Workshop Solutions to Sections 2.1 and $2.2^{(1.1 \& 1.2)}$

1) Find the domain of the function $f(x) = 9 - x^2$.	2) Find the range of the function $f(x) = 9 - x^2$.
Solution:	Solution:
Since $f(x)$ is a polynomial, then	$R_f = (-\infty, 9]$
$D_f = \mathbb{R} = (-\infty, \infty)$	
Note: The domain of any polynomial is \mathbb{K} .	
3) Find the domain of the function $f(x) = 6 - 2x$.	4) Find the range of the function $f(x) = 6 - 2x$.
$\frac{\text{Solution:}}{\text{Since }f(x) \text{ is a nature minimum integral theory}}$	Solution: Since $f(x)$ is a network of degree and (i, c) is after odd
Since $f(x)$ is a polynomial, then $D = \mathbb{P} = (-\infty, \infty)$	Since $f(x)$ is a polynomial of degree one (<i>i</i> . <i>e</i> . is of an odd degree), then
$D_f = \mathbb{I} \mathbb{I} = (-\infty, \infty)$	$R_{\epsilon} = \mathbb{R} = (-\infty, \infty)$
5) Find the domain of the function $f(x) = x^2 - 2x - 3$.	6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$.
Solution:	Solution:
Since $f(x)$ is a polynomial, then	Since $f(x)$ is a polynomial, then
$D_f = \mathbb{R} = (-\infty, \infty)$	$D_f = \mathbb{R} = (-\infty, \infty)$
7) Find the domain of the function $f(x) = 5$.	8) Find the range of the function $f(x) = 5$.
Solution:	Solution:
Since $f(x)$ is a polynomial, then	$R_f = \{5\}$
$D_f = \mathbb{R} = (-\infty, \infty)$	
9) Find the domain of the function $f(x) = x - 1 $.	10) Find the domain of the function $f(x) = x + 5 $.
Solution: Since $f(x)$ is an absolute value of a polynomial then	Solution: Since $f(x)$ is an absolute value of a networmial, then
Since $f(x)$ is an absolute value of a polynomial, then $D_x = \mathbb{R} = (-\infty, \infty)$	Since $f(x)$ is an absolute value of a polynomial, then $D_x = \mathbb{R} = (-\infty, \infty)$
$D_f = \mathbb{I} \mathbb{I} = (-\infty, \infty)$	$D_f = \mathbb{I} \mathbb{I} = (-\infty, \infty)$
Note: The domain of an absolute value of any polynomial	
is \mathbb{R} .	
11) Find the domain of the function $f(x) = x $.	12) Find the range of the function $f(x) = x $.
Solution:	Solution:
Since $f(x)$ is an absolute value of a polynomial, then	$R_f = [0, \infty)$
$D_f = \mathbb{R} = (-\infty, \infty)$	
	Note: The range of an absolute value of any polynomial
	is always $[0,\infty)$.
13) Find the domain of the function $f(x) = 3x - 6 $.	14) Find the domain of the function $f(x) = 9 - 3x $.
Solution:	Solution:
Since $f(x)$ is an absolute value of a polynomial, then	Since $f(x)$ is an absolute value of a polynomial, then
$D_f = \mathbb{R} = (-\infty, \infty)$	$D_f = \mathbb{R} = (-\infty, \infty)$
x + 2	x - 2
$f(x) = \frac{x+1}{x-3}$	$f(x) = \frac{1}{x+3}$
Solution	
<u>30101011.</u>	Solution:
$f(x)$ is defined when $x - 3 \neq 0 \implies x \neq 3$. So,	Solution: $f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So,
$\frac{3010001}{f(x)}$ $f(x) \text{ is defined when } x - 3 \neq 0 \implies x \neq 3. \text{ So,}$ $D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$	Solution: $f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So, $D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$
$\frac{\text{Solution}}{f(x) \text{ is defined when } x - 3 \neq 0 \implies x \neq 3. \text{ So,}$ $D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$	Solution: $f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So, $D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$
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$\frac{\text{Solution}}{f(x) \text{ is defined when } x - 3 \neq 0 \implies x \neq 3. \text{ So,}$ $D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$	Solution: $f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So, $D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$

17) Find the domain of the function	18) Find the domain of the function
$f(x) = \frac{x+2}{x+2}$	$f(x) = \frac{x+2}{x+2}$
Solution: $x^2 - 9$	$x^2 - 5x + 6$
<u>Solution</u> . $f(x)$ is defined when $x^2 - 0 \neq 0 \implies x^2 \neq 0 \implies x \neq +3$	Solution: $f(x)$ is defined when $x^2 - 5x + 6 \neq 0$
$f(x)$ is defined when $x = j \neq 0 \implies x \neq j \implies x \neq \pm 3$.	$\Rightarrow (r-2)(r-3) \neq 0 \Rightarrow r \neq 2 \text{ or } r \neq 3 \text{ So}$
$D_{\epsilon} = \mathbb{R} \setminus \{-3,3\} = (-\infty, -3) \cup (-3,3) \cup (3,\infty)$	$\rightarrow (x - 2)(x - 3) \neq 0 \rightarrow x \neq 2 \text{ or } x \neq 3.30,$
	$D_f = \mathbb{R} \setminus \{2,3\} = (-\infty, 2) \cup (2,3) \cup (3,\infty)$
19) Find the domain of the function	20) Find the domain of the function
f(x) = x + 2	f(x) = x + 2
$f(x) = \frac{1}{x^2 - x - 6}$	$f(x) = \frac{1}{x^2 + 9}$
Solution:	Solution:
$f(x)$ is defined when $x^2 - x - 6 \neq 0$	$f(x)$ is defined when $x^2 + 9 \neq 0$ but for any value x the
$\Rightarrow (x+2)(x-3) \neq 0 \Rightarrow x \neq -2 \text{ or } x \neq 3. \text{ So},$	denominator $x^2 + 9$ cannot be 0. So,
$D_f = \mathbb{R} \setminus \{-2,3\} = (-\infty, -2) \cup (-2,3) \cup (3,\infty)$	$D_f = \mathbb{R} = (-\infty, \infty)$
21) Find the domain of the function	22) Find the domain of the function
$f(x) = \sqrt[3]{x-3}$	$f(x) = \sqrt{x-3}$
Solution:	Solution:
$D_f = \mathbb{R} = (-\infty, \infty)$	$f(x)$ is defined when $x - 3 \ge 0 \implies x \ge 3$ because $f(x)$
	is an even root. So,
Note: The domain of an odd root of any polynomial	$D_f = [3, \infty)$
is ${\mathbb R}$.	
23) Find the domain of the function	24) Find the domain of the function
$f(x) = \sqrt{3 - x}$	$f(x) = \sqrt{x+3}$
Solution:	Solution:
$f(x)$ is defined when $3 - x \ge 0 \implies -x \ge -3 \implies x \le 3$	$f(x)$ is defined when $x + 3 \ge 0 \implies x \ge -3$ because
because $f(x)$ is an even root. So,	f(x) is an even root. So,
$D_f = (-\infty, 3]$	$D_f = [-3, \infty)$
25) Find the domain of the function	26) Find the range of the function
$f(x) = \sqrt{-x}$	$f(x) = \sqrt{-x}$
Solution: $f(x)$ is defined when $x > 0 \rightarrow x < 0$ because $f(x)$ is	Solution: $P = [0, \infty)$
$f(x)$ is defined when $-x \ge 0 \implies x \le 0$ because $f(x)$ is	$R_f = [0, \infty)$
$D_c = (-\infty \ 0]$	
D_f (D_f , D_f)	NOLE: The range of an even root is always ≥ 0 .
27) Find the domain of the function $\frac{1}{2}$	28) Find the domain of the function $x + 2$
$f(x) = \sqrt{9 - x^2}$	$f(x) = \frac{x+2}{\sqrt{x-2}}$
<u>Solution</u> : $f(x)$ is defined when $9 - x^2 > 0 \rightarrow -x^2 > -9 \rightarrow -x^2$	Solution: $\sqrt{x-5}$
$y(x)$ is defined when $y - x \ge 0 \implies -x \ge -y \implies$	$f(x)$ is defined when $x - 3 > 0 \implies x > 3$. So,
$x \leq 9 \rightarrow \forall x^2 \leq \sqrt{9} \rightarrow x \leq 3 \rightarrow -3 \leq x \leq 3$.	$D_f = (3, \infty)$
$D_{\ell} = [-3.3]$	
29) Find the domain of the function	30) Find the domain of the function
x+2	$f(x) = \sqrt{x^2 - 9}$
$f(x) = \frac{1}{\sqrt{9 - x^2}}$	Solution:
Solution:	$f(x)$ is defined when $x^2 - 9 \ge 0 \implies x^2 > 9$
$f(x)$ is defined when $9 - x^2 > 0 \implies -x^2 > -9$	$\Rightarrow \sqrt{x^2} > \sqrt{9} \Rightarrow x > 3 \Rightarrow x > 3 \text{ or } x < -3$
$\Rightarrow x^2 < 9 \Rightarrow \sqrt{x^2} < \sqrt{9} \Rightarrow x < 3 \Rightarrow -3 < x < 3.$	
So,	$D_f = (-\infty, -3] \cup [3, \infty)$
$D_f = (-3,3)$	· · · · · · · ·

