مراجعة في المشتقات

1

Sec 12-7

the slope of the tangent of $f(x)$ at $(a, f(a))$ is

$m = f'(a)$ = derivative of $f(x)$ at a

instanteous rate of change

$$m = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\text{or} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Sec 12-8

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

If $f(x) = x^2$, then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

3

If $f'(a)$ exists $\Rightarrow$

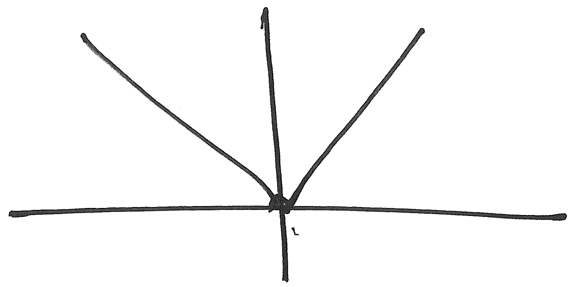
$f(x)$ is differentiable at $x=a$

(a,b) if $f(x)$ is differentiable at any $x \in (a,b)$

If $f(x)$ is differentiable at $x=c$, then $f(x)$ is continuous at $x=c$ ✓

If $f(x)$ is continuous at $x=c$ then $f(x)$ is differentiable at $x=c$ (false) ✗

4



$f(x) = |x|$

$x = 0$

$\therefore$ $f(x)$ is not differentriable at $x = 0$

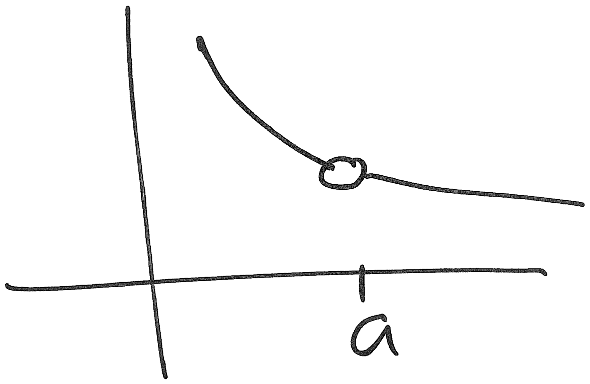
$f(x) = |x-1| + |x+3|$

$x = 1$ $\quad$ $x = -3$

$f(x)$ is not differentriable at

$x = 1, \quad x = -3$

5



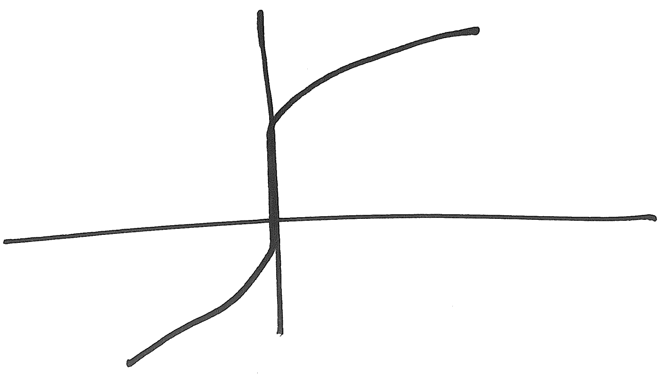
Ex $f(x) = \dfrac{x}{x-2}$

$x - 2 \neq 0$

$x \neq 2$

$\therefore f(x)$ is not differentiable

at $x = 2$



Sec 3-1

$f(x)$	$f'(x)$
a	0
π^3	0
x	1
x^n	$n x^{n-1}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
e^x	e^x
$f \pm g$	$f' \pm g'$
$x^3 \pm x^2$	$3x^2 \pm 2x$
$c\, f(x)$	$c\, f'(x)$
$5e^x$	$5e^x$

