السعدي ALSAADI

نسخة جديدة منقحة

1433/32

13

Ch. 5 - Part 1

- Introduction.
- Probability Distributions.
- Mean , Variance , Standard Deviation and Expectation.

STAT.110

جمال السعدي

رياضيات - إحصاء

Ch. 5 Part. 1

Discrete Probability Distributions

Probability Distributions

- **A random variable is a variable whose values are determined by chance.**
- Variables that can assume all values in the interval between any two given values are called **continuous variables**. For example, if the temperature goes from 60° to 70°.
- If a variable can assume only a specific number of values, such as the outcomes for the roll of a die or the outcomes for the toss of a coin, then the variable is called a **discrete variable**.
- **For these Exercises, state whether the variable is discrete or continuous.**

1. The speed of a jet airplane. **(Continuous)**

2. The number of cheeseburgers a fast-food restaurant serves each day. **(Discrete)**

3. The number of people who play the state lottery each day. **(Discrete)**

4. The weight of a Siberian tiger. **(continuous)**

5. The time it takes to complete a marathon. **(continuous)**

6. The number of mathematics majors in your school. **(Discrete)**

7. The blood pressures of all patients admitted to a hospital on a specific day. **(Discrete)**

A L S A A D I

Example:

Construct a probability for rolling a single die.

Solution

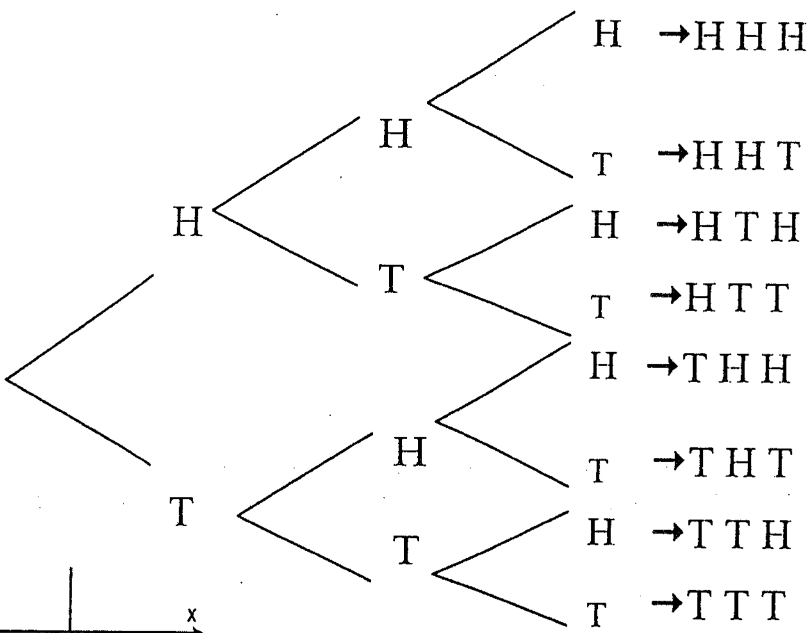
Outcome x	1	2	3	4	5	6
Probability: P(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

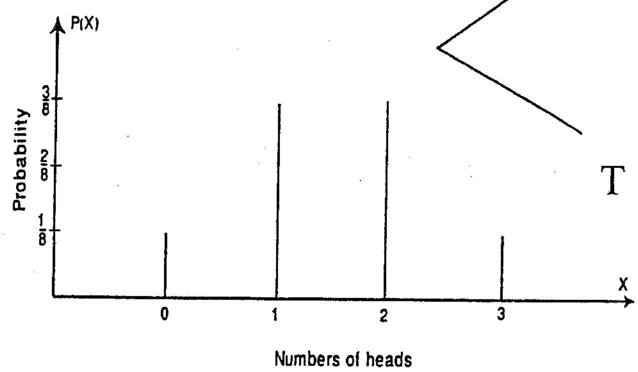
Example:

Represent graphically the probability distribution for the sample space for tossing three coins.

Number of heads x	0	1	2	3
Probability: P (x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Solution





شرطين أساسين
Two Requirements For a Probability Distribution

1. The Sum of the probabilities of all the events in the sample space must be equal 1 $\sum P(X) = 1$
2. The probability of each event in the sample space must be between or equal to 0 and 1. $0 \leq P(X) \leq 1$.

Example:

Determine whether each distribution is a probability distribution.

(a)

X	0	5	10	15	20
P (X)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Yes, it is a probability distribution.

(b)

X	0	2	4	6
P (X)	- 1.0	1.5	0.3	0.2

No. It is not a probability distribution, since P (x) cannot be 1.5 or – 1.0

(c)

X	1	2	3	4
P (X)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{9}{16}$

Yes, it is a probability distribution.