31) Find the range of the function	32) Find the domain of the function
$f(x) = \sqrt{x^2 - 9}$	$f(x) = \frac{x+2}{x+2}$
Solution:	$\int (x) = \sqrt{x^2 - 9}$
$R_f = [0, \infty)$	Solution:
	$f(x)$ is defined when $x^2 - 9 > 0 \implies x^2 > 9$
	$\Rightarrow \sqrt{x^2} > \sqrt{9} \Rightarrow x > 3 \Rightarrow x > 3 \text{ or } x < -3.$
	So,
	$D_f = (-\infty, -3) \cup (3, \infty)$
33) Find the domain of the function	34) Find the domain of the function
$f(x) = \sqrt{9 + x^2}$	$f(x) = \sqrt[4]{x^2 - 25}$
Solution:	Solution:
$f(x)$ is defined when $9 + x^2 \ge 0$ but it is always true for	$f(x)$ is defined when $x^2 - 25 \ge 0 \implies x^2 \ge 25$
any value x . So,	$\Rightarrow \sqrt{x^2} \ge \sqrt{25} \Rightarrow x \ge 5 \Rightarrow x \ge 5 \text{ or } x \le -5.$
$D_f = \mathbb{R}$	So,
	$D_f = (-\infty, -5] \cup [5, \infty)$
35) Find the domain of the function	36) Find the range of the function
$f(x) = \sqrt[6]{16 - x^2}$	$f(x) = \sqrt{16 - x^2}$
Solution:	Solution:
$f(x)$ is defined when $16 - x^2 \ge 0 \implies -x^2 \ge -16 \implies$	We know that $f(x)$ is defined when $16 - x^2 \ge 0$
$x^2 \le 16 \implies \sqrt{x^2} \le \sqrt{16} \implies x \le 4 \implies -4 \le x \le 4$.	$\Rightarrow -x^2 \ge -16 \Rightarrow x^2 \le 16 \Rightarrow \sqrt{x^2} \le \sqrt{16}$
So,	\Rightarrow $ x \le 4 \Rightarrow -4 \le x \le 4$. So,
$D_f = [-4, 4]$	$D_f = [-4, 4]$
	Using D_f we find the outputs vary from 0 to 4 . Hence,
	$R_f = [0,4]$
Find the domain of the function	38) Find the domain of the function
$f(x) = \frac{x + x }{x + x }$	$\left(-\frac{1}{2}\right)$ $x < 0$
$f(x) = \frac{x + x }{x}$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0 \\ 0, & x < 0 \end{cases}$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) = \frac{x + x }{x}$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_x = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $2 = \sqrt{x}$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x-1} + \sqrt{x+2}$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{2 - \sqrt{x}}}$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution:
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ Solution:	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <u>Solution:</u> $f(x) \text{ is defined when}$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: $f(x) \text{ is defined when}$ $1 - x - 1 > 0 \implies x > 1 \implies D \xrightarrow{r - 1} = [1, \infty)$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <u>Solution:</u> $f(x) \text{ is defined when}$ $1 = x \ge 0 \implies D = [0, \infty)$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x - 1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x - 1}} = [-3, \infty)$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <u>Solution:</u> $f(x) \text{ is defined when}$ $1 - x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ $2 - x^2 + 1 \ge 0 \text{ but this is always true for all } x$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x - 1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x + 3}} = [-3, \infty)$ Hence
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <u>Solution:</u> $f(x) \text{ is defined when}$ $1 - x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ $2 - x^2 + 1 > 0 \text{ but this is always true for all } x$ $\implies D \xrightarrow{=} \mathbb{R}.$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: $f(x) \text{ is defined when}$ $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x - 1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x + 3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x - 1}} \cap D_{\sqrt{x - 1}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <u>Solution:</u> $f(x) \text{ is defined when}$ $1 - x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ $2 - x^2 + 1 > 0 \text{ but this is always true for all } x$ $\implies D_{\sqrt{x^2 + 1}} = \mathbb{R}.$ Hence	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x - 1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x + 3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x - 1}} \cap D_{\sqrt{x + 3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <u>Solution:</u> $f(x) \text{ is defined when}$ $1 - x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ $2 - x^2 + 1 > 0 \text{ but this is always true for all } x$ $\implies D_{\sqrt{x^2 + 1}} = \mathbb{R}.$ Hence, $D_f = D = 0 \text{ for } 0 D = [0, \infty) \cap \mathbb{R} = [0, \infty)$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x - 1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x + 3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x - 1}} \cap D_{\sqrt{x + 3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <u>Solution:</u> $f(x) \text{ is defined when}$ $1 - x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ $2 - x^2 + 1 > 0 \text{ but this is always true for all } x$ $\implies D_{\sqrt{x^2 + 1}} = \mathbb{R}.$ Hence, $D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2 + 1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$ 41) The function $f(x) = 3x^4 + x^2 + 1 \text{ is a polynomial}$	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x-1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$ 42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function
$f(x) = \frac{x + x }{x}$ <u>Solution:</u> $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <u>Solution:</u> $f(x) \text{ is defined when}$ $1 - x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ $2 - x^2 + 1 > 0 \text{ but this is always true for all } x$ $\implies D_{\sqrt{x^2 + 1}} = \mathbb{R}.$ Hence, $D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2 + 1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$ 41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function.	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x-1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$ 42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function.
$f(x) = \frac{x + x }{x}$ Solution: $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ Solution: $f(x) \text{ is defined when}$ $1 - x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ $2 - x^2 + 1 > 0 \text{ but this is always true for all } x$ $\implies D_{\sqrt{x^2+1}} = \mathbb{R}.$ Hence, $D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2+1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$ 41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function. 43) The function $f(x) = -3x^2 + 7$ is a quadratic	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: $f(x) \text{ is defined when}$ $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x-1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$ 42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function. 44) The function $f(x) = 2x + 3$ is a linear function.
$f(x) = \frac{x + x }{x}$ Solution: $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ Solution: $f(x) \text{ is defined when}$ 1- $x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ 2- $x^2 + 1 > 0$ but this is always true for all x $\implies D_{\sqrt{x^2 + 1}} = \mathbb{R}.$ Hence, $D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2 + 1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$ 41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function. 43) The function $f(x) = -3x^2 + 7$ is a quadratic function.	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x-1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$ 42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function. 44) The function $f(x) = 2x + 3$ is a linear function.
$f(x) = \frac{x + x }{x}$ Solution: $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ Solution: $f(x) \text{ is defined when}$ $1 - x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ $2 - x^2 + 1 > 0 \text{ but this is always true for all } x$ $\implies D_{\sqrt{x^2 + 1}} = \mathbb{R}.$ Hence, $D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2 + 1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$ 41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function. 43) The function $f(x) = -3x^2 + 7$ is a quadratic function.	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x-1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$ 42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function. 44) The function $f(x) = 2x + 3$ is a linear function.
$f(x) = \frac{x + x }{x}$ Solution: $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ Solution: $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ Solution: $f(x) \text{ is defined when}$ 1- $x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ 2- $x^2 + 1 > 0$ but this is always true for all x $\implies D_{\sqrt{x^2 + 1}} = \mathbb{R}$. Hence, $D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2 + 1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$ 41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function. 43) The function $f(x) = -3x^2 + 7$ is a quadratic function. 45) The function $f(x) = x^7$ is a power function.	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x-1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$ 42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function. 44) The function $f(x) = 2x + 3$ is a linear function. 46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function.
$f(x) = \frac{x + x }{x}$ Solution: $f(x) \text{ is defined when } x \neq 0. \text{ So,}$ $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ 39) Find the domain of the function $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ Solution: $f(x) \text{ is defined when}$ 1- $x \ge 0 \implies D_{\sqrt{x}} = [0, \infty)$ 2- $x^2 + 1 > 0$ but this is always true for all x $\implies D_{\sqrt{x^2 + 1}} = \mathbb{R}.$ Hence, $D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2 + 1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$ 41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function. 43) The function $f(x) = -3x^2 + 7$ is a quadratic function. 45) The function $f(x) = x^7$ is a power function. 47) The function $f(x) = \frac{x-3}{x+2}$ is a rational function and we	$f(x) = \begin{cases} -\frac{1}{x}, & x < 0\\ x, & x \ge 0 \end{cases}$ Solution: It is clear from the definition of the function $f(x)$ that $D_f = \mathbb{R} = (-\infty, \infty)$ 40) Find the domain of the function $f(x) = \sqrt{x-1} + \sqrt{x+3}$ Solution: f(x) is defined when $1 - x - 1 \ge 0 \implies x \ge 1 \implies D_{\sqrt{x-1}} = [1, \infty)$ $2 - x + 3 \ge 0 \implies x \ge -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$ Hence, $D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$ 42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function. 44) The function $f(x) = 2x + 3$ is a linear function. 46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function. 48) The function $f(x) = \sin x$ is a trigonometric function.

49) The function $f(x) = e^x$ is a natural exponential function	50) The function $f(x) = 3^x$ is a general exponential function
$[1011 cm cm cm 2 + \sqrt{m 2} tm m m m m m m m m m m m m m m m m m m$	1011ction. E2) The function $f(x) = -2$ is a constant function
51) The function $f(x) = x^2 + \sqrt{x} - 2$ is an algebraic function.	52) The function $f(x) = -3$ is a constant function.
53) The function $f(x) = \log_3 x$ is a general logarithmic function.	54) The function $f(x) = \ln x$ is a natural logarithmic function.
55) The function $f(x) = 3x^4 + x^2 + 1$ is	56) The function $f(x) = 9 - x^2$ is
Solution:	Solution:
$f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$	$f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$
Hence,	Hence,
f(x) is an even function.	f(x) is an even function.
57) The function $f(x) = x^5 - x$ is	58) The function $f(x) = 2 - \sqrt[3]{x}$ is
Solution:	Solution:
$f(-x) = (-x)^3 - (-x) = -x^3 + x$	$f(-x) = 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} = 2 + \sqrt[5]{x}$
$= -(x^3 - x) = -f(x)$	$=-(-2-\sqrt[5]{x})$
Terret, $f(x)$ is an odd function	Hence,
	f(x) is neither even nor odd.
59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is	60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is
Solution:	Solution:
$\frac{1}{2}$ 2 2 2	3 3 $()$
$f(-x) = 3(-x) + \frac{1}{\sqrt{(-x)^2 + 9}} = -3x + \frac{1}{\sqrt{x^2 + 9}}$	$f(-x) = \frac{1}{\sqrt{(-x)^2 + 9}} = \frac{1}{\sqrt{x^2 + 9}} = f(x)$
$\left(\begin{array}{c} 2 \end{array} \right)$	Hence,
$=-\left(3x-\frac{1}{\sqrt{x^2+9}}\right)$	f(x) is an even function.
Hence,	
f(x) is neither even nor odd.	
61) The function $f(x) = \sqrt{4 + x^2}$ is	62) The function $f(x) = 3$ is
Solution:	Solution:
$f(-x) = \sqrt{4 + (-x)^2} = \sqrt{4 + x^2} = f(x)$	Since the graph of the constant function 3 is symmetric
Hence,	about the $y - axis$, then
f(x) is an even function.	f(x) is an even function.
63) The function $f(x) = \frac{9-x^2}{1-x^2}$ is	64) The function $f(x) = \frac{x^2 - 4}{x^2}$ is
Solution	Solution
<u>Solution:</u> $0 - (-x)^2 = 0 - x^2$	<u>Solution:</u> $(-x)^2 - 4 = x^2 - 4$
$f(-x) = \frac{9 - (-x)}{(-x)^2} = \frac{9 - x}{-x^2}$	$f(-x) = \frac{(-x)^2 - 4}{(-x)^2 + 4} = \frac{x^2 - 4}{x^2 + 4} = f(x)$
(-x) - 2 - x - 2 $(0 - x^2)$	$(-x)^2 + 1 x^2 + 1$
$=-\left(\frac{y-x}{y+2}\right)$	f(x) is an even function
(x+2)	
Therefore, $f(x)$ is poither over period	
(x) is hereic even nor odd. (5) The function $f(x) = 3 x $ is	(66) The function $f(x) = x^{-2}$ is
Solution: $f(x) = S[x]$ is	Solution: $f(x) = x$ is
f(-r) = 3 (-r) = 3 r = f(r)	
Hence,	$f(x) = x^{-2} = \frac{1}{x^2}$
f(x) is an even function.	$\int_{\alpha}^{\alpha} 1 1$
	$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$
	Hence, $f(x)$ is an even function.

