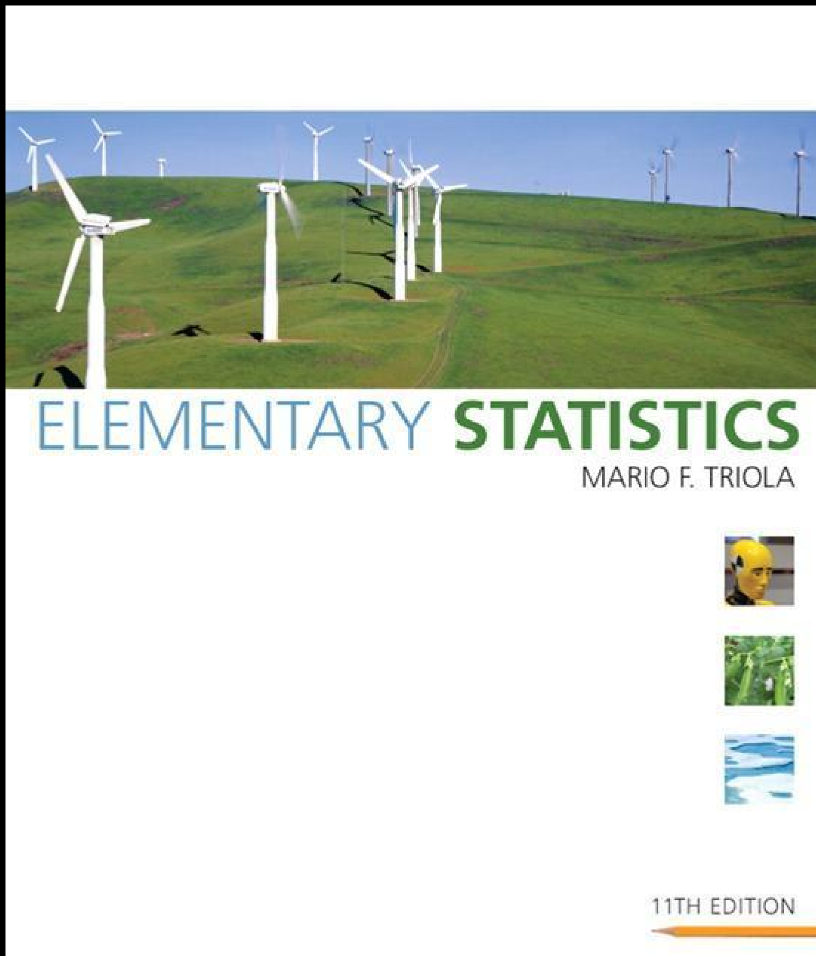
Lecture Slides



Elementary Statistics

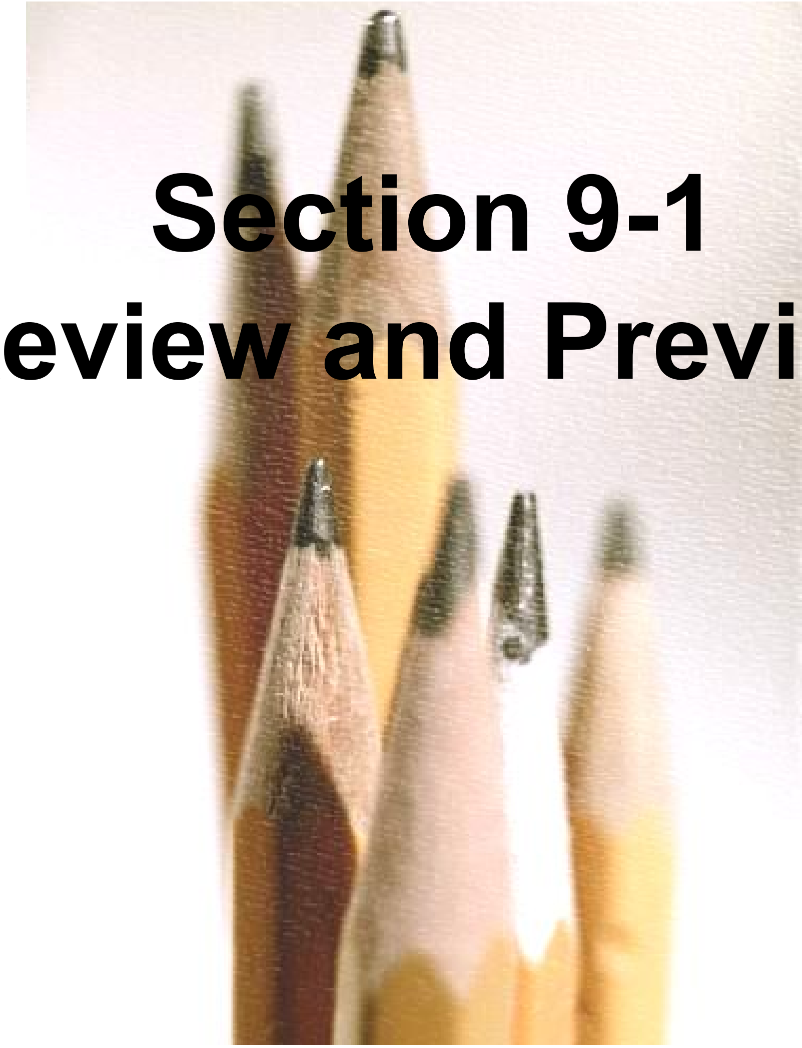
Eleventh Edition

and the Triola Statistics Series

by Mario F. Triola

Chapter 9
Inferences from Two Samples

Section 9-1
Review and Preview



Review

In Chapters 7 and 8 we introduced methods of *inferential statistics*. In Chapter 7 we presented methods of constructing confidence interval estimates of population parameters. In Chapter 8 we presented methods of testing claims made about population parameters. Chapters 7 and 8 both involved methods for dealing with a sample from a single population.

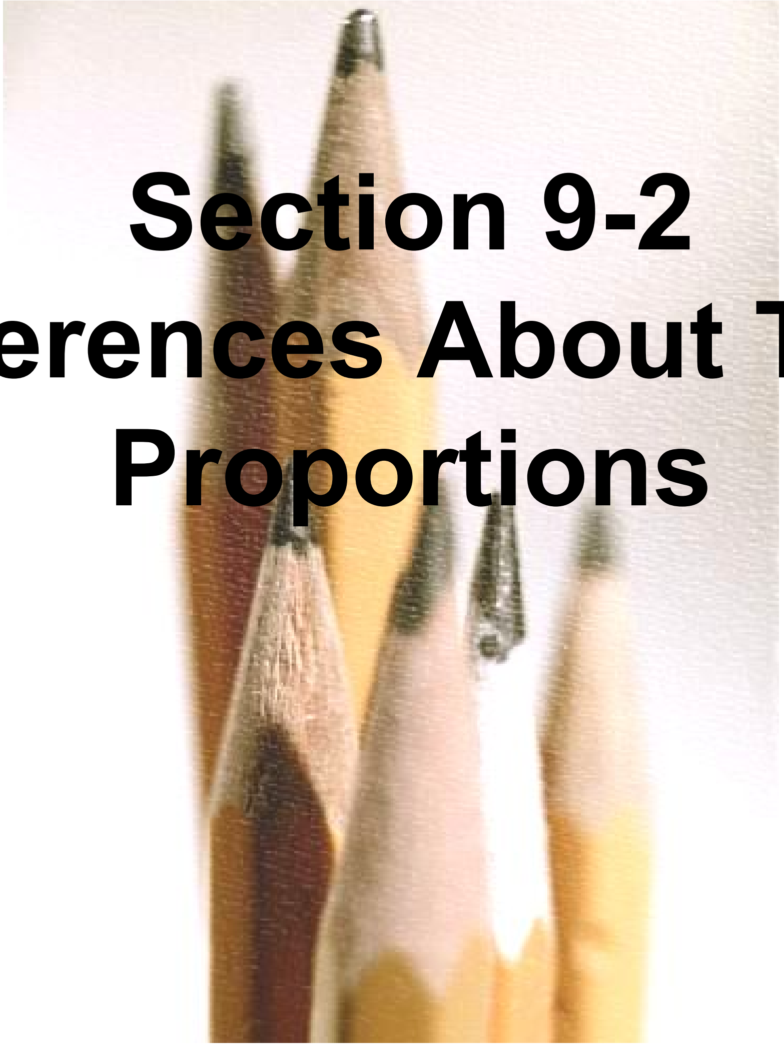
Preview

The objective of this chapter is to extend the methods for estimating values of population parameters and the methods for testing hypotheses to situations involving two sets of sample data instead of just one. The following are examples typical of those found in this chapter, which presents methods for using sample data from two populations so that inferences can be made about those populations.

Preview

- Test the claim that when college students are weighed at the beginning and end of their freshman year, the differences show a mean weight gain of 15 pounds (as in the "Freshman 15" belief).
- Test the claim that the proportion of children who contract polio is less for children given the Salk vaccine than for children given a placebo.
- Test the claim that subjects treated with Lipitor have a mean cholesterol level that is lower than the mean cholesterol level for subjects given a placebo.

Section 9-2
Inferences About Two Proportions



Key Concept

In this section we present methods for (1) testing a claim made about the two population proportions and (2) constructing a confidence interval estimate of the difference between the two population proportions. This section is based on proportions, but we can use the same methods for dealing with probabilities or the decimal equivalents of percentages.