7

Ex If $f(x) = 5x^4 - 2x^3 + 7x + \pi^2 - e^3$

find $f'(x)$

$$f'(x) = 20x^3 - 6x^2 + 7 + 0 - 0$$

$$= 20x^3 - 6x^2 + 7$$

Ex If $f(x) = \frac{1}{x^2}$, find $f'(x)$

$f(x) = x^{-2}$

$f'(x) = -2x^{-3} = \frac{-2}{x^3}$

Ex If $y = \sqrt[3]{x^2}$, find y'

$y = \sqrt[3]{x^2} = x^{2/3}$

$\frac{2}{3} - \frac{1}{1}$

$= \frac{2-3}{3}$

$y' = \frac{2}{3} x^{\frac{2}{3} - \frac{1}{1}} =$

$= \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3x^{1/3}}$

$= \frac{2}{3\sqrt[3]{x}}$

Ex If $f(x) = e^x - x$, find $f''(x)$

$$f(x) = e^x - x$$

$$f'(x) = e^x - 1$$

$$f''(x) = e^x$$

$$f'''(x) = e^x$$

$$f^{(4)} = e^x$$

$$\vdots$$

$$f^{(n)}(x) = e^x$$

Ex If $f(x) = 3x^4 + 4x^3 - 2x + 7$

find $f^{(5)}(x)$

$f(x) = 3x^4 + 4x^3 - 2x + 7$

$f'(x) = 12x^3 + 12x^2 - 2$

$f''(x) = 36x^2 + 24x$

$f'''(x) = 72x + 24$

$f^{(4)}(x) = 72$

$f^{(5)}(x) = \underline{\underline{0}}$

Sec 3-2

y	y'
$y = f(x)\ g(x)$	$y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$y = (x-1)(x+3)$	$y' = (1)(x+3) + (x-1)(1)$ $y = x+3+x-1$ $= 2x+2$
$y = \dfrac{f(x)}{g(x)}$	$y' = \dfrac{f(x)\,g(x) - g'(x)\,f(x)}{(g(x))^2}$
$y = \dfrac{x+5}{x-4}$	$y' = \dfrac{(1)(x-4) - (1)(x+5)}{(x-4)^2}$ $y' = \dfrac{\cancel{x}-4-\cancel{x}-5}{(x-4)^2}$ $= \dfrac{-9}{(x-4)^2}$

(11)

(Ex) If $f(x) = xe^x$, find $f^{(n)}(x)$

$$f(x) = xe^x$$

$$f'(x) = (1)\underline{\underline{e^x}} + x\underline{\underline{e^x}}$$

$$f'(x) = (x+1)\, e^x$$

$$f''(x) = (1)\underline{\underline{e^x}} + (x+1)\,\underline{\underline{e^x}}$$

$$= (x+2)\, e^x$$

$$f'''(x) = (1)\underline{\underline{e^x}} + (x+2)\,\underline{\underline{e^x}}$$