67) The function $f(x) = x^3 - 2x + 5$ is	68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is
Solution:	Solution:
$f(-x) = (-x)^3 - 2(-x) + 5 = -x^3 + 2x + 5$	$f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) = -\sqrt[3]{x^5} + x^3 - x$
$= -(x^3 - 2x - 5)$	$ \left(\frac{3}{\sqrt{x^5}} - \frac{x^3}{x^3} + x \right) f(x)$
Hence,	$= -\left(\sqrt{x} - x + x\right) = -j(x)$
f(x) is neither even nor odd.	Hence,
	f(x) is an odd function.
69) The function $f(x) = 7$ is	70) The function $f(x) = \frac{x^3 - 4}{3 - 4}$ is
<u>Solution:</u>	Solution: x^{3+1}
Since the graph of the constant function 7 is symmetric	$(-r)^3 - 4 - r^3 - 4 r^3 + 4$
about the $y - axis$, then	$f(-x) = \frac{(-x)^{-1}}{(-x)^{3} + 1} = \frac{x^{-1}}{x^{3} + 1} = -\frac{x^{-1}}{x^{3} + 1}$
	$(-x)^{\circ} + 1 - x^{\circ} + 1 - x^{\circ} + 1$
f(x) is an even function.	f(x) is not there even nor odd
71) The function $f(x) = \frac{x^2 - 1}{x^3 + 2}$ is	72) The function $f(x) = x^6 - 4x^2 + 1$ is
Solution:	Solution:
$(-x)^2 - 1$ $x^2 - 1$ $x^2 - 1$	$f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$
$f(-x) = \frac{(-x)^{2}}{(-x)^{3}+2} = \frac{x^{2}}{x^{3}+2} = -\frac{x^{2}}{x^{3}+2}$	Hence,
$(-x)^2 + 3 - x^2 + 3 x^2 - 3$	f(x) is an even function.
f(x) is poither over period	
7(x) is heither even not odd. 72) The function $f(x) = x^2$ is increasing on (0, so)	74) The function $f(u) = u^2$ is decreasing on $(-\infty, 0)$
73) The function $f(x) = x^2$ is increasing on $(0, \infty)$.	74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$.
75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$.	76) The function $f(x) = x^3$ is not decreasing at all.
77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$.	78) The function $f(x) = \sqrt{x}$ is not decreasing at all.
79) The function $f(x) = \frac{1}{x}$ is not increasing at all.	80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty)$.

Workshop Solutions to Sections 2.3 and 2.4(1.3 & app D)

1) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f + g)(x) = x^2$	2) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f+g} =$
Solution:	Solution:
$(f+g)(x) = x^2 + \sqrt{4-x}$	$D_f = \mathbb{R}$
	$g(x)$ is defined when $4 - x \ge 0 \iff x \le 4$. Thus,
	$D_g = (-\infty, 4]$
	$D_{f+g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
3) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f-g)(x) = x^2$	4) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f-g} =$
Solution:	Solution:
$(f-g)(x) = x^2 - \sqrt{4-x}$	$D_f = \mathbb{R}$
	$g(x)$ is defined when $4 - x \ge 0 \iff x \le 4$. Thus,
	$D_g = (-\infty, 4]$
	$D_{f-g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
5) If $f(x) = x^2$ and $g(x) = \sqrt{4 - x}$, then $(fg)(x) =$	6) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{fg} =$
Solution:	Solution:
$(fg)(x) = x^2\sqrt{4-x}$	$D_f = \mathbb{R}$
	$g(x)$ is defined when $4 - x \ge 0 \iff x \le 4$. Thus,
	$D_g = (-\infty, 4]$
	$D_{fg} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
7) If $f(x) = x^2$ and $g(x) = \sqrt{4 - x}$, then $(f \circ g)(x) =$	8) If $f(x) = x^2$ and $g(x) = \sqrt{4 - x}$, then $D_{f \circ g} =$
Solution:	Solution:
$(f \circ g)(x) = f(g(x))$	$(f \circ g)(x) = f(g(x))$
$= f(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4-x$	$= f(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4-x$
	$D_g = (-\infty, 4]$
	$D_{f(g(x))} = \mathbb{R}$
	$D_{f \circ g} = D_g \cap D_{f(g(x))} = (-\infty, 4] \cap \mathbb{R} = (-\infty, 4]$
9) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(g \circ f)(x) = $	10) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{g \circ f} =$
Solution:	Solution:
$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{4 - x^2}$	$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{4 - x^2}$
	$D_f = \mathbb{R}$
	$D_{g(f(x))} = [-2,2]$
	$D_{g \circ f} = D_f \cap D_{g(f(x))} = \mathbb{R} \cap [-2,2] = [-2,2]$
11) If $f(x) = x^2$, then $(f \circ f)(x) =$	12) If $f(x) = x^2$, then $D_{f \circ f} =$
$\frac{\text{Solution:}}{(f-f)(f-f)}$	$\frac{\text{Solution:}}{(5-5)(2)} = (5(2)) = $
$(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$	$(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$
	$D_f = \mathbb{R}$
	$D_{f(f(x))} = \mathbb{R}$
·	$D_{f \circ f} = D_f \cap D_f(f(x)) = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

13) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{f}{a}\right)(x) =$	14) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\underline{f}} =$
Solution:	Solution:
$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$	$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$
	$D_f = \mathbb{R}$ $a(x)$ is defined when $4 - x > 0 \iff x < 4$. Thus,
	$D_g = (-\infty, 4]$
	$D_{\frac{f}{q}} = \left\{ x \in D_f \cap D_g g(x) \neq 0 \right\}$
	$y = \mathbb{R} \cap (-\infty, 4) = (-\infty, 4)$
15) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{g}{f}\right)(x) =$	16) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{g}{f}} =$
Solution:	Solution:
$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$	$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$
	$D_f = \mathbb{R}$ $a(r)$ is defined when $4 - r > 0 \iff r < 4$. Thus,
	$D_g = (-\infty, 4]$
	$D_{\frac{g}{f}} = \left\{ x \in D_f \cap D_g f(x) \neq 0 \right\}$
	$f = \mathbb{R} \setminus \{0\} \cap (-\infty, 4] = (-\infty, 0) \cup (0, 4]$
17) If $f(x) = 9 - x^2$ and $g(x) = 10$, then	18) If $f(x) = 9 - x^2$ and $g(x) = 10$, then (f - a)(x) =
Solution:	Solution:
$\frac{d}{(f+g)(x)} = (9-x^2) + (10) = 9-x^2 + 10$ = 19 - x ²	$(f-g)(x) = (9-x^2) - (10) = 9-x^2 - 10$ = $-x^2 - 1$
19) If $f(x) = 9 - x^2$ and $g(x) = 10$, then	20) If $f(x) = 9 - x^2$ and $g(x) = 10$, then
(g-f)(x) =	(fg)(x) =
$\frac{g(g-f)(x)}{(g-f)(x)} = (10) - (9 - x^2) = 10 - 9 + x^2$ $= 1 + x^2$	$(fg)(x) = (9 - x^2)(10) = 90 - 10x^2$
21) If $f(x) = 9 - x^2$ and $g(x) = 10$, then	22) If $f(x) = 9 - x^2$ and $g(x) = 10$, then
$(f \circ g)(x) =$	$(g \circ f)(x) =$ Solution:
$(f \circ g)(x) = f(g(x)) = f(10)$ = 9 - 10 ² = 9 - 100 = -91	$\overline{(g \circ f)(x)} = g(f(x)) = g(9 - x^2) = 10$
23) If $f(x) = 9 - x^2$ and $g(x) = 10$, then	24) If $f(x) = 9 - x^2$ and $g(x) = 10$, then
$(f \circ f)(x) =$	$(g \circ g)(x) =$
Solution: $(f \circ f)(x) = f(f(x)) = f(9 - x^2)$ $= 9 - (9 - x^2)^2$	$(g \circ g)(x) = g(g(x)) = g(10) = 10$
25) If $f(x) = 9 - x^2$, $g(x) = \sin x$ and $h(x) = 3x + 2$,	26) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then
then $(f \circ g \circ h)(x) =$	(f+g)(x) =
$\frac{\text{Solution:}}{(f \circ a \circ h)(x)} = f(a(h(x)))$	$(f+a)(x) = \sqrt{25 + x^2} + x^3$
= f(a(3x+2))	
$= f(\sin(3x+2))$	
$= 9 - (\sin(3x+2))^2$	
$= 9 - \sin^2(3x + 2)$	

27) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then	28) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then
(f-g)(x) =	(fg)(x) =
<u>Solution:</u> $(f - c)(x) = \sqrt{25 + x^2} - x^3$	50100000000000000000000000000000000000
$\frac{(f-g)(x) = \sqrt{25 + x^2}}{20} \text{ if } f(x) = \sqrt{25 + x^2} \text{ and } g(x) = x^3 \text{ then}$	$(y)(x) = x \sqrt{25 + x^2}$ 20) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$ then
(f) (x) = $(x^2 - x^2) + x^2$ and $y(x) = x^2$, then	$(f \circ a)(x) = (x - x) + x - and y(x) - x , then (f \circ a)(x) = (x - x) + (x -$
$\left(\frac{-g}{g}\right)(x) =$	Solution:
Solution: $\sqrt{25 \pm r^2}$	$(f \circ g)(x) = f(g(x)) = f(x^3) = \sqrt{25 + (x^3)^2}$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{25 + x^{-1}}}{x^3}$	$=\sqrt{25+x^6}$
31) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then	32) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$, then $(f \circ g)(x) =$
$(g \circ f)(x) =$	Solution:
$\frac{\text{Solution:}}{\left(\sqrt{2}\right)^3}$	$(f \circ g)(x) = f(g(x)) = f(x - 2) = \sqrt{x - 2}$
$(g \circ f)(x) = g(f(x)) = g(\sqrt{25} + x^2) = (\sqrt{25} + x^2)$	
$= \sqrt{(25 + x^2)^2}$	24) If $f(x) = \sqrt{x}$ and $q(x) = x - 2$ then $(q \circ q)(x) = 1$
Solution: $y(x) = \sqrt{x}$ and $y(x) = x = 2$, then $(y = y)(x) = 1$	Solution:
$\frac{g(g \circ f)(x)}{(g \circ f)(x)} = g(f(x)) = g(\sqrt{x}) = \sqrt{x} - 2$	$\frac{g(g)(x) = g(g(x)) = g(x-2) = (x-2) - 2}{= x - 2 - 2 = x - 4}$
35) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$, then $(fg)(x) = x - 2$	36) If $f(x) = \sin 5x$ and $g(x) = x^2 + 3$, then
Solution:	$(f \circ g)(x) =$
$(fg)(x) = (\sqrt{x})(x-2) = (x-2)\sqrt{x}$	Solution: $(f - 2)(x) = f(x^2 + 2) = \sin f(x^2 + 2)$
27) If $f(x) = \sin 5x$ and $g(x) = x^2 + 3$ then	$(f \circ g)(x) = f(g(x)) = f(x^2 + 3) = \sin 5(x^2 + 3)$ $32! \text{ If } f(x) = \sin 5x \text{ and } g(x) = x^2 + 3 \text{ then}$
$(a \circ f)(x) =$	(fg)(x) =
Solution:	Solution:
$(g \circ f)(x) = g(f(x)) = g(\sin 5x) = (\sin 5x)^2 + 3$ = $\sin^2 5x + 3$	$(fg)(x) = (\sin 5x)(x^2 + 3) = (x^2 + 3)\sin 5x$
39) If $f(x) = \sqrt{x}$ and $g(x) = \cos x$, then $(g \circ f)(x) =$	40) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$, then
Solution:	$(f \circ g)(x) =$
$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \cos \sqrt{x}$	Solution:
	$(f \circ g)(x) = f(g(x)) = f(1 - x^2) = (1 - x^2) + \frac{1}{1 - x^2}$
41) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$, then	42) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$, then
$(g \circ f)(x) =$	(fg)(x) =
Solution:	Solution:
$(g \circ f)(x) = g(f(x)) = g\left(x + \frac{1}{x}\right) = 1 - \left(x + \frac{1}{x}\right)$	$(fg)(x) = (x + \frac{1}{x})(1 - x^2)$
43) If the graph of the function $f(x) = x^2$ is shifted a	44) If the graph of the function $f(x) = x^2$ is shifted a
distance 2 units upwards, then the new graph	distance 2 units downwards , then the new graph
Solution:	Solution:
$x^2 + 2$	$x^2 - 2$
45) If the graph of the function $f(x) = x^2$ is shifted a	46) If the graph of the function $f(x) = x^2$ is shifted a
distance 2 units to the right , then the new graph	distance 2 units to the left , then the new graph
represented the graph of the function is	represented the graph of the function is
$(r-2)^2 = r^2 - 4r + 4$	$(x+2)^2 = x^2 + 4x + 4$