Notation for Two Proportions

For population 1, we let:

p_1 = population proportion

n_1 = size of the sample

x_1 = number of successes in the sample

$\hat{p}_1 = \dfrac{x_1}{n_1}$ (the sample proportion)

$\hat{q}_1 = 1 - \hat{p}_1$

The corresponding notations apply to p_2, n_2 , x_2 , $\hat{p}_2$. and $\hat{q}_2$, which come from population 2.

Pooled Sample Proportion

- The **pooled sample proportion** is denoted by $\bar{p}$ and is given by:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

- We denote the complement of $\bar{p}$ by $\bar{q}$, so $\bar{q} = 1 - \bar{p}$

Requirements

1. We have proportions from two independent simple random samples.
2. For each of the two samples, the number of successes is at least 5 and the number of failures is at least 5.

Test Statistic for Two Proportions

For $H_0: p_1 = p_2$

$H_1: p_1 \neq p_2, \quad H_1: p_1 < p_2, \quad H_1: p_1 > p_2$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}$$

Test Statistic for Two Proportions - cont

For $H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$, $H_1: p_1 < p_2$, $H_1: p_1 > p_2$

where $p_1 - p_2 = 0$ **(assumed in the null hypothesis)**

$$\hat{p}_1 = \frac{x_1}{n_1} \quad \text{and} \quad \hat{p}_2 = \frac{x_2}{n_2}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{and} \quad \bar{q} = 1 - \bar{p}$$

Test Statistic for Two Proportions - cont

P-value: Use Table A-2. (Use the computed value of the test statistic z and find its *P*-value by following the procedure summarized by Figure 8-5 in the text.)

Critical values: Use Table A-2. (Based on the significance level α, find critical values by using the procedures introduced in Section 8-2 in the text.)

Confidence Interval Estimate of $p_1 - p_2$

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

where $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$

Example:

The table below lists results from a simple random sample of front-seat occupants involved in car crashes. Use a 0.05 significance level to test the claim that the fatality rate of occupants is lower for those in cars equipped with airbags.

	Airbag Available	No Airbag Available
Occupant Fatalities	41	52
Total number of occupants	11,541	9,853

Example:

Requirements are satisfied: two simple random samples, two samples are independent; Each has at least 5 successes and 5 failures (11,500, 41; 9801, 52).
Use the *P*-value method.

Step 1: Express the claim as $p_1 < p_2$.

Step 2: If $p_1 < p_2$ is false, then $p_1 \geq p_2$.

Step 3: $p_1 < p_2$ does not contain equality so it is the alternative hypothesis. The null hypothesis is the statement of equality.

Example:

H_0: $p_1 = p_2$ H_a: $p_1 < p_2$ (original claim)

Step 4: Significance level is 0.05

Step 5: Use normal distribution as an approximation to the binomial distribution. Estimate the common values of p_1 and p_2 as follows:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{41 + 52}{11{,}541 + 9{,}853} = 0.004347$$

With $\bar{p} = 0.004347$ **it follows** $\bar{q} = 0.995653$

Example:

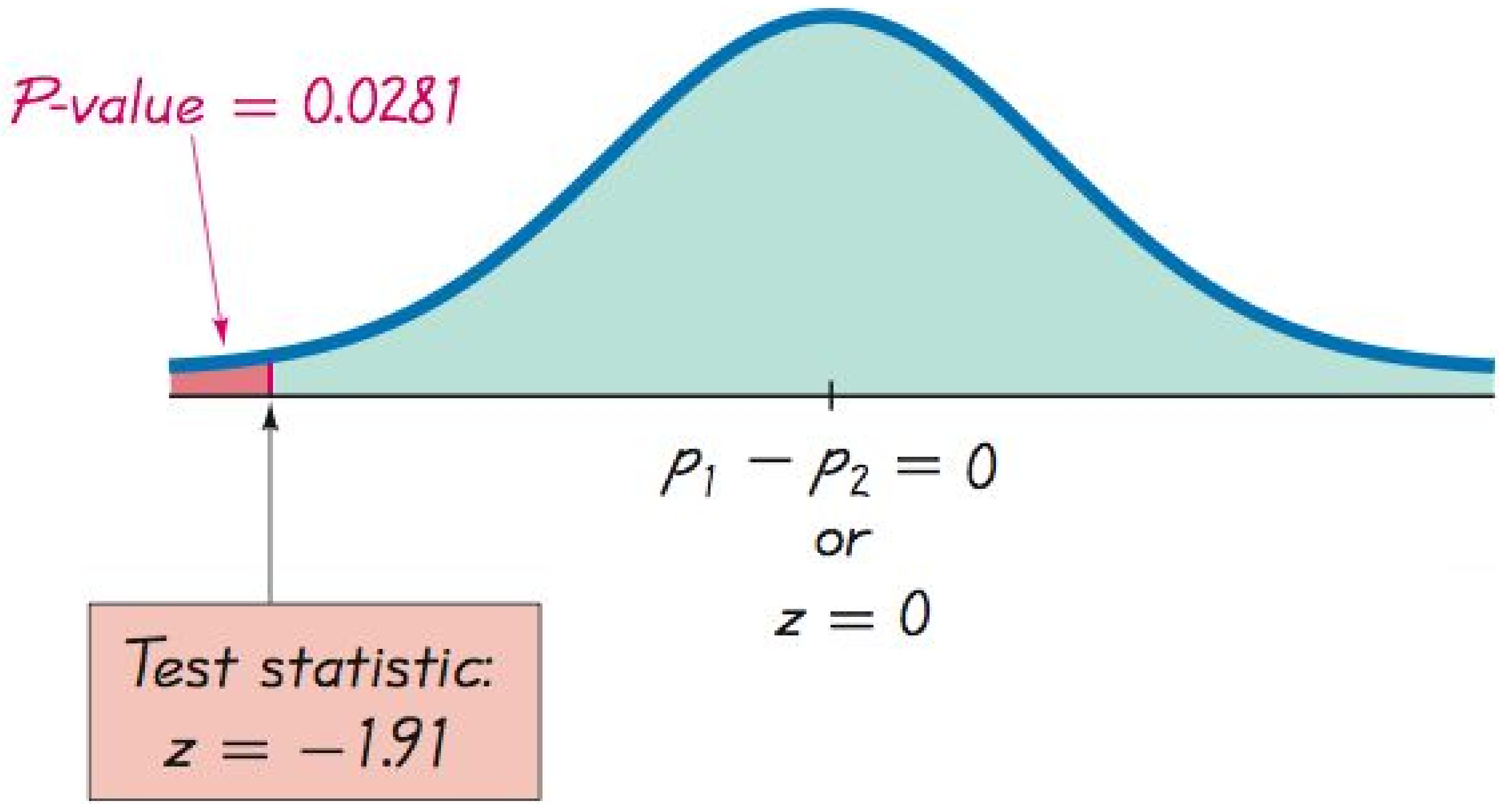
Step 6: Find the value of the test statistic.

$$z=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\overline{p}\overline{q}}{n_1}+\frac{\overline{p}\overline{q}}{n_2}}}$$

$$=\frac{\left(\frac{41}{11,541}-\frac{52}{9,853}\right)-0}{\sqrt{\frac{(0.004347)(0.995653)}{11,541}+\frac{(0.004347)(0.995653)}{9,853}}}$$

$$z=-1.91$$

Example:



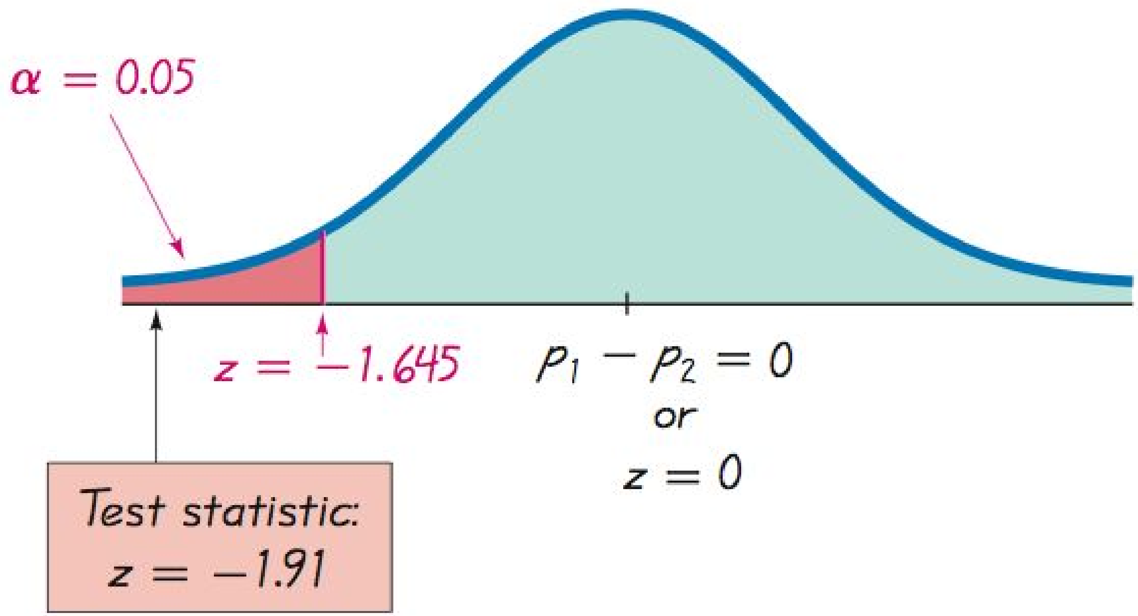
Left-tailed test. Area to left of $z = -1.91$ is 0.0281 (Table A-2), so the *P*-value is 0.0281.

