(-10.4 \text{ m/s}^2\right)\hat{\text{j}} - (20.0 \text{ N})\hat{\text{i}} \\ &= (-32.0 \text{ N})\hat{\text{i}} - (20.8 \text{ N})\hat{\text{j}}.\end{aligned}$$

(b) The magnitude of $\vec{F}_2$ is

$$|\vec{F}_2| = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-32.0\text{ N})^2 + (-20.8\text{ N})^2} = 38.2\text{ N}.$$

(c) The angle that $\vec{F}_2$ makes with the positive x axis is found from

$$\tan\theta = (F_{2y}/F_{2x}) = [(-20.8\text{ N})/(-32.0\text{ N})] = 0.656.$$

Consequently, the angle is either 33.0° or 33.0° + 180° = 213°. Since both the x and y components are negative, the correct result is 213°. An alternative answer is $213° - 360° = -147°$.

••6 While two forces act on it, a particle is to move at the constant velocity $\vec{v} = (3\text{ m/s})\hat{\text{i}} - (4\text{ m/s})\hat{\text{j}}$. One of the forces is $\vec{F}_1 = (2\text{ N})\hat{\text{i}} + (-6\text{ N})\hat{\text{j}}$. What is the other force?

6. Since $\vec{v}$ = constant, we have $\vec{a} = 0$, which implies

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = m\vec{a} = 0\,.$$

Thus, the other force must be

$$\vec{F}_2 = -\vec{F}_1 = (-2\text{ N})\,\hat{\text{i}} + (6\text{ N})\,\hat{\text{j}}.$$

••10 A 0.150 kg particle moves along an x axis according to $x(t) = -13.00 + 2.00t + 4.00t^2 - 3.00t^3$, with x in meters and t in seconds. In unit-vector notation, what is the net force acting on the particle at $t = 3.40$ s?

10. To solve the problem, we note that acceleration is the second time derivative of the position function, and the net force is related to the acceleration via Newton's second law. Thus, differentiating

$$x(t) = -13.00 + 2.00t + 4.00t^2 - 3.00t^3$$

twice with respect to t, we get

$$\frac{dx}{dt} = 2.00 + 8.00t - 9.00t^2, \qquad \frac{d^2x}{dt^2} = 8.00 - 18.0t$$

The net force acting on the particle at $t = 3.40$ s is

$$\vec{F} = m\frac{d^2x}{dt^2}\hat{\text{i}} = (0.150)\left[8.00 - 18.0(3.40)\right]\hat{\text{i}} = (-7.98\text{ N})\hat{\text{i}}$$

•19 In Fig. 5-38, let the mass of the block be 8.5 kg and the angle θ be 30°. Find (a) the tension in the cord and (b) the normal force acting on the block. (c) If the cord is cut, find the magnitude of the resulting acceleration of the block. SSM WWW

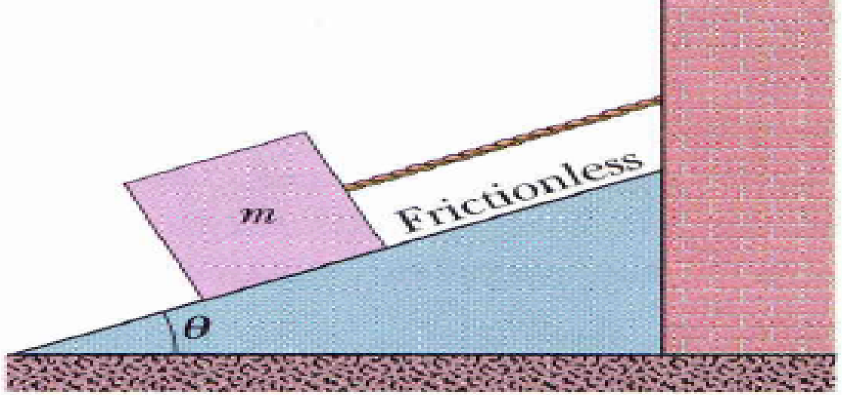


FIG. 5-38 Problem 19.

19. (a) Since the acceleration of the block is zero, the components of the Newton's second law equation yield

$$T - mg\sin\theta = 0$$
$$F_N - mg\cos\theta = 0.$$

Solving the first equation for the tension in the string, we find

$$T = mg\sin\theta = (8.5\text{ kg})(9.8\text{ m/s}^2)\sin 30° = 42\text{ N}.$$

(b) We solve the second equation in part (a) for the normal force F_N:

$$F_N = mg\cos\theta = (8.5\text{ kg})(9.8\text{ m/s}^2)\cos 30° = 72\text{ N}.$$

(c) When the string is cut, it no longer exerts a force on the block and the block accelerates. The x component of the second law becomes $-mg\sin\theta = ma$, so the acceleration becomes

$$a = -g\sin\theta = -(9.8\text{ m/s}^2)\sin 30° = -4.9\text{ m/s}^2.$$

The negative sign indicates the acceleration is down the plane. The magnitude of the acceleration is 4.9 m/s^2.

•14 A block with a weight of 3.0 N is at rest on a horizontal surface. A 1.0 N upward force is applied to the block by means of an attached vertical string. What are the (a) magnitude and (b) direction of the force of the block on the horizontal surface?

14. Three vertical forces are acting on the block: the earth pulls down on the block with gravitational force 3.0 N; a spring pulls up on the block with elastic force 1.0 N; and, the surface pushes up on the block with normal force F_N. There is no acceleration, so

$$\sum F_y = 0 = F_N + (1.0\text{ N}) + (-3.0\text{ N})$$

yields $F_N = 2.0$ N.

(a) By Newton's third law, the force exerted by the block on the surface has that same magnitude but opposite direction: 2.0 N.

(b) The direction is down.

•15 Figure 5-36 shows an arrangement in which four disks are suspended by cords. The longer, top cord loops over a frictionless pulley and pulls with a force of magnitude 98 N on the wall to which it is attached. The tensions in the shorter cords are $T_1 = 58.8$ N, $T_2 = 49.0$ N, and $T_3 = 9.8$ N. What are the masses of (a) disk A, (b) disk B, (c) disk C, and (d) disk D?

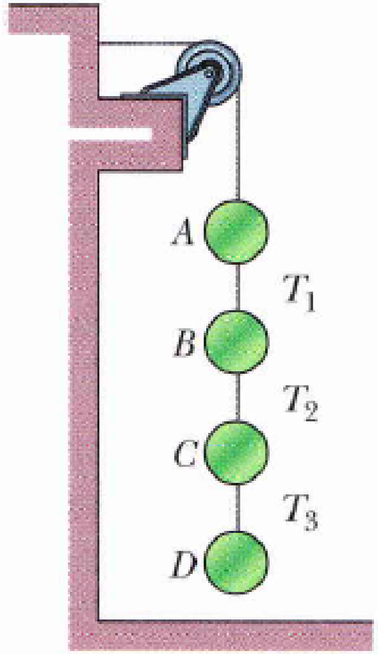


FIG. 5-36 Problem 15.

15. (a) From the fact that $T_3 = 9.8$ N, we conclude the mass of disk D is 1.0 kg. Both this and that of disk C cause the tension $T_2 = 49$ N, which allows us to conclude that disk C has a mass of 4.0 kg. The weights of these two disks plus that of disk B determine the tension $T_1 = 58.8$ N, which leads to the conclusion that $m_B = 1.0$ kg. The weights of all the disks must add to the 98 N force described in the problem; therefore, disk A has mass 4.0 kg.

(b) $m_B = 1.0$ kg, as found in part (a).

(c) $m_C = 4.0$ kg, as found in part (a).

(d) $m_D = 1.0$ kg, as found in part (a).

••59 A block of mass $m_1 = 3.70$ kg on a frictionless plane inclined at angle $\theta = 30.0°$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_2 = 2.30$ kg (Fig. 5-55). What are (a) the magnitude of the acceleration of each block, (b) the direction of the acceleration of the hanging block, and (c) the tension in the cord? ILW

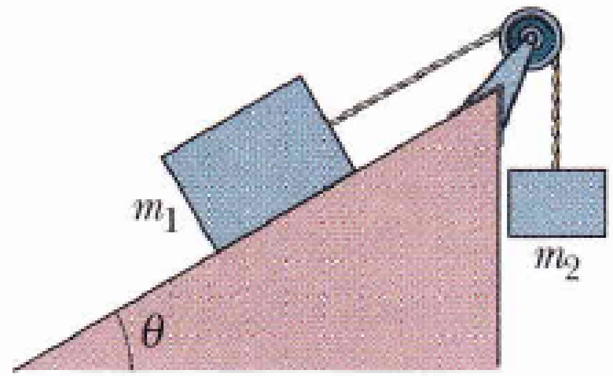
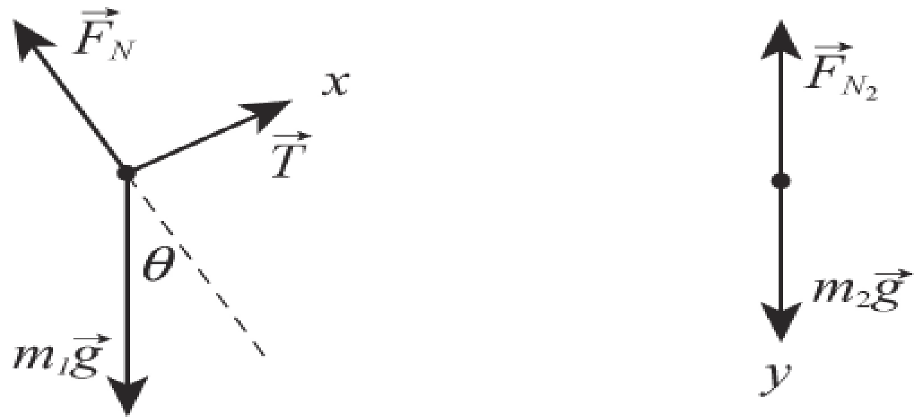


FIG. 5-55 Problem 59.

59. The free-body diagram for each block is shown below. T is the tension in the cord and $\theta = 30°$ is the angle of the incline. For block 1, we take the $+x$ direction to be up the incline and the $+y$ direction to be in the direction of the normal force $\vec{F}_N$ that the plane exerts on the block. For block 2, we take the $+y$ direction to be down. In this way, the accelerations of the two blocks can be represented by the same symbol a, without ambiguity. Applying Newton's second law to the x and y axes for block 1 and to the y axis of block 2, we obtain

$$T - m_1 g \sin\theta = m_1 a$$
$$F_N - m_1 g \cos\theta = 0$$
$$m_2 g - T = m_2 a$$

respectively. The first and third of these equations provide a simultaneous set for obtaining values of a and T. The second equation is not needed in this problem, since the normal force is neither asked for nor is it needed as part of some further computation (such as can occur in formulas for friction).



(a) We add the first and third equations above:

$$m_2 g - m_1 g \sin\theta = m_1 a + m_2 a.$$

Consequently, we find

$$a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2} = \frac{[2.30\text{ kg} - (3.70\text{ kg})\sin 30.0°](9.80\text{ m/s}^2)}{3.70\text{ kg} + 2.30\text{ kg}} = 0.735\text{ m/s}^2.$$

(b) The result for a is positive, indicating that the acceleration of block 1 is indeed up the incline and that the acceleration of block 2 is vertically down.

(c) The tension in the cord is

$$T = m_1 a + m_1 g \sin\theta = (3.70\text{ kg})(0.735\text{ m/s}^2) + (3.70\text{ kg})(9.80\text{ m/s}^2)\sin 30.0° = 20.8\text{N}.$$

••54 In Fig. 5-52, three ballot boxes are connected by cords, one of which wraps over a pulley having negligible friction on its axle and negligible mass. The three masses are $m_A = 30.0$ kg, $m_B = 40.0$ kg, and $m_C = 10.0$ kg. When the assembly is released from rest, (a) what is the tension in the cord connecting B and C, and (b) how far does A move in the first 0.250 s (assuming it does not reach the pulley)?

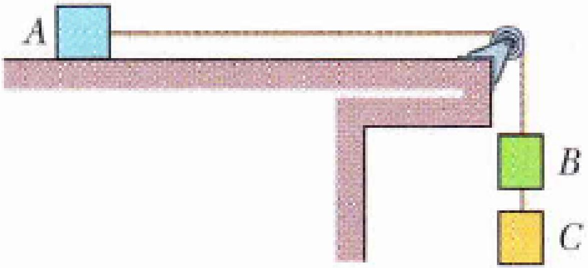


FIG. 5-52 Problem 54.

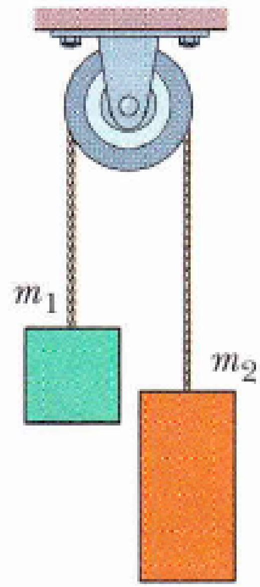
54. (a) The net force on the *system* (of total mass $M = 80.0$ kg) is the force of gravity acting on the total overhanging mass ($m_{BC} = 50.0$ kg). The magnitude of the acceleration is therefore $a = (m_{BC}\, g)/M = 6.125$ m/s^2. Next we apply Newton's second law to block C itself (choosing *down* as the $+y$ direction) and obtain

$$m_C g - T_{BC} = m_C a.$$

This leads to $T_{BC} = 36.8$ N.

(b) We use Eq. 2-15 (choosing *rightward* as the $+x$ direction): $\Delta x = 0 + \frac{1}{2}at^2 = 0.191$ m.