47) If the graph of the function $f(x) = \cos x$ is	48) If the graph of the function $f(x) = \cos x$ is
stretched vertically by a factor of 2, then the new graph	compressed vertically by a factor of $\frac{1}{2}$, then the new graph
represented the graph of the function is	represented the graph of the function is
Solution:	Solution:
$2\cos x$	$\frac{1}{-\cos r}$
	$\frac{2}{2}$
49) If the graph of the function $f(x) = \cos x$ is	So in the graph of the function $f(x) = \cos x$ is stretched
graph represented the graph of the function is	horizontally by a factor of $\frac{1}{2}$, then the new graph
Solution.	represented the graph of the function is
$\cos 2x$	Solution:
	$\cos\frac{1}{2}$
51) The graph of the function $f(x) = \sqrt{x}$ is reflected	52) The graph of the function $f(x) = \sqrt{x}$ is reflected
about the $x - axis$ if	about the $y - axis$ if
Solution:	Solution:
$f(x) = -\sqrt{x}$	$f(x) = \sqrt{-x}$
53) If the graph of the function $f(x) = e^x$ is shifted a	54) If the graph of the function $f(x) = e^x$ is shifted a
distance 2 units upwards , then the new graph	distance 2 units downwards, then the new graph
represented the graph of the function is	Solution:
$\frac{5000001}{\rho^{x}+2}$	$e^x - 2$
55) If the graph of the function $f(x) = e^x$ is shifted a	56) If the graph of the function $f(x) = e^x$ is shifted a
distance 2 units to the right , then the new graph	distance 2 units to the left , then the new graph
represented the graph of the function is	represented the graph of the function is
Solution:	Solution:
<i>e^{x-2}</i>	<u>e^{x+2}</u>
57) $\frac{2\pi}{3}$ rad $=\frac{2\pi}{3} \times \frac{180^{\circ}}{\pi} = 120^{\circ}$	58) $\frac{5\pi}{6}$ rad $=\frac{5\pi}{6} \times \frac{180}{\pi} = 150^{\circ}$
59) $\frac{7\pi}{6}$ rad $=\frac{7\pi}{6} \times \frac{180^{\circ}}{\pi} = 210^{\circ}$	60) $\frac{3\pi}{2}$ rad $=\frac{3\pi}{2} \times \frac{180^{\circ}}{\pi} = 270^{\circ}$
61) $120^{\circ} = 120 \times \frac{\pi}{180^{\circ}} = \frac{2\pi}{3}$ rad	62) $270^{\circ} = 270 \times \frac{\pi}{180^{\circ}} = \frac{3\pi}{2}$ rad
63) $\frac{5\pi}{12}$ rad $=\frac{5\pi}{12} \times \frac{180^{\circ}}{\pi} = 75^{\circ}$	64) $\frac{5\pi}{6}$ rad $=\frac{5\pi}{6} \times \frac{180^{\circ}}{\pi} = 150^{\circ}$ (Repeated)
65) $150^{\circ} = 150 \times \frac{\pi}{180^{\circ}} = \frac{5\pi}{6}$ rad	66) $210^{\circ} = 210 \times \frac{\pi}{180^{\circ}} = \frac{7\pi}{6}$ rad
$67) \ \frac{1}{\sec x} = \cos x$	$68) \ \frac{1}{\csc x} = \sin x$
$69) \ \frac{1}{\cot x} = \tan x$	70) $\frac{\sin x}{\cos x} = \tan x$
71) $\frac{\cos x}{\sin x} = \cot x$	
72) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\cot x =$	73) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\tan x =$
$\frac{\text{Solution:}}{3}$ adi 5	3 adi
$\cos x = \frac{5}{5} = \frac{uay}{hm}$	$\cos x = \frac{5}{5} = \frac{aay}{hyp}$
Now, we should find the length of the opposite side using	Now, we should find the length of the opposite side using
the Pythagorean Theorem, so	the Pythagorean Theorem, so
$ opposite = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$	$ opposite = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
1 adj 3	1 opp 4
$\therefore \cot x = \frac{1}{\tan x} = \frac{1}{\cos p} = \frac{1}{4}$	$\therefore \tan x = \frac{1}{\cot x} = \frac{1}{adj} = \frac{1}{3}$

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74) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\sin x =$ 75) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\csc x =$ Solution: $\cos x = \frac{3}{5} = \frac{adj}{hyp}$ $\cos x = \frac{3}{5} = \frac{adj}{hvp}$ Now, we should find the length of the opposite side using Now, we should find the length of the opposite side using the Pythagorean Theorem, so the Pythagorean Theorem, so $|opposite| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ $|opposite| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ $\therefore \csc x = \frac{1}{\sin x} = \frac{hyp}{opp} = \frac{5}{4}$ $\therefore \sin x = \frac{opp}{hvp} = \frac{4}{5}$ 76) $\sin\left(\frac{5\pi}{\epsilon}\right) =$ 77) $\cos\left(\frac{5\pi}{6}\right) =$ Solution Solution $\frac{5\pi}{6}$ rad $=\frac{5\pi}{6} \times \frac{180^{\circ}}{\pi} = 150^{\circ}$ $\frac{5\pi}{6}$ rad $=\frac{5\pi}{6} \times \frac{180^{\circ}}{\pi} = 150^{\circ}$ So, we deduce now that $\cos\left(\frac{5\pi}{6}\right)$ is in the second quarter. So, we deduce now that $\sin\left(\frac{5\pi}{6}\right)$ is in the second quarter. $\sin\left(\frac{5\pi}{6}\right) = \sin(150^{\circ}) = \sin(180^{\circ} - 30^{\circ}) = \sin(30^{\circ}) =$ $\cos\left(\frac{5\pi}{6}\right) = \cos(150^{\circ}) = \cos(180^{\circ} - 30^{\circ})$ sin*π6=12* $= -\cos(30^{\circ}) = -\cos(\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$ 79) $\cot\left(\frac{5\pi}{\epsilon}\right) =$ 78) $\tan\left(\frac{5\pi}{2}\right) =$ Solution $\frac{5\pi}{6}$ rad $=\frac{5\pi}{6} \times \frac{180^{\circ}}{\pi} = 150^{\circ}$ $\frac{5\pi}{6}$ rad $=\frac{5\pi}{6} \times \frac{180^{\circ}}{\pi} = 150^{\circ}$ So, we deduce now that $\tan\left(\frac{5\pi}{6}\right)$ is in the second So, we deduce now that $\cot\left(\frac{5\pi}{6}\right)$ is in the second quarter. quarter. $\cot\left(\frac{5\pi}{6}\right) = \cot(150^{\circ}) = \cot(180^{\circ} - 30^{\circ})$ $\tan\left(\frac{5\pi}{6}\right) = \tan(150^\circ) = \tan(180^\circ - 30^\circ)$ $= -\cot(30^{\circ}) = -\cot\left(\frac{\pi}{6}\right) = -\sqrt{3}$ $= -\tan(30^{\circ}) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$ 80) If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $\sec x = \frac{1}{\sqrt{3}}$ 81) If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $\csc x =$ $\frac{\text{Solution:}}{\sin x = \frac{2}{3} = \frac{opp}{hyp}}$ $\frac{\text{Solution:}}{\sin x = \frac{2}{3} = \frac{opp}{hyp}}$ 2 Now, we should find the length of the adjacent side using Now, we should find the length of the adjacent side using the Pythagorean Theorem, so the Pythagorean Theorem, so $|adjacent| = \sqrt{3^2 - 2^2} = \sqrt{9 - 4} = \sqrt{5}$ $|adjacent| = \sqrt{3^2 - 2^2} = \sqrt{9 - 4} = \sqrt{5}$ $\therefore \sec x = \frac{1}{\cos x} = \frac{hyp}{adj} = \frac{3}{\sqrt{5}}$ 82) If $\sin x = \frac{3}{4}$ and $0 < x < \frac{\pi}{2}$, then $\cos x = \frac{\pi}{2}$ $\therefore \csc x = \frac{1}{\sin x} = \frac{hyp}{opp} = \frac{3}{2}$ 83) If $\sin x = \frac{3}{4}$ and $0 < x < \frac{\pi}{2}$, then $\cot x =$ Solution Solution: 3 $\sin x = \frac{3}{4} = \frac{opp}{hyp}$ $\sin x = \frac{3}{4} = \frac{opp}{hyp}$ Now, we should find the length of the adjacent side using Now, we should find the length of the adjacent side using the Pythagorean Theorem, so the Pythagorean Theorem, so $|adjacent| = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$ $|adjacent| = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$ $\therefore \cos x = \frac{adj}{hym} = \frac{\sqrt{7}}{4}$ $\therefore \cot x = \frac{1}{\tan x} = \frac{adj}{adm} = \frac{\sqrt{7}}{3}$

84) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cos x =$	85) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\sec x =$
Solution:	Solution:
5 1 hyp 3	5 1 hyp
$\csc x = \frac{1}{3} = \frac{1}{\sin x} = \frac{1}{opp}$	$\csc x = \frac{1}{3} = \frac{1}{\sin x} = \frac{1}{opp}$
Now, we should find the length of the adjacent side using	Now, we should find the length of the adjacent side using
the Pythagorean Theorem, so	the Pythagorean Theorem, so
$ adjacent = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$	$ adjacent = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
adj 4	$\frac{1}{1} - \frac{hyp}{5} = \frac{5}{1}$
$\therefore \cos x = \frac{1}{hyp} = \frac{1}{5}$	$\frac{1}{\cos x} = \frac{1}{\cos x} = \frac{1}{adj} = \frac{1}{4}$
86) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cot x =$	87) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\tan x =$
Solution:	Solution:
5 1 hyp	5 1 hyp
$\csc x = \frac{1}{3} = \frac{1}{\sin x} = \frac{1}{opp}$	$\csc x = \frac{1}{3} = \frac{1}{\sin x} = \frac{1}{opp}$
Now, we should find the length of the adjacent side using	Now, we should find the length of the adjacent side using
the Pythagorean Theorem, so	the Pythagorean Theorem, so
$ adjacent = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$	$ adjacent = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
1 <i>adj</i> 4	1 opp 3
$\therefore \cot x = \frac{1}{\tan x} = \frac{1}{\cos p} = -\frac{1}{3}$	$\therefore \tan x = \frac{1}{\cot x} = \frac{1}{adj} = -\frac{1}{4}$
88) If $f(x) = \sin x$, then $D_f = \mathbb{R}$	89) If $f(x) = \cos x$, then $D_f = \mathbb{R}$
88) If $f(x) = \sin x$, then $R_f = [-1,1]$	88) If $f(x) = \sin x$, then $R_f = [-1,1]$

Workshop Solutions to Section 2.5 (1.5)

How to find the domain and range of the exponential function $f(x) = a^x$?

