KINGDOM OF SAUDI ARABIA
MINSTRY OF HIGHER EDUCATION
JAZAN UNIVERSITY
Preparatory Year Deanship
Department of *Science-mathematics school*

المملكـة العربيـة السـعـودية
وزاره التعـليم العــالـــي
جـامـعـــة جــــــازان
عمادة السنة التحضيرية
قسم العلوم - شعبة الرياضيات

Questions Bank for Faculty of Engineering Students
119 Math. Level I

Chapter 1

LIMITS AND CONTINUITY

1.1 Limits (An Intuitive Approach)

1.2 Computing Limits

1.3 Limits at Infinity; End Behavior of a Function

1.4 Limits (Discussed More Rigorously)

1.5 Continuity

1.6 Continuity of Trigonometric, Exponential, and Inverse Functions

(1) Determine the following limits:

(1) $\lim_{x\to 0}\frac{2-\sqrt{x+4}}{x^2-2x}$

(2) $\lim_{x\to -2}\frac{x^2-4}{x^2-2x-8}$

(3) $\lim_{x\to 0}\frac{\sqrt{x+25}-5}{x^3-8x}$

(4) $\lim_{x\to -3}\frac{x^2+2x-3}{2x+1}$

(5) $\lim_{x\to 0}\frac{\sin 2x}{x}$

(6) $\lim_{x\to 0}\frac{\sin(5x)}{\sin(10x)}$

(7) $\lim_{x\to 0}\frac{\sin(5x)}{\sin(6x)}$

(8) $\lim_{x\to 0}\frac{\sin(x)}{5x}$

(9) $\lim_{x\to 0}\frac{\sin(8x)}{\sin(9x)}$

(10) $\lim_{x\to 2}\frac{x^2-4x+4}{x^2-x-2}$

(11) $\lim_{x\to 0}\frac{\sin(36x)}{\sin(6x)}$

(12) $\lim_{x\to \infty}\frac{8x^{12}+3x-4}{6x-3x^{13}}$

(13) $\lim_{x\to 1}\frac{x^2-2x+1}{2x+1}$

(14) $\lim_{x\to 2}\frac{x^2-4}{x^2+x-6}$

(19) $\lim_{x\to -2}\frac{x^2-4}{x^2-x-6}$

(20) $\lim_{x\to 5}\frac{x^2-2x+1}{2x+1}$

(21) $\lim_{x\to 2}\frac{x^4-16}{x^3-8}$

(22) $\lim_{x\to 0}\frac{\sqrt{x+25}-5}{x^3-8x}$

(23) $\lim_{x\to 2}\frac{x^2-4}{x^2+x-6}$

(24) $\lim_{x\to 0}\frac{\sin(5x)}{3x}$

(25) $\lim_{x\to \infty}\frac{3x^2-2}{3e^{2x}}$

(26) $\lim_{x\to 2}\frac{x^5-32}{x^2-4}$

(27) $\lim_{x\to 3}\frac{x^4-81}{x^2-9}$

(28) $\lim_{x\to 0}\frac{4x^3+2x}{\sqrt{x+100}-10}$

(29) $\lim_{x\to 5}\frac{x^2-25}{x^2-3x-10}$

(30) $\lim_{x\to 0}\frac{\sqrt{x+49}-7}{x^2+5x}$

(31) $\lim_{x\to 0}\frac{\sqrt{x+25}-5}{2x}$

36) $\lim_{x\to 0}\frac{2x-12}{x^4-5x+3}$

37) $\lim_{x\to 2}\frac{x^2+4x-12}{x-2}$

38) $\lim_{x\to 0}\frac{\sqrt{x+25}-5}{x}$

39) $\lim_{x\to -\infty}\frac{6x^3+x^2-1}{3x^4+4x^2+8}$

40) $\lim_{x\to 0}\frac{x-9}{x^2-4x+3}$

41) $\lim_{x\to -3}\frac{x^2-x-12}{x+3}$

42) $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$

