LIFE INSURANCE

COMPILED BY

Dr. MOHAMED GOUDA HOZAIEN

Professor 0F Risk Management and Insurance

Dr . Zeinab Abd el hamid Mohamed

Faculty of commerce

Cairo University

ACKNOWLEDGMENT

To my lovely sons:

Abd El-Rahman

Youssef

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Chapter(1)

RISK

<u>Objectives</u>

After studing this unit you should be able to :

- 1- Define Risk .
- Explain the Degree of Risk .
- Describe the classification of Risk .
- Understand the growing number and variety of Pure Risks .

<u>Contents:</u>

- The Concept of Risk .
- Definition Of Risk .
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CHAPTER (1)

RISK

A fire engine with its siren screaming roars down the street. A building in your neighborhood burns or you see an ambulance racing to the hospital. All these tragic events arouse your interest and emotions. After the noise and excitement have died down, you are grateful that the loss did not happen to you and you may even feel sorry for whoever suffered the loss.

The fact that these losses or similar events could happen to you, and the fact that you can't tell for sure whether or not they will, is a condition we call risk.

The Concept Of Risk

It would seen on the surface that the term *risk* is a simple enough notion. When someone states that there is risk in a particular situation, the listener understands what is meant: that in the given situation there is uncertainty about the outcome and the possibility exists that the outcome will be unfavorable. This loose, which implies a lack of knowledge about the future and the possibility of some adverse consequence, is satisfactory for conversational usage, but for our purpose a somewhat more rigid definition is desirable.

Economists, statisticians, decision theorists, and insurance theorists have long discussed the concepts of risk and uncertainty in an attempt to construct a definition of risk that is useful for analysis in each field of investigation. They have not been able to agree on single definition. A definition of risk that is suitable for the economist or statistician may be worthless as an analytic tool for the insurance theorist. Although the statistician, and the insurance theorist all use the term risk, they may each mean something entirely different.

Current Definitions Of Risk

If we were to survey the best-known insurance text-books, we would find a general lack of agreement concerning the definition of risk. Although the insurance theorists have not agreed on a universal definition, there are common elements in all the definitions: indeterminacy and loss.

The notion of an indeterminate outcome is implicit in all definitions of risk: the outcome must be in question. When risk is said to exist, there must always be at least two possible outcomes. If we know for certain that a loss will occur, there is no risk. At least one of the possible outcomes is undesirable. This may be a loss in the generally accepted sense, in which something the individual possesses is lost, or it may be a gain smaller than the gain that was possible.

Our Definition Of Risk

We define *risk* as a condition of the real world in which there is an exposure to adversity. More specifically.

“Risk is a condition in which there is a possibility of an adverse deviation from a desired outcome that is expected or hoped for”.

Note first that in this definition risk is a condition of the real world; it is a combination of circumstances in the external environment. Note also that in this combination of circumstances, there is a possibility of loss. When we say that an event is possible, we mean that it has a probability between zero and one: it is neither impossible nor definite. Note also that there is no requirement that the possibility be measurable: only that it must exist. We may or may not be able to measure the degree of risk, but the probability of the adverse outcome must be between zero and one.

The undesirable event is described as “an adverse deviation from a desired outcome that is expected or hoped for.” The reference to a desired outcome that is either expected or hoped for contemplates both individual and aggregate loss exposures. The individual hopes that adversity will not occur, and it is the possibility that this hope will not be met that constitutes risk. If you own a house, you hope that it will not catch fire. When you make a wager, you hope that the outcome will be favorable. The fact that the outcome in either event may be something other than what you hope constitutes the possibility of loss or risk.

In the case of an insurer, actuaries predict some specified number and amount of losses and charge a premium based on this expectation. The amount of predicted losses is the desired outcome that is expected by the insurer. For the insurer, risk is the possibility that losses will deviate adversely from what is expected.

Because the term *uncertainty* is often used in connection with the term *risk*, it seems appropriate to explain the relationship between the terms *risk* and *uncertainty*. The most widely held meaning of uncertainty refers to a state of mind characterized by doubt, based on a lack of knowledge about what will or will not happen in the future. Uncertainty, then, is simply a psychological reaction to the absence of knowledge about the future. The existence of risk is a condition or combination of circumstances in which there is a possibility of loss-creates uncertainty on the part of individuals when that risk is recognized.

Uncertainty varies with the knowledge and attitudes of the person. Different attitudes are possible for different individuals under identical conditions of the real world. When there is a possibility of loss, risk exists. Whether or not the person exposed to loss is aware of the risk.

The Degree Of Risk

It is intuitively obvious that there are some situations in which the risk is greater than in other situations. Just as we should agree on what we mean when we use the term risk, we should agree on the way (s) in which risk can be measured. Precisely what is meant when we say that one alternative involves “more risk” or “less risk” than another?. It would seem that the most commonly accepted meaning of degree of risk is related to the likelihood of occurrence. We intuitively consider those events with a high probability of loss to be “riskier” than those with a low probability. The degree of risk is measured by the probability of the adverse deviation. In the case of the individual, the hope is that no loss will occur, so that the probability of a deviation from what is hoped for (which is the measure of risk) varies directly with the probability that a loss will occur. The higher the probability that an event will occur, the greater the likelihood of a deviation from the outcome that is hoped for and the greater the risk, as long as the probability of loss is less than one.

In the case of a large number of exposure units, estimates can be made about the likelihood that a given number of losses will occur, and predictions can be made on the basis of these estimates. Here the expectation is that the predicted number of losses will occur. In the case of aggregate exposures, the degree of risk is not the probability of a single occurrence or loss; it is the probability of some outcome different from that predicted or expected. Insurance companies make predictions about losses that are expected to occur and charge a premium based on this prediction. For the insurance company, then, the risk is that its prediction will not be accurate. Suppose that based on past experience, an insurer estimates that 1 out of 1000 houses will burn if the company insures 100.000 houses, it might predict that 100 houses will burn out of the 100.000 insured, but it is highly unlikely that 100, and only 100, houses will burn. The actual experience will undoubtedly deviate from the expectation, and insofar as this deviation is unfavorable, the insurance company faces risk. Therefore, the insurance company makes a prediction not only with respect to the number of houses that will burn, but also estimates the range of error. The prediction might be that 100 losses will occur and that the range of possible deviation will be plus or minus 10. Some number of houses between 90 and 110 are expected to burn, and the possibility that the number will be more than 100 is the insurer’s risk. Students who have studied statistics will note that when one of the standard measures of dispersion (such as the standard deviation) is used, risk is measurable, and we can say that more risk or less risk exists in a given situation, depending on the standard deviation.

Risk Distinguished From Peril And Hazard

A *peril* is a cause of a loss. We speak of the peril of fire, or windstorm, or hail, or theft. Each of these is the cause of the loss that occurs. A *hazard*, on the other hand, is a condition that may create or increase the chance of a loss arising from a given peril, it is possible for something to be both a peril and a hazard. For instance, sickness is a peril causing economic loss, but it is also a hazard that increases the chance of loss from the peril of premature death. Hazards are normally classified into three categories:

Physical hazards consist of those physical properties that increase the chance of loss from the various perils. Examples of physical hazards that increase the possibility of loss from the peril of fire are the type of construction, the location of the property, and the occupancy of the building.

Moral hazard refers to the increase in the probability of loss that results from dishonest tendencies in the character of the insured person. A dishonest person, in the hope of collecting from the insurance company, may intentionally cause a loss or may exaggerate the amount of a loss in an attempt to collect more than the amount to which he or she is entitled.

***Morale hazard*,** not to be confused with moral hazard, acts to increase losses where insurance exists, not necessarily because of dishonesty, but because of a different attitude toward losses that will be paid by insurance. When people have purchased insurance, they may have a more careless attitude toward preventing losses or may have a different attitude toward the cost of restoring damage.

In addition to these three traditional types of hazard, a fourth hazard - the legal hazard – should be recognized. Legal hazard refers to the increase in the frequency and severity of loss that arises from legal doctrines enacted by legislatures and created by the courts.

Although legal hazard is greatest in the field of legal liability, it also exists in the case of property exposures. In jurisdictions where building codes require that new buildings conform to statutory requirements, the destruction of a building that dose not meet the requirements may force an owner to incur additional costs in reconstruction, thereby increasing the exposure to loss.

Classification Of Risk

Risks may be classified in many ways; however, there are certain distinctions that are particularly important for purposes. These include the following.

financial and nonfinancial Risks: in its broadest context, the term risk includes all situations in which there is an exposure to adversity. In some cases this adversity involves financial loss, while in others it dose not. There is some element of risk in every aspect of human endeavor, and many of these risks have no (or only incidental) financial consequences. In this text we are concerned whit those risks that involve a financial loss.

***Static and Dynamic Risk*s:** A second important distinction is between static and dynamic risks. Dynamic risk are those resulting from changes in the economy. Changes in the price level consumer tastes, income and output, and technology may cause financial loss to members of the economy. These dynamic risks normally benefit society over the long run, since they are the result of adjustments to misallocation of resources. Although these dynamic risks may affect a large number of individuals, they are generally considered less predictable than static risks, since they do not occur with any precise degree of regularity.

Static risks involve those losses that would occur even if there were no changes in the economy. If we could hold consumer tastes, output and income, and the level of technology constant, some individuals would still suffer financial loss. These losses arise from causes other than the changes in the economy, such as the perils of nature and the dishonesty of other individuals. Unlike dynamic risks, static risks are not a source of gain to society. Static losses involve either the destruction of the asset or a change in its possession as a result of dishonesty or human failure. Static losses tend to occur with a degree of regularity over time and, as a result, are generally predictable. Because they are predictable, static risks are more suited to treatment by insurance than are dynamic risks.

Fundamental and Particular Risks**:** The distinction between fundamental and particular risks is based on the difference in the origin and consequences of the losses. Fundamental risks involve losses that are impersonal in origin and consequence. They are group risks, caused for the most part by economic, social, and political phenomena, although they may also result from physical occurrences. They affect large segments or even all of the population. Particular risks involve losses that arise out of individual events and are felt by individuals rather than by the entire group. They may be static or dynamic. Unemployment, war, inflation, earthquakes, and floods are all fundamental risks. The burning of a house and the robbery of a bank are particular risks .

Since fundamental risks are caused by conditions more or less beyond the control of the individuals who suffer the losses and since they are not the fault of anyone in particular, it is held that society rather than the individual has a responsibility to deal with them. Although some fundamental risks are dealt with through private insurance, usually, some from of social insurance or government transfer program is used to deal with fundamental risks.Unemployment and occupational disabilities are fundamental risks treated through social insurance. Flood damage or earthquakes make a district a disaster area eligible for federal funds .

Pure and Speculative Risks: One of the most useful distinctions is that between pure risk and speculative risk. Speculative risk describes a situation in which there is a possibility of loss, but also a possibility of gain. Gambling is a good example of speculative risk. Pure risk, in contrast, is used to designate those situations that involve only the chance of loss or no loss. For example the possibility of loss surrounding the ownership of property.

The distinction between pure and speculative risk is an important one, because normally only pure risks are insurable. Insurance is not concerned with the protection of individuals against those losses arising out of speculative risks. Speculative risk is voluntarily accepted because of its two-dimensional nature, which includes the possibility of gain .

Classification of pure risks: These are also static risk. Pure risks that exist for individuals and business firms can be classified under one the following:

1. *personal risks.* These consist of the possibility of loss of income or assets as a result of the loss of the ability to earn income. In general, earning power is subject to four perils: (a) premature death, (b) dependant old age, (c) sickness or disability, and (d) unemployment.

2. *property risks.* Anyone who owns property faces property risks simply because such possessions can be destroyed or stolen. Property risks embrace two distinct types of losses: direct loss and indirect or "consequential" loss. Direct loss is the simplest to understand. If a house is destroyed by fire, the owner loses the value of the house. This is a direct loss. The property owner no longer has a place to live and during the time required to rebuild the house, it is likely that the owner will incur additional expenses living somewhere else. This loss of use of the destroyed asset is an indirect, or "consequential" loss.

   Property risks, then can involve two types of Losses: (a) the loss of the property and (b) loss of use of the property resulting in lost income or additional expenses.

3. *Liability risks.* The basic peril in the liability risk is the unintentional injury of other persons or damage to their property through negligence or carelessness; however, liability may also result from intentional injuries or damage. Under our legal system, the laws provide that one who has injured another, or damaged another's property through negligence or otherwise, can be held responsible for the harm caused. Liability risk therefore involve the possibility of loss of present assessed or future income as a result of damages assessed or legal liability arising out of either intentional or unintentional torts.

The Burden Of Risk

Regardless of the manner in which risk is defined, the greatest burden in connection with risk is that some losses will actually occur. When a house is destroyed by fire, or money is stolen, or a wage earner dies, there is a financial loss. When someone is negligent and that negligence results in injury to a person or damage to property, there is a financial loss. These losses are the primary burden of risk and the primary reason that individuals attempt to avoid risk or alleviate its impact.

In addition to the losses themselves, there are other detrimental aspects of risk. The uncertainty as to whether the loss will occur requires the prudent individual to prepare for its possible occurrence. In the absence of insurance, one way this can be done is to accumulate a reserve, fund to meet the losses if they do occur. Accumulation of such a reserve fund carries an opportunity cost, for funds must be available at the time of the loss and must therefore be held in a highly liquid state. the return on such funds will presumably be less than if they were put to alternate uses. If each property owner accumulates his or her own fund, the amount of funds held in such reserves will be greater than if the funds are amassed collectively.

Furthermore, the existence of risk may have a deterrent effect on economic growth and capital accumulation. Progress in the economy is determined to a large extent by the rate of capital accumulation, but the investment of capital involves risk that is distasteful. Investors as a class will incur the risks of a new undertaking only if the return on the investment is sufficiently high to compensate for both the dynamic and static risks. The cost of capital is higher in those situations where the risk is greater, and the consumer must pay the resulting higher cost of the goods and services or they will not be forthcoming.

Finally, the uncertainty connected with risk usually produces a feeling of frustration and mental unrest. This is particularly true in the case of pure risk. Speculative risk is attractive to many individuals. The gambler obviously enjoys the uncertainty connected with wagering more than the certainty of not gambling- otherwise he or she would not gamble. But here it is the possibility of gain or profit, which exists only in the speculative risk category, that is attractive. In the case of pure risk, where there is no compensating chance of gain, risk is distasteful. Most people hope that misfortunes will not befall them and that their present state of well- being will continue. While they hope that no misfortune will occur, people are nevertheless likely to worry about possible mishaps. This worry, which induces a feeling of diminished well- being, is an additional burden of risk.

The Growing Number And Variety Of Pure Risks

From the dawn of civilization, humans have faced the possibility of loss. Our ancestors confronted an environment characterized by incredible perils and hazards. The earliest perils giving rise to risk were those of nature and predators (including not only savage beasts but human predators as well). Humans learned to anticipate and prepare for adversity, both collectively and individually. They built shelter and they saved for the future. This provided protection from the elements and savage beasts, but it created new risks. Structures constructed for protection were vulnerable to damage and saving meant accumulation of wealth, which inevitably created new risks. Those who saved were exposed to the predatory inclinations of those who did not (an exposure that continues to the present day). Despite progress in learning how to deal with risks, the challenge of dealing with risk continued to grow. As new ways of addressing risk are found, new risks appear, often as a result of progress.

Increasing Severity Of Losses

As might be expected, with the increasing array of risks, the dollar amount of losses arising from accidents has also increased. Interestingly, however, the increasing dollar amount of losses is not solely a function of the increasing number of risks. Even those losses that arise from the perils of nature- windstorms, earthquakes and floods- have exhibited an increasing severity. Nor is the increasing severity merely a reflection of inflation; the dollar total from these losses continues to increase even when adjusted for inflation. Although the number of earthquakes, floods, and windstorms occur at essentially the same rate as in the past, each new catastrophe seems to exceed previous losses. The reason, of course, is that there is simply more wealth, more investment, and more assets exposed to loss. As business has become more capital intensive, as the technology of production equipment becomes more costly, capital investment increases. With the growth in capital investment, the risk of financial loss also increases.

Questions For Review

Q1: multiple choice : select the best answer a, b or c:

1) Risk is :

a- The cause of loss .

b- Condition in which there is a possibility of an adverse deviation from a desired outcome that is expected.

c- A state of mind characterized by doubt, based on lack of knowledge about what will or will not happen in the future.

2) pure risks are different from speculate risks in that :

a- They are more difficult to insure.

b- They involve both the possibility of loss and gain.

c- They involve only the possibility of loss.

3) moral hazard :

a- Involves dishonesty.

b- Is only carelessness about protecting exposures.

c - None of the above.

4) Morale hazard :

a- Involves dishonesty.

b- Is really the same as moral hazard.

c- None of the above.

5) Hazard is :

a- The cause of loss.

b- Increases the probability of loss or its severity.

c- All of the above.

6) Law of large numbers is an important concept in insurance because :

a- It results in reduced individual risk.

b- It permits insurers to predict future losses more accurately.

c- None of the above.

Q2: Indicate whether each of the following statement is true or false and correct the false one (S) :

1- The statistician and the insurance theorist use the term risk, with the same meaning.

2- Moral hazard has no effect on risks.

3- peril is a synonym for hazard.

4- the major difference between a pure risk and a speculative risk is that the former doesn't involve the possibility of gain while the later does.

5- The degree of risk is measured by the probability of the adverse deviation.

6- The higher the probability that an event will occur, the lesser the likelihood of a deviation from the outcome that is hoped for and the greater the risk.

7- Physical hazard is a condition present in the exposure that increases the chance of loss.

8- Dynamic risks involve those losses that would occur even if there were no changes in the economy.

9- peril is defined as a catastrophic occurrence.

10- degree of risk relates most closely to loss distribution.

Chapter (2)
INTRODUCTION TO RISK MANAGEMENT

Objectives

After studing this unit you should be able to :

- Define Risk Management.
- Explain the Characteristics Of Risk Management .
- Describe Risk Management Tools .
- Understand the Risk Management Process .

Contents:

- Risk Management Defined .
- Risk Management Tools .
- Distinguishing Characteristics Of Risk Management .
- The Risk Management Process .

CHAPTER (2)

INTRODUCTION TO RISK MANAGEMENT

Risk management is a scientific approach to the problem of pure risk, which has as its objective the reduction and elimination of pure risks facing the business firm. Risk management evolved from the field of corporate insurance buying, and is now recognized as a distinct and important function for all businesses and organizations. Many business firms have highly trained individuals who specialize in dealing with pure risk. In some cases this is a fulltime job for one person, or even for an entire department within the company. Those who are responsible for the entire program of pure risk management are risk managers.

Risk Management Defined

As a relatively new discipline, risk management has been defined in a variety of ways by different writers and users of the term. Although they vary in detail, most definitions offered thus far stress two points: first, that risk management is concerned principally with pure risk, and, second, that it is a process or function that involves managing those risks. We propose the following definition of risk management.

Risk management is a scientific approach to dealing with pure risks by anticipating possible accidental losses and designing and implementing procedures that minimize the occurrence of loss or the financial impact of the losses that do occur .

Note first that risk management is described as a "scientific approach" to the problem of pure risks . It must be admitted that risk management is not a science in the same sense as the physical sciences are, any more than management itself is a science . A science is a body of knowledge based on laws and principles that can be used to predict outcomes.

Risk management derives its rules (laws) from the general knowledge of experience, through deduction, and from precepts drawn from other disciplines, particularly decision theory. Although risk management is not a science, it uses a scientific approach to the problem of pure risk. This scientific approach that distinguishes risk management from earlier approaches to risk decisions can be illustrated by contrasting it with those earlier approaches .

Risk Management Tools

Our definition of risk management states that it deals with risk by designing and implementing procedures that minimize the occurrence of loss or the financial impact of the losses that do occur. This indicates the two broad techniques that are used in risk management for dealing with risks. In the terminology of modern risk management, the techniques for dealing with risk are grouped into two broad approaches: risk control and risk financing. Risk control focuses on minimizing the risk of loss to which the firm is exposed, and includes the techniques of avoidance and reduction. Risk financing concentrates on arranging the availability of funds to meet losses arising from the risks that remain after the application of risk control techniques, and includes the tools of retention and transfer .

Risk control :

Broadly defined, *risk control* consists of those techniques that are designed to minimize, at the least possible costs, those risks to which the organization is exposed. Risk control methods include risk avoidance, and the various approaches at reducing risk through loss prevention and control efforts .

Risk Avoidance :

Technically, avoidance takes place when decisions are made that prevent a risk from even coming into existence. Risks are avoided when the organization refuses to accept the risk. Even for an instant. The classic example of risk avoidance by a business firm is a decision not to manufacture a particularly dangerous product of the inherent risk. Given the potential for liability claims that may result if a consumer is injured by a product, some firms judge that the risk is not worth the potential gain.

Risk avoidance should be used in those instances in which the exposure has catastrophic potential and the risk cannot be reduced or transferred. Generally, these conditions will exist in the case of risks for which both the frequency and the severity are high and neither can be reduced .

Although avoidance is the only alternative for dealing with some risks, it is a negative rather than a positive approach. Personal advancement of the individual and progress in the economy both require risk taking. If avoidance is used extensively, the firm may not be able to achieve its primary objectives .

Risk Reduction :

Risk reduction consists of all techniques that are designed to reduce the likelihood of loss, or the potential severity of those losses that do occur. It is common to distinguish between those efforts aimed at preventing losses from occurring and those aimed at minimizing the severity of loss if it should occur, referring to them respectively as loss prevention and loss control. As the designation implies, the emphasis of loss prevention is on preventing the occurrence of loss, that is, on controlling the frequency. Other risk reduction techniques focus on lessening the severity of those losses that actually do occur, such as, the installation of sprinkler systems. These are loss control measures. Other methods of controlling severity include segregation or dispersion of assets and salvage efforts. Dispersion of assets will not reduce the number of fires or explosions that may occur, but it can limit the potential severity of the losses that do occur .

Another distinction is sometimes made between the "engineering approach" to loss prevention and control, in which the principal emphasis is on the removal of hazards that may cause accidents, and the human behavior approach, "in which the elimination of unsafe acts is stressed. This distinction is based on the focus of control measures and represents two schools of thought regarding the emphasis in loss prevention and control. The human behavior approach is based on the view that since most accidents result from human failure, the most effective approach to loss prevention is to change people's behavior. The engineering approach, in contrast, emphasizes systems analysis and mechanical design, aimed at protecting people from careless acts that are viewed as perhaps inevitable. National Safety council ads on television and in-print media urging drivers not to drink typify the human behavior approach. Air bags in automobiles, which are activated without human intervention, typify the engineering approach .

A final way of classifying risk reduction measures is by the timing of their application, which may be prior to the loss event, at the time of the event, or after the loss event. Safety inspections and drivers training classes illustrate measures that are designed to prevent the occurrence before losses occur. Seat belts and air bags are designed to minimize the amount of damage at the time an accident occurs. Post-event loss prevention measures related to auto accidents include negotiating with injured persons for an out-of-court settlement or a stern defense in litigation .

Risk Financing :

Risk financing, in contrast to risk control, consists of those techniques that focus on arrangements designed to guarantee the availability of funds to meet those losses that do occur. Fundamentally, risk financing takes the form of retention or transfer. All risks that cannot be avoided or reduced must, by definition, be transferred or retained. Frequently, transfer and retention are used in combination for a particular risk, with a portion of the risk retained and a part transferred .

Risk Retention :

Risk retention is perhaps the most common method of dealing with risk. Individuals, like organization, face an almost unlimited number of risks, in most cases, nothing is done about them. Risk retention may be conscious or unconscious (i.e.

intentional or unintentional). Because risk retention is the "residual" or "default" risk management technique, any exposures that are not avoided, reduced, or transferred are retained. This means that when nothing is done about a particular exposure, the risk is retained. Unintentional (unconscious) retention occurs when a risk is not recognized. The individual or organization unwittingly and unintentionally retains the risk of loss arising out of the exposure. Unintentional retention can also occur in those instances in which the risk has been recognized, but when the measures designed to deal with it are improperly implemented. If, for example, the risk manager recognizes the exposure to loss in connection with a particular exposure and intends to transfer that exposure through insurance, but then acquires an insurance policy that does not fully cover the loss, the risk is retained .

Unintentional risk retention is always undesirable. Because the risk is not perceived, the risk manager is never afforded the opportunity to make the decision concerning what should be done about it on a rational basis. Also, when the unintentional retention occurs as a result of improper implementation of the technique that was designed to deal with the exposure, the resulting retention is contrary to the intent of the risk manager .

Risk retention may be voluntary or involuntary. Voluntary retention results from a decision to retain risk rather than to avoid or transfer it. Involuntary retention occurs when it is not possible to avoid retention occurs when it is not possible to avoid reduce, or transfer the exposure to an insurance company. Uninsurable exposures are an example of involuntary retention.

A final distinction that may be drawn is between funded retention and unfunded retention. In a funded retention program, the firm earmarks assets and holds them in some liquid or semi liquid form against the possible losses that are retained. The need for segregated assets to find the retention program will depend on the firm's each flow and the size of the losses that may result from the retained exposure .

Risk Transfer :

Transfer may be accomplished in a variety of ways. Transfer of risk through the purchase of insurance contracts is, of course, a primary approach to risk transfer. In consideration of a specific payment (the premium) by one party, the second party contracts to indemnify the fist party up to a certain limit for the specified loss that may or may not occur. Risk transfer may also take the form of contractual arrangements such as hold-harmless agreements, in which one individual assumes another's possibility of loss .

Distinguishing Characteristics Of Risk Management

A better understanding of the risk management function and its place in the organization can be gained by distinguishing risk management from general management and from insurance management .

Risk Management Distinguished from General Management : Initially, risk management should be distinguished from general management. Risk management differs from general management in its scope. Risk management deals with risk, but so too does general management. The difference, of course, is in the type of risks with which general management and risk management deal. General management is responsible for dealing with all risks facing the organization, including both speculative and pure risks .

The risk manager's area of responsibility is narrower in scope, and is limited primarily to pure risks only . Thus while the responsibility of "general" management is to conserve the assets of the organization and maximize profit, the risk management is responsible for a part of this general responsibility. More specifically, the risk manager is responsible for that segment of general management's mission that relates to pure risks .

Risk Management Distinguished from Insurance Management

In addition to its relationship to general management, risk management should also be distinguished from insurance management. Risk management is broader than insurance management, in that it deals with both insurable and uninsurable risks and the choice of the appropriate techniques for dealing with these risks. Because risk management evolved from insurance management, the focus of some risk managers has been primarily with insurable risk. Properly, the focus should include all pure risks, insurable and uninsurable .

The difference is obviously one of emphasis. The insurance management philosophy views insurance as the accepted norm, and retention or no insurance must be justified by a premium reduction that is, in some sense or another, "big enough". Under the risk management philosophy, it is insurance that must be justified. Since the cost of insurance must generally exceed the average losses of those who are insured, the risk manager believes that insurance is a last resort, and should be used only when necessary .

The Risk Management Process

The risk management process can be divided into a series of individual steps that must be accomplished in managing risks. Identifying these individual steps helps to guarantee that important phases in the process will not be overlooked. Although it is useful for the purpose of analysis to discuss each of these steps separately, it should be understood that in actual practice the steps tend to merge with one another. The six steps in the risk management process are :

1 – Determination of objectives .

2 – Identification of risks .

3 – Evaluation of risks .

4 – Considering alternatives and selecting the risk treatment device.

5 – Implementing the decision .

6 – Evaluation and review .

1- Determination of Objectives:

The first step in the risk management process is the determination of the objectives of the risk management program : deciding precisely what it is that the organization would like its risk management program to do. Despite its importance, determining the objectives of the program is the step in the risk management process that is most likely to be overlooked. As a consequence, the risk management efforts of many firms are fragmented and inconsistent. Many of the defects in risk management programs stem from an ambiguity regarding the objectives of the program.

The primary Objective of Risk Management: The first objective of risk management, like the first law of nature, is survival-to guarantee the continuing existence of the organization as an operating entity in the economy. The primary goal of risk management is not to contribute directly to the other goals of the organization-whatever they may be. Rather it is to guarantee that the attainment of these other goals will not be prevented by losses that might arise out of pure risks. This means that the most important objective is not to minimize costs or to contribute to the profit of the organization. Nor is it to comply with legal requirements or to meet some nebulous responsibility related to social responsibility of the firm. It can and does do all of these things, but they are not the principal reason for its existence. The main objective of risk management is to preserve the operating effectiveness of the organization.

The risk management objective must reflect the uncertainty inherent in the risk management situation. Because one cannot know what losses will occur and what the amount of such losses will be, the arrangements made to guarantee survival in the event of loss must reflect the worst possible combination of outcomes. If a loss occurs and, as a result, the organization is prevented from pursuing its other objectives, it is clear that the risk management objective has not been achieved. While not immediately obvious. It is equally true that the risk management objective has not been achieved when there are unprotected loss exposures that could prevent the organization from pursuing its other objectives should the loss occur, even it the loss does not occur .

Once the objectives have been identified, they should be formally recognized in a risk management policy. A formal risk management policy statement provides a basis for achieving a logical and consistent program by offering guidance for those responsible for programming and buying the firm's insurance .

2- Identifying risk exposures:

Obviously, before anything can be done about the risks an organization faces, someone must be aware of them. Risk identification is the most difficult step in the risk management process. It is difficult because it is a continual process and because it is virtually impossible to know when it has been done completely. It is difficult to generalize about the risks that a given organization is likely to face because differences in operations and conditions give rise to differing risks .

Most risk managers use some systematic approach to the problem of risk identification .

Risk Identification Techniques : the first step in risk identification is to gain as thorough a knowledge as possible of the organization and its operations. The risk manager needs a general knowledge of the goals and functions of the organization: what it does and where it does it. This knowledge can be gained through inspections, interviews with appropriate persons within and outside the organization, and by an examination of internal records and documents .

Analysis of the firm's financial statements, in particular, can aid in the process of risk identification. The asset listing in the balance sheet may alert the risk manager to the existence of assets that might otherwise be overlooked. The income and expense classification in the income statement may likewise indicate areas of operation of which the risk manager was unaware .

Another tool that is useful in risk identification is a flow chart. A flow chart of an organization's internal operations views the firm as a processing unit and seeks to discover all the contingencies that could interrupt its processes. These might include damage to a strategic asset located in a bottleneck within the firm's operations or the loss of the services of a key individual or group through disability, death, or resignation. When extended to include the flow of goods and services to and from customers and suppliers, the flow chart approach to risk identification can highlight potential accidents that can disrupt the firm's activities and its profits .

Internal Communication System: To identify new risks, the risk manager needs a far-reaching information system that yields current information on new developments that may give rise to risk .

Tools Of Risk Identification: Exposure identification is an essential phase of both risk management and insurance management. Because insurance management is the older field, the technique of identifying insurable exposures was already highly developed when the risk management movement began. Insurance companies created insurance policy checklists, which identify the various risks for which they offered coverage .

A few of the more important tools used in risk identification include risk analysis questionnaires, exposure checklists, and insurance policy checklists. These, combined with a vivid imagination and a thorough understanding of the organization's operations, can help to guarantee that important exposures are not overlooked .

Risk analysis questionnaires are designed to assist in identifying risks facing an organization. They do this by leading the user through a series of penetrating questions, the answers to which indicate hazards and conditions that give rise to risk. A risk exposure checklist, which is simply a listing of common exposures.

Insurance Policy Checklists: Include a catalogue of the various policies or types of insurance that a given business might need .

3- Evaluating Risks :

Once the risks have been identified, the risk manager must evaluate them. Evaluation implies some ranking in terms of importance, and ranking suggests measuring some aspect of the factors to be ranked. In the case of loss exposures, two facets must be considered : the possible severity of loss, and the possible frequency or probability of loss. Evaluation involves measuring the potential size of the loss and probability that the loss is likely to occur .

A Priority Ranking Based On Severity: One of the techniques used by scientists and engineers in the U.S. space program was criticality analysis-an attempt to distinguish the truly important factors from the overwhelming mass of unimportant ones. Given the wide range of losses that can occur, from the minute to the catastrophic, it seems logical that exposures be ranked according to their criticality. Certain risks, because of the severity of the possible loss, will demand attention prior to others, and in most instances there will be a number of exposures that are equally demanding .

Probability and priority Rankings: Although the potential severity is the most important factor in ranking exposures, an estimate of the probability may also be useful in differentiating among exposures with relatively equal potential severity. Other things being equal, exposures characterized by high frequency should receive attention before exposures in which the loss frequency is low. Exposures that exhibit a high loss frequency are often susceptible to improvement through risk control measures .

4- Consideration Of Alternatives And Selection Of The Risk Treatment Device :

Once the risks have been identified and evaluated, the next step is consideration of the approaches that may be used to deal with risks and the selection of the technique that should be used for each one .

The Choice: This phase of the risk management process is primarily a problem in decision making, more precisely, it is deciding which of the techniques available should be used in dealing with each risk. The extent to which the risk management personnel must make these decisions on their own varies from organization to organization. Sometimes the organization's risk management policy establishes the criteria to be applied in the choice of techniques, outlining the rules within which the risk manager may operate. If the risk management policy is rigid and detailed, there is less latitude in the decision making done by the risk manager. He or she becomes an administrator of the program rather than a policy maker. In other instances, where there is no formal policy or where the policy has been loosely drawn to permit the risk manager a wide range of discretion, the position carries much greater responsibility .

The risk manager considers the size of the potential loss, its probability, and the resources that would be available to meet the loss if it should occur. The benefits and costs in each approach are evaluated, and then, on the basis of the best information available and under the guidance of the corporate risk management policy, the decision is made .

5-Implementation of the Decision:

The decision is made to retain a risk. This may be accomplished with or without a reserve and with or without a fund. If the plan is to include the accumulation of a fund, proper administrative procedure must be set up to implement the decision. If loss prevention is selected to deal with a particular risk, the proper loss-prevention program must be designed and implemented. The decision to transfer the risk through insurance must be followed by the selection of an insurer, negotiations, and placement of the insurance .

6- Evaluation and Review

Evaluation and review must be included in the program for two reasons. First, the risk management process does not take place in a vacuum. Things change : new risks arise and old risks disappear. The techniques that were appropriate last year may not be the most advisable this year, and constant attention is required. Second, mistakes are sometimes made. Evaluation and review of the risk management program permit the risk manager to review decisions and discover mistakes, ideally before they become costly .

How does one review a risk management program? Basically, by repeating each of the steps in the risk management process to determine whether past decisions were proper in the light of existing conditions and if they were properly executed. The risk manager reevaluates the program's objectives, repeats the identification process to ensure, insofar as possible, that it was performed correctly, and then evaluates the risks that have been identified and-verifies that the decision on how to address each risk was proper. Finally, the implementation of the decisions must be verified to make sure they were executed as intended .

Quantitative performance standards: Ideally, standards should be quantified whenever possible. One quantifiable measure of risk management performance that is frequently suggested is the cost of risk, which is the total expenditures for risk management, including insurance premiums paid and retained losses, expressed as a percentage of revenues. RIMS publishes annual studies on the cost of risk, which makes it convenient for the risk manager to compare the risk management costs of the organization with other firms in the same industry. The cost of risk varies from industry to industry, yet it generally averages in the neighborhood of 1 percent of revenues. Although the cost of risk may fluctuate because of factors over which the risk manager has no control, it is a useful standard when properly interpreted .

Quantitative performance standards are more prevalent in the area of risk control than for risk financing functions. Motor vehicle accident rates and other frequency and severity rates are useful benchmarks in measuring risk control measures.

Questions For Review

Q1: Indicate whether each of the following statement is true or false and correct the false one (s):

1- Risk management is a scientific approach to the problem of speculative risk.

2- risk management is a science as physical sciences.

3- The techniques for dealing with risk are grouped into two broad approaches: risk control and risk financing.

4- risk control consists of those techniques that are designed to minimize those risks to which the organization is exposed.

5- risk are not avoided when the organization refuses to accept risk.

6- risk reduction should be used in those instances in which the exposure has catastrophic potential and the risk cannot be reduced or transferred.

7- risk avoidance consists of all techniques that are designed to reduce the probability of loss, or the potential severity of losses that occur.

8- risk retention is the most common method of dealing with risk.

9- voluntary retention results from decision to retain risk rather than to avoid or transfer.

10- risk management process can be divided into steps that must be accomplished in managing risks.

11- risk may be retained by people who prefer not to do so.

Q2: Multiple choice : select the best answer a, b, or c :

1) Direct property losses would include each of the following except .

a - losses to owned building & inventory .

b- losses to leased building & inventory .

c- losses of income after inventory is destroyed by fire .

2) Risk Avoidance :

a- Means measures are taken to eliminate the loss exposure .

b- Means measures are taken to reduce loss severity

c- Means insurance has been purchased and the risk transferred to an insurance company .

3) Typical source of liability losses include all of the following except .

a- Losses caused to workers injured on the job .

b- Losses caused to real property of others .

c- A worker allows a machine to be destroyed by improper maintenance .

4) To estimate the value of potential property losses, risk managers should focus on :

a- The purchase price of the property .

b- The property's replacement cost .

c- The property's fair market value .

CHAPTER (3)
THE INSURANCE DEVICE

The Nature And Functions Of Insurance

There are a number of ways of dealing with risk, the most formal of these is insurance device.

Risk Sharing and Risk Transfer

Insurance has two fundamental characteristics:

- Transferring or shifting risk from one individual to a group.
- Sharing losses, on some equitable basis, by all members of the group.

Insurance Defined From the Viewpoint of the individual

Based on the preceding description. We may define insurance from the individual's viewpoint as follows:

"From an individual point of view, insurance is an economic device whereby the individual substitutes a small certain cost (the premium) for a large uncertain financial loss (the contingency insured against) that would exist if it were not for the insurance".

From the individual's point of view, the purchase of an adequate amount of insurance on a house eliminates the uncertainty regarding a financial loss in the event that the house should burn down. Basically, the insurance device is a method of loss distribution. What would be a devastating loss to an individual is spread in an equitable manner to all members of the group. And it is on this basis that insurance can exist.

Risk Reduction Through pooling :

In addition to eliminating risk at the level of the individual through transfer, the insurance mechanism reduces risk (and the uncertainty related to risk) for society as a whole. The risk the insurance company faces is not merely a summation of the risks transferred to it by individuals; the insurance company is able to do something that the individual cannot, and that is to predict within rather narrow limits the amount of losses that will actually occur. If the insurer could predict future losses with absolute precision, it would face no possibility of loss. It would collect each individual's share of the total losses and expenses of operation and use these funds to pay the losses and expenses as they occur. If the predictions are not accurate, the premiums that the insurer has charged may be inadequate. The accuracy of the insurer's predictions is based on the law of large numbers. By combining a sufficiently large number of homogeneous exposure units, the insurer is able to make predications for the group as a whole. This is accomplished through the theory of probability.

***Probability Theory and the law of Large Numbers*:** probability theory is the body of knowledge concerned with measuring the likelihood that something will happen and making predictions on the basis of this likelihood. It deals with random events and is based on the premise that, while some events appear to be a matter of chance, they actually occur with regularity over a large number of trials. The likelihood of an event is assigned a numerical value between 0 and 1, with those that are impossible assigned a value of 0 and those that are inevitable assigned a value of 1 Events that may or may not happen are assigned a value between 0 and 1, with higher values assigned to those estimated to have a greater likelihood or "probability" of occurring.

At this point, it may be useful to distinguish between two interpretations of probability:

- The relative frequency interpretation. The probability assigned to an event signifies the relative frequency of its occurrence that would be expected, given a large number of separate independent trials. In this interpretation, only events that may be repeated for a "long run" may be governed by probabilities.
- The subjective interpretation. The probability of an event is measured by the degree of belief in the likelihood of the

given incident's occurrence. For example, the coach of a football team may state that his team has a 70 percent chance of winning the conference title, a student may state that she has a 50:50 chance of getting a B in a course, or the weather forecaster may state that there is a 90 percent chance of rain.

