Don't look back,
you're not going
that way.

ملخص الرياضيات

فاينل

Dr.Afnan 🌟
Eng.dhoom

Chapter 2

(a)

$$= \frac{15+3i+10i-2}{25+1}$$

$$= \frac{13+13i}{26} = \frac{13}{26} + \frac{13}{26} i$$

$$= \frac{1}{2} + \frac{1}{2} i$$

(b)

$$\frac{3}{i} \cdot \frac{-i}{-i} = \frac{-3i}{-i^2}$$

$$= -3i$$

قاعدة

i^x

$x \div 4 = \ldots 0 \rightarrow 1$ عدد صحيح

$x \div 4 = \ldots .25 \rightarrow i$

$x \div 4 = \ldots ,50 \rightarrow -1$

$x \div 4 = ,75 \rightarrow -i$

HOMEWORK 3 Dividin

Write each quotient in st

(a) $\frac{3+2i}{5-i}$

Powers of i can be si

$i^2 = -$

Consider the following p

$i^1 = i$

$i^2 = -1$

$i^3 = i^2 \cdot i = (-1) \cdot i$

$i^4 = i^2 \cdot i^2 = (-1)(-$

Powers of i cycle through
i^4 has the same multipl
exponent that is a multip

EXAMPLE 4 Simplifying

Simplify each power of i.

(a) i^{15} $-i$

SOLUTION

(a) Since $i^4 = 1$, write th

$i^{15} = i$

(b) Multiply i^{-3} by 1 in
for i.

i^{-3}

2.2 Exercises

$$i^{-23} = \frac{1}{i^{23}} = \frac{1}{-i} \cdot \frac{i}{i} = i$$

Concept Check Determine

1. Every real number is a
2. No real number is a pu
3. Every pure imaginary r
4. A number can be both
5. There is no real numbe
6. A complex number mi

Identify each number as re
one of these descriptions w

7. -4 real
8. $13i$ Pure im

1. If solving a linear equation leads to a true statement such as $0 = 0$, the equation is an **identity.** Its solution set is **{all real numbers}.** (See Example 2(a).)

2. If solving a linear equation leads to a single solution such as $x = 3$, the equation is **conditional.** Its solution set consists of a single element. (See Example 2(b).)

3. If solving a linear equation leads to a false statement such as $-3 = 7$, the equation is a **contradiction.** Its solution set is $\emptyset$. (See Example 2(c).)

متناقض

صلبية (null set) أو (empty set) أو Ø

Solving for a Specified Variable (Literal Equations)

A formula is an example of a **literal equation** (an equation involving letters).

HOMEWORK 2 Solving for a Specified Variable

Solve for the specified variable.

(b) $A - P = Prt$, for P

(Note that

Square rc
number syster
tions and later
extensively in
Complex

Complex N

If a and b
complex n
is the **imag**

Two com
parts are equa
only if $a = c$

For a cor
number. Thus
If $a = 0$ and
ber. For exam
number such
A complex n
form. (The fo
could be mist
The relat
Figure 1.

* In some texts, th

$a+bi$ | $a+bi$
$b=0$ | $b \neq 0$
$\downarrow$ | $\downarrow$
real no. | non real no.

لما نقول جد قيمة x باعتبار انه
العدد تخيلي نقي
$\boxed{(2x+1)}+5i$

$\downarrow$

هذا معناه
يكون = 0

$2x+1=0$

$\frac{2x}{2}=\frac{-1}{2}$

$\boxed{x=\frac{-1}{2}}$

$x^2 - k$

$(x - \sqrt{k})(x + \sqrt{k}$

$x - \sqrt{k} = 0$ or

$x = \sqrt{k}$ or

This proves the **square root pr**

$x^2 = k \Rightarrow x = \pm\sqrt{k}$

$k > 0$ | $k = 0$ | $k < 0$

$x = \pm\sqrt{k}$ | $x = 0$ | $x = \pm i\sqrt{k}$

Square Root Property

If $x^2 = k$, then $x = \sqrt{k}$

That is, the solution set of x^2

$\{\sqrt{k}, -\sqrt{k}\}$, w

Both solutions $\sqrt{k}$ and $-\sqrt{k}$ $k < 0$. If $k < 0$, then we wri

دايماً دام فيه تربيع
على الـ X يكون
جذر موجب أو سالب
لا ~~تنسى~~ ..

If $k = 0$, then there is only or **solution.**

HOMEWORK 1 Using the S

Solve each quadratic equatio

(a) $x^2 = 17$ **(b)** x

$x^2 = \sqrt{25}$

$x = \pm\sqrt{25}$

$x = 5$, $x = -5$

Completing the Square

of **completing the square,** s

* نفرض التأكد ..

$5^2 = 25$ ✓

$(-5)^2 = 25$ ✓

عشان كذا نكتب + و -

(إكمال المربع)

*لازم يكون معامل لـ x^2 واحد
∴ اقسم جميع حدود المعادلة عليه
عشان يصير (1)

*ننقل العدد الثابت للطرف الثاني

*اضيف نصف معامل x للطرفين
(اربعه) = يعني ارفعه للأس (2)

* اضا الجذر الاول والاخير
واحط اشارة الحد الاوسط

Solving a Quadratic Equation by Completing the Square

To solve $ax^2 + bx + c = 0$, where $a \neq 0$, by completing the square, use these steps.

Step 1 If $a \neq 1$, divide both sides of the equation by a.

Step 2 Rewrite the equation so that the constant term is alone on one side of the equality symbol.

Step 3 Square half the coefficient of x, and add this square to each side of the equation.

Step 4 Factor the resulting trinomial as a perfect square and combine like terms on the other side.

Step 5 Use the square root property to complete the solution.

EXAMPLE 2 **Using Completing the Square ($a = 1$)**

Solve $x^2 - 4x - 14 = 0$.

SOLUTION $x^2 - 4x - 14 = 0$

Step 1 This step is not necessary since $a = 1$.

Step 2	$x^2 - 4x = 14$	Add 14 to each side.
Step 3	$x^2 - 4x + 4 = 14 + 4$	$[\frac{1}{2}(-4)]^2 = 4$; Add 4 to each side.
Step 4	$(x - 2)^2 = 18$	Factor. (Section 1.4) Combine like terms.
Step 5	$x - 2 = \pm\sqrt{18}$	Square root property
Take *both* roots.	$x = 2 \pm \sqrt{18}$	Add 2 to each side.
	$x = 2 \pm 3\sqrt{2}$	Simplify the radical.

each side

4) Combine like te

ty

1.

$ax^2 + bx + c = 0$

he constants $a, b,$

equation.

tep 1)

e. (Step 2)

$\frac{b^2}{4a^2}$.

* يمديني اختصر بالكسر
حتى لو كان عندي جمع وطرح
بشرط اني اختصر
على قسم العدد ومن جميع
الحدود مثلاً

$$\frac{\cancel{4}^{2} \pm \cancel{6}^{3}\sqrt{2}}{\cancel{2}_{1}}$$

$$2 \pm 3\sqrt{2}$$

Quadrati

The solu
are given

EXAMPL

Solve x^2

SOLUTION

The fraction
extends unde

Factor firs

The s

CA
un

d, we use the set of comple
egree 2 or greater.

l Complex Solutions)

cause the greatest degree of

ormula.

Factor as a sum of cubes. (Section 1.4)

Zero-factor property

number and type of solutions based on the value of the discrimin in the following table.

Discriminant	Number of Solutions	Type of Solutio
Positive, perfect square	Two	Rational
Positive, but not a perfect square ($\sqrt{48}$)	Two	Irrational
Zero	One (a double solution)	Rational
Negative	Two	Nonreal comp

rational لحل ,ديكو
بشرط ان جميع
معاملات المعادلة
أعداد صحيحة

CAUTION ***The restriction on a, b, and c is important.***

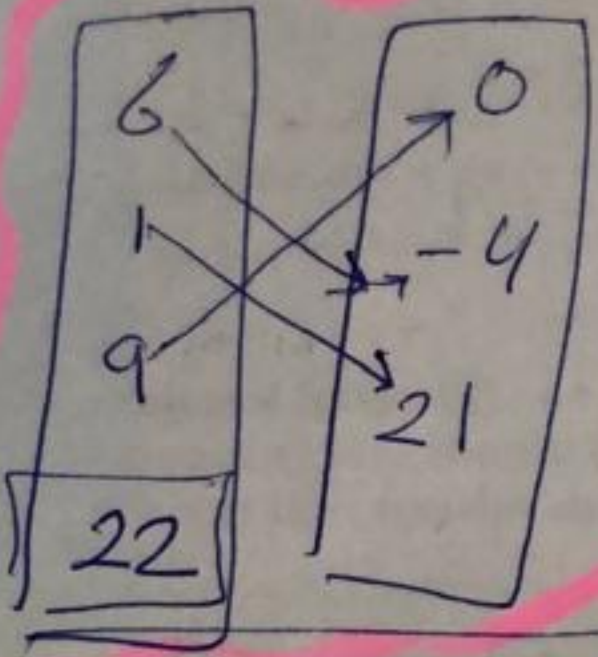
$$1x^2 - \sqrt{5}x - 1 = 0 \quad \text{has discriminant} \quad b^2 - 4ac =$$

which would indicate two rational solutions *if the coefficie*

By the quadratic formula, the two solutions $\frac{\sqrt{5} \pm 3}{2}$ are ir

Chapter 3

3.1 Functions



ليست دالة لا يوجد
عنصر لم يرتبط بعنصر
من (y)

* التكرار في المدى عادي
ولكن في المجال --
(not function)

Relations and Fun

of another.

- The letter grad
numerical scor
- The amount yo
number of gallo
- The dollars spe

We used ordered p
(3, $10.50) indica
you pay *depends* o
called the *depende*
independent varia
Generalizing,
of the first compon
dent variable.

A set of ordere
a *relation*. A speci
ematics and its ap

Relation and F

A **relation** is a
each distinct v
exactly one valu

NOTE The r
number of gallo
each x-value is

EXAMPLE 1 De

domain ... gallons pumped, and ... consists of amounts from the purchaser's wallet. Dividing the dollar amount by the number of gallons pumped gives the exact price of gasoline that day. Was this pump fair? (Later we will see that this price is an example of the slope m of a linear function of the form $y = mx$.)

function

EXAMPLE 2

Give the don...

(a)

Domai , Range تكتبا:

في المجموعات { [

من اليسار إلى اليمين = المجال

من الأسفل إلى الأعلى = المدى

$-\infty$

وجود الأسهم يعني انه من

$(-\infty , \infty)$

(c)

∞

$-\infty$

funct...

SOLUTION

(a) The ... y-val...

(b) The ... 4, in...

(c) The ... wel... real...

*ليس بالضرورة أن تكون العلاقة دالة حتى يكون لديها مجال ومدى..

*كل العلاقات لديها

المعادلات الخطية ..
- تقبس دوال ✓
- مجالها R
- مداها R

*معادلات الجذر التربيعي ..
*تقبس دواله ..
* مجالها مابداخل الجذر أكبر من او يساوي الصفر
* مداها من K إلى +∞

(d) The ar… well … nds indefi… Because th… than or

…ons, suc… of a rela… on the do…

Agre…

Unless spe… otherwise, th… of a relation i… real numbers that produce real numbers when substitut… dent variable.

لأي عدد حقيقي عند التعويض عن الـx به ينتج لنا قيم حقيقي

To illustrate this agreement, since any real number ca… ment for x in $y = 2x + 3$, the domain of this function is t… bers. As another example, the function defined by $y = \frac{1}{x}$ *except* 0 as domain, since y is undefined if $x = 0$.

- مجال الجذر (فقط الزوجي) هو جميع الأعداد الحقيقية ماعدا القيم التي تجعل مابداخل الجذر قيمة (سالبة)

In general, the domain of a function defined by an a… all real numbers, except those numbers that lead to … even root of a negative number.

