NAME: ID: Group Number:

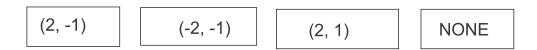
244 Final Exam, May 2013

I) Choose the correct answer:

(a) If If
$$u = (0, 1, 3)$$
, $v = (1, -1, 2)$ and $w = (1, 1, -3)$, then $\frac{u \cdot v}{w \cdot w} w$ equals



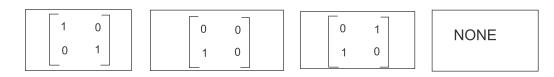
(b) The coordinate vector of (2,0) relative to the basis $B = \{(1,-1),(0,2)\}$ is



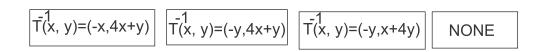
(c) If $\lambda = 2$ is an eigenvalue for a matrix A, then an eigenvalue for A^3 is



(d) If $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ is a reflection about the *y*-axis and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ is a rotation operator through $\theta = \frac{\pi}{2}$, then the matrix $[T_1 \circ T_2]$ is



(e) The inverse of the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (4x + y, -x)$ is



- II) Decide if the following statements are true (T) or false (F). Justify your answer.
 - (a) If A is an $n \times n$ matrix and det A = 2, then det $(A^T A) = 4$.



(b) If A and B are $n \times n$ invertible matrices of the same size and AB = BA, then A and B^{-1} commute.



(c) The set $S = \{A \in M_{22} : \det A = 0\}$ is a subspace of M_{22} .

Т	

F

(d) If V is a vector space of dimension n, then any set of n-1 vectors of V is linearly dependent.



(e) $T: \mathbb{R}^2 \to \mathbb{R}^2, \, T(x,y) = (x,y^2)$ is a linear transformation.



(f) The linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, T(x, y) = (4x - y, x) is one-to-one.

F	

(g) If $\lambda = 0$ is an eigenvalue of a matrix A, then A is not invertible.



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(h) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $a + c = b + d = \lambda$, then λ is an eigenvalue of A.



(i) If A and B are square matrices with the same dimension, then rank $(BA) = \operatorname{rank} (A^T B^T)$.



F

(j) If A is a square matrix, such that AA^T is singular, then so is A.

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F	

$$\begin{cases} x + 2y + 3z = 1\\ 2x + 5y + 3z = 2\\ x + 8z = 1 \end{cases}$$

- (a) Write down the linear system in matrix form Ax = b;
- (b) Find the determinant of the coefficient matrix A, by row operations;
- (c) Use the answer of (b) to prove that the column vectors of A are linearly independent;
- (d) Without solving the system, determine if it has a unique solution;
- (e) Solve the system.

-white page for the solution of Question #III)-

IV) (a) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}.$$

- (b) Find a basis of the eigenspace corresponding to the eigenvalue $\lambda = 2$ of A;
- (c) Is A invertible?

V) Find the rank and the nullity of the matrix

$$A = \left[\begin{array}{rrr} 2 & -1 & 3 \\ 4 & -2 & 1 \\ 2 & 1 & 0 \end{array} \right].$$

VI) Show that $T: \mathbb{R}^3 \to \mathbb{R}^3$, T(x, y, z) = (2x + 3y - z, x + z, -x + 7y) is a linear transformation.

VII) Define the linear operator $T: \mathbb{R}^3 \to \mathbb{R}^3$, through T(u) = Au, where

$$A = \left[\begin{array}{rrrr} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{array} \right].$$

- (a) Find $T(e_1)$, $T(e_2)$ and $T(e_3)$, where e_1 , e_2 , e_3 are the vectors in the standard basis of \mathbb{R}^3 ;
- (b) Find $T(3e_1 4e_2 + 6e_3)$.