Both these interpretations are used in the insurance industry, but for the moment let us concentrate on the relative frequency interpretation.

Determining the Probability of an Event To obtain an estimate of the probability of an event in the relative frequency interpretation, one of two methods can be used. The first is to examine the underlying conditions that cause the event. For example, if we say that the probability of getting a "head" when tossing a coin is .5 or ½ , we have assumed or determined that the coin is perfectly balanced and that there is no interference on the part the "tosses". If we ignore the absurd suggestion that the coin might land on its edge, there are only two possible outcomes, and these are equally likely. Therefore we know that the probability is .5. Because they are determined before an experiment in this manner (i.e., on the basis of causality), they are called a priori probabilities.

These a priori probabilities are not of great significance for us except insofar as they can be used to illustrate the operation of the law of large numbers. Even though we know that the probability of flipping a head is .5, we also know that we cannot use this knowledge to predict whether a given flip will result in a head or a tail. We know that the probability has little relevance for a single trial. Given a sufficient number of flips, however, we would expect the result to approach one-half heads and one-half tails. We feel that this is true even though we may not have the inclination to test it. This commonsense notion that the probability is meaningful only over a large number of trials is an intuitive recognition of the law of large numbers, which in its simplest form states that.

“The observed frequency of an event more nearly approaches the underlying probability of the population as the number of trials approaches infinity”.

When we do not know the underlying probability of an event and cannot deduce it from the nature of the event, we can estimate it on the basis of past experience. We are told that the probability that a 21-year-old male will die before reaching age 22 is .00191. what does this mean? It means that someone has examined mortality statistic and discovered that, in the past, 191 men out of every 100.000 alive at age 21 have died before reaching age 22. It also means that, barring changes in the causes of these deaths, we can expect approximately the same proportion of 21-year-olds to die in the future.

Here, the probability is interpreted as the relative frequency resulting from a long series of trials or observations, and it is estimated after observation of the past rather than from the nature of the event as in the case of a priori probabilities. These probabilities, computed after a study of past experience, are called a posteriori or empirical probabilities. They differ from a priori probabilities, such as those observed in flipping a coin, in the method by which they are determined, but not in their interpretation. In addition while the probability computed prior to the flipping of a coin can be considered to be exact, those computed on the basis of past experience are only estimates of the true probability.

The law of large numbers, which tells us that “a priori estimates are meaningful only over a large number of trials, is the basis for the a posteriori estimates”. Since the observed frequency of an event approaches. The underlying probability of the population as the number of trials increases, we can obtain a notion of the underlying probability by observing events that have occurred. After observing the proportion of the time that the various outcomes have occurred over a long period of time under essentially the same conditions, we construct an index of the relative frequency of the occurrence of each possible outcome. This index of the relative frequency of each of all possible outcomes is called a probability distribution, and the probability assigned to the event is the average rate at which the outcome is expected to occur.

In making probability estimates on the basis of past experience or historical data, we make use of the techniques of statistical inference, which is to say that we make inference about the population based on sample data. It is not usually possible to examine the entire population, so we must be content with a sample. We take a sample to draw a conclusion about some measure of the population (referred to as a parameter) based on a sample value (called a sample statistic). In attempting to estimate the probability of an event, the parameter of the population in which we are interested is the mean or average frequency of occurrence, and we attempt to estimate this value based on our sample. Because only partial information is available, we confront the possibility that our estimate of the mean of the population (the probability) will be wrong.

We know that the observed frequency of an event will approach the underlying probability as the number of trials increases. It therefore follows that the greater the number of trials examined, the better will be our estimate of the probability. The larger the sample on which our estimate of the probability is based, the more closely our estimate should approximate the true probability.

In the case of empirical probabilities, the requirement of a large number has dual application:

- To estimate the underlying probability accurately, the insurance company must have a sufficiently large sample. The larger the sample, the more accurate will be the estimate of the probability.
- Once the estimate of the probability has been made, it must be applied to a sufficiently large number of exposure units to permit the underlying probability to work itself out.
- In this sense, to the insurance company, the law of large numbers means that the larger the number of cases examined

in the sampling process, the better the chance of making a good estimate of the probability; the larger the number of exposure units to which the estimate is applied, the better the chance that actual experience will approximate a good estimate of the probability.

Insurance Defined from the Viewpoint of Society

In addition to eliminating risk for the individual through transfer, the insurance device reduces the aggregate amount of risk in the economy by substituting certain costs for uncertain losses. These costs are assessed on the basis of the predictions made through the use of the law of large numbers. We may now formulate a second definition of insurance:

"From the social point of view, insurance is an economic device for reducing and eliminating risk through the process of combining a sufficient number of homogeneous exposure into a group to make the losses predictable for the group as a whole".

Insurance dose not prevent losses, nor does it reduce the cost of losses to the economy as a whole. The existence of insurance encourages some losses for the purpose of defrauding the insurer and people are less care and may exert less effort to prevent loss than they might if the insurance did not exist.

Insurance: Transfer Or Pooling

The two definitions of insurance-from the viewpoint of the individual and from the viewpoint of society- reflect two different views of insurance, views that have divided insurance scholars for at least the past half century. The first view- that insurance is a device through which the individual substitutes a small certain cost for a large uncertain loss- emphasizes the transfer of risk. It does not attempt to explain how the risk is handled by the transferee. The second view- that insurance is a device for reducing and eliminating risk through pooling- emphasizes the role of insurance in reducing risk in the aggregate, which it does by pooling.

Actually, both definitions are useful. The definition of insurance from the individual's perspective defines the essence of insurance, based on its essential component, transfer. The definition of insurance from the perspective of society is a functional definition, and explains how insurance usually achieves the transfer function.

Elements Of An Insurable Risk

Although is theoretically possible to insure all possibilities of loss, some are not insurable at a reasonable price. For practical reasons, insurers are not willing to accept all the risks others may wish to transfer to them. To be considered a proper subject for insurance, certain characteristics should be present. The four prerequisites listed next represent the “ideal” elements of an insurable risk. Although it is desirable that the risk have these characteristics, it is possible for certain risks that do not have them to be insured.

1- *there must be a sufficiently large number of homogeneous exposure units to make the losses reasonably predictable.* Insurance, as we have seen, is based on the operation of the law of large numbers. A large number of exposure units enhances the operation of an insurance plan by making estimates of future losses more accurate.

2- *The loss produced by the risk must be definite and measurable*. It must be a type of loss that is relatively difficult to counterfeit, and it must be capable of financial measurement. In other words, we must be able to tell when a loss has taken place, and we must be able to set some value on the extent of it.

3- *The loss must be fortuitous or accidental.* The loss must be the result of a contingency; that is, it must be something that may or may not happen. It must not be something that is certain to happen. If the insurance company knows that an event in the future is inevitable, it also knows that it must collect a premium equal to the certain loss that it must pay, plus an additional amount for the expenses of administering the operation. Depreciation, which is a certainty, cannot be insured; it is provided for through a sinking fund. Furthermore, the loss should be beyond the control of the insured. The law of large numbers is useful in making predictions only if we can reasonably assume that future occurrences will approximate past experience. Since we assume that past experience was a result of chance happening, the predictions concerning the future will be valid only if future happenings are also a result of chance.

4- *The loss must not be catastrophic*. It must be unlikely to produce loss to a very large percentage of the exposure units at the same time. The insurance principle is based on a notion of sharing losses, and inherent in this idea is the assumption that only a small percentage of the group will suffer loss at any one time. Damage that results from enemy attack would be catastrophic in nature. There are additional perils, such as floods, which, while they would not affect everyone in the society, would affect only those who had purchased insurance. The principle of randomness in selection is closely related to the requirement that the loss must not be catastrophic.

5- *Economic feasibility* Sometimes, an additional attribute is listed as a requirement of an insurable risk- that the cost of the insurance must not be high in relation to the possible loss or that the insurance must be economically feasible. We can hardly call this a requirement of an insurable risk in view of the fact that the principle is widely violated in the insurance industry today. The four elements of an insurable risk are characteristics of certain risks that permit the successful operation of the insurance principle.

Self-Insurance

Under some circumstance, it is possible for a business firm or other organization to engage in the same types of activates as a commercial insurer dealing with its own risks. When these activities involve the operation of the law of large numbers and predictions regarding future losses, they are commonly referred to as “self-insurance”. To be operationally dependable, such programs must possess the following characteristics:

- The organization should be big enough to permit the combination of a sufficiently large number of exposure units so as to make losses predictable. The program must be based on the operation of the law of large numbers.
- The plan must be financially dependable. In most cases, this will require the accumulation of funds to meet losses that occur, with a sufficient accumulation to safeguard against unexpected deviations from predicted losses.
- The individual units exposed to loss must be distributed geographically in such a manner as to prevent a catastrophe. A loss affecting enough units to result in severe financial loss should be impossible.

The Fields Of Insurance

Insurance may be divided and subdivided into classifications based on the perils insured against or the fundamental nature of the particular program. Basically, the primary distinction is between private insurance and social insurance.

Private insurance consists for the most part of voluntary insurance programs available to the individual as a means of protection against the possibility of loss. This voluntary insurance is usually provided by private firms, but, in some instances, it is also offered by the government. The distinguishing characteristics of private insurance are that it is usually voluntary and that the transfer of risk is normally accomplished by means of a contract. Social insurance, on the other hand, is compulsory insurance, usually operated by the government, whose benefits are determined by law and in which the primary emphasis is on social adequacy. In general, the benefits under social insurance programs attempt to redistribute income based on some notion of “social adequacy”.

Private (Voluntary) insurance :

Private insurance may be classified into three broad categories :

- Life insurance.
- Health insurance.
- Property and liability insurance.

Life Insurance**:** Life insurance is designed to provide protection against two distinct risks: premature death and superannuation. As a matter of personal preference, death at any age is probably premature, and superannuation (living too long) does not normally strike one as an undesirable contingency. From a practical point of view, however, a person can, and sometimes does, die before adequate preparation has been made for the future finical requirements of dependants. In the same way, a person can, and often does, outlive income-earning ability. Life insurance, endowments, and annuities protect the individual and his or her dependents against the undesirable financial consequences of premature death and superannuation .

Health insurance**:** Accident and health insurance (or, more simply, health insurance) is defined as "insurance against loss by sickness or accidental bodily injury". The "loss" may be the loss of wages caused by the sickness or accident or it may be expenses for doctor bills, hospital bills, medicine, or the expenses of long-term car. Included within this definition are forms of insurance that provide lump-sum or periodic payments in the event of loss occasioned by sickness or accident, such as disability income insurance and accidental death and dismemberment insurance.

Property and Liability Insurance: Property and liability insurance consists of those forms of insurance designed to protect against losses resulting from damage to or loss of property and losses arising from large liability. It includes the following types of insurance.

Property insurance: also sometimes referred to as fire insurance, is designed to indemnify the insured for loss of, or damage to, buildings, furniture, fixtures, or other personal property as a result of fire, lightning, windstorm, hail, explosion, and a long list of other perils. Originally, fire was the only peril insured, but the number of perils insured against has gradually been expanded over the years. Today, two basic approaches are taken with respect to the perils for which coverage is provided. Under the first approach, called named-peril coverage, the specific perils against which protection is provided are listed in the policy, and coverage applies only for damage arising out of the listed perils. Under the second approach, called open-peril coverage, the policy lists the perils for which coverage is not provided, and loss from any peril not excluded is covered. Coverage may be provided for both direct loss (i.e., the actual loss represented by the destruction of the asset) and indirect loss (i.e., the loss of income and/or the extra expense that is the result of the loss of the use of the asset protected).

Marine insurance: like fire insurance, is designed to protect against financial loss resulting from damage to owned property, except that here the perils are primarily those associated with transportation. Marine insurance is divided into two classifications: ocean marine and inland marine.

Ocean marine insurance policies provide coverage on all types of oceangoing vessels and their cargoes. Policies are also written to cover the ship owner's liability. Originally, ocean marine policies covered cargo only after it was loaded onto the ship. Today the policies are usually endorsed to provide coverage from "warehouse to warehouse", thus protecting against overland transportation hazards as well as those on the ocean.

Inland marine insurance seems like a contradiction in terms. The field developed as an outgrowth of ocean marine insurance and "warehouse-to-warehouse" coverage. Originally, inland marine developed to cover goods being transported by carries such as railroads, motor vehicles, or ships and barges on the inland waterways and in coastal trade. It was expanded to cover instrumentalities of transportation and communication such as bridges, tunnels, pipelines, power transmission lines, and radio and television communication equipment. Eventually, it was expanded to include coverage on various types of property that is not the course of transportation, but that is away from the owner’s premises.

Automobile insurance provides protection against several types of losses. First, it protects against loss resulting from legal liability arising out of the ownership or use of an automobile. In addition, the medical payments section of the automobile policy consists of a special form of health and accident insurance that provides for the payment of medical expenses incurred as a result of automobile accidents. Coverage is also provided against loss resulting from theft of the automobile or damage to it from many different causes .

Liability insurance embraces a wide range of coverages. The form with which most students are familiar is automobile liability insurance, but there are other liability hazards as well. Coverage is available to protect against nonautombile liability exposures such as ownership of property, manufacturing and construction operations, the sale or distribution of products, and many other exposures .

Burglary, robbery, and theft insurance protect the property of the insured against loss resulting from criminal acts of others. Because a standard clause in these crime policies excludes acts by employees of the insured, they are referred to as "nonemployee crime coverage’s". Protection against criminal acts by employees is provided under fidelity bonds.

Chapter (4)
Life insurance product
Types of contracts

Objectives

After studing this unit you should be able to :

- Define term insurance .
- Describe Whole life insurance.
- Describe Endowment insurance.
- Understand Life annuities.

Contents:

- Term Insurance
  - **Special features in term policies**
  - **Uses of term insurance**
- Whole- Life Insurance
  - **Types of whole- life insurance**
- Endowment Insurance
  - **Types and uses Of Endowment Insurance .**
- Annuities **- Classifications Of Annuities**

It is possible to identify four distinct types of life insurance contracts :

(1) term insurance.

(2) Whole life insurance.

(3) Endowment insurance.

(4) Life annuities.

Other classifications, based on method of premium payment, period of coverage, method of distribution of proceeds and combinations of these basic contract are possible, but the classification above seems sufficient as a background for understanding the various life insurance contracts.

First : term insurance

A term policy may be defined as a contract that furnishes life insurance protection for a limited number of years, the face value of the policy being payable only if death occurs during the stipulated term and nothing being paid in case of survival. Customarily term insurance policies are written for periods of 5,10,15, or 20 years. It may provide protection up to age 65 or 70. Such policies may be renewed for successive term periods at the option of the insured and without evidence of insurability. The premium for term insurance is relatively low, despite the facts that expenses are a larger proportion of the outlay and the rate is increased to allow for adverse selection.

This is due to the fact that term insurance protection is temporary, and no charge is necessary to cover the high death rates at the older ages. Term insurance insures against a contingency only, whereas whole-life contracts insure against a certainty.

Special features in term policies :

Renewable feature: Many five-and ten-year term policies contain an option to renew for limited number of additional periods of protection. This option permits the insured, at the expiration of the first term period, to renew the policy without a medical examination and irrespective of the insured’s health at the time of renewal. The renewal of the policy can be effected by the insured by paying the premium for the age then attained without furnishing any evidence of insurability. Usually, the companies limit the age at which such renewable term policies may be issued and in some instances, the number of renewable permitted is limited.

It should be noted that premium, while level for a given period, increases with each renewal and is based on the attained age of the insured at the time of renewal. Thus, although the scale of rates in contract is guaranteed, the rate increases with each renewal because of the higher attained age of the insured. Adverse selection against the company develops at an increasing rate. Naturally, resistance to the higher premiums will cause many policyholders in good health to fail to renew, while the great majority of those in poor health will renew even in the face of the higher premium. As a result, the mortality experience among surviving insured policyholders will become increasingly unfavorable.

The limitation of coverage to age 55,60,or 65 and the limited number of renewals permitted are additional recognition of the increasing adverse selection evidenced in the face of the rising premium outlay necessary to continue such protection with increasing age.

Convertible feature: Most term policies also include a convertible feature- that is, the privilege on the part of the insured to convert the policy into a permanent in the premium charge, without evidence of insurability. Most companies extended this privilege for only a limited number of Years. This was done in order to minimize adverse selection. Conversion into regular whole- life or endowment insurance is usually allowed. The exchange is usually allowed at any time during the period when conversion is permitted, and may be effected as of the “attained age” or as of the “original age”.

The attained-age method of conversion involves the issuance of a permanent policy of the from currently being issued at the date of conversion, and the premium rate for the new policy is that required at the attained age of the insured.

The original- age method involves a retroactive conversion, with the permanent policy bearing the date and premium rate that would have been paid had the permanent policy been taken out originally, and taking the from of the permanent contract that was being issued at the original date. It should be apparent that this from of contract may be more or less liberal than the from being issued at the date of conversion and that the premium rate will be lower owing to the younger effective age. Most companies require that retroactive conversions take place within 5 years of the date of issue of the term contact. In the case of an original-age conversion, most companies require the insured to pay the difference between the premiums that would have been issued at the same time as the original policy, and the premiums paid thereunder for the same amount of insurance, with interest on such difference at a certain stipulated annual rate, the purpose of the adjustment is to place the insurance company in the same financial position it would have held had the permanent policy been issued in the first place.

In making a choice between the two bases of conversion, the insured may be motivated by lower premium rate payable under the original- age conversion, or the availability of more liberal contract at the original date, but other factors will affect such a decision. Some companies issue term policies that provide the automatic conversion to a specific permanent plan, but the continuation of the insurance is still optional with the insured. There may be some sales and administrative advantage involved, but it is doubtful that such a provision is very effective in reducing adverse selection.

Long-period term contracts: There are two types of contracts providing the same protection. (1) life expectancy (2) term to 60 or 65. the term to expect any provides protection for the life expectancy of the insured. With the premium being level throughout this period. The exact number of years the contract will run depends on the age and the sex of the insured at the date of application and the mortality assumption. The contract contains automatic conversion of the contract into a form of whole life at the end of the period. The term to 60 or 65 contract usually provides Protection for a somewhat shorter period than does the life-expectancy policy and consequently has a slightly lower premium. 65 is the normal retirement age.

Decreasing term insurance (decreasing in face amount) is widely used as arider for permanent contracts and as a separate policy to provide mortgage protection. The policy year by year declines in the face amount, becoming zero at the end of the term of the contract. The premium is very low. Increasing term insurance is never sold as a separate contract. Its primary use in connection with package policies where it is included as part of a contract, to provide benefit that increases with time and is payable on death.

Uses of term insurance: term insurance are designed to afford protection against contingencies that require only taking out of temporary insurance or call for largest amount of insurance protection for the time being at the lowest possible outlay of funds.

Pure term insurance seems justified under the following circumstances :-

- For persons with small income for the present, with family obligations.
- For persons who have placed substantially all their resources in the material assets of a new business that is still in its formative stages, and where premature death of the human factor in that business would spell serious loss.
- To protect one’s insurability. Many young men recognize the need for additional life insurance. As their incomes grow, their need for life insurance becomes greater, term insurance through its conversion feature can serve as a hedge against this possibility.

- As a hedge against loss in the estate already sustained, where the owner of the estate is without current means, where a little time is required to repair the damage. Such owners lack the funds to buy high-premium insurance, they are certainly served by term insurance.
- As additional protection for loans. Term insurance has been a boon during depression years to many who stood in need of loans and whose current income was small.
- As means of protecting mortgage obligations. One of the most common use of term insurance is to provide proceeds from which a mortgage may be cancelled, redeemed, or retired upon the death of the breadwinner.

Second : Whole- Life Insurance

It provides for the payment of the face value upon the death of the insured, regardless of when it may occur.

Types of whole- life insurance:

(1) ordinary life insurance policy is based on the assumption that premiums will be paid throughout the lifetime of the insured. The insured purchases the policy with no intention of paying premiums as long as he or she lives. One's intention may be to use dividend refunds to pay up the whole- life policy in shorter period, or to surrender the policy at retirement for an annuity. The point is that the ordinary life insurance policy is a very flexible contract, and it is important to recognize that the insured is not committed to paying premiums as long as he or she lives.

Besides its moderate cost and the permanent character of the protection afforded, the ordinary life policy furnishes the further advantage of combining saving with insurance. In term insurance there is no savings element, and nothing is paid to the insured in case of survival at the expiration the term. As constructed with this shortcoming, the ordinary life policy presents an entirely different situation. The annual level premium is much in excess of the amount required to pay the current cost of the insurance protection, the balance being retained by company as a reserve and improved at compound interest at an agreed rate for the purpose of making good the deficiency in the later years of life, when the annual level premium is no loner sufficient to pay for the actual cost of insurance. This is the savings or investment element in the policy. Thus, the ordinary life insurance policy is a combination of protection and saving.

The ordinary life contract provides a high degree of flexibility. Certain policy provisions provide this flexibility both before and after maturity of the policy. After maturity, the contract provides for various alternative ways in which the proceeds may he paid to designated beneficiaries. Before maturity, a number of contract provision permit adjustment to meet changing circumstance. For example, the non forfeiture options permit the insured to surrender the policy for its them cash value or to stop Premium payments and accept the equivalent in the form of either a paid-up policy a reduced amount or term insurance for the full face amount, for a limited period of time. Participating policies provide for dividend option that give the insured the option of taking dividends in cash, applying them to reduce premiums, applying them to purchase paid-up units of whole-life insurance, or accumulating them at interest. Surrender values can also be used to purchase an annuity or retirement income. Many people purchase ordinary life insurance to protect their families during the child-rearing period, with the specific objective of using the cash values for their own retirements.

Disadvantages of premium payments for life: the chief objection usually advanced against ordinary life insurance is the continued payment of the premium throughout life. This objection may be obviated by allowing the annual dividends to accumulate with the company with view to either shortening the premium-payment period or hastening the maturity of the contract. The cash surrender value and other options allowed under an ordinary life policy may make desirable a discontinue of premium payments. The insured may surrender the policy to the company for its cash value or, the insured may choose the option of stopping premium payments and taking a paid-up policy for a reduced amount, payable upon death to a designated beneficiary.

(2) limited-payment polices: under the terms of limited-payment policies, the face of the policy is not payable until death, but premiums are charged for a limited number of years only, after which the policy becomes paid up for its full amount. The limitation may be expressed in terms of a number of annual premiums or the age to which annual premiums must be paid-instead of continuing until death, premium payments may be fixed at almost any number of years (ten, fifteen or twenty years or even more). If premiums are limited to twenty years, for example, the policy is known as twenty-payment life policy. Companies also make available

contracts that limit premiums on the basis of a terminal age, such as 60, 65. the objective is to appeal to the buyer with the idea of paying up the policy during his or her working lifetime.

Since limited-payment policies require the payment of premiums during a term that is less than the term of the contract, it follows that the annual level premium under this plan must be larger than that necessary when premium payments continue throughout the life of the policy. The purpose of the plan is to have the policyholder pay an extra amount annually during the fixed premium paying period so that after the termination of this period the policy may remain in force and be carried to successful completion without further financial obligation on the part of the insured. Owing to the higher premiums, the limited payment plan is not well adapted to those whose income is small and whose need for insurance protection is so great as to require emphasis on the amount of protection rather than the accumulation of a fund with the company, especially when there is reason to believe that the income out of which premiums may conveniently be paid will be much greater in the future than it is at present.

The disadvantage of higher premiums is offset by the availability of a larger savings or investment element. The higher premiums produce greater cash values, which are available for use in an emergency and at retirement, there is no presumptive financial advantage of one form over the other. All limited-payment policies contain the same nonforfeiture options, dividend options, settlement options, and other features of the ordinary life insurance policy that provide flexibility for the insured. It is important to remember that all permanent forms of life insurance are made up of protection and savings, and the major distinction between the many plans is the relative importance of these two elements. The single premium whole life policy is an extreme form of limited-payment life insurance. Under this form the savings element is the predominating feature, and the protection element is substantially less than the face of the policy. Consequently, such contracts are purchased primarily for investment purposes. As the number of premiums payments increases, the annual premium and the cash value or saving element become correspondingly smaller. The choice depends upon circumstances and personal preference.

(3) modified life policies:- it is an ordinary life contract under which premiums redistributed so that they are lower than normal during the first three or five years and higher than normal thereafter. Thus, one company under a modified five "may set the premium during the first five years so that it will double thereafter. Other redistributions are also used. During the preliminary period, the premium is always something more than the level term premium for such a period, but less than the ordinary life premium at date of issue. After the preliminary period, the premium is always something larger than the ordinary life premium at the date of issue but less than the ordinary life premium at the insured's attained age at the end of the preliminary period. Regardless of the redistribution arrangement utilized, the company receives the actuarial equivalent of the regular ordinary life premiums.

Modified life insurance policies are designed to overcome resistance of life insurance prospects to the premium outlay necessary to purchase permanent life insurance. It is appropriate for young family or those completing their education, whose need for protection is great but whose income is insufficient to afford regularly ordinary life-it should not be noted at this point that level term insurance with an automatic conversion feature may be considered an extreme case of premium redistribution. Under this arrangement, no part of the premium goes toward savings, and consequently, at the end of the preliminary period, the premium would simply be the ordinary life premium at the insured's attained age. This payment would be higher than premium under the usual modified like plan after the preliminary period. Under all modified life plans, the cash values are smaller in the earlier years than they would be normally and are not payable at as early a date, but in all other respects, the contract is usually identical with the ordinary life policy.

(4) variable life insurance:- Inflation in our economy has become so accepted a fact of life in the minds of most people that it seems to be becoming the controlling influence in how individuals of organize their financial affairs, in response to this perceived need, the life insurance is developing an insurance product has a guaranteed minimum death benefit and the potential of increased insurance benefits without necessarily requiring the spending of more premium dollars. In choosing such a variable-benefit policy, the policyholder will obtain a contract right to have the net investment return in excess of the assumed rate of return applied to increase his or her policy benefits. On the other hand, one will give up the guarantee of cash values and the elements of a fixed-benefit policy that depend on guaranteed cash values specifically, the minimum nonforfeiture benefits and traditional policy loan. The advent of variable life insurance has a potential for working for reaching changes in the life insurance business.

Third: Endowment Insurance

Nature of endowment insurance: endowment policies provide for not only the payment of the face of the policy upon the death of the insured during a fixed term of years, but also the payment of the full face amount at the end of said term if the insured is living. Whereas policies payable only in the event of death are taken out chiefly for the benefit of others, endowment policies, afford protection to other against the death of the insured during the fixed term, usually revert to the insured if he survives the endowment period. Such policies have therefore become popular in recent years as a convenient means of accumulating a fund that will afterwards become available for the use of policyholder.

There are two ways of looking at endowment insurance:-

(1) Mathematical concept.

(2) economic concept.

Mathematical concept : There are two promises made by the company under endowment insurance: (1) to pay the face amount in the event the insured dies during the endowment period, and (2) to pay the face amount in the event the insured survives to the end of the endowment period. The first promise is identical with that made under a level term policy for an equivalent amount and period. The second introduces, a new concept, ***"The pure endowment"*** Just as a company may estimate how many will die, it can also estimate the number who will live to attain a certain age. Thus, A *pure endowment* is defined as a contract that promises to pay the face amount only if the insured is living at the end of a specified period, nothing being paid in case of prior death. In order to provide a death benefit during the endowment period, only term insurance for the same period need be added. It can be seen that these two elements, (1) level term insurance (2) a pure endowment will together meet the two promises made under endowment insurance.

Economic concept: This analysis divides endowment insurance into two parts, decreasing term insurance and increasing investment. The investment part of the contract is not considered, and in fact is not, a pure endowment, all of which is lost in case of death before the end of the term. Rather, it is a savings accumulation that is available to the insured at any time after the first year or two, through surrender of or loan upon the policy. This investment feature is supplemented by term insurance, which is not, however, level term insurance, for $1000 throughout the term of the contract, but insurance for a constantly decreasing amount, which, when added to the investment accumulation at the date of death, will make the amount payable under the policy equal to its face, or $1000 the insurance portion of the contract, therefore, is for a decreasing amount, being almost $1000 in the early years of the contract and gradually decreasing throughout the term. Thus, if at particular time a $1000 endowment policy has an investment accumulation of $150, the insured will be protected against death by $ 580 of decreasing term insurance, but when the accumulation reaches $900, there will remain decreasing term insurance for only the difference between $1000 and $900, or $100.

Finally, when the investment portion of the contract reaches 100 percent of the face of the policy at the time of its maturity, the decreasing term insurance will have declined to zero. At any particular time, the accumulated savings fund on the one side and the decreasing term insurance on the other will always equal the face of the contract. Upon death, the company pays to the policyholder's estate the face amount of the policy, which is made up of (1) the investment standing to his credit and (2) decreasing term insurance equal to difference between the investment accumulation and the face of the contract. The purpose of the decreasing term insurance is always equal that portion of the policy that the policyholder intended to save if he had lived, but that was not saved because the premature death. The premium paid for the policy may be divided into three parts: one for the investment accumulation, another in payment for the cost of the decreasing term insurance, and still another small part to meet the cost of managing the installment plan of the investment and other expenses incident to the insurance business.

Types of endowment insurance policies :

An examination of the contracts issued by different companies shows many variations in the use of the endowment insurance principle. Such policies may be made. Payable in ten, fifteen, Twenty, twenty-five, thirty, or more years, or the length of the term may be so arranged as to cause the policy to mature at certain ages such as 60, 65, 70, and so forth. When the policy is written for short term, its purpose is usually to combine immediate protection with heavy savings. Whereas if written for along term, or to mature at an advanced age its object is usually to combine protection with old ages provision usually. The contracts are paid for by premiums (payable annually, semi annually. Quarterly, or monthly)continuing throughout the term but, if desired, the premiums may be paid on the limited payment plan as, for example, a thirty-Years endowment policy paid up in twenty years, or an endowment at age 65. paid up in twenty years.

Besides the standard contracts, other applications of the endowment principle are sometimes made, as for example, in the case of the "retirement income" or "semi endowments" in the case of the retirement income policy, it provides that the amount payable upon survival is greater than the face amount, and the amount payable at death is the face amount or cash value, whichever is greater. The semi- endowment pays upon survival only half the sum payable in the event of death during the endowment period. Various kinds of "juvenile endowment policies" are also issued by certain companies. These include endowments maturing at specified ages for educational purposes. A return- of- premium benefit is offered that provides for the return of the premiums paid in the event of the child's death before reaching the endowment age.

An extra benefit frequently added to juvenile endowments is payor rider, which, upon the death or disability of the premium payor. Usually the father or mother, provides that premium payments shall cease. The policy be coming fully paid in the event of death or the premiums waived during the payor 's disability.

Uses Of Endowment Insurance :

Briefly stated. Endowment insurance may be useful in four main ways:-

1- as an incentive to save. Endowment insurance is advanced as a method of systematic saving in that it provides for the laying away of a moderate sum each year with a view to having all the accumulations available at the end of a fixed period. This era is recognized as one of particular extravagance, and vast numbers of people because of extravagant habits, never save in dollar even though they receive good incomes, for such people an endowment policy generally turns out to be a means of forcing thrift, since it compels them to do what if it were left entirely to their own option, would remain undone. By requiring the payment of specific sums at regular intervals during a period of years. Endowment insurance enables many to save a worth while sum without being conscious of the sacrifice, whereas haphazard methods of the saving seldom achieve this result "such a policy" it has been said "gives a person a definite aim- one must save just so much every year, and experience soon teaches that one can do it easily "it should also be emphasized that in many instances, the difference between the premium for an endowment policy and that for a life insurance contract requiring a smaller payment is saved only because of the voluntarily assumed sacrifice of paying the higher rate- Endowment insurance, therefore as it concerns those who find it difficult to save, represents a means of using the by- product of their earning- the small sums other wise wasted in needless expenditures- for the accumulation of a competence Endowment insurance also furnishes the advantage of an absolutely safe plan of investment.

2- As Means of providing for old age: endowment insurance, if the term is so selected as to make the policy mature at an age like 60. 65. or 70, may serve as an excellent method of accumulating a fund for support in old age. Many who oppose endowments maturing at earlier periods because of their greater cost are ardent supporters of long- term endowments maturing at an age when an individual's earning capacity usually ceases and when he or she naturally expects to retire from actual work. Relatively few individuals succeed in laying up a decent competence by the time this age is reached. Most people are therefore confronted with two contingencies: (1) an untimely death may leave their families unprotected and (2) in case of survival until old age. They may lack the means of proper support. Both these contingencies may be conveniently provided for by a long- term endowment. If death should occur at any time during the term. The insurance proceeds revert to the family, but should the insured survive to old age, when the need for insurance for family protection in the usual sense has largely or altogether passed, he or she will receive the proceeds of the fund that prudence and foresight enabled him or her to accumulate, to be used it for his or her own support and comfort, through either the ordinary channels of investment or the exercise of the available options in the policy.

3- As a means of hedging against the possibility of the saving period's being cut short by death. Quite often we plan and actually start a savings program for a specific purpose to be completed by a specific date. It might be a plan to provide money for a college education for our children, or an amount we wish to donate to a worth while charity at a definite future date. We may divide the amount by the number of years and set aside each year (considering interest) and put our plan into operation. If death occurred before the completion of the period, we would perhaps fall short of our goal. But by putting the annual savings into an endowment policy to run for the definite period we would be assured that, live or die, our goal would be fully attained.

4- As a means of accumulating a fund for specific purposes endowment insurance may serve admirably in accumulating a fund for specific purposes. For instance, it lends it self to the accumulation of a fund for the benefit of such institutions as colleges churches, hospitals and so forth. Again endowment insurance may be used to accumulate a retirement fund for the policy holder. It also may serve some special family purpose especially as regards the making of proper and certain provision for starting children in life. It is to accomplish this purpose in the most convenient manner for parents or guardians that companies issue the various forms of educational and “children’s” endowments already enumerated. By means of such policies, small savings, which would otherwise probably be wasted, may be accumulated into a fund to be used for any specific purpose, with assurance that the objective will be attained despite the premature death or disability of the policyholder.

Fourth : Life Annuities

Nature of annuities:

an annuity may be defined as a periodic payment made during a fixed period or the duration of a designated life or lives. If the payments are made with reference to life contingencies, the contract is known as a life annuities may be either temporary (payable for a fixed period or until the death of the annuitant whichever is earlier) or whole. The term “life” in the title of an annuity simply indicates that payments are contingent upon the continued existence of one or more lives in the following discussion, reference to life annuities will mean whole- life annuities unless specifically indicated otherwise.

In one sense the life annuity may be described as the opposite of insurance protection against death. In its pure from (that is when not used in connection with some insurance arrangement) a life annuity may be defined as a contract whereby for a cash consideration, one party (the insurer) agrees to pay the other (the annuitant) a stipulated sum (the annuity) periodically throughout life, the understanding being that the principal sum standing to the credit of the annuitant shall be considered liquidated immediately upon the death of the recipient of the annuity payments. The purpose of the annuity, it is seen, is to protect against a risk. The outliving of one’s income- which is just the opposite of that confronting a person who desires life insurance as protection against the loss of income through premature death insurance is a pooling arrangement whereby a group of individuals make contributions so that the dependents of the unfortunate few who die each year may be indemnified for loss of the breadwinner's income, whereas the annuity is a pooling arrangement whereby those who die prematurely and do not need further income make a contribution so that those who live beyond their expectancy may receive more income than their contributions alone would provide from an economic standpoint, life insurance and annuities have been regarded as vastly different from one another. Life insurance protects against the absence of income in the event of premature death or disability whereas the annuity protects (insures) against the absence of income on the part at those "afflicted" with undue longevity. Both mean dependable protection to two unfortunate groups. the one dying too soon and the other living too long. They are both insurance arrangements, the one pertaining to the years of ascendancy and the other to the years of decline. When coupled together the two forms of insurance complete the economic program from start to finish on a basis of financial dependability.

Classification Of Annuities:

Annuities may be classified in a number of ways, but the following descriptive groups are generally used in describing an annuity's function. Understanding them will be helpful in understanding the life annuity contract. They are (1) number of lives (2) method of premium payment, (3) time when income commences, (4) presence or absence of a refund feature and (5) units in which benefits are expressed.

Number of lives: This classification involves simply the question of whether annuity payments are made with reference to a single life or more than one life. If the contract covers two or more lives, it is known generally as a joint- life annuity.

Method of premium payment: Both single- life and joint-life annuities may be purchased with either single premiums or level periodic premiums. Thus, a single life annuity could be purchased with a lump sum accumulated through savings, inheritance, or other media, or an individual may choose to spread the cost over a specified period by paying periodic (usually annual) premiums.

Time when income payments commence: in terms of this classification. There are two categories: immediate and deferred. An immediate annuity is one under which the first benefit payment is due one payment interval (monthly, annually, or other) from the date of purchase. This type of life annuity must always be purchased with a single premium. It is basic to life annuity underwriting that no benefit payments are ever made until the entire purchase price of the annuity is in the hands of the insurance company. The deferred annuity, on the other hand may be purchased with either a single premium or a periodic level premium under a deferred life annuity, there must be a period longer than one benefit- payment interval before benefit payments begin the longer the deferred period, the more flexibility may be permitted in premium payments. With the basic underwriting rule mentioned above still being met. Normally, a number of years elapse before benefit payments commence. The majority of individual deferred annuity contracts are sold on a level- premium basis, with premiums continuing until the stated date for commencement of benefit payments or until prior death of the annuitant. In the case of deferred life annuities purchased under a group annuity pension contract, the single- premium method of purchase is utilized almost exclusively.

Presence of refund feature: under this basis of classification the presence or absence of a refund feature is considered as well as the various forms the refund benefit may take.

Units in which benefits are expressed: traditionally, annuity benefits have been expressed in fixed dollars. With the advent of the variable annuity, the units in which payout benefits are expressed forms another basis of classification.

Nature of insurance companies' obligation: This discussion classification is based on the benefits payable in the event of death prior to following, the commencement of income payments under the contract, that is, during the accumulation period or the liquidation period.

Accumulation period: the immediate annuity is always purchased with a single premium, and the income commences at the end of the first benefit interval following the purchase of the contract. Obviously, there is no obligation on the company before it receives the purchase price. Under a deferred annuity, however, there may be an obligation to pay some benefit before the income begins, that is, during the accumulation period. A deferred annuity may be written on a pure or refund basis in this connection. Under the pure deferred annuity, the premiums are discounted for mortality during the accumulation period, and in event of death prior to the commencement of income payments paid, since they would be fully earned. Where a refund feature is applicable during the accumulation period, premiums with or without interest would be refunded if death occurred before annuity income begins. Naturally, a deferred annuity with such a refund feature would cost more than the pure deferred annuity mentioned above.

Liquidation period: the following discussion of the nature of the company's obligation during the liquidation period is applicable to both immediate and deferred annuities. The discussion is simply a description of the various arrangements under which a given sum can be liquidated on the basis of life contingencies.

There are two arrangements: (1) pure and (2) refund. The pure life annuity, frequently referred to as a straight life annuity, provides periodic income payments that continue as long as the annuitant lives, but terminate upon the annuitant's death. The annuity premium is considered fully earned upon the death of the annuitant, no matter how soon that may occur after the commencement of income, and no refund is payable to the annuitant's estate or any beneficiary. Under this form of income, the entire purchase price is applied to provide income to the annuitant, no part of it going to pay for any refund benefit following the annuitant's death. Thus, the pure life annuity provides the maximum income per dollar of outlay, and is appropriate where only a limited amount of capital is available. It should be pointed out, however, that at the younger ages, because of the high probability of survival the difference in income between a pure life annuity and one with a refund feature is too small to take a chance on an early death.