1- If $f(x) = c \cdot a^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R}$$
 and $R_f = (\pm k, \infty)$

2- If $f(x) = -c \cdot a^{\pm x} \pm k$ where c and k are positive constants, then

 $D_f = \mathbb{R}$ and $R_f = (-\infty, \pm k)$

3- If $f(x) = c \cdot e^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R}$$
 and $R_f = (\pm k, \infty)$

4- If $f(x) = -c.e^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R}$$
 and $R_f = (-\infty, \pm k)$

1) Find the domain of the function $f(x) = 4^x$.	2) Find the range of the function $f(x) = 4^x$.
Solution:	Solution:
From Step (1) above, we deduce that	From Step (1) above, we deduce that
$D_f = \mathbb{R}$	$R_f = (0, \infty)$
3) Find the domain of the function $f(x) = 4^x - 3$.	4) Find the range of the function $f(x) = 4^x - 3$.
Solution:	Solution:
From Step (1) above, we deduce that	From Step (1) above, we deduce that
$D_f = \mathbb{R}$	$R_f = (-3, \infty)$
5) Find the domain of the function $f(x) = 5 - 3^x$.	6) Find the range of the function $f(x) = 5 - 3^x$.
Solution:	Solution:
From Step (2) above, we deduce that	From Step (2) above, we deduce that
$D_f = \mathbb{R}$	$R_f = (-\infty, 5)$
7) Find the domain of the function $f(x) = 3^{-x} + 1$.	8) Find the range of the function $f(x) = 3^{-x} + 1$.
Solution:	Solution:
From Step (1) above, we deduce that	From Step (1) above, we deduce that
$D_f = \mathbb{R}$	$R_f = (1, \infty)$
9) Find the domain of the function $f(x) = e^x$.	10) Find the range of the function $f(x) = e^x$.
Solution:	Solution:
From Step (3) above, we deduce that	From Step (3) above, we deduce that
$D_f = \mathbb{R}$	$R_f = (0, \infty)$
11) Find the domain of the function $f(x) = e^x - 3$.	12) Find the range of the function $f(x) = e^x - 3$.
Solution:	Solution:
From Step (3) above, we deduce that	From Step (3) above, we deduce that
$D_f = \mathbb{R}$	$R_f = (-3, \infty)$
13) Find the domain of the function $f(x) = e^x + 1$.	14) Find the domain of the function $f(x) = \frac{1}{1-x}$.
Solution:	Solution:
From Step (3) above, we deduce that	$f(x)$ is defined when $1 - e^x \neq 0$
$D_f = \mathbb{R}$	$rac{}{}$ $rac{}$ $rac{}$ $rac{}$ $rac{} rac{} rac{} rac{} rac{} r$
, , , , , , , , , , , , , , , , , , ,	$ \Rightarrow r \neq 0 $
	$\therefore p_{1} - \mathbb{R} \setminus \{0\}$
	11 1/c = 100 0.001/c

15) Find the domain of the function $f(x) = \frac{1}{1+e^x}$.	16) Find the domain of the function $f(x) = \sqrt{1+3^x}$.
Solution:	Solution:
$f(x)$ is defined when $1 + e^x \neq 0$.	$f(x)$ is defined when $1 + 3^x \ge 0$.
But there is no value of x makes $1 + e^x = 0$. Therefore.	But $1 + 3^x > 0$ always. Therefore,
$D_f = \mathbb{R}$	$D_f = \mathbb{R}$
17) If $4^{(x+1)} = 8$, then $x =$	18) If $4^{(x-1)} = 8$, then $x =$
Solution:	Solution:
$4^{(x+1)} = 8$	$4^{(x-1)} = 8$
$(2^2)^{(x+1)} = 2^3$	$(2^2)^{(x-1)} = 2^3$
$2^{2(x+1)} = 2^3$	$2^{2(x-1)} = 2^3$
2(x+1) = 3	2(x-1) = 3
2x + 2 = 3	2x - 2 = 3
2x = 3 - 2 = 1	2x = 3 + 2 = 5
$\cdot r = 1$	5
$\frac{x-\frac{1}{2}}{2}$	$\therefore x - \overline{2}$
19) If $9^{(x+1)} = 27$, then $x =$	20) If $9^{(x-1)} = 27$, then $x =$
Solution:	Solution:
$9^{(x+1)} = 27$	$9^{(x-1)} = 27$
$(3^2)^{(x+1)} = 3^3$	$(3^2)^{(x-1)} = 3^3$
$3^{2(x+1)} = 3^3$	$3^{2(x-1)} = 3^3$
2(x+1) = 3	2(x-1) = 3
2x + 2 = 3	2x - 2 = 3
2x = 3 - 2 = 1	2x = 3 + 2 = 5
$\cdot r - \frac{1}{2}$	$\cdot r = \frac{5}{2}$
	$\frac{1}{2}$
21) If $5^{2(x-1)} = 125$, then $x =$	22) If $5^{2(x+1)} = 125$, then $x =$
Solution:	Solution:
$5^{2(x-1)} = 125$	$5^{2(x+1)} = 125$
$5^{2(x-1)} = 5^3$	$5^{2(x+1)} = 5^3$
2(x-1) = 3	2(x+1) = 3
2x - 2 = 3	2x + 2 = 3
2x = 3 + 2 = 5	2x = 3 - 2 = 1
$\therefore r = \frac{5}{2}$	$\therefore r = \frac{1}{2}$
$\cdots = \frac{1}{2}$	

Workshop Solutions to Section 2.6(1.6)

1) The inverse of the function	2) Find the inverse of the function $f(x) = 2x + 3$
$f = \{(0,3), (-2,1), (3,4), (5, -2), (1,7)\}$ is	Solution:
$f^{-1} = \{(3,0), (1,-2), (3,1), (3,-2), (1,7)\}$	$\int 35000000000000000000000000000000000000$
$\int -\{(3,0), (1,-2), (4,3), (-2,3), (7,1)\}$	2r - y - 3
	2x - y - 3
	$x = \frac{1}{2}$
	Now, change x with $y (x \Leftrightarrow y)$
	$y = \frac{x-3}{2}$
	x - 3
	$\therefore f^{-1}(x) = \frac{1}{2}$
3) Find the inverse of the function $f(x) = 3 - 2x$.	4) Find the inverse of the function $f(x) = 3 - \frac{x}{2}$.
Solution:	Colution:
Let $y = 3 - 2r$	Solution:
2r = 3 - y	Let $y = 3 - \frac{1}{2}$
2x = 3 y	2y = 6 - x
$x = \frac{1}{2}$	x = 6 - 2y
Now, change x with y ($x \Leftrightarrow y$)	Now, change x with $y (x \Leftrightarrow y)$
$y = \frac{3-x}{2}$	y = 6 - 2x
3-x	$\therefore f^{-1}(x) = 6 - 2x$
$\therefore f^{-1}(x) = \frac{1}{2}$	
5) Find the inverse of the function $f(x) = \sqrt{2x-3}$.	6) Find the inverse of the function $f(x) = \sqrt[3]{3-2x}$.
Solution:	Solution:
Let $v = \sqrt{2x - 3}$ by squaring both sides	Let $v = \sqrt[3]{3-2x}$ by cubing both sides
$y^2 = 2x - 3$	$y^3 = 3 - 2x$
$2x = y^2 + 3$	$2x = 3 - \gamma^3$
$y^2 + 3$	$x = \frac{3 - y^3}{2}$
$x = \frac{1}{2}$	$\frac{\lambda}{2}$
Now, change x with y (x \Leftrightarrow y)	Now, change x with y (x \Leftrightarrow y)
$y = \frac{x^2 + 3}{2}$	$y = \frac{3 - x^2}{2}$
$x^2 + 3$	$3 - x^3$
$\therefore f^{-1}(x) = \frac{1}{2}$	$\therefore f^{-1}(x) = \frac{1}{2}$
7) Find the inverse of the function	8) Find the inverse of the function $f(x) = -(x-3)^3$.
$f(x) = (2x+3)^2, x \in [0,\infty)$.	Solution:
Solution:	Let $y = -(x - 3)^3$
Let $y = (2x + 3)^2$	$-y = (x - 3)^3$
Take the square root for both sides	Take the cubic root for both sides
$\sqrt{y} = 2x + 3$	$\sqrt[3]{-y} = x - 3$
$2x = \sqrt{y} - 3$	$x = \sqrt[3]{-y} + 3$
\sqrt{y} -3	Now, change x with $v(x \Leftrightarrow v)$
$x = \frac{1}{2}$	$y = \sqrt[3]{-x} + 3$
Now, change x with y (x \Leftrightarrow y)	<i>y y x</i> + 0
$y = \frac{\sqrt{x-3}}{2}$	$f^{-1}(x) = \sqrt[3]{-x} + 3$
$\sqrt{r}-3$	$(x) = \sqrt{x+3}$
$\therefore f^{-1}(x) = \frac{\sqrt{x-3}}{2}$	
9) Find the inverse of the function $f(x) = \frac{x}{x}$	10) Find the inverse of the function $f(r) = \frac{x-3}{x-3}$
Solution:	Solution:
Solution.	$\frac{5000000}{x-3}$
Let $y = \frac{1}{x-3}$	Let $y = \frac{1}{x}$
y(x-3) = x	xy = x - 3
xy - 3y = x	0
0	xy - x = -3
xy - x = 3y	xy - x = -3 x(y - 1) = -3
xy - x = 3y $x(y - 1) = 3y$	$ \begin{aligned} xy - x &= -3 \\ x(y - 1) &= -3 \\ x &= \frac{-3}{y - 1} = -\frac{3}{y - 1} = \frac{3}{-(y - 1)} = \frac{3}{1 - y} \end{aligned} $
xy - x = 3y x(y - 1) = 3y $x = \frac{3y}{y-1}$	xy - x = -3 x(y - 1) = -3 $x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$ Now, change x with y (x \Leftrightarrow y)
xy - x = 3y x(y - 1) = 3y $x = \frac{3y}{y-1}$ Now, change x with y (x \Leftrightarrow y)	xy - x = -3 x(y - 1) = -3 $x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3}{y-1} = \frac{3}{y-1} = \frac{3}{1-y}$
xy - x = 3y x(y - 1) = 3y $x = \frac{3y}{y-1}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3x}{y-1}$	xy - x = -3 x(y - 1) = -3 $x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3}{1-x}$
xy - x = 3y x(y - 1) = 3y $x = \frac{3y}{y-1}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3x}{x-1}$ 3x	xy - x = -3 x(y - 1) = -3 $x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3}{1-x}$ $\therefore f^{-1}(x) = \frac{3}{1-x}$
xy - x = 3y x(y - 1) = 3y $x = \frac{3y}{y-1}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3x}{x-1}$ $\therefore f^{-1}(x) = \frac{3x}{x-1}$	xy - x = -3 x(y - 1) = -3 $x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3}{1-x}$ $\therefore f^{-1}(x) = \frac{3}{1-x}$
xy - x = 3y x(y - 1) = 3y $x = \frac{3y}{y-1}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3x}{x-1}$ $\therefore f^{-1}(x) = \frac{3x}{x-1}$	xy - x = -3 x(y - 1) = -3 $x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$ Now, change x with y (x \Leftrightarrow y) $y = \frac{3}{1-x}$ $\therefore f^{-1}(x) = \frac{3}{1-x}$