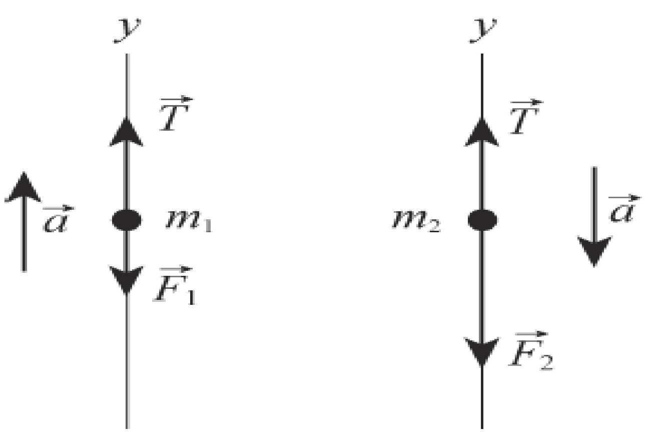
••55 Figure 5-53 shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). The arrangement is known as *Atwood's machine*. One block has mass $m_1 = 1.30$ kg; the other has mass $m_2 = 2.80$ kg. What are (a) the magnitude of the blocks' acceleration and (b) the tension in the cord? GO



55. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$



which can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

Substituting the result back, we have

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$

(a) With $m_1 = 1.3$ kg and $m_2 = 2.8$ kg, the acceleration becomes

$$a = \left(\frac{2.80\,\text{kg} - 1.30\,\text{kg}}{2.80\,\text{kg} + 1.30\,\text{kg}}\right)(9.80\ \text{m/s}^2) = 3.59\ \text{m/s}^2.$$

(b) Similarly, the tension in the cord is

$$T = \frac{2(1.30\,\text{kg})(2.80\,\text{kg})}{1.30\,\text{kg} + 2.80\,\text{kg}}(9.80\ \text{m/s}^2) = 17.4\ \text{N}.$$

•••65 Figure 5-57 shows three blocks attached by cords that loop over frictionless pulleys. Block B lies on a frictionless table; the masses are m_A = 6.00 kg, m_B = 8.00 kg, and m_C = 10.0 kg. When the blocks are released, what is the tension in the cord at the right?

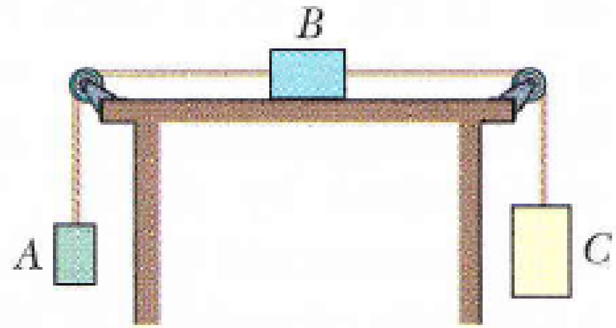


FIG. 5-57 Problem 65.

65. First we analyze the entire *system* with "clockwise" motion considered positive (that is, downward is positive for block C, rightward is positive for block B, and upward is positive for block A): $m_C g - m_A g = Ma$ (where M = mass of the *system* = 24.0 kg). This yields an acceleration of

$$a = g(m_C - m_A)/M = 1.63 \text{ m/s}^2.$$

Next we analyze the forces just on block C: $m_C g - T = m_C a$. Thus the tension is

$$T = m_C g(2m_A + m_B)/M = 81.7 \text{ N}.$$

•••66 Figure 5-58 shows a box of mass $m_2 = 1.0$ kg on a frictionless plane inclined at angle $\theta = 30°$. It is connected by a cord of negligible mass to a box of mass $m_1 = 3.0$ kg on a horizontal frictionless surface. The pulley is frictionless and massless. (a) If the magnitude of horizontal force $\vec{F}$ is 2.3 N, what is the tension in the connecting cord? (b) What is the largest value the magnitude of $\vec{F}$ may have without the cord becoming slack?

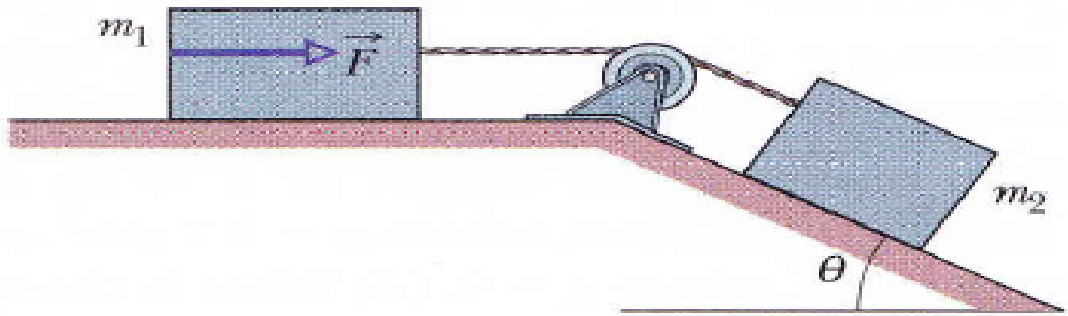
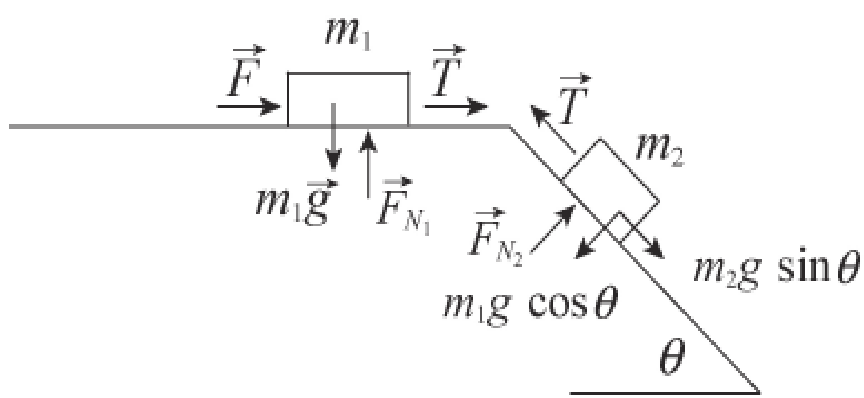


FIG. 5-58 Problem 66.

66. The $+x$ direction for m_2=1.0 kg is "downhill" and the $+x$ direction for m_1=3.0 kg is rightward; thus, they accelerate with the same sign.



(a) We apply Newton's second law to the x axis of each box:

$$m_2 g \sin\theta - T = m_2 a$$
$$F + T = m_1 a$$

Adding the two equations allows us to solve for the acceleration:

$$a = \frac{m_2 g \sin\theta + F}{m_1 + m_2}$$

With $F = 2.3$ N and $\theta = 30°$, we have $a = 1.8$ m/s^2. We plug back and find $T = 3.1$ N.

(b) We consider the "critical" case where the F has reached the *max* value, causing the tension to vanish. The first of the equations in part (a) shows that $a = g \sin 30°$ in this case; thus, $a = 4.9$ m/s^2. This implies (along with $T = 0$ in the second equation in part (a)) that

$$F = (3.0\text{ kg})(4.9\text{ m/s}^2) = 14.7\text{ N} \approx 15\text{ N}$$

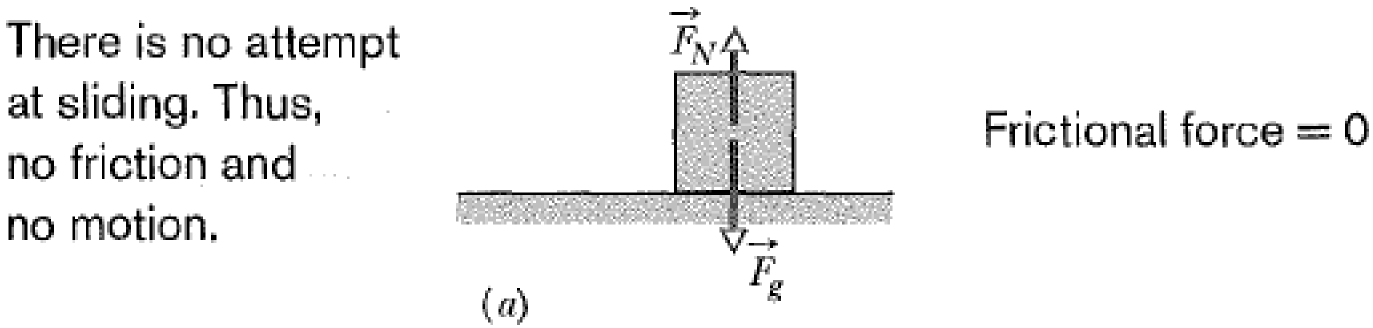
in the critical case.

Friction and Motion

A block rests on a tabletop with the gravitational force $\vec{F}_g$

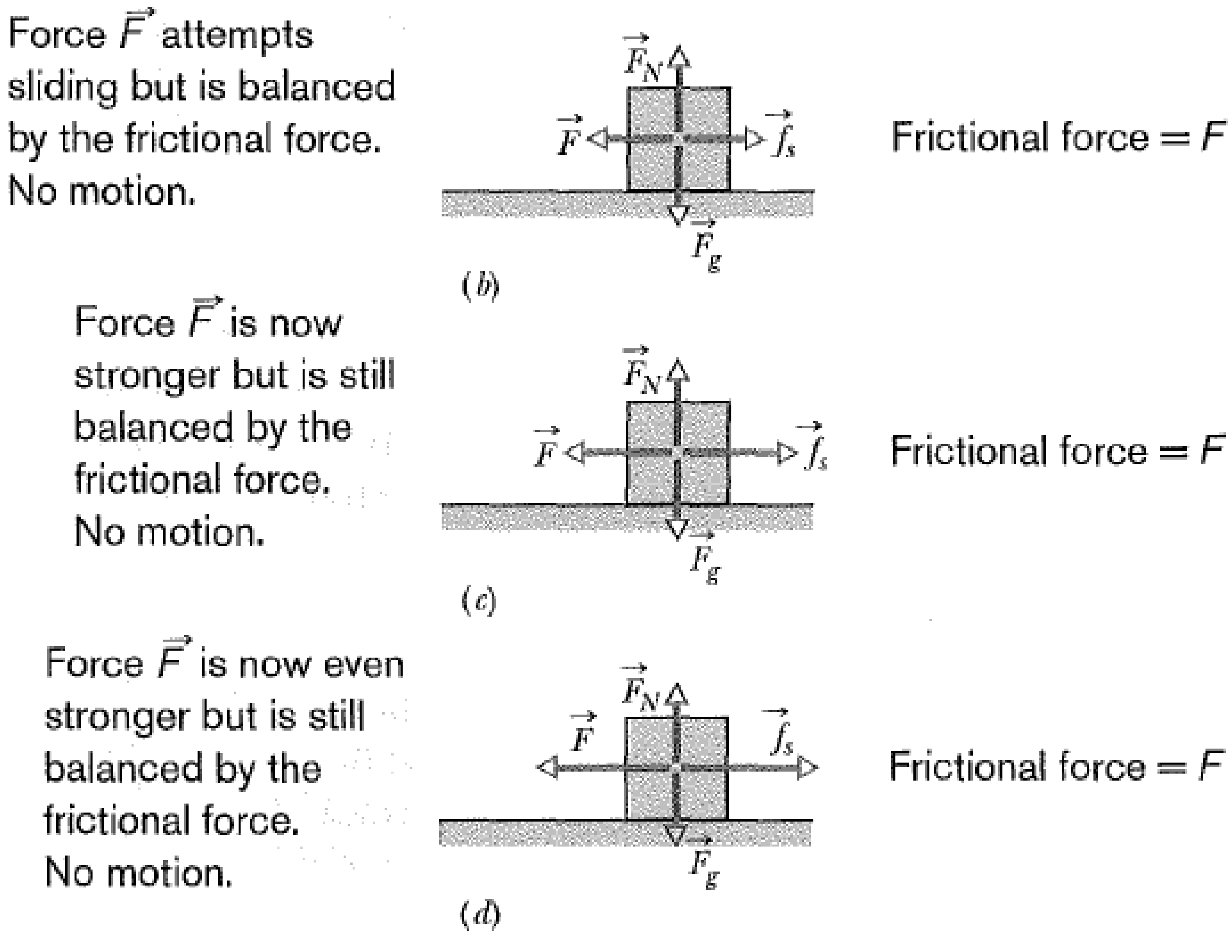
balanced by a normal force $\vec{F}_N$

One exerts a force on the block $\vec{F}$



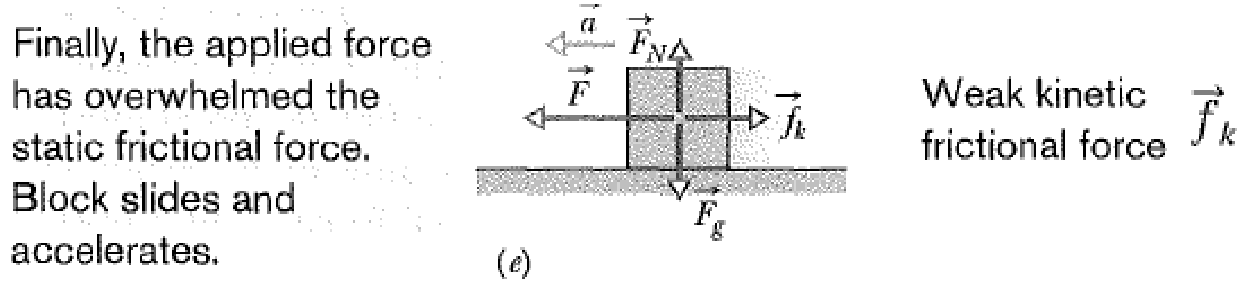
Friction and Motion

An external force *F*, applied to the block, is balanced by a **static frictional force** f_s. As *F* increases, f_s also increases, until f_s reaches a certain maximum value.

