مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

النموذج الأول

Fundamental of Integral Calculus - MATH200 – 1st Exam	
Name:	**ID:**
Instructor:	**Section:**

Answer All Questions:

Question	Degree
1	/7
2	/6
3	/7
4	/5
Sum	/25

Question 1: Evaluate the following indefinite integrals:

1. (2 marks). $\int (x^2 + x)\, dx$.

$$\int x^2 + x \, dx = \int x^2 dx + \int x \, dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + C$$

2. (2 marks). $\int \frac{sinx}{cos^2 x} dx$.

في المحاضرات:

$$3. \int \frac{\cos x}{\sin^2 x} dx$$

$$= \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} dx$$

$$= \int \cot x \cdot \csc x \, dx$$

$$= -\csc x + C$$

$$\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$$

$$= \int \tan x \cdot \sec x \, dx$$

$$= \sec x + C$$

3. (3 marks). $\int x\sqrt{x-1}\, dx$, $(Let\ u = x - 1)$.

$$\int x \sqrt{x-1} \, dx$$

let $u = x - 1 \Rightarrow du = dx$

$x = u - 1$

The integral becomes:

$$\int (u+1)^2 \cdot u^{1/2} \, du$$

$$= \int (u^2 + 2u + 1)\, u^{1/2} \, du$$

$$= \int (u^{2+1/2} + 2u^{1+\frac{1}{2}} + u^{1/2})\, du$$

$$= \int (u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{1/2})\, du$$

$$= \frac{2}{7} u^{\frac{7}{2}} + \frac{4}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{7} (x-1)^{\frac{7}{2}} + \frac{4}{5} (x-1)^{\frac{5}{2}} + \frac{2}{3} (x-1)^{3/2} + C$$

Question 2: Evaluate the following :

1. (2 marks). $\frac{d}{dx}\left[\int_1^x u^3\,du\right]$

$$\frac{d}{dx}\left[\int_1^x u^3\,du\right] = x^3$$

By SFT:

$$\frac{d}{dx}\left[\int_1^x t^3\,dt\right] = x^3$$

2. (4 marks). $\int_0^6 f(x)\,dx$ if $f(x) = \begin{cases} x^2, & x < 2 \\ 3x-2, & x \geq 2 \end{cases}$.

Example: Evaluate $\int_0^6 f(x)\,dx$ if

$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 3x-2 & \text{if } x \geqslant 2 \end{cases}.$$

$$\int_0^6 f(x)\,dx = \int_0^2 x^2\,dx + \int_2^6 (3x-2)\,dx$$

$$= \left.\frac{x^3}{3}\right|_0^2 + \left.\frac{3}{2}x^2 - 2x\right|_2^6$$

$$= \left(\frac{2^3}{3} - 0\right) + (42 - 2)$$

Question 3: Calculate the following definite integrals:

1. (4 marks). $\int_0^{ln3} e^x(1+e^x)^{1/2}\,dx$, $[\,u = 1 + e^x].$

$$\int_0^{\ln 3} e^{x}(1+e^{x})^{1/2}\,dx$$

Let $u = 1+e^x \Rightarrow du = e^x\,dx$

If $x = 0 \Rightarrow u = 1+e^0 = 1+1 = 2$

$x = \ln 3 \Rightarrow u = 1+e^{\ln 3} = 1+3 = 4$

The integral becomes:

$$\int_2^4 u^{1/2}\,du = \left.\frac{u^{3/2}}{3/2}\right|_2^4$$

$$= \frac{2}{3}\left[4^{3/4} - 2^{3/2}\right]$$

$$= \frac{2}{3}\left[2^3 - (\sqrt{2})^3\right]$$

$$= \frac{2}{3}\left[8 - 2\sqrt{2}\right]$$

$$= \frac{16 - 4\sqrt{2}}{3}$$

في المحاضرات:

$$\int_4^9 x^2\sqrt{x}\,dx = \int_4^9 x^2 \cdot x^{1/2}\,dx$$

$$= \int_4^9 x^{5/2}\,dx$$

$$= \frac{2}{7}\left. x^{7/2}\right|_4^9$$

$$= \frac{2}{7}\left[9^{7/2} - 4^{7/2}\right]$$

$$= \frac{2}{7}\left[3^7 - 2^7\right]$$

$$= \frac{4118}{7}$$

2. (3 marks). $\int_4^9 x\sqrt{x}\,dx.$

$$\int_4^9 x\sqrt{x}\,dx = \int_4^9 x \cdot x^{1/2}\,dx =$$

$$\int_4^9 x^{3/2}\,dx = \left.\frac{x^{5/2}}{5/2}\right|_4^9$$

$$= \frac{2}{5}\left[9^{\frac{5}{2}} - 4^{\frac{5}{2}}\right]$$

$$= \frac{2}{5}\left[(\underbrace{9^{1/2}}_{\sqrt{9}})^5 - (\underbrace{4^{1/2}}_{\sqrt{4}})^5\right]$$

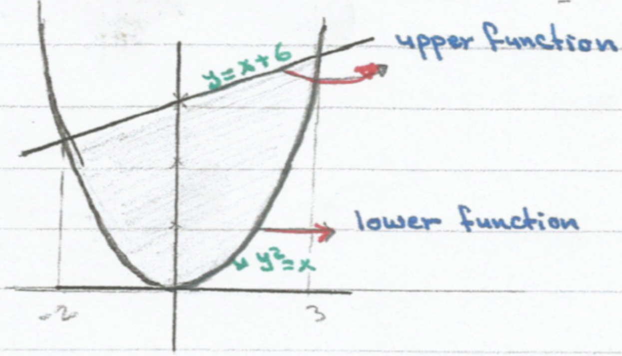
$$= \frac{2}{5}\left[3^5 - 2^5\right]$$

$$= \frac{2}{5}\left[211\right] = \frac{422}{5} = 84.4$$

***Question 4*:** (5 marks)

Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

2. Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$



First we have to find the limits of integration:

$$y = x^2 \rightarrow (1)$$

$$y = x + 6 \rightarrow (2)$$

By substitute (1) in (2), we get

$$x^2 = x + 6 \Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -2$$

Thus:

$$A = \int_{-2}^{3} (x+6) - (x^2)\, dx$$

$$= \frac{x^2}{2} + 6x - \frac{x^3}{3} \Big|_{-2}^{3}$$

$$= \frac{125}{6}$$

النموذج الثاني

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Answer All Questions:

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1	/7
2	/6
3	/7
4	/5
Sum	/25

Question 1: Evaluate the following indefinite integrals:

1. (2 marks) $\int (\sqrt{x} + x)dx.$

$$\int(\sqrt{x} + x)\,dx = \int \sqrt{x}\,dx + \int x\,dx$$

$$= \int x^{1/2}\,dx + \int x\,dx$$

$$= \frac{x^{3/2}}{3/2} + \frac{x^2}{2} + C$$

$$= 2/3\, x^{3/2} + \frac{1}{2}x^2 + C$$

2. (2 marks) $\int \frac{secx}{cosx} dx.$

$$\int \frac{\sec x}{\cos x}\,dx = \int \frac{\frac{1}{\cos x}}{\frac{\cos x}{1}}\,dx \qquad \sec x = \frac{1}{\cos x}$$

$$= \int \frac{1}{\cos x} \cdot \frac{1}{\cos x}\,dx$$

$$= \int \frac{1}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos x}\right)^2 dx = \int \sec^2 x\,dx = \tan x + C$$

3. (3 marks) $\int \frac{x^2}{x^2+1} dx.$

$$5.\ \int \frac{x^2}{x^2+1}\,dx$$

$$= \int \frac{x^2+1-1}{x^2+1}\,dx$$

$$= \int \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right)dx$$

$$= \int dx - \int \frac{1}{x^2+1}\,dx$$

$$= x - \tan^{-1} x + C$$

Question 2: Evaluate the following :