Example:

Step 7: Because the P-value of 0.0281 is less than the significance level of $\alpha = 0.05$, we reject the null hypothesis of $p_1 = p_2$.

Because we reject the null hypothesis, we conclude that there is sufficient evidence to support the claim that the proportion of accident fatalities for occupants in cars with airbags is less than the proportion of fatalities for occupants in cars without airbags. Based on these results, it appears that airbags are effective in saving lives.

Example: Using the Traditional Method



With a significance level of α = 0.05 in a left- tailed test based on the normal distribution, we refer to Table A-2 and find that an area of α = 0.05 in the left tail corresponds to the critical value of *z* = –1.645. The test statistic of does fall in the critical region bounded by the critical value of *z* = –1.645. We again reject the null hypothesis.

Caution

When testing a claim about two population proportions, the *P*-value method and the traditional method are equivalent, but they are *not* equivalent to the confidence interval method. If you want to test a claim about two population proportions, use the *P*-value method or traditional method; if you want to estimate the difference between two population proportions, use a confidence interval.

Example:

Use the sample data given in the preceding Example to construct a 90% confidence interval estimate of the difference between the two population proportions. (As shown in Table 8-2 on page 406, the confidence level of 90% is comparable to the significance level of α = 0.05 used in the preceding left-tailed hypothesis test.) What does the result suggest about the effectiveness of airbags in an accident?

Example:

Requirements are satisfied as we saw in the preceding example.

90% confidence interval: $z_{\alpha/2} = 1.645$
Calculate the margin of error, E

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$= 1.645\sqrt{\frac{\left(\frac{41}{11{,}541}\right)\left(\frac{11{,}500}{11{,}541}\right)}{11{,}541} + \frac{\left(\frac{52}{9{,}853}\right)\left(\frac{9801}{9{,}853}\right)}{9{,}853}}$$

$$= 0.001507$$

Example:

Construct the confidence interval

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

$$(0.003553 - 0.005278) - 0.001507$$
$$< (p_1 - p_2) <$$
$$(0.003553 - 0.005278) + 0.001507$$

$$-0.00323 < (p_1 - p_2) < -0.000218$$

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Example:

The confidence interval limits do not contain 0, implying that there is a significant difference between the two proportions. The confidence interval suggests that the fatality rate is lower for occupants in cars with air bags than for occupants in cars without air bags. The confidence interval also provides an estimate of the amount of the difference between the two fatality rates.

Why Do the Procedures of This Section Work?

The distribution of $\hat{p}_1$ can be approximated by a normal distribution with mean p_1, standard deviation $\sqrt{p_1 q_1 / n_1}$, and variance $p_1 q_1 / n_1$.

The difference $\hat{p}_1 - \hat{p}_2$ can be approximated by a normal distribution with mean $p_1 - p_2$ and variance

$$\sigma^2_{(\hat{p}_1 - \hat{p}_2)} = \sigma^2_{\hat{p}_1} + \sigma^2_{\hat{p}_2} = \frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}$$

The variance of the *differences* between two independent random variables is the *sum* of their individual variances.

Why Do the Procedures of This Section Work?

The preceding variance leads to

$$\sigma_{(\hat{p}_1 - \hat{p}_2)} = \sqrt{\frac{\bar{p}_1 \bar{q}_1}{n_1} + \frac{\bar{p}_2 \bar{q}_2}{n_2}}$$

We now know that the distribution of $p_1 - p_2$ is approximately normal, with mean $p_1 - p_2$ and standard deviation as shown above, so the z test statistic has the form given earlier.

Why Do the Procedures of This Section Work?

When constructing the confidence interval estimate of the difference between two proportions, we don't assume that the two proportions are equal, and we estimate the standard deviation as

$$\sigma = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Why Do the Procedures of This Section Work?

In the test statistic

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}}$$

use the positive and negative values of z (for two tails) and solve for $p_1 - p_2$. The results are the limits of the confidence interval given earlier.

Recap

In this section we have discussed:

- **Requirements for inferences about two proportions.**
- **Notation.**
- **Pooled sample proportion.**
- **Hypothesis tests.**

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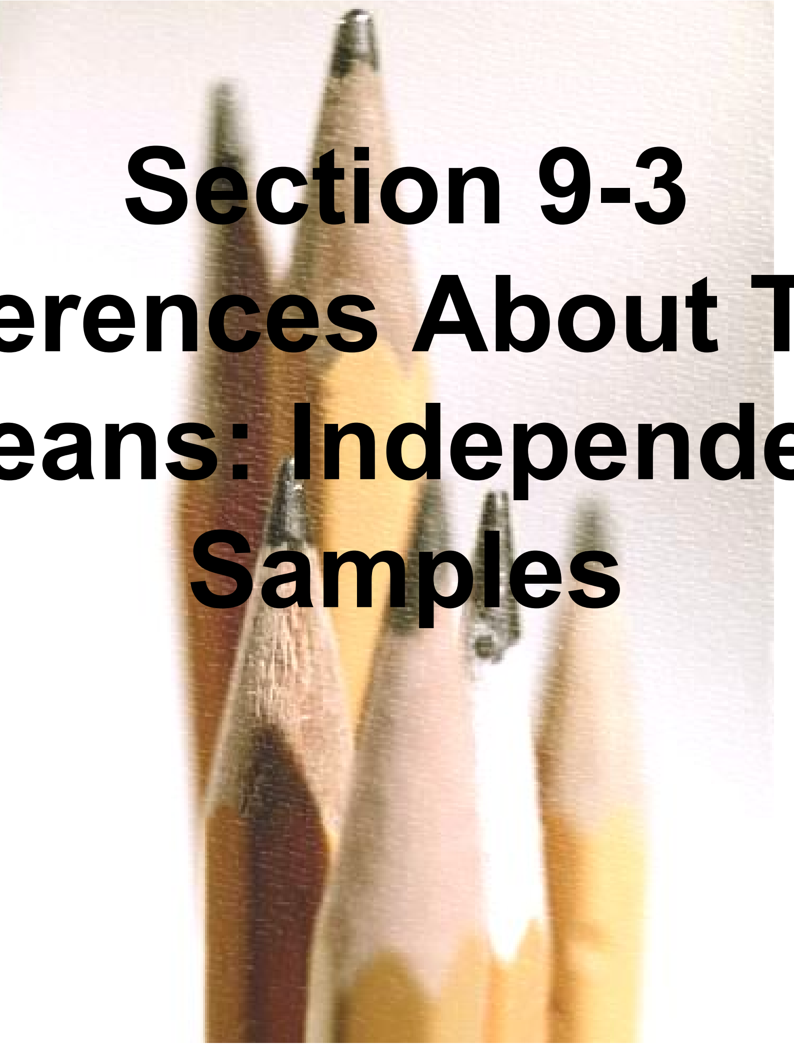
Section 9-3 Inferences About Two Means: Independent Samples



Key Concept

This section presents methods for using sample data from two independent samples to test hypotheses made about two population means or to construct confidence interval estimates of the difference between two population means.

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Key Concept

In Part 1 we discuss situations in which the standard deviations of the two populations are unknown and are not assumed to be equal. In Part 2 we discuss two other situations: (1) The two population standard deviations are both known; (2) the two population standard deviations are unknown but are assumed to be equal. Because is typically unknown in real situations, most attention should be given to the methods described in Part 1.

Part 1: Independent Samples with σ_1 and σ_2 Unknown and Not Assumed Equal

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Definitions

Two samples are **independent** if the sample values selected from one population are not related to or somehow paired or matched with the sample values from the other population.

Two samples are **dependent** if the sample values are *paired*. (That is, each pair of sample values consists of two measurements from the same subject (such as before/after data), or each pair of sample values consists of matched pairs (such as husband/wife data), where the matching is based on some inherent relationship.)

Notation

μ_1 = population mean

σ_1 = population standard deviation

n_1 = size of the first sample

$\overline{x}_1$ = sample mean

s_1 = sample standard deviation

Corresponding notations for μ_2, σ_2, s_2, $\overline{x}_2$ and n_2 apply to population 2.

Requirements

1. σ_1 an σ_2 are unknown and no assumption is made about the equality of σ_1 and σ_2 .
2. The two samples are independent.
3. Both samples are simple random samples.
4. Either or both of these conditions are satisfied: The two sample sizes are both large (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions.

Hypothesis Test for Two Means: Independent Samples

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

(where $\mu_1 - \mu_2$ is often assumed to be 0)

Hypothesis Test - cont

Test Statistic for Two Means: Independent Samples

Degrees of freedom: In this book we use this simple and conservative estimate: df = smaller of $n_1 – 1$ and $n_2 – 1$.

***P*-values:** Refer to Table A-3. Use the procedure summarized in Figure 8-5.

Critical values: Refer to Table A-3.