11) Find the inverse of the function $f(x) = \frac{x+2}{x-3}$. 12) Find the inverse of the function $f(x) = \sqrt{x} + 5$. Solution: Solution: Let $y = \frac{x+2}{2}$ Let $y = \sqrt{x} + 5$ x-3 $\sqrt{x} = y - 5$ by squaring both sides y(x-3) = x+2 $x = (y - 5)^2$ xy - 3y = x + 2xy - x = 3y + 2Now, change x with $y (x \Leftrightarrow y)$ x(y-1) = 3y + 2 $x = \frac{3y+2}{y-1}$ $y = (x - 5)^2$ Now, change x with $y (x \Leftrightarrow y)$ $f^{-1}(x) = (x-5)^2$ $y = \frac{3x+2}{x-1}$ $\therefore f^{-1}(x) = \frac{3x+2}{x-1}$ 13) Find the inverse of the function $f(x) = \sqrt[3]{x^5}$. 14) Find the inverse of the function $f(x) = 2x^3 - 5$. Solution: Solution: Let $y = 2x^3 - 5$ Let $y = \sqrt[3]{x^5}$ $2x^3 = y + 5$ $v = x^{\frac{5}{3}}$ $x^3 = \frac{y+5}{2}$ take the cubic root for both sides $y^{\frac{3}{5}} = (x^{\frac{5}{3}})^{\frac{3}{5}}$ $x = \sqrt[3]{\frac{y+5}{2}}$ Now, change x with $y (x \Leftrightarrow y)$ $x = \sqrt[5]{v^3}$ $y = \sqrt[3]{\frac{x+5}{2}}$ Now, change x with y ($x \Leftrightarrow y$) $f^{-1}(x) = \int_{-1}^{3} \frac{x+5}{2}$ $y = \sqrt[5]{x^3}$ $\therefore f^{-1}(x) = \sqrt[5]{x^3}$ 15) Find the inverse of the function $f(x) = \sqrt[3]{\frac{x+2}{5}}$. 16) Evaluate $2^{\log_2(5x+3)}$ Solution: Let $y = \sqrt[3]{\frac{x+2}{5}}$ by cubing both sides Solution: $2^{\log_2(5x+3)} = 5x + 3$ $y^3 = \frac{x+2}{5}$ $5y^3 = x+2$ 17) Evaluate $log_2 2^{(5x+3)}$ $x = 5v^3 - 2$ Solution: $log_2 2^{(5x+3)} = 5x + 3$ Now, change x with $y (x \Leftrightarrow y)$ $y = 5x^3 - 2$ $\therefore f^{-1}(x) = 5x^3 - 2$ 18) $log_2 64 - log_2 32 + log_2 2 = log_2 \frac{64 \times 2}{32}$ 19) $log_3 27 - log_3 81 + 5 log_3 3 = log_3 \frac{27 \times 3^5}{81}$ $= log_2 4 = log_2 2^2$ $= log_3 81 = log_3 3^4$ $= 2log_2 2$ $=4log_33$ $= 2 \times 1 = 2$ $= 4 \times 1 = 4$ OR OR $log_264 - log_232 + log_22 = log_22^6 - log_22^5 + log_22$ $log_3 27 - log_3 81 + 5 log_3 = log_3 3^3 - log_3 3^4 + 5 \times 1$ = 6 - 5 + 1 = 2= 3 - 4 + 5 = 420) $log_3 54 - log_3 2 = log_3 \frac{54}{2}$ 21) If $log_2(6+2x) = 1$, then x =Solution: $= log_3 27 = log_3 3^3 = 3$ $log_2(6+2x) = 1$ $2^{\log_2(6+2x)} = 2^1$ 6 + 2x = 22x = 2 - 6 = -4x = -222) If ln(x+3) = 5, then x =23) If ln(x) = 5, then x =Solution: Solution: ln(x + 3) = 5ln(x) = 5 $e^{\ln(x+3)} = e^5$ $e^{\ln(x)} = e^5$ $x + 3 = e^5$ $x = e^5$ $x = e^5 - 3$

24) If $e^{(2x-3)} = 5$, then $x =$	25) $log_3 2 = \frac{ln2}{ln3}$
Solution:	<i>u</i> LS
$e^{(2x-3)} = 5$	
$lne^{(2x-3)} = ln5$	26) $\log 25 + \log 4 = \log(25 \times 4)$
2x-3=ln5	$= \log 100 = \log 10^2$
2x = ln5 + 3	= 2
$x = \frac{ins+3}{2}$	
27) $log_3 18 - log_3 6 = log_3 \frac{18}{6}$	28) $log_2 6 - log_2 15 + log_2 20 = log_2 \frac{6 \times 20}{15}$
$= log_3 3$	$= log_2 8 = log_2 2^3$
= 1	= 3
29) $e^{3ln2} = e^{ln2^3} = 2^3 = 8$	31) Find the inverse of the function $f(x) = 5 + lnx$.
	Solution:
30) If $3^{2-x} = 6$, then $x =$	Let $y = 5 + lnx$
Solution:	lnx = y - 5
$3^{2-x} = 6$	$e^{\ln x} = e^{y-5}$
$log_3 3^{2-x} = log_3 6$	$x = e^{y-5}$
$2 - x = \log_3 6$	Now, change x with y (x \Leftrightarrow y)
$x = 2 - \log_3 6 = 2 - \log_3 (3 \times 2)$	$y = e^{x-5}$
$= 2 - (log_3 + log_3 2) = 2 - (1 + log_3 2)$	$\therefore f^{-1}(x) = e^{x-5}$
$= 2 - 1 - \log_3 2$	
$= 1 - \log_3 2$	
32) Find the domain of the function	33) Find the domain of the function
$f(x) = \sin^{-1}(3x + 5) .$	$f(x) = \cos^{-1}(3x - 5)$.
Solution:	Solution:
We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$. So,	We know that the domain of $\cos^{-1}(x)$ is $[-1,1]$. So,
$-1 \le 3x + 5 \le 1$	$-1 \leq 3x - 5 \leq 1$
$-6 \le 3x \le -4$	$4 \leq 5x \leq 0$
$-2 \le x \le -\frac{4}{2}$	$\frac{4}{2} \le x \le 2$
г <u>4</u> 1	3 [4] 1
$\therefore D_f = \left\lfloor -2, -\frac{1}{3} \right\rfloor$	$\therefore D_f = \begin{bmatrix} -3 & 2 \end{bmatrix}$
34) Find the domain of the function	
$f(x) = 2\sin^{-1}(x) + 1$.	
Solution:	
We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$. So,	
$\therefore D_f = [-1,1]$	

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Before proceeding to the questions 35-55, we should be aware of the following well-known right triangles:



42) If $\alpha = \cos^{-1}\left(\frac{4}{r}\right)$, then $\cot \alpha =$ 41) If $\alpha = \cos^{-1}\left(\frac{4}{r}\right)$, then $\csc \alpha =$ Solution: Solution: $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ 3 $\cos \alpha = \frac{4}{5} = \frac{\frac{3}{adj}}{\frac{h}{h}}$ $\cos \alpha = \frac{4}{5} = \frac{adj}{hyn}$ Now, we should find the length of the opposite side using Now, we should find the length of the opposite side using the Pythagorean Theorem, so the Pythagorean Theorem, so $|opposite| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ $|opposite| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ $\therefore \csc \alpha = \frac{1}{\sin \alpha} = \frac{hyp}{opp} = \frac{5}{3}$ 43) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\tan \alpha =$ $\therefore \cot \alpha = \frac{1}{\tan \alpha} = \frac{adj}{opp} = \frac{4}{3}$ 44) If $\alpha = \cos^{-1}\left(\frac{4}{r}\right)$, then $\sin \alpha =$ Solution: Solution: $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ $\cos \alpha = \frac{4}{5} = \frac{adj}{hyp}$ $\cos \alpha = \frac{4}{5} = \frac{adj}{hvp}$ Now, we should find the length of the opposite side using Now, we should find the length of the opposite side using the Pythagorean Theorem, so the Pythagorean Theorem, so $|opposite| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ $|opposite| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ $\therefore \sin \alpha = \frac{opp}{hyp} = \frac{3}{5}$ 46) $\tan \left(\cos^{-1} \left(\frac{4}{5} \right) \right) = \frac{5}{5}$ $\therefore \tan \alpha = \frac{1}{\cot \alpha} = \frac{opp}{adj} = \frac{3}{4}$ 45) $\sin\left(\cos^{-1}\left(\frac{4}{r}\right)\right) =$ Solution: Solution: Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ $\cos \alpha = \frac{4}{5} = \frac{adj}{hyp}$ $\cos \alpha = \frac{4}{5} = \frac{adj}{hyp}$ Now, we should find the length of the opposite side using Now, we should find the length of the opposite side using the Pythagorean Theorem, so the Pythagorean Theorem, so $|opposite| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ $|opposite| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ $\therefore \sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \sin(\alpha) = \frac{opp}{hyp} = \frac{3}{5}$ 47) $\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) =$ $\therefore \quad \tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \tan(\alpha) = \frac{opp}{adj} = \frac{3}{4}$ 48) $\cos(\tan^{-1} x) =$ Solution: Solution: x 2 Let $\alpha = \tan^{-1} x$ Let $\alpha = \sin^{-1}\left(\frac{2}{5}\right)$ $\tan \alpha = x = \frac{opp}{adj}$ $\sin \alpha = \frac{2}{5} = \frac{1}{5} \frac{1}{5}$ VZI Now, we should find the length of the hypotenuse side Now, we should find the length of the adjacent side using using the Pythagorean Theorem, so the Pythagorean Theorem, so $|hypotenuse| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$ $|adjacent| = \sqrt{5^2 - 2^2} = \sqrt{25 - 4} = \sqrt{21}$ $\cos(\tan^{-1} x) = \cos(\alpha) = \frac{adj}{hyn} = \frac{1}{\sqrt{\alpha^2 + 1}}$ $\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin\left(2\alpha\right)$ Now, use the identity $sin(2x) = 2 sin x \cdot cos x$. Thus, $\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ $= 2 \times \frac{2}{5} \times \frac{\sqrt{21}}{5} = \frac{4\sqrt{21}}{25}$ 49) $\sin(\tan^{-1} x) =$ 50) $\csc(\tan^{-1} x) =$ Solution: Solution: Let $\alpha = \tan^{-1} x$ $\tan \alpha = x = \frac{opp}{adj}$ Let $\alpha = \tan^{-1} x$ $\tan \alpha = x = \frac{opp}{adj}$ Now, we should find the length of the hypotenuse side Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so using the Pythagorean Theorem, so $|hypotenuse| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$ $|hypotenuse| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$ $\sin(\tan^{-1} x) = \sin(\alpha) = \frac{opp}{hyp} = \frac{x}{\sqrt{x^2 + 1}}$ $\csc(\tan^{-1} x) = \csc(\alpha) = \frac{1}{\sin \alpha} = \frac{hyp}{opp} = \frac{\sqrt{x^2 + 1}}{x}$

