NAME:
ID:
Group Number:

244
Final Exam, May 2013

I) Choose the correct answer:

(a) If If $u = (0, 1, 3)$, $v = (1, -1, 2)$ and $w = (1, 1, -3)$, then $\frac{u \cdot v}{w \cdot w} w$ equals

$\frac{5}{11}$(1, 1, -3)	-18	$\frac{6}{11}$(1, 1, -3)	NONE

(b) The coordinate vector of $(2, 0)$ relative to the basis $B = \{(1, -1), (0, 2)\}$ is

(2, -1)	(-2, -1)	(2, 1)	NONE

(c) If $\lambda = 2$ is an eigenvalue for a matrix A, then an eigenvalue for A^3 is

6	8	9	NONE

(d) If $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ is a reflection about the y-axis and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ is a rotation operator through $\theta = \frac{\pi}{2}$, then the matrix $[T_1 \circ T_2]$ is

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	NONE

(e) The inverse of the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (4x + y, -x)$ is

$T^{-1}(x, y)=(-x,4x+y)$	$T^{-1}(x, y)=(-y,4x+y)$	$T^{-1}(x, y)=(-y,x+4y)$	NONE

II) Decide if the following statements are true (T) or false (F). Justify your answer.

(a) If A is an $n \times n$ matrix and $\det A = 2$, then $\det(A^T A) = 4$.

T		F

(b) If A and B are $n \times n$ invertible matrices of the same size and $AB = BA$, then A and B^{-1} commute.

T		F

(c) The set $S = \{A \in M_{22} : \det A = 0\}$ is a subspace of M_{22}.

T		F

(d) If V is a vector space of dimension n, then any set of $n-1$ vectors of V is linearly dependent.

T

F

(e) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x, y^2)$ is a linear transformation.

T

F

(f) The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (4x - y, x)$ is one-to-one.

T

F

(g) If $\lambda = 0$ is an eigenvalue of a matrix A, then A is not invertible.

T

F

(h) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $a + c = b + d = \lambda$, then λ is an eigenvalue of A.

T

F

(i) If A and B are square matrices with the same dimension, then rank $(BA) =$ rank $(A^T B^T)$.

T

F

(j) If A is a square matrix, such that AA^T is singular, then so is A.

T

F

III) Consider the linear system

$$\begin{cases} x + 2y + 3z = 1 \\ 2x + 5y + 3z = 2 \\ x + 8z = 1 \end{cases}.$$

(a) Write down the linear system in matrix form $Ax = b$;

(b) Find the determinant of the coefficient matrix A, by row operations;

(c) Use the answer of (b) to prove that the column vectors of A are linearly independent;

(d) Without solving the system, determine if it has a unique solution;

(e) Solve the system.

-white page for the solution of Question #III)-

IV) (a) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}.$$

(b) Find a basis of the eigenspace corresponding to the eigenvalue $\lambda = 2$ of A;

(c) Is A invertible?

V) Find the rank and the nullity of the matrix

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 1 \\ 2 & 1 & 0 \end{bmatrix}.$$

VI) Show that $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(x, y, z) = (2x + 3y - z, x + z, -x + 7y)$ is a linear transformation.

VII) Define the linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$, through $T(u) = Au$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}.$$

(a) Find $T(e_1)$, $T(e_2)$ and $T(e_3)$, where e_1, e_2, e_3 are the vectors in the standard basis of $\mathbb{R}^3$;

(b) Find $T(3e_1 - 4e_2 + 6e_3)$.