(There are also exceptions for logarithmic and trig… They are covered in further treatment of precalculus math…

- وبالنسبة للكسر .. المجال هو جميع القيم ماعدا التي تجعل (المقام = صفر)

Determining Whether Relations Are Functions Since e… only one value of y in a function, any vertical line must i… most one point. This is the **vertical line test** for a function…

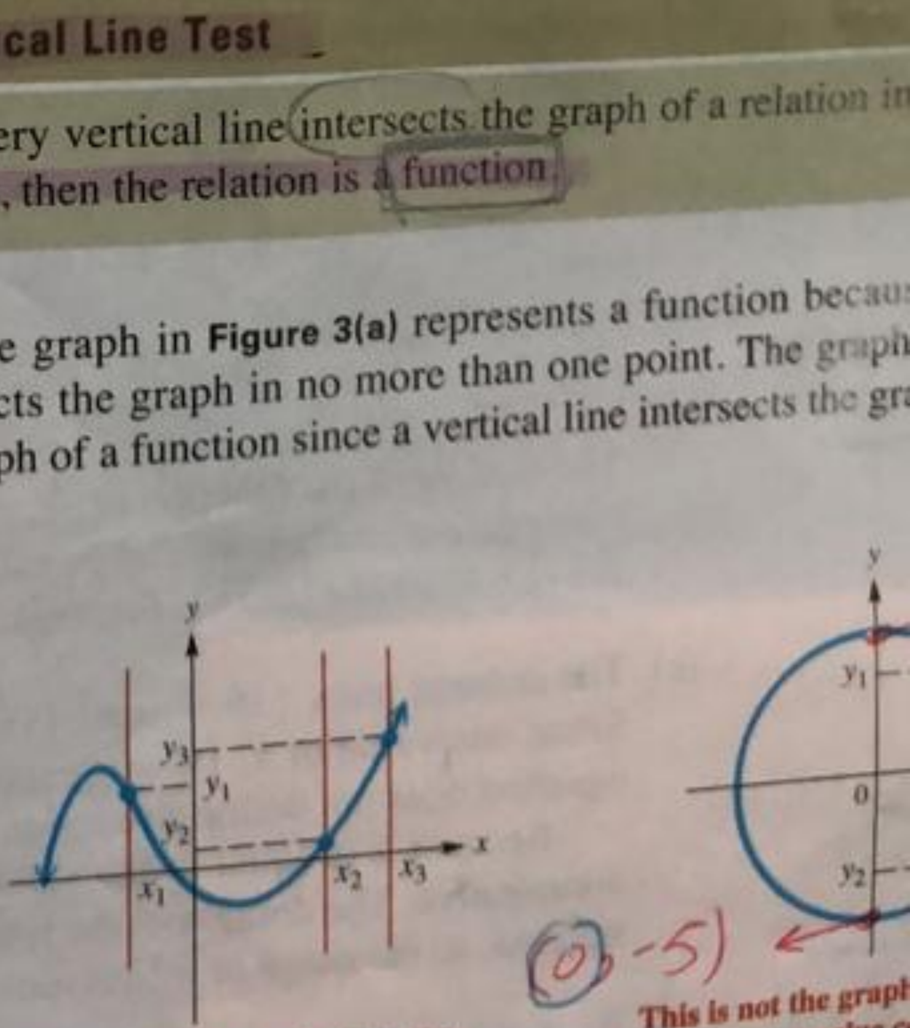
Vertical Line Test

If every vertical line intersects the graph of a relation i… point, then the relation is a function.

The graph in **Figure 3(a)** represents a function becau… intersects the graph in no more than one point. The graph… the graph of a function since a vertical line intersects the gr… point.

• لما يقطع الخط بنقطة واحدة متكون دالة ..

• اذا قطع الخط أكثر من نقطة ماحتكون دالة ..



This is the graph of a function. Each x-value corresponds to only one y-value.
(a)

This is not the graph… The same x-value c… two different y…
(b)

Figure 3

* الـ x ارتبطت بأعداد كثيرة من الـ y
∴ ليست دالـــة

tion defines ...
domain is

$\{x \mid x$ is a ...

* المتباينات ليست دوال ..
مجالها R , مداها R

* المعادلات الكسرية دوال ..
مجالها R باستثناء القيم التي تجعل المقام صفراً
ومداها
اذا كان البسط عدد ثابت يكون على R باستثناء الصفر ← R\{0}
= = متغيرات او عمليات يكون على R باستثناء القيم التي تجعله صفراً

more
in th
e for
unctic
dical
$1 \geq$
$2x \geq$
$x \geq$
unctio
as x
∞).

in Example 2

ction defined
an inequality
difficult. The

in and range.

الجذور الغريبة مجالها ومداها

(R)

* الدالة العكسية تنطبق على Range مدى.

all satisfy the inequality, so y is not a function of x here. Any number can be used for x or for y, so the domain and the range of this relation are both the set of real numbers, $(-\infty, \infty)$.

* مجال الدالة العكسية هو مدى الدالة.

(e) Given any value of x in the domain of

$$y = \frac{5}{x-1},$$

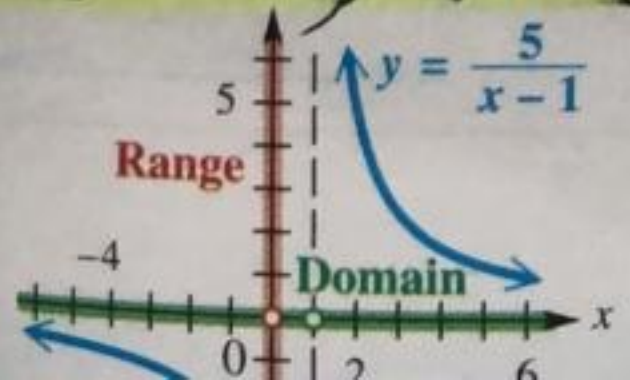
$\boxed{y^2 = x}$

* أس ال y زوجي ليست دالة

$R = (-\infty, \infty)$, $D = [0, \infty)$

we find y by subtracting 1 from x, and then dividing the result into 5. This process produces exactly one value of y for each value in the domain, so this equation defines a function.

The domain of $y = \frac{5}{x-1}$ includes all real numbers except those that make the denominator 0. We find these numbers by setting the denominator equal to 0 and solving for x.



$$x - 1 = 0$$

$$x = 1 \quad \text{Add 1. (Section 2.1)}$$

der an equation involving x and y. Assume that y can be expressed as tion f of x. To find an expression for $f(x)$, use the following steps.

Solve the equation for y.

Replace y with $f(x)$.

WORK 4 Writing Equations Using Function Notation

e that y is a function f of x. Rewrite each equation using function nota- hen find $f(-2)$ and $f(a)$.

$= x^2 + 1$ **(b)** $x - 4y = 5$

easing, Decreasing, and Constant Functions Informally speaking, a func- *icreases* on an interval of its domain if its graph rises from left to right on terval. It *decreases* on an interval of its domain if its graph falls from left ht on th ontal on For exa 1] bec larly, th lways 5 5] becau *e x-valu* The form

ncreasing, Decreasing, and Constant Functions

مهمة

uppose that a function f is defined over an interval I and x_1 and x_2 are in I.

a) f **increases** on I if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$. نفس — أكبر ← أكبر / أصغر ← أصغر

b) f **decreases** on I if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$. عكس — أكبر ← أصغر / أصغر ← أكبر

c) f **is constant** on I if, for every x_1 and x_2, $\boxed{f(x_1) = f(x_2)}$. → لازم المساواة

Figure 9 illustrates these ideas.

Constant? * أي عدد بدون متغير هو ←

$-\sqrt{2}$ ✓ * أي عدد بدون متغير حتى لو كان سالب ↑

$\frac{1}{2}$ ✓

Increasing * أي عدد مع متغير هو ↗

$2x$

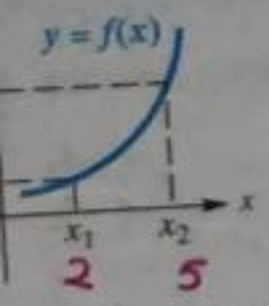
$\frac{1}{2}x$

$\sqrt{2}x$

decreasing * أي عدد مع متغير و سالب هو ↘

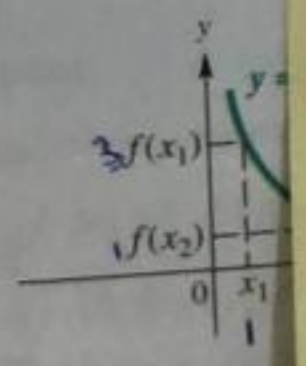
$-2x$

$-\sqrt{2}x$



x_2, and $f(x_1) < f(x_2)$. ncreasing.

(a)



Whenever $x_1 < x_2$, an f is decrea

(b)

Figure

NOTE To decide whether a fun stant on an interval, ask yourself, *right?"*

lue: (a) $f(-2)$.

a curve at a given point. The derivative is used to find the slope of the desired line, and then the slope and the given point are used in the point-slope form to solve the problem.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - 1 = -3[x - (-4)] \qquad x_1 = -4,\ y_1 = 1,\ m = -3$$

Be careful with signs.

$$y - 1 = -3(x + 4)$$

$$y - 1 = -3x - 12 \qquad \text{Distributive property (Section 1.2)}$$

$$y = -3x - 11 \qquad \text{Add 1. (Section 2.1)}$$

is (a) increasing.

المعطى نقطتين ← اجيب

الميل مع نقطة

HOMEWORK 1 **Using the Point-Slope Form (Given Two Points)**

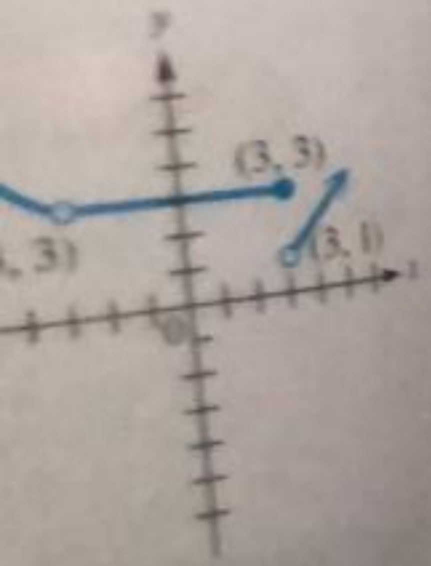
Write an equation of the line through $(-3, 2)$ and $(2, -4)$. Write the result in standard form $Ax + By = C$.

بالصيغة القياسية

معاملات الـ x و y

تكون كلها صحيحة Z

يعني ما في كسر



NOTE The lines in **Example 1 and Homework 1** both have negative slopes. Keep in mind that a slope of the form $-\frac{A}{B}$ may be interpreted as either $\frac{-A}{B}$ or $\frac{A}{-B}$.

(صيغة الميل المقطع)

Slope-Intercept Form As a special case of the point-slope form of the equation of a line, suppose that a line passes through the point $(0, b)$, so the line has y-intercept b. If the line has slope m, then using the point-slope form with $x_1 = 0$ and $y_1 = b$ gives the following.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - b = m(x - 0) \qquad x_1 = 0,\ y_1 = b$$

$$\boxed{y = mx + b} \qquad \text{Solve for } y.$$

Slope m; y-intercept b

line. We n

Since this result shows the slope of the line and the y-intercept, it is called the

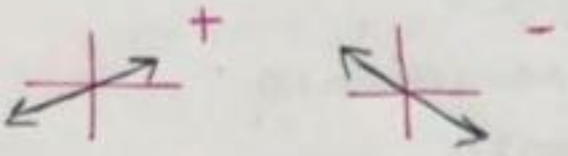
(صيغة ميل ومقطع)

$\boxed{m = \frac{-A}{B}}$

$\boxed{y\text{-intercept} = \frac{C}{B}}$

$\boxed{x\text{-intercept} = \frac{C}{A}}$

* الميل + من اليسار طالع
* الميل - من اليسار نازل



* لازم نفس النقطة اذا
أخذت منها y
لازم ال x تكون منها

Slope-Intercept Form

The **slope-intercept form** of the
y-intercept b is

$y =$

EXAMPLE 2 **Finding the Slope and y-I**

Find the slope and y-intercept of the li

SOLUTION Write the equation in slop

$$4x + 5y = -10$$
$$5y = -4x$$
$$y = -\frac{4}{5}$$
$$m$$

The slope is $-\frac{4}{5}$ and the y-interc

NOTE Generalizing from Exam

Ax

is $-\frac{A}{B}$, and the y-intercept b is $\frac{C}{B}$.

HOMEWORK 2 **Using the Slope-Inter**

Write an equation of the line throu
using the slope-intercept form.

EXAMPLE 3 **Finding an Equation fro**

Use the graph of the linear functi
following.

(a) Find the slope, y-intercept, and x

(b) Write the equation that defines f

SOLUTION

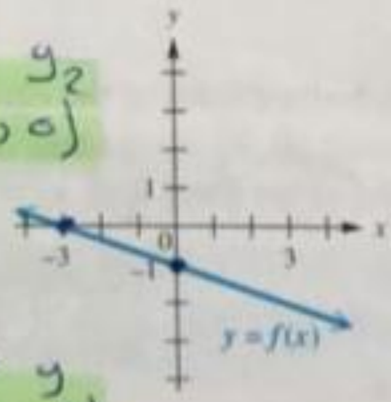
(a) The line falls 1 unit each time t
the slope is $\frac{-1}{3} = -\frac{1}{3}$. The grap
and intersects the x-axis at the p
-1 and the x-intercept is -3.

(b) The slope is $m = -\frac{1}{3}$, and the y

$$y = f(x) = mx + b$$

$$f(x) = -\frac{1}{3}x - 1$$

$x_2 \quad y_2$
$(-3, 0)$



$x \quad y$
$(0, -1)$

Figure 13

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 + 1}{-3 - 0} = \boxed{\frac{1}{-3}}$$

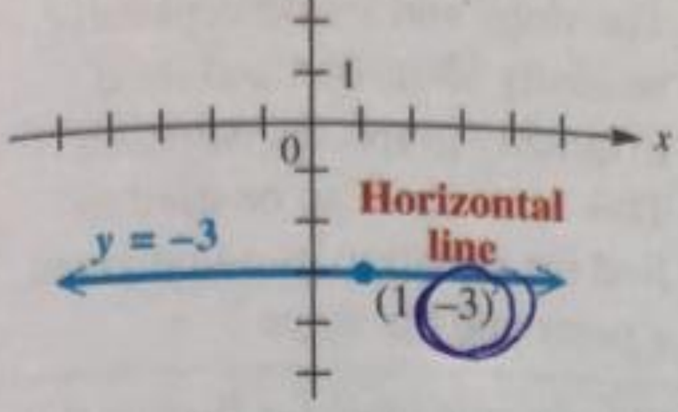
the graph of

Points)

Then graph the line

e 13 to complete the

by 3 units. Therefore,
s at the point $(0, -1)$
ore, the y-intercept is



(b)

Figure 14

slope of vertical line

= undifind

slope of horizontal line

= 0

* كل الخطوط الرأسية متوازية

* الأفقي والأفقي دائماً متوازية

on of

Equations o

An equation
An equation

Parallel and P
they should have
are parallel. The
and only if q" m

Parallel Line

Two distinct
slope.