Regardless of the questions of equity and technical soundness, most people are opposed to placing a substantial sum of money into a contract that promise little or no return if they should die shortly after income payments commence. Therefore, most companies have added a refund feature to life annuities in order to make them more salable. In contrast to the pure life annuity, not all of the purchase price goes to provide income payments to the annuitant. Part of the purchase price is applied to meet the cost of guaranteeing a minimum amount of benefits, whether or not the annuitant lives to receive them. Thus, for a given premium outlay, a smaller periodic income payment will be available under a refund life annuity than would be available under the pure life annuity form. The minimum benefit guarantee or refund feature may be stated either in terms of a guaranteed number of annuity payments whether the annuitant lives or dies, or in terms of a refund of the purchase price in the event of the annuitant's early death.

the first form of life annuity with a refund feature mentioned above is named, "life annuity certain and continuous" "life annuity with installments certain" and life annuity with minimum guaranteed return". This arrangement calls for a guaranteed number of monthly (or annual) payments to be made whether annuitant lives or dies, with payments to continue for the whole of the annuitant's life if he or she should live beyond the guaranteed period. Contracts are usually written with payments guaranteed for five, ten, fifteen or twenty years. The size of income payments available from a given principal is less for the longer guarantee period.

The variable Annuity: In times of deflation, the value of such a fixed and guaranteed annuity income increases because of a falling price structure, that is, the purchasing power of fixed dollar income tends to go up in periods of falling prices. On the other hand, when inflation produces a rising price level, when inflation a rising price level, the purchasing power of the same fixed-dollar income tends to fall off. Consequently, in times of deflation, the fixed-dollar annuity finds much popularity, whereas in periods of inflation, annuity incomes providing a decreasing amount of purchasing power tend to cause wide criticism. The impact of inflation had led to search for a way of providing a guaranteed life annuity with a reasonable stable purchasing power. The variable life annuity, based on the equity type of investments, has been advanced as a possible answer to this problem.

Nature of the variable annuity: during the accumulation period, a level annual premium is paid to the insurance company and, along with other variable annuity premiums, placed in a special "variable annuity account". The funds in this account are invested separately from the company's other assets, mostly in common stocks, based on current investment results, is valued at $10, a level premium of $100 after expenses will purchase ten units is changed, the level premium of $100 would purchase more or less than 10 units. This procedure would continue until the maturity of the contract. At that time, the accumulated total number of units credited would be applied, according to actuarial principles and based on current valuation of a unit, to convert credited units to a retirement income of so many units, to be valued annually for the lifetime of the annuitant.

Instead of providing for the payment each month of a fixed number of dollars, the variable annuity provides for the payment each month or year of the current value of a fixed number of annuity units. Thus, the dollar amount of each payment would depend on the dollar value of an annuity unit when the payment is made. The valuation assigned to unit would depend on the investment results of the special account, which would be invested mostly in common stocks.

Investment risk assumed by annuitant: Under a convential annuity, the insurance company assumes the mortality, expense, and investment risks. The company invests the assets behind convential annuities mostly in stable fixed-dollar investments.

This is necessary to implement their guarantee of fixed-dollar incomes. Under a variable annuity, the insurance co. assumes only the risk of fluctuations due to mortality and expenses. the assets behind variable annuities are invested in equity type investments, and dollar income is permitted to fluctuate accordingly on the theory that the dollar income, while varying, will provide a more stable amount of purchasing power. The annuitant is assuming the investment risk, but it is important to remember that the insurance company is still absorbing expense fluctuations and guaranteeing that the annuitant will not outlive his or her income. Equity- type investments it is asserted by proponents of the plan, yield more under normal business conditions, and the annuitant has a more than he would receive under guaranteed annuity. On the other hand, in time of depressed business conditions, the variable annuitant will receive somewhat less in dollars of income than in the case of a conventional fixed dollar annuity.

CHAPTER 5
MORTALITY TABLES

PROBABILITY

In the life insurance business, the subject of *probability* has its application in calculating and using the *probabilities of living and dying.* The concern is with predicting future deaths, based on past experience. In so doing, it is usually expedient to use "mortality tables". This chapter deals with the subject of *probability* and its use in connection with "mortality tables". Succeeding chapters will show how *compound interest* and *probability* are combined in life insurance calculations.

In general, the probability, or likelihood, of some event occurring is expressed mathematically as a fraction or a decimal. This indicates how many times the event may be expected to occur out of a certain number of *opportunities* for it to occur. For example, the probability in one attempt of drawing an ace at random out of a deck of 52 playing cards (four of which are aces) is $\frac{4}{52}$, or simply $\frac{1}{13}$. This means that an ace can be expected to be drawn one time out of every 13 attempts.

If a very large number of such attempts were made, it would actually happen that very nearly $\frac{1}{13}$ of the attempts produce aces. This statement can be made with certainty, because of the statistical concept known as the *law of large numbers*. In this particular example, the probability given

(namely $\frac{1}{13}$) could be derived either by:

1) **Exact mathematical calculation, since it is known that $\frac{1}{13}$ of the cards in the deck are aces; or**
2) **By observation of a large number of attempts, and calculation of the ratio of aces drawn to total draws.**

A second example might be the probability of a person now age 75 dying within the next year. This probability might be 0.07337. This decimal may be expressed as the fraction $\frac{7.337}{100.000}$. This means that persons now age 75 can be expected to die within the next year out of every 100,000 such persons now alive. In this particular example, the probability can be derived only by the observation of a large number of persons age 75. It cannot be calculated exactly by prior knowledge, as was the case with the aces in a deck of cards.

DERIVING PROBABILITIES

The most important probability in life insurance mathematics is the probability that a particular person will die within one year. It has been found that many characteristics of the particular person affect that probability, but the most important such characteristic is the person's age. Therefore, a different probability exists for each age. Chart 7-1 shows examples of such probabilities.

CHART-1

Person's Age	*Probability That the Person Will Die within One Year*
20	**0.00179**
21	**0.00183**
22	**0.00186**
.	**.**
.	**.**
85	**0.16114**

It has been pointed out that probabilities of dying can be derived only by observing a large number of persons. In actual practice, insurance companies usually observe all of the persons they are currently insuring, but over a very limited period of time. By this means, persons of all ages are observed within this period, and probabilities of dying within one year are calculated for each age.

Basically, the probability of dying within one year at each age, called the *rate of mortality,* is equal to the ratio of the number dying at that age to the number who are exposed to the risk of dying at that age. (The number dying at a certain age includes those persons who die within the year starting at that exact age and before their next birthday.) The rate of mortality for a certain age is calculated by dividing the number dying at that age by the number so exposed. For example, if 910 persons are being observed who are all age 54, and 12 of them die during the

year of observation, the probability that a person age 54 will die within a year (the rate of mortality at age 54) may be calculated as

$$\frac{12}{910}\text{, or .01319 (rounded off)}$$

To Illustrate- **A certain group of insured persons all the same age has been observed over a period of years. The group contains 4,112 persons celebrating their 64th birthday. Calculate the rate of mortality at age 63, if it was observed that 87 out of this same age group had died the previous year.**

Solution- **Since 87 of the group died the year before, there were actually 4,112 + 87 = 4,199 persons attaining age 63 (one year previous). Hence, the rate of mortality at age 63 may be calculated as**

$$\text{Rate of Mortality at Age 63} = \frac{\text{Number Age 63 Dying within the Year}}{\text{Number Exposed at Age 63}}$$

Substituting 87 for number *dying*, 4199 for number exposed

$$= \frac{87}{4{,}199}$$

$$= .02072$$

Most of the practical problems faced by an insurance company in deriving these probabilities are associated with the calculation of the number so exposed. For example,

while the period of time being observed may be a calender year (from January 1 to December 31) birth dates occur over the entire year. If a particular person attained the age of 45 on July 1 that year, he would be exposed to death as a person age 44 for half the year and as a person age 45 for half the year Adjustments also have to be made for those persons who enter the group or terminate at some time during the observation period It would be wrong to exclude them completely from the number exposed, because had they died while under observation, they would certainly have been included in the number dying.

The actual rates of mortality for each age experienced by a single insurance company will show considerable fluctuation from year to year. To produce valid and reliable estimates of the mortality rates and to minimize accidental fluctuations, it is usual to base the rates of mortality on the experience of a number of years, rather than on that of a single year. However, the number of years used is small enough to reflect current experience. In addition, it is common to combine the experience of a number of companies.

Even with a large volume of experience, the actual rates of mortality will not vary from age to age exactly in the manner that would be theoretically expected, because of accidental fluctuations in the experience. These rates of mortality are then adjusted slightly to correct for accidental fluctuations, in order to obtain the theoretically proper relationship between mortality rates for the various ages.

The mathematical process of adjusting the experience rates to produce a theoretically consistent mortality table is called *graduation.*

In addition to age, there are other important **characteristics which affect the probability of dying. For a life insurance company, the most important are**

1- The person's sex;

2- The health status of the person at the tine he became insured; and

3- The length of time since he became insured.

Therefore, separate probabilities of dying within one year (age by age) are often derived for each of these characteristics.

In practice, safety margins are usually added to the computed rates of mortality before the rates are used for certain insurance calculations, such as setting premiums. These provide for unpredicted increases in mortality or for temporary adverse mortality fluctuations.

TYPES OF MORTALITY TABELS

A mortality table, as the name implies, is a tabulation of the probabilities of dying during the year at each age, i.e., the rates of mortality. Generally, it also includes related information which can be derived from these rates.

There are two principal types of modality tables,

depending on the origin of the data used in deriving. The rates of mortality:

1) **Tables derived from population statistics. These are generally prepared by the National Office of Vital Statistics, based upon data collected during a regular census and registered deaths. An example is the "1960 U.S. Life fables".**
2) **Tables derived from data on insured lives. These generally represent the pooled experience of a number of life insurance companies. These tables are classified into two types:**

   a) **Annuity mortality tables, for use with anntuity contracts (benefits payable only if the contract-holder is alive). An example is the "Annuity Table for 1949" which is printed in the Appendix of this text in Table II.**

   b) **Insurance mortality tables, for use with life insurance contracts (benefits payable when the contract-holder dies). An example is the "1958 C.S.O. Table" which is printed in the Appendix of this text in Table III.**

Experience has shown that the rates of mortality for persons buying annuity contracts are lower, age by age, than for those buying life insurance contracts. This apparently results from the fact that some persons base their selection of one or the other type of contract upon the knowledge that their own probabilities of dying are belier or worse than the

average. Therefore, life insurance companies use different mortality tables for life insurance and for annuities.

In both annuity and insurance operations, it is important that the rates of mortality assumed be conservative. In life insurance, this requires that the table used should exhibit higher rates of mortality than will probably be experienced. This is needed so that the company will not be required to pay death benefits sooner than was anticipated. The converse is true in the selection of a conservative table to use for annuity contracts. Here the rates of mortality assumed should be lower than the expected rates, so that the company will not be required to pay annuity benefits for a longer period of time than was assumed in the calculations.

In general, there has been an observed trend over a period of many years toward lower mortality rates. This has been a result of our economic and medical advances. Therefore, conservative insurance modality tables have tended to become more conservative. On the other hand, conservative annuity mortality tables have tended to become less conservative (because more people are living longer).

It was pointed out in Section 7.2 that separate probabilities of dying are often derived for each sex. Experience has shown that the rates of mortality for females are lower, .age by age, than for males. At many ages this difference is very substantial. This accounts for the fact that the Annuity Table for 1949 is actually two separate tables:

one for males and one for females. Only the male Table is printed in Table II.

The 1958 Commissioners' Standard Ordinary Table, which is used in connection with life insurance, was developed from experience for both male and female lives. However, it is customary (and permitted by law) to assume that the rates of mortality contained in this table apply only to males. When using this table for females, the usual procedure is to subtract three years from the true age of the female. For example, a woman age 25 is considered to be subject to the rate of mortality applicable to a man age 22. This is known as "using a 3-year setback," or a "3-year rating down in age." In Table III, the label "male" is used, as a reminder that it is customary to make an adjustment where a female life is involved. At the very young and very old ages, a 3-year setback is not appropriate, however, and other types of adjustments are customary.

The Annuity Table for 1949 (Table II) and the 1958 C.S.O. Table (Table III) will be used for all calculations in this text. However, the principles discussed are general and can be applied to any mortality table.

STRUCTIURE OF A MORTALITY TABELS

A mortality table is generally shown with four basic columns. The beginning and ending portions of the 1958 C.S.O. Table, which is printed in full in Table III, appear in

Chart 7-2. This shows the columns to be described, and will be used later to explain the interrelationships among them.

CHART-2

Portions of the 1958 C.S.O. Table

Age ***x***	***Number Living at Age x*** l_x	***Number Dying Between Age x and*** **Age *(x+1)*** d_x	***Rate of Mortality*** q_x
0	**10,000,000**	**70,800**	**0.00708**
1	**9,929,200**	**17,475**	**0.00167**
2	**9,911,725**	**15,066**	**0.00152**
.	.	.	.
.	.	.	.
.	.	.	.
97	**39,787**	**18,456**	**0.48842**
98	**19,331**	**12,916**	**0.66815**
99	**6,415**	**6,415**	**1.00000**

COLUMNS FOR AGE AND OF MORTALITY. **The first column represents the age. Ages are very often shown starting with age zero (a person's first year of life).**

Another column contains the rates of mortality. The rate of mortality shown opposite each age represents the assumed probability of dying within one year for a person

who is that particular age. It is customary to assume that the probability of dying is a certainty (equals 1) at some very high age, such as 99. This age, then, is the highest age shown in the first column.

The rate of mortality, or the probability of dying, at age *x* is represented by the symbol. q_x

In this symbol, the letter *x*, shown to the right and slightly lower than the q, is a "subscript." Hence, it is a pad of the whole symbol, and q, does not mean "*q* multiplied by *x*". The symbol is read "*q* sub *x*" or simply "*q x*". An example would be q_{27}

Which is read "*q* sub 27" or "*q* 27". It means the rate of mortality at age 27, or the probability that a person age 27 will die within a year (before he reaches age 28).

This symbol q_x appears at the top of the rate of mortality column in a mortality table. Since the letter *x* is used therein as a general representation for age, the letter *x* appears at the top of the age column.

GRAPHIC PRESENTATION OF *q* **Relative values of q_x can be seen on the graph, Figure-1. The two curved lines represent values of q_x. (for males) from the Annuity Table for 1949 and the 1958 C.S.O. Table. Ages from age 0 to age 70 are shown along the bottom of the graph. For each age, the distance up to one of the lines indicates the value of q_x at that age, as set forth in the particular mortality table. Space limitations make it impossible to show the graph to the**

highest age in the tables. The value of q_x is 1.000 at age 99 in the 1958 C.S.O. Table and at age 109 in the Annuity Table for 1949. This gives an indication of how rapidly the death rates increase after age 70. This graph would have to be 20 times as tall as it is to show values for all ages up through the highest age.

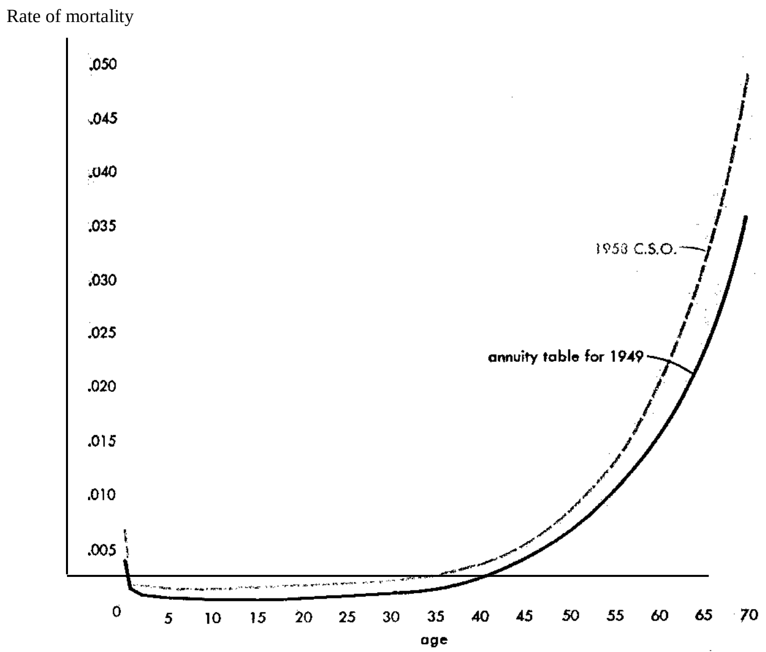
It is interesting to note that the rates of mortality actually decrease age by age from age 0 to approximately age 10. From this point, the rates increase very slowly until about age 40. It should also be noted that the graph clearly portrays that rates of mortality for those buying annuity contracts are lower, age by age, than for those buying life insurance contacts.

COLUMNS FOR NUMBER LIVING AND DYING. **The other two basic columns of a mortality table are set up to show what happens each year to a large group of people all the same age, starting when they are all a certain low age, such as age zero. An arbitrary number of people, such as 10,000,000, are assumed to be alive at this time. During the first year a certain number will die, leaving the remaining persons to begin the second year. Then a certain number of these will die before reaching the end of the second year, etc.**

FIGURE-1

Values of q_x

(black line is Annuity Table for 1949-colored line Is 1958 C.S.O. Table)



One column shows how many persons (out of the original group) are assumed to still be alive at each age. The number shown opposite each age represents the assumed number out of the original group who are still living at that particular age.

The number of this group who are still alive at age *x* is represented by the symbol. l_x

That is, *l* with a subscript *x*. It is read "ι sub *x*" or simply "*lx*." An example would be l_{20}

Which is read "ι sub 20" or"*l*20". It means the assumed number out of the original group who are still living at age '20.

The other column shows how many persons out of this group are expected to die at each age. The number shown opposite each age represents the number of persons who are expected to die while they are that particular age, that is, in the year after reaching that age but before reaching the next age.

The number out of this group who die in the year they are age *x* is represented by the symbol d_x**.**

That is, *d* with a subscript *x*. It is read *"d* sub *x*" or simply *"dx*." An example would be d_{74}

Which is read "*d* sub 74" or "*d* 74." It means the number of the group who are expected to die while age 74 (in the year after reaching age 74 but before reaching age 75).

Referring to the basic columns shown above in the portion of the 1958 C.S.O. Table, the ι_x**, column shows that 10,000,000 persons (in this hypothetical group) start out life together at age zero. This is the same as saying that** ι_0 **=**

10,000,000. The same column shows that it is assumed that 9,929,200 of them will still be alive at age one, and 9,911,725 of them will still be alive at age two. That is, the table shows

l_1 = 9,929,200

l_2 = 9,911,725

The d_x column shows that 70,800 of the persons in this group die while they are age zero (during the year after birth but before reaching age one). This is the same as saying that d_0 = 70,800. The same column shows that it is expected that 17,475 of them will die during the year they are age one, and 15,066 will die during the year they are age two. That is, the table shows

d_1= 17,475

d_2= 15,066

The probability of dying while age *x*, that is, during the year between age *x* and age (*x* + 1), is sometimes called the probability of dying at age *x*. Likewise, the number so dying is sometimes called the number dying at age *x*.

EQUATIONS FOR INTERRELATIONSHIP. **In general terms, it may be said that if the number dying in a given year (d_x) is subtracted from the number living at the beginning of that year (l_x), the result will represent the number still alive at the next higher age (l_{x+1}). In equation form, this is written:**

$$l_{x+1} = l_x - d_x$$

To Illustrate- **Calculate the value of l_{98} for the 1958 C.S.O. Table using the above equation.**

Solution

Basic equation

$$l_{x+1} = l_x - d_x$$

Substituting 97 for *x* (the age)

$$l_{98} = l_{97} - d_{97}$$

Substituting the values for l_{97} and d_{97} from the table

=37,787 - 18,456

= 19,331

This value of l_{98} agrees with that given in the table.

In general terms, it may be said that if the number living at a certain age (l_x) is multiplied by the rate of mortality at that age (q_x), the result will represent the number expected to die during the year they are that age (d_x). In equation form, this is written

$$d_x = l_x q_x$$

To Illustrate- **Calculate the value of & for the 1958 C.S.O. Table using the above equation a**

Solution

Basic equation

$$d_x = l_x . q_x$$

Substituting 97 for x (the age)

$$d_{97} = l_{97}\, q_{97}$$

Substituting values for l_{97} and q_{97} from the table

= (37,787)(.48842)

Multiplying; rounding to nearest whole number

= 18,456

This value of d_{97} agrees with that given in the table.

In general terms, it may be said that if the number expected to die during the year in which they are a certain age (d_x) is divided by the number living at that age (l_x), the result will represent the rate of mortality at that age (q_x).

The equation, $d_x = l_x . q_x$, may be solved for q_x by dividing both sides by l_x, giving this relationship in equation form:

$$\frac{d_x}{l_x} = q_x$$

To Illustrate- **Calculate the value of q_2 for the 1958 C.S.O. Table using the above equation.**

Solution

Basic equation

$$q_x = \frac{d_x}{l_x}$$

Substituting 2 for x (the age)

$$q_2 = \frac{d_2}{l_2}$$ **Substit**

uting values for d_2 and l_2 from the table

$$= \frac{15,066}{9,911,725}$$

= 0.00152

This value of q_2 agrees with that given in the table.

It will be observed that the value for q_{99} shown in the 1958 C.S.O. Table is *certainty*, i.e., 1. This is done arbitrarily for the purpose of conveniently ending the table, and not because it was observed that everybody who reaches age 99 dies before reaching 100.

CONSTRUCTION OF A MORTALITY TABLE

After the rate of mortality, q_x, is established for each age, the other columns can be constructed. The youngest age in the table should be the youngest age for which it is expected the table will be used. In most cases, tables begin with age zero. The entire mortality table can be constructed by the following steps:

1- Assume an initial value for l_x (at the youngest age in the table). This is usually some large round number, such as 1,000,000 or 10,000,000.

2- Calculate the number of deaths between this age, x, and the next age, $(x + 1)$. This is done using the equation

$$d_X = l_x q_x$$

3- Calculate the number living at the second age $(x + 1)$, using the equation

$$l_{x+1} = l_x - d_x$$

4- Repeat steps 2 and 3, successively, for each higher age.

By way of example, it can be seen how the columns of the 1958 C.S.O. Table were constructed by this process after the rates of mortality were known. The portions of the table appearing in Section 7.4 indicate that an initial value of l_x was chosen to be 10,000,000 at age zero. In other words,

$$L_0 = 10{,}000{,}000$$

The number of deaths between age zero and age one was calculated by applying the basic equation:

$$d_x = l_x\, q_x,$$

Substituting 0 for x

$$d_0 = l_0\, q_0$$

Substituting 10,000,000 for l_0, and the value for q_0 from the table

$$= (10{,}000{,}000)(.00708)$$

Multiplying; rounding to nearest whole number

$$= 70{,}800$$

Next, the number living at age one was calculated as follows:

Basic equation

$$l_{x+1} = l_x - d_x$$

Substituting 0 for x

$$l_1 = l_0 - d_0$$

Substituting 10,000,000 for l_0, and the value of d_0 calculated above

= 10,000,000- 70,800

= 9,929,200

Repeating this process, the number of deaths at age one (between age one and age two) was calculated.

Basic equation

$$d_x = l_x q_x$$

Substituting 1 for x

$$d_1 = l_1 q_1$$

Substituting the value of l_1 calculated above, and the value for q_1

= (9,929,200)(.00 176)

= 17,475

The number living at age two was then calculated as follows:

Basic equation

$$l_{x+1} = l_x - d_x$$

Substituting 1 for x

$$l_2 = l_1 - d_1$$

Substituting the values for l_1 and d_1 calculated above

= 9,929,200 - 17,475

= 9,911,725

All these values agree with those in the portion of the table shown earlier in this chapter. This process was repeated successively until the entire 1958 C.S.O. Table was constructed.

In constructing the Annuity Table for 1949, age 10 was chosen as the lowest age. As shown in Table II, 10,000,000 was chosen as the value of ‘10 However, this table was later extended to begin at age 0. The values of l_x and d_x for ages under age 10 were then calculated in such a way as to produce this same value for l_{10}, namely 10,000,000. As a result, the initial figure, l_0, is 10,104,755.

EXERCISES

1- According to the Annuity Table for 1949 (see Table 11 in the Appendix of this book), what is the probability that a man age 60 will die before reaching age 61?

2- According to the 1958 C.S.O. Table (see Table III in the Appendix of this book), what is the rate of mortality at age 60? Express the answer as a fraction.

3- If the rate of mortality at a certain age is.00742, and the number of persons living at that age is 107,412, how many of them may be expected to die within a year?

4- Using Table III, calculate the probability that a man age 79 will live to age 80. (Hint: It is a certainty that he will either live or die that year.)

5- Using a "3-year setback" for females, what is the female rate of mortality at age 29, according to Table 111? (Hint: Consider that a female is subject to the rate of mortality for a male 3 years younger.)

6- If a mortality table shows l_{18} = 994,831 and d_{18} = 1,094, calculate the value of l_{19}.

7- If a mortality table shows l_{42} = 9,408,108 and l_{43} = 9,374,239, calculate the value of d_{42}.

8- If a mortality table shows l_{36} = 951,003 and q_{36} = .0022, calculate the value of d_{75}.

9- Using the figures in Exercise 8, calculate the value of l_{37}.

10- If a mortality table shows l_{75} = 4,940,810 and d_{75} = 361,498, calculate the value for q_{75}.

11- Calculate the missing items in the following portion of a mortality table:

Age x	l_x	d_x	q_x
20	**92,637**	**…**	**…**
21	**91,914**	**…**	**…**
22	**91,192**	**…**	**…**
23	**90,471**		

PROBABILITIES OF LIVING AND DYING

PROBABILITIES OF LIVING OR DYING IN ONE *YEAR.* **The probability that a person age x will live to reach (*x*+1) is represented by the symbol**

P_x

That is, *p* with a subscript *x*. It is read "*p* sub *x*'' or simply ''*p x*'' An example would be

p_{43}

Which is read "*p* sub 43" or "*p* 43". It means the probability that a person age 43 will live to reach age 44, that is, will be alive for at least one whole year.

In general terms, it may be said that if the number living at age (*x*+1) is divided by the number living at age *x*, the result will he the probability that a person age *x* will live to reach age (*x*+1). In equation form, this is written

$$p_x = \frac{l_{x+1}}{l_x}$$

To Illustrate- **Using the Annuity Table for 1949 (Table II), and the above equation, calculate the value of p_{95}.**

Solution

Basic equation

$$p_x = \frac{l_{x+1}}{l_x}$$

Substituting 95 for *x*

$$p_{95} = \frac{l_{96}}{l_{95}}$$

Substituting the values for l_{96} and l_{95}, from Table II

$$= \frac{150{,}429}{220{,}194}$$

= .683166

The result shows that, according to this particular table, the probability that a person age 95 will live for at least one whole year is .683166.

It is a certainty that a person will either live for one year or die within that year. Since only one of those two events can occur, Rule 1 in Section-1 is applicable: the probability that one of the events will happen is the *total* of the probabilities of each individual event happening.

$$\begin{pmatrix}\text{Probability of} \\ \text{Living 1 Year}\end{pmatrix} + \begin{pmatrix}\text{Probability of} \\ \text{Dying within} \\ \text{1 Year}\end{pmatrix} = \begin{pmatrix}\text{Probability of} \\ \text{Either Living or} \\ \text{Dying That Year}\end{pmatrix}$$

Symbols can be substituted for each of the above expressions, as follows:

Substitute $\boxed{p_x}$ **For** $\begin{pmatrix}\text{Probability of} \\ \text{Living 1 Year}\end{pmatrix}$

Substitute $\boxed{q_x}$ **For** $\begin{pmatrix} \text{Probability of} \\ \text{Dying Within 1 Year} \end{pmatrix}$

Substitue 1 (certainty) for $\begin{pmatrix} \text{Probability of Either} \\ \text{Living or Dying That Year} \end{pmatrix}$

Consequently, the equation is

$$\boxed{P_x + q_x = 1}$$

As an example, according to the 1958 C.S.O. Table, q_{21}=.00183.

Expressed as a fraction, this is $\frac{183}{100,000}$ **This means that, out of 100,000 persons all age 21, there will be 183 deaths during the year. If 183 persons out of 100.000 can be expected to die between the ages of 21 and 22, then 100,000-183=99,817 will survive to age 22. Therefore, the probability that a person age 21 will be living at age 22 is**

$\frac{99,817}{100,000}$ **or .99817**

In other words:

p_{21} = .99817

Using these figures, it can he shown that the equation $p_x + q_x = 1$ is applicable:

Basic equation

$$\boxed{P_x + q_x = 1}$$

Substituting 21 for *x*

$p_{21} + q_{21} = 1$

Substituting the values given above for p_{21}, and q_{21}

.99817 + .00183 = 1

Adding; result verifies the equation

1.00000 = 1

The equation $p_x + q_x$, = 1 can be used to calculate either p_x or q_x when the value of only one of these probabilities is known.

To Illustrate- **Given that p_{46} = .995138, how many persons age 46 can be expected to die before reaching age 47 out of a group of 1,000,000? Out of a group of 100,000? Out of a group of 10,000?**

Solution

Basic equation

$p_x + q_x = 1$

Substituting 46 for *x*

$P_{46} + q_{46} = 1$

Substituting the given value for p_{46}

.995 138 + p_{46} = 1

Subtracting .995138 from both sides

q_{46} **= .004862**

This result can also be written:

$$q_{46} = \frac{4{,}852}{1{,}000{,}000}$$

Which means that out of 1,000,000 persons age 46, 4,862 can be expected to die in the succeeding year, before reaching age 47.

To find the number out of a group of 100,000, the numerator and denominator are each divided by 10. This is done by moving the decimal points one place to the left:

$$q_{46} = \frac{486.2}{100{,}000.0}$$

The numerator would seem to imply a number of persons dying which is not a whole number. This need not be confusing if it is remembered that such numbers are approximations for the exact number predicted to die, or averages based on observations of more than one year. Hence, this expression means that, out of 100,000 persons age 46, approximately 486 can be expected to die in the succeeding year.

To find the number out of a group of 10,000, the numerator and denominator are each divided by 10 again by the method of moving the decimal points one place to the left:

$$q_{46} = \frac{48.62}{10{,}000.00}$$

This expression means that, out of 10,000 persons age 46, about 48 or 49 can be expected to die in the succeeding year.

PROBABILITIES OF LIVING OR DYING *IN* n *YEARS*,

The concepts presented above can be extended to include the probabilities of a person living for any number of years, or dying within any number of years. The probability that a person age x will live at least *ii* more years, or that he will reach age (*x+n),* is represented by the symbol $_np_x$.

That is, *p* with subscripts of *n* preceding and *x* following. It is read *"n p x".* An example would be $_{15}p_{20}$

Which is read "15 p 20". It represents the probability that a person age 20 will live at least 15 more years, that is that he will reach age 35.

In general terms, the probability that a person age *x* will live at least it more years ($_nP_x$) is found by dividing the number living at age *(x+n*) by the number living at age *x*. In equation form this is written

$$_n p_x = \frac{l_{x+n}}{l_x}$$

To Illustrate- **Using Table II, calculate the probability that a man age 48 will live at least 6 more years. Show the answer to 5 decimal places.**

Solution

Basic equation

$$_n p_x = \frac{l_{x+n}}{l_x}$$

Substituting 48 for x, 6 for *n*

$$ {}_{6}p_{48} = \frac{l_{48+6}}{l_{48}} $$

$$ = \frac{l_{54}}{l_{48}} $$

Substituting the values for *15]* and b~ from Table II

$$ = \frac{9{,}103{,}034}{9{,}493{,}401} $$

= .95888

To Illustrate Again- **Using Table Ill with a "3-year setback" for females, calculate the probability that a woman age 36 will live to reach age 46. Show the answer to 5 decimal places.**

Solution

Three years must be subtracted from the ages before using the table. This means using the table as if calculating the probability of a male age 33 reaching age 43. The number of years involved, *n*, is 10.

Basic equation

$$ {}_{n}p_{x} = \frac{l_{x+n}}{l_{x}} $$

Substituting values 33 for *x*, 10 for *n*

$$ {}_{10}p_{33} = \frac{l_{33+10}}{l_{33}} $$

$$ = \frac{l_{43}}{l_{33}} $$

Substituting values for l_{43} and l_{33} from Table III

$$= \frac{9,135,122}{9,418,208}$$

= .96994

The probability that a person age *x* will die within *n* **years, or will die before reaching age (*x*+*n*), is represented by the symbol $_nq_x$.**

That is, *q* with subscripts of *it* preceding and *x* following. It is read *"n q x"* An example would be $_{12}q_{65}$.

Which is read "12 *q* 65". t means the probability that a person age 65 will die within the next 12 years, that is, that he will die before reaching age 77.

In general terms, the probability that a person age x will die within *n* years ($_nq_x$) found by dividing the difference between the number living at ages *x* and (*x*+*n*) by the number living at age *x*. This is expressed in equation form as

$$_nq_x = \frac{l_x - l_{x+n}}{l_x}$$

The numerator equals the number who *die* between ages *x* and (*x*+*n*) because the number living at age *x* is reduced by all those who die in the interval in order to arrive at the number still living at age (*x*+*n*).

To Illustrate- Using Table II, calculate the probability that a man age 30 will die within the next 20 years. Show the answer to five decimal places.

Solution

Basic equation

$$ {}_{n}q_{x} = \frac{l_x - l_{x+n}}{l_x} $$

Substituting 30 for *x*, 20 for *n*

$$ {}_{20}q_{30} = \frac{l_{30} - l_{30+20}}{l_{30}} $$

$$ = \frac{l_{30} - l_{50}}{l_{30}} $$

Substituting values from Table II

$$ = \frac{9{,}870{,}777 - 9{,}388{,}071}{9{,}870{,}777} $$

$$ = \frac{482{,}706}{9{,}870{,}777} $$

= .04890

The number in the numerator, namely 482,706, is the number who die between ages 30 and 50. It is equal to the total of the numbers in the d_x column, beginning with d_{30} and ending with d_{49}.

It is a certainty that a person age *x* will either live at least *n* years or else die within *n* years. Therefore, the total of these two individual probabilities is equal to 1. In equation form, this is written as

$$ {}_{n}p_{x} + {}_{n}q_{x} = 1 $$

This equation is similar to that discussed above for the relationship between the probabilities of living and/or dying for one year.

SOLVING FOR OTHER UNKNOWNS. **The above equations for $_np_x$ or $_nq_x$, namely,**

$$_n p_x = \frac{l_{x+n}}{l_x}$$

$$_n q_x = \frac{l_x - l_{x+n}}{l_x}$$

Can be solved for any desired unknown value which appears therein.

To Illustrate- **Using Table III, to what age does a man age 24 have a 50-50 chance of living?**

Solution

The question may be stated in another way by asking, "For what value of n is $_np_{24}$ equal to .50?"

Basic equation

$$_n p_x = \frac{l_{x+n}}{l_x}$$

Substituting .50 for $_np_x$, 24 for x

$$0.50 = \frac{l_{24+n}}{l_{24}}$$

Substituting the value for l_{24} from Table III

$$0.50 = \frac{l_{24+n}}{9{,}593{,}960}$$

Solving for l_{24+n} by multiplying both sides by 9,593,960

4,796,980 = l_{24+n}

The problem asks for the *age* for which l_x 4,796,980; this age will equal (24+*n)*. Reference to Table III shows that l_{73} 4,731,089 is the nearest to the desired value. Hence, age 73 is the sought-after age. Since 73-24 = 49, the sought-after number of years, *n* equals 49.

<u>*PROBABILITIES INVOLVING MORE THAN ONE EVENT*</u>. **Probabilities involving the happening of more than one event may be calculated using the rules given in Section 7.1.**

To Illustrate- **Using Table II, calculate the probability that a man age 30 will die either at age 50 or at age 51.**

Solution

Since only one of the events can occur, Rule I given in Section 7.1 is applicable: the two individual probabilities are added, The probability that a man age 30 will die during the year he is age 50 is equal to the number so dying divided by the number living at age 30:

Basic equation

$$\begin{pmatrix}\text{Poebabilit y of} \\ \text{Dying at 50}\end{pmatrix} = \frac{d_{50}}{l_{30}}$$

Substituting the values for d_{50} and l_{30} from Table II

$$= \frac{61{,}558}{9{,}870{,}777}$$

= .00624

Similarly, the probability that a man age 30 will die

during the year he is age 51 is

Basic equation

$$\begin{pmatrix}\text{Poebabilit y of}\\ \text{Dying at 51}\end{pmatrix} = \frac{d_{51}}{l_{30}}$$

Substituting the values for d_{51} and l_{30} from Table II

$$= \frac{67{,}869}{9{,}870{,}777}$$

= .00688

The desired probability equals the total of the two individual probabilities:

Basic equation

$$\begin{pmatrix}\text{Poebabilit y of}\\ \text{Dying at 50 or 51}\end{pmatrix} = \begin{pmatrix}\text{Probabilit y of}\\ \text{Dying at 50}\end{pmatrix} + \begin{pmatrix}\text{Probabilit y of}\\ \text{Dying at 51}\end{pmatrix}$$

= .00426 + .00688

= .01312

To Illustrate Again- Using Table III, calculate the probability that a man, age 50, and his son, age 20, will both live at least 15 more years.

Solution

Since the happening of one event has no effect upon the happening of the other, Rule 2 in Section 7.1 is applicable: the two individual probabilities are multiplied.

For the man:

Basic equation

$$_n p_x = \frac{l_{x+n}}{l_x}$$

Substituting .50 for *x*, 15 for *n*

$$_{15} p_{50} = \frac{l_{50+15}}{l_{50}}$$

$$= \frac{l_{65}}{l_{50}}$$

Substituting the value for l_{65} from Table III

$$= \frac{6,800,531}{8,762,306}$$

= .77611

For the son:

Basic equation

$$_n p_x = \frac{l_{x+n}}{l_x}$$

Substituting 20 for *x*, 15 for *n*

$$_{15} p_{20} = \frac{l_{20+15}}{l_{20}}$$

$$= \frac{l_{35}}{l_{20}}$$

Substituting the value for l_{35} from Table III

$$= \frac{9,373,807}{9,664,994}$$

= .96987

The desired probability that both will live at least 15 years equals the product of two individual probabilities multiplied together:

Basic equation

$$\begin{pmatrix}\text{Poebabilit y of} \\ \text{Both Live}\end{pmatrix} = \begin{pmatrix}\text{Probabilit y} \\ \text{Man Lives}\end{pmatrix}\begin{pmatrix}\text{Probabilit y} \\ \text{Son Lives}\end{pmatrix}$$

Substituting the probabilities calculated above

= (.77611)(.96987)

= .75273

Questions For Review

(Use Table III for Exercises 1 to 11)

1) **What is the probability that a man age 20 will live for 1 year?**
2) **What is the probability that a man age 20 will live for 25 years?**
3) **What is the probability that a man age 30 will be living at age 50?**
4) **Using a "3-year setback" for females, calculate the probability that a female age 20 will survive to age 45.**
5) **How many men out of 10,000 men age 35 can be expected to live to age 65?**
6) **What is the probability that a man age 30 will die before reaching age 65?**
7) **Using a "3-year setback" for females, calculate the number of females out of 100,000 females age 27 who can be expected to die within 10 years.**
8) **To what age does a man age 21 have a $\frac{1}{3}$ chance of living?**
9) **What is the probability that two men, ages 30 and 40, will both survive 10years?**
10) **What is the probability that two men ages 30 and 40, will both die in the next 10 years?**
11) **What is the probability that a man age 20 will die either during the year he is age 70 or during the year he is age 80?**

CHAPTER 6

LIFE ANNUITIES

INTRODUCTION

It was pointed out that there are two types of annuities: *annuities certain,* which involve a fixed number of payments, and annuities where the continuation of the payments depends upon the occurrence of some event. A *life annuity* is an example of the second type. Each payment in a life annuity is made only if a designated person is alive to pay or receive it.

Accumulated and present values of life annuities can be calculated in a way similar to the method used for annuities certain in financial math. Accumulated and present values of just one payment will be considered first, as was done for payments certain in financial math. To ask the question:

"How much should a man now age 35 pay for the right to receive $100 at age 60 if he is then alive to receive it?"