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Confidence Interval Estimate of $\mu_1 - \mu_2$: Independent Samples

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where $$E = t_{\alpha/2}\sqrt{\frac{s_1^{\,2}}{n_1} + \frac{s_2^{\,2}}{n_2}}$$

where df = smaller $n_1 - 1$ and $n_2 - 1$

Caution

Before conducting a hypothesis test, consider the context of the data, the source of the data, the sampling method, and explore the data with graphs and descriptive statistics. Be sure to verify that the requirements are satisfied.

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Example:

A headline in *USA Today* proclaimed that "Men, women are equal talkers." That headline referred to a study of the numbers of words that samples of men and women spoke in a day. Given below are the results from the study. Use a 0.05 significance level to test the claim that men and women speak the same mean number of words in a day. Does there appear to be a difference?

Number of Words Spoken in a Day	
Men	**Women**
$n_1 = 186$	$n_2 = 210$
$\bar{x}_1 = 15{,}668.5$	$\bar{x}_2 = 16{,}215.0$
$s_1 = 8632.5$	$s_2 = 7301.2$

Example:

Requirements are satisfied: two population standard deviations are not known and not assumed to be equal, independent samples, simple random samples, both samples are large.

Step 1: Express claim as $\mu_1 = \mu_2$.

Step 2: If original claim is false, then $\mu_1 \neq \mu_2$.

Step 3: Alternative hypothesis does not contain equality, null hypothesis does.

$H_0 : \mu_1 = \mu_2$ (original claim) $\quad H_a : \mu_1 \neq \mu_2$

Proceed assuming $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$.

Example:

Step 4: Significance level is 0.05

Step 5: Use a *t* distribution

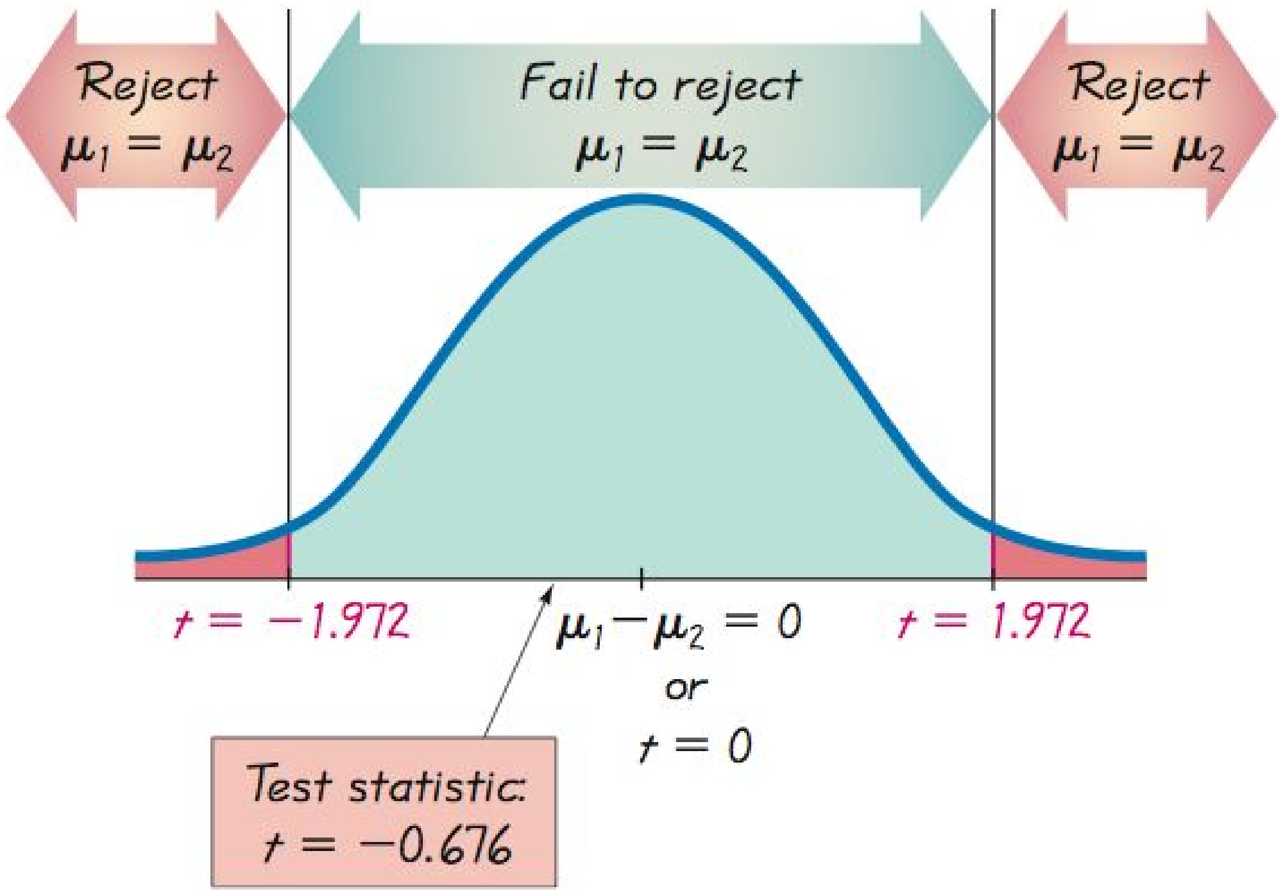
Step 6: Calculate the test statistic

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

$$= \frac{\left(15{,}668.5 - 16{,}215.0\right) - 0}{\sqrt{\dfrac{8632.5^2}{186} + \dfrac{7301.2^2}{210}}} = -0.676$$

Example:

Use Table A-3: area in two tails is 0.05, df = 185, which is not in the table, the closest value is
$t = \pm 1.972$



Example:

Step 7: Because the test statistic does not fall within the critical region, fail to reject the null hypothesis:
$\mu_1 = \mu_2$ (or $\mu_1 - \mu_2 = 0$).

There is not sufficient evidence to warrant rejection of the claim that men and women speak the same mean number of words in a day. There does not appear to be a significant difference between the two means.

Example:

Using the sample data given in the previous Example, construct a 95% confidence interval estimate of the difference between the mean number of words spoken by men and the mean number of words spoken by women.

Number of Words Spoken in a Day	
Men	Women
$n_1 = 186$	$n_2 = 210$
$\bar{x}_1 = 15{,}668.5$	$\bar{x}_2 = 16{,}215.0$
$s_1 = 8632.5$	$s_2 = 7301.2$

Example:

Requirements are satisfied as it is the same data as the previous example.

Find the margin of Error, *E*; use $t_{\alpha/2}$ = 1.972

$$E = t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 1.972\sqrt{\frac{8632.5^2}{186}+\frac{7301.2^2}{210}} = 1595.4$$

Construct the confidence interval use *E* = 1595.4 and $\bar{x}_1 = 15{,}668.5$ and $\bar{x}_2 = 16{,}215.0$.

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

$$-2141.9 < (\mu_1 - \mu_2) < 1048.9$$

Example:

Step 4: Significance level is 0.05

Step 5: Use a *t* distribution

Step 6: Calculate the test statistic

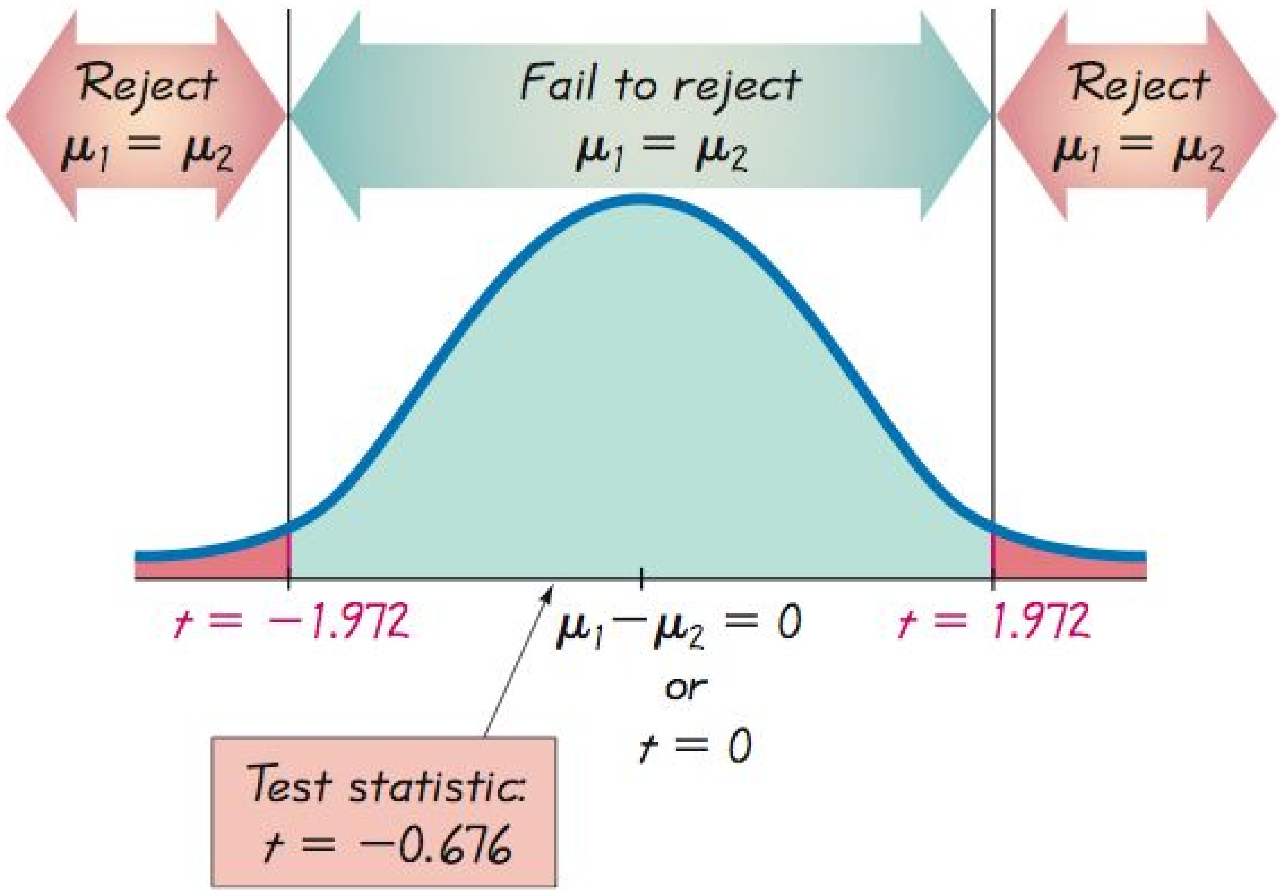
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