When two li

Perpendicul

Two lines, nei
slopes have a
ther of which i

$M_1 = \frac{-1}{m_2}$

For example
to it is $\frac{4}{3}$, since
rocals of each o

NOTE Beca
the *mathematic*
know that all ve
tal line are perp

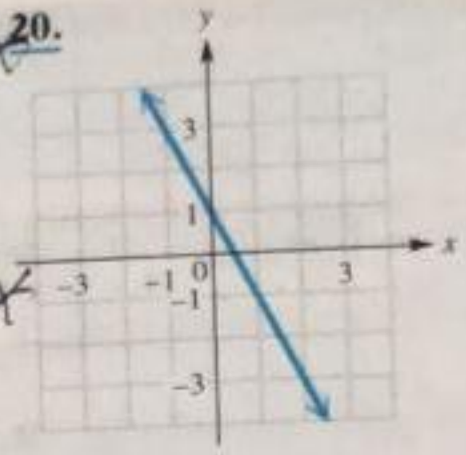
20.

✗ لما يكون $\parallel$ اخذ نفس المكتوب

$y = y \qquad x = x$

✗ لما يكون $\perp$ اخذ عكس المكتوب

$y = x \qquad x = y$



In Exercises 23–26, write an
form for the line described. Se

23. through $(-1, 4)$, parallel

24. through $(1, 6)$, perpendic

25. through $(4, 1)$, parallel to

26. through $(-5, 6)$, perpenc

27. Find k so that the line thr
   (a) parallel to $3y + 2x =$

$(4,1) \perp y = -5$

$\boxed{x = 4}$ الحل

$(-5, 6) \parallel x = -2$

$\boxed{x = -5}$

26) $(-5, 6) \perp x = -2$

$\boxed{y = 6}$

$(2, 5) \perp x = 3$

$\boxed{y = 5}$

Relating Concepts

For individual or collaborat

In this section we state t
which is vertical, are per
their slopes ha
28–34, we outli
where the two
these exercises
needed.

By the converse

then triangle F

28. Find an ex

29. Find an expression fo

30. Find an expression fo

31. Use your results fror
   the Pythagorean theo

32. Factor $-2x_1x_2$ from

33. Use the property tha
   Exercise 32, showin

34. State your conclusio

ins

wing.

x) and its domain

denominator

$g(x)$ undefined.
ession for $g(x)$,
es $g(x) = 3$ and

As **Home Work 4** shows, ***it is not always true that $f \circ g = g \circ f$***. In fact, the composite functions $f \circ g$ and $g \circ f$ are equal only for a special class of functions.

In calculus it is sometimes necessary to treat a function as a composition of two functions. The next example shows how this can be done.

هو المقدار اللي $g(x)$
يتكرر

1] وتشيل المقدار المتكرر
2] اضع مكانه X
وتكون هي ال $f(X)$

EXAMPLE 5 Finding Functions That Form a Given Composite

Find functions f and g such that

$$(f \circ g)(x) = (x^2 - 5)^3 - 4(x^2 - 5) + 3.$$

SOLUTION Note the repeated quantity $x^2 - 5$. If we choose $g(x) = x^2 - 5$ and $f(x) = x^3 - 4x + 3$, then we have the following.

$$\begin{aligned}
(f \circ g)(x) &= f(g(x)) && \text{By definition} \\
&= f(x^2 - 5) && g(x) = x^2 - 5 \\
&= (x^2 - 5)^3 - 4(x^2 - 5) + 3 && \text{Use the rule for } f.
\end{aligned}$$

There are other pairs of functions f and g that also satisfy these conditions. Here is another such pair.

$$f(x) = (x - 5)^3 - 4(x - 5) + 3 \quad \text{and} \quad g(x) = x^2$$

Chapter 4

Vertex
Quadratic Models

where a_n, a_n

LOOKING AHEAD TO CALCULUS In calculus, polynomial functions are used to approximate more complicated functions. For example, the trigonometric function $\sin x$ is approximated by the polynomial

$$x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}.$$

أي عدد حقيقي درجته صفر والصفر ليس له درجة

When we are anal
coefficient a_n play ar
$a_n x^n$. The table provi

Polynomial Funct
$f(x) = 2$
$f(x) = 5x - 1$
$f(x) = 4x^2 - x + 1$
$f(x) = 2x^3 - \frac{1}{2}x +$
$f(x) = x^4 + \sqrt{2}x^3$

The function $f(x) =$

Quadratic Functio
functions. Polynom

$a > 0 \rightarrow \cup$

$a < 0 \rightarrow \cap$

Quadratic Func

A function f is a

where a, b, and

$|a| > 1 \rightarrow$ أضيق من x^2

$|a| < 1 \rightarrow$ أعرض من x^2

The simplest

$f(x) = x^2,$

as shown in **Figu**
parabola. Ever
defined over th
graph that is a pa
The doma
$(-\infty, \infty)$, and th
lowest point on
origin $(0, 0)$. Th
on th

ading
term

a_n

mial

x^2

x

es

- Opens up if $a > 0$
- Opens down if $a < 0$
- Vertically stretched (narrower) if $|a| > 1$ تضييق رأسي
- Vertically shrunk (wider) if $0 < |a| <$ توسيع رأسي

$\boxed{1}$ لإيجاد مجال الدالة التربيعية هو (R)

$\boxed{2}$ لإيجاد مدى الدالة التربيعية لابد من ايجاد قيمة ال $\boxed{K}$ وذلك بإيجاد ال $\boxed{h}$ ثم نعوض بالدالة فيها فيصبح المدى من K إلى $+\infty$

$[k, \infty)$

* معامل x^2 هو a
* معامل x هو b

EXAMPLE 1 **Graphi**

Graph each function.

(a) $f(x) = x^2 - 4x$

(b) $g(x) = -\frac{1}{2}x^2$ (a

(c) $F(x) = -\frac{1}{2}(x -$

SOLUTION

(a) See the table wit
the range is $[-6$

x	$f(x)$
−1	3
0	−2
1	−5
2	−6
3	−5
4	−2
5	3

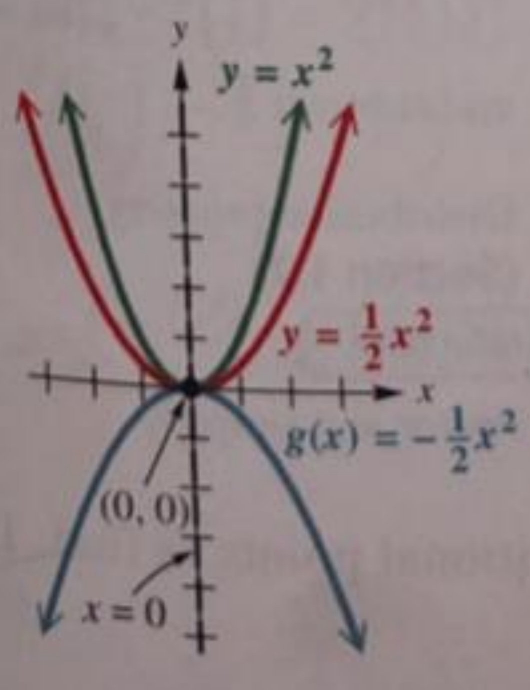


Figure 4

(b) Think of $g(x) =$
version of the gr
of the graph c
(0,0), and the
domain is $(-\infty$

$-\frac{1}{2}x^2$ from part (b)
its to the right and
s of the parabola is
s $(-\infty, 3]$.

ratic function

ola opens up if a is
d, we complete the

$(a = 1)$

ocating the vertex.
creasing.

same side of the
same number to
e in Section 1.3.
ion, we adjusted

$\neq 1)$

locating the vertex.

st be 1.

-3 from the first
s.

$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$, so add
ract $\frac{1}{9}$.

ive property
1.2)

nd simplify.

The

The x-intercepts are found by setting $f(x)$ equal

$0 = -3x^2 - 2x + 1$

$0 = 3x^2 + 2x - 1$

$0 = (3x - 1)(x + 1)$

$x = \frac{1}{3}$ or $x = -1$

Therefore, the x-intercepts are $\frac{1}{3}$ and -1.

*إذا معامل x^2 سالب
تخرج الدالة مقلوبة
ويكون المدى $(-\infty, k]$
مثال توضيحي:

* find the rang of the function
$f(x) = -x^2 + 1$

[1] $h = \frac{-b}{2a} = \frac{0}{2(-1)} = \frac{0}{-2} = \boxed{0}$

[2] $k = f(h) = -(0)^2 + 1 = \boxed{1}$

$\therefore$ rang $= (-\infty, k]$

Figure 6

---

* find the rang of the function $f(x) = -(x+1)^2$
مقلوبة

NOTE It is possible to reverse
quadratic function from its graph
graph are known. Since quadratic

$f(x) =$

substitute the x- and y-values of t

$f(x) = a\left[x - \left(-\right.\right.$

$f(x) = a\left(x + \frac{1}{3}\right.$

---

* $f(x) = 4 - x^2$

$-x^2 + 4$
مقلوبة

Now find the value of a by su
point on the graph, say $(0, 1)$

$1 = a\left(0 + \frac{1}{3}\right)$

$1 = a\left(\frac{1}{9}\right) +$

[1] $h = \frac{-b}{2a} = \frac{0}{2(-1)} = \boxed{0}$

$-\frac{1}{3} = \frac{1}{9}a$

[2] $k = f(h) = -(0)^2 + 4 = \boxed{4}$

$a = -3$

$\therefore$ rang $= (-\infty, 4]$

Verify in Example 2 tha

In the Exercise set, pro
Equations.

the tools for determining that the
enclosed by the parabola and the x-axis
is $\frac{32}{27}$ (square units).

① لايجاد مقطع محور x
اساوي لمعادلة بصفر

② لايجاد مقطع محور y
اعوض كل x بصفر

Thus, the vertex (h, k) can be expre
sary to memorize the expression fo
The following statements summ

Graph of a Quadratic Function

The quadratic function defined by

$$y = f(x) = a$$

where $h = -\frac{b}{2a}$ and

The graph of f has the following

1. It is a parabola with vertex $(h$
2. It opens up if $a > 0$ and dow
3. It is wider than the graph of y
4. The y-intercept is $f(0) = c$.
5. The

* لما يطلب الأصفار أو الجذور
أو الحلول دول نفس
الشيء تاخدها الحاجة.

* لما يطلب العوامل أو يقول
حلل بس اضربه هرضين في المين

factor
$x = \frac{1}{3}$ → عامل

$x = \frac{1}{3} \rightarrow 3x = 1$
$(3x-1)$ → صفر، جذر، حل

roots zero

لما يطلب الاصفار أو الحلول
يعني roots or solutions
ناخدها عكس اشارة الحاجة

$x = -3$ $\quad x = \frac{1}{3}$ $\quad x = -\frac{1}{2}$

$(x+3) \rightarrow$ عامل
جذر
المسألة

$$\begin{array}{r|rrrr} -3 & 6 & 19 & 2 & -3 \\ & & -18 & -3 & 3 \\ \hline & 6 & 1 & -1 & 0 \end{array}$$

factors $= (6x^2 + x - 1)(x+3)$

Factor

For any
and onl

EXAMPL

Determin
(a) $f(x)$
(b) $f(x)$

SOLUTIO

(a) By
divi

Use
for t

The

(b)

Be
mi

Th

W
linear

■ Testing Potential Zeros

* نستخدم القسمة التركيبية
اذا كان مقسوم عليه
من الدرجة الأولى

* اذا كانت اكبر من الدرجة
الأولى نستخدم القسمة
الطويلة

* دائماً درجة خارج القسمة
أقل من درجة المقسوم

Let $f(x)$ and $g(x)$ be polynomials with $g(x)$ of lea
and $g(x)$ of degree 1 or more. There exist unique po
$r(x)$ such that

الباقي خارج القسمة

$$f(x) = g(x) \cdot q(x) + r(x),$$

where either $r(x) = 0$ or the degree of $r(x)$ is less th

For instance, we saw in **Example 6** of **Section 1.3** tha

$$\frac{3x^3 - 2x^2 - 150}{x^2 - 4} = 3x - 2 + \frac{12x - 158}{x^2 - 4}$$

We can express this result using the division algorith

$$\underbrace{3x^3 - 2x^2 - 150}_{f(x)} = \underbrace{(x^2 - 4)}_{g(x)}\underbrace{(3x - 2)}_{q(x)}$$

Dividend = Divisor · Quotient
(original polynomial)

Synthetic Division When a given polynomial
binomial of the form $x - k$, a shortcut method ca

rvals: $(-\infty, 0.92)$,
each interval shows
re than 160 ft above

e quadratic formula

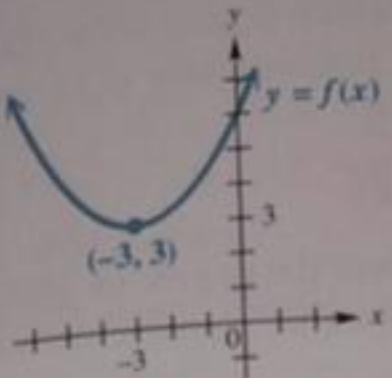
adratic function. Do

e vertex.

ain, and (d) range.
increasing and (f)
Homework 1-2.