1. (4 marks) $\int_0^6 g(t)\,dt$ if $g(t) = \begin{cases} t^2, & t < 2 \\ 3t - 2, & t \geq 2 \end{cases}$

ample: Evaluate $\int_0^6 f(x)\,dx$ if

$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 3x - 2 & \text{if } x \geq 2 \end{cases}.$$

$$\int_0^6 f(x)\,dx = \int_0^2 x^2\,dx + \int_2^6 (3x-2)\,dx$$

$$= \frac{x^3}{3}\Big|_0^2 + \left(\frac{3}{2}x^2 - 2x\right)\Big|_2^6 \longrightarrow$$

طريقة الحساب

$$= \left(\frac{2^3}{3} - 0\right) + (48 - 8)$$

$$\frac{8}{3} + 40 = \frac{128}{3}$$

$$= 42.6$$

$$\frac{3}{2}[6^2 - 2^2] - 2[6-2]$$

$$= \frac{3}{2}[36-4] - 2[4]$$

$$= \frac{3}{2}[32] - 8$$

$$= 48 - 8$$

$$= 40$$

2. (2 marks) $\frac{d}{dx}[\int_1^x t^4\,dt]$.

$$\frac{d}{dx}\left[\int^x t^4\,dt\right] = x^4$$

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Question 3: Calculate the following definite integrals by a u-substitution:

1. (4 marks) $\int_0^8 x\sqrt{1+x}\,dx$, $[u = 1 + x]$.

$u = 1 + x \Rightarrow du = dx$

$x = u - 1$

if $x = 0 \Rightarrow u = 1 + 0 = 1$

$x = 8 \Rightarrow u = 1 + 8 = 9$

$$\int_0^8 x\sqrt{1+x}\,dx = \int_1^9 (u-1)\,u^{1/2}\,du = \int_1^9 u^{\frac{1}{2}+1} - u^{\frac{1}{2}}\,du$$

$$= \int_1^9 u^{\frac{3}{2}} - u^{1/2}\,du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \Bigg|_1^9$$

$$= \frac{2}{5}\left[(9)^{\frac{5}{2}} - (1)^{\frac{5}{2}}\right] - \frac{2}{3}\left[(9)^{3/2} - (1)^{3/2}\right]$$

الحل بالتفصيل في
آخر النموذج

$$= \frac{1192}{15}$$

2. (3 marks) $\int_0^{\pi/8} cos2xsin^5 2xdx$, $[u = sin2x]$.

$$\int_0^{\frac{\pi}{8}} \sin^5 2x \cos 2x\,dx$$

let $u = \sin 2x \Rightarrow du = 2\cos 2x\,dx$

$\Rightarrow \frac{1}{2}du = \cos 2x\,dx$

If $x = 0 \Rightarrow u = \sin 2(0) = \sin(0) = 0$

If $x = \frac{\pi}{8} \Rightarrow u = \sin 2\left(\frac{\pi}{8}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Then

$$\int_0^{\pi/8} \sin^5 2x \cos 2x\,dx = \int_0^{1/\sqrt{2}} u^5 \cdot \left(\tfrac{1}{2}du\right)$$

$$= \frac{1}{2}\,\frac{u^6}{6}\Bigg|_0^{1/\sqrt{2}}$$

$$= \frac{1}{2}\left[\left(\frac{1}{\sqrt{2}}\right)^6 - 0\right] = \frac{1}{96}$$

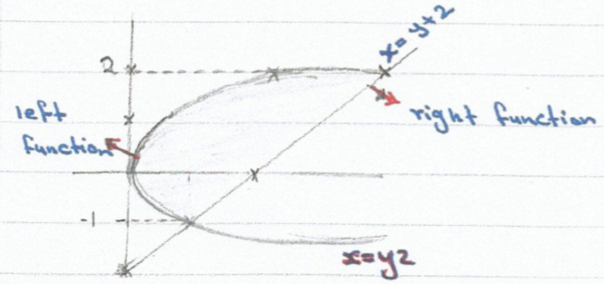
Question 4: (5 marks) $x = y+2$

Find the area of the region enclosed by $x = y^2$ and $y = x - 2$, integrating with respect to y.