43) $\lim_{x\to \infty}\frac{2x^6+x^2+5}{5x^5+x^2-3}$

44) $\lim_{x\to 0}\frac{4x-4}{x^3-3x+8}$

45) $\lim_{x\to 1}\frac{x^2+x-2}{x-1}$

46) $\lim_{x\to 0}\frac{\sqrt{x+16}-4}{x}$

47) $\lim_{x\to \infty}\frac{-3x^5+6x^2+2}{4x^4+x^2-1}$

(15) $\lim_{x\to -3} \frac{3x^2-2x+1}{2x+1}$

(16) $\lim_{x\to 0} \frac{\sin(5x)}{3x}$

(17) $\lim_{x\to 0} \frac{2x^3+6x}{\sqrt{x+81}-9}$

(18) $\lim_{x\to 3} \frac{x^2-9}{x^2+x-12}$

32) $\lim_{x\to 0} \frac{5x-6}{x^3-4x+3}$

33) $\lim_{x\to 4} \frac{x^2-x-12}{x-4}$

34) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$

35) $\lim_{x\to \infty} \frac{2x^3+5x^2+4}{4x^5+3x^2+3}$

(48) $\lim_{x\to 2} \frac{x^2+x+6}{x^2-4x-4}$

(49) $\lim_{x\to 3} \frac{x^2-4x+3}{x^2+x-12}$

(50) $\lim_{x\to \infty} \frac{11x^8+5x^5-4x}{7x^4+9x^9}$

(51) $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{2x}$

(52) $\lim_{x\to -\infty} \frac{x^5+3x^3-4x}{4x^4-9x^5}$

(Q) Find a value of the constant k that will make the function continuous at $x=4$

$$f(x)=\begin{cases} \frac{x^2-16}{x-4}, & x\neq 4 \\ k, & x=4 \end{cases}$$

(Q) Find a value of the constant k that will make the function continuous at $x=3$

$$f(x)=\begin{cases} \frac{x^2-9}{x-3}, & x\neq 3 \\ k, & x=3 \end{cases}$$

(Q) Find a value of the constant k that will make the function continuous at $x = 1$

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ k, & x = 1 \end{cases}$$

(Q) Find a value of the constant k that will make the function continuous at $x = 5$

$$f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5}, & x \neq 5 \\ k, & x = 5 \end{cases}$$

Q) Find the value of the constant K , if possible, that will make the function continuous everywhere.

$$f(x) = \begin{cases} 9 - x^2 & , \ \mathrm{x} \leq 2 \\ 2x + k & , \ \mathrm{x} > 2 \end{cases}$$

Q) Determine where $f(x) = \sqrt{x^2 - 4}$ is continuous

(Q) For what values of x is there a discontinuity in the graph of

$$y = \frac{x^2 - 25}{x^2 + 3x - 10}?$$

(Q) For what values of x is there a discontinuity in the graph of

$$y = \frac{x^2 - 25}{x^2 - 3x - 10}?$$

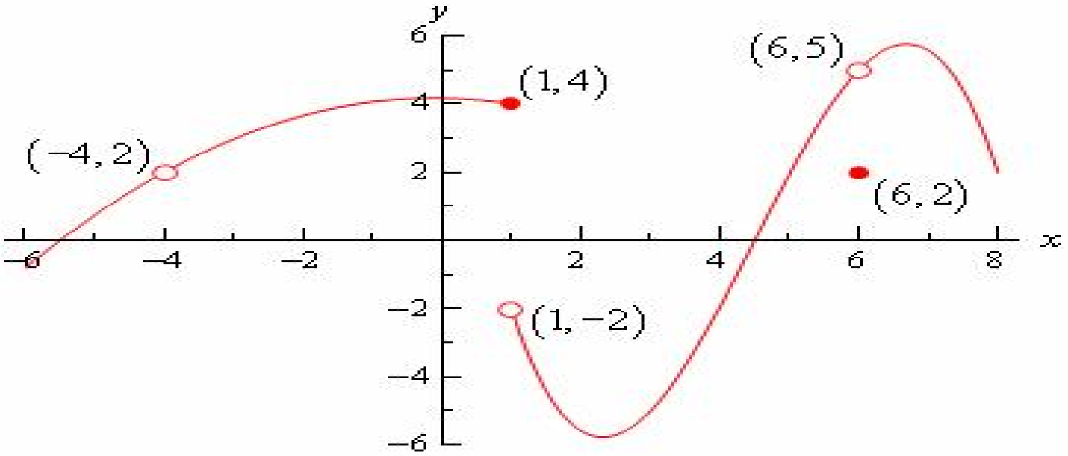
(Q) For what values of x is there a discontinuity in the graph of