51) $\sec(\tan^{-1} x) =$ 52) $\sec(\sin^{-1}\frac{x}{2}) =$ Solution: Solution: Let $\alpha = \tan^{-1} x$ × Let $\alpha = \sin^{-1}\frac{x}{2}$ $\tan \alpha = x = \frac{opp}{adj}$ Now, we should find the length of the hypotenuse side $\sin \alpha = \frac{x}{3} = \frac{opp}{hyp}$ V9-x2 using the Pythagorean Theorem, so Now, we should find the length of the adjacent side using $|hypotenuse| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$ the Pythagorean Theorem, so $\sec(\tan^{-1} x) = \sec(\alpha) = \frac{1}{\cos \alpha} = \frac{hyp}{adi} = \frac{\sqrt{x^2 + 1}}{1} = \sqrt{x^2 + 1}$ $|adjacent| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$ $sec\left(\sin^{-1}\frac{x}{3}\right) = sec(\alpha) = \frac{1}{\cos\alpha} = \frac{hyp}{adj} = \frac{3}{\sqrt{9-x^2}}$ 54) $tan\left(\sin^{-1}\frac{x}{3}\right) =$ 53) $\cot\left(\sin^{-1}\frac{x}{3}\right) =$ Solution: Solution: Let $\alpha = \sin^{-1}\frac{x}{x}$ Let $\alpha = \sin^{-1}\frac{x}{2}$ $\sin \alpha = \frac{x}{3} = \frac{opp}{hyp}$ $\sin \alpha = \frac{x}{3} = \frac{opp}{hyp}$ Now, we should find the length of the adjacent side using Now, we should find the length of the adjacent side using the Pythagorean Theorem, so the Pythagorean Theorem, so $|adjacent| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$ $|adjacent| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$ $\tan\left(\sin^{-1}\frac{x}{3}\right) = \tan(\alpha) = \frac{1}{\cot\alpha} = \frac{opp}{adj} = \frac{x}{\sqrt{9-r^2}}$ $\cot\left(\sin^{-1}\frac{x}{3}\right) = \cot(\alpha) = \frac{1}{\tan\alpha} = \frac{adj}{opp} = \frac{\sqrt{9-x^2}}{x}$ 55) $\cos\left(\sin^{-1}\frac{x}{2}\right) =$ Solution: Let $\alpha = \sin^{-1}\frac{x}{x}$ $\sin \alpha = \frac{x}{3} = \frac{opp}{hyp}$ Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $|adjacent| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$ $\cos\left(\sin^{-1}\frac{x}{3}\right) = \cos(\alpha) = \frac{adj}{hvp} = \frac{\sqrt{9-x^2}}{3}$

Workshop Solutions to Sections 3.4 and 3.5(2.2 & 2.5)

1) $\lim_{r \to 3^+} \frac{2}{r-3} =$	2) $\lim_{r \to 3^{-}} \frac{2}{r-3} =$
Solution: $\frac{1}{10}$ then $u \ge 2$ \rightarrow $u = 2 \ge 0$	Solution: $\frac{1}{10}$ $\frac{1}{10}$
$\begin{array}{c} 11 \ x \rightarrow 5 \ , \text{ then } x > 5 \ \implies x - 5 > 0 \\ 2 \end{array}$	If $x \to 3$, then $x < 3 \implies x - 3 < 0$
$\therefore \lim_{x \to 3^+} \frac{1}{x - 3} = \infty$	$\therefore \lim_{x \to 3^-} \frac{1}{x - 3} = -\infty$
3) $\lim_{x \to -2} \frac{-2}{x-2} =$	4) $\lim_{x \to 0^+} \frac{-2}{x^2 - 2} =$
$\frac{x \rightarrow 3^{+} x - 5}{\text{Solution:}}$	$\frac{x \rightarrow 3}{\text{Solution:}} x = 5$
If $x \to 3^+$, then $x > 3 \implies x - 3 > 0$	If $x \to 3^-$, then $x < 3 \implies x - 3 < 0$
$\therefore \lim_{x \to 3^+} \frac{z}{x-3} = -\infty$	$\therefore \lim_{x \to 3^-} \frac{z}{x-3} = \infty$
5) $\lim_{x \to -\infty} \frac{2}{x} =$	6) $\lim_{x \to -\infty} \frac{2}{x} =$
Solution: $x \rightarrow -3^+ x + 3$	Solution: $x \rightarrow -3^{-}x + 3$
If $x \to -3^+$, then $x > -3 \implies x + 3 > 0$	If $x \to -3^-$, then $x < -3 \implies x + 3 < 0$
$\therefore \lim_{x \to -3^+} \frac{2}{x+3} = \infty$	$\therefore \lim_{x \to -3^-} \frac{2}{x+3} = -\infty$
7) $\lim \frac{3x-1}{x} =$	8) $\lim \frac{3x-1}{x} =$
$\sum_{x \to 2^+}^{x \to 2^+} x - 2$	Solution: $x \to 2^ x - 2^-$
If $x \to 2^+$, then $x > 2 \implies x - 2 > 0$ and $3x - 1 > 0$	If $x \to 2^-$, then $x < 2 \implies x - 2 < 0$ and $3x - 1 > 0$
$\therefore \lim_{x \to 2^+} \frac{3x - 1}{x - 2} = \infty$	$\therefore \lim_{x \to 2^-} \frac{3x - 1}{x - 2} = -\infty$
9) $\lim_{x \to \infty} \frac{1-x}{x} = 1 + \frac{1-x}{x} = 1 +$	10) $\lim_{x \to \infty} \frac{1-x}{x} = 1$
$\int_{x \to -2^+}^{x \to -2^+} (x+2)^2 =$	$x \rightarrow -2^{-} (x+2)^{2}$
If $x \to -2^+$, then $x > -2$	If $x \to -2^-$, then $x < -2$
$\implies 1 - x > 0 \text{ and } (x + 2)^2 > 0$	$\implies 1-x>0 \text{ and } (x+2)^2 > 0$
$\therefore \lim_{x \to -2^+} \frac{1-x}{(x+2)^2} = \infty$	$\therefore \lim_{x \to -2^+} \frac{1-x}{(x+2)^2} = \infty$
11) $\lim_{x \to 1^+} \frac{x-1}{(x+2)^2} =$	12) $\lim_{x \to 0^+} \frac{x-1}{(x+2)^2} =$
$\frac{x \rightarrow -2^{-1} (x + 2)^{-1}}{\text{Solution:}}$	$\frac{x \rightarrow -2}{\text{Solution:}} (x + 2)^{-1}$
If $x \to -2^+$, then $x > -2$	If $x \to -2^-$, then $x < -2$
$\Rightarrow x - 1 < 0 \text{ and } (x + 2)^2 > 0$ $x - 1$	$\Rightarrow x - 1 < 0 \text{ and } (x + 2)^2 > 0$ $x - 1$
$\therefore \lim_{x \to -2^+} \lim_{(x+2)^2} = -\infty$	$\therefore \lim_{x \to -2^-} \frac{1}{(x+2)^2} = -\infty$
13) $\lim \frac{6x-1}{2} =$	14) $\lim_{x \to -\infty} \frac{6x - 1}{x} =$
Solution: $x \rightarrow 2^+ x^2 - 4$	Solution: $x \rightarrow 2^{-} x^{2} - 4$
If $x \to 2^+$, then $x^2 > 4$	If $x \to \overline{2}^-$, then $x^2 < 4$
$\Rightarrow x^2 - 4 > 0 \text{ and } 6x - 1 > 0$	$\Rightarrow x^2 - 4 < 0 \text{ and } 6x - 1 > 0$ $6x - 1$
$\therefore \lim_{x \to 2^+} \frac{1}{x^2 - 4} = \infty$	$\therefore \lim_{x \to 2^+} \frac{1}{x^2 - 4} = -\infty$
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$$\begin{array}{ll} 15 & \lim_{x \to -2} \frac{6x - 1}{x^2 - 4} = \\ & \text{Solution:} \\ \text{if } x \to -2^2, \text{ then } x^2 < 4 \\ & \Rightarrow x^2 - 4 < 0 \text{ and } 6x - 1 < 0 \\ & \therefore & \lim_{x \to -2} \frac{6x - 1}{x^2 - 4} = \infty \end{array}$$

$$\begin{array}{ll} 16 & \lim_{x \to -2} \frac{6x - 1}{x^2 - 4} = \\ & \text{Solution:} \\ \text{if } x \to -2^2, \text{ then } x^2 < 4 \\ & \Rightarrow x^2 - 4 > 0 \text{ and } 6x - 1 < 0 \\ & \therefore & \lim_{x \to -2} \frac{6x - 1}{x^2 - x - 6} = \\ & \text{Solution:} \end{array}$$

$$\begin{array}{ll} f(x) = \frac{6x - 1}{x^2 - x - 6} = \\ & \text{Solution:} \\ f(x) = \frac{6x - 1}{x^2 - x - 6} = \frac{6x - 1}{(x - 3)(x + 2)} \\ \text{if } x \to -2^2, \text{ then } x < -2 \\ & \Rightarrow x - 3 < 0, x + 2 < 0 \text{ and } 6x - 1 < 0 \\ & \therefore & \lim_{x \to -2} \frac{6x - 1}{x^2 - x - 6} = \frac{6x - 1}{(x - 3)(x + 2)} \\ \text{if } x \to -2^2, \text{ then } x > 3 \\ & \Rightarrow x - 3 < 0, x + 2 < 0 \text{ and } 6x - 1 < 0 \\ & \therefore & \lim_{x \to -2} \frac{6x - 1}{x^2 - x - 6} = \frac{-1}{(x - 3)(x + 2)} \\ \text{if } x \to 3^2, \text{ then } x > 3 \\ & \Rightarrow x - 3 < 0, x + 2 > 0 \text{ and } 6x - 1 < 0 \\ & \therefore & \lim_{x \to -2} \frac{6x - 1}{x^2 - x - 6} = \frac{-1}{(x - 3)(x + 2)} \\ \text{if } x \to 3^2, \text{ then } x > 3 \\ & \Rightarrow x - 3 < 0, x + 2 > 0 \text{ and } -1 < 0 \\ & \therefore & \lim_{x \to -2} \frac{1}{x^2 - x - 6} = \frac{-1}{(x - 3)(x + 2)} \\ \text{if } x \to 3^2, \text{ then } x > 3 \\ & \Rightarrow x - 3 < 0, x + 2 > 0 \text{ and } -1 < 0 \\ & \therefore & \lim_{x \to -2^2} \frac{1}{x^2 - x - 6} = \frac{-1}{(x - 3)(x + 2)} \\ \text{if } x \to 3^2, \text{ then } x > 3 \\ & \Rightarrow x - 3 < 0, x + 2 > 0 \text{ and } -1 < 0 \\ & \therefore & \lim_{x \to -2^2} \frac{1}{x^2 - x - 6} = \frac{-1}{(x - 3)(x + 2)} \\ \text{if } x \to 3^2, \text{ then } x > 3 \\ & \Rightarrow x - 3 < 0, x + 2 > 0 \text{ and } -1 < 0 \\ & \therefore & \lim_{x \to -1^2} \frac{1}{x^2 - x - 6} = \infty \end{aligned}$$

$$21) \lim_{x \to +1^2} \lim_{x \to -\infty} \lim_{x \to -1^2/2^2} \lim_{x \to 0} x = \frac{-1}{2^2} \text{ is a vertical asymptote of } f(x) = \frac{1 - x}{2 - 4} = \infty \\ \lim_{x \to -1^2/2^2} \lim_{x \to -1^2} \frac{1 - x}{2 - 4} = \infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty \\ \lim_{x \to -1^2/2^2} \frac{1 - x}{x^2 - 4} = -\infty$$