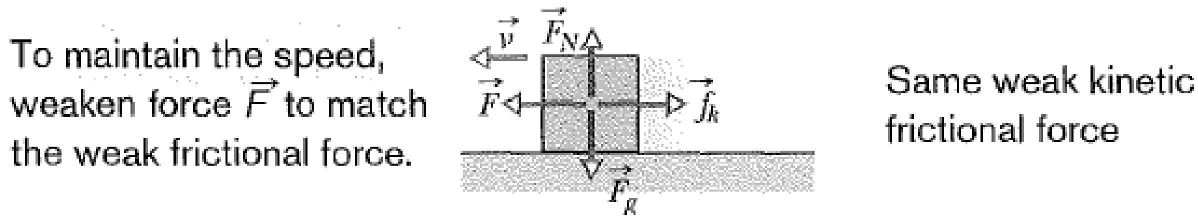


Friction and Motion

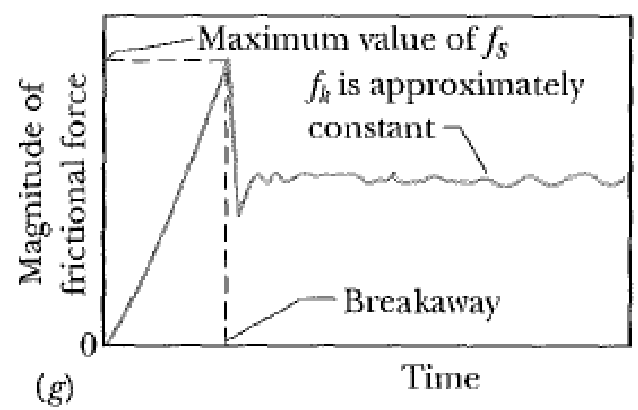
The block then "breaks away," accelerating suddenly in the direction of *F*.



If the block is now to move with constant velocity, *F* must be reduced from the maximum value



Some experimental results for the sequence (a) through (f)



Properties of Friction

Property 1. If the body does not move, then the static frictional force $\vec{f}_s$ and the component of $\vec{F}$ that is parallel to the surface balance each other. They are equal in magnitude, and $\vec{f}_s$ is directed opposite that component of $\vec{F}$.

Properties of Friction

Property 2. The magnitude of $\vec{f}_s$ has a maximum value $f_{s,\max}$ that is given by

$$f_{s,\max} = \mu_s F_N, \tag{6-1}$$

where μ_s is the **coefficient of static friction** and F_N is the magnitude of the normal force on the body from the surface. If the magnitude of the component of $\vec{F}$ that is parallel to the surface exceeds $f_{s,\max}$, then the body begins to slide along the surface.

Properties of Friction

Property 3. If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value f_k given by

$$f_k = \mu_k F_N, \tag{6-2}$$

where μ_k is the **coefficient of kinetic friction.** Thereafter, during the sliding, a kinetic frictional force $\vec{f}_k$ with magnitude given by Eq. 6-2 opposes the motion.

- The coefficients μ_s and μ_k are dimensionless.
- Their values depend on certain properties of both the body and the surface; hence, they are usually referred to with the preposition "between,"

CHECKPOINT

A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor?

(b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it?

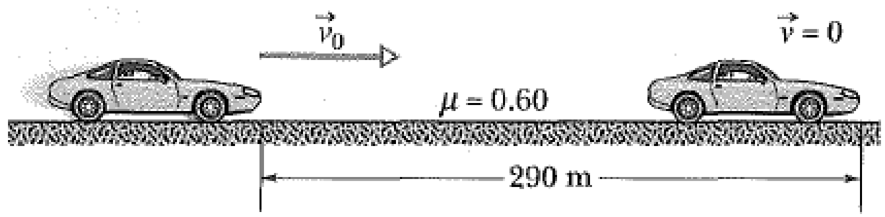
(c) If the maximum value $f_{s,max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N?

(d) If it is 12 N?

(e) What is the magnitude of the frictional force in part (c)?

Sample Problem

If a car's wheels are "locked" during emergency braking, the car slides along the road. Ripped-off bits of tire and small melted sections of road form the "skid marks" that reveal that cold-welding occurred during the slide. The marks were 290 m long! Assuming that μ_k = 0.60 and the car's acceleration was constant during the braking, how fast was the car going when the wheels became locked?



Acceleration *a* was due only to a kinetic frictional force

$$-f_k = ma \qquad f_k = \mu_k F_N$$

$$a = -\frac{f_k}{m} = -\frac{\mu_k m g}{m} = -\mu_k g$$

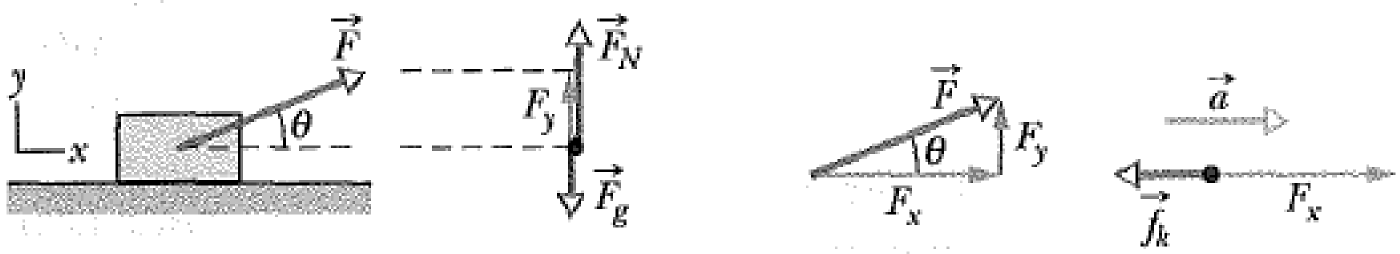
Acceleration is constant
& v = 0
& $x-x_0$ = 290 m

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v_0 = \sqrt{2\mu_k g(x - x_0)}$$
$$= \sqrt{(2)(0.60)(9.8\ \text{m/s}^2)(290\ \text{m})}$$
$$= 58\ \text{m/s} = 210\ \text{km/h}.$$

Sample Problem

A block of mass *m* = 3 kg slides along a floor while a force *F* of magnitude 12.0 N is applied to it at an upward angle θ. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.4$. We can vary θ from 0 to 90° (the block remains on the floor). What θ gives the maximum value of the block's acceleration magnitude *a*?



Along y-direction $F_N + F\sin\theta - mg = m(0)$ $\quad F_N = mg - F\sin\theta$

Along x-direction $F\cos\theta - \mu_k F_N = ma$

Substituting for F_N

$$a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right)$$

Finding a maximum

$$\frac{da}{d\theta} = -\frac{F}{m}\sin\theta + \mu_k\frac{F}{m}\cos\theta = 0 \quad \Rightarrow \quad \tan\theta = \mu_k$$

$$\theta = \tan^{-1}\mu_k = 21.8° \approx 22°$$

The Drag Force and Terminal Speed

- A **fluid** is anything that can flow, generally either a gas or a liquid.
- When there is a relative velocity between a fluid and a body, the body experiences a **drag force** D
- The drag force opposes the relative motion and points in the direction in which the fluid flows relative to the body..

Fall of blunt (like a baseball) in air

The drag force $D = \frac{1}{2}C\rho A v^2$

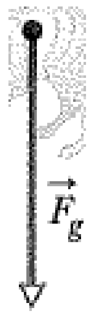
ρ : air density

A: the effective cross-sectional area of the body

$\vec{v}$: body speed

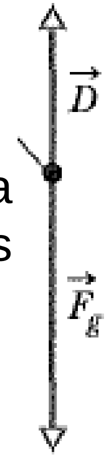
C: drag coefficient C (0.4 to 1.0)

the body when it has just begun to fall and



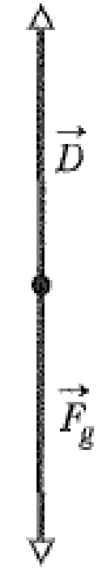
(a)

the free-body after a drag force has developed



(b)

The drag force increases until it balances F_g on the body. The body now falls at constant terminal speed.



(c)

Newton's second law for a vertical *y* axis $F_{net,y} = ma_y$

$$D - F_g = ma \qquad \frac{1}{2}C\rho A v_t^2 - F_g = 0$$

terminal speed $v_t = \sqrt{\dfrac{2F_g}{C\rho A}}$

Sample Problem

A raindrop with radius R = 1.5 mm falls from a cloud that is at height h = 1200 m above the ground. The drag coefficient C for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water *p",* is 1000 kg/m^3, and the density of air ρ_a is 1.2 kg/m^3.

(a) What is the terminal speed?

$$F_g = V\rho_w g = \tfrac{4}{3}\pi R^3 \rho_w g$$

$$v_t = \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}}$$

$$= \sqrt{\frac{(8)(1.5 \times 10^{-3}\ \text{m})(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)}{(3)(0.60)(1.2\ \text{kg/m}^3)}}$$

$$= 7.4\ \text{m/s} \approx 27\ \text{km/h.} \qquad \text{(Answer)}$$

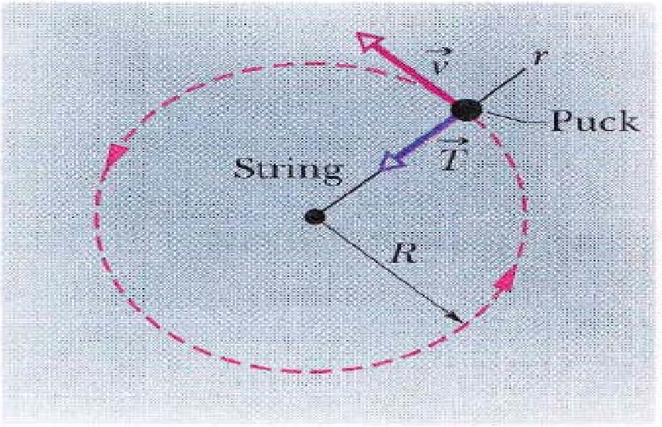
(b) What would be the drop's speed just before impact if there were no drag force?.

The initial velocity v_0=0, displacement $x–x_0$ is $-h$

$$v = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(1200\ \text{m})}$$

$$= 153\ \text{m/s} \approx 550\ \text{km/h.}$$

Uniform Circular Motion



- A particle moves in a circular path with a constant speed v
- It then has centripetal force of

$$a = \frac{v^2}{R} \qquad \text{centripetal acceleartion}$$

From Newton's second law, the centripetal force is:

$$F = ma = m\frac{v^2}{R}$$

- Both velocity and acceleration are constant
- Both *a* and F are directed to the center. They are perpendicular to v (in the direction of the tangent)

Example:

A car moves with a constant speed in a circular motion of radius 50 m and makes a full cycle in 0.2 min. Find the speed and the acceleration of the car.

Since the speed of the car is constant , therefore

$$v = \frac{X}{t} = \frac{2\pi r}{t} = \frac{2\text{x}3.14\text{x}50}{0.2\text{x}60} = 26.2 \text{ m/s}$$

$$a = \frac{v^2}{R} = \frac{26.2^2}{50} = 13.7 \text{ m/s}^2$$

•1 A bedroom bureau with a mass of 45 kg, including drawers and clothing, rests on the floor. (a) If the coefficient of static friction between the bureau and the floor is 0.45, what is the magnitude of the minimum horizontal force that a person must apply to start the bureau moving? (b) If the drawers and clothing, with 17 kg mass, are removed before the bureau is pushed, what is the new minimum magnitude? SSM WWW

The minimum force F_{min} is equal to the maximum static friction $f_{s,max}$

(a)

$$F_{\min} = f_{s,\max} = \mu_s F_N = \mu_s mg$$

$$= 45 \text{x} 0.45 \text{x} 9.8 = 198 \text{ N}$$

(b)

$$F_{\min} = f_{s,\max} = \mu_s F_N = \mu_s mg$$

$$= (45 - 17) \text{x} 0.45 \text{x} 9.8 = 123 \text{ N}$$

wall?

••18 A 4.10 kg block is pushed along a floor by a constant applied force that is horizontal and has a magnitude of 40.0 N. Figure 6-27 gives the block's speed v versus time t as the block moves along an x axis on the floor. The scale of the figure's vertical axis is set by $v_s = 5.0$ m/s. What is the coefficient of kinetic friction between the block and the floor?

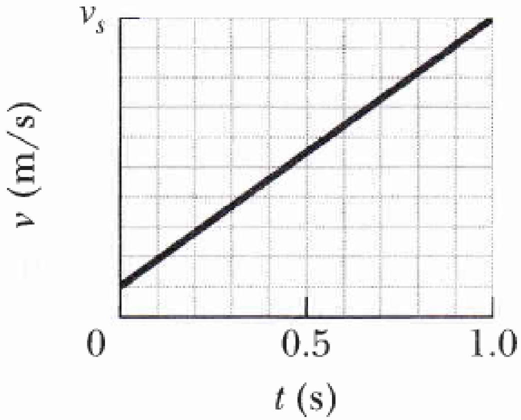


FIG. 6-27 Problem 18.

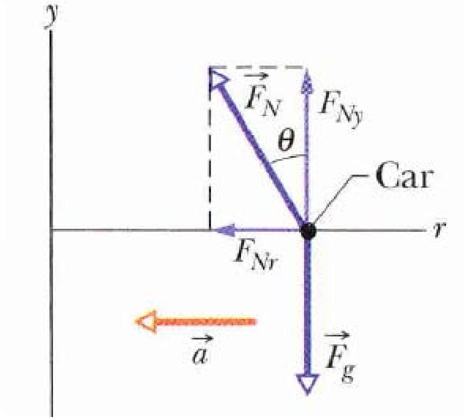
Acceleration = slope = 0.45

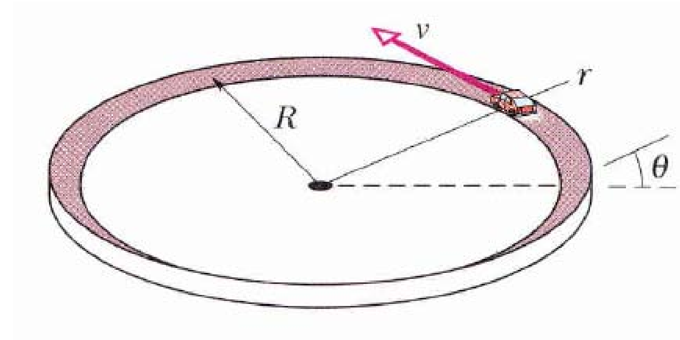
Kinetic friction $\quad f_k = \mu_k F_N = \mu_k mg$

Newton's 2nd law : $\quad F - f_k = ma$

$$F - \mu_k F_N = ma \quad \Rightarrow \quad \mu_k mg = F - ma \quad \Rightarrow \quad \mu_k F_N = \frac{F - ma}{mg} = \frac{40 - 4.1\text{x}4.5}{4.1\text{x}9.8}$$

6-13*a* represents a car of mass m as it moves at a constant speed v of 20 m/s around a banked circular track of radius $R = 190$ m. (It is a normal car, rather than a race car, which means any vertical force from the passing air is negligible.) If the frictional force from the track is negligible, what bank angle θ prevents sliding?