Is the same as asking?

"What is the present value to a man now age 35 of $100 payable at age 60, calculated with benefit of survivorship?'

The phrase *with benefit of survivorship* is used to distinguish this situation from one where only rates of interest are involved, as was the case in financial mathematices. If only rates of interest were involved in finding the present value, the answer would be

Basic equation for present value

$$A = S\upsilon^n$$

Substituting \$100 for S, 25 for *n* because 25 years are involved between age 35 and age 60.

$$= \$100\upsilon^{25}$$

But now the element of survivorship is also involved, because the man must survive in order to receive the payment. *With benefit of survivorship,* then, means that payments will be made only if the designated payor or recipient is alive at the time the payment is due.

To begin solving the problem posed above, it is necessary to con-suit a mortality table. If Table II is used, the number shown as living at each of the two ages involved is

$l_{35} = 9{,}814{,}474$

$l_{60} = 8{,}465{,}043$

This means that if there is a group of 9,814,474 men alive at age 35, then 8,465,043 of this group will still be alive at age 60. In order to solve the problem, it must be assumed that all of these men are individually involved, that is, each one still alive at age 60 will receive \$100. It is desired to know the present value of this money to men when they are age 35 (calculated with benefit of survivorship).

Since \$100 is to be paid to each of the l_{60} men, the total amount that will be paid out altogether is

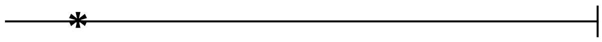
$$\$100(l_{60}) = (\$100)(8{,}465{,}043)$$

$$= \$846{,}504{,}300$$

Twenty-five years earlier, l_{35} men will pay the money in. The original question now may be stated: "How much will each pay?" The total amount paid in is

$$\boxed{\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(l_{35}) = \begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix} + (9{,}814{,}474)}$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(l_{35}) \qquad\qquad \mathbf{\$100(}l_{60}\mathbf{)}$$



age 35 **age 60**

The money paid in will earn interest over the 25-year period. For this example, the rate will be assumed to be $2\frac{1}{2}$%. The basic equation for finding present value can be used to show that *all the money paid in equals the present value of all the money to be paid out* 25 *years later.* The amount each pays in can then be found:

$$\boxed{\mathbf{A=S\upsilon^{n}}}$$

substituting $\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}814{,}474)$ **for *A*, \$846,504,300 for S, and the value of υ^{25} at $2\frac{1}{2}$% from Table I**

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}814{,}474) = (\$846{,}504{,}300)(.539391)$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}814{,}474) = \$456{,}596{,}801$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix} = \$46.52$$

This \$46.52 is less than the present value calculated at interest only. The latter would be

A = $100($\upsilon^{25}$ at $2\frac{1}{2}$%)

= ($100)(.539391)

= $53.94

$53.94 is the amount each man would pay in if all were to receive $100 25 years later (dead or alive). The $46.52 payment with benefit of survivorship is smaller because in that case only those who survive are to receive their $100.

It can be proved that $46.52 is the desired present value, with benefit of survivorship, at age 35 of $100 payable at age 60 as follows:

Total amount paid in = $46.52(135)

= ($46.52)(9,814,474)

= $456,569,330.48

Total amount accumulated at $2\frac{1}{2}$% for 25 years

= ($456,569,330.48)($1.025)^{25}$

= ($456,569,330.48)(1.853944)

= $846,453,970.83

Amount payable to each survivor at age 60 (the accumulated fund divided by the number of survivors)

= $846,453,970.83 ÷ l_{60}

= $846,453,970.83 ÷ 8,465,043

= $99.99

(The missing I cent is due to the fact that $46.52 was rounded off to the nearest cent instead of using more decimal places.)

The present value, with benefit of survivorship, at age 35 of $100 payable at age 60 can be written as

$$\$100\left(\frac{l_{60}}{l_{35}}\right)\upsilon^{25}$$

or as

$$\frac{(\$100)(l_{60}\upsilon^{25})}{l_{35}}$$

Both these expressions permit interesting verbal interpretations. In the first, $\left(\frac{l_{60}}{l_{35}}\right)$ equals the probability that a person age 35 will live to age 60. Hence, the first expression says that the \$100 is multiplied by the probability of surviving and also by the regular discounting factor for finding present values at interest. The second expression says that the \$100 payable to each of l_{60} persons is discounted at interest for 25 years, and this amount is divided among the l_{35} persons to find out how much each must pay in.

In the numerical example, it was shown how compound interest and probability are combined in calculating contingent payments. Using more general terms, the equation is

$$\left(\begin{array}{c}\text{Present Value of \$1 Due in } n \\ \text{Years to a Life Now Age } x\text{, With} \\ \text{Benefit of Survivorship}\end{array}\right) = \$1\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

To Illustrate- Using the 1958 C.S.O. Table (Table III) and 3% interest, calculate the present value at age 20 of \$400 due in 15 years if the person is still alive; also due in 25 years.

Solution

Due in 15 Years

Basic equation

$$\text{Present Value} = \$400\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

Substituting 20 for *x* (the evaluation age), 15 for *n* (the number of years)

$$= \$400\left(\frac{l_{35}\upsilon^{15}}{l_{20}}\right)$$

Substituting the values for the *l*'s from Table III, for υ^{15} from Table I (3%)

$$= \$400\left[\frac{(9{,}373{,}807)(.641862)}{9{,}664{,}994}\right]$$

$$= \$400\left(\frac{6{,}016{,}691}{9{,}664{,}994}\right)$$

= $249.01

Due In 25 Years

Basic equation

$$\text{Present Value} = \$400\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

Substituting 20 for *x*, 25 for *n*

$$= \$400\left(\frac{l_{45}\upsilon^{25}}{l_{20}}\right)$$

Substituting tin, values for the *l*'s from Table 111, for υ^{25} from Table I (3%)

$$= \$400\left[\frac{(9{,}048{,}999)(.477606)}{9{,}664{,}994}\right]$$

$$= \$400\left(\frac{4{,}321{,}856}{9{,}664{,}994}\right)$$

= $178.87

As the number of years *increases* before the payment is to be made, the present value *decreases.* This is because there is a smaller probability that it will have to be paid, and because there will be a greater number of years in which to earn interest.

PRESENT VALUE OF LIFE ANNUITY

CALCU*LATION OF PRESENT VALUES.* An annuity is a series of payments. It is not difficult to find the present value of a series of payments where each payment is made only if the designated payor or recipient is alive to pay or receive it, The present value of the annuity is the *total* of the present values of each of the individual payments. The principles explained above can be used to find the present value of each individual payment.

For example, the present value at age 25 of a life annuity of $100 per year for three years, first payment due at age 26, can be represented by the following line diagram:

	$100	$100	$100
*	|	|	|
age 25	26	27	28
	1	2	3 years

The present value of each of the three payments can be calculated individually, as follows:

The Payment Due at Age 26

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

Substituting 25 for x, 1 for n (Exponent 1 need not be

written)

$$= \$100\left(\frac{l_{26}\upsilon}{l_{25}}\right)$$

***The Payment Due at Age* 27**

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{x+n}\upsilon^{n}}{l_{x}}\right)$$

Substituting 25 for *x*, 2 for *n*

$$= \$100\left(\frac{l_{27}\upsilon^{2}}{l_{25}}\right)$$

***The Payment Due at Age* 28**

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{x+n}\upsilon^{n}}{l_{x}}\right)$$

Substituting 25 for *x*, 3 for *n*

$$= \$100\left(\frac{l_{28}\upsilon^{3}}{l_{25}}\right)$$

The present value at age 25 of this annuity is the total of these three expressions. The common multiplier ($100) can be factored out. The fractions to be added together have a common denominator (l_{25}). Hence, the present value of the annuity can be expressed as

$$\text{Present Value} = \$100\left(\frac{l_{26}\upsilon + l_{27}\upsilon^{2} + l_{28}\upsilon^{3}}{l_{25}}\right)$$

The *numerator* of this expression represents the total to be paid out to the survivors at each age, with each such amount being discounted at interest to the evaluation date.

The *denominator* represents the number of persons alive on the evaluation date, among whom this total present value to be paid in must be allocated.

If, for example, the 1958 C.S.O. Table and 3% interest were being used, the present value of the annuity would be calculated as follows:

From above

$$\text{Present Value} = \$100\left(\frac{l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3}{l_{25}}\right)$$

Substituting the values for the l's from Table III, for the υ's from Table 1(3%)

$$= \$100\left(\frac{\begin{matrix}(9{,}557{,}155)(.970874)\\ +(9{,}538{,}423)(.942596)\\ +(9{,}519{,}442)(.915142)\end{matrix}}{9{,}575{,}636}\right)$$

$$= \$100\left(\frac{9{,}278{,}793 + 8{,}990{,}879 + 8{,}711{,}641}{9{,}575{,}636}\right)$$

$$= \$100\left(\frac{26{,}981{,}313}{9{,}575{,}636}\right)$$

= **$281.77**

It can be verified that the payment of $281.77 by each of the persons age 25 will provide $100 to each of the survivors at ages 26, 27, and 28, as follows (using the 1958 C.SO. Table at 3%):

If each of the l_{25}, or 9,575,636, persons contributes $281.77, a fund is provided of

($281.77)(9,575,636) = $2,698,126,955.72

During one year it will earn, interest of

($2,698, 126,955.72)(.03) = $80,943,808.67

The total amount of money in the fund at the end of one year is then

$2,698,126,955.72+$80,943,808.67=$2,779,070,764.39

Payments of $100 to each of the l_{26} or 9,557,155, survivors will require

($100)(9,557,155) = $955,715,500

This leaves a balance in the fund at the end of one year of

$2,779,070,764.39 – $955,715,500 = $1,823,355,264.39

The continued progress of the fund can be traced in Chart 8-1

CHART-1

(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
Year	***Fund* at *Beginning of* Year**	***Interest for One* Year *(Col.* 2 × .03)**	***Total Fund at End of* Year *before Annuity Payments Are Mode (Col.* 2+ *Col.* 3)**	***Annuity Payments to Survivors***	***Balance in Fund afterAnnuityPayments Are Made (Col. 4-Col.*5)**

1	$2,698,].26.955.72	$80,943,808.67	$2,779,070.764.39	$955715500.00	$i,823,355,264.39
2	1,823,355,264,39	54,700,657.93	1.878,055,922.32	953,842,300.00	924,213,622.32
3	924,213,622.32	27,726408.67	951,940.030,99	951,944.20C.00	(-4,169.01)

The shortage of $4,169.01 represents less than $\frac{1}{20}$th of a cent for each of the survivors, and results from rounding off the individual contribution, $281.77, to two decimal places. If all calculations had been carried to more decimal places, the balance in the fund at the end of the third year would have been even closer to zero.

To Illustrate- **Using the Annuity Table for 1949 and $2\frac{1}{2}$% interest, calculate the present value at age 40 of a life annuity of $25 per year for 4 years, first payment due at age 41.**

Solution-

The line diagram for this life annuity appears as follows:

$25 $25 $25 $25

***———|———|———|———|**

age 40 41 42 43 44

1 2 3 4years

The expression for the present value will have *a numerator* representing the total to be paid out to the

survivors at each age, with each such amount being discounted at interest to the evaluation date:

$\$25(l_{41}\upsilon + l_{42}\upsilon^2 + l_{43}\upsilon^3 + l_{44}\upsilon^4)$

The *denominator* is the number living on the evaluation date *(l_{40}):*

Basic equation

$$\text{Present Value} = \$25\left(\frac{l_{41}\upsilon + l_{42}\upsilon^2 + l_{43}\upsilon^3 + l_{44}\upsilon^4}{l_{40}}\right)$$

Substituting the values for the l's from Table II, for the υ's from Table I($2\frac{1}{2}$ %)

$$= \$25\left(\frac{\begin{array}{r}(9{,}715{,}549)(.975610)\\ +(9{,}693{,}980)(.951814)\\ +(9{,}642{,}815)(.905951)\end{array}}{9{,}735{,}263}\right)$$

$$= \$25\left(\frac{9{,}478{,}587 + 9{,}226{,}866 + 8{,}979{,}486 + 8{,}735{,}918}{9{,}735{,}263}\right)$$

= $93.53

TYPES OF LIFE ANNUITIES. **Life annuities may be either temporary life annuities or whole life annuities. In a *temporary life annuity,* each payment is made only if a designated person is then alive, but the payments are limited to a- fixed number of years. In a *whole life annuity,* the payments continue for the entire lifetime of a designated person.**

The three-year and four-year life annuities calculated above are examples of *temporary life annuities.* Each payment is made only if a designated person is then alive,

but the number of such payments is limited to a definite number The first payment is made one period following the date on which the present value is calculated.

There are also temporary life annuities in which the first payment is made at the *beginning,* that is, on the same date on which the present value is calculated. These are known as *temporary life annuities due.* The use of the word "due" is analogous to its use in annuities certain.

The line diagrams of two five-payment life annuities, one immediate and one due, look like this:

$1 $1 $1 $1 $1

Temporary life annuity *

age *x* *x*+1 *x*+2 *x*+3 *x*+4 *x*+5

$1 $1 $1 $1

Temporary life annuity due *

age *x* *x*+1 *x*+2 *x*+3 *x*+4 *x*+5

To Illustrate- Using the 1958 C.S.O. Table and 3% interest calculate the present value at age 25 of a 3-year life annuity due of $100 per year.

Solution-

The line diagram for this life annuity due appears as follows:

$100 $100 $100

*****

age 25 26 27 28

1 2 3 years

The expression for the present value will have a numerator representing the total to be paid out to the

survivors at each age, with each such amount being discounted at interest to the evaluation date. The first payment is due upon the evaluation date. Hence, its present value is simply $100($l_{25}$); it is not multiplied by any discounting factor. The *denominator* is the number living at the evaluation date:

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{25} + l_{26}\upsilon + l_{27}\upsilon^2}{l_{25}}\right)$$

Substituting the values for the *l*'s from Table III, for the υ's from Table I(3%)

$$= \$100\left(\frac{\begin{matrix}(9{,}557{,}636)\\ +(9{,}557{,}155)(.970874)\\ +(9{,}538{,}423)(.942596)\end{matrix}}{9{,}575{,}636}\right)$$

$$= \$100\left(\frac{9{,}575{,}636 + 9{,}278{,}793 + 8{,}990{,}879}{9{,}575{,}636}\right)$$

= $290.79

The present value of a three-year life annuity identical to this one, except that the first payment was made at the *end* of the first year, was calculated earlier in this section to be $281.77. This value is less than the present value of the life annuity due ($290.79), because each payment in the annuity immediate is paid one year later than its counterpart in the annuity due. Hence, there is a smaller probability that it will have to be paid, and there is a greater number of years in which interest is earned.

Life annuities wherein the payments continue for the entire lifetime of a designated person are known as *whole life*

annuities. **Without the word "due," this name implies that the first payment is made one period following the date on which the present value is calculated. Whole life annuities in which the first payment is made at the *be ginning,* that is, on the same date on which the present value is calculated, are known as *whole life annuities due.***

The present value of whole life annuities is calculated by exactly the same procedure as that shown above for temporary life annuities. In the ease of whole life annuities, the payments are included to the end of the mortality table. It is thus assumed that all will die before a certain age; therefore, the number of payments to include in the calculation is actually a limited number, just as for temporary life annuities.

To Illustrate- **Using the Annuity Table for 1949 and $2\frac{1}{2}$% interest, calculate the present value at age 106 of a whole life annuity of $50 per year.**

Solution-

The first payment is due 1 year after age 106 (at age 107). Payments will continue for the person's entire lifetime. However, the Annuity Table for 1949 (shown in Table II) assumes that no persons live beyond the age of 109. Hence, the line diagram for this annuity appears as follows:

$50 $50 $50

*———|———|———|

age 106 107 108 109

1 2 3 years

The expression for the present value will have a *numerator* representing the total to be paid out to the

survivors at each age, with each such amount being discounted at interest to the evaluation date. The *denominator* is the number living on the evaluation date:

Basic equation

$$\text{Present Value} = \$50\left(\frac{l_{107}\upsilon + l_{108}\upsilon^2 + l_{109}\upsilon^3}{l_{106}}\right)$$

Substituting the values for the l's from Table II, for the υ's from Table I($2\frac{1}{2}$ %)

$$= \$50\left(\frac{\begin{array}{l}(54)(.975610)\\ +(16)(.951814)\\ +(\ \ 4)(.928599)\end{array}}{167}\right)$$

$$= \$50\left(\frac{52.6829 + 15.2290 + 3.7144}{167}\right)$$

= **$21.45**

There is no rule for the number of decimal places to keep in rounding the answers obtained by the actual multiplications in the numerator. It is desirable to keep only a sufficient number of digits to have a meaningful answer. In the above illustration, each multiplication answer was rounded to four decimal places. In previous illustrations (involving much larger numbers for the *l's),* each multiplication answer was rounded to the nearest whole number.

The present value of the annuity calculated in the above illustration ($21.45) is less than the amount of one year's payment ($50). This is a phenomenon encountered when only the very high ages are being used. In this illustration it indicates that a considerable portion of those

paying for the annuity at age 106 will die before receiving even one payment.

The calculation of the present value of whole life annuities at the younger ages would become very laborious if the above procedure were followed. Therefore, in actual practice this calculation is done by using *corn mutation functions.* This method will be explained later in Section 8.7.

A *deferred life annuity* is a life annuity in which the first payment is *postponed* one or more periods. Once payments commence, they may continue for the remaining lifetime of the designated person, or they may be limited to a specified number of payments.

As an example of a deferred life annuity, consider the problem of finding the present value at age 65 of a whole life annuity of $100 per year, the first payment being made 38 years after the evaluation date. This means that the first payment is due at age 65 + 38 = age 103 (if the person is then alive). If the table used were the Annuity Table for 1949, the final payment would be at age 109, because this table shows none living after that age:

$100 $100 $100 $100 $100 $100 $100

*

Age 65········· 103 104 105 106 107 108 109

38 39 40 41 42 43 44years

The total amount to be paid out to the survivors at age 103 would be $100($l_{103}$)

The present value is being calculated as of a time 38

years prior to the date of this payment. Hence, the present value is this total multiplied by the factor for finding present value:

$100($l_{103}\upsilon^{38}$)

The present value of each of the payments is similarly calculated, and the total is divided by the number living at age 65 to derive the amount each must pay in. The expression for the present value of this deferred life annuity is

$$\$100\left(\frac{l_{103}\upsilon^{38}+l_{104}\upsilon^{39}+l_{105}\upsilon^{40}+l_{106}\upsilon^{41}+l_{107}\upsilon^{42}+l_{108}\upsilon^{43}+l_{109}\upsilon^{44}}{l_{106}}\right)$$

To Illustrate- **Using the Annuity Table for 1949 and $2\frac{1}{2}$% interest, calculate the present value at age 40 of a temporary life annuity of \$1,500 per year, first payment at age 50 and the last payment at age 53.**

Solution-

The line diagram for this life annuity appears as follows:

\$1,500 \$1,500 \$1,500 \$1,500

*****

age 40 ········50 51 52 53

········10 11 12 13 years

The amount payable at age 50 is due 10 years after the evaluation date; the amount payable at age 51 is due 11 years after the evaluation date; etc. Following the above procedure of expressing the present value as the total of the

present values of the individual payments:

Basic equation

$$\text{Present Value} = \$1{,}500\left(\frac{l_{50}\upsilon^{10} + l_{51}\upsilon^{11} + l_{52}\upsilon^{12} + l_{53}\upsilon^{13}}{l_{40}}\right)$$

Substituting the values for the *l*'s from Table II the υ's from Table I($2\frac{1}{2}$ %)

$$= \$1{,}500\left(\frac{\begin{matrix}(9{,}388{,}071)(.781198)\\ +(9{,}326{,}513)(.762145)\\ +(9{,}258{,}644)(.743556)\\ +(9{,}184{,}223)(.725420)\end{matrix}}{9{,}735{,}263}\right)$$

$$= \$1{,}500\left(\frac{7{,}333{,}942 + 7{,}108{,}155 + 6{,}884{,}320 + 6{,}662{,}419}{9{,}735{,}263}\right)$$

= **\$4,312.49**

RELATIONSHIPS AMONG LIFE ANNUITIES. **There are certain relationships among types of annuities which are important.**

The first of these is the relationship between a *whole life annuity* and a *whole life annuity due.* The following line diagrams show a whole life annuity and a whole life annuity due, both evaluated at age *x*:

\$1 \$1 \$1 ········ (for life)

whole Life annuity —*—|——|——|————

age x x+1 x+2 x+3 ···············

\$1 \$1 \$1 \$1 ······· (for life)

whole life annuity due *—|——|——|————

age x x+l x+2 x+3

The diagrams show that the only difference between

the two types is the one payment at age *x* in the second annuity. This illustrates that the *present valve of the whole life annuity due is equal to the present valve of the whole life annuity plus the amount of one payment.* In equation form, this is

$$\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity Due}\end{pmatrix}=\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity}\end{pmatrix}+1$$

For a payment of \$10 per year, the equation would be

$$\$10\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity Due}\end{pmatrix}=\$10\left[\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity}\end{pmatrix}+1\right]$$

$$=\$10\begin{pmatrix}\text{Present Value at Age } x\\ \text{of Whole Life Annuity}\end{pmatrix}+\$10$$

To Illustrate- Using the Annuity Table for 1949 and $2\frac{1}{2}$% interest, calculate the present value at age 106 of a whole life annuity due of \$50 per year.

Solution

The line diagram for this life annuity due appears as follows (with 109 being the highest age in this particular mortality table):

```
$50   $50   $50   $50
*------|------|------|
age 106   107   108 109
           1     2   3 years
```

In an earlier illustration, the present value at age 106 of a whole life annuity of \$50 per year (first payment one year following age 106) was calculated to be \$21.45. Hence, the above relationship can be used, with \$21.45 substituted in

the calculation.

Basic equation

$$\$50\begin{pmatrix}\text{Present Value at Age 106}\\ \text{of Whole Life Annuity Due}\end{pmatrix} = \$50\left[\begin{pmatrix}\text{Present Value at Age 106}\\ \text{of Whole Life Annuity}\end{pmatrix}+1\right]$$

$$= \$50\begin{pmatrix}\text{Present Value at Age 106}\\ \text{of Whole Life Annuity}\end{pmatrix} + \$50$$

= $2145 + $50

= **$71.45**

This desired present value could also have been calculated by using the other procedure, as follows (remembering that since the first payment is due upon the evaluation date, it is not multiplied by any present value factor):

Basic equation

$$\text{Present Value} = \$50\left(\frac{l_{106} + l_{107}\upsilon + l_{108}\upsilon^2 + l_{109}\upsilon^3}{l_{106}}\right)$$

Substituting the values for the *l*'s from Table II for the υ's from Table I($2\frac{1}{2}$%)

$$= \$50\left(\frac{\begin{matrix}(167)\\ +(\ \ 54)(.975610)\\ +(\ \ 16)(.951814)\\ +(\ \ \ 4)(.928599)\end{matrix}}{167}\right)$$

$$= \$50\left(\frac{167.0000 + 52.6829 + 15.2290 + 3.7144}{167}\right)$$

= **$71.45**

This answer agrees with that obtained by the use of the

relationship between a whole life annuity and a whole life annuity due.

The second important relationship is that between *temporary life* annuities and *temporary life annuities due.* The following line diagrams show, as a specific example, a $500 19-year life annuity and a $500 20-year life annuity due, both evaluated at age 30:

$500 $500 $500 $500 ··· $500

19-years life annuity *

age 30 31 32 33 34 ······ 49 50

20-

$500 $500 $500 $500 $500 ··· $500

*****

age 30 31 32 33 34 ······ 49 50

The only difference between them is the one payment of $500 on the evaluation date. This example illustrates that *the present value of a temporary life annuity due is equal to the present value of a temporary life annuity having one fewer total periods plus the amount of one payment.* In equation form, this is

$$\begin{pmatrix} \text{Present Value at Age } x \\ \text{of } n\text{-years Life Annuity Due} \end{pmatrix} = \begin{pmatrix} \text{Present Value at Age } x \\ \text{of } (n-1)\text{-years Life Annuity} \end{pmatrix} + 1$$

To Illustrate- Using the 1958 C.S.O. Table and 3% interest calculate the present value at age 25 of a 4-year life annuity due of $100 per year.

Solution

The line diagram for this temporary life annuity due

appears as follows:

\$100 \$100 \$100 \$100

*****

age 25 26 27 28 29

1 2 3 4 years

In an earlier illustration, the present value at age 25 of a 3-year life annuity of \$100 per year (first payment due 1 year following age 25) was calculated to be \$281.77. Using the above relationship and substituting \$281.77, gives Basic equation

$$\$100\begin{pmatrix}\text{Present Value at Age } x \\ \text{of } n\text{-years Life Annuity}\end{pmatrix} = \$100\left[\begin{pmatrix}\text{Present Value at Age } x \\ \text{of } (n-1)\text{-years Life Annuity}\end{pmatrix}+1\right]$$

Substituting 25 for *x*, 4 for *n*

$$\$100\begin{pmatrix}\text{Present Value at Age 25} \\ \text{of 4-years Life Annuity}\end{pmatrix} = \$100\left[\begin{pmatrix}\text{Present Value at Age 25} \\ \text{of 3-years Life Annuity}\end{pmatrix}+1\right]$$

$$= \$100\begin{pmatrix}\text{Present Value at Age 25} \\ \text{of 3-years Life Annuity}\end{pmatrix} + \$100$$

Substituting present value calculated above

= \$281.77 + \$100

= \$381.77

This desired present value could have been calculated by using the other procedure, as follows:

Basic equation

$$\text{Present Value} = \$100\left(\frac{l_{25} + l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3}{l_{25}}\right)$$

Substituting the values for the *l*'s from Table III for the υ's from Table I(3%)

$$= \$100 \left(\frac{\begin{array}{l} (9{,}575{,}636) \\ + (9{,}557{,}155)(.970574) \\ + (9{,}538{,}423)(.942596) \\ + (9{,}519{,}442)(.915142) \end{array}}{9{,}575{,}636} \right)$$

$$= \$100 \left(\frac{9{,}575{,}636 + 9{,}278{,}793 + 8{,}990{,}879 + 8{,}711{,}641}{9{,}575{,}636} \right)$$

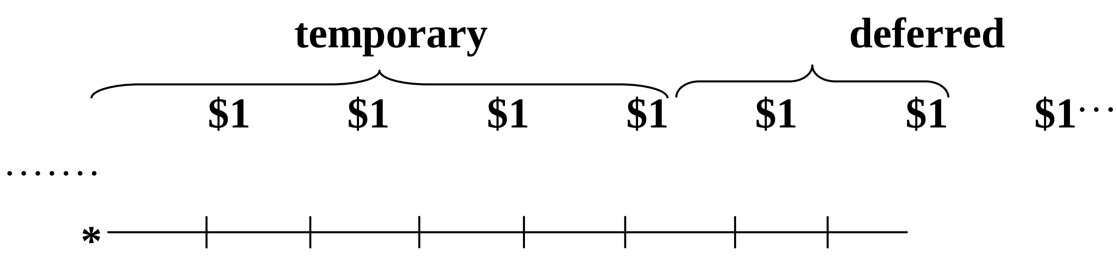
= \$381.77

This answer agrees with that obtained by using the relationship between an *n*-year life annuity due and an *(n*-l)-year life annuity.

The third important relationship involves *whole life annuities, temporary life annuities, and deferred life annuities,* This relationship is as follows: *A temporary life annuity plus a deferred life annuity (deferred the same number of years for which the temporary annuity runs) equals a whole life annuity.* The reasoning here is the same as that for annuities certain ("second method"). In equation form, this is

$$\left(\begin{array}{c} \text{Present Value} \\ \text{at Age } x \text{ of} \\ n\text{-years Temporary} \\ \text{Life Annuity} \end{array} \right) + \left(\begin{array}{c} \text{Present Value} \\ \text{at Age } x \\ \text{of Life Annuity} \\ \text{Deferred } n\text{-years} \end{array} \right) = \left(\begin{array}{c} \text{Present Value} \\ \text{at Age } x \\ \text{of Whole Life} \\ \text{Annuity} \end{array} \right)$$

For example, a 5-year temporary life annuity plus a life annuity deferred 5 years equals a whole life annuity. This is shown in the following line diagram:



Age *x* $\quad$ *x*+1 $\quad$ *x*+2 $\quad$ *x*+3 $\quad$ *x*+4 $\quad$ *x*+5 $\quad$ *x*+6 *x*+7·······

It is useful to know this relationship because published tables of life annuity values usually show only whole life annuities and temporary life annuities. The values of deferred life annuities must be obtained by some other means.

To Illustrate- **If a published table gives the present value of a whole life annuity of \$1 per year to a man age 40 as \$18.80, and the present value of a 15-year temporary life annuity of \$1 per year to a man age 40 as \$12.84, find the present value of a \$100 life annuity deferred for 15 years to a man age 40.**

Solution

Using the present value relationship above:

Basic equation

$$\$100\begin{pmatrix} n\text{ - years} \\ \text{Temporary} \end{pmatrix} + \$100\begin{pmatrix} \text{Deferred} \\ n\text{ - years} \end{pmatrix} = \$100(\text{Whole Life})$$

Substituting 15 for *n*

$$\$100\begin{pmatrix} 15\text{ - years} \\ \text{Temporary} \end{pmatrix} + \$100\begin{pmatrix} \text{Deferred} \\ 15\text{ - years} \end{pmatrix} = \$100(\text{Whole Life})$$

$$\$100(12.84) + \$100\begin{pmatrix} \text{Deferred} \\ 15\text{years} \end{pmatrix} = \$100(18.80)$$

Substituting the present values given

$$\$100(12.84) + \$100\begin{pmatrix} \text{Deferred} \\ 15\text{years} \end{pmatrix} = \$100(18.80)$$

\$100(12.84) from each side

$$\$100\begin{pmatrix}\text{Deferred}\\ \text{15years}\end{pmatrix} = \$100(18.80) - \$100(12.84)$$

= $100 (5.96)

= $596

Both life annuities (first payment at end of' one period) and life annuities due (first payment at beginning) have considerable practical use in life insurance company operations. For example, annuities are widely sold whereby the buyer pays a lump sum to the insurance company, and the company then pays back a periodic income as long as the buyer lives. Here, the first payment is usually made to the buyer at the end of the first period. Hence, this is an example of a life annuity. An example of a life annuity due would be the payment of premiums on a life insurance policy. They constitute a life annuity due because money changes hands only if a designated person is alive at the time each premium is payable, this premium being payable at the beginning of each period.

COMMUTATION FUNCTIONS

FOR EVALUATING A SINGLE PAYMENT. **When dealing with payments *certain*, tabulated values for the present value and accumulated value factors (such as Table I) aid in performing calculations. When benfit of survivorship is involved,, these factors are much more complicated. For example, the factor for the present value of a payment of 1 (with benefit of survivorship)**

$$\frac{l_{x+n}\upsilon^n}{l_x}$$

Is different for each *age* and *number of years.* Tabulated values of this factor, as well as factors for the present value of temporary and whole life annuities, are generally published for the common mortality tables and interest rates. Such tabulations are quite voluminous, however. In practice, another aid in performing calculations is widely used, namely, *commutation functions* (sometimes called *commutation symbols)*.

To see how commutation functions are used and how they simplify the work, consider again the expression for finding present values of a contingent payment due in *n* years to a life now age *x*:

$$\frac{l_{x+n}\upsilon^{n}}{l_{x}}$$

As will be seen later, it will be very useful to have the numerator and denominator look similar to each other. This is accomplished by multiplying numerator and denominator both by υ^{x}:

$$\frac{(l_{x+n}\,\upsilon^{n})(\upsilon^{n})}{(l_{x})(\upsilon^{x})}=\frac{l_{x+n}\upsilon^{n}}{l_{x}\upsilon^{x}}$$

The value of the fraction is unchanged by multiplying both the numerator and the denominator by the same amount. In the numerator, υ^{n} multiplied by υ^{x} equals υ^{x+n} (adding exponents when multiplying). The numerator and denominator above now look similar to each other, since in each case the subscript of the *l* is the same as the exponent of the υ.

Obviously, then, it would be extremely useful to have $l_x \upsilon^x$ already calculated for all values of x (based on a desired mortality table and interest rate). This value of l_x, multiplied by υ^x is represented by the commutation symbol D_x.

The following, then, is the definition of the D_x symbol:

$$D_x = l_x \upsilon^x$$

In Table II and Table IV, columns of D_x are shown for the Annuity Table for 1949 at $2\frac{1}{2}$% and for the 1958 C.S.O. Table at 3%, respectively.

As an example, using age 20 in Table IV, the value of D_{20} can be verified by multiplying l_{20} (from Table III) by υ^{20} at 3% (from Table I):

Basic equation

$$D_x = l_x \upsilon^x$$

Substituting 20 for x

$$D_{20} = l_{20} \upsilon^{20}$$

Substituting the values for l_{20} and υ^{20} from the tables

= (9,664,994)(.553676)

= 5,351,275

The values of D_x in Table IV were derived using more decimal places in υ^n. Nevertheless, the above answer is very close to that shown in the Table for D_{20}.

Above, the factor for finding present values of a contingent payment due in n years to a life now age x was finally expressed as

$$\frac{l_{x+n}\upsilon^{x+n}}{l_x\upsilon^x}$$

The numerator is equal to D_{x+n}, since the definition of a *"D"* is *l* multiplied by υ (the subscript of the *l* being the same as the exponent of the υ, and this then being the subscript of the *D)*. The denominator is equal to D_x. Hence the factor for finding present values may be expressed

$$\begin{pmatrix}\text{Present Value of \$1}\\ \text{Due in } n \text{ Years to a}\\ \text{Life Now Age } x\text{, with}\\ \text{Benefit of Survivorship}\end{pmatrix} = \$1\left(\frac{D_{x+n}}{D_x}\right)$$

In the above expression, the subscript of *D* in the *numerator* is the age when the contingent payment is to be made. The subscript of *D* in the *denominator* is the age at which the present value is being evaluated.

The value of this factor

$$\frac{D_{x+n}}{D_x}$$

Is the same as the value of the factor used previously

$$\frac{l_{x+n}\upsilon^{n}}{l_x}$$

but the *D's* are easier to use in making calculations.

In the example given in Section 8.1, the present value at age 35 of \$100 payable at age 60 (with benefit of survivorship) would be

$$\$100\left(\frac{D_{60}}{D_{35}}\right)$$

Using values of *D* from Table II, this becomes

$$\$100\left(\frac{1{,}923{,}965}{4{,}135{,}535}\right)=\$46.52$$

The answer is the same as before, but the calculation is simplified.

To Illustrate- **Using Table IV, calculate the present value at age 20 of $400 due in 15 years if the person is then still alive.**

Solution

This is the same problem as shown in the illustration in Section 8.1. The solution will now be given using commutation functions.

Basic equation

$$\text{Present Value} = \$400\left(\frac{D_{x+n}}{D_x}\right)$$

Substituting 20 for *x*, 15 for *n*

$$= \$400\left(\frac{D_{35}}{D_{20}}\right)$$

Substituting the values for D_{35} and D_{20} from Table IV

$$= \$400\left(\frac{3{,}331{,}295}{5{,}351{,}273}\right)$$

= $249.01

This answer agrees with that calculated in Section 8.1.

The factor for calculating the *accumulated* value of a single payment by using commutation functions is the

***inverse* of the *present value* factor (i.e., numerator and denominator are switched):**

$$\begin{pmatrix} \text{Accumulated Value of \$1} \\ \text{at End of } n \text{ Years to a} \\ \text{Life Age } x\text{, at the Beginning} \\ \text{with Benefit of Survivorship} \end{pmatrix} = \$1\left(\frac{D_x}{D_{x+n}}\right)$$

Note that in this expression, it is still true that the subscript of *D* in the *numerator* is the age when the contingent payment is to be made. The subscript of *D* in the *denominator* is the age at which the accumulated value is being evaluated.

FOR EVALUATING AN ANNUITY. **The present value at age *x* of a whole life annuity due of $1 per year may be expressed as the total of the present values of the individual payments:**

$$\text{Present Value} = \$1\left(\frac{D_x}{D_x}\right) + \$1\left(\frac{D_{x+1}}{D_x}\right) + \$1\left(\frac{D_{x+2}}{D_x}\right) + \ldots\begin{pmatrix} \text{to the end} \\ \text{of the} \\ \text{mortality table} \end{pmatrix}$$

The common multiplier ($1) can be factored out. The fractions to be added together alt have a common denominator (D_x). Hence, the present value of the annuity can be expressed as follows:

$$\text{Present Value} = \$1\left[\frac{D_x + D_{x+1} + D_{x+2} + \ldots \text{to the end of the mortality table}}{D_x}\right]$$

In order to avoid the necessity of adding together all the *D*'s to the end of the mortality table, this total is also tabulated. This total of the *D*'s to the end of the modality table is represented by the commutation symbol N_x
The subscript of the *N* being the same as that of the first Din

the series.

The following, then, is the definition of the N_x symbol:

$$N_x = (D_x + D_{x+1} + D_{x+2} + \ldots \text{to the end of the mortality table})$$

In Table II and Table IV, columns of N_x are shown for the Annuity Table for 1949 at $2\frac{1}{2}$% and for the 1958 C.S.O. Table at 3%, respectively.

As an example, using age 104 in Table II, the value of N_{104} can be verified by adding the *D*'s starting with D_{104} to the end of the mortality table:

Basic equation

$$N_x = (D_x + D_{x+1} + D_{x+2} + \ldots \text{to the end of the mortality table})$$

Substituting 104 for *x*

$$N_{104} = D_{104} + D_{105} + D_{106} + D_{107} + D_{108} + D_{109}$$

Substituting the values for the D's from Table II

$$= 89 + 35 + 12 + 4 + 1 + 0$$

$$= 141$$

This value agrees with that shown in the Table for N_{104}

The commutation function *Nx* can also be used to simplify the, calculation of *temporary* life annuities. In the example given in Section 8.2, the present value at age 25 of a life annuity of \$100 per year for three years, first payment due at age 26, would be

$$\$100\left(\frac{D_{26} + D_{27} + D_{28}}{D_{25}}\right)$$

Here the total of the D's to the end of the modality table is not needed, but only the total for three years. This can be found by taking N_{26} (the total of the D's from age 26 to the end of the table) and *subtracting* N_{29} (the total of the D's from age 29 to the end of the table). What remains after the subtraction is $D_{26} + D_{27} + D_{28}$ That is,

$$\$100\left(\frac{D_{26}+D_{27}+D_{28}}{D_{25}}\right)=\$100\left(\frac{N_{26}-N_{29}}{D_{25}}\right)$$

Using values of *N* and *D* from Table IV, this becomes

$$\$100\left(\frac{108{,}616{,}223-95{,}729{,}800}{4{,}573{,}377}\right)= \$281.77$$

The answer is the same as that calculated in Section 8.2.

A general statement may be made that the factor to use in evaluating a life annuity will be of the form $\frac{N-N}{D}$, where the subscript of the first *N* is the age when the first payment is due, the subscript of the second *N* is the first age when there are no more payments due (i.e., one greater than the age when the last payment is due), and the subscript of the *D* is the age at which the annuity is being evaluated (or paid for). Also, the difference between the subscripts of the two *N*'s equals the actual number of payments. If the payments are to be made for life, the second *N* does not appear.

To Illustrate- **Using Table II, calculate the present value at age 40 of a deferred temporary life annuity of $1,500 per year, first payment at age 50 and last payment at age 53.**

Solution

This is the same problem as shown in an illustration in Section 8.2. The solution will now be given by using commutation functions:

Basic equation; subscript of first *N* is. age at first payment; subscript of second *N* is one age greater than when last payment is due: subscript of *D* is evaluation age

$$\text{Present Value} = \$1{,}500\left(\frac{N_{50} - N_{54}}{D_{40}}\right)$$

Substituting the Values for N_{30}, N_{34} and D_{40} from Table II

$$= \$1{,}500\left(\frac{51{,}853{,}713 - 41{,}429{,}812}{3{,}625{,}710}\right)$$

= $4,312.49

This answer agrees with that calculated in Section 8.2.