(d)

X	2	3	7
P (X)	0.5	0.3	0.4

No, it is not, since $\sum p(X) = 1.2$

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Mean, Variance, Standard Deviation, and Expectation

- **Formula for the mean of a probability distribution**

The mean of a random variable with a discrete probability distribution

$$\mu = X_1 \, . \, P(X_1) + X_2 \, . \, P(X_2) + X_3 \, . \, P(X_3) + \ldots\ldots\ldots\ldots + X_n \, . \, P(X_n)$$

$$\mu = \sum X \, . P(X)$$

- **Formula for the variance of a probability distribution**

$$\sigma^2 = \sum [X^2 \, . \, P(X)] - \mu^2$$

- **The standard deviation of a probability distribution is**

$$\sigma = \sqrt{\sigma^2} \quad \text{or} \quad \sqrt{\sum [X^2 \, . \, P(X)] - \mu^2}$$

- **The expected value:**

$$\mu = E(X) = \sum X . \, P(X)$$

- **Remember that variance and standard deviation cannot be negative.**

A L S A A D I

If X is a discrete random variable with $\sum\left[X^2P(X)\right]=6$ and $E(X)=2$. The variance for the probability distribution of X is
A) 1.732 B) 2 C) 4 D) 1.141

$E(x)=2$
$\Rightarrow \mu=2$

$$\sigma^2 = \sum x^2 \cdot P(x) - \mu^2$$
$$= 6 - (2)^2 = 6 - 4 = \boxed{2}$$

Find the mean of the distribution shown.

x	1	2
P(x)	0.40	0.60

A) 1.60 B) 0.87 C) 1.09 D) 1.33

$$\mu = \sum x \cdot P(x) = (1)(0.40) + (2)(0.60) = \boxed{1.6}$$

In a frequency distribution, if the percentages are 20%, 38%, X and 16%, then the percentage X is ...
A) 26% B) 11% C) 16% D) 21%

In frequency distribution : $\sum P(x) = 1$ ← كأعداد عشرية
$\sum P(x) = 100\%$ ← كنسب

$$20\% + 38\% + X + 16\% = 100\%$$
$$74\% + X = 100\%$$
$$X = 100\% - 74\% \Rightarrow \boxed{X = 26\%}$$

Example:

A pizza shop owner determines the number of pizza that are delivered each day. Find the mean variance, and standard deviation for the distribution shown. If the manager stated that 45 pizzas were delivered on one day. Do you think that this is a يستحق تصديقه believable claim?

Number of deliveries X	35	36	37	38	39
Probability: P (X)	0.1	0.2	0.3	0.3	0.1

Solution

X	P (x)	X. P (x)	X^2 . P (x)
35	0.1	3.5	122.5
36	0.2	7.2	259.2
37	0.3	11.1	410.7
38	0.3	11.4	433.2
39	0.1	3.9	152.1
		$\sum$ x. P (x) = 37.1	$\sum$ x^2. p (x) = 1377.7

- Mean: $\mu = \sum \ x \ . p(x) = 20.8$
- Variance: $\sigma^2 = \sum \ x^2 . p(x) - \mu^2$

  $= 1377.7 - (37.1)^2$

  $= 1.29$
- Standard deviation: $\sigma = \sqrt{\sigma} = \sqrt{1.29} = 1.1$

Example: متجر بيع بالتجزئة

The number of suits sold per day at a retail store is shown in the table, with the corresponding probabilities. Find the mean, variance, and standard deviation of the distribution.

Number of suits sold X	19	20	21	22	23
Probability P (X)	0.2	0.2	0.3	0.2	0.1

If the manager of the retail store wants to be sure that he has enough suits for the next 5 days, how many should the manager purchase ?

Solution

X	P (x)	X. P (x)	X^2 . P (x)
19	0.2	3.8	72.2
20	0.2	4	80
21	0.3	6.3	132.3
22	0.2	4.4	96.8
23	0.1	2.3	52.9
		$\sum$ x. P (x) = 20.8	$\sum$ x^2. P (x) = 434.2

- Mean. $\mu = \sum x \cdot p(x) = 37.1$
- Variance: $\sigma^2 = \sum x^2 \cdot p(x) - \mu^2$

  $= 434.2 - (20.8)^2$

  $= 1.56$
- Standard deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{1.56} = 1.2$
- The number of suits = (20.8) × (5) = 104 suits

Example:

From past experience, a company has found that in cartons of transistors, 92 % contain no defective transistors, 3% contain one defective transistor, 3% contain two defective transistors, and 2% contain three defective transistors. Find the mean, variance, and standard deviation. For the defective transistors.