$$= \frac{(15{,}668.5 - 16{,}215.0) - 0}{\sqrt{\dfrac{8632.5^2}{186} + \dfrac{7301.2^2}{210}}} = -0.676$$

Example:

Use Table A-3: area in two tails is 0.05, df = 185, which is not in the table, the closest value is

$t = \pm 1.972$



Example:

Step 7: Because the test statistic does not fall within the critical region, fail to reject the null hypothesis:

$\mu_1 = \mu_2$ (or $\mu_1 - \mu_2 = 0$).

There is not sufficient evidence to warrant rejection of the claim that men and women speak the same mean number of words in a day. There does not appear to be a significant difference between the two means.

Part 2: Alternative Methods

Independent Samples with σ_1 and σ_2 Known.

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Requirements

1. The two population standard deviations are both known.

2. The two samples are independent.

3. Both samples are simple random samples.

4. Either or both of these conditions are satisfied: The two sample sizes are both large (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions.

Hypothesis Test for Two Means: Independent Samples with σ_1 and σ_2 Both Known

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

***P*-values** and **critical values**: **Refer to Table A-2.**

Confidence Interval: Independent Samples with σ_1 and σ_2 Both Known

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where $E = z_{\alpha/2} \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Methods for Inferences About Two Independent Means

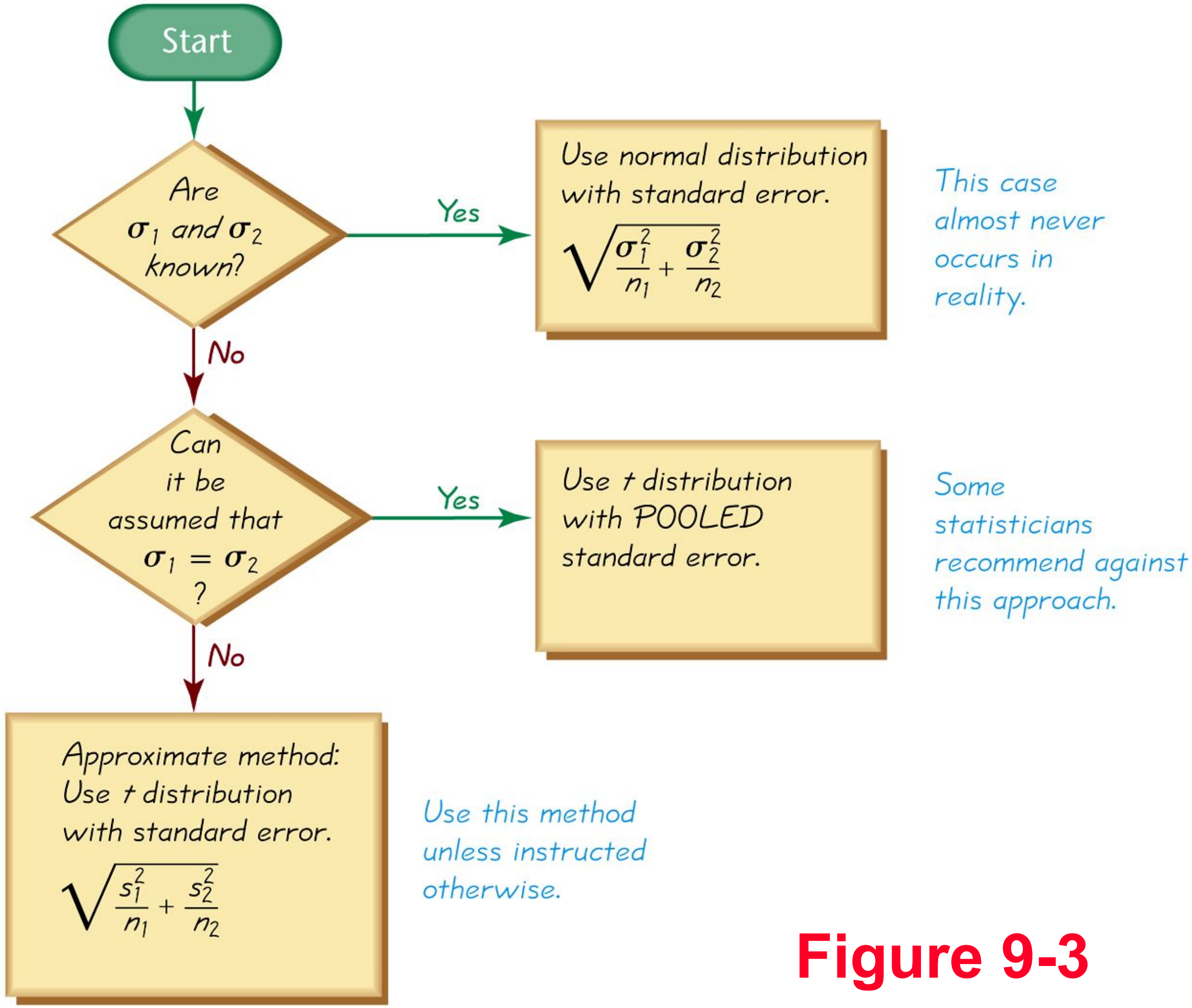


Figure 9-3

Assume that $\sigma_1 = \sigma_2$ and Pool the Sample Variances.

Requirements

1. The two population standard deviations are not known, but they are assumed to be equal. That is $\sigma_1 = \sigma_2$.
2. The two samples are independent.
3. Both samples are simple random samples.
4. Either or both of these conditions are satisfied: The two sample sizes are both large (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions.

Hypothesis Test Statistic for Two Means: Independent Samples and $\sigma_1 = \sigma_2$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}}$$

Where

$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

and the number of degrees of freedom is df = $n_1 + n_2 - 2$

Confidence Interval Estimate of $\mu_1 - \mu_2$: Independent Samples with $\sigma_1 = \sigma_2$

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where $$E = t_{\alpha/2} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

and number of degrees of freedom is df = $n_1 + n_2 - 2$

Strategy

Unless instructed otherwise, use the following strategy:

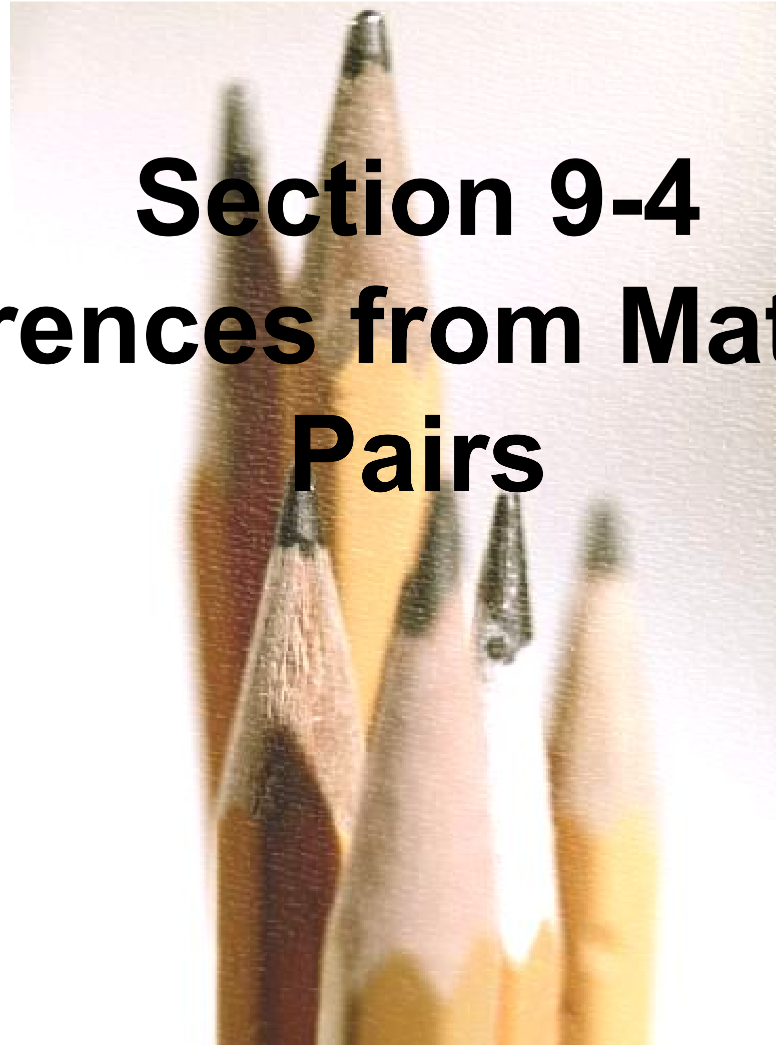
Assume that σ_1 and σ_2 are unknown, do not assume that $\sigma_1 = \sigma_2$, and use the test statistic and confidence interval given in Part 1 of this section. (See Figure 9-3.)