One commutation symbol standing by itself has no usefulness. When commutation functions are used, they must be involved in a fraction; that is, one or more commutation functions must be divided by one or more other commutation functions. The reason underlying this is the fact that present values or accumulated values (with benefit of survivorship) always involve probabilities of living or dying. Such probabilities are calculated by dividing some number of persons living (or dying) by some number of persons living.

Questions For Review

(Use Table II, unless specified differently)

1- **Write an expression (using commutation functions) for the present value at age 24 of $1,000 payable at age 62 (if alive); calculate the value.**

2- **Write all expression (us n g commutation functions) for the accumulated value, with benefit of surv~vorship, at age 63 of $500 deposited with an insurance company when a man is age 21; calculate the value.**

3- **Calculate the values of $\frac{D_x}{D_{25}}$ for x = 35, 40, 45 and 50. Compare the results with the corresponding values of the present value factor, υ^n for n = 10, 15, 20 and 25 at $2\frac{1}{2}$% using Table I.**

4- **If females will exhibit the same mortality as males who are 5 years younger, calculate the amount which a woman now age 30 would receive back 20 years later if she deposits $100 now, to accumulate with benefit of survivorship. (Hint: The .5-year "setback" means that instead of using $\frac{D_{30}}{D_{50}}$, use $\frac{D_{25}}{D_{45}}$**

5- **Write expressions (us big commutation functions) for the present value at age 23 of a whole life annuity of $1 per year, with the first payment at age 23; age 24; age 55. Calculate the values.**

6- **Write an expression (using commutation functions) for the present value at age 36 of a deferred 10-year life annuity of 8100 per year, first payment due at age 46; calculate the value.**

7- Calculate the value at age :35 of a whole life annuity due of $1,500 per year.

8- Calculate the amount a man age 45 should pay for a life annuity due of $1,000 per year, which has payments for 10 years only.

9- What is the present value to a man age 64 of a temporary life annuity of $100 per year for 3 years?

10- Construct a schedule showing that the value found in Exercise will provide the benefits specified.

11- An insurance company has sold a life insurance policy to a man age 25 with premiums of $18.09 payable every year for life. What is the present value at the time the policy is sold of all the future premiums the company will receive?

12- State in words what each of the following expressions represents: $\$100\left(\frac{N_{69}}{D_{20}}\right)$ $\qquad$ $\$100\left(\frac{N_{69}-N_{79}}{D_{20}}\right)$

CHART-3

Present Value at Age *x* of Life Annuity of 1 per Year

Type of Life Annuity	*Symbol for Present Value*	*Present Value Using Commutation Functions*
Whole Life Annuity	a_x	$\frac{N_{x+1}}{D_x}$
Whole Life Annuity Due	$\ddot{a}_x$	$\frac{N_x}{D_x}$
Temporary Life Annuity for *n* Years	$a_{x:\overline{n}\rceil}$	$\frac{N_{x+1}-N_{x+n+1}}{D_x}$
Temporary Life Annuity Due for *n* Years	$\ddot{a}_{x:\overline{n}\rceil}$	$\frac{N_x-N_{x+n}}{D_x}$

Whole Life Annuity, Deferred for *m* Years	${}_{m\|}a_x$	$\frac{N_{x+m+1}}{D_x}$
Whole Life Annuity Due, Deferred for *m* Years	${}_{m\|}\ddot{a}_x$	$\frac{N_{x+m}}{D_x}$
Temporary Life Annuity for *n* years, Deferred for *m* Years	${}_{m\|}a_{x:\overline{n}\|}$	$\frac{N_{x+m+1} - N_{x+m+n+1}}{D_x}$
Temporary Life Annuity Due for *n* Years, Deferred for *m* Years	${}_{m\|}\ddot{a}_{x:\overline{n}\|}$	$\frac{N_{x+m+1} - N_{x+m+n}}{D_x}$

Chart-3 displays certain internationally used symbols, each of which represents the present value at age x of a life annuity of 1 per year. (These present values are also shown as they would be calculated using commutation functions.)

The use of "a" with the number of years under an "angle" is analogous with the symbol $a_{\overline{n}|i}$ given in Chapter 6 for the present value of an annuity certain. For life annuities, however, the age at which the present value is calculated also becomes a part of the symbol. For example, $a_{\overline{n}|i}$ (read "*a* sub *x* angle *n*") represents the present value at age *x* of an *n*-year life annuity.

The use of two dots over the "*a*" is also analogous to the usage in annuities certain, indicating an annuity due.

Life annuity symbols wherein a *bar* is placed over the "*a*" represent annuities payable continuously. For example: $\bar{a}_x$

is the internationally used symbol for the present value at age *x* of a whole life annuity of 1 per year payable continuously.

CHAPTER 7

NET SINGLE PREMIUMS

payments were discussed which are made only if a designated person is alive. Life insurance involves a payment to be made only when a designated person dies. The amount of such a payment is called the amount of insurance or the death benefit.

It is common in life insurance for alt of the figures quoted in connection with such insurance to be based upon a death benefit of $1,000. Accordingly, for a policy with a $25,000 death benefit, all such figures would be multiplied by 25. This was seen in connection with Tables of Settlement Options in Section 8.5. In discussing premiums in this book, it will also be assumed that the unit policy is $1,000.

NET SINGLE PREMIUM FOR ONE YEAR OF LIFE INSURANCE – THE NATURAL PREMIUM

The present value of the benefits offered by a particular insurance policy is equal to the *net single premium.* This amount is calculated using a designated mortality table and a specified interest rate. The net single premium does not include any amount for expenses or profits.

It may be desired, for example, to calculate the net single premium that a man age 25 should pay for $1,000 of life insurance covering a one-year period. Under such insurance, if he dies before reaching age 26, $1,000 will be paid to his beneficiary. To begin solving the problem, it is necessary to consult a mortality table, If Table LII is used,

the numbers shown living and dying at age 25 are

l_{25} **= 9,575,636**

d_{25} **= 18,481**

This means that if there is a group of 9,575,636 men alive at age 25, then 18,481 men of this group may be expected to die during the year (before reaching age 26). In order to find the net single premium, it is assumed that all of these men are individually involved, that is, that each one who dies that year will receive $1,000 (to be paid to the recipient designated). For purposes of simplifying some of the calculations, it is customary to assume that all such payments are made at the end of the year in which death occurs, In actual practice, however, such payments are made very soon after death occurs. The methods of calculation used when payments are assumed to be made "at the moment of death" will be in Section 9.7.

Since a $1,000 benefit is to be paid for each of the d_{25} **men, the total amount that will he paid out as benefits is**

$$\$1{,}000 d_{25} = (\$1{,}000)(18{,}481)$$

$$= \$18{,}481{,}000$$

One year earlier, l_{25} **men will pay the money in. The original problem may now be stated: "How much will each pay?" The total amount paid in is**

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(l_{25}) = \begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}(9{,}575{,}636)$$

$$\begin{pmatrix}\text{Amount each}\\ \text{pays in}\end{pmatrix}(9{,}575{,}636) \qquad\qquad (\$1{,}000)(18{,}481) = \$18{,}481{,}000$$

*————————————————|

age 25 **age 26**

The money paid in will earn interest over the one-year period. For this example, the rate will be assumed to be 3%. The basic equation for finding present value can In, used to show that *all the money paid in equal the present value of all the money to be paid out one year later.* The "amount each pays in' can then be solved for:

$\boldsymbol{A} = \boldsymbol{S\upsilon^n}$

Substituting $\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}$ **(9,575,636) for *A*, $18,481,000 for *S* and the value of υ^1 at 3% from the table**

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}\mathbf{(9{,}575{,}636) = (\$18{,}481{,}000)(.970874)}$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}\mathbf{(9{,}575{,}636) = \$17{,}942{,}722}$$

$$\begin{pmatrix}\text{Amount Each}\\ \text{Pays In}\end{pmatrix}\mathbf{=\$1.87}$$

It can be demonstrated that $1.87 is the desired net single premium at age 25 to provide $1,000 of insurance for a one-year period, as follows:

Total amount paid in = $1.87($l_{25}$)

= $1.87(9,575,636)

= $17,906,439.32

Total amount accumulated at 3% to the end of the year

= $17,906,439.32(1 + *i*)

= \$17,906,439.32(1.03)

= \$18,443,632.50

Amount payable for each who dies during the year (the accumulated fund divided by the number who die)

= \$18,443,632.50 ÷ d_{25}

= \$18,443,632.50 ÷ 18,481

= \$998 approximately

(The missing \$2 results from rounding off \$1.87 to the nearest cent, instead of using more decimal places.)

This net single premium for one year of life insurance at age 25 is also called the "natural premium" at age 25. It could be written as

$$\$1{,}000\left(\frac{d_{25}}{l_{25}}\right)\upsilon \qquad \textbf{Or as} \qquad \frac{\$1{,}000\, d_{25}\, \upsilon}{l_{25}}$$

Both these expressions for the natural premium permit interesting verbal interpretations. In the first, $\left(\frac{d_{25}}{l_{25}}\right)$ equals the probability that a person age 25 will die before reaching age 26 (q_{25}). The first expression says that the \$1,000 is multiplied by the probability of dying and also by the regular factor for finding present values at interest. The second expression says that the \$1,000 payable to each of d_{25} persons is discounted at interest for one year, and this amount is divided among the l_{25} persons to find out how much each must pay in.

Using more general terms, the equation for the natural premium is

$$\begin{pmatrix}\text{Net Single Premium for} \\ \text{\$1,000 Death Benefit} \\ \text{to a Life Age } x\text{, If} \\ \text{Death Occurs in 1 Year}\end{pmatrix} = \$1{,}000\left(\frac{d_x \upsilon}{l_x}\right)$$

To Illustrate- **Using the 1958 C.S.O. Table (Table III) and 3% interest, calculate the net single premium for 1 year of life insurance of $1,000 at age 40; also at age 60; also at age 80.**

At *Age* 40

Basic equation

$$\begin{pmatrix}\text{Net Single} \\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_x \upsilon}{l_x}\right)$$

Substituting 40 for *x*

$$= \$1{,}000\left(\frac{d_{40} \upsilon}{l_{40}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{(32{,}622)(.970874}{9{,}241{,}359}\right)$$

= $3.43

At *Age* 60

$$\begin{pmatrix}\text{Net Single} \\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_x \upsilon}{l_x}\right)$$

Substituting 60 for *x*

$$= \$1{,}000\left(\frac{d_{60} \upsilon}{l_{60}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{(156{,}592)(.970874}{7{,}698{,}698}\right)$$

= $19.75

At *Age* 80

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_x \upsilon}{l_x}\right)$$

Substituting 80 for *x*

$$= \$1{,}000\left(\frac{d_{80} \upsilon}{l_{80}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{(288{,}848)(.970874}{2{,}626{,}372}\right)$$

= $10678

It can be seen that the net single premium for one year of insurance, the natural premium, increases sharply at the older ages. This is similar to the age-by-age increase in the values of q~, (the probability of dying within one year) shown in Chapter 7.

NET SINGLE PREMIUMS FOR TERM INSURANCE

Life insurance which provides a benefit if death occurs during a specified period of years is known as *term insurance.* The one-year insurance considered in the above

section is one-year term insurance.

To determine the net single premiums for life insurance for longer periods, the procedure is basically the same. For example, it may be desired to find the net single premium at age 25 for $1,000 of insurance during a period of three years (between ages 25 and 28), i.e., for three-year term insurance. The total amount to be paid out for those who die during the first year is $1,000 d_{25}

Assuming such payments are made at the *end of the year,* the present value at age 25 of those payments is $1,000 $d_{25}\upsilon$

The total amount to be paid out for those who die during the second year is $1,000 d_{26}

Assuming such payments are made at the end of the year, the present value at age 25 of those payments is

$1,000 $d_{25}\upsilon^2$

The total amount to be paid out for those who die during the third year is $1,000 d_{27}

Assuming such payments are made at the end of the year, the present value at age 25 of those payments is

$1,000 $d_{27}\upsilon^3$

(In each case the exponent of the υ is the number of years between age 25 and the date when the death benefit is paid.)

The present value at age 25 of all the death benefits paid during the three-year period is the total of the three individual present values. The common multiplier ($1,000) can be factored out:

Present Value = \$1,000 ($d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3$)

This amount is paid in at age 25 by the 125 persons. Hence, the above expression should be divided by l_{25} to find out how much each must pay in (the net single premium):

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3}{l_{25}}\right)$$

The *numerator* of this expression represents the total to be paid out for those who die in each of the three years, with each such amount being discounted at interest from the end of the year of death to the evaluation date. The *denominator* represents the number of persons alive on the evaluation date, among whom this total present value to be paid in must be allocated.

If, for example, the 1958 C.S.O. Table and 3% interest are used, the value of this net single premium can be calculated as follows:

From above

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3}{l_{25}}\right)$$

Substituting values from the tables

$$= \$1{,}000\left(\frac{\begin{array}{c}(18{,}481)(.970874)\\ +(18{,}732)(.942596)\\ +(18{,}981)(.915142)\end{array}}{9{,}575{,}636}\right)$$

$$= \$1{,}000\left(\frac{17{,}943 + 17{,}657 + 17{,}370}{9{,}575{,}636}\right)$$

= **\$5.53**

It can be demonstrated that if each of the *I,~* persons

pays $5.53,

The resulting fund will provide $1000 (at the end of the year of death) for all who die before age 28. At the beginning of the first year, the amount paid in is

$5.53 l_{25} = ($5.53)(9,575,636)

= $52,953,267.08

At the end of one year, interest earned on the fund is equal to

($52,953,267.08)(.03) = $1,588,598.01

Hence, the total fund at that time is

$52,953,267.08 + $1,588,598.01 = $54,541,865.09

From this total fund, $1,000 is deducted for each person who has died during the first year:

$1,000 d_{25} = ($1,000)(18,481)

= $18,481,000

This leaves a balance in the fund of

$54,541,865.09 - $18,48l,000 = $36,060,865.09

The continued operation of the fund for succeeding years may be traced in the accompanying schedule (Chart 9-1). The final shortage in the fund represents less than is for each person dying the final year, and results from rounding off the'net single premium to two decimal places.

CHART-1

(1) Year	(2) *Fund at Beginning of year (Col. 6 of Previous Yeas)*	(3) *Interest for One Year (Col. 2 × .03)*	(4) *Total Fund at End of Year before Payment of Death Claims (Col.2+Col.3)*	(5) *Claims Paid at End of Year (Number of Deaths × $1000)*	(6) *Balance of Fund at End of Year After Payment of Claims (Col.4-Col.5)*
1	**$52,953,267.08**	**$1,588,598.01**	**$54,541,865.09**	**$18,481,000**	**$36,060,865.09**
2	**36,060,865.09**	**1,081,825.95**	**37,142,691.04**	**18,732,000**	**18,410,691.04**
3	**18,410,691.04**	**552,320.73**	**18,963,011.77**	**18,981,000**	**-17,988.23**

To Illustrate- **Using the 1958 C.S.O. Table (Table III) and 3% interest, calculate the net single premium at age 50 for $5,000 of 2-year term insurance.**

Solution

The line diagram for this life insurance appears as follows:

$\$5{,}000d_{50}$ $\$5{,}000d_{50}$

*

age 50 51 52

1 2 years

The expression for the net single premium is a fraction with a *numerator* equal to the total of the amounts to be paid out for those who die in each of the 2 years, discounted at

interest from the end of each year of death to the evaluation date:

$\$5000 d_{50} \upsilon + \$5000 d_{51} \upsilon^2$

or

$\$5000\ (d_{50} \upsilon + d_{51} \upsilon^2$

The *denominator* of the fraction is the number living on the evaluation date at age 50:

Basic equation

$$\begin{pmatrix}\text{Net Single} \\ \text{Premium}\end{pmatrix} = \$5{,}000\left(\frac{d_{50}\upsilon + d_{51}\upsilon^2}{l_{50}}\right)$$

Substituting values from the tables

$$= \$5{,}000\left(\frac{\begin{array}{c}(72{,}902)(.970874) \\ +(79{,}160)(.942596)\end{array}}{8{,}762{,}306}\right)$$

$$= \$5{,}000\left(\frac{70{,}779 + 74{,}616}{8{,}762{,}306}\right)$$

= $82.97

NET SINGLE PREMIUMS FOR WHOLE LIFE INSURANCE

Under whole life insurance, the death benefit will be paid whenever death occurs; that is, the period of years covered by the insurance extends to the end of the mortality table. Thus, whole life insurance may be looked upon as term insurance covering a period of years equal to those remaining in the mortality table.

The calculation of the net single premium for whole life insurance follows exactly the same procedure as that shown above for term insurance. In the case of whole life insurance, the years included extend to the end of the mortality table.

To Illustrate- **Using the 1958 C.S.O Table and 3% interest, calculate the net single premium at age 96 for $1,000 of whole life insurance.**

Solution

The period of years covered by this insurance extends for the person s entire lifetime. However, since the 1958 C.S.O. Table assumes that no persons live beyond the age of 100, it is assumed that the insurance ends at age 100. The line diagram for this whole life insurance appears as follows:

$\$1,000d_{96}$ $\$1,000d_{97}$ $\$1,000d_{98}$ $\$1,000d_{99}$

*****

age96 97 98 99

100

1 2 3 4years

The expression for the net single premium is a fraction with a *numerator* representing the total to be paid out for those who die in each of the years, with each such amount being discounted at interest from the end of the year of death to the evaluation date. The *denominator* of the fraction is the number living on the evaluation date:

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1,000\left(\frac{d_{96}\upsilon + d_{97}\upsilon^2 + d_{98}\upsilon^3 + d_{99}\upsilon^4}{l_{96}}\right)$$

Substituting values from the tables

$$= \$1,000 \left(\frac{\begin{array}{l} (25,250)(.970874) \\ + (18,456)(.942596) \\ + (12,916)(.915142) \\ + (\ 6,415)(.888487) \end{array}}{63,037} \right)$$

$$= \$1,000 \left(\frac{24,515 + 17,397 + 11,820 + 5,700}{63,037} \right)$$

= $942.81

The calculation of net single premiums for whole life insurance at the younger ages would become very laborious if the above procedure were used. Therefore, in actual practice this calculation is usually done by using commutation functions. The commutation functions which apply in net single premium calculations will be explained in Section 9.8.

NET SINGLE PREMIUMS FOR A PURE ENDOWMENT

A *pure endowment* is an amount which is paid on a certain date only if a designated person is then alive to receive it. It is, therefore, the opposite of life insurance. It is, in fact, the same as a payment which is made with benefit of survivorship, as described in Chapters. Pure endowments are often combined with life insurance, as will be shown in Section 9.5. In that context, the term "net single premium for a pure endowment" is used instead of "present value of a single payment with benefit of survivorship".

Since the principles and equations for such a payment were presented in Section 8.1, the following equation will be

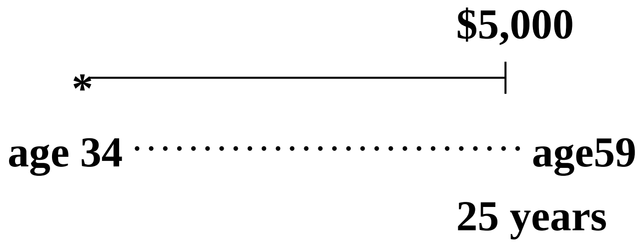
given here without further explanation:

$$\begin{pmatrix}\text{Net Single Premium}\\ \text{\$1,000 Pure Endowment}\\ \text{to a Life Age } x\text{, Due}\\ \text{at the End of } n \text{ Years}\end{pmatrix} = \$1{,}000\left(\frac{l_{x+n}\,\upsilon^{n}}{l_x}\right)$$

To Illustrate- Using the 1958 C.S.O. Table and 3% interest, calculate the net single premium for a female age 34 for a $5,000 pure endowment due in 25 years, using a ‘‘3-year setback’’ for females.

Solution

The line diagram for this pure endowment appears as follows:



The female’s age at the date the pure endowment is due is 34+25=59. The use of a "3-year setback" means that 3 years must be subtracted from the age before using the Table. The problem must be treated as if the age were 34-3=31, and the pure endowment were payable at age 59-3 = 56:

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$5{,}000\left(\frac{l_{x+n}\upsilon^{n}}{l_x}\right)$$

Substituting 31 for *x*, 25 for *n*

$$= \$5{,}000\left(\frac{l_{56}\upsilon^{25}}{l_{31}}\right)$$

Substituting values from the tables

$$= \$5{,}000\left(\frac{(8{,}223{,}010)(.477606}{9{,}460{,}165}\right)$$

= **$2,075.73**

NET SINGLE PREMIUMS FOR ENDOWMENT INSURANCE

Endowment insurance means that the benefit will be paid if death occurs during a specified number of years, or the benefit will be paid at the end of that period if the person is then alive. Therefore, endowment insurance is a combination of two benefits already presented: *term insurance* and *pure endowment.* The payment on death constitutes term insurance, while the payment on survival constitutes a pure endowment.

To Illustrate- **Using the 1958 C.S.O. Table and 3% interest, calculate the net single premium at age 62 for a $7,500 endowment-at-age-6^{5} insurance policy.**

Solution

The policy provides that $7,500 will be paid if death occurs during the period between ages 62 and 65. It also provides that $7,500 will be paid at age 65 if the person is then alive. The line diagram for this endowment insurance policy appears as follows:

$\$7{,}500\ l_{65}$

$\$7{,}500d_{62}$ $\$7{,}500d_{63}$ $\$7{,}500d_{64}$

*

age.62 63 64 65

1 2 3years

The expression for the net single premium for the *term insurance part* has a *numerator* representing the total to be paid out for those who die each year, with each such amount being discounted at interest from the end of the year of death to the evaluation date. The *denominator* is the number living on the evaluation date. The common multiplier ($7,500) can be factored out

$$\begin{pmatrix}\text{Net Single Premium} \\ \text{for Term} \\ \text{Insurance Part}\end{pmatrix} = \$7{,}500\left(\frac{d_{62}\upsilon + d_{63}\upsilon^2 + d_{64}\upsilon^3}{l_{62}}\right)$$

The expression for the net single premium for the *pure endowment part* follows from the equation given in Section 9.4:

$$\begin{pmatrix}\text{Net Single Premium for} \\ \text{Pure Endowment Part}\end{pmatrix} = \$7{,}500\left(\frac{l_{x+n}\upsilon^n}{l_x}\right)$$

The expression for the net single premium for the entire *endowment insurance policy* is the total of the above two expressions. The two expressions can be readily added, since they already have a common denominator (l_{62}). The common multiplier ($7,500) can be factored out:

Adding the above expressions

$$\begin{pmatrix}\text{Net Single} \\ \text{Premium}\end{pmatrix} = \$7{,}500\left(\frac{d_{62}\upsilon + d_{63}\upsilon^2 + d_{64}\upsilon^3 + d_{65}\upsilon^3}{l_{62}}\right)$$

Substituting values from the tables

$$= \$7{,}500\left(\frac{\begin{array}{l}(179{,}271)(.970874)\\+(191{,}174)(.942596)\\+(203{,}394)(.915142)\\+(\ 6{,}800{,}531)(.915142)\end{array}}{7{,}374{,}370}\right)$$

$$= \$7{,}500\left(\frac{174{,}050+180{,}200+186{,}134+6{,}223{,}452}{63{,}037}\right)$$

= $6,879.06

It should be noted that the exponents on the last two υ's are the same in the expression above. *(d*$_{64}$ and *l*$_{65}$ are both multiplied by υ^3.) This is done because the two benefits are payable on the same date: the death benefit for those who die during the final year, and the pure endowment benefit for those still alive at the end of the final year.

The two separate parts of the net single premium for endowment insurance (term insurance and pure endowment) can be calculated separately if it is desired to know the relative contribution of each to the total net premium. For example, Chart 9-2 shows such figures (according to the 1958 C,S,O. Table at 3%) for 20-year endowment insurance issued at ages 20, 40, and 60.

CHART-2

Net Single Premiums per $1,000

	Age 20	*Age* 40	*Age* 60
20-Year Tern ………………………………..	**$ 3177**	**$115.08**	**$474.22**

20-Year Pure Endowment	**529.41**	**461.25**	**188.88**
TOTAL = 20-Year Endowment Pnhcy..........	**$561.18**	**$576.33**	**$663.10**

COMMUTATION FUNCTIONS

FOR ONE YEAR TERM INSURANCE. **To see how commutation functions are used in calculating net single premiums, consider again the expression for the net single premium for a death benefit of 1 to a life age *x* if death occurs in one year (the natural premium):**

$$\frac{d_x \upsilon^1}{l_x}$$

The same procedure is followed here as was shown in Chapter 8, namely, both the numerator and the denominator are multiplied by υ^x:

$$\frac{(d_x \upsilon^1)(\upsilon^x)}{(l_x)(\upsilon^x)} = \frac{d_x \upsilon^{x+1}}{l_x \upsilon^x}$$

The value of the fraction is unchanged by multiplying both the numerator and the denominator by the same amount. In the numerator, & multiplied by υ^x equals υ^{x+l} (adding exponents when multiplying).

Looking at the numerator, it is seen that it would be useful to have $d_x\upsilon^{x+1}$ already calculated for all values of *x* (based on a desired mortality table and interest rate). This value of d_x multiplied by υ^{x+1} is represented by the

commutation symbol: C_x

The following, then, is the definition of the C_x symbol:

$C_x = d_x \upsilon^{x+1}$

In Table IV, columns of C_x are shown for the 1958 C.S.O. Table at 3%. As an example, using age 20 in Table IV, the value of C_{20} can be verified by multiplying d_{20} (from Table III) by υ^{21} at 3% (from Table I):

Basic equation

$$C_x = d_x \upsilon^{x+1}$$

Substituting 20 for *x*

$C_{20} = d_{20} \upsilon^{21}$

Substituting values from the tables

= (17,300)(.537549)

= 9,300

This agrees with the value given in Table IV

Above, the factor for finding the net single premium for one year of insurance to a life age *x* was finally expressed as

$$\frac{d_x \upsilon^{x+1}}{l_x \upsilon^x}$$

The numerator is equal to C_x (because C_x is defined as d_x multiplied by υ^{x+1}). The denominator is equal to D_x (because D_x is defined as l_x multiplied by υ^x) Hence, the factor for finding the net single premium for one-year term insurance may be expressed

$$\begin{pmatrix}\text{Net Single Premium for}\\ \text{\$1,000 Death Benefit to}\\ \text{a Life Age } x\text{, If Death}\\ \text{Occurs in 1 Year}\end{pmatrix} = \$1{,}000\left(\frac{C_x}{D_x}\right)$$

The value of this factor $\frac{C_x}{D_x}$

is the same as the value of the factor for the natural premium used previously:

$$\frac{d_x \upsilon^1}{l_x}$$

but the commutation functions are easier to use in making calculations.

In the example given in Section 9.1, the net single premium at age 25 for $1,000 of one-year term insurance would be

$$\$1{,}000\left(\frac{C_{25}}{D_{25}}\right)$$

Using values for the commutation functions from Table IV, this becomes

$$\$1{,}000\left(\frac{8{,}570}{4{,}573{,}377}\right) = \$1.87$$

The answer is the same as before, but the calculation is simplified.

To Illustrate- **Using Table IV, calculate the net single premium at age 40 for one year of life insurance of $1,000.**

Solution

This is the same problem as shown in the illustration in Section 9.1. The solution will now be given by using commutation functions:

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{C_x}{D_x}\right)$$

$$= \$1{,}000\left(\frac{C_{40}}{D_{40}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{9{,}709}{2{,}833{,}002}\right) = \$3.43$$

This answer agrees with that calculated in Section 9.1

FOR OTHER BENEFITS **The net single premium at age** ***x*** **for $1,000 of** ***whole life insurance*** **may be expressed as the following total of the net single premiums for the individual years insurance (with the subscripts of the** ***D's*** **all being the age at the evaluation date):**

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{C_x}{D_x}\right) + \$1{,}000\left(\frac{C_{x+1}}{D_x}\right) + \$1{,}000\left(\frac{C_{x+2}}{D_x}\right) + \ldots..\begin{pmatrix}\text{to the end of the}\\ \text{mortality table}\end{pmatrix}$$

The common multiplier ($1,000) can be factored out. The fractions to be added together all have a common denominator (D_x). Accordingly, the net single premium for the whole life insurance can be expressed as

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left[\frac{C_x + C_{x+1} + C_{x+2} + \text{...to the end of the mortality table}}{D_x}\right]$$

In order to avoid the necessity of adding together all the *C*'s to the end of the mortality table, this total is also tabulated. The total of the C's to the end of the mortality table is represented by the commutation symbol M_x

the subscript of the *M* being the same as that of the first *C* in the series. The following, then, is the definition of the M_x **symbol:**

$M_x = (C_x + C_{x+1} + C_{x+2} + \ldots$ **to the end of the mortality table)**

In Table IV, columns of M_x **are shown Dir the 1958 C.S.O. Table at 3%. The commutation function** M_x **can also be used to simplify the calculation of *term insurance* and *endowment insurance* net single premiums. In the example given in Section 9.2, the net single premium at age 25 for $1,000 of three-year term insurance is**

$$\$1{,}000\left(\frac{C_{25} + C_{26} + C_{27}}{D_{25}}\right)$$

Here the total of the C's to the end of the mortality table is not needed, but only the total for three years. This total can be found by taking M_{25} **(the total of the C's from age 25 to the end of the table) and subtracting** M_{28} **(the total of the C's from age 28 to the end of the table). What remains after the subtraction is** $C_{25}+C_{26}+C_{27}$**. That is,**

$$\$1{,}000\left(\frac{C_{25} + C_{26} + C_{27}}{D_{25}}\right) = \$1{,}000\left(\frac{M_{25} - M_{28}}{D_{25}}\right)$$

Using values of *M* and ID from Table IV, this becomes

$$\$1,000\left(\frac{1,276,590-1,251,291}{4,573,377}\right)=\$5.53$$

The answer is the same as that calculated in Section 9.2.

A general statement may be made that the factor to use in calculating a net single premium or an accumulated cost of insurance will be of the form $\frac{M-M+D}{D}$**, where the subscript of the first *M* is the age when the insurance coverage begins; the subscript of the second *M* is the age at which the insurance coverage stops (i.e., one greater than the last age covered); the subscript of the *D* in the numerator is the age at which a pure endowment would be paid; and the subscript of the *D* in the denominator is the age at which this net single premium or accumulated cost of insurance is paid. The difference between the subscripts of the Ms equals the actual number of years of insurance coverage. If there is no pure endowment involved, the *D* in the numerator does not appear. If the insurance is for the whole of life, the second *M* does not appear.**

To Illustrate- **Using Table IV, calculate the net single premium at age 50 for $5,000 of** ^{2}**-year term insurance.**

Solution

This is the same problem as shown in the illustration in Section 9.2. The solution will now be given by using commutation functions. In the general expression given above, the *D* in the numerator will not appear, because there is no pure endowment involved:

Basic equation (subscripts of the *M*'s define the period of coverage; subscript of *D* is the evaluation age)

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$5{,}000\left(\frac{M_{50} - M_{52}}{D_{50}}\right)$$

Substituting values from Table IV

$$= \$5{,}000\left(\frac{1{,}028{,}986 - 995{,}821}{1{,}998{,}744}\right) = \$82.96$$

This answer is only 1 cent different from that calculated in Section 9.2.

To Illustrate Again- **Using Table IV, calculate the net single premium at age 96 for $1,000 of whole life insurance.**

Solution

This is the same problem as shown in the illustration in Section 9.3. The solution will now be given by using commutation functions. In the general expression for net single premiums, the second *M* will not appear, because the insurance is for the whole of life. Also, the *D* in the numerator will not appear, because there is no pure endowment involved:

Basic equation

$$\begin{pmatrix}\text{Net Single}\\ \text{Premium}\end{pmatrix} = \$1{,}000\left(\frac{M_{96}}{D_{96}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{3{,}481}{3{,}692}\right)$$

= $942.85

This answer is only 4 cents different from that calculated in Section 9.3.

To Illustrate Again- **Using Table IV, calculate the net single premium at age 62 for a $7,500 endowment-at-age-65 insurance policy.**

Solution

This is the same problem as shown in the illustration in Section 9.5. The solution will now be given by using commutation functions:

Basic equation (subscripts of the *M*'s define the period of coverage; subscript of *D* in numerator is age of pure endowment; subscript of *D* in denominator is evaluation age)

$$\begin{pmatrix}\text{Net Single}\\\text{Premium}\end{pmatrix} = \$7{,}500\left(\frac{M_{62} - M_{65} + D_{65}}{D_{62}}\right)$$

Substituting values from Table IV

$$= \$7{,}500\left(\frac{773{,}206 - 686{,}750 + 995{,}688}{1{,}179{,}823}\right)$$

= $6,879.07

This answer is only one cent different from that calculated in Section 9.5.

The form $\frac{M - M + D}{D}$ **is also used to compute the**

accumulated cost of insurance. However, the evaluation date for the accumulated cost of insurance is at the *end* of the term of coverage, whereas the net single premium is evaluated at the *beginning.* This is because the accumulated cost of insurance represents the amount that would have to be paid by the survivors at the end of the term of coverage, while the net single premium is the amount to be paid by those living at the beginning of the term of coverage. Therefore, when the accumulated cost is computed, the subscript of the ID in the denominator is the highest age.

To Illustrate- **Using Table IV, calculate the accumulated cost of insurance at age 28 for $1,000 of 3-year term insurance.**

Solution

This is the same S-year term policy for which the net single premium was calculated on page 208 to be $5.53. There the denominator used in the calculation was D_{25} because the net single premium is evaluated at the beginning of the insurance. Now to calculate the accumulated cost of insurance, the denominator is D_{28} because the evaluation date is at the end. (The *D* in the numerator will not appear, because there is no pure endowment involved.)

Basic equation (subscripts of the *M*'s define the period of coverage; subscript of *D* is the evaluation age)

$$\begin{pmatrix}\text{Accumulated Cost}\\ \text{of Insurance}\end{pmatrix} = \$1{,}000\left(\frac{M_{25} - M_{28}}{D_{28}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{1{,}276{,}590 - 1{,}251{,}291}{4{,}160{,}727}\right)$$

= $6.08

This answer agrees with that calculated in Section 9.6 for the same policy without the use of commutation functions.

It should be noted that the accumulated cost of insurance ($6.08) is higher than the net single premium ($5.53). This is as expected since the number of persons paying in at the beginning of the term of coverage is greater than the number of survivors who would pay at the end.

Questions For Review

(Use Table IV for all of the following. Assume insurance is payable at the end of the year unless otherwise indicated.)

1- Write an expression (using commutation functions) for the net single premium at age 10 for $10,000 of 1-year term insurance. Calculate the value.

2- Write an expression (using commutation functions) for the net single premium at age 10 for $10,000 of term-to-age-$4^0$ insurance. Calculate the value.

3- Write an expression (using commutation functions) for the net single premium at age 65 for $1,000 of whole life insurance. Calculate the value.

4- Write an expression (using commutation functions) for the net single premium at age 5 for a $2,000 pure endowment due 25 years thereafter. Calculate the value.

5- Write an expression (using commutation functions) for the net single premium at age 40 for a $5,000 30-year endowment insurance policy. Calculate the value,

6- Write an expression (using commutation functions) for the accumulated cost at age 30 for a $10,000 12-year term insurance policy, Calculate the value.

7- Write an expression (using commutation functions) for the net single premium at age 15 for a $1,000 terni-fo-age-65 insurance policy, assuming the insurance is payable at the moment of death, Calculate the value.

8- Describe in words what is represented by each of the following expressions:

a) $\$1{,}000\left(\frac{C_{43}}{D_{43}}\right)$

b) $\$1{,}000\left(\frac{C_{43}+C_{44}+C_{45}+C_{46}}{D_{43}}\right)$

c) $\$1{,}000\left(\frac{M_{43}+M_{47}}{D_{43}}\right)$

d) $\$5{,}000\left(\frac{M_{62}}{D_{62}}\right)$

e) $\$15{,}000\left(\frac{M_{25}-M_{50}+D_{50}}{D_{25}}\right)$

f) $\$1{,}500\left(\frac{D_{65}}{D_{25}}\right)$

g) $\$..{,}500\left(1+\frac{1}{2}i\right)\left(\frac{M_{0}-M_{40}}{D_{0}}\right)$

SYMBOLS FOR NET SINGLE PREMIUMS

Chart 9-3 displays certain internationally used symbols, each of which represents the net single premium at age *x* for $1 of life insurance. (These net single premiums are also shown as they would he calculated using commutation functions.)

The capital letter *"A"* is used with a subscript for the evaluation age, and the number of years under an "angle." In general, the symbol $A_{x:\overline{n}|}$. refers to *n*-year endowment insurance. If the reference is to term insurance or to a pure endowment, then a "1" is placed over the age or the number of years, respectively.

CHART-3

Net Single Premium at Ago *x* for $1 of Life Insurance

Type of Life Insurance	*Symbol for Net Single Premium*	*Net Single Premium Using Commutation Functions*
Whole Life Insurance	A_x	$\frac{M_x}{D_x}$
***n*-Year Term Insurance**	$A^1_{x:\overline{n}|}$	$\frac{M_x - M_{x+n}}{D_x}$
Pure Endowment Due in *n* Years	$A_{x:\overline{n}|}^{\;\;1}$	$\frac{D_{x+n}}{D_x}$
***n*-Year Endowment Insurance**	$A_{x:\overline{n}|}$	$\frac{M_x - M_{x+n} + D_{x+n}}{D_x}$

Net single premium symbols wherein a *bar* is placed over the *"A"* represent insurance payable at the moment of death. For example,

$$\overline{A}_x$$

is the internationally used symbol for the net single premium at age *x* for $1 of whole life insurance payable at the moment of death.

CHAPTER 8
ANNUAL PREMIUMS

INTRODUCTION

The purchase of life insurance by the payment of a single premium when the policy is issued is relatively uncommon, because few people are financially able to do so. The purchase of life insurance by payment of the one-year term insurance premium each year is also uncommon, because this premium increases sharply at the older ages.

A more practical procedure has been devised for paying premiums whereby the premium is paid annually but its amount is the same each year. The calculation of such annual level premiums is based on this principle: at *the date the policy is issued, the present value of the premiums must be equal to the present value of the benefits.*

Annual premiums so calculated are called *net annual premiums.* The word "net" means that the premium calculation involves only rates of interest and modality, with no consideration of expenses or profits. (Annual premiums which do include an amount for expenses and profits are called *gross annual premiums.* These will be considered in Section 9.7.)

Life insurance policies may be issued on any date during a calendar year. The first annual premium is due on that date, the second annual premium is due one year later, etc. The period of time between such anniversaries is known as a *policy year,* to distinguish it from a calendar year (i.e., January 1 to December 31). In this book, references to "years" in connection with insurance policies will mean *policy years.*

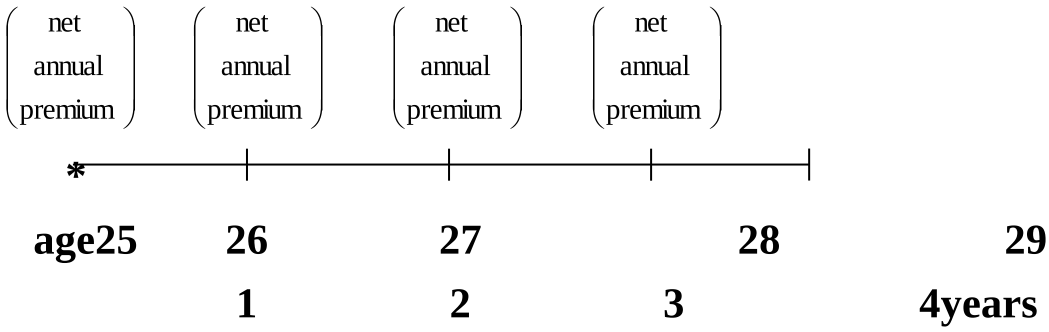
Annual premiums are always paid at the beginning of the year, such payments taking place each year only if the

person insured is then alive to pay. Therefore, annual premiums for a policy constitute a ***life annuity due.*** Such premiums may be paid for the same number of years as the insurance benefit covers, or a fewer number of years. Thus, they may constitute either a ***whole life annuity due*** or a ***temporary life annuity due.*** In describing a certain policy, if the premium paying period is not specified, it is generally understood that premiums are payable for as long as there is life insurance coverage.