$$y = \frac{x^2 - 25}{x^2 + 9x - 10}?$$

(Q) For what values of x is there a discontinuity in the graph of

$$y = \frac{x^2 - 25}{x^2 + 7x + 10}?$$

Q1) Given the following graph



compute each of the following

(a) $\lim_{x \to -4^+} f(x) =$..............................

(b) $\lim_{x \to -4^-} f(x) =$..............................

(c) $\lim_{x \to -4} f(x) =$..............................

(d) $\lim_{x \to 1^+} f(x) =$..............................

Chapter 2 and 3

THE DERIVATIVE

TOPICS IN DIFFERENTIATION

(2) Find the first derivative $y' = \frac{dy}{dx}$ for the following functions:

$(a)\ y = \tan(5x^3) + L\,n(x^3 + 3x)$

$(b)\ y = (3x^3 + 4)(x^2 + 1)^5$

$(c)\ y = e^{\sin(2x)+1}$

$(d)\ y = \sqrt[3]{(x^2 + 1)^5}$

$(e)\ y = \tan^{-1}(x)$

$(f)\ y = \frac{1}{x^7} + \sec(x^3)$

$(g)\ y = (x^3 + 3)^4.\cos ec(x)$

$(h)\ y = (x^2 + 1)^6.(\cos(x) - 5)^2$

$(i)\ y = \cot x$

$(j)\ y = (x^2 + 1).\tan^{-1}(x)$

$(k)\ y = \sin(\sqrt{x})$

$(l)\ y = -\frac{1}{x^5} + \sin(\sqrt{5x^3})$

$(m)\ y = \sqrt{(3x^3 + x)^6}.\sin(x)$

$(n)\ y = (\sin x + 1)^4.(2x - 5)^2$

$(o)\ y = \frac{\cos(x)}{x + 3}$

$p)\ y = (x^2 + 1).\tan^{-1}(x)$

$(q)\ y = \sec(4x)$

$(r)\ y = 5e^{3x^2} + \tan(5x^3)$

$(s)\ y = \ln(3x^3 + x).\cos(5x)$

$(t)\ y = (x^2 + 1)^6.(2x - 5)^2$

$(a')\ y = \frac{1}{x^3} + \tan(2x)$

$(b')\ y = \sin(3x).\cos(5x)$

$(c')\ y = (4x^5 + 1)^6.(2x^3 - 5)^2$

$(d')\ y = \frac{x + 1}{x + 3}$

$(e')\ y = \cos^5(x)$

$(f')\ y = \frac{1}{x} + \tan(5x^3)$

$(g')\ y = (3x^3 + x)^6.\cos(5x)$

$(h')\ y = (x^2 + 1)^6.(2x - 5)^2$

$(i')\ y = \frac{\sin(3x)}{x^3 + 3}$

$(j')\ y = (x^2 + 1).\tan^{-1}(x)$

$(k')\ y = \sin^4(x)$

$(l')\ y = \frac{x^5}{x^3 - 2}$

$(m')\ y = \sin^3(x^2) + \ln(x^2)$

$(n')\ y = \frac{\sin(3x)}{x^3 + 3}$

$(o')\ y = (x^2 + 1).\tan^{-1}(x)$

$(p')\ y = x^x$

(q') if $y = e^{ax}$ find $y^{(n)}$?

$(r')\ y = (\cosh x)^{\sinh x}$

$(s')\ y = 5^x$

$(t')\ y = \sin^{-1}(x)$

Find the limits by using L'Hopital rule :

Q1) (1) $\lim_{x\to\infty}\frac{e^{3x}}{x^2}$ (2) $\lim_{x\to 0} x^x$ (3) $\lim_{x\to 0}\frac{\cos x+2x-1}{3x}$.