Recap

In this section we have discussed:

- **Independent samples with the standard deviations unknown and not assumed equal.**
- **Alternative method where standard deviations are known**
- **Alternative method where standard deviations are assumed equal and sample variances are pooled.**

Section 9-4
Inferences from Matched Pairs



Key Concept

In this section we develop methods for testing hypotheses and constructing confidence intervals involving the mean of the differences of the values from two dependent populations.

With dependent samples, there is some relationship whereby each value in one sample is paired with a corresponding value in the other sample.

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Key Concept

Because the hypothesis test and confidence interval use the same distribution and standard error, they are equivalent in the sense that they result in the same conclusions. Consequently, the null hypothesis that the mean difference equals 0 can be tested by determining whether the confidence interval includes 0. There are no exact procedures for dealing with dependent samples, but the *t* distribution serves as a reasonably good approximation, so the following methods are commonly used.

Notation for Dependent Samples

d = individual difference between the two values of a single matched pair

μ_d = mean value of the differences d for the **population** of paired data

$\bar{d}$ = mean value of the differences d for the paired **sample** data (equal to the mean of the $x - y$ values)

s_d = standard deviation of the differences d for the paired **sample** data

n = number of **pairs** of data.

Requirements

1. **The sample data are dependent.**

2. **The samples are simple random samples.**

3. **Either or both of these conditions is satisfied: The number of pairs of sample data is large (n > 30) or the pairs of values have differences that are from a population having a distribution that is approximately normal.**

Hypothesis Test Statistic for Matched Pairs

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

where degrees of freedom = n **– 1**

P-values and Critical Values

Use Table A-3 (*t*-distribution).

Confidence Intervals for Matched Pairs

$$\bar{d} - E < \mu_d < \bar{d} + E$$

where $E = t_{\alpha/2} \frac{s_d}{\sqrt{n}}$

Critical values of $t_{\alpha/2}$: Use Table A-3 with $n - 1$ degrees of freedom.

Example:

Data Set 3 in Appendix B includes measured weights of college students in September and April of their freshman year. Table 9-1 lists a small portion of those sample values. (Here we use only a small portion of the available data so that we can better illustrate the method of hypothesis testing.) Use the sample data in Table 9-1 with a 0.05 significance level to test the claim that for the population of students, the mean change in weight from September to April is equal to 0 kg.

Example:

Table 9-1 Weight (kg) Measurements of Students in Their Freshman Year

April weight	66	52	68	69	71
September weight	67	53	64	71	70
Difference d = (April weight) − (September weight)	−1	−1	4	−2	1

Requirements are satisfied: samples are dependent, values paired from each student; although a volunteer study, we'll proceed as if simple random sample and deal with this in the interpretation; STATDISK displays a histogram that is approximately normal

Example:

Weight gained = April weight – Sept. weight

μ_d denotes the mean of the "April – Sept." differences in weight; the claim is $\mu_d = 0$ kg

Step 1: claim is $\mu_d = 0$ kg

Step 2: If original claim is not true, we have $\mu_d \neq 0$ kg

Step 3: H_0: $\mu_d = 0$ kg original claim
H_1: $\mu_d \neq 0$ kg

Step 4: significance level is $\alpha = 0.05$

Step 5: use the student t distribution

Example:

Step 6: find values of d and s_d

differences are: –1, –1, 4, –2, 1

d = 0.2 and s_d = 2.4

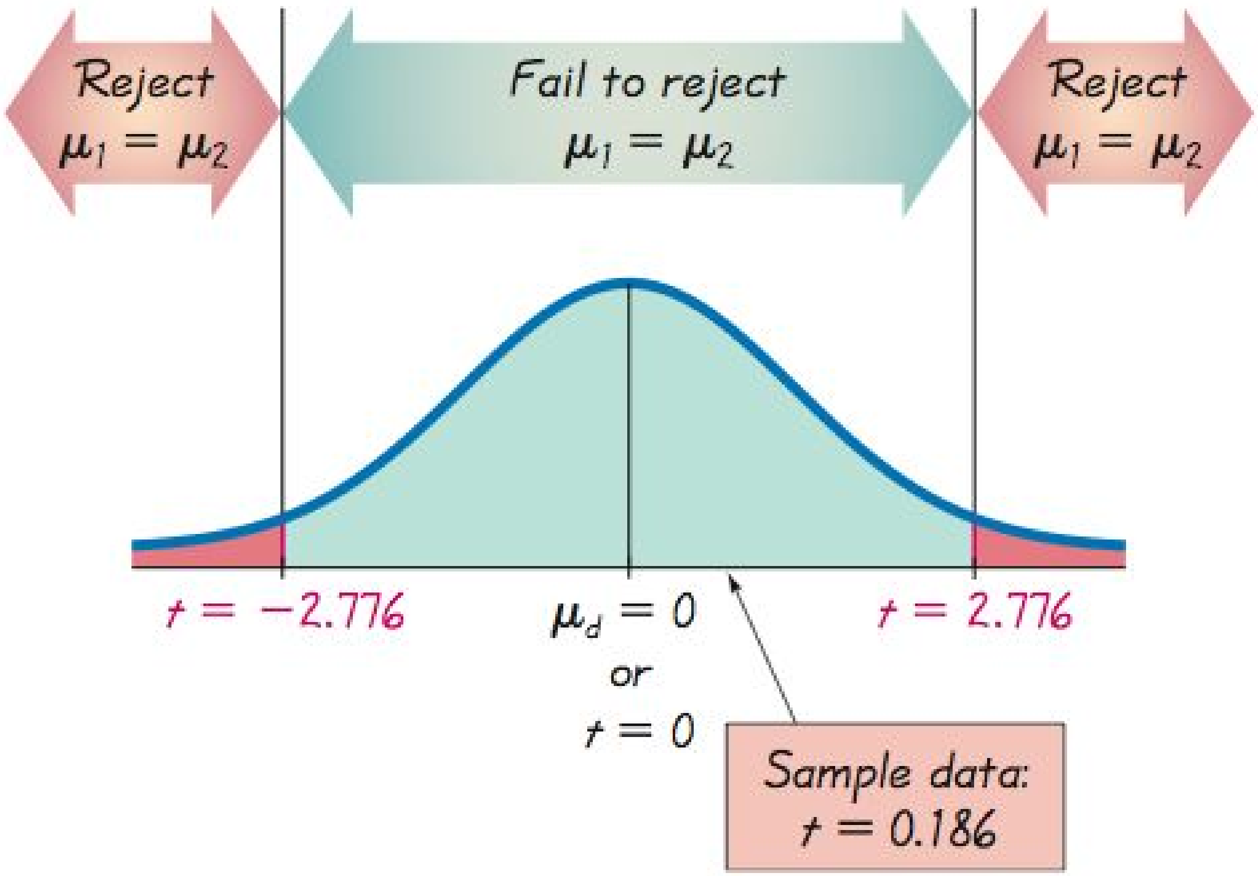
now find the test statistic

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{0.2 - 0}{\frac{2.4}{\sqrt{5}}} = 0.186$$

Table A-3: df = n – 1, area in two tails is 0.05, yields a critical value t = ±2.776

Example:

Step 7: **Because the test statistic does not fall in the critical region, we fail to reject the null hypothesis.**



Example:

We conclude that there is not sufficient evidence to warrant rejection of the claim that for the population of students, the mean change in weight from September to April is equal to 0 kg. Based on the sample results listed in Table 9-1, there does not appear to be a significant weight gain from September to April.

Example:

The conclusion should be qualified with the limitations noted in the article about the study. The requirement of a simple random sample is not satisfied, because only Rutgers students were used. Also, the study subjects are volunteers, so there is a potential for a self-selection bias. In the article describing the study, the authors cited these limitations and stated that "Researchers should conduct additional studies to better characterize dietary or activity patterns that predict weight gain among young adults who enter college or enter the workforce during this critical period in their lives."

Example:

The *P*-value method:

Using technology, we can find the *P*-value of 0.8605. (Using Table A-3 with the test statistic of t = 0.186 and 4 degrees of freedom, we can determine that the P-value is greater than 0.20.) We again fail to reject the null hypothesis, because the *P*-value is greater than the significance level of α = 0.05.

Example:

Confidence Interval method:

Construct a 95% confidence interval estimate of μ_d, which is the mean of the "April–September" weight differences of college students in their freshman year.