NET ANNUAL PREMIUMS FOR TERM INSURANCE

A term life insurance policy which covers a stated number of years normally requires an annual premium payable at the beginning of each of those years. The calculation of the net annual premiums is based upon the principle stated above: at *the date the policy is issued, the present value of the net premiums must be equal to the present value of the benefits.*

For example, it may be desired to calculate the net annual premium (per $1,000 of insurance) for a four-year term insurance policy issued to a person age 25. Since no premium-paying period is specified, it is understood that these premiums are payable for four years. In line diagram form, this series of net annual premiums appears as follows:



The net annual premiums constitute a *temporary life annuity due.* Their present value, at the date the policy is issued, may be calculated by consulting a mortality table and assuming that all persons enumerated therein are individually involved. At age 25, their present value is

$$\begin{pmatrix}\text{Present Value of} \\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix}(l_{25} + l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3)$$

This equation shows that the present value of the net annual premiums equals the total of each of the net annual premiums paid by survivors at each age, with each such amount being discounted at interest to the evaluation date. The first item inside the parentheses, namely l_{25}, is not multiplied by any υ factor because it represents those net annual premiums which are payable upon the evaluation date.

The above expression would be divided by the number living on the evaluation date (l_{25}) to find the present value per person (as was done in Chapter 8). However, in this case, this step is not necessary. The above expression represents a total present value for all the l_{25} persons. Calculations which follow will demonstrate how it4s used to find a net annual premium per person.

Similarly, the present value of the *benefits* may be calculated. In line diagram form, the death benefits to be paid appear as follows:

	\$1,000d_{25}	**\$1,000d_{26}**	**\$1,000d_{27}**	**\$1,000d_{28}**
*	|	|	|	|
age 25	**26**	**27**	**28**	**29**

1 2 3 4years

At age 25, the present value is

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix} = \$1{,}000(d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3 + d_{28}\upsilon^4)$$

That is, the $1,000 death benefit is paid for those who die at each age, with each such amount being discounted at interest from the end of the year of death to the evaluation date.

The above expression would be divided by the number living on the evaluation date (l_{25}) to find this present value per person, i.e., the net single premium (as was done in Chapter 9). In this case, this step is not necessary. The above expression represents a total net single premium for all the l_{25} persons. It will be used in the calculation below to find a net annual premium per person.

The expression for the present value of net annual premiums is equal to the expression for the present value of the benefits:

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}$$

$$(l_{25} + l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3)$$

$$= \$1{,}000(d_{25}\upsilon + l_{26}\upsilon^2 + d_{27}\upsilon^3 + d_{28}\upsilon^4)$$

The equation can be solved for the Net Annual Premium, which will be the net annual premium per person. If, for example, the 1958 C.S.O. Table and 3% interest were being used to calculate the above net annual premiums, the present value of the net annual premiums (the left side)

would be evaluated as follows;

From above

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{25} + l_{26}\upsilon + l_{27}\upsilon^2 + l_{28}\upsilon^3)$$

Substituting the values for the l's from Table III, for the υ's from Table 1(3%)

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}(9,575,636)\\ +(9,557,423)(.970874)\\ +(9,538,423)(9.42596)\\ +(9,519,442)(.915142)\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}9,575,636 + 9,278,793\\ +8,990,879 + 8,711,641\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[36,556,949]$$

The same mortality table and interest rate used to calculate the present value of the net annual premiums are used to calculate the present value of the benefits. The right side of the equation would be evaluated as follows;

From above

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix} = \$1,000(d_{25}\upsilon + d_{26}\upsilon^2 + d_{27}\upsilon^3 + d_{28}\upsilon^4)$$

Substituting the values for the d's from Table III, for the υ's from Table 1(3%)

$$= \$1,000\begin{bmatrix}(18,481)(.970874)\\ +(18,732)(.942596)\\ +(18,981)(.915142)\\ +(19,324)(.888487)\end{bmatrix}$$

= \$1,000(17,943+17,657+17,370+17,169)

= **\$70,139,000**

The net annual premium per person can then be found:

Basic equation

$$\begin{pmatrix}\text{Present Value of} \\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value} \\ \text{of Benfits}\end{pmatrix}$$

Substituting values calculated above

$$\begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix}[36{,}556{,}949] = \$70{,}139{,}000$$

$$\begin{pmatrix}\text{Net Annual} \\ \text{Premium}\end{pmatrix} = \$1.92$$

In line diagram form, this series of net annual premiums appears as follows:

\$1.92 \$1.92 \$1.92 \$1.92

age 25 26 27 28 29

It can be demonstrated that the payment of these net annual premiums will provide \$1,000 (at the end of the year of death) for all who die between ages 25 and 29. Using Table III, the amount of premium paid in at the beginning of the first year is

$$\$1.92(l_{25}) = \$1.92(9{,}575{,}636)$$

$$= \$18{,}385{,}221.12$$

At the end of one year, the accumulated value of this amount is

$$(\$18{,}385{,}221.12)(1.03) = \$18{,}936{,}777.75$$

From this fund is deducted \$1,000 for each who have died during the first year:

$1,000($d_{25}$) = $1,000(18,481)

= $18,481,000

This leaves a balance in the fund of

$18,936,777.75 - $18,481,000 = $455,777.75

The continued operation of the fund for succeeding years may be traced in the accompanying schedule (Chart 10-1). The final excess in the fund represents less than $3 for each person dying the final year, and results from rounding off the net annual premium to two decimal places.

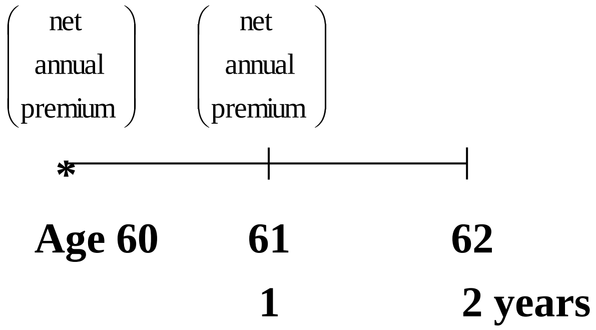
CHART-1

(1) Year	*(2)* Premiums Paid at Beginning of Year	*(3)* Total Fund at Beginning of Year (Col.6, Previous Year, plusCol.2)	*(4)* Fund Accumulated for One Year (Col 3× 1.03)	*(5)* Claims Paid at End of Year (Number of Deaths× $1,000)	*(6)* Balance in Fund at End of Year after Payment of Claims (Col 4- Col.5)
1	$18,385,221.12	$18,385,221.12	$18,936,777.75	$18,481,000	$455,777.75
2	18,349,737.60	18,805,515.35	19,369,680.81	18,732,000	637,680.81
3	18,313,772.16	18,951.452.97	19,519,996.56	18,981,000	538,996.56
4	18,277,328.64	18,816,325.20	19,380,814.96	19,324,000	56,814.96

To Illustrate- **Using Table III and** 3% **interest, calculate the net annual premium (per $1,000) for a 2-year term insurance policy issued at age 60.**

Solution

In line diagram form, the *net annual premiums* for this policy appear as follows:



Their total present value is equivalent to the net annual premiums paid by the survivors at each age, with each such amount being discounted at interest to the evaluation date.

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{60} + l_{61}\upsilon)$$

Substituting the values for the Tables

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}(7{,}698{,}698)\\ +(7{,}452{,}106)(.970874)\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[7{,}698{,}698 + 7{,}322{,}435]$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[15{,}021{,}133]$$

In line diagram form, the *benefits* for this policy appear as follows:

$1,000$d_{60}$ $1,000$d_{61}$

Age 60 61 62

1 2 years

Their total present value is equivalent to the amounts paid for those who die at each age, with each amount being discounted at interest from the end of the year of death to the evaluation date.

Basic equation

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix} = \$1{,}000(d_{60}\upsilon + d_{61}\upsilon^2)$$

Substituting the values for the Tables

$$= \$1{,}000\begin{bmatrix}(156{,}592)(.970874)\\ +(167{,}736)(.942596)\end{bmatrix}$$

= $1,000(152,031 + 158,107)

= $310,138,000

The net annual premium is then found by substituting these values in the basic equation:

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}$$

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[15{,}021{,}133] = \$310{,}138{,}000$$

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix} = \$20.65$$

The net annual premiums for a whole life insurance policy may be paid either

1. **For the entire lifetime of the person insured. They thus constitute a *whole life annuity due.* This kind of insurance policy is known as an *ordinary life* policy.**

or

2. **For a number of years which is *less* than the entire lifetime, say for *n* years. They thus constitute a *temporary life annuity due.* This kind of insurance is known as an *n-payment life* policy.**

To Illustrate- **Using the 1958 C.S.O. Table and 3% interest, calculate the net annual premium (per $1,000) for an ordinary life insurance policy issued at age 96; also for a 2-payment life policy issued at that age.**

Solution

The *present value of the benefits* is the same for both policies because both provide insurance coverage for the whole of life. The line diagram for these benefits appears as follows (remembering that all persons die before age 100, according to the 1958 C.S.O. Table):

	$\$1,000d_{96}$	$\$1,000d_{97}$	$\$1,000d_{98}$	$\$1,000d_{99}$
*				
age 96	**97**	**98**	**99**	**100**
	1	**2**	**3**	**4years**

Their total present value is equivalent to the amounts of benefits paid for those who die at each age, with each such amount being discounted at interest from the end of the year of death to the evaluation date.

Basic equation

$$\begin{pmatrix}\text{Present Value} \\ \text{of Benfits}\end{pmatrix} = \$1{,}000(d_{96}\upsilon + d_{97}\upsilon^2 + d_{98}\upsilon^3 + d_{99}\upsilon^4)$$

Substituting the values form the Tables

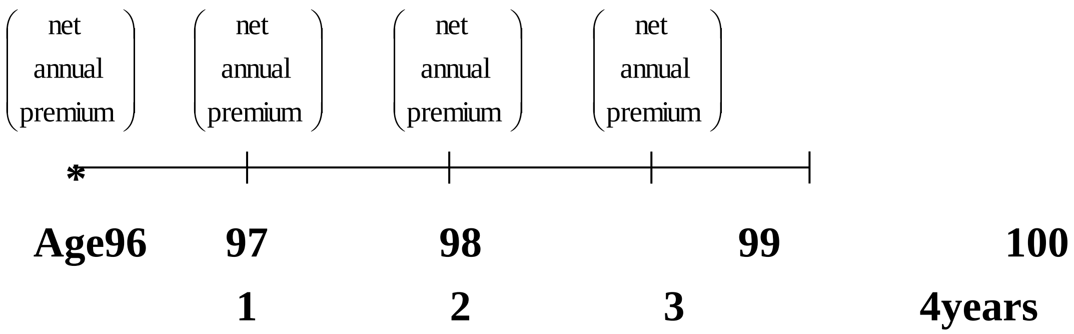
$$= \$1{,}000\begin{bmatrix} (25{,}250)(.970874) \\ +(18{,}456)(.942596) \\ +(12{,}916)(.915142) \\ +(\ \ 6{,}415)(.888487) \end{bmatrix}$$

= $1,000(24,515+17,397+11,820+5,700)

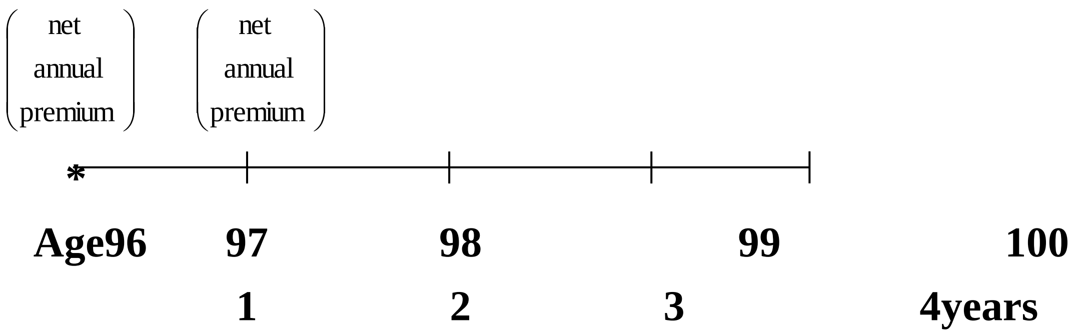
= $59,432,000

The line diagrams for the *net annual premiums* for these two policies appear as follows:

Ordinary Life



Two-Payment Life



In each case, their total present value is equivalent to the net annual premiums paid by the survivors at each age, with each such amount being discounted at interest to the evaluation date.

Ordinary Life

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{96} + l_{97}\upsilon + l_{98}\upsilon^2 + l_{99}\upsilon^3)$$

Substituting values from the tables

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}(63{,}037)\\ +(37{,}787)(.970874)\\ +(19{,}331)(9.42596)\\ +(\ 6{,}415)(.915142)\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[63{,}037 + 36{,}686 + 18{,}221 + 5{,}871]$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[123{,}815]$$

Two-Payment Life

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{96} + l_{97}\upsilon)$$

Substituting values from the tables

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}(63{,}037)\\ +(37{,}787)(.970874)\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[63{,}037 + 36{,}686]$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[99{,}723]$$

The net annual premium for each policy is then solved for, as follows:

Ordinary Life

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix}=\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}$$

Substituting values calculated above

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[123{,}815]=\$59{,}432{,}000$$

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}=\$480.01$$

Two-Payment Life

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix}=\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}$$

Substituting values calculated above

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[99{,}723]=\$59{,}432{,}000$$

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}=\$595.97$$

As the premium-paying period is shortened, the amount of the net annual premium for the same benefits increases. This can be seen by comparing the net annual premium for the ordinary life policy above ($480.01) with the net annual premium for the two-payment life policy ($595.97).

The calculation of net annual premiums for whole life insurance at the younger ages would become very laborious if the procedure described were used. In actual practice, calculations for all kinds of net annual premiums are usually simplified by using commutation functions. How commutation functions are used to compute net annual premiums will be explained in Section 9.6.

NET ANNUAL PREMIUMS FOR ENDOWMENT

INSURANCE

The net annual premiums for an endowment insurance policy may be paid either

1. For the same number of years as the insurance covers, say for *n* years. They thus constitute a *temporary life annuity due for n years.* This kind of insurance policy is known as an *n-year endowment* policy.

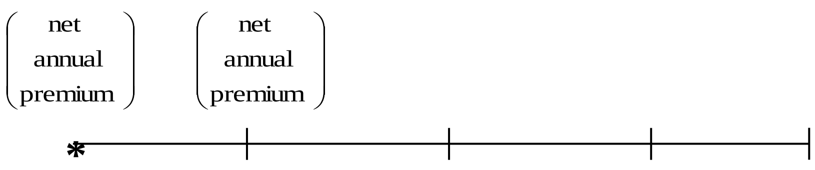
or

2. For a number of years which is *less* than the period that the insurance covers, say for *m* years (with the insurance coverage for *n* years). They thus constitute a *temporary life annuity due for m years.* This kind of insurance policy is known as an *in-payment n-year endowment* policy.

To Illustrate- **Using the 1958 C.S.O. Table and 3% interest, calculate the net annual premium for a $1,000 2-payment endowment-at age-65 policy issued to a man age 61. Construct a schedule proving that this premium will provide the benefits of the policy.**

Solution

The number of years of insurance equals 65 - 61 = 4. Hence, this is a 2-payment 4-year endowment policy. In line diagram form, the *net annual premiums* appear as follows:



Age61 62 63 64 65

1 2 3 4years

Their total present value is equivalent to the net annual

premiums paid by the survivors at each age, with each such amount being discounted at interest to the evaluation date.

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}(l_{61} + l_{62}\upsilon)$$

Substituting values from the tables

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}\begin{bmatrix}(7{,}452{,}106)\\ +(7{,}374{,}370)(.970874)\end{bmatrix}$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[7{,}542{,}106 + 7{,}159{,}584]$$

$$= \begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[14{,}701{,}690]$$

In line diagram form, the *benefits* for this policy appear as follows:

				$1,000$l_{65}$
	$1,000$d_{61}$	**$1,000$d_{62}$**	**$1,000$d_{63}$**	**$1,000$d_{64}$**
*				
age 61	**62**	**63**	**64**	**65**
	1	**2**	**3**	**4years**

The amounts paid for those who die at each age are payable at the end of the year of death. The pure endowment is paid to the survivors at the end of the 4-year period. All amounts paid are discounted at interest to the evaluation date. Notice that in the expression below, the exponents on the last two υ's are the same. Both d_{64} and d_{65} are multiplied by υ^4. This is because the two benefits are payable on the same date: the death benefit for those who die during the final year, and the pure endowment benefit to those still alive at the end of the final year.

Basic equation

$$\begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix} = \$1{,}000(d_{61}\upsilon + d_{62}\upsilon^2 + d_{63}\upsilon^3 + d_{64}\upsilon^4 + l_{65}\upsilon^4)$$

Substituting the values for the Tables

$$= \$1{,}000\begin{bmatrix}(167{,}736)(.970874)\\ +(179{,}271)(.942586)\\ +(191{,}174)(.915142)\\ +(203{,}394)(.888487)\\ +(6{,}800{,}531)(.888487\end{bmatrix}$$

=$1,000(162,854 + 168,980 + 174,951

+ 180,713 + 6,042,183)

= $6,729,678,000

The net annual premium is then found:

Basic equation

$$\begin{pmatrix}\text{Present Value of}\\ \text{Net Annual Premiums}\end{pmatrix} = \begin{pmatrix}\text{Present Value}\\ \text{of Benfits}\end{pmatrix}$$

Substituting values calculated above

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}[14{,}701{,}690] = \$6{,}729{,}678{,}000$$

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix} = \$457.75$$

The accompanying schedule (Chart 10-2) demonstrates that this premium will provide the benefits of the policy.

CHART-2

(1) Year	(2) Premiums Paid at Beginning of Year	(3) Total Fund at Beginning of Year	(4) Fund Accumulated for One Year (Col.3×	(5) Claims Paid at End of Year (Number	(6) Balance in Fund at End of Year after

		(Col.6 Previous Year, plus Col.2)	1.03)	of Death × $1,000)	Payment of Claims (Col.4 - Col.5)
1	$3,452,399,021.50	$3,452,399,021.50	$3,555,970,992.15	$167,736,000	$3,388,234,992.15
2	3,375,617,867.50	6,763,852,859.65	6,966,768,445.44	179,271,000	6,787,497,445.44
3	None	6,787,497,445.44	6,991,122,368.80	191,174,000	6,799,948,368.80
4	None	6,799,948,368.80	7,003,946,819.86	203,394,000	6,800,552,819.86

At the end of the four years, the number of survivors is l_{65} = 6,800,531

Therefore, the total pure endowments to be paid to the survivors at that time is

$ 1,000(6,800,531) = $6,800,531,000

This leaves a balance in the hind, after the pure endowments are paid, equal to

$6,800,552,819.86 - $6,800,531,000 = $21,819.86

This represents only about $\frac{1}{3}$ of a cent for each of the survivors, and is the result of rounding off the net annual premium to 2 decimal places.

COMMUTATION FUNCTIONS

The commutation functions explained in Chapters 8 and 9 can be used to calculate net annual premiums and will generally simplify the work.

The basis for the calculation is the equation given in Section 10.5:

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\begin{pmatrix}\text{Annuity}\\ \text{Factor}\end{pmatrix}=\begin{pmatrix}\text{Net Single}\\ \text{Premiums}\end{pmatrix}$$

The \$1,000 four-year term insurance policy issued at age 25, discussed in Section 10.2, will be used as an example. The left-hand side of the above equation may be written

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\left(\frac{N_{25}-N_{29}}{D_{25}}\right)$$

This follows from the general statement made in Chapter 8 that the factor to use in evaluating a *life annuity* will be of the form $\frac{N-N}{D}$ where the subscript of the first *N* is the age when the first payment is due, the subscript of the second *N* is the first age when there are no more payments due, and the subscript of the *D* is the age at which the annuity is being evaluated.

The right-hand side of the above equation (Net Single Premium) may be written

$$\$1{,}000\left(\frac{M_{25}-M_{29}}{D_{25}}\right)$$

This follows from the general statement made in Chapter 9 that the factor to use in calculating a *net single premium* will be of the form $\frac{M-M+D}{D}$, where the subscript of the first M is the age when the insurance coverage begins, the subscript of the second *M* is the age at which the insurance coverage stops, the subscript of the *D* in the

numerator is the age at which a pure endowment would be paid, and the subscript of the *ID* in the denominator is the age at which this net single premium is evaluated. (In this particular case, the *D* in the numerator does not appear, because no pure endowment is involved.)

The entire equation then appears as follows:

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}\left(\frac{N_{25}-N_{29}}{D_{25}}\right)=\$1{,}000\left(\frac{M_{25}-M_{29}}{D_{25}}\right)$$

The commutation symbol D_{25} appears in the denominator on both sides of the equation. If both sides are multiplied by D_{25}, the denominators are eliminated. The equation can then be solved for (Net Annual Premium) by dividing both sides by $(N_{25} - N_{29})$. The result is

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}=\$1{,}000\left(\frac{M_{25}-M_{29}}{N_{25}-N_{29}}\right)$$

Using values of *M* and *N* from Table IV, this becomes

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix}=\$1{,}000\left(\frac{1{,}276{,}590-1{,}243{,}091}{113{,}189{,}600-95{,}729{,}800}\right)$$

= $1.92

The answer is the same as that calculated in Section 10.2.

A general statement may be made that the factor to use in calculating a *net annual premium* will be of the form $\frac{M-M+D}{N-N}$ where the subscripts in the numerator define the benefits and follow the rule given in Chapter 9 for calculating net single premiums, and the subscripts of the N's in the denominator define the premium-paying period and follow the rule given in Chapter 8 for calculating life annuity factors.

To Illustrate- Using Table IV, calculate the net

annual premium (per $1,000) for a 2-year term insurance policy issued at age 60.

Solution

This is the same problem as the illustration in Section 10.2. The solution will now be given by using commutation functions. In the general expression given above, the *D* in the numerator will not appear, because there is no pure endowment involved.

Basic equation; subscripts of the *M*'s define the period of coverage; subscripts of the *N*'s define the premium-paying period

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix} = \$1{,}000\left(\frac{M_{60} - M_{62}}{N_{60} - N_{62}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{825{,}847 - 773{,}206}{16{,}510{,}076 - 13{,}960{,}493}\right)$$

$$= \$1{,}000\left(\frac{52{,}641}{2{,}549{,}583}\right)$$

= $20.65

This answer agrees with that calculated in Section 10.2.

To Illustrate Again- **Using Table IV, calculate the net annual premium (per $1,000) for an ordinary life policy issued at age 96.**

Solution

This is the same problem as the illustration in Section 10.3. The solution will now be given by using commutation functions. In the general expression for net annual premiums, the second *M* in the numerator will not appear, because the insurance is for the whole of life. Also, the *D* in the numerator will not appear, because there is no pure

endowment involved. In the denominator, the second N will not appear, because the premium payments are to be made for life.

Basic equation

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix} = \$1{,}000\left(\frac{M_{96}}{N_{96}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{3{,}481}{7{,}251}\right)$$

= $480.07

This answer is only 6 cents different from that calculated in Section 10.3. The difference is due to the fact that the commutation functions as shown in the tables are rounded off to the nearest whole number.

To Illustrate Again- **Using Table IV, calculate the net annual premium for a $1,000 2-payment endowment-at-age-65 policy issued to a man age 61.**

Solution

This is the same problem as the illustration in Section 10.4. The solution will now be given by using commutation functions.

Basic equation; subscripts of the *M*,s define the period of coverage; subscript of *D* is age of pure endowment; subscripts of the *N*'s define the premium-paying period

$$\begin{pmatrix}\text{Net Annual}\\ \text{Premiums}\end{pmatrix} = \$1{,}000\left(\frac{M_{61} - M_{65} + D_{65}}{N_{61} - N_{63}}\right)$$

Substituting values from Table IV

$$= \$1{,}000\left(\frac{800{,}042 - 686{,}750 + 995{,}688}{15{,}203{,}325 - 12{,}780{,}670}\right)$$

$$= \$1{,}000\left(\frac{1{,}108{,}980}{2{,}422{,}682}\right)$$

= $457.75

This answer agrees with that calculated in Section 10.4.

Questions For Review

(Use Table IV for all of the following.)

1- **Write an expression (using commutation functions) -for the *net annual premium* (per $1,000) for each of the following policies. (If *a* student wishes, he can calculate the value of each.).**

a) **A 20-year term insurance policy issued at age 25.**

b) **A 1-year term insurance policy issued at age 65.**

c) **A tenn-to-age-65 insurance policy issued at age 40.**

d) **An ordinary life policy issued at age 0.**

e) **A 30-payment life policy issued at age 21.**

f) **A whole life insurance policy issued at age 30, wherein premiums stop at age 70 (i.e., last premium is payable at age 69).**

g) **A 25-year endowment insurance policy issued at age 28.**

h) **A 20-payment 30-year endowment insurance policy issued at age 15.**

i) **A 30-payment endowment-at-age-70 policy issued at age 22.**

2- **Calculate the net annual premium for a $20,000 ordinary life insurance policy issued to a girt age 15. Use a "3-year setback" for females.**

3- **Calculate the net annual premium for a $1,000 20-payment life policy issued at age 5, assuming the insurance is payable at the moment of death.**

4- **State in words what each of the following represents:**

a) $\$1{,}000\left(\frac{M_5 - M_{35} + D_{35}}{N_5 - N_{25}}\right)$

b) $\$1{,}000\left(\frac{M_{60}}{N_{60} - N_{80}}\right)$

c) $\$1{,}000\left(\frac{M_{14}-M_{24}}{N_{14}-N_{19}}\right)$

d) $(\$5{,}000)(1.015)\left(\frac{M_{25}-M_{65}}{N_{25}-N_{65}}\right)+\$5{,}000\left(\frac{D_{65}}{N_{25}-N_{65}}\right)$

GROSS ANNUAL PREMIUMS

The *net* annual premiums considered thus far are sufficient, in terms of mortality and assumed interest rates, to provide the benefits guaranteed in the policy. However, they make no provision for the life insurance company's expenses of conducting business. Therefore, an amount called the *loading* must be added to these net premiums to provide for expenses, profits, and the possibility of unforeseen adversities. The total of the net premium and the loading is known as the *gross premium.* It is the gross premium which the policyowner pays to the insurance company.

Where the policy is "participating" (receives policy dividends), it is not necessary to have great refinement in the calculation of the loading. Savings from operations can be returned to the policyowners of participating policies in the form of dividends, and the dividend calculations can be changed when necessary to meet changing conditions. However, the amount of the loading must be reasonably conservative.

Where a policy is "nonparticipating," however, very detailed analysis of probable expenses is employed in calculating the loading. Because no policy dividends are returned to policyowners, the insurance company has no means of adjusting its income to allow for changes in expenses subsequent to issue of the policy.

The gross annual premium may be calculated by a variety of methods. Companies use many different formu1as for computing loadings, some very simple, some complex. As

a result, the equations for calculating the gross premiums vary.

The expenses of a life insurance company fall into three principal categories:

1- **Those expenses which are relatively *constant for each policy* regardless of the amount of the policy. These include the cost of issuing the policy, collecting the premiums, paying the claims, etc.**

2- **Those expenses which *vary with the amount of the premium.* These include state premium taxes and agents' commissions.**

3- **Those expenses which *vary with the amount of insurance,* i.e., those expenses which are usually higher for larger amount policies. These include costs of establishing whether applicants are in good health (such as medical examiners' fees), drawing up directions for payment of proceeds, etc.**

Expenses which are relatively constant regardless of policy size (category 1) are frequently provided for by adding a certain charge for each policy, regardless of the amount of the policy. This is known as a "policy charge" or "policy fee." However, prior to about 1957, this method of loading was generally considered illegal because state laws prohibiting "discrimination between different policyholders of the same class" were interpreted to prohibit such a method of calculation. Therefore, a method of calculation was used in those days which was based on the determination of the amount of an average-size policy. The gross premiums per $1,000 were so calculated that such an

average-size policy would yield the amount needed to pay for these particular expenses. The result was that large policies yielded more than enough to cover their expenses, while small policies yielded an insufficient amount. In total, however, approximately the correct amount was collected.

A compromise between the "old" and "new" methods of providing for these expenses which are relatively constant regardless of policy size is sometimes used. Under this method, gross premiums per $1,000 are quoted which vary according to the size group into which the policy falls. For example, these groups might be

Policies of less than $5,000 face amount

Policies of $5,000 to $9,999 face amount

Policies of $10,000 to $24,999 face amount

Policies of $25,000 to $99,999 face amount

Policies of $100,000 and over face amount

Within each such size group, an average-size policy is used to calculate the gross premium per $1,000 for that group, such that the loading for this average-sized policy will cover the particular expenses referred to in category 1. The result is that the smaller-sized policies require a larger gross premium per $1,000 than the larger policies. This method is known as "band grading," or simply "banding."

In providing for those expenses which vary with the amount of the premium (category 2), a simple method is to add a percentage of the gross annual premium per $1,000 to the net annual premium. In equation form, this would be

Gross = Net + Percent of Gross

To Illustrate- **Calculate the gross annual premium per $1,000 for a policy for which the net annual premium per $1,000 is $12.49, and a loading is needed of 25% of the gross annual premium. What gross annual premium would the policyowner pay for a $5,000 policy, assuming a charge is also made of $7.50 per policy?**

Solution

Before computing the premium which the policyowner pays, it is necessary to find the gross premium *per $1,000* of insurance. The policy owner's gross premium is this gross premium per $1,000 multiplied by the number of thousands of insurance, plus any policy charge which is added by the company.

The calculation is first made to find the gross annual premium per $1,000:

Basic equation

Gross = Net + Percent of Gross

Substituting $12.49 for net, .25 for percent

Gross = $12.49 + (.25)(Gross)

Subtracting (.25)(Gross) from each side

Gross = (.25)(Gross) = $12.49

Gross (1- .25) = $12.49

Gross (.75) = $12.49

Gross = $16.65

For the $5,000 policy, the gross annual premium would

be 5 times $16.65, plus the $7.50 charge:

$$\begin{pmatrix}\text{Gross Premium} \\ \text{For \$5,000}\end{pmatrix} = (5)(\$16.65) + \$7.50$$

$$= \$90.75$$

A common method of providing for those expenses which vary with the amount of insurance (category 3) is to use a constant amount per $1,000 of insurance. The expenses referred to in category 2, which vary with the amount of the premium, are then provided for by a percentage of gross annual premium. In equation form, this total would be

Gross = Net + Constant + Percent of Gross

To Illustrate- **Calculate the gross annual premium per $1,000 for a policy for which the net annual premium per $1,000 is $31.28, if it is to be loaded $3 per $1,000 plus 20% of the gross annual premium. What would the gross annual premium be for a $15,000 policy, assuming a charge is also made of $10 per policy?**

Solution

The calculation is first made to find the gross annual premium per $1,000:

Basic equation

Gross = Net + Constant + Percent of Gross

Substituting $31.28 for net, $3 for constant, and .20 for percent

Gross = $31.28 + $3.00 + (.20)(Gross)

Gross - (.20)(Cross) = $31.28 + $3.00

Gross (1- .20) = $31.28 + $3.00

Gross (.80) = $34.28

Gross = $42.85

For the $15,000 policy, the gross annual premium paid by the policy-owner would be 15 times $42.85, plus the $10 charge:

$$\begin{pmatrix}\text{Gross Premium}\\ \text{For \$15,000}\end{pmatrix} = (15)(\$42.85) + \$1.00$$

= $652.75

In this case, the total $652.75 gross annual premium which the policy-owner pays is made up of the net annual premium plus loading, as follows:

$\begin{pmatrix}\text{Net Annual}\\ \text{Premium}\end{pmatrix}$	**= (15)($31.28)**	**= $469.20**
$\begin{pmatrix}\text{Expenses Constant}\\ \text{per Policy}\end{pmatrix}$	**= $10.00**	**= 10.00**
$\begin{pmatrix}\text{Expenses Varying}\\ \text{with Premium}\end{pmatrix}$	**= (15)(20)($42 85)**	**= 128.55**
$\begin{pmatrix}\text{Expenses Varying}\\ \text{with Amount}\\ \text{of Insurance}\end{pmatrix}$	**= (15)($3.00)**	**= 45.00**
		$652.75 Total

Since each insurance company determines its own method for calculating the loading, not all insurance companies use loading formulas which add loadings of each of the types described above. Companies sometimes use loading methods involving only one or two of the types of additions described, rather than all three, or they may use very complex methods of loading which are not given in this book.

EXERCISES

(Use Table II and $2\frac{1}{2}$% interest, unless specified differently)

1- **Write an expression (using symbols) for the present value at age 10 of $250 due in 25 years, with benefit of survivorship.**

2- **Write an expression (using symbols) for the present value at age 65 of a 4-year temporary life annuity due of $50 per year.**

3- **Write an expression (using symbols) for the present value at age 20 of a deferred life annuity having 3 payments of $750 each, the first one of which is payable at age 42.**

4- **Write an expression (using symbols) for the present value at age 9Gof a whole life annuity of $1,000 per year, assuming the mortality table which will be used is the 1958 C.S.O, Table.**

5- **Calculate the present value at age 70 of a $40 payment due at age 80, with benefit of survivorship.**

6- **If it is assumed that females will always show the same mortality experience as males 3 years younger, calculate the value for a female at age 25 of $100 due 15 years later, with benefit of survivorship. (Hint: The 3-year "setback" means that instead of using l_{25} and l_{40}, use l_{22} and l_{37})**

7- **Calculate the amount that a man age 22 should pay for the right to receive $10 per year, first payment due at age 30 and last payment due at age 33.**

8- **Calculate the present value at age 100 of a deferred life annuity of $1,000 per year, first payment at age 107 (payments continue for life).**

9- **Using the 1958 C.S.O. Table (Table III)•and 3% interest, calculate the present value at age 97 of a whole life**

annuity due of $100 per year.

10- Calculate the present value at age 40 of a whole life annuity, deferred for 10 years, of $15 per year, using the following present value factors for I per year:
***Present Value at Age 40 of Whole Life Annuity* = 18.713**
***Present Value at Age 40 of 10-year Life Annuity* = 8.509**

SYMBOL FOR NET ANNUAL PREMIUMS

Chart-3 displays certain internationally used symbols, each of which represents the net annual premium for $1 of life insurance issued at age *x*. (These net annual premiums are also shown as they would be calculated using commutation functions.)

The capital letter *"P"* is used with a subscript for the issue age, and the number of years under an "angle." In this respect, the subscripts are identical to those shown in Section 9,9 which appear with *"A"* for the various types of net single premiums. However, there is an additional subscript shown at the lower *left* of the *"F"* whenever the premium-paying period is shorter than the benefit period. For example, the symbol $_{m}p_{x:\overline{n}|}$ represents the net annual premium for a $1 *m*-payment *n*-year endowment policy issued at age *x*.

CHART-3

Net Annual Premium for $1 of Life Insurance Issued at Age *x*

Type of Life Insurance	*Symbol for Net Annual Premium*	*Net Annual Premium Using Commutation Functions*
Ordinary Life	P_x	$\frac{M_x}{N_x}$
***m*-Payment Life**	${}_mP_x$	$\frac{M_x}{N_x - N_{x+m}}$
***n*-Year Term Insurance**	$P^1_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n}}{N_x - N_{x+n}}$
***m*-Payment n-Year Term Insurance**	${}_mP^1_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n}}{N_x - N_{x+m}}$
***n*-Year Endowment Insurance**	$P_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}}$
***m*-Payment *n*-Year Endowment Insurance**	${}_mP_{x:\overline{n}\rceil}$	$\frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+m}}$

Questions For Review

Q1: Complete The Following Statements With The Suitable Word(S):-

1- ……….. is a periodical payments of equal size, payable over a period of years.

2- ……….. option allows the insured to buy some type of cash value insurance without proving insurability.

3- ……….. policy provides the payment of sum insured at the death of the insured person any time death may occur.

4- ……….. it is a life insurance policy where the annual premium is payable for life.

5- ……….. offers insurance protection for a fixed period of time.

6- ……….. is considered as term insurance plus pure endowment.

7- ……….. Policies provide protection against the financial problem associated with premature death .

8- Term insurance policy should not be used when the need for life insurance is ………..

9- The whole life insurance premiums are………..than term life insurance premiums .

10- The level – premium method of paying for whole life insurance produces a savings value, called the ………..

11- Level premium whole life insurance policies require insured to pay the same premium as long as they ………..

12- ……….. policies allow the insured to continue the coverage up to a specified age regardless of the status of the insured's health .

13- ……….. policies allow the insured the option of converting the policy to a whole life policy .

14- ……….. insurance policy should not be used when the need for life insurance is permanent .

15- ………. Is used by insurers in order to improve predictions.

16-……. It is a life insurance policy where the annual premium is payable for life.

17-…………. Is a periodical payments of equal size, payable over a period of years.

18-…………. offers insurance protection for a fixed period of time.

19-………… is considered as term insurance plus pure endowment.

20-………. Insurance policy can he used when the need for life insurance is temporary.

21-…….... Life insurance policies are useful to people wanting to combine savings with their life insurance purchase.

22-……… is a contract in which the insurer promises the insured a series of periodic payments.

23-……… is the cause of loss.

24-……… is the person whose death causes the insurer to pay the claim.

25-we call the payment the insurer receives a ……....

26-……….. policies that allow the insured to continue the coverage up to specified age regardless the status of the insured's health.

27-………. Insurance policy should not be used when the need for life insurance is permanent.

Q2: Multiple choice: Select the best answer a,b,or c:

1) Purchasing term life insurance is advisable when:

a- You want life insurance to meet part of your investment needs.

b- The need for protection is temporary.

c- All of the above.

2) An ordinary life insurance implies:

a- The premiums will be paid as long as the insured live.

b- The payment of only single premium.

c- None of the above.

3) Paying level premium for life insurance:

a- Results in savings or investment element.

b- Results in a declining protection element.

c- All of the above.

4) Limited- payment life insurance:

a- Has a higher premiums than ordinary life.

b- Has a larger investment element than ordinary life insurance.

c- All of the above.

5) Endowment life insurance:

a- Is the same as a pure endowment.

b- Has a relatively low cash value.

c- May be considered as term insurance and pure endowment.

6) Renew ability option in term life insurance:-

a- Protects the insurability of the insured for the period specified.

b- Has no effect on the cost the policy.

c- None of the above.

7) Convertibility option in term life insurance:-

a- Allows the insured to buy some type of cash value insurance without proving insurability.

b- Is found only in endowment policies.

c- None of the above.

8) The most simple form of life insurance is :-

a- Single premium whole life .

b- Term life insurance .

c- Variable life insurance .

Q3: Indicate whether each of the following statements is true or false and correct the false one (s):-

1- Under an endowment policy, the insurer pays the face amount of the policy only if the insured dies during the period.

2- Premiums for endowment policies are comparatively higher than of a term insurance.

3- Renewable term insurance is a type of contact under which the insured may renew a policy before its expiration date, providing the medical examination can still be based.

4- A term insurance policy offers insurance protection for a fixed period of time in which little or no cash value is accumulated.