تكامل بالنسبة لy

التكامل هنا بالنسبة ل y اذا نوجد الحدود على محور y وكما ذكرنا في المحاضرات لابد تكون الداول التي يتم التعويض عنها في القانون على الشكل X =

3. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$ integrating with respect to y.



First we have to find the limits of integration.

$y = x - 2 \rightarrow (1)$

$y^2 = x \rightarrow (2)$

by substitute (2) in (1). we get

$y = y^2 - 2 \Rightarrow y^2 - y - 2 = 0$

$\Rightarrow (y-2)(y+1) = 0$

$\Rightarrow y = 2$ or $y = -1$

Thus:

$$A = \int_{-1}^{2} \left(\begin{matrix}\text{right}\\ \text{function}\end{matrix}\right) - \left(\begin{matrix}\text{left}\\ \text{function}\end{matrix}\right) dy$$

$$= \int_{-1}^{2} (y+2) - y^2 \, dy$$

$$= \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_{-1}^{2}$$

$$= \frac{9}{2}$$

$$\int_0^8 x\sqrt{1+x}\,dx$$

$u = 1+x \Rightarrow du = dx$ $\quad$ if $x=0 \Rightarrow u = 1+0=1$

$x = u-1$ $\quad$ if $x=8 \Rightarrow u = 1+8=9$

$$\therefore \int_0^8 x\sqrt{1+x}\,dx = \int_1^9 (u-1)\,u^{1/2}\,du = \int_1^9 u^{\frac{1}{2}+1} - u^{1/2}\,du$$

$$= \int_1^9 u^{3/2} - u^{1/2}\,du$$

$$= \left.\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right|_1^9$$

$$= \frac{2}{5}\left[(9^{\frac{1}{2}})^5 - (1^{1/2})^5\right] - \frac{2}{3}\left[(9^{1/2})^3 - (1^{1/2})^3\right]$$

$$= \frac{2}{5}\left[3^5 - 1\right] - \frac{2}{3}\left[3^3 - 1\right]$$

$$= \frac{2}{5}\left[242\right] - \frac{2}{3}\left[26\right]$$

$$= \frac{484}{5} - \frac{52}{3}$$

$$= \frac{3(484) - 5(52)}{15}$$

$$= \frac{1452 - 260}{15}$$

$$= \frac{1192}{15}$$

النموذج الثالث

Fundamental of Integral Calculus - MATH200 – 1st Exam	
Name:	**ID:**
Instructor:	**Section:**

Answer All Questions:

Question	Degree
1	/8
2	/9
3	/8
Sum	/25

Question 1: Evaluate the following indefinite integrals:

1. (2 marks) $\int x^4\,dx$

$$\int x^4 dx = \frac{x^5}{5} + c$$

2. (2 marks) $\int \frac{2}{3x^5}\,dx$

$$\int \frac{2}{3x^5} dx = \frac{2}{3}\int \frac{1}{x^5} dx = \frac{2}{3}\int x^{-5} dx = \frac{2}{3}\frac{x^{-4}}{-4} + c$$

$$= -\frac{2}{12} x^{-4} + c$$

$$= -\frac{1}{6} x^{-4} + c$$

3. (2 marks) $\int 4\cos x\,dx$

$$\int 4\cos x\, dx = 4\int \cos x\, dx = 4\sin x + c$$

4. (2 marks) $\int x^{\frac{1}{3}}\,dx$

$$\int x^{1/3} dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c$$

$$= \frac{x^{\frac{4}{3}}}{4/3} + c = \frac{3}{4} x^{\frac{4}{3}} + c$$

Question 2: Answer the following questions:

1. (3 marks) $\int e^{\sin x}\cos x\,dx\,;\,u=\sin x$

$u=\sin x \Rightarrow du=\cos x\,dx$

$\therefore \int e^{\sin x}\cos x\,dx=\int e^{u}\,du= e^{u}+c = e^{\sin x}+c$

2. (3 marks) $\int_{-e}^{-1}\frac{1}{x}\,dx$

$\int_{-e}^{-1}\frac{1}{x}\,dx = \ln|x|\Big|_{-e}^{-1}$

$= \ln|-1| - \ln|-e|$

$= \ln(1) - \ln(e) = -1$

3. (3 marks) $\int\frac{dx}{1+3x^2}\,;\,u=\sqrt{3}\,x$

$\int\frac{dx}{1+3x^2} = \int\frac{dx}{1+(\sqrt{3}x)^2}$

Let $u=\sqrt{3}x \Rightarrow du=\sqrt{3}\,dx$

$\Rightarrow dx=\frac{1}{\sqrt{3}}\,du.$

The integral becomes:

$\int\frac{du/\sqrt{3}}{1+u^2} = \frac{1}{\sqrt{3}}\int\frac{1}{1+u^2}\,du$

$= \frac{1}{\sqrt{3}}\tan^{-1}u + c$

$= \frac{1}{\sqrt{3}}\tan^{-1}\sqrt{3}x + c$

Question 3:

1. (4 marks) Find the total area between the curve $y = 1 - x^2$ and the $x - axis$ over the interval [0,2] (See Figure) ;
   total area = $\int_0^2 |f(x)|\,dx$

3. Find the area between the curve $y = 1 - x^2$ and the x-axis over the interval $[0,2]$

$$\text{Total Area} = \int_0^1 (1-x^2)\,dx + \int_1^2 |1-x^2|\,dx$$

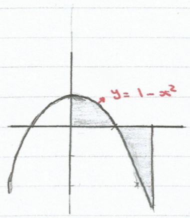
$$= \left[x - \frac{x^3}{3}\right]_0^1 + \left|\left[x - \frac{x^3}{3}\right]_1^2\right|$$

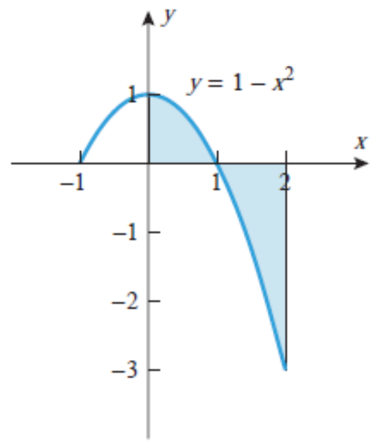
$$= \left(1 - \frac{1}{3}\right) + \left|\left(2 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)\right|$$

$$= \frac{2}{3} + \left|-\frac{2}{3} - \frac{2}{3}\right|$$

$$= \frac{2}{3} + \left|\frac{-4}{3}\right|$$

$$= \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2 \text{ units}^2.$$





2. (4 marks) Evaluate $\int_0^2 x(1+x^2)^3 \; dx \; ; u = 1 + x^2$

$$\int_0^2 x(x^2+1)^3\,dx$$

let $u = x^2 + 1 \Rightarrow du = 2x\,dx$

$\Rightarrow x\,dx = \frac{1}{2} du.$

If $x = 0 \Rightarrow u = 1$

$x = 2 \Rightarrow u = 5$

Then

$$\int_0^2 x(x^2+1)^3\,dx = \frac{1}{2}\int_1^5 u^3\,du$$

$$= \frac{1}{2}\,\frac{u^4}{4}\Big|_1^5$$

$$= \frac{1}{8}(5^4 - 1^4)$$

$$= 78.$$

تمنياتي للجميع بالتوفيق

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Mada Altiary