First Semester 1438/1439	Student Name:	اسم الطالب:
Midterm Exam / Stat. 101	Student ID:	رقم الطالب:
Exam Time: 13:00 — 15:00	No. Section:	رقم الشعبة:
Exam day: Sunday 01/03/1439	Trainer Name:	اسم المدرّب:

توقيع المدقق	توقيع المصحح	المجموع	10	9	8	7	6	5	4	3	2	1	السؤال
													الدرجة

عدد صفحات الأسئلة (4) عدد الأسئلة (10)

أجب عن جميع الأسئلة الآتية في الفراغات المخصّصة لها

(1.5 marks)

Question 1: Classify each variable as Qualitative or Quantitative.	The answer
The variable that record ID of students in an exam.	Qualitative
The variable that record weights of children in a school.	Quantitative
The variable that record colors of cars.	Qualitative

(1.5 marks)

Question 2: Classify each variable as Continuous or Discrete.	The answer
The variable that record heights of people.	Continuous
The variable that record numbers of children in schools of Riyadh city.	Discrete
The variable that record weight of books.	Continuous

(1.5 marks)

Question 3: Determine which of the following statements are True or False.	The answer
$\lim_{x \to \infty} F_X(x) = 0$	False
The mean is sensitive to extreme values.	True
Two events A and B are independent if $P(A \cup B) = P(A) + P(B)$.	False

(1.5 marks)

Question 4: Put the right word or symbol in its proper position: discrete space, continuous space, parameter, statistic , permutation, mutually independent, mutually exclusive.
Two events A and B are **mutually exclusive** if they cannot occur at the same time.
Any an arrangement of r distinct objects from a set of n different objects, is called a **permutation.**
If a space Ω consists uncountable number of outcomes, then Ω is called a **continuous space.**

0.25 أ.د.كانوي

(6 marks)
Question 5: Let 3, 7, 4, 6, 5, 12, 5, 6 be data of a sample. Then:

a) Calculate the mean for the given data.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{3+7+4+6+5+12+5+6}{8} = \frac{48}{8} = 6 \qquad 0.25+0.5$$

b) Calculate the median for the given data.

First we order the data $\underbrace{3}_{x_1}, \underbrace{4}_{x_2}, \underbrace{5}_{x_3}, \underbrace{5}_{x_4}, \underbrace{6}_{x_5}, \underbrace{6}_{x_6}, \underbrace{7}_{x_7}, \underbrace{12}_{x_8}$. Now because the number of data is an even, then we have: 0.5

$$\tilde{x} = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2} = \frac{x_4 + x_5}{2} = \frac{5+6}{2} = 5.5 \qquad 0.25+0.5$$

c) How much of modes we have in the given data, and then determine it.
The mode is the value that occurs most often in data set, therefore we have two modes:

$\hat{x}_1 = 5$ and $\hat{x}_2 = 6$ $\qquad$ ~~0.25~~+0.25

0.5 + 0.5

d) Calculate the range for the given data.

$R = x_\ell - x_s = 12 - 3 = 9$ $\qquad$ ~~0.25+0.25~~

e) Calculate Q_1, D_6 and P_{85} for the given data.

For Q_1: we have the rank of Q_1 is: $q_r = \frac{r(n+1)}{4} \Rightarrow q_1 = \frac{(8+1)}{4} = \underbrace{2}_{k}.\underbrace{25}_{s}$. $\qquad$ 0.25

Therefore, we get that: $Q_1 = x_k + s(x_{k+1} - x_k) = x_2 + 0.25(x_3 - x_2) = 4 + 0.25(5-4) = 4.25$ $\qquad$ 0.5

For D_6: we have the rank of D_6 is: $d_r = \frac{r(n+1)}{10} \Rightarrow d_6 = \frac{6(8+1)}{10} = \underbrace{5}_{k}.\underbrace{4}_{s}$. $\qquad$ 0.25

Therefore, we get that: $D_6 = x_k + s(x_{k+1} - x_k) = x_5 + 0.4(x_6 - x_5) = 6 + 0.4(6-6) = 6$ $\qquad$ 0.5

For P_{85}: we have the rank of P_{85} is: $p_r = \frac{r(n+1)}{100} \Rightarrow p_{85} = \frac{85(8+1)}{100} = \underbrace{7}_{k}.\underbrace{65}_{s}$. $\qquad$ 0.25

Therefore, we get that: $P_{85} = x_k + s(x_{k+1} - x_k) = x_7 + 0.65(x_8 - x_7) = 7 + 0.65(12-7) = 10.25$ $\qquad$ 0.5

f) If the variance of the given data is $S^2 = 8.6436$, then calculate the standard score for the value 7.