5- Premiums for a 20- year endowment life insurance policy, are higher than for a 20 payment life insurance policy, assuming same issue age and amount of insurance.

6- Whole life insurance covers the subject of the insurance for an entire life.

7- Ordinary life insurance is life insurance issued in amounts of 1000 or more.

8- Under the limited payment plan for whole life insurance the insured pays premiums for certain number of years, after which time no further premium payments need to be made.

9- The variable life annuity in theory is a hedge against inflation.

10- The three basic types of life insurance (term, whole life and endowment) are sometimes combined in various ways to from special purpose policy forms.

11- Payments under a variable annuity will vary according to investment results and the age of the annuitant.

12- Under a straight life annuity as periodic income is paid to the annuitant until death.

13- Renewable term insurance is a type of contract under which the insured may convert a policy to another type of insurance.

14- Under a whole life insurance policy the insured person receives an annual payment as long as he/she lives.

15- Under the limited payment plan for a whole life insurance the annual premium is higher than in an ordinary life assuming the same age and amount of insurance.

16- A term insurance policy offers insurance protection for an entire life.

Solutions

Q1:- Complete:-

(1) Annuities

(2) Convertibility

(3) Whole life insurance.

(4) An ordinary life insurance.

(5) Term insurance.

(6) Endowment insurance

(7) Term insurance.

(8) Permenant

(9) Larger

(10) Reserves

(11) Lives or reach age 100

(12) Renewability

(13) Convertability

(14) Term insurance.

Q2: Multiple choice:

1- b	**2- a**	**3- a**
4- c	**5- c**	**6- a**
7- a	**8- b**	

Q3:- true / false

No. of statement	T/F	Correction of the false one
1	**F**	**Under endowment insurance policy, the insurer pays the face amount of the policy if the insured dies during the policy period or still alive to the end of the time of the policy.**
2	**T**	
3	**F**	**Renewable term insurance is a type of contract under which the insured may renew a policy before its expiration date, without the medical**

No. of statement	T/F	Correction of the false one
		examination.
4	**T**	
5	**T**	
6	**T**	
7	**T**	
8	**T**	
9	**T**	
10	**T**	
11	**T**	
12	**T**	
13	**F**	**Convertable term insurance is a type of contract under which the insured may convert a policy to another type of insurance.**
14	**F**	**Under a whole life annuity the insured person receives an annual payment as long as he/she lives.**
15	**T**	

No. of statement	T/F	Correction of the false one
16	**F**	**A whole live insurance policy offers insurance protection for an entire life.**

Question one:

a) Select the best answer a, b, c or d

1- Renewability option in term insurance:-

a) Should be added to all whole life policies

b) protects the insurability of the insured for the period specified.

C) has no effect on the cost of the policy.

d) None of the above

2- The probability that a person aged 45 will die the following year is

(a) q_{45} **(b)** e_{45} **(c)** $1/q_{45}$ **(d)** p_{45}

3-The probability that a person aged 22 will live the next 18years and die the following year is:

(a) q_{22} **(b)** $^{10}/q_{22}$ **(c)** $^{18}/^{1}q_{22}$ **(d)** $^{18}q_{22}$

4- Is defined as the probability that an event will occur:-

(a) peril **(b) chance of loss**

(c) hazard (d) risk.

5-Law of large numbers is an important concept in insurance because:

a- It results in reduced individual risk.

b- It permits insurers to predict future losses more accurately.

c- None of the above.

6-Pooling is used by insurers in order to:-

a- encourage self- insurance.

b- improve predictions.

c- None of the above.

7-An ordinary life insurance implies:-

a- The premiums will be paid as long as the insured live.

b- The payment of only single premium.

c- None of the above.

Question two:

Indicate whether each of the following statements is true or false:

a- A whole life insurance policy offers insurance protection for a short period of time

b- Chance of loss is computed by dividing the number of exposure units by the number of losses.

c- Renewable term insurance is a type of contract under which the insured may renew a policy before

its expiration date, providing the medical examination can still be based.

d- Under the limited payment plan for a whole life insurance, the insured pays premiums for certain number of years, after which time no further premiums need to be made.

e- Polling is used by insures in order to improve predication.

f- An ordinary life insurance implies that the payment of only single premium

g- Because prediction for future losses is based on past experience, losses must be accidental.

h- Unless the possible loss can be measured in economic terms, there is no risk.

i- Insurance is the sharing of risk and transfer of loss.

j- Because of the law of large numbers, losses to be insured must be definite in time, place and amount.

k- Pooling is used by insurers in order to make losses fortuitous.

l- A term insurance policy offers insurance protection for a fixed period of time in which a large cash value is accumulated.

m- Under a whole life insurance policy the insured person receives an annual payment as long as he lives.

n- Under the limited, payment plan for a whole life insurance the annual premium is higher than in an ordinary life assuming the same age and amount of insurance.

o- Renewable term insurance is a type of contract under which the insured may convert a policy to another type of insurance.

p- A term insurance policy offers insurance protection for an entire life.

q- Renewable term insurance is a type of contract under which the insured may renew a policy before its expiration date, providing the medical examination can still be passed.

r- The annual premium for an ordinary life insurance is lower than for a 10- payment life assuming same issue age and amount of insurance.

s- Premiums for a 20-year endowment life insurance are higher than for a 20- payment life insurance policy, assuming same issue age and amount of insurance.

t- A term insurance policy offers insurance protection for a fixed period of time in which a large cash value is accumulated.

u- Pooling is used by insurers in order to improve predictions.

v- Endowment insurance is considered as term insurance plus pure endowment.

Solutions

1	b
2	A
3	C
4	B
5	B
6	B
7	A
A	F
B	F

C	F
D	T
E	T
F	F
G	T
H	T
I	F
J	T
K	F
L	F
M	F
N	T
O	F
P	F
Q	F
R	T
S	T
T	F
U	T
V	T

The Life Table

1) State in words the probabilities represented by the following symbols:

P_{33}, ${}_{10}P_{30}$, q_{40}, ${}_{20}q_{53}$, ${}_{10/20}q_{34}$, and ${}_{10/}q_{30}$.

2) Of a group of 1000 25-year old persons, what are the predicted numbers that will die before reaching age 45? If ${}_{20}P_{25} = 0.80$.

3) Find the numerical values of each of the following probabilities:

a) The probability that (25) will live to age 60

b) The probability that (20) will live at least 60 more years.

c) The probability that (27) will die within a year after his 41st birthday.

d) The probability that (65) will live at least 10 years but not more than 15 years.

4) If $\ell x = k\ (185 - 2x)$

${}_{5/5}q_{70}$Find P_{80}, q_{75}, ${}_{5/}q_{75}$,

5) The probability that a man aged 25 and another aged 45 will both survive a period of twenty years is 0.7. The probability that a man aged 25 will survive ten years is 0.9. Find the probability that a man aged 35 will live to age 65.

6) If the probabilities of surviving ten years are 0.8, 0.7, and 0.6 for persons of ages 30, 40 and 50, respectively, what is the probability that an individual now age 30 will die between ages 50 and 60?

7) Given that $\ell 25 = 10{,}000$, and the rates of mortality at ages 25, 26, 27, 28 and 29 are 0.003, 0.0031, 0.0033, 0.0036, and 0.004 respectively, compute columns of ℓx, d_x, and p_x for these ages.

8) Complete the following table

Age x	No. of Livings ℓx	No. of deaths d_x	Probability of survival p_x	Probability of death q_x
41	150,000		0.98	
42		3234		
43				
44	140043			0.035
45		5406		

From the table calculate the values of $_2p_{42}$, $_{2\backslash}q_{42}$.

9) Prove the following identities:

a) $_m p_x = \dfrac{_{m/n}q_x}{_n q_{x+m}}$

b) $_{m+n}p_x = {}_n p_x \cdot {}_m p_{x+n}$

c) $_{m\backslash n}q_x = {}_m p_x - {}_{m+n}p_x$

10) If $\ell x = k\,(96 - x)$, where x is a constant, show that:

$$, \quad q_x = \frac{1}{96-x} \quad p_x = \frac{95-x}{96-x} \qquad , d_x = k$$

Solutions

1) P_{33}: The probability that a person aged now 33 will survive the next year.

$_{10}P_{30}$: The probability that a person aged now 30 will survive the next 10 years.

q_{40}: The probability that a person aged now 40 will die within the next year.

$_{20}q_{53}$: The probability that a person aged now 53 will die within the next 20 years.

$_{10\backslash 20}q_{34}$: The probability that a person aged now 34 will survive the next 10 years and then die within the following 20 years (between the ages 44 & 64).

$_{10\backslash}q_{30}$: The probability that a person aged now 30 will survive the next 10 years and then die within the following year. (between the ages 40 & 41).

2) $\ell_{25} = 1000$

$$_{20}p_{25} = \frac{\ell_{45}}{\ell_{25}} = 0.8 \rightarrow \ell_{45} = 0.8\ell_{25} = 0.8(1000) = 800$$

The No. of deaths predicted = $\ell_{25} - \ell_{45}$ = 1000 - 800 = 200

3)

a) $35p_{25} = \frac{\ell_{60}}{\ell_{25}}$

b) $60p_{20} = \frac{\ell_{80}}{\ell_{20}}$

c) $14/q_{27} = \frac{\ell_{41} - \ell_{42}}{\ell_{27}}$

d) $_{10\backslash 5}q_{65} = \dfrac{\ell_{75} - \ell_{80}}{\ell_{65}}$

4) $\ell x = k\,(185 - 2x)$

$$P_{80} = \frac{\ell_{81}}{\ell_{80}} = \frac{k(185-2(81))}{k(185-2(80))} = \frac{23k}{25k} = \frac{23}{25} = 0.92$$

$$q_{75} = \frac{\ell_{75} - \ell_{76}}{\ell_{75}} = \frac{k(185-2(75) - k(185-2(76)))}{k(185-2(75))}$$

$$= \frac{35k - 33k}{35k} = \frac{2k}{35k} = \frac{2}{35} = 0.05714$$

$$5\backslash q_{75} = \frac{\ell_{80} - \ell_{81}}{\ell_{75}} = \frac{25k - 23k}{35k} = \frac{2}{35} = 0.05714$$

$$5\backslash 5q_{70} = \frac{\ell_{75} - \ell_{80}}{\ell_{70}} = \frac{k(185-2(75)) - k(185-2(80))}{k(185-2(70))}$$

$$= \frac{35k - 25k}{45k} = \frac{10k}{45k} = \frac{2}{9} = 0.22$$

5) $_{20}p_{25} \times {}_{20}p_{45} = 0.7$

$\rightarrow \ell_{65} = 0.7\,\ell_{25}$ $\qquad \dfrac{\ell_{45}}{\ell_{25}} \times \dfrac{\ell_{65}}{\ell_{45}} = \dfrac{\ell_{65}}{\ell_{25}} = 0.7$

$_{10}p_{25} = 0.9$

$$\frac{\ell_{35}}{\ell_{25}} = 0.9 \rightarrow \ell_{35} = 0.9\,\ell_{25}$$

$$_{30}p_{35} = \frac{\ell_{65}}{\ell_{35}} = \frac{0.7\,\ell_{25}}{0.9\,\ell_{25}} = \frac{0.7}{0.9} = 0.778$$

$$_{20/10}q_{30} = ?? = \frac{\ell_{50} - \ell_{60}}{\ell_{30}}$$

6) **$_{10}p_{30} = 0.8$**

$_{10}p_{40} = 0.7$

$_{10}p_{50} = 0.6$

$$\frac{\ell_{40}}{\ell_{30}} = 0.8 \quad \rightarrow \ell_{30} = \ell_{40} \div 0.8 = 1.25\ell_{40}$$

$$\frac{\ell_{50}}{\ell_{40}} = 0.7 \quad \rightarrow \ell_{50} = 0.7\ell_{40}$$

$$\frac{\ell_{60}}{\ell_{50}} = 0.6 \quad \rightarrow \ell_{60} = 0.6\ell_{50} = (0.6)\left(0.7\ell_{40}\right) = 0.42\ell_{40}$$

$$_{20\backslash 10}q_{30} = \frac{\ell_{50} - \ell_{60}}{\ell_{30}} = \frac{0.7\ell_{40} - 0.42\ell_{40}}{1.25\ell_{40}} = \frac{0.28\ell_{40}}{1.25\ell_{40}} = 0.224$$

7)

x	ℓx	d_x	p_x
25	10,000	30	= 1-0.003 = 0.997
26	9970	31	= 1-0.0031 = 0.9969
27	9939	33	= 1-0.0033 = 0.9967
28	9906	36	0.9964
29	9870	40	0.996

8)

x	ℓ x	d_x	p_x	q_x
41	150,000	3000	0.98	0.02
42	147,000	3234	0.978	0.022
43	143,766	3723	0.9741	0.0259
44	140,043	4901	0.965	0.035
45	135,142	5406	0.96	0.04

$q_{41} = 1 - p_{41} = 0.02$, $d_{41} = \ell_{41} \times q_{41} = 3000$

$\ell_{42} = \ell_{41} - d_{41} = 147{,}000$, $q_{42} = d_{42} / \ell_{42} = 0.022$

$p_{42} = 1 - q_{42} = 0.978$, $\ell_{43} = \ell_{42} \times p_{42} = 143{,}766$

$p_{43} = \ell_{44} / \ell_{43} = 0.9741$, $d_{43} = \ell_{43} - \ell_{44} = 3723$

$d_{44} = \ell_{44} \times q_{44} = 4901$

$$ {}_2p_{42} = \frac{\ell_{44}}{\ell_{42}} = \frac{140,043}{147,000} = 0.95267 $$

$$ {}_2\backslash q_{42} = \frac{\ell_{44} - \ell_{45}}{\ell_{42}} = \frac{140,043 - 135,142}{147,000} = 0.03334 $$

9) a) $\text{LHS} = \frac{\ell_{x+m}}{\ell_x}$

$$ \text{RHS} = \frac{\ell_{x+m} - \ell_{x+m+n}}{\ell_x} \div \frac{\ell_{x+m} - \ell_{x+m+n}}{\ell_{x+m}} $$

$$ = \frac{\left(\ell_{x+m} - \ell_{x+m+n}\right)}{\ell_x} \times \frac{\ell_{x+m}}{\left(\ell_{x+m} - \ell_{x+m+n}\right)} $$

$$ = \frac{\ell_{x+m}}{\ell_x} = \text{LHS} $$

b) $\text{LHS} = \frac{\ell_{x+m+n}}{\ell_x}$

$$ \text{RHS} = \frac{\ell_{x+n}}{\ell_x} . \frac{\ell_{x+m+n}}{\ell_{x+n}} = \frac{\ell_{x+m+n}}{\ell_x} = \text{LHS} $$

c) $\text{LHS} = \frac{\ell_{x+m} - \ell_{x+m+n}}{\ell_x}$

$$ \text{RHS} = \frac{\ell_{x+m}}{\ell_x} - \frac{\ell_{x+m+n}}{\ell_x} = \frac{\ell_{x+m} - \ell_{x+m+n}}{\ell_x} = \text{LHS} $$

$$d_x = \ell x - \ell x+1 \qquad 10)$$

$$= k\,(96 - x) - k\,(96 - (x+1))$$

$$= k\,(96 - x) - k\,(96 - x - 1)$$

$$= k\,(96 - x) - k\,(95 - x)$$

$$= 96\,k - xk - 95k + x\,k$$

$$= 96\,k - 95k = k$$

$$P_x = \frac{\ell_{x+1}}{\ell_x} = \frac{k(96-(x+1))}{k(96-x)}$$

$$= \frac{96-x-1}{96-x} = \frac{95-x}{96-x}$$

$$q_x = \frac{d_x}{\ell_x} = \frac{k}{k(96-x)} = \frac{1}{96-x}$$

Net Single Premium

Life Annuities

1) **Two payments of \$1000 each are to be received at the end of five and ten years, respectively. Find their present value if they are to be received only if (45) is alive to receive them.**

2) **A man aged 45 is to receive 25,000 L.E. from an insurance company at age 60 if he is alive. Compute his net single premium.**

3) **A man aged 30 deposited 5,000 L.E. with an insurance company as a net single premium of a 25-year pure endowment policy. Find the amount of the benefit.**

4) **Find the net single premium for a whole life ordinary annuity of 10,000 L.E. per annum issued to a man who was 50 years old.**

5) **Find the net single premium for a whole life annuity due of 3,000 L.E. per year issued to a man aged 35.**

6) **What annual income for life can a man aged now 45 buy with 6,000 L.E., the first payment due at age 60.**

7) **A man dies leaving his widow a 200,000 L.E. insurance policy. If she was 60 years old when he died and chooses to receive annual income for life with the 1^{st} payment to be made now, what is the size of her annual income?**

8) **A man aged 45 has a contract for a deferred whole life annuity of L.E. 1000 a year, first payment at age 65. He wishes to replace this by a twenty-year-temporary life annuity due (first**

payment to be made at once). What will be the size of the new payment.

9) **A woman has 15,000 L.E. if she was 40 years old and used the amount to buy a temporary life annuity of 10 annual payments; what would be the size of each payment if the 1st payment is to be made when she is:**

c) 50 years old b) 41 years old a) 40 years old

10) **A man aged 56 is granted a pension of 3000 L.E. a year for 5 years beginning at once, and 600 L.E. a year thereafter for the rest of his life. If all payments are contingent on his survival, find their expected present value.**

11) **State the meaning and the numerical values of**

$A^{1}_{35:\overline{20}|}$, $15/\ddot{a}_{50}$, $a_{50:\overline{20}|}$.

12) **Prove the following identities:**

a) $D_{x+1} = v.\, p_x \,.\, D_x$

b) ${}_{m+n}E_x = {}_mE_x \,.\, {}_nE_{x+m}$.

c) $a_x = v.p_x + v^2.\,{}_2p_x\,.\ddot{a}_{x+2}$.

13) **Derive by the mutual fund method the net single premium in terms of commutation symbols for a 15-year pure endowment policy for an amount $2000 issued at age 35.**

14) **Derive by the mutual fund method the net single premium in terms of commutation symbols for a 20-year ordinary life annuity issued on the life of a man aged 40 pays $500 per year.**

15) Derive by the mutual fund method the expected present value of a $200 whole life annuity due issued to a person aged 50.

16) **Derive by the mutual fund method the net single premium in terms of commutation symbols for a 20-year life annuity issued on the life of a man aged 25, pays $1500 every year, first payment at age 60.**

17) **a) Show that:** $\ddot{a}_x = 1 + v\, p_x\, \ddot{a}_{x+1}$

b) Use the identity of the previous problem to find $\ell 40$ and $\ell 41$,

if at 3%, a_{40} = 19.374 , a_{41} = 19.026,

a_{42} = 18.672, $\ell 42$ = 9,173,375

Solutions

1) x = 45   1st payment at age 50 & the other at age 55

S = 1000

Expected present value $(EPV) = 1000\, A^{\;1}_{45:\overline{5}|} + 1000\, A^{\;1}_{45:\overline{10}|}$

$$= 1000 \left[\frac{D_{50} + D_{55}}{D_{45}} \right]$$

2) x = 45   S = 25,000   n = 15

Net Single Premium $(NSP) = S.\, A^{\;1}_{x:\overline{n}|} = 25{,}000\, A^{\;1}_{45:\overline{15}|}$

$$= 25{,}000 \frac{D_{60}}{D_{45}}$$

3) x = 30   NSP = 5,000   n = 25   S = ??

$$NSP = S.\, A^{\;1}_{x:\overline{n}|}$$

$$S = A^{\;1}_{x:\overline{n}|}$$

$$= 5000 \div \frac{D_{55}}{D_{30}}$$

$$= 5000 \times \frac{D_{30}}{D_{55}}$$

4) x = 50   R = 10,000

$$NSP = R\, a_x$$

$$= 10{,}000 \frac{N_{51}}{D_{50}}$$

5) R = 3000   x = 35

$$NSP = R\, \ddot{a}x$$

$$=3000\frac{N_{35}}{D_{35}}$$

k = 15   NSP = 6000 (deferred)   x = 45   6) R = ??

$$NSP = R.k\backslash\ddot{a}_x$$
$$R = NSP \div k\backslash\ddot{a}_x$$
$$= 6000 \div 15\backslash\ddot{a}_{45}$$
$$= 6000 \div \frac{N_{60}}{D_{45}}$$
$$R = 6000 \times \frac{D_{45}}{N_{60}}$$

R = ??   x = 60   7) NSP = 200,000

$$NSP = R.\ddot{a}_x$$
$$R = NSP \div \ddot{a}_x$$
$$= 200{,}000 \div \frac{N_{60}}{D_{60}}$$
$$= 200{,}000 \times \frac{D_{60}}{N_{60}}$$

k = 20   R = 1000   8) x = 45

R_2 = ??   Change that with   n = 20

$$1000_{20}\backslash\ddot{a}_{45} = R_2 .\ddot{a}_{45:\overline{20}|}$$

$$1000\frac{N_{65}}{D_{45}} = R\,\frac{N_{45}-N_{65}}{D_{45}}$$

$$R = 1000 \times \frac{N_{65}}{D_{45}} \times \frac{D_{45}}{N_{45}-N_{65}} = 1000 \times \frac{N_{65}}{N_{45}-N_{65}}$$

9) NSP = 15000   x = 40   n = 10   R = ??

a) $NSP = R.\frac{N_{40}-N_{50}}{D_{40}}$

$$R = 15000 \div \frac{N_{40} - N_{50}}{D_{40}}$$

$$= 15000 \times \frac{D_{40}}{N_{40} - N_{50}}$$

b) $R = 15000 \times \frac{D_{40}}{N_{41} - N_{51}}$

c) $R = 15000 \times \frac{D_{40}}{N_{50} - N_{60}}$

10) x = 56 R_1 = 3000 R_2 = 600

$$NSP = 3000 \frac{N_{56} - N_{61}}{D_{56}} + 600 \frac{N_{61}}{D_{56}}$$

11) $A_{35:\overline{20}|}^{\;\;1} = \frac{D_{55}}{D_{20}}$

The net single premium of a 20-years pure endowment policy issued on the life of a person aged 35.

* $15 \backslash \ddot{a}_{50} = \frac{N_{65}}{D_{50}}$

The expected present value of a whole life annuity deferred for 15-years, contingent on the life of a person aged 50.

* $a_{50:\overline{20}|} = \frac{N_{51} - N_{71}}{D_{50}}$

The expected present value of a 20-year ordinary life annuity contingent on the life of a person aged 50.

12) a) LHS = $v^{x+1} . \ell x+1$

$$\text{RHS} = v.\frac{\ell_{x+1}}{\ell_x}.v^x.\ell_x$$

$$= v^{x+1}.\ell_{x+1} = \text{LHS}$$

b) $\text{LHS} = \frac{D_{x+m+n}}{D_x}$

$$\text{RHS} = \frac{D_{x+m}}{D_x} \times \frac{D_{x+m+n}}{D_{x+m}}$$

$$= \frac{D_{x+m+n}}{D_x} = \text{LHS}$$

c) $\text{LHS} = \frac{N_{x+1}}{D_x}$

$$\text{RHS} = v.\frac{\ell_{x+1}}{\ell_x} + v^2\frac{\ell_{x+2}}{\ell_x}.\frac{N_{x+2}}{D_{x+2}} \qquad \times \frac{v^x}{v^x}$$

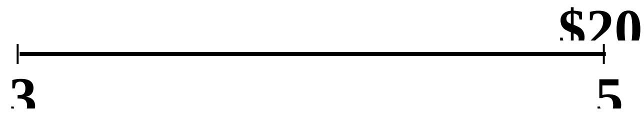
$$\text{RHS} = \frac{v^{x+1}.\ell_{x+1}}{v^x\,\ell_x} + \frac{v^{x+2}.\ell_{x+2}}{v^x.\ell_x}.\frac{N_{x+2}}{D_{x+2}}$$

$$= \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x}.\frac{N_{x+2}}{D_{x+2}}$$

$$= \frac{D_{x+1} + N_{x+2}}{D_x}$$

$$= \frac{N_{x+1}}{D_x} = \text{LHS}$$

13)



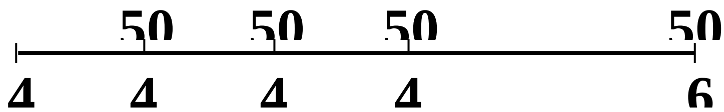
Net single premium of 15-year pure endowment policy issued at age 35 = $A_{35:\overline{15}|}^{\;\;1}$ **= 2000**

$$\ell_{35}\; A_{35:\overline{15}|}^{\;\;1} = 2000\, \ell_{50}\, v^{15}$$

$$A_{35:\overline{15}|}^{\;\;1} = 2000\, \frac{\ell_{50}\,.v^{15}}{\ell_{35}} \times \frac{v^{35}}{v^{35}}$$

$$A_{35:\overline{15}|}^{\;\;1} = 2000\, \frac{\ell_{50}\,.v^{50}}{\ell_{35}\,.v^{35}}$$

$$A_{35:\overline{15}|}^{\;\;1} = 2000\, \frac{D_{50}}{D_{35}}$$



14)

$$\ell_{50}\, a_{40:\overline{20}|} = 500\,\ell_{41}\, v + 500\,\ell_{42}\, v^2 + 500\,\ell_{43}\, v^3 + \cdots + 500\,\ell_{60}\, v^{20}$$

$$a_{40:\overline{20}|} = 500\, \frac{\ell_{41}.v + \ell_{42}.v^2 + \ell_{43}.v^3 + \cdots + \ell_{60}.v^{20}}{\ell_{40}} \times \frac{v^{40}}{v^{40}}$$

$$= 500\, \frac{\ell_{41}.v^{41} + \ell_{42}.v^{42} + \ell_{43}.v^{43} + \cdots + \ell_{60}.v^{60}}{\ell_{40}.v^{40}}$$

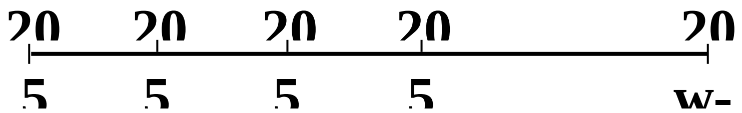
$$= 500\, \frac{D_{41} + D_{42} + D_{43} + \cdots + D_{60}}{D_{40}}$$

$N_{41} = D_{41} + D_{42} + \ldots + D_{60} + D_{61} + D_{62} + \ldots + D_{w-1}$ Since

$D_{61} + D_{62} + \ldots + D_{w-1}$ $\qquad$ $N_{61} =$

$N_{41} - N_{61} = D_{41} + D_{42} + \ldots + D_{60}$ Then

$$a_{40:\overline{20|}} = 500\,\frac{N_{41} - N_{61}}{D_{40}}$$ **Then**



15)

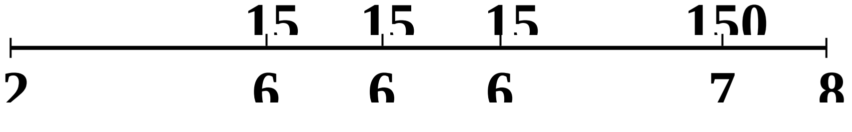
$$\ell_{50}\,\ddot{a}_{50} = 200\ell_{50} + 200\ell_{51}.v^1 + 200\ell_{52}.v^2 + \cdots + 200\ell_{w-1}.v^{w-1-50}$$

$$\ddot{a}_{50} = 200\,\frac{\ell_{50} + \ell_{51}.v^1 + \ell_{52}.v^2 + \cdots + \ell_{w-1}.v^{w-1-50}}{\ell_{50}} \times \frac{v^{50}}{v^{50}}$$

$$\ddot{a}_{50} = 200\,\frac{\ell_{50}.v^{50} + \ell_{51}.v^{51} + \ell_{52}.v^{52} + \cdots + \ell_{w-1}.v^{w-1}}{\ell_{50}.v^{50}}$$

$$\ddot{a}_{50} = 200\,\frac{D_{50} + D_{51} + D_{52} + \cdots + D_{w-1}}{D_{50}}$$

$$\ddot{a}_{50} = 200\,\frac{N_{50}}{D_{50}}$$



16)

$$\ell_{25}\;.\;{}_{35/}\ddot{a}_{25:\overline{20|}} = 1500.\ell_{60}.v^{35} + 1500.\ell_{61}.v^{36} + 1500.\ell_{62}.v^{37}$$
$$+ \cdots + 1500\ell_{79}.v^{54}$$

$$ {}_{35}/\ddot{a}_{25:\overline{20}|} = 1500\ \frac{\ell_{60}.v^{35} + \ell_{61}.v^{36} + \ell_{62}.v^{37} + \cdots + \ell_{79}.v^{54}}{\ell_{25}} \times \frac{v^{25}}{v^{25}} $$

$$ = 1500\ \frac{\ell_{60}.v^{60} + \ell_{61}.\mathrm{v}^{61} + \ell_{62}.\mathrm{v}^{62} + \cdots + \ell_{79}.\mathrm{v}^{79}}{\ell_{25}.\mathrm{v}^{25}} $$

$$ {}_{35}/\ddot{a}_{25:\overline{20}|} = 1500\ \frac{D_{60} + D_{61} + D_{62} + \cdots + D_{79}}{D_{25}} $$

$N_{60} = D_{60} + D_{61} + \ldots. + D_{79} + D_{80} + D_{81} + \ldots + D_{w-1}$ Since

$D_{80} + D_{81} + \ldots + D_{w-1}$ $\qquad$ $N_{80} =$

$N_{60} - N_{80} = D_{60} + D_{61} + \ldots + D_{79}$

$$ {}_{35}/\ddot{a}_{25:\overline{20}|} = 1500\ \frac{N_{60} - N_{80}}{D_{25}} $$

17) a) $\mathrm{LHS} = \frac{N_x}{D_x}$

$$ \mathrm{RHS} = 1 + v\frac{\ell_{x+1}}{\ell_x}.\frac{N_{x+1}}{D_{x+1}} \qquad \times \frac{v^x}{v^x} $$

$$ = 1 + \frac{v^{x+1}}{v^x}.\frac{\ell_{x+1}}{\ell_x}.\frac{N_{x+1}}{D_{x+1}} $$

$$ = 1 + \frac{D_{x+1}}{D_x}.\frac{N_{x+1}}{D_{x+1}} = 1 + \frac{N_{x+1}}{D_x} $$

$$ = \frac{D_x}{D_x} + \frac{N_{x+1}}{D_x} = \frac{D_x + N_{x+1}}{D_x} $$

$$ = \frac{N_x}{D_x} = \mathrm{LHS} $$

$\ddot{a}_x = 1 + a_x$ $\qquad$ **b- Since**

$\ddot{a}_{40} = 1 + a_{40} = 20.374$ $\qquad$ **Then**

$\ddot{a}_{41} = 1 + a_{41} = 20.026$

$$\ddot{a}_{42} = 1 + a_{42} = 19.672$$

i = 3%   **&Now from the last identity:** $\ddot{a}_x = 1 + v P_x \ \ddot{a}_{x+1}$

***v* = (1.03)$^{-1}$ Then**

$$\ddot{a}_{41} = 1 + v . P_{41} . \ddot{a}_{42}$$

$$20.026 = 1 + (1.03)^{-1} . \frac{\ell_{42}}{\ell_{41}} . (19.672)$$

$$20.026 = 1 + (1.03)^{-1} . \frac{9,173,375}{\ell_{41}} . (19.672)$$

$$20.026 - 1 = \frac{175202556.3}{\ell_{41}}$$

$$\ell_{41} = \frac{175202556.3}{19.026} = 9208585.9$$

$$\ell_{41} = 9,208,586$$

$$\ddot{a}_{40} = 1 + v . P_{40} . \ddot{a}_{41}$$

$$20.374 = 1 + (1.03)^{-1} . \frac{\ell_{41}}{\ell_{40}} . (20.026)$$

$$20.374 - 1 = \frac{(1.03)^{-1} (9,208,586)(20.026)}{\ell_{40}}$$

$$19.374 = \frac{179039944.9}{\ell_{40}}$$

$$\ell_{40} = \frac{179039944.9}{19.374} = 9241248.3$$

$$\ell_{40} = 9,241,248$$

Net Single Premium

Life Insurance Policies

1) **Find the net single premium for a 10,000 L.E. whole life insurance policy for a boy aged 18 years old at the issue of the policy.**

2) **Find the net single premium for a 25 year, 100,000 L.E. term insurance policy for a man aged 45.**

3) **How much 35 years endowment insurance can a man aged now 30 buy now with 120,000 L.E.?**

4) **A life insurance contract issued on the life of a man now ages 40 provides the payment of 20,000 L.E. if death occurs during the next 5 years, 30,000 L.E. if death occurs during the following 15 years, and a pure endowment of 60,000 L.E if the insured person still alive to attain age 60. Compute the net single premium for this contract.**

5) **A life insurance policy issued at age 35 that provides the payment of $5000 if death occurs before age 50, $10000 if death occurs between age 50 and 60; and $1000 a year for life first payment to be made at age 60. Compute the net single premium.**

6) **A life insurance policy issued on the life of a man aged 45 provides the payment of 500 L.E a year for the next 10 years, 7500 L.E. if death occurs between ages 55 and 60, and 1000 a whole life annuity first payment to be made at age 60. Find the net single premium.**

7) Find the net single premium for a policy issued at age 40 which provides:

- **$ 20,000 for death benefits if death occurs before age 55.**
- **20 years life annuity of $50,000 a year first amount at age 55.**
- **Pure endowment of $30,000 if he survives to age 75.**

8) Prove the following identities:

a) $A_x = v\ddot{a}_x - a_x$

b) $A_x = v\,(q_x + p_x \,.\, A_{x+1})$

c) $A^{\;1}_{x:\overline{n}|} = A_{x:\overline{n}|} - A_{x:\overline{n}|}^{\quad\;\;1}$

9) State in words the meaning of the following formula:

$$\frac{5000\left(M_{30}-M_{35}\right)+7500\left(M_{35}-M_{50}\right)+10,000\,D_{50}+2000N_{61}}{D_{30}}$$

10) Derive by the mutual fund method the net single premium in terms of commutation symbols for a $20,000 whole life insurance policy issued on the life of a man aged 40.

11) Derive by the mutual fund method the net single premium in terms of commutation symbols for a $ 35000 20-year term insurance policy issued on the life of a person aged 30.

12) Derive by the mutual fund method the net single premium in terms of commutation symbols for a $50,000 25-year endowment insurance policy issued on the life of a person aged 40.

13) Derive by the mutual fund method the net single premium in terms of commutation symbols for a life insurance policy issued on the life of a man aged 30, provides the following benefits:

- $50,000 in case of death between the ages 30 and 45,

- $75,000 in case of death between the ages 45 and 65,

- $85,000 in case of survival to the age 65.

14) A certain single premium contract provides for a pure endowment of $2,500 payable if the person aged 45 survives for 15 years. In event of the death before age 60, the single premium is to be returned without interest.

Find the net single premium for that policy.

15) Show that:

$$\frac{1-i\ a_{x:\overline{n-1}|}}{1+i}=\frac{M_x-M_{x+n}+D_{x+n}}{D_x}$$

Solutions

1) x = 18 S = 10,000

NSP = S. A_x

$$=10,000\frac{M_{18}}{D_{18}}$$

2) x = 45 S = 100,000 n = 25

$$NSP = 100,000\, A^{1}_{45:\overline{25}|}$$

$$=100,000\frac{M_{45}-M_{70}}{D_{45}}$$

3) S = ?? x = 30 n = 35 NSP = 120,000

$$NSP = S.\, A_{x:\overline{n}|}$$

$$120,000 = S.\frac{M_{30}-M_{65}+D_{65}}{D_{30}}$$

$$S = 120,000 \div \frac{M_{30}-M_{65}+D_{65}}{D_{30}}$$

$$=120,000\times\frac{D_{30}}{M_{30}-M_{65}+D_{65}}$$

4) x = 40

$$NSP = 20,000\frac{M_{40}-M_{45}}{D_{40}}+30,000\frac{M_{45}-M_{60}}{D_{40}}+60,000\frac{D_{60}}{D_{40}}$$

5) x = 35

$$NSP = 5000\frac{M_{35}-M_{50}}{D_{35}}+10,000\frac{M_{50}-M_{60}}{D_{35}}+1000\frac{N_{60}}{D_{35}}$$

6) x = 45

$$NSP = 500\frac{N_{46}-N_{56}}{D_{45}}+7,500\frac{M_{55}-M_{60}}{D_{45}}+1000\frac{N_{60}}{D_{45}}$$

7) x = 40

$$NSP = 20{,}000\,\frac{M_{40} - M_{55}}{D_{40}} + 5000\,\frac{N_{55} - N_{75}}{D_{40}} + 30{,}000\,\frac{D_{75}}{D_{40}}$$

8) a) $LHS = 1 - d\,\ddot{a}_x$

$RHS = v.\ddot{a}x - ax$ Since $v = 1 - d$

$= (1-d)\ddot{a}x - (\ddot{a}x - 1)$ $ax = \ddot{a}x - 1$

$\ddot{a}x - d\ddot{a}x - \ddot{a}x + 1 = 1 - d\,\ddot{a}x = LHS$

b) $LHS = \frac{M_x}{D_x}$

$$RHS = v\left(\frac{d_x}{\ell_x} + \frac{\ell_{x+1}}{\ell_x}.\frac{M_{x+1}}{D_{x+1}}\right)$$

$$= \frac{v.d_x}{\ell_x} + \frac{v.\ell_{x+1}}{\ell_x}.\frac{M_{x+1}}{D_{x+1}} \quad \times \frac{v^x}{v^x}$$

$$= \frac{v^{x+1}.d_x}{v^x.\ell_x} + \frac{v^{x+1}.\ell_{x+1}}{v^x.\ell_x}.\frac{M_{x+1}}{D_{x+1}}$$

$$= \frac{C_x}{D_x} + \frac{D_{x+1}}{D_x}.\frac{M_{x+1}}{D_{x+1}} = \frac{C_x + M_{x+1}}{D_x} = \frac{M_x}{D_x} = LHS$$

c) $LHS = \frac{M_x - M_{x+n}}{D_x}$

$$RHS = \frac{M_x - M_{x+n} + D_{x+n}}{D_x} - \frac{D_{x+n}}{D_x} = \frac{M_x - M_{x+n}}{D_x} = LHS$$

9) The net single premium of a life insurance policy issued on the life of a man aged (30) provides the following benefits:

a) 5000 incase of death prior to age 35.

b) 7500 in case of death before the age 50.

c) 10,000 in case of survival to the age 50.

d) 2000 per annum for the rest of his life; first payment at age 61.