About how many extra transistors per day would the company need to replace the defective ones if it used 10 cartons per day?

Solution

X	P (x)	X. P (x)	$X^2 . P(x)$
0	0.92	0	0
1	0.03	0.03	0.03
2	0.03	0.06	0.12
3	0.02	0.06	0.18
		$\sum x.P(x) = 0.15$	$\sum x^2.P(x) = 0.33$

- Mean. $\mu = \sum x.p(x) = 0.15$
- Variance: $\sigma^2 = \sum x^2 . p(x) - \mu^2$

  $= 0.33 - (0.15)^2$

  $= 0.3075$
- Standard deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.3075} = 0.555$
- Number of extra transistors = (0.15) . (10) = 1.5 is ≅ 2.

A L S A A D I

What is the sample size for the following probability distribution?

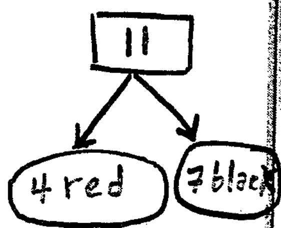
X	1	3	5	7	9
$P(X)$	1/7	1/7	2/7	2/7	1/7

A) It cannot be determined B) 25 C) 5 D) 1

* لا يمكن أن يستدل على حجم العينة sample size من جدول التوزيع الاحتمالي .

A box contains 4 red balls and 7 black balls. 5 balls are selected with replacement. The standard deviation of the number of red balls that will be obtained is ...
A) 0.968. B) 1.08 C) 0.938 D) 1.16

4 red , 7 black , 5 are selected $\therefore n = 5$
To find standard deviation for red.



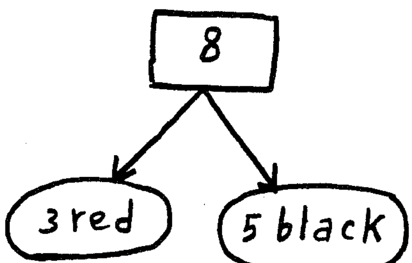
$\therefore p = \frac{4}{11}$, $q = \frac{7}{11}$

$$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{5\left(\frac{4}{11}\right)\left(\frac{7}{11}\right)}$$
$$= 1.075 \simeq \boxed{1.08}$$

A box contains 3 red balls and 5 black balls. 4 balls are selected with replacement. The standerd deviation of the number of red balls that will be obtained is
A) 0.938 B) 5 C) 4 D) 0.968

3 red , 5 black , 4 are slected $\therefore n = 4$
To find standard deviation for red balls.

$$\sigma = \sqrt{n \cdot p \cdot q}$$



$\therefore p = \frac{3}{8}$, $q = \frac{5}{8}$

$$= \sqrt{4 \cdot \left(\frac{3}{8}\right) \cdot \left(\frac{5}{8}\right)}$$
$$= 0.968$$

Example:

A person decides to invest \$ 50.000 in a gas well. Based on history, the Probabilities of the outcomes are as follows.

Outcome x	P (x)
\$ 80.000 (Highly successful)	0.2
\$ 40.000 (Moderately successful	0.7
- \$ 50.000 (Dry well) خسارة كبيرة	0.1

استثمار

- Find the expected value of the investment.

  Would you consider this a good investment?

Solution

$E(x) = \sum x \cdot P(x)$

$= (80000)(0.2) + (40000)(0.7) + (-50000)(0.1)$

$= \$\ 39000$

This a good investment.

If X is a discrete random variable with $\sum[X^2P(X)]=4$ and $E(X)=-2$. The standard deviation for the probability distribution of X is

A) 8 B) 1.41 C) 2.828 D) 0

$\sum x^2 P(x) = 4$, $\mu = E(x) = -2$, σ ?

$\sigma^2 = \sum x^2 P(x) - \mu^2$

$= 4 - (-2)^2 = 4 - 4 = 0$

$\therefore \sigma^2 = 0 \Rightarrow \sigma = \sqrt{\sigma^2} = \sqrt{0} = \boxed{0}$

If X is a discrete random variable with $\sum[X^2P(X)]=7$ and $\sigma^2=2$, then the mean for the probability distribution of X is ...

A) 2.24. B) 5 C) 1.141 D) 2

$\sigma^2 = \sum x^2 \cdot P(x) - \mu^2$ ← قانون

$2 = 7 - \mu^2$ حل معادلة

$\mu^2 = 7 - 2 = 5$

$\therefore \mu = \sqrt{5} = 2.236 \simeq \boxed{2.24}$