Q2) Find The limits by using L'Hopital rule :

(a) $\lim_{x\to 0}\frac{\sin 2x}{x}$ (b) $\lim_{x\to \pi/2}\frac{1-\sin x}{\cos x}$ (c) $\lim_{x\to 0}\frac{e^x-1}{x^3}$

(d) $\lim_{x\to 0^-}\frac{\tan x}{x^2}$ (e) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ (f) $\lim_{x\to +\infty}\frac{x^{-4/3}}{\sin(1/x)}$

Q3) Find The limits by using L'Hopital rule :

(a) $\lim_{x\to +\infty}\frac{x}{e^x}$ (b) $\lim_{x\to 0^+}\frac{\ln x}{\csc x}$

Q4)

Evaluate

(a) $\lim_{x\to 0^+} x\ln x$ (b) $\lim_{x\to \pi/4}(1-\tan x)\sec 2x$

Exercise

Find the limits..

1. $\lim_{x \to 0} \frac{e^x - 1}{\sin x}$

2. $\lim_{\theta \to 0} \frac{\tan \theta}{\theta}$

3. $\lim_{x \to 0} \frac{\sin 2x}{\sin 5x}$

4. $\lim_{x \to +\infty} \frac{x^{100}}{e^x}$

5. $\lim_{x \to +\infty} \frac{\ln x}{x}$

6. $\lim_{x \to +\infty} xe^{-x}$

7. $\lim_{x \to +\infty} x \sin \frac{\pi}{x}$

8. $\lim_{x \to 0^+} \tan x \ln x$

9. $\lim_{x \to 0} (1 + 2x)^{-3/x}$

10. $\lim_{x \to 0} (e^x + x)^{1/x}$

Q) Use the definition of derivative to find $f'(x)$ for $f(x) = 8x + 5$.

Q) Find $f''(1)$ for $f(x) = x^{\frac{2}{7}} + 2x^{\frac{5}{4}} + 8$.

Chapter 5

INTEGRATION

5.2 The Indefinite Integral
5.3 Integration by Substitution
5.4 The Definition of Area as a Limit
5.5 The Definite Integral
5.9 Evaluating Definite Integrals by Substitution

Chapter 7

PRINCIPLES OF INTEGRAL EVALUATION

7. 2 Integration by Parts
7. 3 Integrating Trigonometric Functions
7. 4 Trigonometric Substitutions
7. 5 Integrating Rational Functions by Partial Fractions

Evaluate the following integrals:

(1) $\int_{-2}^{3} (6x^2 - 5)dx$

(2) $\int (e^{2x} + \sinh 4x)dx$

(3) $\int_{-1}^{1} (x^4 + 3x^2 + 1)dx$

(4) $\int [(e^{\sin x} + 7).\cos x]dx$

(5) $\int \frac{x^2 + 2x}{x^3 + 3x^2 + 5}dx$

(6) $\int [\cos(2x).e^{\sin(2x)} - \frac{1}{x^5}]dx$

(7) $\int [\cosh(2x) + \sin(3x)]dx$

(8) $\int x.e^x dx$ (using integration by parts)

(9) $\int \frac{1}{\sqrt{1 - x^2}} dx$

(10) $\int (4x + 1)^4 dx$

(11) $\int \sec x\, dx$

(12) $\int_{-1}^{5} (3x^3 + 2x + 5)dx$

(13) $\int_{0}^{\frac{®}{2}} \cos x\, dx$

(14) $\int_{-\frac{®}{2}}^{\frac{®}{2}} x^2.\sin(x)\, dx$

(15) $\int x \sin x\, dx$ (using integration by parts)

(16) $\int_{-2}^{2} (3x^2 + 2x - 1)dx$

(17) $\int x \ln x dx$ (using integration by parts)

Final Exams Sheets

Sheet (1)

Q1 Determine whether each of the statements are **True or False** ,write the answers in the table

(1) $\int_1^2 3t^2\,dt = 7.$

(2) $\frac{d}{dx}[\ln(f(x))] = \frac{1}{f(x)}.$

(3) $\int e^x\,dx = e^{\frac{x^2}{2}} + c.$

(4) All continuous functions are polynomials.