10) $\ell_{40}\,A_{40} = 20{,}000\,d_{40}.v + 20{,}000\,d_{41}.v^2 + \cdots + 20{,}000\,d_{w-1}.v^{w-40}$

$$A_{40} = 20{,}000\,\frac{d_{40}.v + d_{41}.v^2 + d_{42}.v^3 + \cdots + d_{w-1}.v^{w-40}}{\ell_{40}} \times \frac{v^{40}}{v^{40}}$$

$$= 20{,}000\,\frac{d_{40}.v^{41} + d_{41}.v^{42} + d_{42}.v^{43} + \cdots + d_{w-1}.v^{w}}{\ell_{40}.v^{40}}$$

$$= 20{,}000\,\frac{C_{40} + C_{41} + C_{42} + \cdots + C_{w-1}}{D_{40}}$$

$$A_{40} = 20{,}000\,\frac{M_{40}}{D_{40}}$$

11) $\ell_{30} A^{\;1}_{30:\overline{20|}} = 35{,}000\,d_{30}.v^1 + 35{,}000\,d_{31}.v^2 + \cdots + 35{,}000\,d_{49}.v^{20}$

$$A^{\;1}_{30:\overline{20|}} = 35{,}000\,\frac{d_{30}.v^1 + d_{31}.v^2 + \cdots + d_{49}.v^{20}}{\ell_{30}} \times \frac{v^{30}}{v^{30}}$$

$$A^{\;1}_{30:\overline{20|}} = 35{,}000\,\frac{d_{30}.v^{31} + d_{31}.v^{32} + \cdots + d_{49}.v^{50}}{\ell_{30}.v^{30}}$$

$$A^{\;1}_{30:\overline{20|}} = 35{,}000\,\frac{C_{30} + C_{31} + \cdots + C_{49}}{D_{30}}$$

$$A^{\;1}_{30:\overline{20|}} = 35{,}000\,\frac{M_{30} - M_{50}}{D_{30}}$$

12) $\ell_{40}.A_{40:\overline{25|}} = 50{,}000\,d_{40}.v + 50{,}000\,d_{41}.v^2 + \cdots + 50{,}000\,d_{64}.v^{25} + 50{,}000\,\ell_{65}.v^{25}$

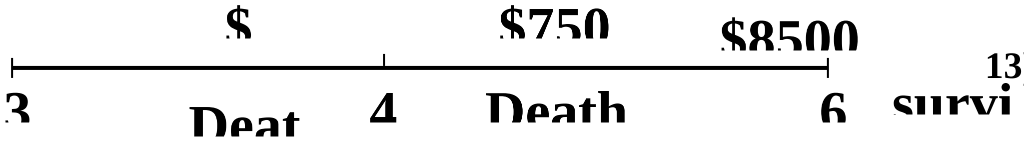
$$A_{40:\overline{25|}} = 50{,}000\,\frac{d_{40}.v + d_{41}.v^2 + \cdots + d_{64}.v^{25} + \ell_{65}.v^{25}}{\ell_{40}} \times \frac{v^{40}}{v^{40}}$$

$$A_{40:\overline{25|}} = 50{,}000\,\frac{d_{40}.v^{41} + d_{41}.v^{42} + \cdots + d_{64}.v^{65} + \ell_{65}.v^{65}}{\ell_{40}.v^{40}}$$

$$A_{40:\overline{25}|} = 50,000\,\frac{C_{40}+C_{41}+\cdots+C_{64}+D_{65}}{D_{40}}$$

$$A_{40:\overline{25}|} = 50,000\,\frac{M_{40}-M_{65}+D_{65}}{D_{40}}$$

13)



$$\boldsymbol{\ell_{30}\ .\ NSP = 50,000\,(d_{30}\,.\,v^{1} + d_{31}\,.\,v^{2} + \ldots + d_{44}\,.\,v^{15})}$$

$$\boldsymbol{+\ 75,000\,(d_{45}\,.\,v^{16} + d_{46}\,.\,v^{17} + \ldots + d_{64}\,.\,v^{35})}$$

$$\boldsymbol{+\ 85,000\ .\ \ell_{65}\ .\ v^{35}}$$

$$NSP = \frac{\begin{array}{l}50,000\left(d_{30}.v^{1}+d_{31}.v^{2}+\cdots+d_{44}.v^{15}\right)\\ +75,000\left(d_{45}.v^{16}+d_{46}.v^{17}+\cdots+d_{64}.v^{35}\right)\\ +85,000.\ell_{65}.v^{35}\end{array}}{\ell_{30}} \times \frac{v^{30}}{v^{30}}$$

$$NSP = \frac{\begin{array}{l}50,000\left(d_{30}.v^{31}+d_{31}.v^{32}+\cdots+d_{44}.v^{45}\right)\\ +75,000\left(d_{45}.v^{46}+d_{46}.v^{47}+\cdots+d_{64}.v^{65}\right)\\ +85,000.\ell_{65}.v^{65}\end{array}}{\ell_{30}.v^{30}}$$

$$NSP = \frac{50,000\left(C_{30}+C_{31}+\cdots+C_{44}\right)+75,000\left(C_{45}+C_{46}+\cdots+C_{64}\right)+85,000\,D_{65}}{D_{30}}$$

$$NSP = \frac{50,000\left(M_{30}-M_{45}\right)+75,000\left(M_{45}-M_{65}\right)+85,000\,D_{65}}{D_{30}}$$

S = $2500 n = 15 14) x = 45

$$NSP = 2500\ A_{45:\overline{15}|}^{\ \ \ 1} + NSP\,A^{1}_{45:\overline{15}|}$$

$$NSP = 2500\,\frac{D_{60}}{D_{45}} + NSP\ .\ \frac{M_{45}-M_{60}}{D_{45}}$$

$$NSP - NSP\,\frac{M_{45}-M_{60}}{D_{45}} = 2500\,\frac{D_{60}}{D_{45}}$$

$$NSP\left[1-\frac{M_{45}-M_{60}}{D_{45}}\right] = 2500\;\frac{D_{60}}{D_{45}}$$

× D45

$$NSP\left[\frac{D_{45}}{D_{45}}-\frac{M_{45}-M_{60}}{D_{45}}\right] = 2500\,\frac{D_{60}}{D_{45}}$$

$$NSP\left[D_{45}-M_{45}+M_{60}\right] = 2500\,D_{60}$$

$$NSP = 2500\;\frac{D_{60}}{D_{45}-M_{45}+M_{60}}$$

15) $$RHS = A_{x:\overline{n}|} = 1-d\ddot{a}_{x:\overline{n}|}$$

$$LHS\quad = \frac{1}{1+i}-\frac{i}{1+i}\,.\,a_{x:\overline{n-1}|}$$

$$= v - d\,a_{x:\overline{n-1}|}$$

$$= (1-d) - d\,a_{x:\overline{n-1}|}$$

$$= 1-d-d\,a_{x:\overline{n-1}|}$$

$$= 1-d\left(1+a_{x:\overline{n-1}|}\right)$$

$$= 1-d\left(1+\frac{N_{x+1}-N_{x+1+n-1}}{D_x}\right)$$

$$= 1-d\left(\frac{D_x+N_{x+1}-N_{x+n}}{D_x}\right)$$

$$= 1-d\left(\frac{N_x-N_{x+n}}{D_x}\right) = 1-d\,\ddot{a}_{x:\overline{n}|} = RHS$$

Net Annual Premium

1) State in words the meaning of the following symbols:

a) ${}_{5}P^{\ 1}_{35:\overline{15|}}$

b) $P_{40:\overline{20|}}$

c) $\dfrac{10{,}000(M_{25}-M_{45})+25000(M_{45}-M_{60})+1500(N_{60}-N_{75})}{N_{25}-N_{50}}$

2) In each of the following problems, assume 5000 policy issued at age 45; compute the net annual premium

a) An ordinary life policy.

b) 10-payment life policy.

c) 10-year term policy.

d) 15-payment, 20-year term policy.

e) 20-year endowment policy.

f) 15-payment, 20-year endowment policy.

3) Compute the net annual premium for a 40,000 twenty year endowment insurance issued at age 25,45 and 65.

4) How large a term insurance policy maturing in 30 years can be purchased by a net annual premium payable for 10 years of 2200 L.E, If the policy issued at age 40.

5) Find the size of the annual premium which paid by means of 15 annual payments for a person aged 35 years who has a life insurance policy that provides him with 2500 L.E if he dies within the next 15 years, 220 L.E a year if he lives during the following 10 years, and 170 L.E a year for the rest of his life the first payment starts at age 60.

6) Find the net annual premium payable for fifteen years for a policy issued at age 60 which provides:

- $20,000 if death occurs before age 70.

- 10 years life annuity of $5,000 a year first amount at age 70.

- Pure endowment of $30,000 if he survives to age 80.

7) A certain life insurance policy issued at age 30, provides for twenty annual premiums L.E. 1000 in event of death of the insured between ages 30 and 40, L.E. 2000 in event of death between ages 40 and 50, and L.E. 3000 in event of death thereafter. Find the net annual premium.

8) Compute the net annual premium payable for twenty years for a policy issued at age 35 which provides for a death benefit of L.E. 20,000 if death occurs between ages 35 and 50, L.E. 10.000 if death occurs between 50 and 60 and a pure endowment of L.E. 15,000 if the insured survives to age 60.

Solutions

1) a) ${}_{5}P^{1}_{35:\overline{15}|}$: The net annual premium of a 5-payment 15-year term insurance issued on the life of a man aged 35.

b) $P_{40:\overline{20}|}$: The net annual premium of 20-year endowment insurance issued of the life of a man aged 40.

c) The net annual premium payable for 25 years of a life insurance policy issued on the life of a person aged 25 provides the following benefits:

- 10,000 in case of death between the ages 25 & 45.
- 25,000 in case of death between the ages 45 & 60.
- 1500 a year for the 15-years, first payment due at age 60.

2) S = 5000 , x = 45

(a) $NAP = 5000\dfrac{M_{45}}{N_{45}} = P_{45}$

(b) $NAP = 5000\dfrac{M_{45}}{N_{45} - N_{55}} = {}_{10}P_{45}$

(c) $NAP = 5000\dfrac{M_{45} - M_{55}}{N_{45} - N_{55}} = P^{1}_{45:\overline{10}|}$

(d) $NAP = 5000\dfrac{M_{45} - M_{65}}{N_{45} - N_{60}} = {}_{15}P^{1}_{45:\overline{20}|}$

(e) $NAP = 5000\dfrac{M_{45} - M_{65} + D_{65}}{N_{45} - N_{65}} = P_{45:\overline{20}|}$

(f) $NAP = 5000\dfrac{M_{45} - M_{65} + D_{65}}{N_{45} - N_{60}} = {}_{15}P_{45:\overline{20}|}$

3) S = 40,000 n = 20

$$x = 25 \rightarrow \text{NAP} = 40{,}000 \frac{M_{25} - M_{45} + D_{45}}{N_{25} - N_{45}}$$

$$x = 45 \rightarrow \text{NAP} = 40{,}000 \frac{M_{45} - M_{65} + D_{65}}{N_{45} - N_{65}}$$

$$x = 65 \rightarrow \text{NAP} = 40{,}000 \frac{M_{65} - M_{85} + D_{85}}{N_{65} - N_{85}}$$

4) S = ?? n = 30 x = 40 NAP = 2200

$$\text{NAP} = S \times \frac{M_{40} - M_{70}}{N_{40} - N_{50}}$$

$$S = 2200 \div \frac{M_{40} - M_{70}}{N_{40} - N_{50}}$$

$$S = 2200 \times \frac{N_{40} - N_{50}}{M_{40} - M_{70}}$$

5) x = 35

$$\text{NAP} = \frac{2500\left(M_{35} - M_{50}\right) + 220\left(N_{50} - N_{60}\right) + 170\,N_{60}}{N_{35} - N_{50}}$$

6) x = 60

$$\text{NAP} = \frac{20{,}000\left(M_{60} - M_{70}\right) + 5000\left(N_{70} - N_{80}\right) + 30{,}000\,D_{80}}{N_{60} - N_{75}}$$

7) x = 30

$$\text{NAP} = \frac{1000\left(M_{30} - M_{40}\right) + 2000\left(M_{40} - M_{50}\right) + 3000\,M_{50}}{N_{30} - N_{50}}$$

8) x = 35

$$\text{NAP} = \frac{20{,}000\left(M_{35} - M_{50}\right) + 10{,}000\left(M_{50} - M_{60}\right) + 15000\,D_{60}}{N_{35} - N_{55}}$$

Gross Annual Premium

1) A life insurance company estimates that the following expenses will be needed for a 150,000 L.E. ordinary life policy issued on the life of a person aged 45 years:

a. Agent commission: 40% of the first premium, 10% of the second premium, 7% of the third through the tenth premium, followed by a "service fee" of 4% on all premiums collected thereafter.

b. Premium taxes: 2.75% of each gross premium.

c. Administrative expenses: 300 L.E. in the first year, and 125 L.E. in each subsequent year.

d. Cost of settlement: 200 L.E. at the time of the death claim or maturity.

Required: Compute the gross annual premium of this policy.

2) A life insurance company estimates that the following expenses will be needed for a 250,000 L.E. fifteen years term insurance policy issued to a person aged 35 years:

a. Agent commission: 45% of the first premium and 6% of all premiums collected thereafter.

b. Premium taxes: 3% of each gross premium.

c. Administrative expenses: 100 L.E. in the first year, and 30 L.E. in each subsequent year.

d. Cost of settlement: 150 L.E. at the time of the death claim or maturity.

Required: Compute the gross annual premium of this policy.

3) A life insurance company estimates that the following expenses will be needed for a 120,000 L.E. twenty-five years endowment insurance policy issued on the life of a person aged 25 years:

a. Agent commission: 30% of the first premium, 14% of the second premium, 5% of the third through the tenth premium, followed by a "service fee" of 2% on all premiums collected thereafter.

b. Premium taxes: 2.5% of each gross premium.

c. Administrative expenses: 180 L.E. in the first year, and 60 L.E. in each subsequent year.

d. Cost of settlement: 120 L.E. at the time of the death claim or maturity.

Required: Compute the gross annual premium of this policy.

4) A life insurance company estimates that the following expenses will be needed for a 200,000 L.E. thirty years pure endowment insurance policy issued to a person aged 30 years:

a. Agent commission: 55% of the first premium, 15% of the second premium, 4% of the third through the tenth premium, followed by a "service fee" of 1% on all premiums collected thereafter.

b. Premium taxes: 1.5% of each gross premium.

c. Administrative expenses: 250 L.E. in the first year, and 150 L.E. in each subsequent year.

d. Cost of settlement: 700 L.E. at the end of the policy.

Required: Compute the gross annual premium of this policy.

5) A life insurance company estimates that the following expenses will be needed for a 250,000 L.E. fifteen payments, thirty years pure endowment insurance policy issued to a person aged 35 years:

a. Agent commission: 40% of the first premium, 12% of the second premium, 6% of the third through the tenth premium, followed by a "service fee" of 2% on all premiums collected thereafter.

b. Premium taxes: 3.5% of each gross premium.

c. Administrative expenses: 105 L.E. in the first year, and 65 L.E. in each subsequent year.

d. Cost of settlement: 350 L.E at the end of the policy.

Required: Compute the gross annual premium of this policy.

6) A life insurance company estimates that the following expenses will be needed for a 160,000 L.E. ten payments, twenty years term insurance policy issued to a person aged 40 years:

a. Agent commission: 40% of the first premium, 15% of the second premium, 4% of all gross premiums collected thereafter.

b. Premium taxes: 2.5% of each gross premium.

c. Administrative expenses: 200 L.E. in the first year, and 95 L.E. in each subsequent year.

d. Cost of settlement: 250 L.E. at the time of the death claim or maturity.

Required: Compute the gross annual premium of this policy.

7) A life insurance company estimates that the following expenses will be needed for a 140,000 L.E. twenty payments life insurance policy issued to a person aged 40 years:

a. Agent commission: 35% of the first premium, 9% of the second premium, and 3% of all premiums collected thereafter.

b. Premium taxes: 2% of each gross premium.

c. Administrative expenses: 100 L.E. in the first year, and 40 L.E. in each subsequent year.

d. Cost of settlement: 200 L.E. at the time of the death claim or maturity.

Required: Compute the gross annual premium of this policy.

8) A life insurance company estimates that the following expenses will be needed for a 195,000 L.E. fifteen payments, thirty years endowment insurance policy issued to a person aged 35 years:

a. Agent commission: 45% of the first premium and 7% of all premiums collected thereafter.

b. Premium taxes: 1.5% of each gross premium.

c. Administrative expenses: 120 L.E. in the first year, and 50 L.E. in each subsequent year.

d. Cost of settlement: 450 L.E. at the end of the policy.

Required: Compute the gross annual premium of this policy.

9) A certain life insurance policy provides for $35,000 in event of death before age 70 with $50,000 cash payment if the insured survives to age 70. Assuming the policy is issued at age (40) and provides for twenty-five annual premiums. It is estimated that the following expenses will be incurred:

a. Commission: 55% of the first gross premium, 20% of the second gross premium, 5% of all gross premiums thereafter.

b. Premium Taxes: 3% of each gross premium.

c. Administrative Expenses: $160 in the first year, and $35 thereafter.

d. Cost of Settlement: $8.

Required: **Compute the minimum gross premium.**

10) A Special ten-payment life insurance policy issued at age 35 provides a death benefit of $25,000 for the first fifteen years, and $45,000 for the next ten years, and $60,000 thereafter for the reminder of life. It is estimated that the following expenses will be incurred.

a. Commission: 60% of the first year gross premium, 25% of the second year gross premium, 10% of all gross premiums thereafter.

b. Premium Taxes: 2.5% of each gross premium.

c. Administrative Expenses: $200 in the first year and $40 in each succeeding year.

d. Cost of Settlement: $10.

Required: Compute the minimum gross premium for this policy.

Solutions

1) x = 45

- Benefit: Whole life insurance $150{,}000 \times A_{45}$

- Premium: Whole life annuity due $G\,\ddot{a}_{45}$

- Expenses

a. Agent commission:

$$= 0.40G + 0.10G\left(\frac{D_{46}}{D_{45}}\right) + 0.07G\left(\frac{N_{47} - N_{55}}{D_{45}}\right) + 0.04G\left(\frac{N_{55}}{D_{45}}\right)$$

b. Premium taxes: $0.0275G\,\ddot{a}_{45}$

c. Administrative expenses $= 300 + 125\left(\frac{N_{46}}{D_{45}}\right)$

d. Cost of settlement: It will be added to the amount of insurance as it will be paid at the same time. (The benefit will be $150{,}200 \times A_{45}$)

- Now using the equation of value:

The EPV of the Premium = The EPV of the Benefits
+ The EPV of the Expenses

$$G\,\ddot{a}_{45} = 150{,}200 \times A_{45}$$

$$+ 0.40G + 0.10G\left(\frac{D_{46}}{D_{45}}\right) + 0.07G\left(\frac{N_{47} - N_{55}}{D_{45}}\right) + 0.04\,G\left(\frac{N_{55}}{D_{45}}\right)$$

$$+\ 0.0275G\,\ddot{a}_{45} + 300 + 125\left(\frac{N_{46}}{D_{45}}\right)$$

Then:

$$G = \frac{150{,}200 \times A_{45} + 300 + 125\left(\frac{N_{46}}{D_{45}}\right)}{\ddot{a}_{45} - 0.40 - 0.10\left(\frac{D_{46}}{D_{45}}\right) - 0.07\left(\frac{N_{47} - N_{55}}{D_{45}}\right) - 0.04\left(\frac{N_{55}}{D_{45}}\right) - 0.0275\,\ddot{a}_{45}}$$

2) x = 35

- Benefit: Term insurance (***n*** = 15) $250{,}000 \times A^{1}_{35:\overline{15}|}$

- Premium: Temporary life annuity due (***n*** = 15) $G\,\ddot{a}_{35:\overline{15}|}$

- Expenses

a. Agent commission: $= 0.45G + 0.06G\left(\frac{N_{36} - N_{50}}{D_{35}}\right)$

b. Premium taxes: $= 0.03G\,\ddot{a}_{35:\overline{15}|}$

c. Administrative expenses: $= 100 + 30\left(\frac{N_{36} - N_{50}}{D_{35}}\right)$

d. Cost of settlement: It will be added to the amount of insurance as it will be paid at the same time. (The benefit will be $250{,}150 \times A^{1}_{35:\overline{15}|}$)

- Now using the equation of value:

The EPV of the Premium = The EPV of the Benefits + The EPV of the Expenses

$$G\,\ddot{a}_{35:\overline{15}|} = 250{,}150 \times A^{1}_{35:\overline{15}|} + 0.45G + 0.06G\left(\frac{N_{36} - N_{50}}{D_{35}}\right) + 0.03G\,\ddot{a}_{35:\overline{15}|} + 100 + 30\left(\frac{N_{36} - N_{50}}{D_{35}}\right)$$

Then:

$$G = \frac{250{,}150 \times A^{\,1}_{35:\overline{15}|} + 100 + 30\left(\frac{N_{36} - N_{50}}{D_{35}}\right)}{\ddot{a}_{35:\overline{15}|} - 0.45 - 0.06\left(\frac{N_{36} - N_{50}}{D_{35}}\right) - 0.03\,\ddot{a}_{35:\overline{15}|}}$$

3) x = 25

- Benefit: Endowment insurance (***n***= 25) $120{,}000 \times A_{25:\overline{25}|}$

- Premium: Temporary life annuity due (***n*** = 25)

- Expenses

a. Agent commission:

$$= 0.30G + 0.14G\left(\frac{D_{26}}{D_{25}}\right) + 0.05G\left(\frac{N_{27} - N_{35}}{D_{25}}\right) + 0.02G\left(\frac{N_{35} - N_{50}}{D_{25}}\right)$$

b. Premium taxes: $0.025G\ \ddot{a}_{25:\overline{25}|}$

c. Administrative expenses: $180 + 60\left(\frac{N_{26} - N_{50}}{D_{25}}\right)$

d. Cost of settlement: It will be added to the amount of insurance as it will be paid at the same time. (The benefit will be $120{,}120 \times A_{25:\overline{25}|}$

- Now using the equation of value:

The EPV of the Premium = The EPV of the Benefits

+ The EPV of the Expenses

$$G\,\ddot{a}_{25:\overline{25}|} = 120{,}20 \times A_{25:\overline{25}|}$$

$$+0.30G + 0.14G\left(\frac{D_{26}}{D_{25}}\right) + 0.05G\left(\frac{N_{27}-N_{35}}{D_{25}}\right)$$

$$+0.02G\left(\frac{N_{35}-N_{50}}{D_{25}}\right) + 0.025G\,\ddot{a}_{25:\overline{25}|} + 180 + 60\left(\frac{N_{26}-N_{50}}{D_{25}}\right)$$

Then:

$$G = \frac{250{,}150 \times A_{25:\overline{25}|} + 180 + 60\left(\frac{N_{26}-N_{50}}{D_{25}}\right)}{\ddot{a}_{25:\overline{25}|} - 0.30 - 0.14\left(\frac{D_{26}}{D_{25}}\right) - 0.05\left(\frac{N_{27}-N_{35}}{D_{25}}\right) - 0.02\left(\frac{N_{35}-N_{50}}{D_{25}}\right) - 0.025\,\ddot{a}_{25:\overline{25}|}}$$

4) x = 30

- Benefit: Pure endowment insurance (***n*** = 30) $200{,}000 \times A_{30:\overline{30}|}^{\;\;1}$

- Premium: Temporary life annuity due (***n*** = 30) $G\,\ddot{a}_{30:\overline{30}|}$

- Expenses

a. Agent commission:

$$= 0.55G + 0.15G\left(\frac{D_{31}}{D_{30}}\right) + 0.04G\left(\frac{N_{32}-N_{40}}{D_{30}}\right) + 0.01G\left(\frac{N_{40}-N_{60}}{D_{30}}\right)$$ b.

Premium taxes: $= 0.015G\,\ddot{a}_{30:\overline{30}|}$

c. Administrative expenses: $= 250 + 150\left(\frac{N_{31}-N_{60}}{D_{30}}\right)$

d. Cost of settlement: It will be added to the amount of insurance as it will be paid at the same time. (The benefit will be $200{,}700 \times A_{30:\overline{30}|}^{\;\;1}$)

- Now using the equation of value:

The EPV of the Premium = The EPV of the Benefits
+ The EPV of the Expenses

$$G\,\ddot{a}_{30:\overline{30|}} = 200{,}700\times A^{\;\;1}_{30:\overline{30|}}$$

$$+0.55G+0.15G\left(\frac{D_{31}}{D_{30}}\right)+0.04G\left(\frac{N_{32}-N_{40}}{D_{30}}\right)+0.01G\left(\frac{N_{40}-N_{60}}{D_{30}}\right)$$

$$+0.015G\,\ddot{a}_{30:\overline{30|}}+250+150\left(\frac{N_{31}-N_{60}}{D_{30}}\right)$$

Then:

$$G=\frac{200{,}700\times A^{\;\;1}_{30:\overline{30|}}+250+150\left(\frac{N_{31}-N_{60}}{D_{30}}\right)}{\ddot{a}_{30:\overline{30|}}-0.55-0.15\left(\frac{D_{31}}{D_{30}}\right)-0.04\left(\frac{N_{32}-N_{40}}{D_{30}}\right)-0.01\left(\frac{N_{40}-N_{60}}{D_{30}}\right)-0.015\,\ddot{a}_{30:\overline{30|}}}$$

5) x = 35

- Benefit: Pure endowment insurance (***n*** = 30) $250{,}000\times A_{35:\overline{30|}}^{\;\;\;1}$

- Premium: Temporary life annuity due (***t*** = 15) $G\;\ddot{a}_{35:\overline{15|}}$

- Expenses

a. Agent commission:

$$=0.40G+0.12G\left(\frac{D_{36}}{D_{35}}\right)+0.06G\left(\frac{N_{37}-N_{45}}{D_{35}}\right)+0.02G\left(\frac{N_{45}-N_{50}}{D_{30}}\right)$$

b. Premium taxes: $=0.035G\;\ddot{a}_{35:\overline{15|}}$

c. Administrative expenses: $=105+65\left(\frac{N_{36}-N_{65}}{D_{35}}\right)$

d. Cost of settlement: It will be added to the amount of insurance as it will be paid at the same time. (The benefit will be $250{,}350\times A_{35:\overline{30|}}^{\;\;\;1}$)

- Now using the equation of value:

The EPV of the Premium = The EPV of the Benefits
+ The EPV of the Expenses

$$G\,\ddot{a}_{35:\overline{15}|} = 250{,}350 \times A^{\;1}_{35:\overline{30}|}$$

$$+0.40G + 0.12G\left(\frac{D_{36}}{D_{35}}\right) + 0.06G\left(\frac{N_{37} - N_{45}}{D_{35}}\right) + 0.02G\left(\frac{N_{45} - N_{50}}{D_{30}}\right)$$

$$+.035G\,\ddot{a}_{35:\overline{15}|} + 105 + 60\left(\frac{N_{36} - N_{65}}{D_{35}}\right)$$

Then:

$$G = \frac{250{,}350 \times A^{\;1}_{35:\overline{30}|} + 105 + 65\left(\frac{N_{36} - N_{65}}{D_{35}}\right)}{\ddot{a}_{35:\overline{15}|} - 0.40 - 0.12\left(\frac{D_{36}}{D_{35}}\right) - 0.06\left(\frac{N_{37} - N_{45}}{D_{35}}\right) - 0.02\left(\frac{N_{45} - N_{50}}{D_{35}}\right) - 0.035\,\ddot{a}_{35:\overline{15}|}}$$

6) x = 40

- Benefit: Term insurance (***n*** = 20) $160{,}000 \times A^{\;1}_{40:\overline{20}|}$

- Premium: Temporary life annuity due (***t*** = 10) $G\,\ddot{a}_{40:\overline{10}|}$

- Expenses

a. Agent commission:

$$= 0.40G + 0.15G\left(\frac{D_{41}}{D_{40}}\right) + 0.04G\left(\frac{N_{42} - N_{50}}{D_{40}}\right)$$

b. Premium taxes: $= 0.025 G\, \ddot{a}_{40:\overline{10|}}$

c. Administrative expenses: $= 200 + 95\left(\dfrac{N_{41} - N_{60}}{D_{40}}\right)$

d. Cost of settlement: It will be added to the amount of insurance as it will be paid at the same time. (The benefit will be $160{,}250 \times A^{1}_{40:\overline{20|}}$)

- Now using the equation of value:

The EPV of the Premium = The EPV of the Benefits
+ The EPV of the Expenses

$$G\, \ddot{a}_{40:\overline{10|}} = 160{,}250 \times A^{1}_{40:\overline{20|}}$$
$$+ 0.40G + 0.15G\left(\frac{D_{41}}{D_{40}}\right) + 0.04G\left(\frac{N_{42} - N_{50}}{D_{40}}\right) + 0.025G\, \ddot{a}_{40:\overline{10|}}$$
$$+ 200 + 95\left(\frac{N_{41} - N_{60}}{D_{40}}\right)$$

Then:

$$G = \frac{160{,}250 \times A^{1}_{40:\overline{20|}} + 200 + 95\left(\dfrac{N_{41} - N_{60}}{D_{40}}\right)}{\ddot{a}_{40:\overline{10|}} - 0.40 - 0.15\left(\dfrac{D_{41}}{D_{40}}\right) - 0.04\left(\dfrac{N_{42} - N_{50}}{D_{40}}\right) - 0.025\, \ddot{a}_{40:\overline{10|}}}$$

7) x = 40

- Benefit: Whole Life Insurance $140{,}000 \times A_{40}$

- Premium: Temporary life annuity due (***n*** = 20) $G\, \ddot{a}_{40:\overline{20|}}$

- Expenses

a. Agent commission:

$$= 0.35G + 0.09G\left(\frac{D_{41}}{D_{40}}\right) + 0.03G\left(\frac{N_{42} - N_{60}}{D_{40}}\right)$$

b. Premium taxes: $= 0.02\,G\,\ddot{a}_{40:\overline{20}|}$

c. Administrative expenses: $= 100 + 40\left(\frac{N_{41}}{D_{40}}\right)$

d. Cost of settlement: It will be added to the amount of insurance as it will be paid at the same time. (The benefit will be $140{,}200 \times A_{40}$)

- Now using the equation of value:

The EPV of the Premium = The EPV of the Benefits + The EPV of the Expenses

$$G\,\ddot{a}_{40:\overline{20}|} = 140{,}200 \times A_{40}$$
$$+ 0.35G + 0.09G\left(\frac{D_{41}}{D_{40}}\right) + 0.03G\left(\frac{N_{42} - N_{60}}{D_{40}}\right) + 0.02\,G\,\ddot{a}_{40:\overline{20}|}$$
$$+ 100 + 40\left(\frac{N_{41}}{D_{40}}\right)$$

Then:

$$G = \frac{140{,}200 \times A_{40} + 100 + 40\left(\frac{N_{41}}{D_{40}}\right)}{\ddot{a}_{40:\overline{20}|} - 0.35 - 0.09\left(\frac{D_{41}}{D_{40}}\right) - 0.03\left(\frac{N_{42} - N_{60}}{D_{40}}\right) - 0.02\,\ddot{a}_{40:\overline{20}|}}$$

8) x = 35

- Benefit: Endowment insurance (***n*** = 30) $195{,}000 \times A_{35:\overline{30}|}$

- Premium: Temporary life annuity due (***t*** = 15) $G\ \ddot{a}_{35:\overline{15}|}$

- Expenses

a. Agent commission: $= 0.45G + 0.07G\left(\dfrac{N_{36} - N_{50}}{D_{35}}\right)$

b. Premium taxes: $0.015G\ \ddot{a}_{35:\overline{15}|}$

c. Administrative expenses: $120 + 50\left(\dfrac{N_{36} - N_{65}}{D_{35}}\right)$

d. Cost of settlement: It will be added to the amount of insurance as it will be paid at the same time. (The benefit will be $195{,}450 \times A_{35:\overline{30}|}$)

- Now using the equation of value:

The EPV of the Premium = The EPV of the Benefits + The EPV of the Expenses

$$G\ \ddot{a}_{35:\overline{15}|} = 195{,}450 \times A_{35:\overline{30}|} + 0.45G + 0.07G\left(\frac{N_{36} - N_{50}}{D_{35}}\right)$$
$$+ 0.015G\,\ddot{a}_{35:\overline{15}|} + 120 + 50\left(\frac{N_{36} - N_{65}}{D_{35}}\right)$$

Then:

$$G = \frac{195{,}450 \times A_{35:\overline{30}|} + 120 + 50\left(\dfrac{N_{36} - N_{65}}{D_{35}}\right)}{\ddot{a}_{35:\overline{15}|} - 0.45 - 0.07\left(\dfrac{N_{36} - N_{50}}{D_{35}}\right) - 0.015\,\ddot{a}_{35:\overline{15}|}}$$

9) **Premium:** $G\ddot{a}_{40:\overline{25}|}$

Benefit: $35{,}000\,A^{1}_{40:\overline{30}|}+50{,}000\,A_{40:\overset{\;1}{\overline{30}|}}$

Commission: $0.55G+0.2G\,D_{41}/D_{40}+0.05G\,\dfrac{N_{42}-N_{65}}{D_{40}}$

Tax: $0.03G\ddot{a}_{40:\overline{25}|}$

Administrative: $160+35a_{40:\overline{29}|}$

$$G\ddot{a}_{40:\overline{25}|}=35{,}008\,A^{1}_{40:\overline{30}|}+50{,}008\,A_{40:\overset{\;1}{\overline{30}|}}+0.55G+0.2G\,D_{41}/D_{40}$$
$$+0.05G\,\frac{N_{42}-N_{65}}{D_{40}}+0.03G\ddot{a}_{40:\overline{25}|}+160+35a_{40:\overline{29}|}$$

Then:

$$G=\frac{35{,}008\,A^{1}_{40:\overline{30}|}+50{,}008\,A_{40:\overset{\;1}{\overline{30}|}}+160+35a_{40:\overline{29}|}}{\ddot{a}_{40:\overline{25}|}-0.55-0.2D_{41}/D_{40}-0.05\dfrac{N_{42}-N_{65}}{D_{40}}-0.03\ddot{a}_{40:\overline{25}|}}$$

10) **Premium: Ten-payment:** $G\ddot{a}_{35:\overline{10}|}$

Benefit: $\dfrac{25{,}000(M_{35}-M_{50})+45{,}000(M_{50}-M_{60})+60{,}000M_{60}}{D_{35}}$

Commission: $0.60G+0.25G\,D_{36}/D_{35}+0.10G\,\dfrac{N_{37}-N_{45}}{D_{35}}$

Premium taxes: $0.025G\ddot{a}_{35:\overline{10}|}$

Administrative: 200 + 40 a_{35}

S_3 = 60,010 **S_2 = 45,010** **Cost of settlement: S_1 = 25,010**

EPV (Gross Premium) = EPV (Benefit) + EPV (Expenses)

$$G\,\ddot{a}_{35:\overline{10}|} = \frac{25{,}010(M_{35} - M_{50}) + 45{,}010(M_{50} - M_{60}) + 60{,}010M_{60}}{D_{35}}$$

$$+0.6G + .025\,G\,D_{36}/D_{35} + 0.10G\,\frac{N_{37} - N_{45}}{D_{35}}$$

$$+0.025\,G\,\ddot{a}_{35:\overline{10}|} + 200 + 40\,a_{35}$$

Then:

$$G = \frac{\dfrac{[25{,}010(M_{35} - M_{50}) + 45{,}010(M_{50} - M_{60}) + 60{,}010M_{60}]}{D_{35}} + 200 + 40\,a_{35}}{\ddot{a}_{35:\overline{10}|} - 0.6 - 0.25\,D_{36}/D_{35} - 0.10\,\dfrac{(N_{37} - N_{45})}{D_{35}} - 0.025\,\ddot{a}_{35:\overline{10}|}}$$

التاريخ: 2015/6/2	المادة: تأمينات حياة E	جامعة القاهرة
الوقت: 12 – 2	الفرقة الثالثة – مجموعة (ج، د، متميزة)	كلية التجارة

Answer the following questions:

Question One: (4 Marks)

a) A woman has L.E. 20000 if she was 33 years old and used the amount to buy a life annuity of 9 payments. What would be the size of the payment if the 1st payment is to be made at once?

Since: $N_{34} = 731118.1$ $N_{43} = 433018.8$

$N_{33} = 772562.6$ $D_{42} = 27271.4$

b) The probability that a person aged 40 will live the next 5 years is 0.8, and the probability that a person aged 45 will die within the next year is 0.05. If there are 1250 persons alive at age 40, Find the probability that a person aged 40 will die within one year after his 45th birthday.

Question Two: (6 Marks)

a) Derive the following using the mutual fund method:

1- $\ddot{a}_x = 1 + a_x$

2- The net annual premium payable for 6 years for an insurance policy issued on the life of a person aged 42 providing the following benefits:

- L.E. 30000, in case of death before age 54,
- L.E. 4000, annually thereafter, last payment at age 62, and
- L.E. 45000, if he survives to the age 63.

b) Given that:

$M_{x+1} = 8758.208$, $C_x = 101.253$, $D_x = 34074.85$, **and the interest rate is 5%.** Find **the value of** a_x **.**

Question Three: (7 Marks)

a) Show that:

$$A_{x:\overline{n}|} = v - d \cdot a_{x:\overline{n-1}|}$$

$$a_x \div (1-d)\, p_x = 1 + a_{x+1}$$

$$v \cdot N_x - N_{x+1} = D_x - d \cdot N_x$$

$$d_x + \boldsymbol{I}_{x+1} \cdot A_{x+1} = \boldsymbol{I}_x (1+i) A_x$$

b) State in words the meaning of the following formula:

$$\frac{2000\ (N_{36} - N_{40}) + 40000\ (M_{40} - M_{50}) + 4800\ N_{62}}{D_{35}}$$

Q

uestion Four: (3 Marks)

A special 20-payments life insurance policy issued for a person aged 35 provides a death benefit of L.E. 10000 for the first 15 years, and a death benefit of L.E 30000 thereafter.

The premiums payable for the first 6 years are exactly one-third of the ultimate premium.

It is estimated that the following expenses will be incurred:

a. Agent commission; 60% of the first year gross premium, 10% of the second year gross premium, 6% of the next 9 gross premiums, and 3% of all gross premiums collected thereafter.

b. Premium Taxes; 4% of each gross premium.

c. Administrative expenses; L.E. 7 in the first year, and L.E. 2 in each succeeding year.

d. Cost of settlement; L.E. 4 at the time of the death claim or maturity.

Required: Find the gross annual premium (in symbols only) of this policy.

التاريخ: 2016/5/29	المادة: تأمينات حياةE	جامعة القاهرة
الوقت: 1 – 3	الفرقة الثالثة – مجموعة (ب)	كلية التجارة

Answer the following questions:

Question One:

a) Show that:

$$v + d = \frac{-A_x + A_{x+k}}{d + d \cdot a_x} + \frac{1 + a_{x+k}}{\ddot{a}_x}$$

$$C_x + M_{x+1} = -\left(N_x - D_x\right) + v \cdot N_x$$

b) Derive in terms of commutation symbols the net single premium payable for an insurance policy issued on the life of a man aged 35, providing the following benefits:

- **L.E. 50000 if he dies before reaching age 50, and**
- **L.E. 5000 a year thereafter, last payment at age 60.**

Question Two:

a) A man aged 29 has a 15 years Due life annuity policy of L.E. 2000. He wishes to replace this by an annuity, first payment to be made at age 40. What will be the size of the new payment? (Answer just in symbols only).

b) Prove the following identity:

$$\left(1-p_x\right)\cdot\left(1+i\right)^{-1}=\frac{A_x-A_{x+1}\cdot v}{1-A_{x+1}}$$

c) If $\ddot{a}_x$ = 14.26 and A_x = 0.19283, Find the interest rate (i)?

<u>Question Three:</u>

a) The probability that a person aged 25 and another aged 45 will both survive 20 years is 0.8, and the probability of a person aged 25 will survive 10 years is 0.89.
Find the probability that a person aged 35 will live at least 30 years.

b) State in words the meaning of the following formula:

$$\frac{35000\left(M_{20}-M_{30}\right)+1500\left(N_{30}-N_{35}\right)+40000D_{35}+2000N_{41}}{N_{20}-N_{25}}$$

c) <u>Complete the following statements:</u>

1. ……… means that the loss must be definite as to cause, time, place, and amount.
2. ……… is an undesired reduction of economic value arising from chance.
3. Term insurance policy should not be used when the need for life insurance is ………
4. The level premium method of paying for whole life insurance produces a savings value called the ………

Question Four:

A life insurance company estimates that the following expenses will be needed for a L.E. 200000 whole life insurance policy issued to a person aged 25 years.

a. Agent commission: 55% of the first premium, 10% of the second premium, 5% of the third through the seventh premium, followed by a "service fee" of 3% on all premiums collected thereafter.

b. Premium taxes: 4% of each gross premium.

c. Administrative expenses: L.E. 150 in the first year, and L.E. 80 in each succeeding year.

d. Cost of settlement: L.E. 400 at the time of the death claim or maturity.

Required: Compute the gross annual premium of this policy.

(Answer just in symbols only).