$(0)^2 - 2(0)(2) + 4 = 4$

$f(x) = y$



17 vertex $= (2, -1)$

$\boxed{h=2}$, $\boxed{k=-1}$

x y
(0,0)

$f(x) = a(x-h)^2 + k$

$f(0) = a(0-2)^2 + 1$

$0 = 4a - 1$

$\frac{4a}{4} = \frac{1}{4}$ $a = \frac{1}{4}$

$\boxed{f(x) = \frac{1}{4}(x-2)^2 - 1}$

18 $f(x) = a(x-h)^2 + k$

vertex $= (1, 4)$

$\boxed{h=1}$, $\boxed{k=4}$

x y
(0,2)

$f(0) = a(0-1)^2 + 4$

$2 = a + 4$

$\boxed{a = -2}$

$\boxed{f(x) = -4(x-1)^2 + 4}$

5. $f(x) = (x-2)^2$

7. $f(x) = -\frac{1}{2}(x+1)^2 - 3$

9. $f(x) = x^2 - 10x + 21$

11. $f(x) = -\frac{1}{2}x^2 - 3x - \frac{1}{2}$

Concept Check *The figure shows th work Exercises 12–13.*

12. What is the minimum value of

13. How many real solutions are th

Concept Check *Several possible g*

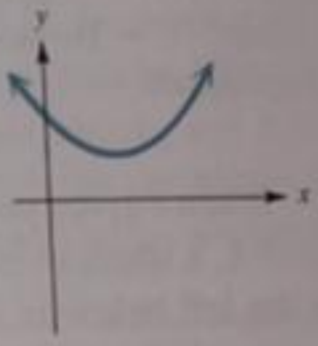
f

are shown below. For the restricti corresponding graph from choice necessary.)

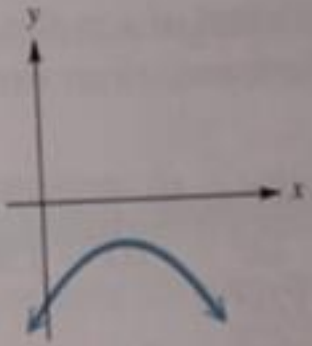
14. $a < 0$; $b^2 - 4ac = 0$

16. $a > 0$; $b^2 - 4ac > 0$

A.

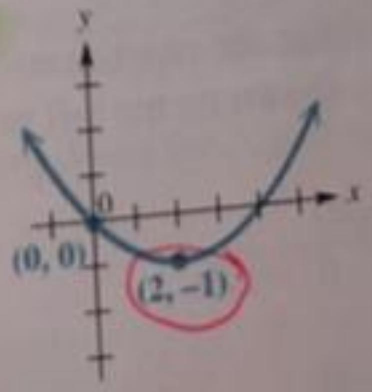


D.



Connecting Graphs with Eq whose graph matches the one

17.



Chapter 5

ent is used only
nterchanging the

of F. For a func-

nly one y-value,

y-value 7 corre-
and $(3, 7)$ both

in the domain

one to
e function

gic, the last line

-one in the next

$25 - x^2$

* اي دالة درجتها فردية
تكون 1-1
حتى لو كانت تحت جذر

* بينما اي كثيرة حدود
درجتها زوجية
تكون not 1-1
حتى وان كانت تحت جذر

(b) For the function $f(x) = \sqrt{25 - x^2}$, if we choose $a = 3$ and $b = -3$, then $3 \neq -3$, but

$$f(3) = \sqrt{25 - 3^2} = \sqrt{25 - 9} = \sqrt{16}$$

and

$$f(-3) = \sqrt{25 - (-3)^2} = \sqrt{25 - 9} =$$

Here, even though $3 \neq -3$, $f(3) = f(-3) = 4$ ... a one-to-one function.

* اي دالة تزايدية او تناقصية
تكون one to one function
لان الخط الافقي يقطعها
في كل مرة بنقطة واحدة
فقط

As illustrated in **Example 1(b)**, a way to show ... to-one is to produce a pair of different domain ... function value. There is also a useful graphical test ... tells whether or not a function is one-to-one.

Horizontal Line Test

A function is one-to-one if every horizontal ... function at most once.

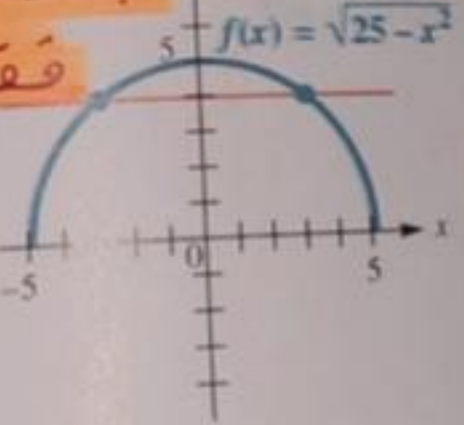


Figure 3

NOTE In **Example 1(b)**, the graph of ... shown in **Figure 3**. Because there is at least ... the graph in more than one point, this fun...

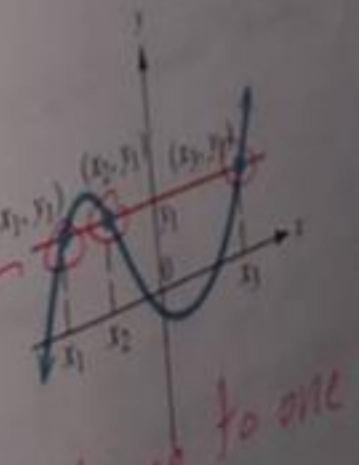
$(x - 2)^2$

* Not 1-1 function
* but it function

HOMEWORK 1 **Using the Horizontal Line**

Determine whether each graph is the gra...

(a)

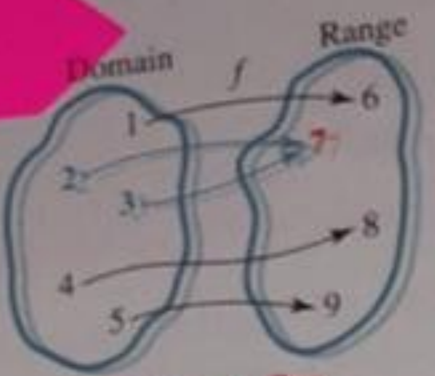


قطعها في اكثر من نقطة

Not one to one

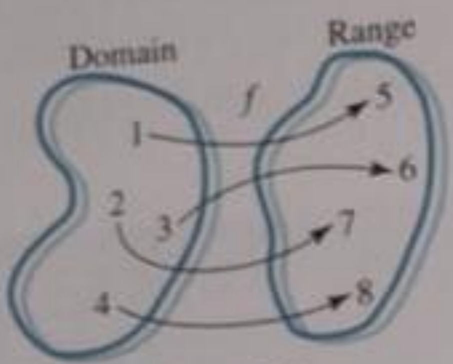
...the function grap...

- One-to-One Functions
- Inverse Functions
- Equations of Inverses
- An Application of Inverse Functions to Cryptography



Not One-to-One

Figure 1



One-to-One

الأصول **Figure 2** الصور

$f(a) = f(b) \Rightarrow a = b$

ممكن يجي في نفس السؤال

* not function
* 1-1 function

(3,5), (3,4), (5,6)

① المطلوب 1-1

② function ولا or not

* one to one ماعنديش تكرار في ال (y)
* function ماعنديش تكرار في ال (x)

$F = \{($

(Notice that we have d
once.) We can form an
x- and y-values of each

$G = \{($

To show that these two
tion f to have an inver

In a one-to-one fu
and each y-value

The function f shown
sponds to *two* x-values
belong to the function.

One-to-One Functi

A function f is a on
of f.

$\sqrt{a} = \sqrt{b}$

Using the concep
in the preceding box

We use this stateme
examples.

EXAMPLE Decid

Decide whether eac

(a) $f(x) = -4x +$

SOLUTION

(a) We must show

-4

By the definiti

Domain $f(x)$ = Range $f^{-1}(x)$

Range $f(x)$ = Domain $f^{-1}(x)$

CAUTION ***Do not confuse the −1 in f^{-1} with a negative exponent.*** The symbol $f^{-1}(x)$ does not represent $\frac{1}{f(x)}$. It represents the inverse function of f.

inverse

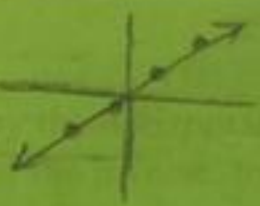
ON

e graph o
st, f is a o
e steps in t

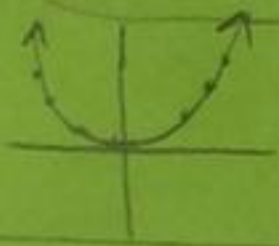
hus, $f^{-1}(x$
$= 2x + 5$
y 2, and ad
The for
and then a
unction do

he equatio
orizontal l
$= 3$ and
^{2}-term, th
-value. Th
ne and do
The ste

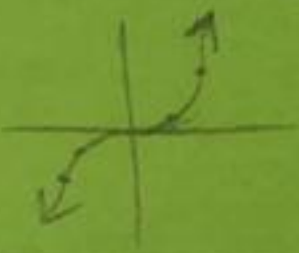
* $f(x) = x \rightarrow$



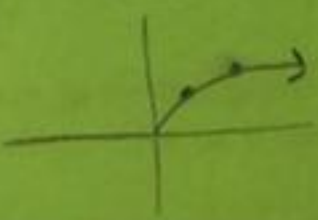
* $f(x) = x^2 \rightarrow$



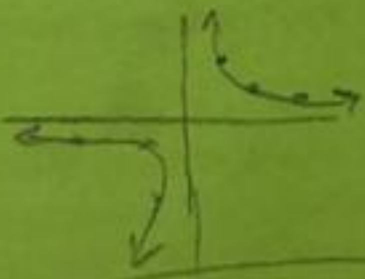
* $f(x) = x^3 \rightarrow$



* $f(x) = \sqrt{x} \rightarrow$



* $f(x) = \frac{1}{x} \rightarrow$



* $f(x) = |x| \rightarrow$

ember
roots.

$x - 2 = y^2$ Solve for y.

$\pm\sqrt{x-2} = y$ Square root property **(Section 2.3)**

The last step shows that there are two y-values for each choice of x greater than 2, so the given function is not one-to-one and cannot have an inverse.

Figure 7 shows that the horizontal line test assures us that this horizontal translation of the graph of the cubing function is one-to-one.

$f(x) = (x-2)^3$ Given function

Replace $f(x)$ with y.

1 $f(x) \rightarrow y$

2 $y \rightarrow x$

3 solve for y

4 $y \rightarrow f^{-1}(x)$

[one to one function] لو ماكانت

ما فيك تعملها عكسي

doesn't have inverse

on

functio

SOLUTION

(a) The graph o
test, f is a o
the steps in t

Thus, $f^{-1}($
$y = 2x + 5$
by 2, and a

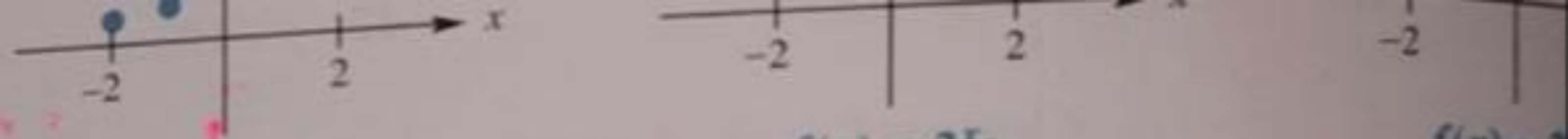
The fo
5 and then
function d

(b) The equat
horizonta
$x = 3$ an
x^2-term,
y-value.
one and

The

Remember both roots.

The las
greater
an inve



$f(x) = 2^x$;
integers as domain

$f(x) = 2^x$;
selected rational numbers
as domain

$f(x) = 2$
real numb
as doma

Figure 10

Using this interpretation of real exponents, all rules and theore nents are valid for all real number exponents, not just rational ones to the rules for exponents presented earlier, we use several new prop chapter. These properties are generalized below. Proofs of the prope given here, because they require more advanced mathematics.

$(-2)^{\frac{1}{2}} = \sqrt{-2}$
undefined

Additional Properties of Exponents

For any real number $a > 0$, $a \neq 1$, the following statements are

(a) a^x **is a unique real number for all real numbers** x.

(b) $a^b = a^c$ **if and only if** $b = c$.

(c) If $a > 1$ **and** $m < n$, **then** $a^m < a^n$.