(5) The function $f(x) = \sqrt{x-1}$ is continuous at $x = 0.$

(6) If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$ then $\lim_{x \to a} \frac{f(x)}{g(x)} = 0.$

(7) The derivative of a polynomial is a polynomial.

(8) $\frac{d}{dx}[®^3] = 3®^2.$

(9) $\int \frac{x}{1+x^2}\,dx = \ln(1+x^2) + c.$

(10) If $f(x) = 5x^2 - 7$ then $f'(2) = 13.$

1	2	3	4	5	6	7	8	9	10

:**Q2) Find the following limits**

$(a)\ \lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3}}{x}$

$(b)\ \lim_{x\to 1} \dfrac{2(x^2-1)}{x^2+7x-8}$

$(c)\ \lim_{x\to 0} \dfrac{1-\cos^2(x)}{2x^2}$

$(d)\ \lim_{x\to \infty} \dfrac{x^5+4x^4-3}{5x^3+3x+3}$

Q3) find $\dfrac{dy}{dx}$ for

$(a)\ y = \tan(\sin(x))$

$(b)\ y = \cos(\ln x)$

$(c) 3y^2+6x^2 = xy^2$

$(d)\ y = (e^{2x}).\cos^{-1}(x^3)$

Q4) Evaluate the following integrals:

$(a) \int e^{3\sqrt{x}}.\sqrt{x}\,dx$

$(b) \int 3\ln(x)dx$

$(c) \int [e^{3x}.\sin(e^{3x})]dx$

$(d) \int \dfrac{t^2}{t^2+1}dt$

$(e) \int \sec(x).\cos^2(x)dx$

$(f) \int_{-1}^{1} (3x^2+2x-1)dx$

$(g) \int_{0}^{\frac{\text{®}}{4}} (2\sin(x)-\cos(x))dx$

Sheet (2)

Q1) Choose the correct answer:

1) If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$ then $\lim_{x \to a} \frac{f(x)}{g(x)}$

a) must exist

b) not enough information

c) does not exist

d) approach to 1

2) Which of the following is also true?

a) If f(x) is a polynomial, then f(x) is continuous.

b) If f(x) is not a polynomial, then it is not continuous.

c) If f(x) is continuous, then it is a polynomial.

d) If f(x) is not continuous, then it is not a polynomial.

3) If $f'(a)$ exists then $\lim_{x \to a} f(x)$

a) equal a

b) equal $f(a)$

c) equal $f'(a)$

d) not exists

4) $\frac{d}{dx}(e^7) =$

a) $7e^6$

b) $6e^7$

c) 0

d) Not exists

5) We know $f(1) = 3$ and $f'(1) = 2$ then $\frac{d}{dx}(x^2 . f(x))$ at x=1 equal

a) 6

b) 2

c) 8

d) -6

Q2) Find the following limits:

$(a)\ \lim_{x \to 0} \frac{5x}{\sqrt{5+x} - \sqrt{5}}$

$(b)\ \lim_{x\to1} 9.\left(\frac{x^2-1}{x^2+8x-9}\right)$

$(c)\ \lim_{x\to0} \frac{12\sin(5x)}{10x}$

$(d)\ \lim_{x\to\infty} \frac{1}{x^3+4x+3}$

Q3) find $\frac{dy}{dx}$ for

$(a)\ y = \tan(3e^{2x+1})$

$(b)\ y = \ln(\ln x)$

.$(c) 3y^2 - 2xy + 6x^2 = x^2y^2$

$(d)\ y = (2^{x^2}).\tan^{-1}(\sin x)$

Q4) Evaluate the following integrals:

$(a) \int e^x.\cos(x)dx$

$(b) \int [\tan^{-1}(x)]dx$

$(c) \int [e^{3x}.\sin(e^{3x})]dx$

$(d)\int \frac{2x^3 + x}{x^4 + x^2 + 5}dx$

$(e)\int \sec(x).\cos(x)dx$

.

Q4) Evaluate the following definite integrals:

$(a)\int_{-1}^{4} (2x^3 + 6x + 5)dx$

.

