ENG. Mahdi Alsiddig
0563938879

1.6 Continuity of Functions

- شروط اتصال الدالة عند قيمة معينة :

1. $f(c)$ exists
2. $\lim_{x \to c} f(x)$ exists
3. $f(c) = \lim_{x \to c} f(x)$

Problem Set 1.6

State whether the indicated function is continuous at 3:

(3) $h(x) = \frac{3}{x-3}$

Solution

$$1.\quad \boldsymbol{h}(\mathbf{3}) = \frac{3}{3-3} = \frac{3}{0} = \infty$$

$$\therefore \boldsymbol{h(x)\ not\ continous\ at\ x = 3}$$

(9) $h(x) = \frac{x^2-9}{x-3}$

Solution

$$1.\quad \boldsymbol{h}(\mathbf{3}) = \frac{9-9}{3-3} = \frac{\mathbf{0}}{\mathbf{0}}$$

$$\therefore \boldsymbol{h(x)\ not\ continous\ at\ x = 3}$$

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(13) $f(t) = \begin{cases} t-3, & if \;\; t \leq 3 \\ 3-t, & if \;\; t > 3 \end{cases}$

Solution

$$1.\, f(3) = t - 3 = 3 - 3 = \mathbf{0}$$

$$2.\, \lim_{x \to 3^+} 3 - t = 3 - 3 = \mathbf{0}$$

$$3.\, \lim_{x \to 3^-} 3 - t = 3 - 3 = \mathbf{0}$$

$$\boldsymbol{f(3) = \lim_{x \to 3^+} = \lim_{x \to 3^-} = 0}$$

$$\boldsymbol{\therefore f(t) \text{ is continous at } t = 3}$$

<u>Example 1 :</u> $f(x) = \frac{x^2-4}{x-2}$, How should f be defined at $x = 2$ in order to make it continuous there?

Solution

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x - 2} = \lim_{x \to 2}(x + 2) = 2 + 2 = \mathbf{4}$$

$$\boldsymbol{\therefore now\ we\ define\ f(2) = 4}$$

Example 3 : Determine all points of discontinuity of $f(x) = \frac{\sin x}{x(1-x)}$, $x \neq 0,1$. Classify each point of discontinuity as removable or nonremovable.

Solution

$\boldsymbol{x = 0}$:

$$\lim_{x\to 0} \frac{\sin x}{x(1-x)} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{(1-x)} = 1 \cdot 1 = \mathbf{1}$$

$\therefore$ ***we define*** $\boldsymbol{f(0) = 1}$ ***, Thus*** $\boldsymbol{x = 0}$ ***is a removeable discontiuty***

$\boldsymbol{x = 1}$:

$$\lim_{x\to 1^+} \frac{\sin x}{x(1-x)} = -\infty \qquad and \quad \lim_{x\to 1^-} \frac{\sin x}{x(1-x)} = \infty$$

$\therefore$ $\boldsymbol{x = 1}$ ***is a nonremoveable discontiuty***

Example 4 : Show that $h(x) = |\, x^2 - 3x + 6 \,|$ is continuous at each real number.

Solution

Domain of $h(x) = \boldsymbol{R}$

$\therefore$ $\boldsymbol{h(x)}$ **is continuous at each real number**

Example 5 : Show that $h(x) = \sin\frac{x^4-3x+1}{x^2-x-6}$ is continuous except at 3 and -2.

Solution

$\boldsymbol{x = 3}$:

$$h(3) = \sin\frac{81-9+1}{9-3-6} = \sin\frac{73}{0} = \sin\infty = \infty$$

$\therefore$ $\boldsymbol{h(x)}$ **is not continuous at x=3**

$\boldsymbol{x = -2}$:

$$h(3) = \sin\frac{16-6+1}{4+2-6} = \sin\frac{11}{0} = \sin\infty = \infty$$

$\therefore$ $\boldsymbol{h(x)}$ **is not continuous at x=-2**

Example 7 : What is the largest interval over which the function defined by $g(x) = \sqrt{4-x^2}$ is continuous ?

Solution

Domain of $g(x) = [-\mathbf{2}, \mathbf{2}]$

$\therefore$ $\boldsymbol{The\ largest\ interval}$ is $[-2, 2]$