$$= (x+3)\, e^x$$

$$f^{(n)}(x) = (x+n)\, e^x$$

Ex) If $f(t)=\sqrt{t}\,(a+bt)$, find $f'(t)$

$$f(t)=\underline{\underline{\sqrt{t}}}\,\underline{\underline{(a+bt)}}$$

$$f'(t)=\frac{1}{2\sqrt{t}}(a+bt)+\sqrt{t}\,(b)$$

$$=\frac{a+bt}{2\sqrt{t}}+\frac{b\sqrt{t}}{1}\cdot\frac{2\sqrt{t}}{2\sqrt{t}}$$

$$=\frac{a+bt+2bt}{2\sqrt{t}}$$

$$=\frac{a+3bt}{2\sqrt{t}}$$

Ex If $f(x) = \sqrt{x} \cdot g(x)$ where $g(4) = 2$, $g'(4) = 3$, find $f'(4)$

$$f(x) = \sqrt{x}\, g(x)$$

$$f'(x) = \frac{1}{2\sqrt{x}} \cdot g(x) + \sqrt{x} \cdot g'(x)$$

$$f'(4) = \frac{1}{2\sqrt{4}} \cdot g(4) + \sqrt{4} \cdot g'(4)$$

$$= \frac{1}{2\cancel{4}} (\cancel{2}) + 2 \cdot (3)$$

$$= \frac{1}{2} + \frac{6}{1} = \frac{1+12}{2}$$

$$= \frac{13}{2}$$

Ex If $f(x) = e^x g(x)$, where $g(0) = 2$, $g'(0) = 5$, find $f'(0)$

$$f(x) = e^x g(x)$$

$$f'(x) = e^x \cdot g(x) + e^x g'(x)$$

$$f'(0) = e^0 g(0) + e^0 g'(0)$$

$$= (1)(2) + (1)(5)$$

$$= 2 + 5 = 7$$

(15)

(Ex) If $y = \dfrac{x^2+x-2}{x^3+6}$, find y'

$$y' = \frac{(2x+1)(x^3+6) - 3x^2(x^2+x-2)}{(x^3+6)^2}$$

$$= \frac{2x^4 + 12x + x^3 + 6 - 3x^4 + 6x^2}{(x^3+6)^2}$$

$$= \frac{-x^4 + x^3 + 6x + 12x + 6}{(x^3+6)^2}$$

Ex If $h(2)=4$ and $h'(2)=-3$,

find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2}$

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right) = \frac{h'(x)\cdot x-(1)\,h(x)}{x^2}$$

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2} =$$

$$\frac{h'(2)\cdot 2-(1)\,h(2)}{(2)^2}$$

$$= \frac{(-3)(2)-(1)(4)}{4}$$

$$= \frac{-6-4}{4} = \frac{-10}{4} = -\frac{5}{2}$$

(17)

Ex) suppose that $f(2) = -3$,

$g(2) = 4$, $f'(2) = -2$ and $g'(2) = 7$

find $h'(2)$, if

x)

(1) $h(x) = 5f(x) - 4g(x)$

$h'(x) = 5f'(x) - 4g'(x)$

$h'(2) = 5f'(2) - 4g'(2)$

$= 5(-2) - 4(7)$

$= -10 - 28 = -38$

—

(19)

(2) $h(x) = f(x) \cdot g(x)$

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$h'(2) = f'(2) \cdot g(2) + f(2)\, g'(2)$$

$$= (-2)(4) + (-3)(7)$$

$$= -8 - 21 = -29$$

(19)

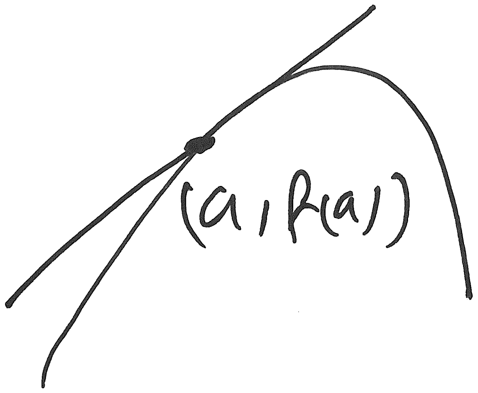
(3) $h(x) = \dfrac{f(x)}{g(x)}$

$$h'(x) = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{(g(x))^2}$$

$$h'(2) = \frac{f'(2)\, g(2) - g'(2)\cdot f(2)}{(g(2))^2}$$

$$= \frac{(-2)(4) - 7(-3)}{(4)^2}$$

$$= \frac{-8+21}{16} = \frac{13}{16}$$



(x_1, y_1)