(d) If $0 < a < 1$ **and** $m < n$, **then** $a^m > a^n$.

تحويل داخلي
$f(x) = 2^{x+3}$

تحويل خارجي
$f(x) = 2^x + 3$

تحويل داخلي
تحويل خارجي
$f(x) = 2^{x-2} - 1$

★ إذا كان التحويل داخلي ⇐
نتحرك يمين ويسار بعكس إشارة
(h)

★ إذا كان التحويل خارجي ⇐
نتحرك لأعلى وأسفل بنفس إشارة
(k)

★ المدى يتأثر بالتحويل الخارجي
فقط..
$R = (K, \infty)$
وإن لم يوجد تحويل خارجي يكون
$R = (0, \infty)$

- $f(x) =$
$(-\infty, \infty$
- The x-ax
- The grap

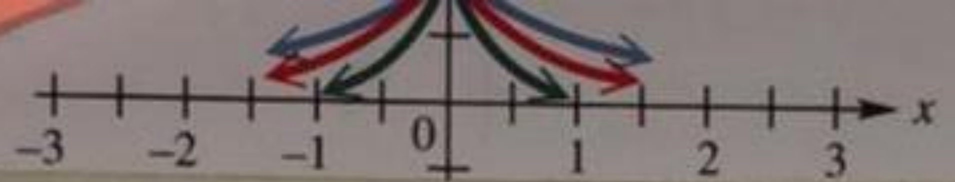
For $f($

- $f(x) =$
domain,
- The x-ax
- The grap

The graph
y-axis. Thus, w

If $f(x$

This is support
The graph
larger values o
to the graph in
similar to the g
several typical



مثال خارجي

$f(x) = 2^{x-3} + 7$	$f(x) = -2^{-x+1} - 5$
inc.	inc.
$D = (-\infty, \infty)$	$D = (-\infty, \infty)$
$R = (7, \infty)$	$R = (-\infty, -5)$
$y = 7$	$y = -5$

w-

$$\left(x^{\cancel{\frac{4}{3}}}\right)^{\cancel{\frac{3}{4}}} = (81)^{\frac{3}{4}}$$

$$x = (3^{\cancel{4}})^{\frac{3}{4}}$$

$$x = 3^3$$

$$x = \boxed{27}, \boxed{-27}$$

± ← الجذر الزوجي

+ بس ← الجذر الفردي

HOMEWOR

Solve $\left(\frac{1}{3}\right)^{x}$

EXAMPLE

Solve 2^{x+4}

SOLUTION

Check by

Late
equation
possible.
like

since it
same ba

HOME

Solve x

Home Work [1]

★ للتحويل من معادلة لوغاريتم لمعادلة أسية احل بعكس عقارب الساعة ابتداءً من الأساس (a)

★ = = أسية = = لوغاريتم ⟸

لوغاريتم الأساس للناتج يساوي الأس.

Logarithmic Form	Exponential Form
$\log_2 8 = 3$	$2^3 = 8$
$\log_{1/2} 16 = -4$	$\left(\frac{1}{2}\right)^{-4} = 16$
$\log_{10} 100{,}000 = 5$	$10^5 = 100{,}000$
$\log_3 \frac{1}{81} = -4$	$3^{-4} = \frac{1}{81}$
$\log_5 5 = 1$	$5^1 = 5$
$\log_{3/4} 1 = 0$	$\left(\frac{3}{4}\right)^0 = 1$

Logarithmic Equations The definition of lo[g] **logarithmic equation,** which is an equation w[ith] term. Many logarithmic equations can be solved [...] exponential form.

HOMEWORK 1 **Solving Logarithmic Equations**

Solve each equation.

(a) $\log_x \frac{8}{27} = 3$ (b) $\log_4 x = \frac{5}{2}$

★ لايجاد قيمة عبارة لوغاريتم

1 أساوي العبارة بـ y

2 استخدم تعريف اللوغاريتم ($x = a^y$)

3 أحل المعادلة الاسية

مثال خارجي:-

1] $\log_{16} 4 = y$

2] $x = a^y$

$4 = 16^y$

$4 = (4^2)^y$

$4 = 4^{2y}$

$2y = 1 \Rightarrow \boxed{y = \frac{1}{2}}$

Logarithmic Functions We define the logari[thmic function as] follows.

Logarithmic Function

If $a > 0$, $a \neq 1$, and $x > 0$, then

$f(x) = \log_a x$

defines the **logarithmic function with base** a.

Exponential and logarithmic functions are inve[rses] ... $y = 2^x$ is shown in red in **Figure 17.** The graph of ... ing the graph of $y = 2^x$ across the line $y = x$. The ... defined by $y = \log_2 x$, shown in blue, has the y-a[xis] ...

x	2^x
−2	0.25
−1	0.5
0	1
1	2
2	4

x	$\log_2 x$
0.25	−2
0.5	−1
1	0
2	1
4	2

$\boxed{\text{* مسلمات في الدالات اللوغاريتمية}}$

* $\log_b 1 = 0$
* $\log_b b = 1$
* $\log_b b^x = x$
* $\log_b(-b) = \text{undefind}$

* المجال في الدالة اللوغاريتمية يتأثر بالتحويل الداخلي (h) فيكون مجالها (h, ∞)

* المدى في الدالة الأسية يتأثر بالتحويل الخارجي فيكون مداها (k, ∞)

* محور تقارب الدالة الأسية x-axis
معادلته $\Leftarrow$ $\boxed{y = 0}$

* محور تقارب الدالة اللوغاريتمية y-axis
معادلته $\Leftarrow$ $\boxed{x = 0}$

Since the dom
the range of a logar
same way, both th
rithmic function ar

Thus, lo

Logarithmi

For $f(x) = \log_2$

x
$\frac{1}{4}$
$\frac{1}{2}$
1
2
4
8

- $f(x) = \log_a$ domain, $(0, \infty$
- The y-axis is a
- The graph pass

For $f(x) = \log_{1/2}$

x
$\frac{1}{4}$
$\frac{1}{2}$
1
2
4
8

- $f(x) = \log_a x$, entire domain,
- The y-axis is a
- The graph passe

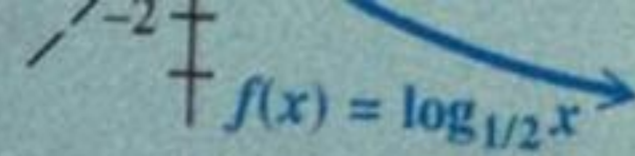


Figure 20

$$\boxed{\begin{aligned} f(x) &= \log_a x \\ f^{-1}(x) &= a^x \end{aligned}}$$

its

مثال خارجي

$f(x) = \log_2 x$

$f^{-1}(x) = 2^x$

$f(x) = 5^x$

$f^{-1}(x) = \log_5 x$

(b) Another way to graph a logarithmic f in exponential form as $x = 3^y$, and th sponding x-values. Several selected o the graph in **Figure 21.**

CAUTION If you write a logarithmi graph, as in **Example 2(b)**, start *first* sponding x-values. ***Be careful to write the correct order.***

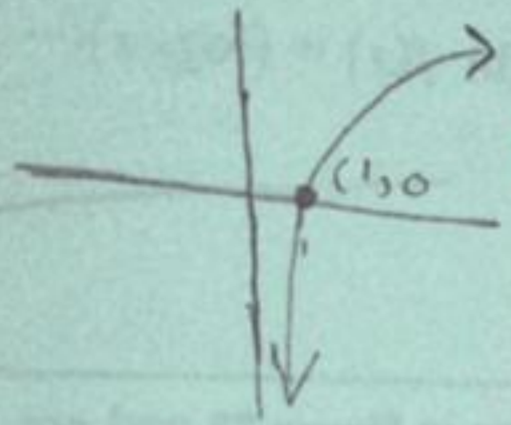
ers,
the
ga-

b] D = (0, ∞)

R = (−∞, ∞)

a > 1 ⇒ inc.

y-axis ⇒ x = 0

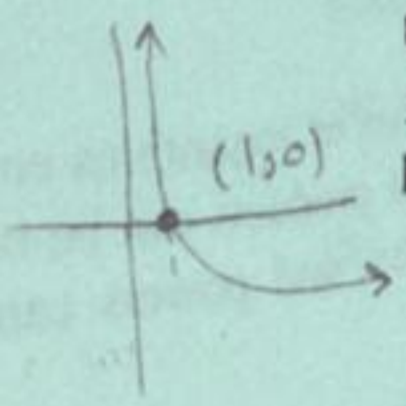


a] D = (0, ∞)

R = (−∞, ∞)

0 < a < 1 ⇒ dec.

y-axis ⇒ x = 0



The graphs in Figures 18 and 19 and the infor following generalizations about the graphs of loga $f(x) = \log_a x$.

Characteristics of the Graph of $f(x) = \log_a x$

1. The points $\left(\frac{1}{a}, -1\right)$, $(1, 0)$, and $(a, 1)$ are
2. If $a > 1$, then f is an increasing function decreasing function.
3. The y-axis is a vertical asymptote.
4. The domain is $(0, \infty)$, and the range is $(-$

(h,∞)

EXAMPLE 2 Graphing Logarithmic Functions

Graph each function.

(a) $f(x) = \log_{1/2} x$

SOLUTION

(a) One approach is to first graph $y = \left(\frac{1}{2}\right)^x$, wh of f, by plotting points. Some ordered pairs graph shown in red in Figure 20. The graph o tion of the graph of $y = \left(\frac{1}{2}\right)^x$ across the line $y = \log_{1/2} x$ are found by interchanging the pairs for $y = \left(\frac{1}{2}\right)^x$. See the graph in blue in Fig

x	$y = \left(\frac{1}{2}\right)^x$
−2	4
−1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
4	[illegible]

x	$f(x) = \log_{1/2} x$
4	−2
2	−1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2
$\frac{1}{16}$	4

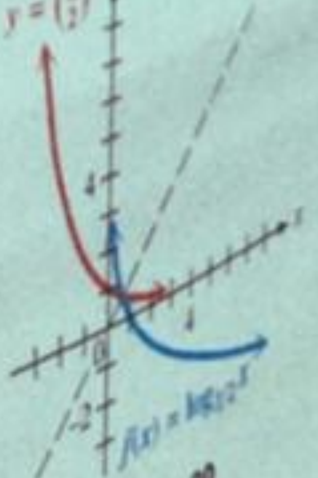


Figure 20

EXAMPLE 1 **Solving an Exponential Equation**

Solve $7^x = 12$. Give the solution to the neares

SOLUTION The properties of exponents given solve this equation, so we apply the precedi any appropriate base b can be used, the best p We choose base e (natural) logarithms here.

$$7^x = 12$$

$$\ln 7^x = \ln 12 \quad \text{Property of}$$

$$x \ln 7 = \ln 12 \quad \text{Power prop}$$

This is exact.

$$x = \frac{\ln 12}{\ln 7} \quad \text{Divide by}$$

$$x \approx 1.277$$

This is approx

The solution set is $\{1.277\}$.

* لحل معادلة أسية لدينا حالتين:

1. base = base
الأساس = الأساس

★ لحلها نخضع الأس = الأس

2. base ≠ base
نأخذ log أو ln للطرفين
طبقاً للمسألة

$2x-1$ $x+2$

Homework[1]

en evaluating a quotien
ent with $\ln \frac{12}{7}$, which ca
quotient of two logarit

$$\frac{\ln 12}{\ln 7} \neq \ln$$

HOMEWORK 1 **Solving an Exponential Equat**

Solve $3^{2x-1} = 0.4^{x+2}$. Give the solution to th

olving Base e Exponential Equa

ns to the ne

F.

28. $f(x) = \log_3(x-1)+2$

31, write an equation for the
se 2 or 3, translated and/or

31.

n. Simplify the result (if p
See Example 3.

34. $\log_4(2x\,5y)$

37. $\log_3 \frac{\sqrt{x}\cdot\sqrt{y}}{w^2\sqrt{z}}$

. Assume all variables

$\log_a t$

Solve $3^{2x-1} = 0.4^{x+2}$. Give the solution to the nearest thousandth.

quations in earlier sectio
d on the property hol
s are one-to-one.

wing holds

$\log_a y$

EXAMPLE 2 **Solving Base *e* Exponential Equations**

Solve each equation. Give solutions to the nearest thousandth.

اذا كان الأساس e بنختار
ناخذ ln

(a) $\boxed{e^{x^2}} = 200$

(b) $e^{2x+1} \cdot e^{-4x} = 3e$

SOLUTION

لازم e يكون في طرف لوحده

(a)

$$\begin{aligned} e^{x^2} &= 200 \\ \ln e^{x^2} &= \ln 200 && \text{Take the natural logarithm on each side.} \\ x^2 &= \ln 200 && \ln e^{x^2} = x^2 \text{ (Section 5.3)} \\ x &= \pm\sqrt{\ln 200} && \text{Square root property (Section 1.3)} \\ x &\approx \pm 2.302 && \text{Use a calculator.} \end{aligned}$$

Remember both roots.