$\overline{d}$ **= 0.2, s_d = 2.4, n = 5, $t_{a/2}$ = 2.776**

Find the margin of error, E

$$E = t_{\alpha/2} \frac{s_d}{\sqrt{n}} = 2.776 \cdot \frac{2.4}{\sqrt{5}} = 3.0$$

Example:

Construct the confidence interval:

$$\bar{d} - E < \mu_d < \bar{d} + E$$

$$0.2 - 3.0 < \mu_d < 0.2 + 3.0$$

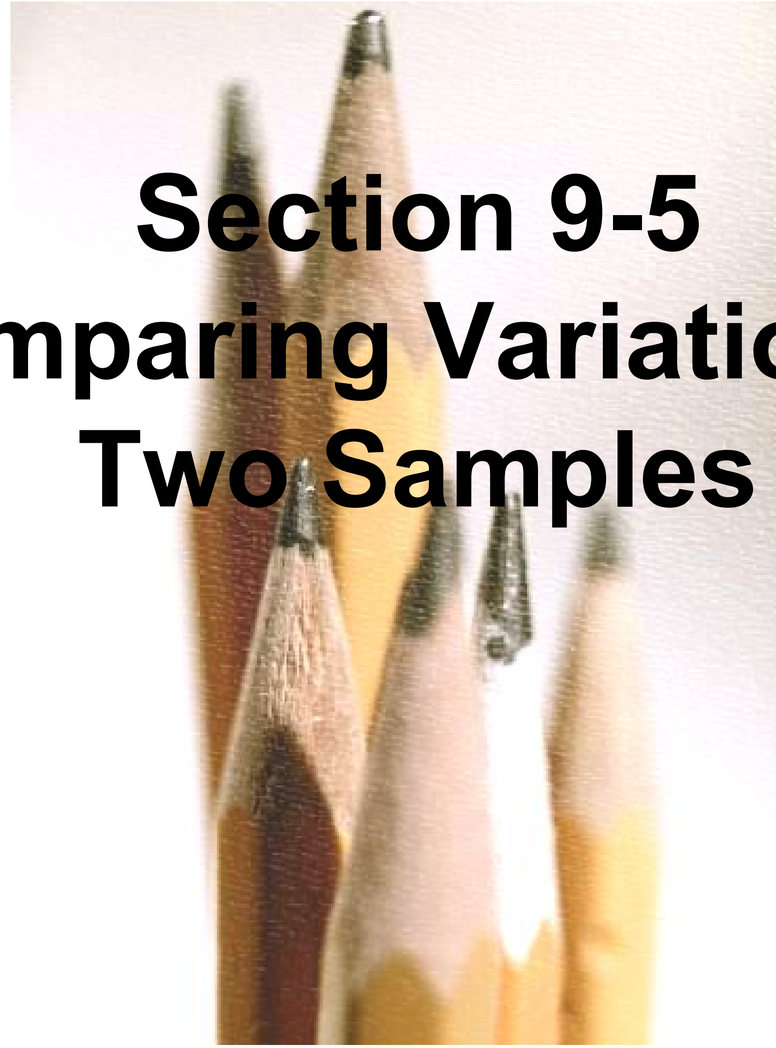
$$-2.8 < \mu_d < 3.2$$

We have 95% confidence that the limits of 2.8 kg and 3.2 kg contain the true value of the mean weight change from September to April. In the long run, 95% of such samples will lead to confidence interval limits that actually do contain the true population mean of the differences.

Recap

In this section we have discussed:

- **Requirements for inferences from matched pairs.**
- **Notation.**
- **Hypothesis test.**
- **Confidence intervals.**



Section 9-5
Comparing Variation in Two Samples

Key Concept

This section presents the *F* test for comparing two population variances (or standard deviations). We introduce the *F* distribution that is used for the *F* test.

Note that the *F* test is very sensitive to departures from normal distributions.

Part 1

F test for Comparing Variances

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Notation for Hypothesis Tests with Two Variances or Standard Deviations

s_1^2 = *larger* of two sample variances

n_1 = size of the sample with the *larger* variance

σ_1^2 = variance of the population from which the sample with the *larger* variance is drawn

$s_2^2, n_2,$ and σ_2^2 are used for the other sample and population

Requirements

1. The two populations are **independent**.
2. The two samples are simple random samples.
3. The two populations are each **normally distributed.**

Test Statistic for Hypothesis Tests with Two Variances

$$F = \frac{s_1^2}{s_2^2}$$

Where s_1^2 is the larger of the two sample variances

Critical Values: Using Table A-5, we obtain critical *F* values that are determined by the following three values:

1. The significance level α
2. Numerator degrees of freedom = $n_1 - 1$
3. Denominator degrees of freedom = $n_2 - 1$

Properties of the *F* Distribution

- The *F* distribution is not symmetric.
- Values of the *F* distribution cannot be negative.
- The exact shape of the *F* distribution depends on the two different degrees of freedom.

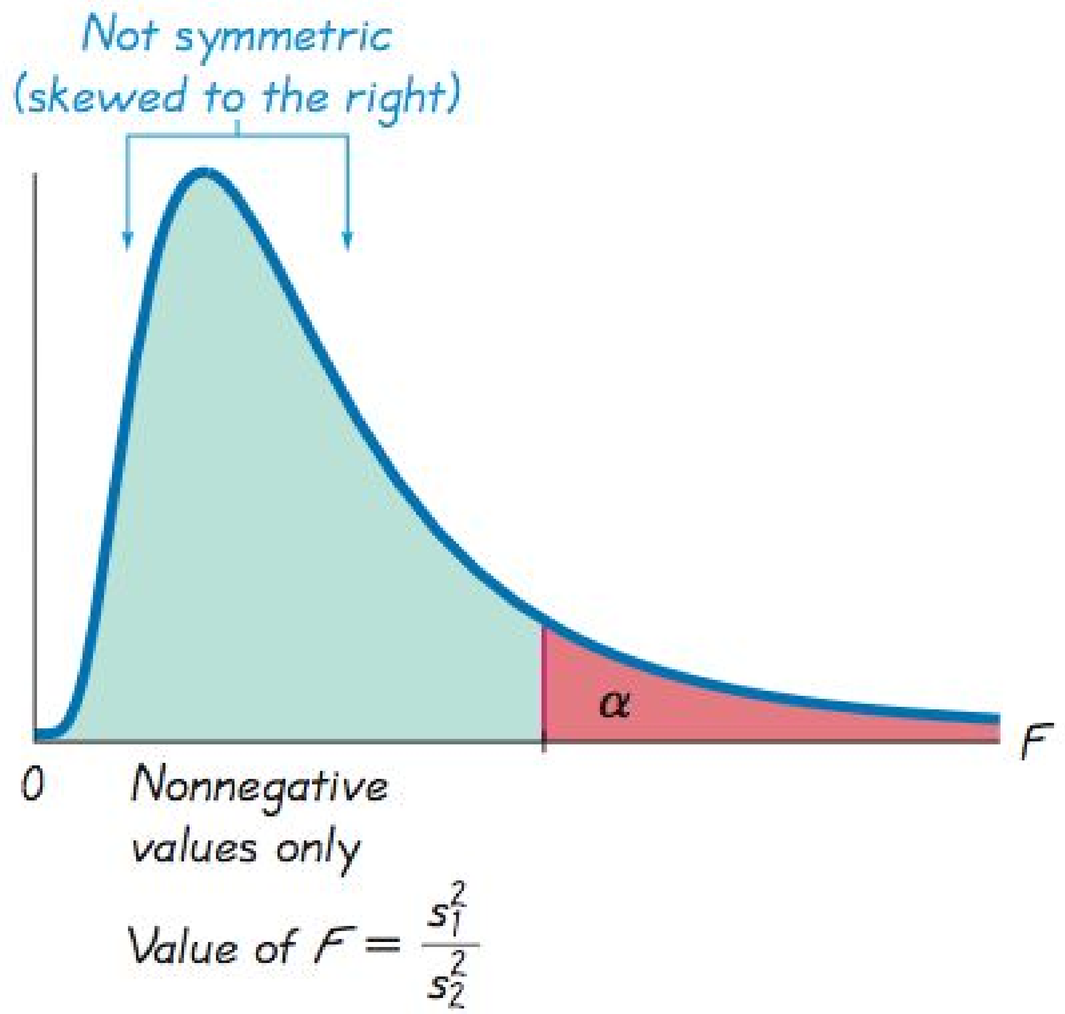
Finding Critical *F* Values

To find a critical *F* value corresponding to a 0.05 significance level, refer to Table A-5 and use the right-tail are of 0.025 or 0.05, depending on the type of test:

Two-tailed test: use 0.025 in right tail

One-tailed test: use 0.05 in right tail

Finding Critical *F* Values



Properties of the *F* Distribution - continued

If the two populations do have **equal variances**, then $F = \frac{s_1^2}{s_2^2}$ will be close to 1 because s_1^2 and s_2^2 are close in value.

Properties of the F Distribution - continued

If the two populations have radically different variances, then F will be a large number.

Remember, the larger sample variance will be s_1^2.

Conclusions from the *F* Distribution

Consequently, a value of F near 1 will be evidence in favor of the conclusion that $\sigma_1^2 = \sigma_2^2$.

But a large value of F will be evidence against the conclusion of equality of the population variances.

Example:

Data Set 20 in Appendix B includes weights (in g) of quarters made before 1964 and weights of quarters made after 1964. Sample statistics are listed below. When designing coin vending machines, we must consider the standard deviations of pre-1964 quarters and post-1964 quarters. Use a 0.05 significance level to test the claim that the weights of pre-1964 quarters and the weights of post-1964 quarters are from populations with the same standard deviation.