$(a, f(a))$

$m = f(a)$

$$\boxed{y = m(x - x_1) + y_1}$$

$y_1 = f(x_1)$

Ex find the equation of the tangent line to the curve

$y = x\sqrt{x}$ at the point $(1,1)$

$$y = x\sqrt{x}$$

$$y' = (1)\sqrt{x} + x \cdot \frac{1}{2\sqrt{x}}$$

$$m = y'\Big|_{x=1} = (1)\sqrt{1} + 1 \cdot \frac{1}{2\sqrt{1}}$$

$$= \frac{1}{1} + \frac{1}{2} = \frac{2+1}{2}$$

$$= \frac{3}{2}$$

11

x_1, y_1

$m = \frac{3}{2}$ $(1, 1)$

$$y = m(x - x_1) + y_1$$

$$y = \frac{3}{2}(x - 1) + 1$$ (2)

$$2y = \cancel{2} \cdot \frac{3}{\cancel{2}}(x - 1) + (1)(2)$$

$$2y = 3x - 3 + 2$$

$$\boxed{2y = 3x - 1}$$

Ex Find an equation of the tangent line to the curve $y = \frac{e^x}{1+x^2}$ at the point $(1, \frac{1}{2}e)$

$$y = \frac{e^x}{1+x^2}$$

$$y' = \frac{e^x(1+x^2) - 2x\,e^x}{(1+x^2)^2}$$

$$m = y'\Big|_{x=1} = \frac{e^1(1+1) - 2(1)e^1}{(1+1)^2}$$

$$= \frac{2e-2e}{(2)^2} = 0$$

24

$m = 0$ $\quad$ $(x_1, y_1) = (1, \frac{1}{2}e)$

$$y = m(x - x_1) + y_1$$

$$y = 0 + \frac{1}{2}e$$

$$\boxed{y = \frac{1}{2}e}$$

Ex If $g(x) = x\,f(x)$, where $f(3)=4$, $f'(3)=2$, find an equation of the tangent line to the graph g at the point where $x=3$

$g(x) = x\,f(x)$

$g'(x) = (1)\,f(x) + x \cdot f'(x)$

$m = g'(3) = (1)\,f(3) + 3\,f'(3)$

$m = g'(3) = 1(4) + 3(2)$

$= 4 + 6 = 10$

(26)

$y = m(x - x_1) + y_1$

$m = 10$ $\quad (3, g(3))$

$(3, 12)$

$g(x) = \cancel{3} f(x)$

$g(3) = 3 f(3)$

$= 3(4) = 12$

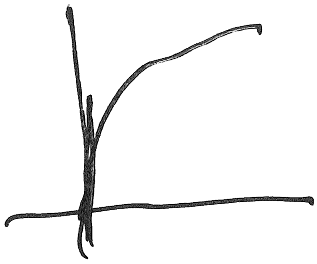
$y = 10(x - 3) + 12$

$y = 10x - 30 + 12$

$\boxed{y = 10x - 18}$

Ex $f(x) = \sqrt{x}$ is differentiable

on



$$f'(x) = \frac{1}{2\sqrt{x}} \longrightarrow x > 0$$

differentiable on $(0, \infty)$

$f(x)$ at $x = 0$ not differentiable

Ex $f(x) = \sqrt{x+4}$

not diff at $x = -4$

diff on $(-4, \infty)$

Sec 3-3

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

Ex If $y = x^2 \sin x$, find y'

$$y = x^2 \sin x$$

$$y' = 2x \cdot \sin x + x^2 \cos x$$

$$= x(2\sin x + x\cos x)$$

$$= x(x\cos x + 2\sin x)$$

Ex Find $D^{27}(\cos x)$, $\dfrac{d^{34}}{dx^{34}}(\cos x)$

$D^{165}(\cos x)$, $\dfrac{d^{144}}{dx^{144}}(\cos x)$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f^{(4)}(x) = \cos x$