The solution set is $\{\pm 2.302\}$.

don 1.3)

(Section 1.3)

c in Form

olve logarithms of

$x^3 - 19) = 3$

al form

Inverse,

لحل اي معادلة لوغاريتمية :

1) اذا كان اللوغاريتم في طرف والعدد الثابت في طرف نحولها للشكل الأسي

2) اذا كان log = log نساوي مابداخل اللوغاريتم

EXAMPLE 4 **Solving a Logarit**

Solve $\log_2[(3x-7)(x-4)]$

SOLUTION $\log_2[(3x-7)(x$

$(3x-7)($

$3x^2 - 19$

$3x^2 - 19$

$(3x-4)($

$3x - 4 = 0$

$x = \frac{4}{3}$

A check is necessary to be s equation is positive. In both $\log_2 8 = 3$ is true. The solut

HOMEWORK 4 **Solving a L**

Solve $\log(3x+2) + \log(x$

$\log_{10}[(3x+2)(x-1)] = 1$

$10^1 = (3x+2)(x-1)$

$10 = 3x^2 - 3x + 2x - 2$

$3x^2 - 3x + 2x - 2 - 10 = 0$

$3x^2 - x - 12 = 0$ بالآلة

$x = \frac{1+\sqrt{145}}{6}$ ✓ , $x = \frac{1-\sqrt{145}}{6}$ ✗ مرفوض

NOTE We could have u first writing

$\log(3x+2) + \log(x$

$\log_{10}[(3x+2)(x$

$(3x+2)(x$

and then continuing as sho

EXAMPLE 5 **Solving a Base**

Solve $\ln e^{\ln x} - \ln(x-3) =$

SOLUTION This logarithmi Homework 4 because the ex

$\ln e^{\ln x} - \ln(x$

$\ln x - \ln(x$

$\ln$

Chapter 6

(b) Convert 34.817° to degrees, minutes, and seconds to the nearest second.

(الوضع القياسي)

(origin point)

1 الرأس هو نقطة الأصل (0,0)

2 خط بداية الزاوية على محور x الموجب

الوضع القياسي

Standard Position An angle is in **standard position** if its vertex is at the origin and its initial side lies on the positive x-axis. The angles in **Figures 8(a) and 8(b)** are in standard position. An angle in standard position is said to lie in the quadrant in which its terminal side lies. An acute angle is in quadrant I (**Figure 8(a)**) and an obtuse angle is in quadrant II (**Figure 8(b)**). **Figure 8(c)** shows ranges of angle measures for each quadrant when $0° < \theta < 360°$.

$x<0, y>0$

+ Sin, CSC

2 1

All +

x and y > 0

3 4

+ tan, cot

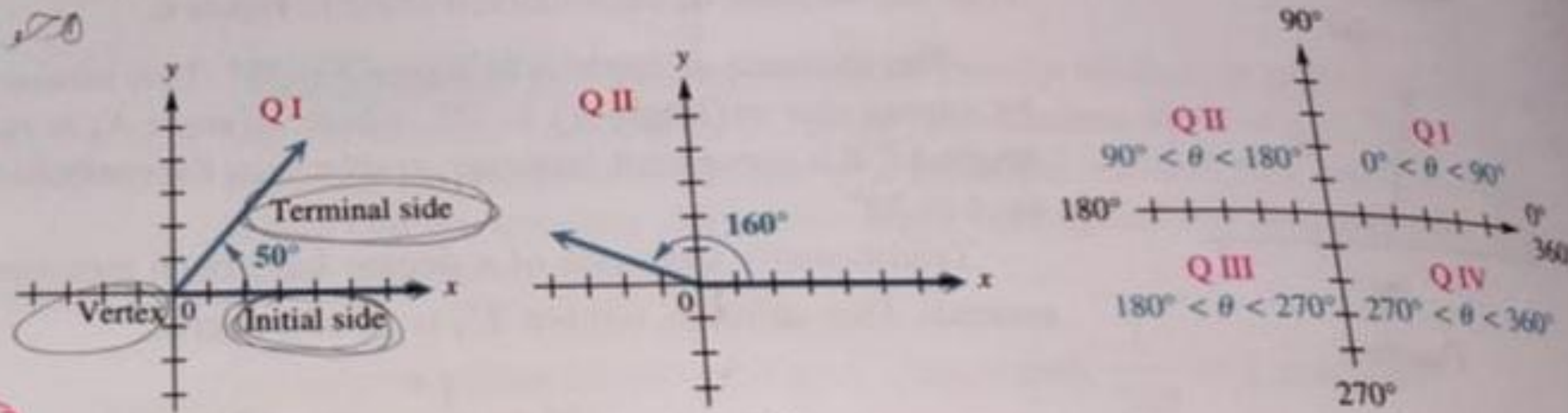
$x<a$

$y<0$

cos, sec +

$x>0$

$y<0$



(a) **(b)** **(c)**

Figure 8

(الزوايا الربعية)

Quadrantal Angles

Angles in standard position whose terminal sides lie on the x-axis or y-axis, such as angles with measures 90°, 180°, 270°, and so on, are **quadrantal angles.**

$r^2 = x^2 + y^2$

Notice that r

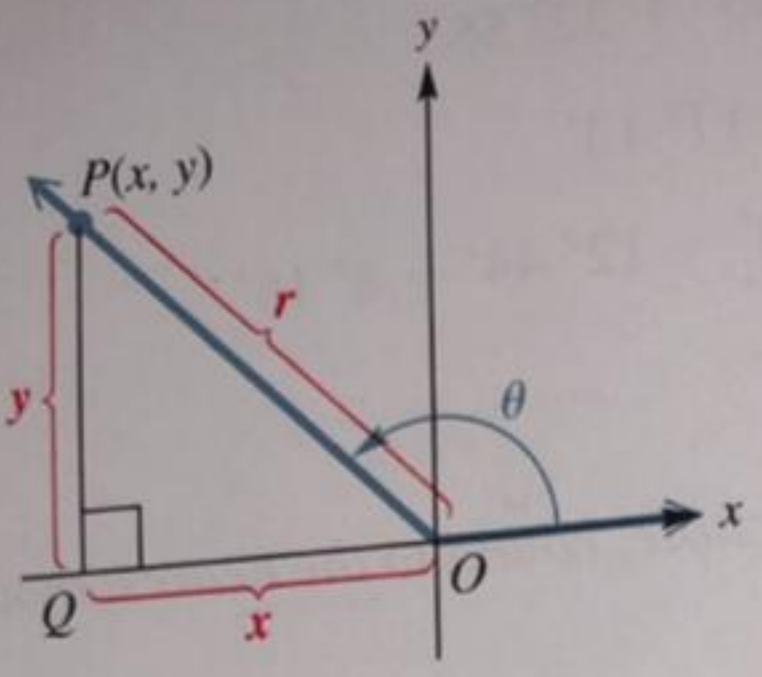
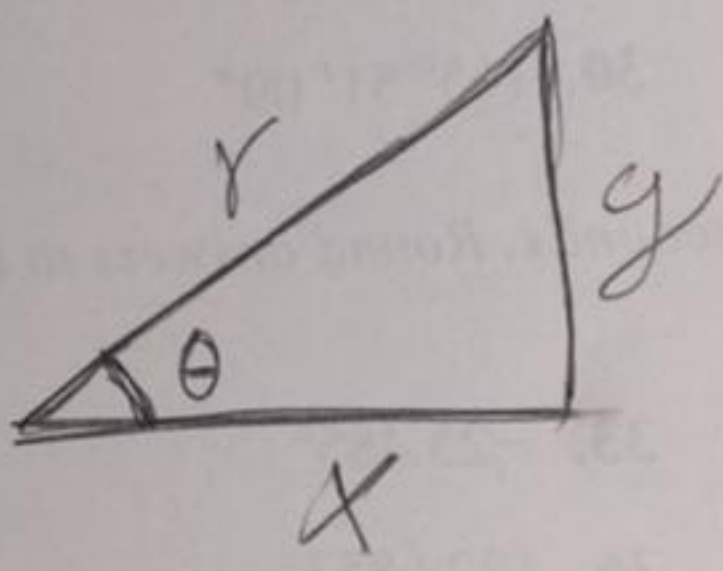


Figure 13



x = adjacent (المجاور)

y = opposite (المقابل)

r = hypotenuse (الوتر)

The six t

gent, secant

see

Trigonom

Let (x, y)

angle θ i

$r = \sqrt{x^2}$

$\sin\theta =$

$\csc\theta$

EXAMP

The term

passes thr

the six trig

SOLUTION

formed b

$(8, 15)$ to

right of t

$x = 8$ an

$r =$

We can n

0)

quadrantal

hen the

Also, $\cos\theta$ and $\sec\theta$ are reciprocals,
identities hold for any angle θ that does not lead to

حاصل ضرب اى دالة في
مقلوبها = 1

Reciprocal Identities

For all angles θ for which both functions are defined, the following identities hold.

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

The reciprocal identities can be written in different forms.

As before, we have given only algebraic transformations produce eq

sides of

$\sin^2\theta + c$

tity.

$= 1 - \sin^2$

ransform

homework 5

LOOKING AHEAD TO CALCULUS

The reciprocal, Pythagorean, and quotient identities are used in calculus to find derivatives and integrals of trigonometric functions. A standard technique of integration called **trigonometric substitution** relies on the Pythagorean identities.

Quotient Identities

Consider the qu

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r} \div \frac{x}{r}$$

Similarly, $\frac{\cos\theta}{\sin\theta} = \cot\theta$, for $\sin\theta \neq 0$.

Quotient Identities

For all angles θ for which the deno identities hold.

$\tan\theta$

$$\tan = \frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}}$$

$$\tan = \frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x} = \boxed{\frac{y}{x}}$$

$$\cot = \frac{\cos\theta}{\sin\theta} = \frac{\frac{x}{r}}{\frac{y}{r}}$$

$$\cot = \frac{x}{\cancel{r}} \cdot \frac{\cancel{r}}{y} = \boxed{\frac{x}{y}}$$

HOMEWORK 5 to Find

Find $\sin\theta$ and $\tan\theta$, given that $\cos\theta$

CAUTION *Be careful to choose th*

EXAMPLE 6 **Using Identities to Find Fun**

Find $\sin\theta$ and $\cos\theta$, given that $\tan\theta$

SOLUTION Since θ is in quadrant III, sin ing to say that since $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and tan This is *incorrect*, because $\sin\theta$ and $\cos\theta$

We use the Pythagorean identity t the reciprocal identity $\cos\theta = \frac{1}{\sec\theta}$ to

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(\frac{4}{3}\right)^2 + 1 = \sec^2\theta$$

$$\frac{16}{9} + 1 = \sec^2\theta$$

$$\tan\theta = \boxed{\frac{y}{x}} = \frac{(-)4}{(-)3}$$

$x = -3,\ y = -4,\ r = ?$

$$x^2 + y^2 = r^2$$

$$(-3)^2 + (-4)^2 = r^2$$

$$9 + 16 = r^2$$

$$\sqrt{25} = \sqrt{r^2}$$

$$\boxed{r = \pm 5}$$

$$\sin\theta = \frac{y}{r} = \boxed{\frac{-4}{5}}$$

$$\cos\theta = \frac{x}{r} = \boxed{\frac{-3}{5}}$$

- Cofunctions
- Trigonometric Function Values of Special Angles
- Reference Angles
- Special Angles as Reference Angles
- Finding Angle Measures

position. The defi
tion values of ang
in **Figure 22** x an
of the right triang
hypotenuse.

The side of
site angle A, an
side adjacent t
these sides to re
the trigonometri
the following rig

$\neq \sin\theta = \cos\theta$ of it's complemen

ساين لزاوية = كساين متممها

Right-Triang

Let A represe

$\sin A = \frac{y}{r}$

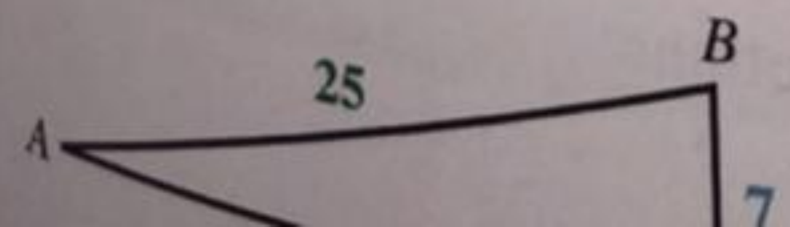
$\cos A = \frac{x}{r}$

$\tan A = \frac{y}{x}$

NOTE W
"side oppos

EXAMPLE 1

Find the sine



Chapter 7

ts of Functions

ng

rem

In order to speak meaningfully about rates of change, tangent lines, and areas bounded by curves, we have to investigate the process of finding limits. Indeed, the concept of *limit* is the cornerstone on which the development of calculus rests. Before we try to give a definition of a limit, let us look at more examples.