Radial calculation:

$$-F_N \sin\theta = m\left(-\frac{v^2}{R}\right).$$

Vertical calculations:

$$F_N \cos\theta - mg = m(0),$$

from which

$$F_N \cos\theta = mg.$$

Combining results:

$$\theta = \tan^{-1}\frac{v^2}{gR}$$

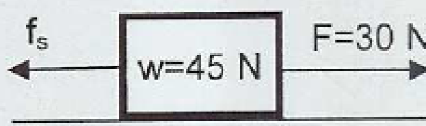
$$= \tan^{-1}\frac{(20\ \text{m/s})^2}{(9.8\ \text{m/s}^2)(190\ \text{m})} = 12^\circ. \quad \text{(Answer)}$$

Q.16 A 60 kg person weighs 100 N on the moon. The acceleration of gravity on the moon is:

(A) zero (B) 4.9 m/s^2 (C) 19.6 m/s^2 (D) 1.67 m/s^2 (E) 9.8 m/s^2

$$W = mg_{moon} \qquad g_{moon} = \frac{W}{m} = \frac{100}{60} = 1.67 \text{ m/s}^2$$

Q.17 A block slides on a rough surface (see Figure). The block starts to slide when a parallel force of 30 N is applied. The coefficient of static friction μ_s is:



(A) 1 (B) 0.4 (C) 0.33 (D) 0.67 (E) Zero

$$F_N = 45 \text{ N}$$

$$f_{s,\max} = 30 \text{ N} = \mu_s F_N \qquad \mu_s = \frac{30}{45} = 0.67$$

Q.18 A cable holds a ball of weight 250 N in static equilibrium. The tension in the cord is:

(A) 500 N (B) 9.8 N (C) 250 N (D) zero (E) 50 N

Q.19 The formula for the centripetal force is:

(A) $a = \frac{v^2}{R}$ (B) F=ma (C) F=mg (D) $F = m\frac{v^2}{R}$ (E) none of these

Q.24 A car rounds a 20 m radius curve at 10 m/s. The magnitude of its acceleration is:

(A) Zero (B) 5 m/s^2 (C) 2 m/s^2 (D) 4 m/s^2 (E) 6 m/s^2

$$a = \frac{v^2}{R} = \frac{10^2}{20} = 5\,\text{m/s}^2$$

Q.12 The formula for the friction force is:

(A) $f=\mu N$ (B) F=ma (C) w=mg (D) F=N (E) F=2f

Q.29 A boy pulls a wooden box along a rough horizontal Floor at constant speed. Which of the followings must be true?

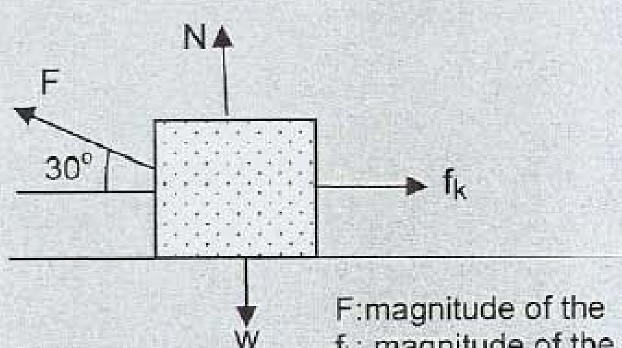
(A) $F\cos\theta > f_k$ and N=W

(B) $F = f_k$ and N > W

(C) $F > f_k$ and N < W

(D) $F\cos\theta = f_k$ and $N = W - F\sin\theta$

(E) none of these



F: magnitude of the Force

f_k: magnitude of the force of friction

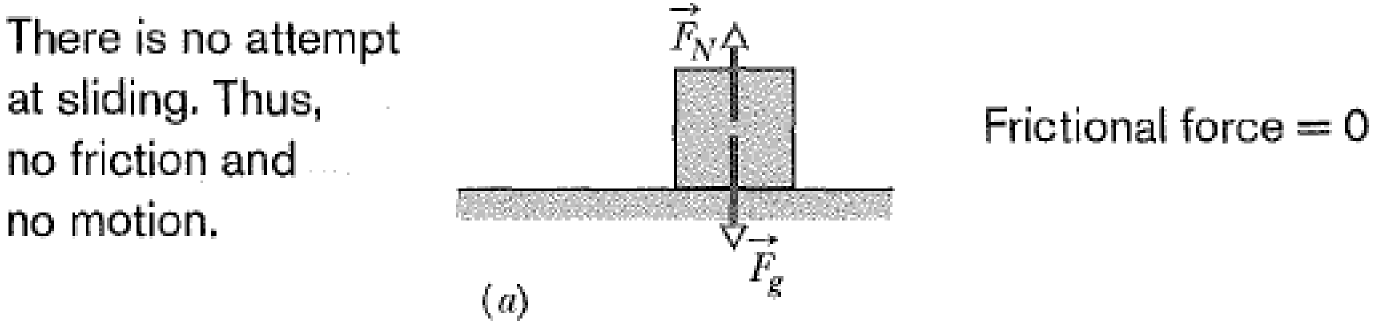
N: magnitude of the normal force

W : weight

Friction and Motion

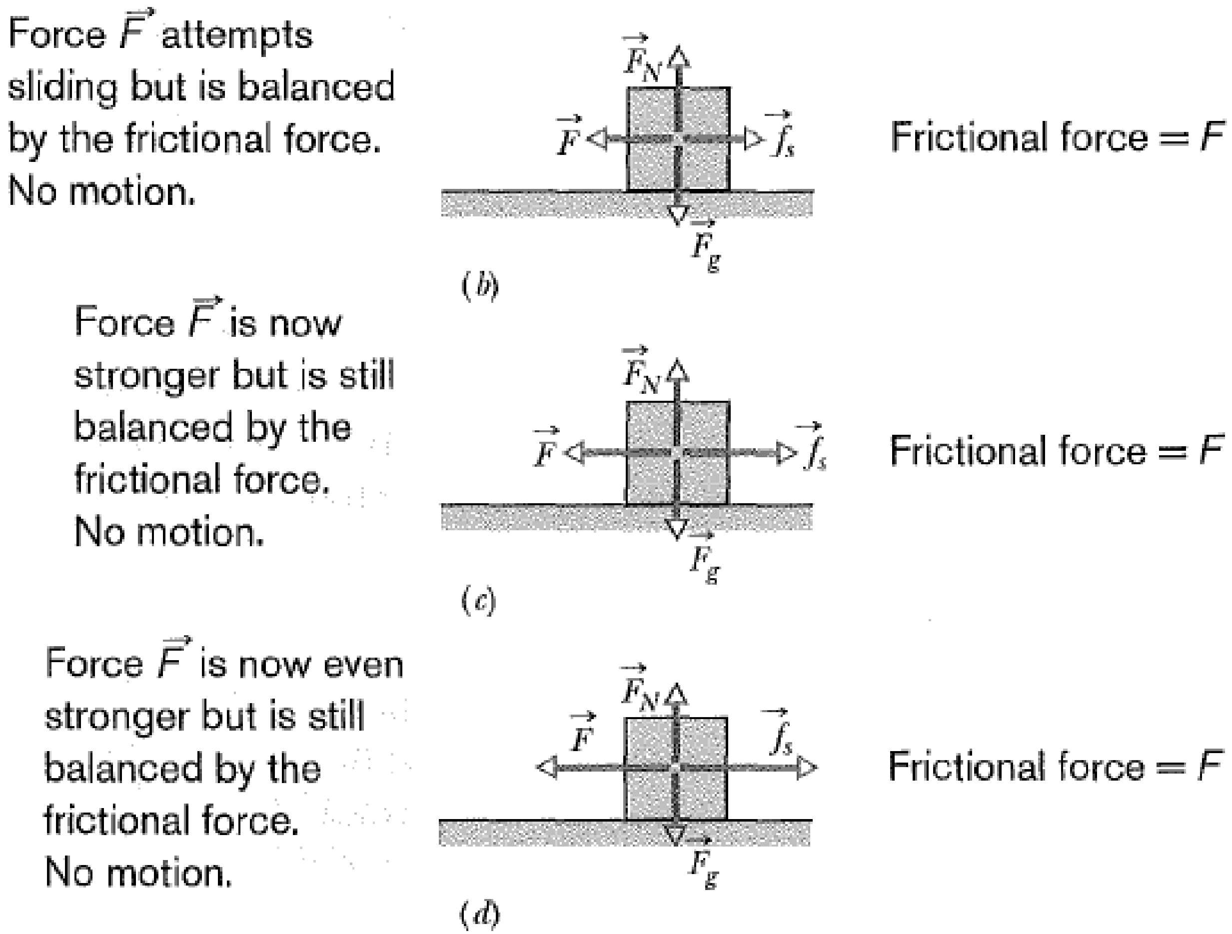
A block rests on a tabletop with the gravitational force $\vec{F}_g$ balanced by a normal force $\vec{F}_N$

One exerts a force on the block $\vec{F}$



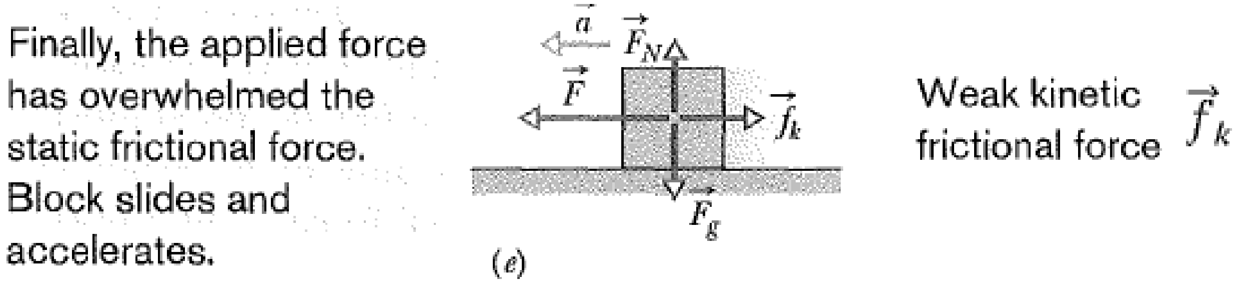
Friction and Motion

An external force *F*, applied to the block, is balanced by a **<u>static frictional force</u>** f_s. As *F* increases, f_s also increases, until f_s reaches a certain maximum value.

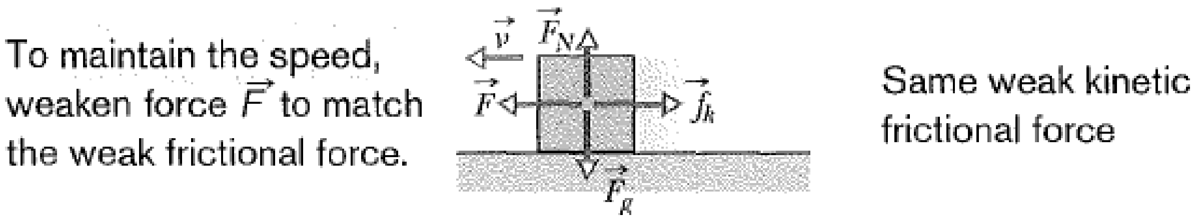


Friction and Motion

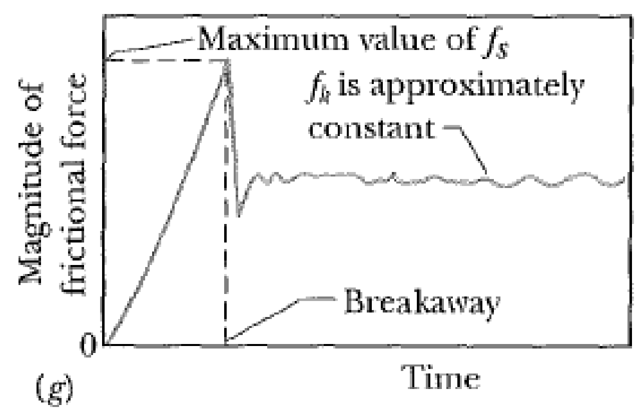
The block then "breaks away," accelerating suddenly in the direction of *F*.



If the block is now to move with constant velocity, *F* must be reduced from the maximum value



Some experimental results for the sequence (a) through (f)



Properties of Friction

Property 1. If the body does not move, then the static frictional force $\vec{f}_s$ and the component of $\vec{F}$ that is parallel to the surface balance each other. They are equal in magnitude, and $\vec{f}_s$ is directed opposite that component of $\vec{F}$.

Properties of Friction

Property 2. The magnitude of $\vec{f}_s$ has a maximum value $f_{s,\max}$ that is given by

$$f_{s,\max} = \mu_s F_N, \tag{6-1}$$

where μ_s is the **coefficient of static friction** and F_N is the magnitude of the normal force on the body from the surface. If the magnitude of the component of $\vec{F}$ that is parallel to the surface exceeds $f_{s,\max}$, then the body begins to slide along the surface.

Properties of Friction

Property 3. If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value f_k given by

$$f_k = \mu_k F_N, \tag{6-2}$$

where μ_k is the **coefficient of kinetic friction.** Thereafter, during the sliding, a kinetic frictional force $\vec{f}_k$ with magnitude given by Eq. 6-2 opposes the motion.

- The coefficients μ_s and μ_k are dimensionless.
- Their values depend on certain properties of both the body and the surface; hence, they are usually referred to with the preposition "between,"

CHECKPOINT

A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor?

(b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it?