$D^{27}(\cos x) = \sin x$

$$\frac{d^{34}}{dx^{34}}(\cos x) = -\cancel{s}\cos x$$

المشتقة تتكرر
كل ٤ مرات

$$\begin{array}{r} 6 \\ 4\,\overline{)\,27} \\ 24 \\ \hline 3 \end{array}$$

201 4c1 601 80

$$D^{1\boxed{65}}(\cos x) = -\sin x$$

$$\begin{array}{r|l} 4 & 65 \\ \hline & 16 \end{array}$$

$$\begin{array}{r} 65 \\ 4 \\ \hline 25 \\ 24 \\ \hline 1 \end{array}$$

$$\frac{d^{1\boxed{44}}}{dx^{144}}(\cos x) = \cos x$$

Ex Find $\frac{d^{77}}{dx^{77}}(\sin x) = \cos x$

60
64
68
72
76

$$\begin{array}{r} 19 \\ 4 \,\overline{)\,77} \\ 4 \\ \hline 37 \\ 36 \\ \hline 1 \end{array}$$

sec 3-4

$y = \sqrt{x}$. $y' = \frac{1}{2\sqrt{x}}$

$y = \sqrt{f(x)}$ $y' = \frac{f'(x)}{2\sqrt{f(x)}}$

34

Ex Find $F'(x)$ if $F(x) = \sqrt{x^2+1}$

$$F(x) = \sqrt{x^2+1}$$

$$F'(x) = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}$$

$$F'(x) = \frac{x}{\sqrt{x^2+1}}$$

Ex If $y = (x^3 - 1)^{100}$, find y'

$$y = x^n \qquad y' = n x^{n-1}$$

$$y = (f(x))^n \qquad y' = n(f(x))^{n-1} f'(x)$$

$$y = (x^3 - 1)^{100}$$

$$y' = 100 (x^3 - 1)^{99} \, 3x^2$$

$$= 300x^2 (x^3 - 1)^{99}$$

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Ex If $f(x) = \dfrac{1}{\sqrt[3]{x^2+x+1}}$, find $f'(x)$

$$f(x) = \frac{1}{(x^2+x+1)^{1/3}}$$

$$= (x^2+x+1)^{-1/3}$$

$-\frac{1}{3} - \frac{1}{1}$

$= \frac{-1-3}{3}$

$$f'(x) = -\frac{1}{3}(x^2+x+1)^{-\frac{1}{3}-1}(2x+1)$$

$$= \frac{-(2x+1)}{3(x^2+x+1)^{4/3}}$$

$$= \frac{-(2x+1)}{3\sqrt[3]{(x^2+x+1)^4}}$$

Ex If $g(t) = \left(\frac{t-2}{2t+1}\right)^9$ find $g'(t)$

$$g(t) = \left(\frac{t-2}{2t+1}\right)^9$$

$$g'(t) = 9\left(\frac{t-1}{2t+1}\right)^8 \cdot$$

$$\frac{(1)(2t+1) - 2(t-2)}{(2t+1)^2}$$

$$= 9\left(\frac{(t-1)}{2t+1}\right)^8 \frac{\cancel{2t}+1 - \cancel{2t}+4}{(2t+1)^2}$$

$$= 9\,\frac{(t-1)^8}{(2t+1)^8} \cdot \frac{5}{(2t+1)^2}$$

$$= \frac{45(t-1)^8}{(2t+1)^{10}}$$

Ex) If $y = (2x+1)^3 (x^3 - x + 1)^4$, find y'

$$y = (2x+1)^3 (x^3 - x + 1)^4$$

$$y' = 3(2x+1)^2 (2) (x^3 - x + 1)^4$$

$$+ (2x+1)^3 (4) (x^3 - x + 1)^3 \cdot (3x^2 - 1)$$

$$= 6(\underline{2x+1)^2} (x^3 - x + 1)^4$$

$$+ 4\underline{(3x^2 - 1)} \underline{(2x+1)^3} (x^3 - x + 1)^3$$

$$= (2x+1)^2 (x^3 - x + 1)^3 [$$

$$6(x^3 - x + 1) + 4(3x^2 - 1)(2x+1)]$$

2

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$$= (2x+1)^2(x^3 - x + 1)^3 [$$

$$(6x^3 - 4x + 1) + 4(3x^2-1)(2x+1)]$$

(40)