Pre-1964 Quarters	Post-1964 Quarters
$n = 40$	$n = 40$
$s = 0.08700$ g	$s = 0.06194$ g

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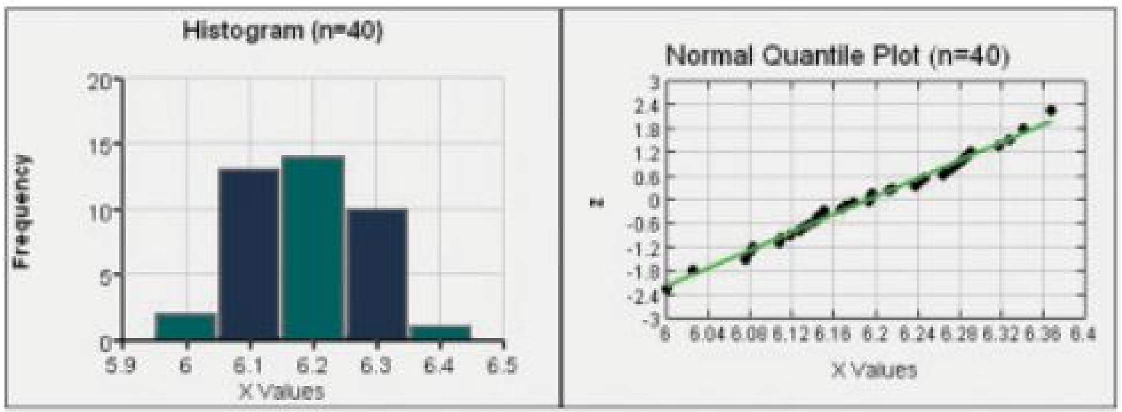
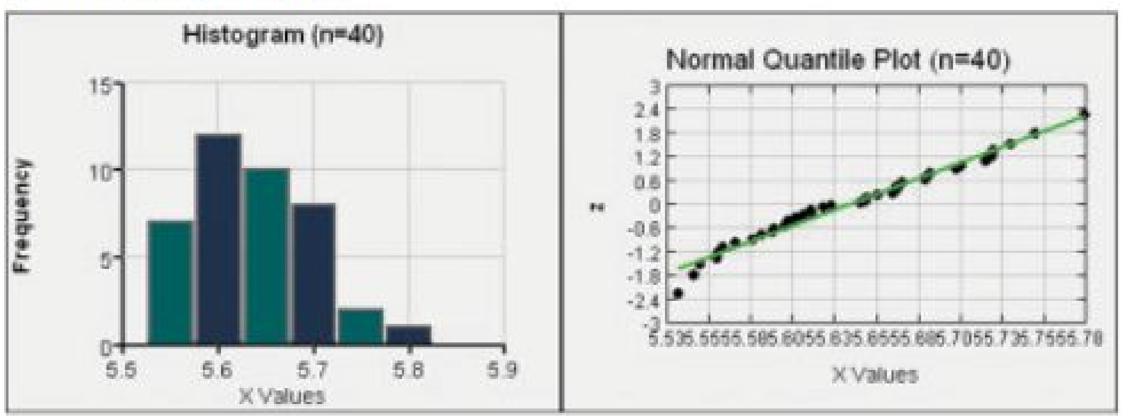
Example:

Requirements are satisfied: populations are independent; simple random samples; from populations with normal distributions

PRE-1964 QUARTERS



POST-1964 QUARTERS



Example:

Use sample variances to test claim of equal population variances, still state conclusion in terms of standard deviations.

Step 1: claim of equal standard deviations is equivalent to claim of equal variances

$$\sigma_1^2 = \sigma_2^2$$

Step 2: if the original claim is false, then

$$\sigma_1^2 \neq \sigma_2^2$$

Step 3: $H_0: \sigma_1^2 = \sigma_2^2$ **original claim**

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Example:

Step 4: significance level is 0.05

Step 5: involves two population variances, use *F* distribution variances

Step 6: calculate the test statistic

$$F = \frac{s_1^2}{s_2^2} = \frac{0.08700^2}{0.016194^2} = 1.9729$$

For the critical values in this two-tailed test, refer to Table A-5 for the area of 0.025 in the right tail. Because we stipulate that the larger variance is placed in the numerator of the F test statistic, we need to find only the right-tailed critical value.

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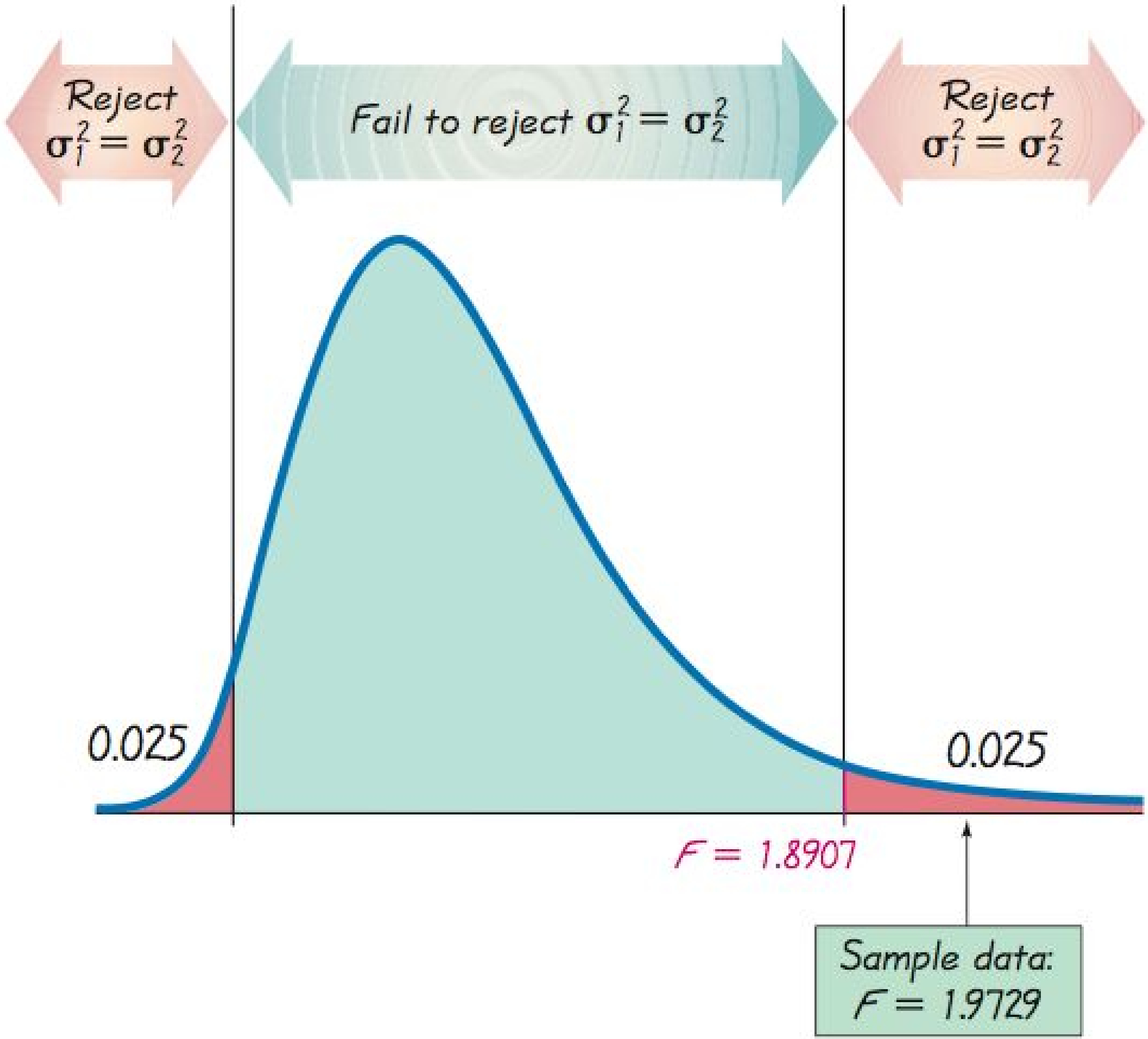
Example:

From Table A-5 we see that the critical value of F is between 1.8752 and 2.0739, but it is much closer to 1.8752. Interpolation provides a critical value of 1.8951, but STATDISK, Excel, and Minitab provide the accurate critical value of 1.8907.

Step 7: The test statistic $F = 1.9729$ does fall within the critical region, so we reject the null hypothesis of equal variances. There is sufficient evidence to warrant rejection of the claim of equal standard deviations.

Example:


Reject
$\sigma_1^2 = \sigma_2^2$
Fail to reject $\sigma_1^2 = \sigma_2^2$
Reject
$\sigma_1^2 = \sigma_2^2$
0.025
0.025
$F = 1.8907$
Sample data:
$F = 1.9729$

Example:

There is sufficient evidence to warrant rejection of the claim that the two standard deviations are equal. The variation among weights of quarters made after 1964 is significantly different from the variation among weights of quarters made before 1964.

Recap

In this section we have discussed:

- Requirements for comparing variation in two samples
- Notation.
- Hypothesis test.
- Confidence intervals.
- *F* test and distribution.