EXAMPLE 1

(2) $\frac{\text{Poly}}{\text{Poly}} \rightarrow$ anly (حلل) $\rightarrow$ lowest term (تبسيط) simplefy $\rightarrow$ sub. (تعويض)

(3) $\sqrt{}\ \pm$ no. $\rightarrow \times \frac{\sqrt{}\ \mp \text{no.}}{\sqrt{}\ \mp \text{no.}}$ (نضرب في المرافق) $\rightarrow$ simple form (تبسيط) $\rightarrow$ sub. (تعويض)

$\lim_{x \to a} f(x) = P(a)$

$\lim_{x \to a} C = C$ نهاية الثابت = ثابت

مثال $\lim_{x \to 2} 3 = 3$

2 − 2

H.W2

$y = f(x)$

=2

=2.001

=1.9

=1.99

Definition 1 An informal definition of limit

If $f(x)$ is defined for all x near a, except possibly at a itself, and if we can ensure that $f(x)$ is as close as we want to L by taking x close enough to a, but not equal to a, we say that the function f approaches the **limit** L as x approaches a, and we write

$$\lim_{x \to a} f(x) = L.$$

تعريف الكلام

$x_3 = 1.001 \quad f(x_3) = 2.001$

$x_4 = 0.9 \quad f(x_4) = 1.9$

$x_5 = 0.09 \quad f(x_5) = 1.99$

$x_6 = 0.009 \quad f(x_6) = 1.999$

Definition 1 An informal definition of limit

If $f(x)$ is defined for all x near a, except possibly at a itself, and if we can ensure that $f(x)$ is as close as we want to L by taking x close enough to a, but not equal to a, we say that the function f approaches the **limit** L as x approaches a, and we write

$$\lim_{x \to a} f(x) = L.$$

$x \neq a$ تقترب لكن لا تساوي →

التعويض مباشر

① sub → no. ✓

→ $\frac{\text{no.}}{0}$ doesn't exist

→ $\frac{0}{0}$ نشكل طريقة لمكونين امبا ش، نستخدم طريقة ثانية، نشوف شكل الدالة

↓ $\frac{0}{\text{no}}$ ↓ 0

(قوانين)

This definition is *informal* because phrases such as *close as we want* and *close enough* are imprecise; their meaning depends on the context. To a machinist manufacturing a piston, *close enough* may mean *within a few thousandths of an inch*. To an astronomer studying distant galaxies, *close enough* may mean *within a few thousand light-years*. The definition should be clear enough, however, to enable us to recognize and evaluate limits of specific functions. A more precise "formal" definition, given in Section 7.4, is needed if we want to *prove* theorems about limits like Theorems 2–4, stated later in this section.

approaches 0; there is no single number L that they approach.

The following example shows that even if $f(x)$ is defined at $x = a$, the limit of $f(x)$ as x approaches a may not be equal to $f(a)$.

BEWARE!

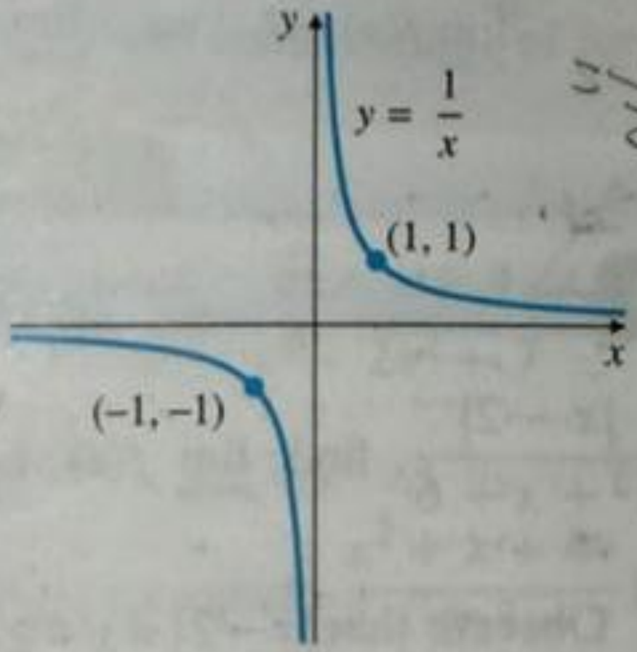
Always be aware that the existence of $\lim_{x \to a} f(x)$ does not require that $f(a)$ exist and does not depend on $f(a)$ even if $f(a)$ does exist. It depends only on the values of $f(x)$ for x near but not equal to a.

EXAMPLE 3

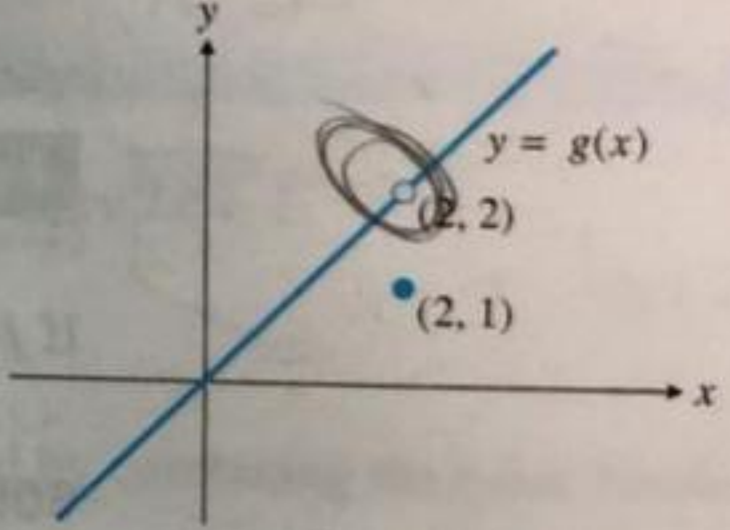
Let $g(x) = \begin{cases} x & \text{if } x \neq 2 \\ 1 & \text{if } x = 2. \end{cases}$ (See **Figure 2(b)**.) Then

$$\lim_{x \to 2} g(x) = \lim_{x \to 2} x = 2, \qquad \text{although} \quad g(2) = 1.$$

$\lim_{x \to 2} g(x) = \lim_{x \to 2} x = 2$



(a)



(b)

Figure 2

(a) $\lim_{x \to 0} \frac{1}{x}$ does not exist

(b) $\lim_{x \to 2} g(x) = 2$, but $g(2) = 1$

** لازم النهاية تكون $\boxed{\text{وحيده}}$

** اذا وجدت نهايتين لازم يكونو $\boxed{\text{متساويتين}}$

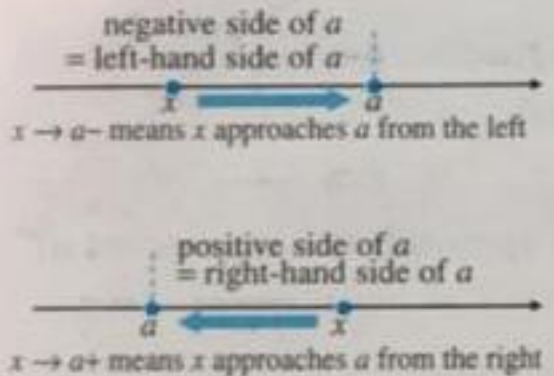


Figure 3 One-sided approach

One-Sided Limits Limits are *unique*; if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} f(x) = M$, then $L = M$. Although a function f can only have one limit at any particular point, it is, nevertheless, useful to be able to describe the behavior of functions that approach different numbers as x approaches a from one side or the other. (See **Figure 3**.)

$$\frac{x}{|x|} = \begin{cases} \frac{x}{x} & x > 0 \\ \frac{x}{-x} & x < 0 \end{cases}$$

$$\frac{x}{|x|} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

Definition 2 Informal definition of left and right limits

If $f(x)$ is defined on some interval (b, a) extending to the left of $x = a$, and if we can ensure that $f(x)$ is as close as we want to L by taking x to the left of a and close enough to a, then we say $f(x)$ has **left limit** L at $x = a$, and we write

$$\lim_{x\to a-} f(x) = L.$$

If $f(x)$ is defined on some interval (a, b) extending to the right of $x = a$, and if we can ensure that $f(x)$ is as close as we want to L by taking x to the right of a and close enough to a, then we say $f(x)$ has **right limit** L at $x = a$, and we write تقريبا

$$\lim_{x\to a+} f(x) = L.$$

الجهة اليمنى موجوده

الجهة اليسرى موجوده

= متساوين يوجد نهاية

≠ غير متساوين لا توجد نهاية

Note the use of the suffix + to denote approach from the right (the *positive* side) and the suffix − to denote approach from the left (the *negative* side).

ex 4

$$\lim_{x\to 2^+} \frac{x-2}{x^2+x-6}$$

$$\lim_{x\to 2^+} \frac{(x-2)}{(x-2)(x+3)} = \boxed{\frac{1}{5}}$$

$$\lim_{x\to 2^-} \frac{|x-2|}{x^2+x-6} = \frac{0}{0}$$

$$= \lim \frac{-(x-2)}{(x-2)(x+3)} = \boxed{\frac{-1}{5}}$$

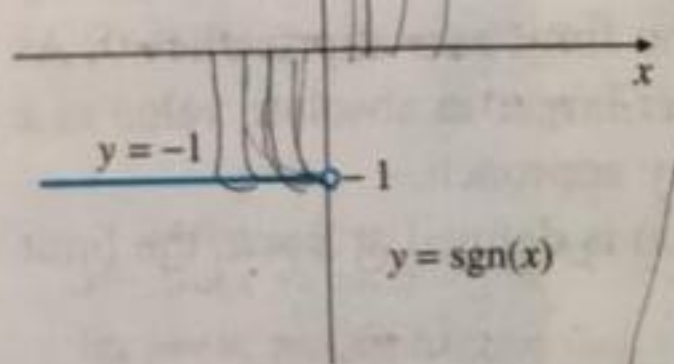


Figure 4 $\lim_{x\to 0} \operatorname{sgn}(x)$ does not exist, because $\lim_{x\to 0-} \operatorname{sgn}(x) = -1$, $\lim_{x\to 0+} \operatorname{sgn}(x) = 1$

$$\lim_{x\to 0-} \operatorname{sgn}(x) = -1 \quad \text{and} \quad \lim_{x\to 0+} \operatorname{sgn}(x) = 1$$

because the values of sgn (x) approach -1 (they *are* -1) if x is negative and approaches 0, and they approach 1 if x is positive and approaches 0. Since these left and right limits are not equal, $\lim_{x\to 0}$ sgn (x) *does not exist.*

$\lim_{x\to 0} \frac{x}{|x|} = \lim_{x\to -0} -1 = -1$

$\lim_{x\to 0} \frac{x}{|x|} = \lim_{x\to 0} 1 = 1$

$\lim_{x\to 0} \frac{x}{|x|} = \begin{cases} \lim_{x\to 0} \frac{x}{|x|} \\ \lim_{x\to 0} \frac{x}{|x|} \end{cases}$

dos not a

As suggested in Home Work 3, the relationship between ordinary (two-sided) limits and one-sided limits can be stated as follows:

Theorem 1: Relationship between one-sided and two-sided limits

A function $f(x)$ has limit L at $x = a$ if and only if it has both left and right limits there and these one-sided limits are both equal to L:

a^- a^+

$$\lim_{x\to a} f(x) = L \iff \lim_{x\to a-} f(x) = \lim_{x\to a+} f(x) = L.$$

لو النهاية موجودة لازمه النهاية اليمين واليسار موجودين وهما متساويين

EXAMPLE 4

If $f(x) = \dfrac{|x-2|}{x^2 + x - 6}$, find: $\lim_{x\to 2+} f(x)$, $\lim_{x\to 2-} f(x)$, and $\lim_{x\to 2} f(x)$.

SOLUTION Observe that $|x - 2| = x - 2$ if $x > 2$, and $|x - 2| = -(x - 2)$ if $x < 2$. Therefore,

1. If $P(x)$ is a polynomial and a is any real number, then

$$\lim_{x \to a} P(x) = P(a).$$

2. If $P(x)$ and $Q(x)$ are polynomials and $Q(a) \neq 0$, then

$$\lim_{x \to a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)}.$$

The Squeeze Theorem The following theorem will enable us to calculate some very important limits in subsequent chapters. It is called the *Squeeze Theorem* because it refers to a function g whose values are squeezed between the values of two other functions f and h that have the same limit L at a point a. Being trapped between the values of two functions that approach L, the values of g must also approach L. (See **Figure 5**.)

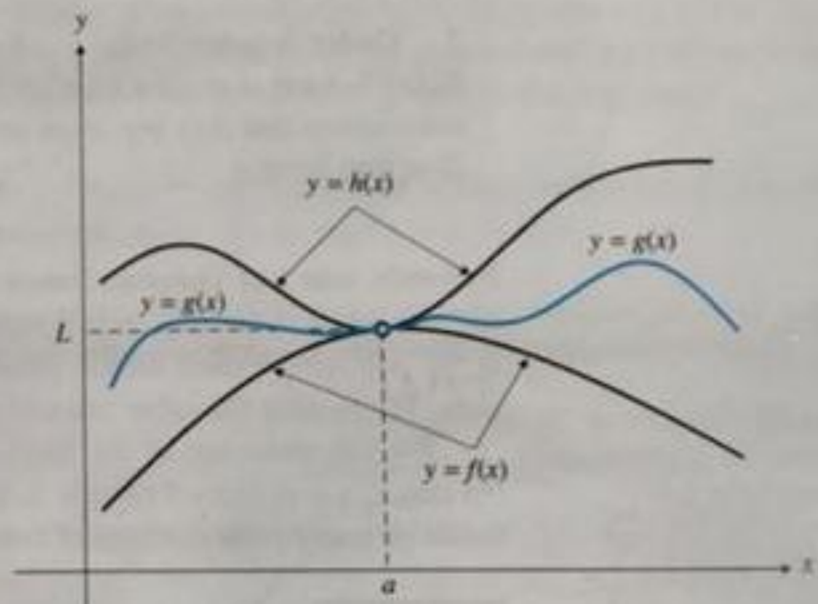


Figure 5 The graph of g is squeezed between those of f and h

$h(x) \leq f(x) \leq g(x)$

نهايتين موجودين ومتساويتين

إذا $f(x)$

متساوي $f(x) = h(x) = g(x)$

الهاية داله كثيرة الحدود تأتي مباشرة بالتعويض المباشر

إذا كان المقام بعد التعويض لا يساوي الصفر فإن النهاية تأتي بالتعويض المباشر.