(Ex)

If $y = e^{\sin x}$, find y'

$y = e^x$ $\qquad$ $y' = e^x$

$y = e^{f(x)}$ $\qquad$ $y' = e^{f(x)} \cdot f'(x)$

$y = e^{\sin x}$

$y' = e^{\sin x} \cdot \cos x$

Ex) If $y = \sin(x^2)$, find y'

$y = \sin x \qquad y' = \cos x$

$y = \underline{\sin}(f(x))$

$y' = \cos(f(x)) \cdot f'(x)$

$y = \sin(x^2)$

$y' = \cos(x^2) \cdot 2x$

$= 2x \cdot \cos(x^2)$

(42)

Ex If $y = \tan(\sin x)$, find y'

$y = \tan x \qquad y' = \sec^2 x$

$y = \tan(f(x))$

$y' = \sec^2(f(x)) \cdot f'(x)$

$y = \tan(\sin x)$

$y' = \sec^2(\sin x) \cdot \cos x$

$y' = \cos x \cdot \sec^2(\sin x)$

Ex If $y = \sin^2 x$, find y'

$$y = \sin^2 x = (\sin x)^2$$

$$y' = 2(\sin x)\cos x$$

$$\boxed{y' = \cancel{\cos}\ \sin 2x}$$

(44)

(Ex) If $y = 5^{x^3}$, find y'

$y = a^x$

$y' = a^x \cdot \ln a$

$y = 5^x \Rightarrow y' = 5^x \cdot \ln 5$

$y = a^{f(x)} \Rightarrow y' = a^{f(x)} \cdot \ln a \cdot f'(x)$

$y = 5^{x^3}$

$y' = 5^{x^3} \cdot \ln 5 \cdot 3x^2$

y	y'
$y = \sqrt{f(x)}$	$y' = \dfrac{f'(x)}{2\sqrt{f(x)}}$
$y = (f(x))^n$	$y' = n\,(f(x))^{n-1} \cdot f'(x)$
$y = e^{f(x)}$	$y' = e^{f(x)} \cdot f'(x)$
$y = a^x$ $y = a^{f(x)}$	$y' = a^x . \ln a$ $y' = a^{f(x)} . \ln a . f'(x)$
$y = \sin(f(x))$ $y = \tan f(x)$ $y = \cos(f(x))$	$y' = \cos(f(x)) \cdot f'(x)$ $y' = \sec^2(f(x)) \cdot f'(x)$ $y' = -\sin(f(x)) \cdot f'(x)$

$\boxed{\text{sec } 3\text{-}5}$

y	y'
$(y)^2$	$2y \cdot y'$
e^y	$e^y \cdot y'$
$\sin y$	$y' \cos y$
$\underline{\underline{x^2}}\,\underline{\underline{y^3}}$	$2x \cdot y^3 + x^2 \cdot 3y^2 y'$

Ex If $x^2+y^2=25$ Find y'

$$x^2+y^2=25$$

نشتق الطرفين بالنسبة لـ x

$$2x+2yy'=0$$

$$2yy'=-2x \qquad (\div 2y)$$

$$y'=\frac{-\cancel{2}x}{\cancel{2}y}$$

$$\boxed{y'=\frac{-x}{y}}$$

Ex Find y' if $\sin(x+y) = y^2 \cos x$

$$\sin(x+y) = y^2 \cos x$$

$$\cos(x+y)(1+y') = 2yy'\cos x + y^2(-\sin x)$$

$$\cos(x+y) + y'\cos(x+y) = 2yy'\cos x - y^2 \sin x$$

$$y'\cos(x+y) - 2yy'\cos x = -\cos(x+y) - y^2 \sin x$$

$$y'(\cos(x+y) - 2y\cos x) = -\cos(x+y) - y^2 \sin x$$

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$\therefore \left(\cos(x+y) - 2y\cos x\right)$