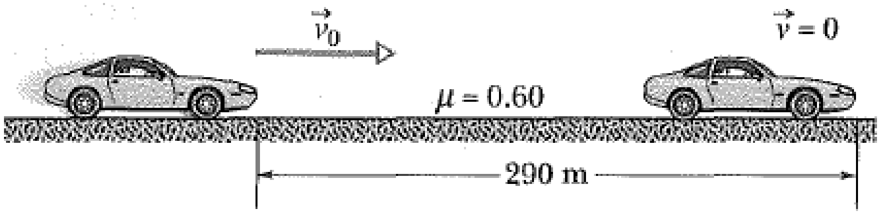
(c) If the maximum value $f_{s,max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N?

(d) If it is 12 N?

(e) What is the magnitude of the frictional force in part (c)?

Sample Problem

If a car's wheels are "locked" during emergency braking, the car slides along the road. Ripped-off bits of tire and small melted sections of road form the "skid marks" that reveal that cold-welding occurred during the slide. The marks were 290 m long! Assuming that μ_k = 0.60 and the car's acceleration was constant during the braking, how fast was the car going when the wheels became locked?



Acceleration *a* was due only to a kinetic frictional force

$$-f_k = ma \qquad f_k = \mu_k F_N$$

$$a = -\frac{f_k}{m} = -\frac{\mu_k mg}{m} = -\mu_k g$$

Acceleration is constant

& v = 0

& $x - x_0$ = 290 m

$$v^2 = v_0^2 + 2a(x - x_0)$$

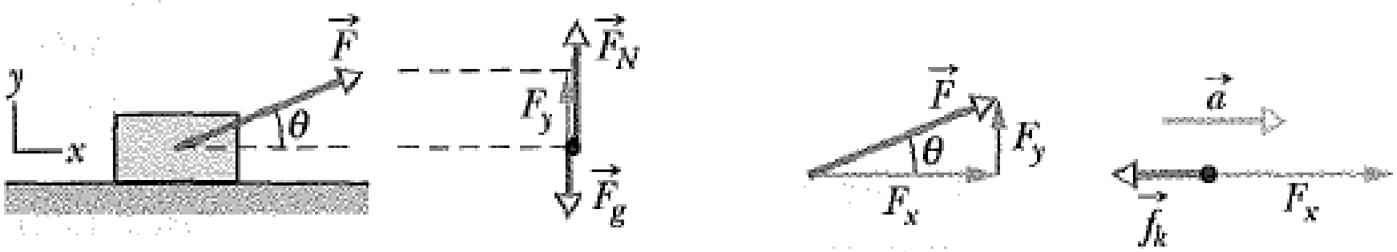
$$v_0 = \sqrt{2\mu_k g(x - x_0)}$$

$$= \sqrt{(2)(0.60)(9.8\ \text{m/s}^2)(290\ \text{m})}$$

$$= 58\ \text{m/s} = 210\ \text{km/h}.$$

Sample Problem

A block of mass m = 3 kg slides along a floor while a force F of magnitude 12.0 N is applied to it at an upward angle θ. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.4$. We can vary θ from 0 to 90° (the block remains on the floor). What θ gives the maximum value of the block's acceleration magnitude a?



Along y-direction $$F_N + F\sin\theta - mg = m(0)$$ $$F_N = mg - F\sin\theta$$

Along x-direction $$F\cos\theta - \mu_k F_N = ma$$

Substituting for F_N $$a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right)$$

Finding a maximum $$\frac{da}{d\theta} = -\frac{F}{m}\sin\theta + \mu_k\frac{F}{m}\cos\theta = 0 \Rightarrow \tan\theta = \mu_k$$

$$\theta = \tan^{-1}\mu_k = 21.8° \approx 22°$$

The Drag Force and Terminal Speed

- A **fluid** is anything that can flow, generally either a gas or a liquid.
- When there is a relative velocity between a fluid and a body, the body experiences a **drag force** *D*
- The drag force opposes the relative motion and points in the direction in which the fluid flows relative to the body..

Fall of blunt (like a baseball) in air

The drag force $D = \frac{1}{2}C\rho A v^2$

ρ : air density

A: the effective cross-sectional area of the body

$\vec{v}$: body speed

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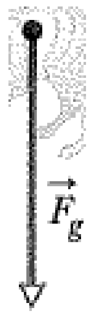
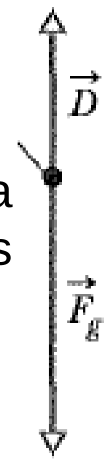
the body when it has just begun to fall and



(a)

the free-body after a drag force has developed



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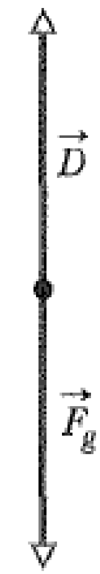

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Newton's second law for a vertical *y* axis $F_{net,y} = ma_y$

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terminal speed $v_t = \sqrt{\dfrac{2F_g}{C\rho A}}$

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A raindrop with radius R = 1.5 mm falls from a cloud that is at height h = 1200 m above the ground. The drag coefficient C for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water *p",* is 1000 kg/m^3, and the density of air ρ_a is 1.2 kg/m^3.

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$$F_g = V\rho_w g = \tfrac{4}{3}\pi R^3 \rho_w g$$

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$$= \sqrt{\frac{(8)(1.5\times 10^{-3}\text{ m})(1000\text{ kg/m}^3)(9.8\text{ m/s}^2)}{(3)(0.60)(1.2\text{ kg/m}^3)}}$$

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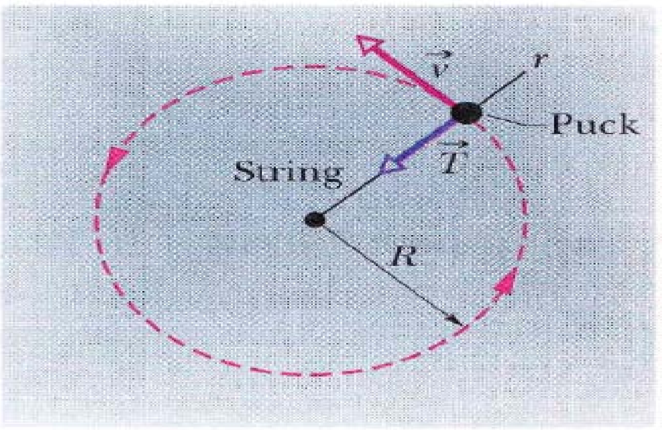

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wall?

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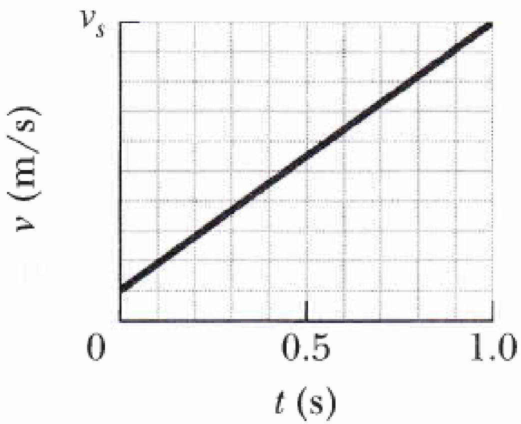


FIG. 6-27 Problem 18.

Acceleration = slope = 0.45

Kinetic friction $f_k = \mu_k F_N = \mu_k mg$

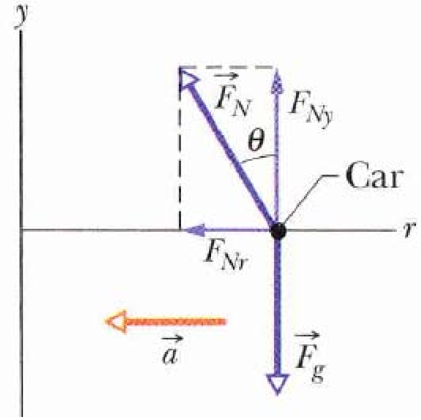
Newton's 2nd law: $F - f_k = ma$

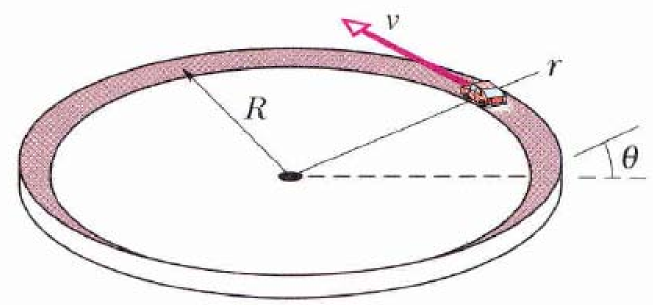
$$F - \mu_k F_N = ma \quad \Rightarrow \quad \mu_k mg = F - ma \quad \Rightarrow \quad \mu_k F_N = \frac{F - ma}{mg} = \frac{40 - 4.1\text{x}4.5}{4.1\text{x}9.8}$$

6-13*a* represents a car of mass m as it moves at a constant speed v of 20 m/s around a banked circular track of radius $R = 190$ m. (It is a normal car, rather than a race car, which means any vertical force from the passing air is negligible.) If the frictional force from the track is negligible, what bank angle θ prevents sliding?

Radial calculation:

$$-F_N \sin\theta = m\left(-\frac{v^2}{R}\right).$$





Vertical calculations:

$$F_N \cos\theta - mg = m(0),$$

from which

$$F_N \cos\theta = mg.$$

Combining results:

$$\theta = \tan^{-1}\frac{v^2}{gR}$$

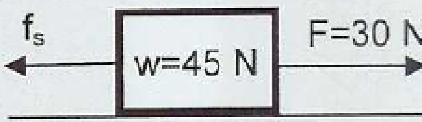
$$= \tan^{-1}\frac{(20\text{ m/s})^2}{(9.8\text{ m/s}^2)(190\text{ m})} = 12^\circ. \quad \text{(Answer)}$$

Q.16 A 60 kg person weighs 100 N on the moon. The acceleration of gravity on the moon is:

(A) zero (B) $4.9\ m/s^2$ (C) $19.6\ m/s^2$ (D) <u>$1.67\ m/s^2$</u> (E) $9.8\ m/s^2$

$$W = mg_{moon} \qquad g_{moon} = \frac{W}{m} = \frac{100}{60} = 1.67\ \mathrm{m/s^2}$$

Q.17 A block slides on a rough surface (see Figure). The block starts to slide when a parallel force of 30 N is applied. The coefficient of static friction μ_s is:



(A) 1 (B) 0.4 (C) 0.33 (D) <u>0.67</u> (E) Zero

$$F_N = 45\ \mathrm{N}$$

$$f_{s,\max} = 30\ \mathrm{N} = \mu_s F_N \qquad \mu_s = \frac{30}{45} = 0.67$$

Q.18 A cable holds a ball of weight 250 N in static equilibrium. The tension in the cord is:

(A) 500 N (B) 9.8 N (C) <u>250 N</u> (D) zero (E) 50 N

Q.19 The formula for the centripetal force is:

(A) $a = \frac{v^2}{R}$ (B) F=ma (C) F=mg (D) $F = m\frac{v^2}{R}$ (E) none of these

Q.24 A car rounds a 20 m radius curve at 10 m/s. The magnitude of its acceleration is:

(A) Zero (B) $5\ m/s^2$ (C) $2\ m/s^2$ (D) $4\ m/s^2$ (E) $6\ m/s^2$

$$a = \frac{v^2}{R} = \frac{10^2}{20} = 5\,\mathrm{m/s^2}$$

Q.12 The formula for the friction force is:

(A) $f = \mu N$ (B) F=ma (C) w=mg (D) F=N (E) F=2f

Q.29 A boy pulls a wooden box along a rough horizontal Floor at constant speed. Which of the followings must be true?

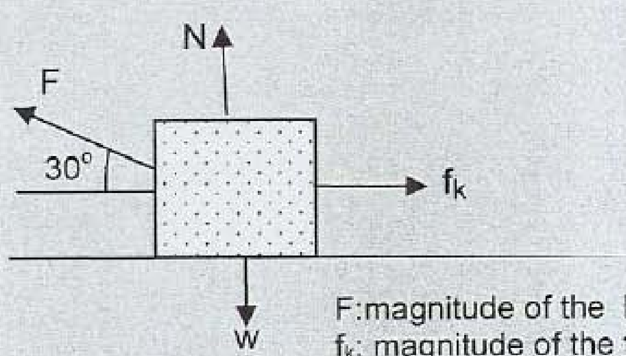
(A) $F\cos\theta > f_k$ and N=W

(B) $F = f_k$ and N >W

(C) $F > f_k$ and N<W

(D) $F\cos\theta = f_k$ and $N = W - F\sin\theta$

(E) none of these



F:magnitude of the Force

f_k: magnitude of the force of friction

N: magnitude of the normal force

W : weight

Kinetic Energy

1 Rank the following velocities according to the kinetic energy a particle will have with each velocity, greatest first: (a) $\vec{v} = 4\hat{i} + 3\hat{j}$, (b) $\vec{v} = -4\hat{i} + 3\hat{j}$, (c) $\vec{v} = -3\hat{i} + 4\hat{j}$, (d) $\vec{v} = 3\hat{i} - 4\hat{j}$, (e) $\vec{v} = 5\hat{i}$, and (f) $v = 5$ m/s at 30° to the horizontal.

$$(a)\ \text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_x^2 + v_y^2\right) = \frac{1}{2}m\left(4\text{x}4 + 3\text{x}3\right) = 12.5m$$

$$(b)\ \text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_x^2 + v_y^2\right) = \frac{1}{2}m\left((\text{-}4)\text{x}(\text{-}4) + 3\text{x}3\right) = 12.5m$$

$$(c)\ \text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_x^2 + v_y^2\right) = \frac{1}{2}m\left((\text{-}3)\text{x}(\text{-}3) + 4\text{x}4\right) = 12.5m$$

$$(d)\ \text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_x^2 + v_y^2\right) = \frac{1}{2}m\left(3\text{x}3 + (\text{-}4)\text{x}(\text{-}4)\right) = 12.5m$$

$$(e)\ \text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}mv_x^2 = \frac{1}{2}m\left(5\text{x}5\right) = 12.5m$$

$$(\text{f})\ \text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_x^2 + v_y^2\right) = \frac{1}{2}m\left[(5\text{xcos}30)^2 + (5\text{xsin}30)^2\right] = \frac{25}{2}m\left[\cos^2 30 + \sin^2 30\right] = 12.5$$

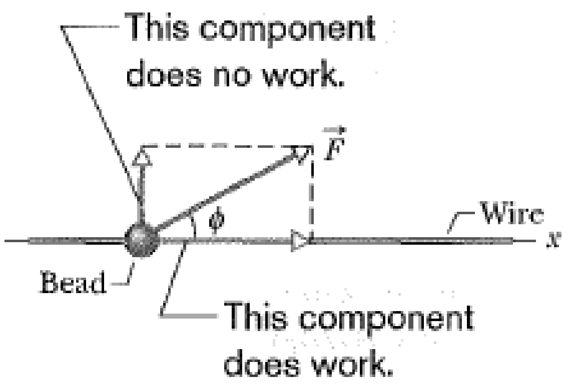
Work

Energy transferred to or from an object by means of a force acting on the object.