يخصم ربع على أي خطأ

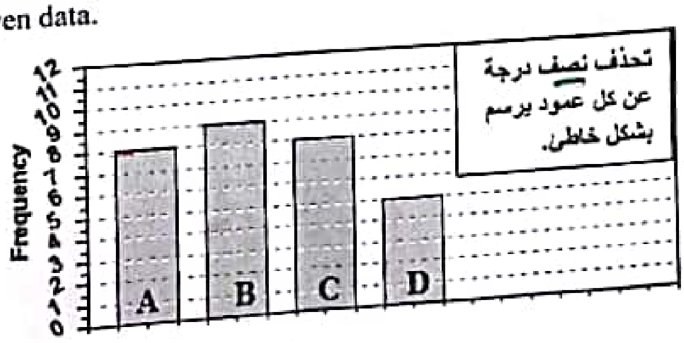
$$z = \frac{x - \bar{x}}{S} = \frac{7-6}{2.94} = 0.34 \qquad 0.25+0.5$$

اذا حسب S و حسب Z بشكل خاطئ

(2 mark)
Question 6: Consider the following data:

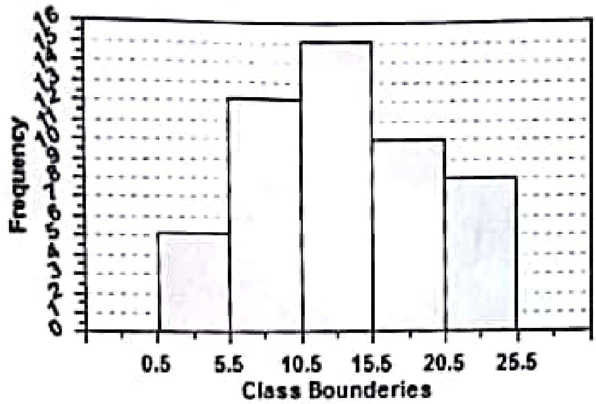
B A A B C A B A C A B C D C B
C D A A D C B A C B D C D B B

Draw the bar graph for the given data.



(5 marks)

Question 7: If we have data with the following histogram:



Then:

a) Complete of the following frequency distribution table for the given data in the previous figure:

Class Limit	Class Boundaries	Midpoint	Frequency	Relative Frequency	Percentage %	A.C.F.
1 − 5	0.5 → 5.5	3	5	5/50 = 0.10	0.10×100 = 10	5
6 − 10	5.5 → 10.5	8	12	0.24	24	5+12 = 17
11 − 15	10.5 → 15.5	13	15	0.30	30	32
16 − 20	15.5 → 20.5	18	10	0.20	20	42
21 − 25	20.5 → 25.5	23	8	0.16	16	50
Sum	-------------------	------------	50	1	100	———

b) Calculate the median for the given data.

ثلاث درجات ونصف للجدول: تحذف ربع درجة عن كل قيمة خاطئة، ويتوقف الحذف عند فناء درجة السؤال.

$$\tilde{x} = \tilde{L} + \frac{\frac{1}{2}\sum f_i - (\bar{F} - \tilde{f})}{\tilde{f}} \times C = \overset{10.5}{\cancel{15}} + \frac{\frac{50}{2} - (32 - 15)}{15} \times 5 = \overset{13.17}{\cancel{17.333}}\ldots$$

0.5 0.5

اذا اخطأ طالب صحح بأقرانه كاملة

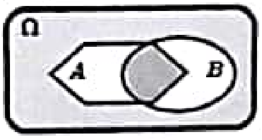
d) Calculate the range for the given data.

$$R = \underset{0.25}{x_k - x_1} = \underset{0.25}{23 - 3 = 20} \qquad ; k = 5$$

0.5

(4 marks)

Question 8: If we have Ω a space of elementary evens, A and $B \in 2^{\Omega}$ with $P(A \setminus B) = 0.25$, $P(B \setminus A) = 0.30$ and $P(A \cap B) = 0.15$. Then calculate the following probabilities:



a) $P(A) = P(A \setminus B) + P(A \cap B) = 0.25 + 0.15 = 0.40$ 0.5+0.5

b) $P(B) = P(B \setminus A) + P(A \cap B) = 0.30 + 0.15 = 0.45$ 0.5+0.5

c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.40 + 0.45 - 0.15 = 0.70$ 0.25+0.25

d) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.70 = 0.30$ 0.25+0.25

e) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.45} = 0.333\ldots$ 0.25+0.25

f) Are the events A and B independent, and why?