$$y' = \frac{-\cos(x+y) - y^2 \sin x}{\cos(x+y) - 2y\cos x}$$

$$\boxed{y' = -\frac{\cos(x+y) + y^2 \sin x}{\cos(x+y) - 2y\cos x}}$$

Ex) Find y' if $x^3 + y^3 = 6xy$

and find the tangent to $x^3 + y^3 = 6xy$ at the point $(3,3)$

$$x^3 + y^3 = \underline{\underline{6xy}}$$

نقل

$$3x^2 + 3y^2 y' = 6y + 6xy'$$

$$3y^2 \underline{\underline{y'}} - 6x\underline{\underline{y'}} = 6y - 3x^2$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$\boxed{y' = \frac{6y - 3x^2}{3y^2 - 6x}}$$

5d

$$m = y'\Big|_{(3,3)} = \frac{6(3) - 3(3)^2}{3(3)^2 - 6(3)}$$

$$= \frac{18 - 27}{27 - 18} = \frac{-9}{9}$$

$$= -1$$

$m = -1$ $(3,3)$

$y = m(x - x_1) + y_1$

$y = (-1)(x - 3) + 3$

$y = -x + 3 + 3$

$$\boxed{y = -x + 6}$$

Ex) Find y'' if $x^4+y^4=16$

$$x^4+y^4=16$$

نشتق

$$4x^3+4y^3\cdot y'=0$$

$$\cancel{4}y^3y'=-\cancel{4}x^3$$

$\div y^3$

$$\boxed{y'=-\frac{x^3}{y^3}}$$

$$y''=\frac{-3x^2\cdot y^3-3y^2y'(-x^3)}{(y^3)^2}$$

$$=\frac{-3x^2y^3+3\cancel{y^2}x^3\cdot\frac{-x^3}{y^3}}{y^6}$$

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$$y'' = \frac{-3x^2y^3 \cancel{+} \frac{x^6}{y}}{y^6}$$

$$y'' = \frac{-3x^2y^3 \cdot y - \cancel{y} \cdot \frac{x^6}{\cancel{y}}}{y^6 \cdot y}$$

$$y'' = \frac{-3x^2y^4 - 3x^6}{y^7}$$

$$= \frac{-3x^2(y^4 + x^4)}{y^7}$$

$$= \frac{-3x^2(16)}{y^7} = \frac{-48x^2}{y^7}$$

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$f(x)$	$f'(x)$
$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1}x$	$\frac{1}{1+x^2}$
$\cot^{-1}x$	$\frac{-1}{1+x^2}$
$\sec^{-1}x$	$\frac{1}{x\sqrt{x^2-1}}$
$\csc^{-1}x$	$\frac{-1}{x\sqrt{x^2-1}}$

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Ex If $y = \tan(x^2)$, find y'

$$y' = \frac{(1)(2x)}{1+(x^2)^2} = \frac{2x}{1+x^4}$$

Ex $y = \sin^{-1}(e^x)$

$$y = \sin^{-1} x \Longrightarrow y' = \frac{1}{\sqrt{1-x^2}}$$

$$y = \sin^{-1}(e^x)$$

$$y' = \frac{e^x}{\sqrt{1-(e^x)^2}} = \frac{e^x}{\sqrt{1-e^{2x}}}$$

Ex If $y = \frac{1}{\sin^{-1} x}$ find y'

$$y = \frac{1}{\sin^{-1} x} = (\sin^{-1}(x))^{-1}$$

$$y' = (-1)\,(\sin^{-1}(x))^{-2} \, \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{-1}{\sqrt{1-x^2}\,(\sin^{-1}(x))^2}$$