- It has the same units as energy and is a scalar quantity

Work and Kinetic Energy

$$F_x = ma_x \qquad v^2 = v_0^2 + 2a_x d \qquad \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d.$$



$$K_f - K_i = W = F_x d$$

K_f & K_i: kinetic energy after & start of displacement

$$W = Fd\cos\phi = \vec{F} \bullet \vec{d} \qquad \text{(work done by constant force)}$$

Units for work: $1\ \mathrm{J} = 1\ \mathrm{kg \cdot m^2/s^2} = 1\ \mathrm{N \cdot m} = 0.738\ \mathrm{ft \cdot lb}.$

$$W = Fd\cos\phi = \vec{F} \bullet \vec{d} \qquad \text{(work done by constant force)}$$

- W = +ve when ϕ<90°, F parallel to d
- W = -ve when ϕ>90°, F opposite to d
- W = 0 when ϕ=0° , F perpendicular to d

Theory of Work – Kinetic Energy

$$\Delta K = K_f - K_i = W, \qquad K_f = K_i + W,$$

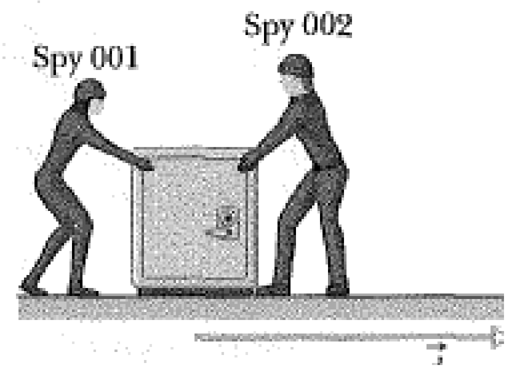
$$\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}$$

CHECKPOINT 1

A particle moves along an *x* axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes
(a) from -3 *mls* to -2 *mls?*
(b) from -2 mls to 2 *mls?*
(c) In each situation, is the work done on the particle positive, negative, or zero?

Sample Problem

Figure 7-4*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



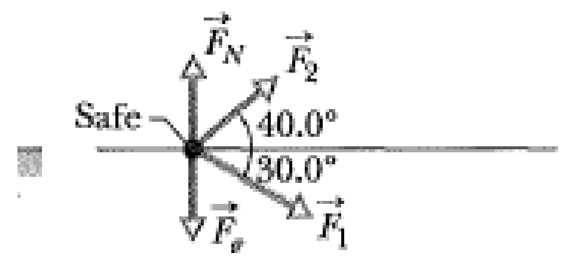
(a) What is the net work done on the safe by forces F_1 and F_2 during the displacement *d*?

$$W_1 = F_1 d \cos\phi_1 = (12.0\text{ N})(8.50\text{ m})(\cos 30.0°) = 88.33\text{ J},$$

$$W_2 = F_2 d \cos\phi_2 = (10.0\text{ N})(8.50\text{ m})(\cos 40.0°) = 65.11\text{ J}.$$

$$W = W_1 + W_2 = 88.33\text{ J} + 65.11\text{ J} = 153.4\text{ J} \approx 153\text{ J}.$$

Only force components parallel to the displacement do work.



(b) During the displacement, what is the work W_g done on the safe by the gravitational force *Fg?* and what is the work W_N done on the safe by the normal force F_N from the floor?

$$W_g = mgd\cos 90° = mgd(0) = 0$$
$$W_N = F_N d\cos 90° = F_N d(0) = 0.$$

(c) The safe is initially stationary. What is its speed v_f *at* the end of the 8.50 m displacement?

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

$v_i = 0 \rightarrow$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\text{ J})}{225\text{ kg}}} = 1.17\text{ m/s}.$$

Sample Problem

If $\vec{d} = (-3.0\text{ m})\hat{\text{i}}$ and $\vec{F} = (2.0\text{ N})\hat{\text{i}} + (-6.0\text{ N})\hat{\text{j}}$ calculate the work done?

$$W = \vec{F}\cdot\vec{d} = [(2.0\text{ N})\hat{\text{i}} + (-6.0\text{ N})\hat{\text{j}}]\cdot[(-3.0\text{ m})\hat{\text{i}}].$$

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{\text{i}}\cdot\hat{\text{i}} + (-6.0\text{ N})(-3.0\text{ m})\hat{\text{j}}\cdot\hat{\text{i}}$$
$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \quad \text{(Answer)}$$

If the kinetic energy at the beginning of displacement is 10 J, what is its kinetic energy at the end of *d?*

$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}.$$

Work Done by the Gravitational Force

For a rising object

$$W_g = mgd\cos 180° = mgd(-1) = -mgd.$$

After the object has reached its maximum height and is falling back down, the angle ¢ between force F_g and displacement d is zero.

$$W_g = mgd\cos 0° = mgd(+1) = +mgd.$$

which becomes opposite when lowering the object.

Work done in lifting an object

$$\Delta K = K_f - K_i = W_a + W_g,$$

It becomes opposite when lowering the object.

If the object is stationary before and after the lift, then $K_f = K_i = 0$

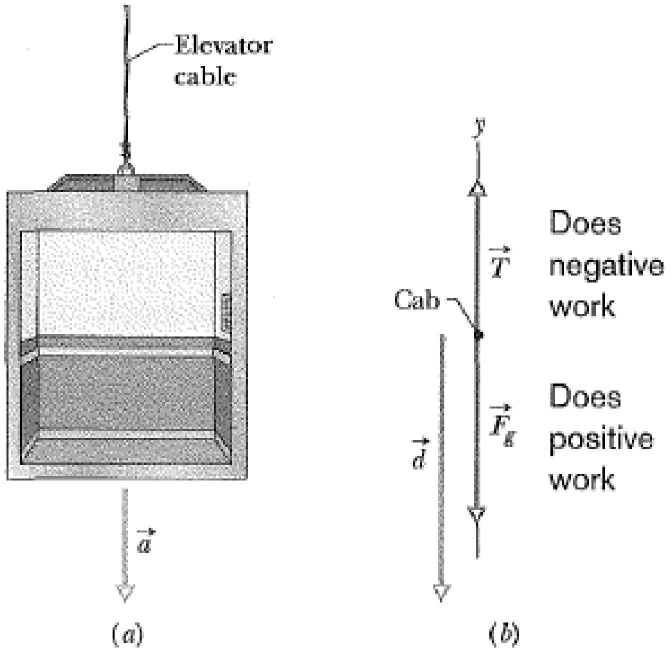
$$W_a + W_g = 0$$
$$W_a = -W_g.$$

Sample Problem

An elevator cab of mass m = 500 kg is descending with speed v_i = 4.0 *mls* when its supporting cable begins to slip, allowing it to fall with constant acceleration $a = gl5$.

(a) During the fall through a distance d = 12 m, what is the work W_g done on the cab by the gravitational force?

$$W_g = mgd\cos 0° = (500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(1)$$
$$= 5.88 \times 10^4\text{ J} \approx 59\text{ kJ}. \quad \text{(Answer)}$$



(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull T of the elevator cable?

$$W_T = Td\cos\phi = m(a+g)d\cos\phi.$$

$$W_T = m\left(-\frac{g}{5} + g\right)d\cos\phi = \frac{4}{5}mgd\cos\phi$$

$$= \frac{4}{5}(500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})\cos 180°$$

$$= -4.70 \times 10^4\text{ J} \approx -47\text{ kJ}.$$

(c) What is the net work *W* done on the cab during the fall?

$$W = W_g + W_T = 5.88 \times 10^4 \text{ J} - 4.70 \times 10^4 \text{ J}$$
$$= 1.18 \times 10^4 \text{ J} \approx 12 \text{ kJ}. \qquad \text{(Answer)}$$

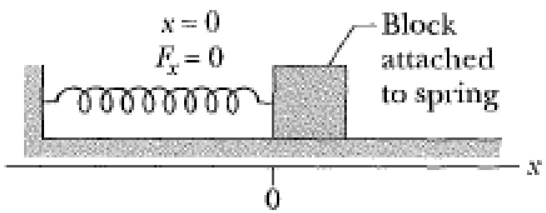
(d) What is the cab's kinetic energy at the end of the 12 m fall?

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500 \text{ kg})(4.0 \text{ m/s})^2 + 1.18 \times 10^4 \text{ J}$$
$$= 1.58 \times 10^4 \text{ J} \approx 16 \text{ kJ}.$$

Work Done by a Spring Force

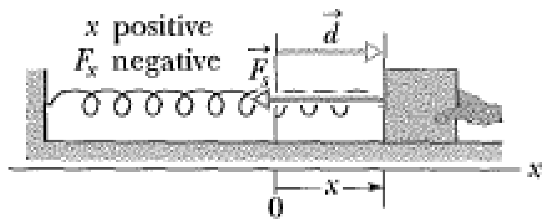
The *spring force* is

$$\vec{F}_s = -k\vec{d} \qquad \text{(Hooke's law)},$$



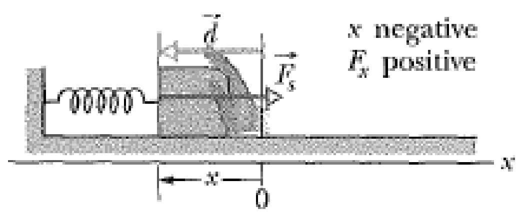
spring in a relaxed state

- The (-) sign is because the direction of F is always opposite to d
- *k* is the spring (or force) constant. It measures of the spring stiffness



block is displaced by *d*
spring is stretched

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force)}.$$



spring is compressed

It is + or –ve depending on whether the *net* transfer of energy is to or from the block as the block moves from x_i to x_f

If $x_i = 0$:

$$W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force)}.$$

The work done by an applied force

If the block is stationary before and after the displacement

$$W_a = -W_s.$$

Work Done by a General Variable Force

(1) One-dimensional analysis

If the force magnitude to vary with position *x*.

$$W = \int_{x_i}^{x_f} F(x)\,dx \qquad \text{(work: variable force).}$$

(2) Three-dimensional analysis

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

Sample Problem

Force $\vec{F} = (3x^2\ \text{N})\hat{\text{i}} + (4\ \text{N})\hat{\text{j}}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

$$W = \int_2^3 3x^2\,dx + \int_3^0 4\,dy = 3\int_2^3 x^2\,dx + 4\int_3^0 dy$$

$$= 3[\tfrac{1}{3}x^3]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3]$$

$$= 7.0\ \text{J}.$$

(Answer

Power

The time rate at which work is done by a force

If a force does an amount of work *W* in an amount of time Δt, the **average power** due to the force during that time interval is

$$P_{avg} = \frac{W}{\Delta t} \qquad \text{(average power).}$$

Instantaneous power *P* is the instantaneous time rate of doing work

$$P = \frac{dW}{dt} \qquad \text{(instantaneous power).}$$

For a particle moving along a straight line and is acted on by a constant force *F* directed at some angle ϕ.

$$P = \frac{dW}{dt} = \frac{F \cos \phi \, dx}{dt} = F \cos \phi \left(\frac{dx}{dt} \right),$$

$$= Fv \cos \phi.$$

$$P = \vec{F} \cdot \vec{v} \qquad \text{(instantaneous power).}$$

CHECKPOINT

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Units of power

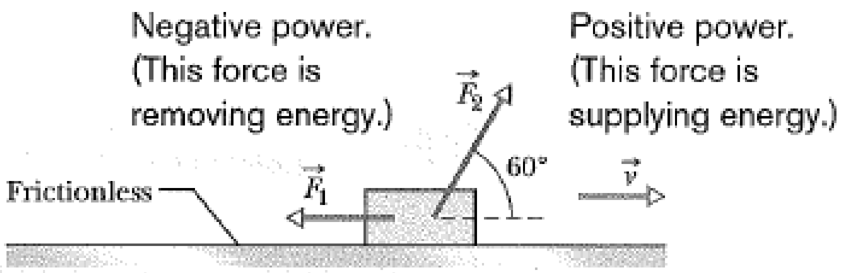
$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}$$

Work is then expressed as power multiplied by time, as in the common unit kilowatt-hour.

$$\begin{aligned} 1 \text{ kilowatt-hour} &= 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) \\ &= 3.60 \times 10^6 \text{ J} = 3.60 \text{ MJ}. \end{aligned}$$

Sample Problem

Figure 7-14 shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?



$$P_1 = F_1 v \cos \phi_1 = (2.0\text{ N})(3.0\text{ m/s}) \cos 180^\circ$$
$$= -6.0\text{ W}. \qquad \text{(Answer)}$$

$$P_2 = F_2 v \cos \phi_2 = (4.0\text{ N})(3.0\text{ m/s}) \cos 60^\circ$$
$$= 6.0\text{ W}. \qquad \text{(Answer)}$$

$$P_{\text{net}} = P_1 + P_2$$
$$= -6.0\text{ W} + 6.0\text{ W} = 0,$$

Q.27 Which one of the following choices is a unit of power?