$(b)\int_{0}^{\frac{®}{2}} (4\sin(x) + 3\cos(x))dx$

Sheet (3)

Question 1 : **(A) Find the following limits:**

(1) $\lim_{x\to 2}(x^3-4x+2)^3$

(2) $\lim_{x\to 1}\frac{3x-3}{x^2+x-2}$

(3) $\lim_{x\to +\infty}\sqrt[3]{\frac{1+2x-8x^4}{2-x^4}}$

(4) $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$

(5) $\lim_{x\to 0}\frac{e^{2x}+e^{-3x}-2}{\sin 2x+\sin 3x}$

(B) (1) Explain why $\lim_{x\to 0}\frac{|2x|}{x}$ does not exist.

(2) Find the values of x, if any at which f is not continuous :

$$f(x)=\frac{x+1}{x^2-9}.$$

Question 2 :

(1) Find $f'(x)$ if $f(x)=2x^4+x^{-2}+4$

(2) Find $\frac{dy}{dx}$ if $y=\frac{2x+1}{3x-2}$

(3) Find $\frac{dy}{dx}$ if $y=\ln(\sqrt{x^2+\sec x})+2^{3x}$

(4) Find $\frac{d^2y}{dx^2}$ if $y=\cos^{10}(3x)$

Question 3 :

(1) Find y' if $3y^4 + x.\sin 5y = 3x^3$

(2) Find y' if $y = \dfrac{x^2\sqrt{e^{\cos x}}}{(2x^2+1)^3}$

(3) Find $\dfrac{dy}{dx}$ if $y = (2x^4-3)^3 e^{x^2+\sin^{-1}x}$

Question 4 **Evaluate the following integrals:**

(1) $\int (3x^2 + 2x^{-3} + 2)dx$

(2) $\int \dfrac{6x^2+4x}{x^3+x^2+1}dx$

(3) $\int (\cos 3x + \sec^2 x)dx$

(4) $\int \dfrac{dx}{1+5x^2}$ (using integration by substitution)

(5) $\int xe^x.dx$ (using integration by parts)

Question 5 **Evaluate the following definite integrals:**

(1) $\int_{-1}^{2} (2x+1)dx$

(2) $\int_{-\frac{®}{2}}^{\frac{®}{2}} \cos x \, dx$

Sheet (4)

Question 1 : **(A) Find the following limits:**

(1) $\lim_{x\to+\infty} \sqrt[3]{\dfrac{2+3x-27x^4}{4-x^4}}$

(2) $\lim_{x\to1} \dfrac{4x-4}{x^2+3x-4}$

(3) $\lim_{x\to0} \dfrac{e^{3x}+e^{-2x}-2}{\sin x+\sin 5x}$

(4) $\lim_{x\to0} \dfrac{\sin 3x}{\sin 6x}$

(B) (1) Explain why $\lim_{x\to0} \dfrac{|x|}{4x}$ does not exist.

(2) Find the values of x, if any at which f is not continuous :

$$f(x) = \frac{x+1}{x^2-4}.$$

Question 2 :

(1) Find $f'(x)$ if $f(x) = 3x^5 + x^{-3} + 2$

(2) Find $\dfrac{dy}{dx}$ if $y = \ln(\sqrt{x^2+\cos x}) + 3^{4x}$

(3) Find $\dfrac{d^2y}{dx^2}$ if $y = \sec(4x)$

Question 3 :

(1) Find y' if $2y^3 + x^2.\sin y = 2x^2$

(2) Find $\frac{dy}{dx}$ if $y=(x^3-1)^5 e^{x^3+\cos x}$

Question 4 **Evaluate the following integrals:**

(1) $\int (4x^3+3x^{-4}+1)dx$

(2) $\int \frac{8x^3+8x}{x^4+2x^2+2}dx$

(3) $\int (\cos 3x+\csc^2 x)dx$

(4) $\int \frac{dx}{1+3x^2}$ (using integration by substitution)

(5) $\int xe^x.dx$ (using integration by parts)

Question 5 **Evaluate the following definite integrals:**

(1) $\int_{-1}^{2} (3x^2-1)dx$