The events A and B are not independent, because we have:

$$\overset{0.25}{P(A \cap B) = 0.15} \neq \underset{0.25}{P(A) \cdot P(B)} = 0.45 \times 0.40 = 0.18$$

0.25+0.25

(4 marks)

Question 9: In a particular population, 30% of people drive Korean cars, 15% of people drive Japanese and the rest (55%) of people drive cars made in other countries. It is known that 10% of people driving Korean cars have accidents, 7% of people driving Japanese cars have accidents, and 12% of people driving cars made other countries have accidents. If we elected randomly one of this population, and we find that he had an accident, what is the probability that this person driving a Korean car?

Answer: We suppose that:

Z_1 is the event that, the person driving a Korean car. Then we have: $P(Z_1) = 0.30$ 0.25

Z_2 is the event that, the person driving a Japanese car. Then we have: $P(Z_2) = 0.15$ 0.25

Z_3 is the event that, the person driving a car made in other countries. Then we have: $P(Z_3) = 0.55$ 0.25

A is the event that, the person had an accident. 0.25

Then we have: $P(A \mid Z_1) = 0.10$, $P(A \mid Z_2) = 0.07$ and $P(A \mid Z_3) = 0.12$. 0.75

The probability to calculate is $P(Z_1 \mid A) = ?$, where we have:

$$P(Z_1 \mid A) = \frac{P(Z_1 \cap A)}{P(A)} = \frac{P(Z_1) \cdot P(A \mid Z_1)}{P(A)}$$ 0.25+0.25

But we note that the events Z_1, Z_2 and Z_3 are a partition of the certain event Ω, Therefore, we can calculate $P(A)$ by using the total probability formula. Where have: 0.5

$$P(A) = \sum_{i \in I} P(Z_i)\, P(A \mid Z_i) = \frac{30}{100}\frac{10}{100} + \frac{15}{100}\frac{7}{100} + \frac{55}{100}\frac{12}{100} = \frac{1065}{10000} = 0.1065$$ 0.25+0.5

Therefore, we have (by using the Bayes' formula):

$$P(Z_1 \mid A) = \frac{P(Z_1)\, P(A \mid Z_1)}{P(A)} = \frac{(30/100)(10/100)}{1065/10000} = \frac{300}{1065} = 0.282$$ 0.25+0.25

(3 marks)

Question 10: Suppose that $\Omega = \{HH, HT, TH, TT\}$, $\mathscr{A} = 2^{\Omega}$ and $P(A) = \dfrac{|A|}{|\Omega|}$. Now, let X be a random variable on the probability space $[\Omega, \mathscr{A}, P]$ defined by $X(\omega) = \begin{cases} 0 & \text{for } \omega = HH \\ 1 & \text{for } \omega = HT, TH \\ 2 & \text{for } \omega = TT \end{cases}$.

Then determine the distribution function F_X.

Answer: We know that $F_X(x) = P(\{\omega \in \Omega \,;\, X(\omega) \le x\})$. But we have from the definition of X: 0.5

$$\{\omega \in \Omega \,;\, X(\omega) \le x\} = \begin{cases} \varnothing & \text{for } x < 0 \\ \{HH\} & \text{for } 0 \le x < 1 \\ \{HH, HT, TH\} & \text{for } 1 \le x < 2 \\ \{HH, HT, TH, TT\} = \Omega & \text{for } x \ge 2 \end{cases}$$ 1

So we get that:

$$F_X(x) = P(\{\omega \in \Omega \,;\, X(\omega) \le x\}) = \begin{cases} P(\varnothing) = 0 & \text{for } x < 0 \\ P(\{HH\}) = 1/4 & \text{for } 0 \le x < 1 \\ P(\{HH, HT, TH\}) = 3/4 & \text{for } 1 \le x < 2 \\ P(\{HH, HT, TH, TT\} = P(\Omega) = 4/4 & \text{for } x \ge 2 \end{cases}$$ 1.5

End of Answers

2.5
1.5