(A) kg · m^2/s (B) N·m/s^3 (C) m/s (D) J/s (E) kg.m/s^2

Q.39 The SI unit of work is:

(A) kg · m^2/s (B) kg · m (C) kg · m/s (D) kg · m^2/s^2 (E) kg · s

Q.19 The SI unit of work is:

(A) kg·m^2/s^2 (B) kg · m (C) kg · m/s (D) kg · m^2/s (E) kg · s

Q.22 The kinetic energy of a moving object is:

(A) mvt^2 (B) $Fd\cos\theta$ (C) ma (D) mgh (E) $\frac{1}{2}mv^2$

Q.7 A man pulls a sled along a rough horizontal surface by applying a constant force $\bar{F}$ at an angle θ above the horizontal. In pulling the sled a horizontal distance d, the work done by the man is:

(A) Fd (B) Fcosθ (C) Fdsinθ (D) Fdtanθ (E) Fdcosθ

$$W = Fd\cos\varphi$$

Q.6 A particle moves 3 m in the positive x direction while being acted upon by a constant force $\vec{F} = (4\hat{i} + 2\hat{j} - 4\hat{k})$ N The work done on the particle by this force is:

(A) 20 J (B) 12 J (C) 30 J (D) -20 J (E) none of these

$$d = 3\hat{i} \qquad W = \vec{F} \bullet \vec{d} = \left(4\text{x}3 + 4\text{x}0 - 4\text{x}0\right) = 12\ \text{J}$$

Q.24 A single constant force $\vec{F} = (2\hat{i} - 5\hat{j})$ N acts on a 4 kg particle. If the particle moves from the origin with vector position $\vec{r} = (3\hat{i} - 5\hat{j})$ m. The work done by this force is:

(A) 19 J (B) 15 J (C) 6 J (D) 31 J (E) 25 J

$$W = \vec{F} \bullet \vec{r} = \left(2\text{x}3 + (-5)\text{x}(-5)\right) = 6 + 25 = 31\,\text{J}$$

Q.30 A moving particle of mass 2 kg, has kinetic energy 16 J. Its speed is:

(A) 4 m/s (B) 9.8 m/s (C) 10 m/s (D) 19.6 m/s (E) 3.16 m/s

$$K = \frac{1}{2}mv^2 \qquad \text{v} = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\text{x}16}{2}} = 4\ \text{m/s}$$

Q.34 A 800 kg car moves from rest to speed of 8 m/s. The net work done is:

(A) 1210 J (B) 2210 J (C) 25600 J (D) 1000 J (E) 10 J

$$W = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}\text{x}800(64-0) = 25600\text{ J}$$

Q.9 A 6 kg cart starts up an incline with a speed of 3 m/s and comes to rest up the incline. The total work done on the cart is:

(A) 6 J (B) 8 J (C) -27 J (D) -18 J (E) none of these

$$W = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}\text{x}6(0-9) = -27\text{ J}$$

Q.8 At time t = 0 a 2 kg particle has a velocity of $(4\hat{i} - 3\hat{j})$m/s . At t = 3s its velocity is $(2\hat{i} + 5\hat{j})$m/s . During this time the work done on it was:

(A) 4 J (B) -4 J (C) -12 J (D) -40 J (E) $(4\hat{i} + 36\hat{j})$J

$$v_i^2 = (4)^2 + (-3)^2 = 25\text{ m/s} \qquad v_f^2 = (2)^2 + (5)^2 = 29\text{ m/s}$$

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}\text{x}2(29-25) = 4\text{ J}$$

Q.20 If the restoring force is 20 N, Then the work done in stretching a spring a distance of 0.5 m is:

(A) 3 J (B) 6 J (C) 9 J (D) 12 J (E) 5 J

$$k = \frac{F}{x} = \frac{20}{0.5} = 40\ \text{N/m} \qquad W = \frac{1}{2}kx^2 = \frac{1}{2}\text{x}40\text{x}(0.5)^2 = 9\ \text{J}$$

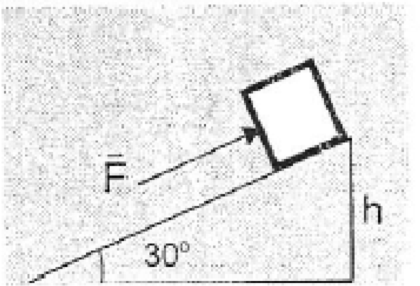
Q.33 A spring has a force constant of 150 N/m. The work done on the spring to stretch it by 0.02 m from its equilibrium position is:

(A) 0.03 J (B) 10 J (C) Zero (D) 1 J (E) 2 J

$$W = \frac{1}{2}kx_f^2 = \frac{1}{2}\text{x}150\text{x}(0.02)^2 = 0.03\ \text{J}$$

Q.23 In the figure a 10 kg box is pushed up a rough incline (μ_k=0.2) angle at 30° to the horizontal, a force of 50 N parallel to the incline is applied. As the box slides 2 m, the work done by the applied force is:

(A) -200 J (B) 50 J (C) 100 J (D) 200 J (E) Zero



$$W_F = Fd = 50\text{x}2 = 100\ \text{J}$$

Q.24 Refer to question 23, the work done by the normal force on the box is:

(A) 49 J (B) 110.84 J (C) Zero (D) 98 J (E) 101.84 J

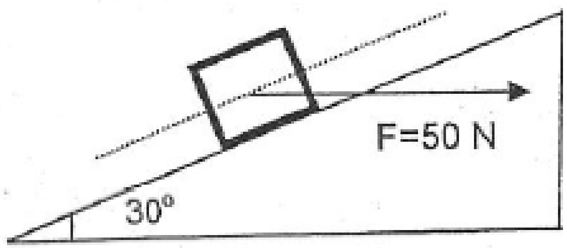
$$W_N = F_N d \cos 90 = 0\ \text{J}$$

Q.25 Refer to question 23, the work done by the gravity force on the box is:

(A) - 98 J (B) -117.6 J (C) 58.8 J (D) 9.8 J (E) Zero

$$W_g = (mg\sin\theta)(d)\cos 180 = -mgd\sin\theta = -10\text{x}9.8\text{x}2\text{x}\sin 30 = -98\ \text{J}$$

Q.36 In the figure a 5 kg crate is pushed up a frictionless inclined plane of 30° above the horizontal, by a horizontal force of 50 N. As the crate moves 2 m, the work done by the force is:



(A) Zero (B) 45.27 J (C) 24 J (D) 12 J (E) 86.60 J

$$W_F = Fd\cos\varphi = Fd\cos\theta = 50\text{x}2\text{x}\cos 30 = 86.6\ \text{J}$$

Q.37 Referring to question (36),the work done by the gravitational force of the crate is:

(A) -49 J (B) -5 J (C) -59 J (D) -980 J (E) Zero

$$W_g = (mg\sin\theta)\,(d)\cos 180 = -mgd\sin\theta = -5\text{x}9.8\text{x}2\sin 30 = -49\ \text{J}$$

Q.38 Referring to question (36), the work done by the normal force is:

(A) Zero (B) 980 J (C) 105 J (D) 49 J (E) 9.8 J

$$W_N = F_N d\cos 90 = 0$$

Q.28 If the work done on a particle is 24 J in 6 s. The power is:

(A) 36 W (B) 2 W (C) 1 W (D) 6 W (E) 4 W

$$P = \frac{W}{t} = \frac{24}{6} = 4\ \text{W}$$

Q.23 A ball of mass 0.25 kg is dropped from a height 35 m above the ground. The work done by gravitational force is:

(A) 5 J (B) 40 J (C) 85.75 J (D) 4 J (E) 1 J

$$W_g = mgd = 0.25\text{x}9.8\text{x}35 = 85.75\ \text{J}$$

Ch 8: Potential Energy and Conservation of Energy

I is energy associated with the configuration of a system of objects that exert forces on one another.

Work and Potential Energy

$$\Delta U = -W.$$

$$\Delta U_g = -W_g = mgd \ \text{(for rising)} = -mgd \quad \text{(for falling)}$$

$$\Delta U_s = -W_s = \frac{1}{2}kx^2$$

$$W = \int_{x_i}^{x_f} F(x)\,dx. \qquad \Delta U = -\int_{x_i}^{x_f} F(x)\,dx.$$

Gravitational Potential Energy

$$\Delta U = mg(y_f - y_i) = mg\,\Delta y.$$

Gravitational potential energy when the particle is at a certain height *y*

$$U - U_i = mg(y - y_i).$$

Usually, $U_i = 0$ and $y_i = 0$

$$U(y) = mgy \qquad \text{(gravitational potential energy).}$$

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

Sample Problem

A 2.0 kg sloth hangs 5.0 m above the ground

(a) What is the gravitational potential energy *U* of the sloth - Earth system if *y* = 0 to be

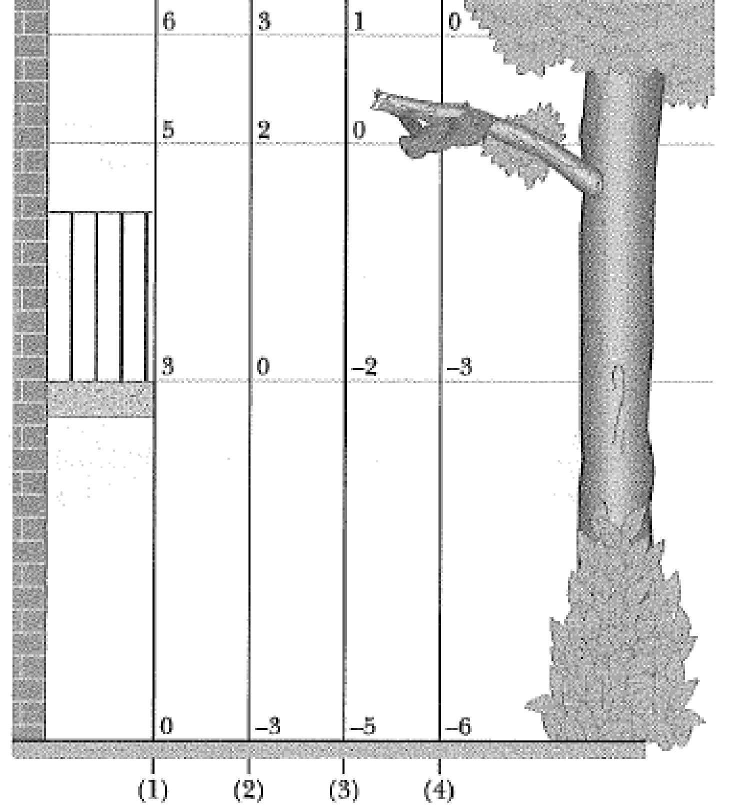
(1) at the ground,

(2) at a balcony floor that is 3.0 m above ground,

(3) at the limb

(4) 1.0 m above the limb?

Take the gravitational potential energy = 0 at *y* = 0



For choice (1) the sloth is at *y* = 5.0 m,

$$U = mgy = (2.0\ \text{kg})(9.8\ \text{m/s}^2)(5.0\ \text{m}) = 98\ \text{J}.$$

For the other choices, the values of *U* are

(2) $U = mgy = mg(2.0\ \text{m}) = 39\ \text{J},$

(3) $U = mgy = mg(0) = 0\ \text{J},$

(4) $U = mgy = mg(-1.0\ \text{m}) = -19.6\ \text{J} \approx -20\ \text{J}.$

(b) The sloth drops to the ground. For each choice of reference point, what is the change *t:..U* in the potential energy of the sloth - Earth system due to the fall?

$$\Delta U = mg\,\Delta y = (2.0\ \text{kg})(9.8\ \text{m/s}^2)(-5.0\ \text{m}) = -98\ \text{J}.$$

Elastic Potential Energy

$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

If $U_i = 0$ and $x_i = 0$, the potential energy associated with the spring at position x

$$U(x) = \tfrac{1}{2}kx^2 \qquad \text{(elastic potential energy).}$$

Q.36 A spring with spring constant of 40 N/m is compressed by a force a distance of 0.4 m. The potenti energy stored in the spring is:

(A) 0.5 J (B) 2.5 J (C) 3.2 J (D) 10 J (E) 200 J

$$U = \frac{1}{2}kx^2 = \frac{1}{2}\text{x}40\text{x}(0.4)^2 = 3.2\text{ J}$$

Q.28 A spring with spring constant of 20 N/m is compressed by force of 10 N. The potential energy stored in the spring is:

(A) 0.5 J (B) 2.5 J (C) 5 J (D) 10 J (E) 200 J

$$x = \frac{F_s}{k} = \frac{10}{20} = 0.5\text{ m} \qquad U_s = \frac{1}{2}kx^2 = \frac{1}{2}\text{x}20\text{x}(0.5)^2 = 2.5\text{ J}$$