$= 0$ تعويض

$= \frac{|x-2|}{x-2} = \frac{|0-2|}{0-2}$

$= \frac{2}{-2} = \boxed{-1}$

$16 - 16 + 1 = \boxed{1}$

$\frac{6}{9} = \boxed{\frac{2}{3}}$

$\frac{0}{2} = \boxed{0}$

$\frac{0}{0}$ تفشل التعويض

$\frac{(x-3)^2}{(x-3)(x+3)} = \frac{x-3}{x+3}$

$\frac{0}{6} = \boxed{0}$

1. Find: (a) $\lim f(x)$ ... shown in **Figure**

(a) 1

(b) 1

(c) 1

In Exercises 2–
given in Figure

2. $\lim_{x \to 1^-} g(x)$

In Exercises 4–

4. $\lim_{x \to 4} (x^2 -$

6. $\lim_{x \to 1} \frac{x^2 -}{x +}$

8. $\lim_{h \to 2} \frac{}{4 -}$

10. $\lim_{x \to \pi}$

$\frac{1}{0}$

$\frac{1}{\infty} = 0$

$\frac{1}{-\infty} = 0$

Limits at Infinity

C

sho

are

and

l la

$\lim_{x \to 0} \frac{1}{x}$ = doesn't exist

$\lim_{x \to \pm\infty} \frac{1}{x} = 0$

$\lim_{x \to 0^+} \frac{1}{x} = \infty$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

Recall that

cannot

لايجاد نهاية كثيرة الحدود عندما x تؤول لـ $+\infty$ أو $-\infty$ نعوض فقط بالـ term الذي يحمل أكبر قوة ..

إيجاد النهاية للدوال الكسرية عند $x \to +\infty$ أو $-\infty$ لدينا ثلاث حالات :

$\boxed{1}$ درجة البسط > درجة المقام
النهاية تكون $+\infty$ أو $-\infty$
بحسب اشارة الحد الذي يحمل أكبر قوة في البسط والمقام ..

$\star \lim_{x \to \infty} \left(\frac{-3x^2+7}{5x+1} \right) = \frac{-3x^2}{5x^1} = - = \boxed{-\infty}$

$\boxed{2}$ درجة البسط < درجة المقام
النهاية = $\boxed{\text{صفر}}$

$\boxed{3}$ درجة البسط = درجة المقام
النهاية تساوي ناتج قسمة أعلى حد في البسط على أعلى حد في المقام

SUMMARY OF LIMITS AT $\pm\infty$ FOR RATIONAL FUNCTIONS

Let $P_m(x) = a_m x^m + \cdots + a_0$ and $Q_n(x) = b_n x^n + \cdots + b_0$ be polynomials of degree m and n, respectively, so that $a_m \neq 0$ and $b_n \neq 0$. Then

$$\lim_{x \to \pm\infty} \frac{P_m(x)}{Q_n(x)}$$

(a) equals zero if $m < n$,
(b) equals $\frac{a_m}{b_n}$ if $m = n$,
(c) does not exist if $m > n$.

Product Rule for limits that lim
use this fact in the following e
limits at $\pm\infty$ for the function $x/$
making a table of values or draw

EXAMPLE 2

Evaluate $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$

SOLUTION Rewrite the express

$$f(x) = \frac{x}{\sqrt{x^2\left(1 + \frac{1}{x^2}\right)}} =$$

$$= \frac{x}{|x|\sqrt{1 + \frac{1}{x^2}}}$$

$$= \frac{\operatorname{sgn} x}{\sqrt{1 + \frac{1}{x^2}}},$$

The factor $\sqrt{1 + (1/x^2)}$ approache
the same limits as $x \to \pm\infty$ as doe

$$\lim_{x \to \infty} f(x) = 1$$

Limits at Infinity for Rational F

limits at $\pm\infty$ are constant ones, F
rational functions. Recall that a rat
The following examples show how
limits at infinity and negative inf
do this is to *divide the numerator*
appearing in the denominator. Th
negative infinity either both fail to

HOMEWORK 1 (Numerator and d

Evaluate $\lim_{x \to \pm\infty} \frac{2x^2 - x + 3}{3x^2 + 5}$.

HOMEWORK 2 (Degree of num

Evaluate $\lim_{x \to \pm\infty} \frac{5x+2}{2x^3 - 1}$.

The limiting behavior of rationa
summarized at the left.

The technique used in the p
general kinds of functions. Th
rational, and the limit seems to
matters by rationalizing the num

The factor $\sqrt{1+(1/x^2)}$ approaches 1 as x approaches ∞ or $-\infty$, so $f(x)$ must have the same limits as $x \to \pm\infty$ as does sgn (x). Therefore (see **Figure 7**),

② إذا كان لدينا جذر ± مقدار نضرب في مرافق الجذر ونقسمه

$\boxed{ex\ 3}$

$$\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{(\sqrt{x^2+x}+x)}$$

$$\frac{\cancel{x^2}+x-\cancel{x^2}}{\sqrt{x^2+x}+x} = \frac{x}{\sqrt{x^2(1+\frac{1}{x})}+x}$$

$$= \frac{x}{x\sqrt{1+\frac{1}{x}}+x}$$

$$= \frac{\cancel{x}}{\cancel{x}(\sqrt{1+\frac{1}{x}}+1)}$$

$$= \frac{1}{\sqrt{1+\frac{1}{x}}+1} = \frac{1}{\sqrt{1+\frac{1}{\infty}}+1} = \frac{1}{2}$$

HOMEWORK 4 (Polynomial behavior at infinit

(a) $\lim_{x\to\infty}(3x^3-x^2+2)=\infty$ (b) lim

(c) $\lim_{x\to\infty}(x^4-5x^3-x)=\infty$ (d) lim

لإيجاد نهاية كثيرة حدود
عند +∞ نعوض x→ نقطة
∞-
بالحد الذي يحمل الكبرى

a) $\lim_{x\to\infty} = 3(\infty)^3 = \boxed{\infty}$

b) $\lim_{x\to-\infty} = 3(-\infty)^3 = \boxed{-\infty}$

c) $\lim_{x\to\infty} = (\infty)^4 = \boxed{\infty}$

d) $\lim_{x\to-\infty} = (-\infty)^4 = \boxed{\infty}$

The highest-degree term of a polynomial dom
large, so the limits of this term at ∞ and $-\infty$
polynomial. For the polynomial in parts (a) ar

$$3x^3-x^2+2=3x^3\Big(1-$$

The factor in the large parentheses approache
havior of the polynomial is just that of its higl

We can now say a bit more about the limits a
rational function whose numerator has higl
Earlier in this section we said that such a limit
can assign ∞ or $-\infty$ to such limits, as the follo

EXAMPLE 5 (Rational functions with nume

Evaluate $\lim_{x\to\infty}\frac{x^3+1}{x^2+1}$.

SOLUTION Divide the numerator and the de
of x in the denominator:

$$\lim_{x\to\infty}\frac{x^3+1}{x^2+1}=\lim_{x\to\infty}\frac{x+\frac{1}{x^2}}{1+\frac{1}{x^2}}= \quad \text{li}$$

A polynomial $Q(x)$ of degree $n>0$ can have
most n different real numbers r for which $Q($
a rational function $R(x)=P(x)/Q(x)$, that func
those finitely many zeros of Q. At each of
infinite limits, or one-sided infinite limits. He

HOMEWORK 5

(a) $\lim_{x\to2}\frac{(x-2)^2}{x^2-4}=\lim_{x\to2}\frac{(x-2)^2}{(x-2)(x+2)}=$

(b) $\lim_{x\to2}\frac{x-2}{x^2-4}=\lim_{x\to2}\frac{x-2}{(x-2)(x+2)}=$

(c) $\lim_{x\to2+}\frac{x-3}{x^2-4}=\lim_{x\to2+}\frac{x-3}{(x-2)(x+2)}=$

(d) $\lim \frac{x-3}{x^2-4}=\lim_{x\to2-}\frac{x-3}{(x-2)(x+2)}$

zero

Sub. $\frac{n}{\infty}, \frac{n}{-\infty} = 0$; $\frac{\infty}{\infty}, \frac{-\infty}{\infty}, \frac{-\infty}{-\infty}$

Product Rule for limits that $\lim_{x\to\pm\infty} 1/x^n = 0$ for any positive integer n. We will use this fact in the following examples. Example 2 shows how to obtain the limits at $\pm\infty$ for the function $x/\sqrt{x^2+1}$ by algebraic means, without resorting to making a table of values or drawing a graph, as we did above.

EXAMPLE 2

Evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for $f(x) = \dfrac{x}{\sqrt{x^2+1}}$.

$\frac{P(x)}{Q(x)}$

SOLUTION Rewrite the expression for $f(x)$ as follows:

$$f(x) = \frac{x}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}} = \frac{x}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}}$$

Remember $\sqrt{x^2} = |x|$.

$x \to +\infty$ $|x| = x$

$$= \frac{x}{|x|\sqrt{1+\frac{1}{x^2}}}$$

$$= \frac{\operatorname{sgn} x}{\sqrt{1+\frac{1}{x^2}}},$$

where $\operatorname{sgn} x = \dfrac{x}{|x|} = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0. \end{cases}$

$x \to -\infty$ $|x| = -x$

The factor $\sqrt{1+(1/x^2)}$ approaches 1 as x approaches ∞ or $-\infty$, so $f(x)$ must have the same limits as $x \to \pm\infty$ as does sgn (x). Therefore (see **Figure 7**),

$f(x)$ if $x = 3$ if

1. $\lim_{x \to 3} f(x) = f(c)$

1) $\lim_{x \to c} f(x) = f(3)$

2) $\lim_{x \to 3} f(x) = -3$

3) $\lim_{x \to 3} f(x) = f(3)$

$f(x)$ is Constant at $x = 3$ if ?

(a) $\lim_{x \to 3} f(x) = 3$

(b) $\lim_{x \to 3} f(x) = f(c)$

(c) $\lim_{x \to 3} f(x) = f(3)$ ✓

(d) $\lim_{x \to c} f(x) = f(3)$

اتصال عند النقطة الداخلية

Definition 4 Continuity at an interior point

We say that a function f is **continuous** at an interior point c of its domain if

① النهاية موجودة ② متساويين ③ قيمة الدالة موجودة عند التعويض

$$\lim_{x \to c} f(x) = f(c).$$

If either $\lim_{x \to c} f(x)$ fails to exist or it exists but is not equal to $f(c)$, then we will say that f is **discontinuous** at c.

In graphical terms, f is continuous at an interior point c of its domain if its graph has no break in it at the point $(c, f(c))$; in other words, if you can draw the graph through that point without lifting your pen from the paper. Consider **Figure 10**. In (a), f is continuous at c. In (b), f is discontinuous at c because $\lim_{x \to c} f(x) \neq f(c)$. In (c), f is discontinuous at c because $\lim_{x \to c} f(x)$ does not exist. In both (b) and (c) the graph of f has a break at $x = c$.

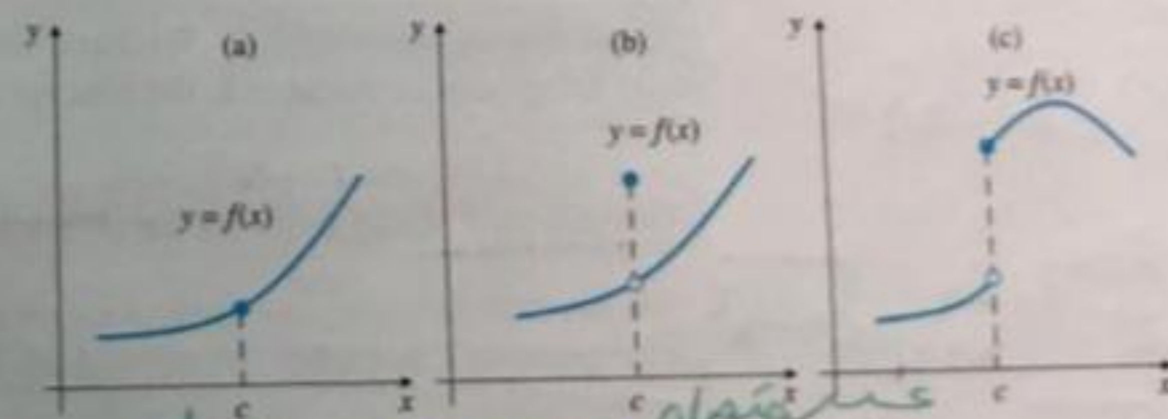


Figure 10
(a) f is continuous at c
(b) $\lim f(x) \neq f(c)$