Ex) If $f(x) = x \arctan \sqrt{x}$ find $f(x)$

$$f(x) = x \tan^{-1} \sqrt{x}$$

$$\boxed{y = \tan^{-1} x \Rightarrow y' = \frac{1}{1+x^2}}$$

$$f'(x) = (1) \cdot \tan^{-1} \sqrt{x} + x \cdot \frac{\frac{1}{2\sqrt{x}}}{1+(\sqrt{x})^2}$$

$$f'(x) = \tan^{-1} \sqrt{x} + \frac{\sqrt{x}\cancel{\sqrt{x}}}{2\cancel{\sqrt{x}}(1+x)}$$

$$\boxed{f'(x) = \tan^{-1} \sqrt{x} + \frac{\sqrt{x}}{2(1+x)}}$$

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Sec 3-6

y	y'
$y = \ln x$	$y' = \frac{1}{x}$
$y = \ln(f(x))$	$y' = \frac{f'(x)}{f(x)}$
$y = \log_a x$	$y' = \frac{1}{x \ln a}$
$y = \log_a f(x)$	$y' = \frac{f'(x)}{f(x) \cdot \ln a}$

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Ex) If $y = \ln(\sin x)$, find y'

$$y' = \frac{\cos x}{\sin x} = \cot x$$

Ex) $y = \ln(\cos x)$, find y'

$$y' = \frac{-\sin x}{\cos x} = -\tan x$$

Ex If $f(x) = \sqrt{\ln x}$, find $f'(x)$

$$f(x) = \sqrt{\ln x}$$

$$f'(x) = \frac{\frac{1}{x}}{2\sqrt{\ln x}}$$

$$f'(x) = \frac{1}{2x\sqrt{\ln x}}$$

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Ex If $f(x) = \log_{10}(2+\sin x)$, find $f'(x)$

$f(x) = \log_{10}(2+\sin x)$

$$\boxed{f'(x) = \frac{\cos x}{(2+\sin x)\cdot \ln 10}}$$

Ex If $f(x) = \ln|x|$, find $f'(x)$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$f(x) = \ln|x| = \begin{cases} \ln x & x > 0 \\ \ln(-x) & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{x} & x > 0 \\ \frac{-1}{-x} = \frac{1}{x} & x < 0 \end{cases}$$

$$f'(x) = \frac{1}{x} \quad \forall x \in R - \{0\}$$

Ex If $y = x^{\sqrt{x}}$, find y'

$$y = x^{\sqrt{x}}$$

$$\ln y = \ln x^{\sqrt{x}}$$

$\underline{\ln}$

$\ln x^{n} = n \ln x$

$$\ln y = \sqrt{x} \cdot \ln x$$

نشتق

$$\frac{y'}{y} = \frac{1}{2\sqrt{x}} \cdot \ln x + \sqrt{x} \cdot \frac{1}{x}$$

نضرب y

$$y' = y \left[\frac{\ln x}{2\sqrt{x}} + \frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}\sqrt{x}} \cdot \frac{2}{2} \right]$$

نعوض عن y

$$y' = x^{\sqrt{x}} \left[\frac{\ln x + 2}{2\sqrt{x}} \right]$$

Ex If $y^x = x^y$, find y'

$$y^x = x^y \qquad \underline{\underline{\ln}}$$

$$\ln y^x = \ln x^y$$

$$x \ln y = y \ln x$$

نشتق

$$1 \cdot \ln y + x \cdot \frac{y'}{y} = y' \cdot \ln x + y \cdot \frac{1}{x}$$

بالضرب في xy

$$xy \cdot \ln y + x\cancel{y} \cdot x \, \frac{y'}{\cancel{y}} = xy \cdot y' \ln x + \cancel{x}y \cdot y \, \frac{1}{\cancel{x}}$$

$$xy \ln y + x^2 y' = xy y' \ln x + y^2$$

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$$x^2 y' = xy y' \ln x = y^2 - xy \ln y$$

$$y'(x^2 - xy \ln x) = y^2 - xy \ln y$$

$$\boxed{y' = \frac{y^2 - xy \ln y}{x^2 - xy \ln x}}$$