Conservation of Mechanical Energy

The **mechanical energy** E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it

$$E_{mec} = K + U \qquad \text{(mechanical energy).}$$

The force transfers energy between K of the object and U of the system

$$\Delta K = W \qquad \Delta U = -W. \qquad \Rightarrow \qquad \Delta K = -\Delta U.$$

$$K_2 - K_1 = -(U_2 - U_1),$$

We can rewrite

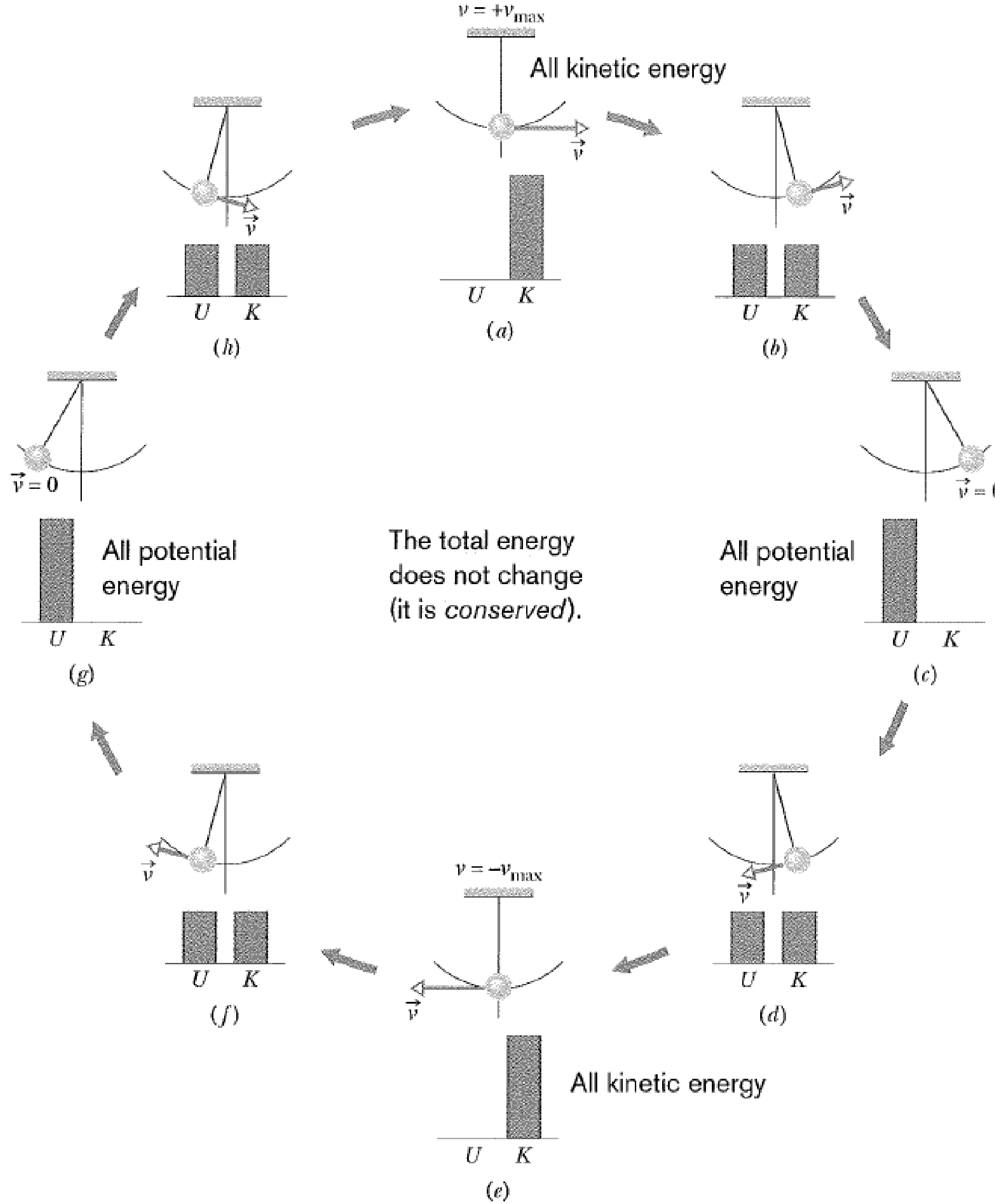
$$K_2 + U_2 = K_1 + U_1 \qquad \text{(conservation of mechanical energy).}$$

$$\begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system}\end{pmatrix} = \begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system}\end{pmatrix},$$

principle of conservation of mechanical energy

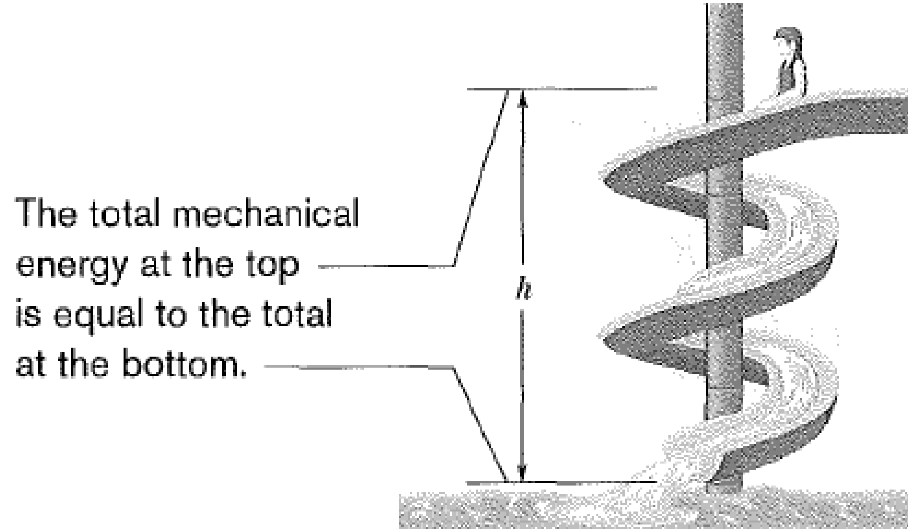
$$\Delta E_{mec} = \Delta K + \Delta U = 0.$$

Principle of conservation of mechanical energy for a pendulum swing The energy of the pendulum - Earth system is transferred back and forth between *K* and gravitational potential energy *U*, with $K + U =$ constant. If we know the gravitational potential energy when the pendulum bob is at its highest point, the kinetic energy is obtained of the bob at the lowest point (Fig. *8-7e*).



Sample Problem

A child of mass *m* is released from rest at the top of a water slide, at height *h* = 8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



Let the mechanical energy be $E_{mec,f}$ When the child is at the top of the slide and $E_{mec,b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{\text{mec},b} = E_{\text{mec},t}.$$

$$K_b + U_b = K_t + U_t,$$

$$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

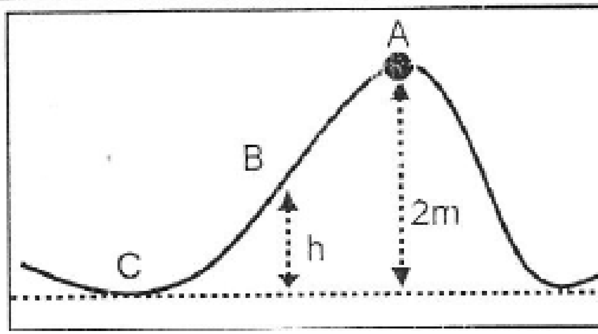
$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(8.5\ \text{m})}$$
$$= 13\ \text{m/s}.$$

Q.29 The potential energy of a falling object of weight w from height h is:

(A) mvt^2 (B) $Fd\cos\theta$ (C) mgh (D) ma (E) $\frac{1}{2}mv^2$

Q.39 A 0.2 kg bead slides from rest on a frictionless curved wire. (see Figure). The speed of the bead at **C** is:

(A) 6.26 m/s (B) 9.9 m/s (C) 2.52 m/s (D) 8.85 m/s (E) 6.6 m/s



$$\text{at A}: \quad v_A = 0 \quad \& \quad E_{mec,A} = U_{max} = mgh_A$$

$$\text{at C}: \quad h = 0 \quad \& \; E_{mec,C} = K_{max} = \frac{1}{2}mv_{max}^2$$

$$E_{mec,A} = E_{mec,C} \qquad mgh_A = \frac{1}{2}mv_{max}^2 \qquad v_{max} = \sqrt{2gh_A} = \sqrt{2x9.8x2} = 6.26\text{ m/s}$$

A 6 kg block is released from rest 80 m above the ground. When it has fallen 60 m its kinetic energy is

(a) 4000 J (b) 400 J (c) 380 J (d) 3528 J

At height h = 80 cm

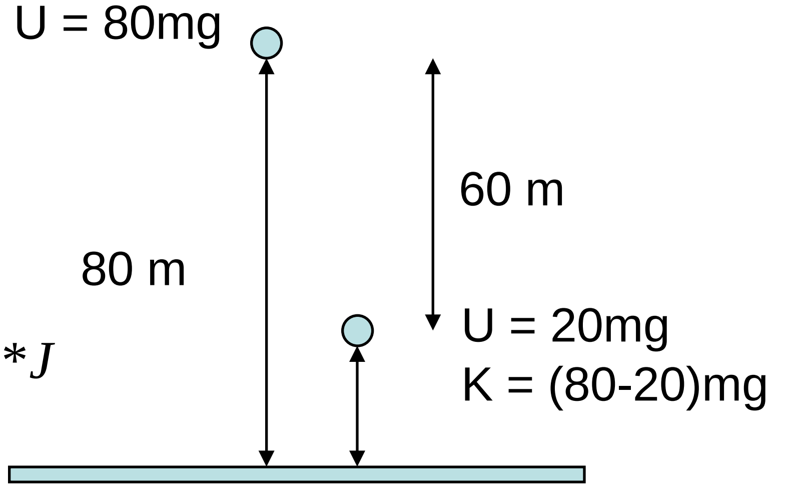
$$E_{mec} = U + K = U_{\max} = mgh = 6\text{x}9.8\text{x}80 = 4704\ \text{J}$$

At height h = 20 cm

$$E_{mec} = 4704\ \text{J} = U + K = mgh + \frac{1}{2}mv^2 = 6\text{x}9.8\text{x}20 + K = 1176 + K$$

$$K = 4704 - 1176 = 3528\ \text{J}$$

$$\text{K} = \text{mg(h - h')} = \text{mg(80 - 20)}$$



$$v_{\max} = \sqrt{2gh} = \sqrt{2x9.8x80} = ***J$$

$$v(at\ 20\ \text{m}) = \sqrt{2g(h-h')} = \sqrt{2x9.8x(80-60)} = ***J$$

Work Done on a System by an External Force

No Friction Involved

$$W = \Delta E_{mec} \quad \text{(work done on system, no friction involved)},$$

Friction Involved

$$Fd = \Delta E_{mec} + f_k d.$$

$$\Delta E_{th} = f_k d \quad \text{(increase in thermal energy by sliding)}.$$

If the friction results in change in the thermal energy

$$\Delta E_{th} = f_k d$$

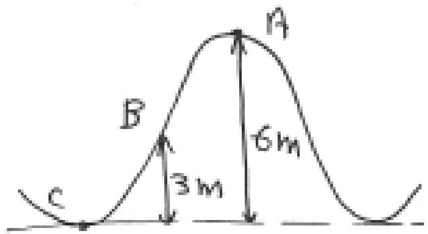
$$Fd = \Delta E_{mec} + \Delta E_{th}.$$

In general

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int},$$

ΔE: change in any other type of internal energy

Ex A 0.2 kg bead (خرزة) slides from rest on a frictionless curved wire. The speed of the bead at B is:



ⓐ 7.7 m/s ⓑ 9.8 m/s ⓒ 2.52 m/s ⓓ 8.85 m/s ⓔ 6.6 m/s

③ The greatest potential energy is at:

ⓐ Point A ⓑ Point B ⓒ Point C

at A: $v_A = 0$ & $E_{mec,A} = U_{max} = mgh_A$

at B: $E_{mec,B} = E_{mec,A} = mgh_A = K + U_B = \frac{1}{2}mv_B^2 + mgh_B$ $\quad \frac{1}{2}mv_B^2 = mgh_A - mgh$

$\frac{1}{2}v_B^2 = mg(h_A - h_B)$ $\quad v_B = \sqrt{2g(h_A - h_B)} = \sqrt{2x9.8x3} = 7.7$ m/s

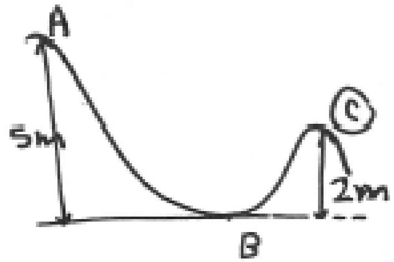
(2) What is the kinetic energy and speed at C?

at C: $h_C = 0$ & $E_{mec,C} = K_C = \frac{1}{2}mv_{max}^2 = E_{mec,A} = mgh_A$

$K_C = mgh_A = 0.2x9.8x6 = 11.8$ J

$v_{max} = \sqrt{2gh_A} = \sqrt{2x9.8x6} = 10.8$ m/s

Ex A 0.5 kg ball slides (يتزلق) from rest at A on a frictionless (أملس) curved (منحني) wire (سلك). Find the speed of the ball at point B



(a) 8 m/s (b) 4.9 m/s
(c) 9.9 m/s (d) 3 m/s

$$v_B = \sqrt{2gh_A} = \sqrt{2x9.8x5} = 9.9 \text{ m/s}$$

Ex In the previous (في السؤال السابق) question, the kinetic energy of the ball at point C is

(a) 10 J (b) 20 J (c) 12 J (d) 14.7 J (e) 0

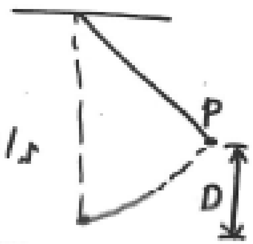
$$K_B = U_A - U_C = mgh_A - mgh_C = mg(h_A - h_C) = 0.5x9.8x3 = 14.7 \text{ J}$$

$$v_B = \sqrt{2g(h_A - h_B)} = \sqrt{2x9.8x3} = 7.7 \text{ m/s}$$

Ex: A simple pendulum consists of 2 kg mass attached to a string. It is released from rest at point p ($D = 1.54$ m) as shown. The speed at the lowest point is:

(a) 5.5 m/s (b) 6 m/s

(c) 6.5 m/s (d) 7 m/s (e) 7.5 m/s



$$\text{at p}: \quad E_{mec} = U_{max} = mgD$$

$$\text{at lowest point}: \quad E_{mec} = mgD = K_{max} = \frac{1}{2} m v_{max}^2$$

$$v_{max} = \sqrt{2gD} = \sqrt{2\text{x}9.8\text{x}1.54} = 5.5\ \text{m/s}$$

Ex: A 4 kg block starts (يبدأ) its motion (حركة) with 100 J of kinetic energy. The maximum (أقصى) distance (مسافة) the block move up a 30° smooth (أملس) incline (سطح مائل) is

ⓐ 5.1 m ⓑ 3.1 m ⓒ 2.55 m ⓓ 1.53 m ⓔ 6 m

$$U_i = 0 \quad K_i = 100\ \text{J} = \text{E}_{\text{mec}}$$

When the body stops: $\text{K}_\text{f} = 0 \quad \text{E}_{\text{mec}} = \text{U}_\text{f} = K_i$

$$100 = \text{mgh} = 4\text{x}9.8\text{xh} \quad \text{h} = \frac{100}{4\text{x}9.8} = 2.55$$

$$\sin\theta = \frac{h}{x} \quad x = \frac{h}{\sin\theta} = \frac{2.55}{\sin 60} = 2.94\ \text{m}$$