26) The vertical asymptote of $f(x) = \frac{7-x}{x^2-5x+6}$ is 25) The vertical asymptote of $f(\overline{x}) = \frac{3-x}{x^2-x-6}$ is Solution Solution: $f(x) = \frac{7 - x}{x^2 - 5x + 6} = \frac{7 - x}{(x - 3)(x - 2)}$ $f(x) = \frac{3-x}{x^2 - x - 6} = \frac{3-x}{(x-3)(x+2)} = \frac{-(x-3)}{(x-3)(x+2)}$ We see that the function f(x) is not defined when x-3=0 or $x-2=0 \implies x=3$ or x=2. x + 2We see that the function f(x) is not defined when Since $\lim_{x \to 3^+} \frac{7 - x}{x^2 - 5x + 6} = \lim_{x \to 3^+} \frac{7 - x}{(x - 3)(x - 2)} = \infty$ $\lim_{x \to 3^-} \frac{7 - x}{x^2 - 5x + 6} = \lim_{x \to 3^-} \frac{7 - x}{(x - 3)(x - 2)} = -\infty$ $x^{2} - x - 6 = 0 \implies (x - 3)(x + 2) = 0$ \Rightarrow x = 3 or x = -2. Since $\lim_{x \to 3} \frac{3-x}{x^2 - x - 6} = \lim_{x \to 3} \frac{3-x}{(x-3)(x+2)}$ $= \lim_{x \to 3} \frac{-(x-3)}{(x-3)(x+2)} = \lim_{x \to 3} \frac{-1}{x+2} = -\frac{1}{5}$ and $\lim_{x \to 2^+} \frac{7 - x}{x^2 - 5x + 6} = \lim_{x \to 2^+} \frac{7 - x}{(x - 3)(x - 2)} = -\infty$ $\lim_{x \to 2^-} \frac{7 - x}{x^2 - 5x + 6} = \lim_{x \to 2^-} \frac{7 - x}{(x - 3)(x - 2)} = \infty$ then, x = 3 is a removable discontinuity. $\lim_{x \to -2^+} \frac{3-x}{x^2 - x - 6} = \lim_{x \to -2^+} \frac{3-x}{(x-3)(x+2)} = \infty$ and then, x = 3 and x = 2 are vertical asymptotes. $\lim_{x \to -2^{-}} \frac{3-x}{x^2 - x - 6} = \lim_{x \to -2^{-}} \frac{3-x}{(x-3)(x+2)} = -\infty$ then, x = -2 is a vertical asymptote only 27) The vertical asymptote of $f(x) = \frac{x-7}{x^2+5x+6}$ is 28) The vertical asymptote of $f(x) = \frac{x-7}{x^2+3x}$ is Solution: Solution: $f(x) = \frac{x-7}{x^2+5x+6} = \frac{x-7}{(x+3)(x+2)}$ $f(x) = \frac{x-7}{x^2+3x} = \frac{x-7}{x(x+3)}$ We see that the function f(x) is not defined when We see that the function f(x) is not defined when x = 0 or $x + 3 = 0 \implies x = 0$ or x = -3. Since x + 3 = 0 or $x + 2 = 0 \implies x = -3$ or x = -2. $\lim_{x \to -3^+} \frac{x-7}{x^2+3x} = \lim_{x \to -3^+} \frac{x-7}{x(x+3)} = \infty$ $\lim_{x \to -3^-} \frac{x-7}{x^2+3x} = \lim_{x \to -3^-} \frac{x-7}{x(x+3)} = -\infty$ Since $\lim_{x \to -3^+} \frac{x-7}{x^2+5x+6} = \lim_{x \to -3^+} \frac{x-7}{(x+3)(x+2)} = \infty$ $\lim_{x \to -3^{-}} \frac{x-7}{x^2+5x+6} = \lim_{x \to -3^{-}} \frac{x-7}{(x+3)(x+2)} = -\infty$ and $\lim_{x \to 0^+} \frac{x-7}{x^2+3x} = \lim_{x \to 0^+} \frac{x-7}{x(x+3)} = -\infty$ $\lim_{x \to 0^-} \frac{x-7}{x^2+3x} = \lim_{x \to 0^-} \frac{x-7}{x(x+3)} = \infty$ and $\lim_{x \to -2^+} \frac{x-7}{x^2 + 5x + 6} = \lim_{x \to -2^+} \frac{x-7}{(x+3)(x+2)} = -\infty$ $\lim_{x \to -2^{-}} \frac{x-7}{x^2 + 5x + 6} = \lim_{x \to -2^{-}} \frac{x-7}{(x+3)(x+2)} = \infty$ then, x = -3 and x = 0 are vertical asymptotes. then, x = -3 and x = -2 are vertical asymptotes. 29) The vertical asymptote of $f(x) = \frac{x-7}{x^2-3x}$ is 30) The vertical asymptotes of $f(x) = \frac{2x^2+1}{x^2-9}$ are Solution: Solution: $f(x) = \frac{x-7}{x^2-3x} = \frac{x-7}{x(x-3)}$ $f(x) = \frac{2x^2 + 1}{x^2 - 9} = \frac{2x^2 + 1}{(x + 3)(x - 3)}$ We see that the function f(x) is not defined when We see that the function f(x) is not defined when x = 0 or $x - 3 = 0 \implies x = 0$ or x = 3. Since $x^2 - 9 = 0 \implies x = \pm 3$. Since $\lim_{x \to 3^+} \frac{x - 7}{x^2 - 3x} = \lim_{x \to 3^+} \frac{x - 7}{x(x - 3)} = -\infty$ $\lim_{x \to 3^+} \frac{2x^2 + 1}{x^2 - 9} = \lim_{x \to 3^+} \frac{2x^2 + 1}{(x + 3)(x - 3)} = \infty$ $\lim_{x \to 3^{-}} \frac{x-7}{x^2-3x} = \lim_{x \to 3^{-}} \frac{x-7}{x(x-3)} = \infty$ $\lim_{x \to 3^{-}} \frac{2x^2 + 1}{x^2 - 9} = \lim_{x \to 3^{-}} \frac{2x^2 + 1}{(x + 3)(x - 3)} = -\infty$ and and $\lim_{x \to 0^+} \frac{x-7}{x^2 - 3x} = \lim_{x \to 0^+} \frac{x-7}{x(x-3)} = \infty$ $\lim_{x \to -3^+} \frac{2x^2 + 1}{x^2 - 9} = \lim_{x \to -3^+} \frac{2x^2 + 1}{(x + 3)(x - 3)} = -\infty$ $\lim_{x \to -3^-} \frac{2x^2 + 1}{x^2 - 9} = \lim_{x \to -3^-} \frac{2x^2 + 1}{(x + 3)(x - 3)} = \infty$ $\lim_{x \to 0^{-}} \frac{x-7}{x^2-3x} = \lim_{x \to 0^{-}} \frac{x-7}{x(x-3)} = -\infty$ then, x = 3 and x = 0 are vertical asymptotes. then, $x = \pm 3$ are vertical asymptotes.

because $x \to y$ $1 - f(2) = \frac{(2) + 1}{(2)^2 - 9} = \frac{3}{-5} = -\frac{3}{5}$ $2 - \lim_{x \to -\frac{1}{2}^2 - 9} = \lim_{x \to -\frac{1}{2}^2 (2)^2 - 9} = \frac{3}{-5} = -\frac{3}{5}$ $3 - \lim_{x \to -\frac{1}{2}^2 - 9} = f(2)$ CR We know that $D_f = \mathbb{R} \setminus \{\pm 3\}$, so $\{2\} \in D_f$. Note: Any function is continuous on its domain. 34) The function $f(x) = \frac{x+1}{2x-9}$ is continuous on its domain which is $D_f = \mathbb{R} \setminus \{\pm 3\}$. 35) The function $f(x) = \frac{x+1}{2x-9}$ is is continuous on its domain which is $D_f = \mathbb{R} \setminus \{\pm 3\}$. 36) The function $f(x) = \frac{x+1}{2x-9}$ is continuous on its domain which is $D_f = \mathbb{R} \setminus \{\pm 3\}$. 37) The function $f(x) = \frac{x^{x+1}}{x-1} = \frac{x^{x+1}}{x-0}$ is discontinuous at $a = 0$ because 1 - $f(0) = 3$ 2 - $\lim_{x \to 0} \frac{\sin 3x}{x} = 3(1) = 3$ 3 - $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \left\{\frac{2x^{2} - 3x + 1}{x-1}, x \neq 1$ is $\frac{1}{1}$ is $x = 1$ because 1 - $f(1) = 7$ 2 - $\lim_{x \to 0} \frac{\sin 3x}{x-0} = 3(1) = 3$ 3 - $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \left\{\frac{2x^{2} - 3x + 1}{x-1}, x \neq 1$ is $\frac{1}{1}$ is $x = 1$ because 1 - $f(1) = 7$ 2 - $\lim_{x \to 0} \frac{\sin 3x}{x-0} = 3(1) = 3$ 3 - $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \left\{\frac{2x^{2} - 3x + 1}{1}, x \neq 1$ is $\frac{1}{1}$ is $\frac{1}{x-1} = \lim_{x \to 1} (2x - 1) = 1$ 3 - $\lim_{x \to 1} f(x) \neq f(1)$ 39) The function $f(x) = \frac{2x^{2} - 3x + 1}{x-1} = \lim_{x \to 1} (2x - 1) = 1$ 31) The function $f(x) = \frac{2x^{2} + 3x + 2}{3x + 1} = \lim_{x \to 1} (2x - 1) = 1$ 31) The function $f(x) = \frac{2x^{2} + 3x + 2}{3x + 1} = \lim_{x \to 1} (2x - 1) = 1$ 31) The function $f(x) = \frac{2x^{2} + 3x + 2}{3x + 1} = \lim_{x \to 1} (2x - 1) = 1$ 31) The function $f(x) = \frac{2x^{2} + 3x + 2}{3x + 1} = \lim_{x \to 1} (2x - 1) = 1$ 32) The function $f(x) = \frac{2x^{2} + 3x + 2}{3x + 1} = \frac{2x^{2}}{3x + 1} = \lim_{x \to 1} (2x - 1) = 1$ 32) The function $f(x) = \frac{2x^{2} + 3x + 2}{3x + 1} = \frac{2x^{2}}{3x + 1} = \frac{2x^{2}}{3x + 1} = \frac{2x^{2} + 3x + 2}{3x + 2} = \frac{2x^{2} + 3x + 2}{3x + 2} = \frac{2x^{2} + 3x^{2} + 3x$
$1 - f(2) = \frac{(2)+1}{(2)^2-9} = \frac{3}{-5} = -\frac{3}{5}$ $2 - \lim_{x\to 3^+} \frac{1}{x^2-9} = \lim_{x\to 2} \frac{(2)+1}{(2)^2-9} = \frac{3}{-5} = -\frac{3}{5}$ $3 - \lim_{x\to 3^+} \frac{1}{x^2-9} = f(2)$ OR Note: Any function is continuous on its domain. 34) The function $f(x) = \frac{x+1}{x^2-3}$ is continuous on its domain. 34) The function $f(x) = \frac{x+1}{x^2-3}$ is continuous on its domain. 35) The function $f(x) = \frac{x+1}{x^2-3}$ is continuous on its domain. 36) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 3, x = 0 \end{cases}$ is continuous at $a = 0$ because $1 - f(0) = 3$ $2 - \lim_{x\to 0^+} \frac{\sin \sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{\sin x}{x} = 3 \lim_{x\to 0^+} \frac{\sin x}{3x} = 3(1) = 3$ $3 - \lim_{x\to 0^+} \frac{x}{x} = 1 = \lim_{x\to 0^+} \frac{(2x-1)(x-1)}{x-1} = \lim_{x\to 1^+} (2x-1) = 1$ $3 - \lim_{x\to 0^+} \frac{x^2}{x-1} = \lim_{x\to 0^+} \frac{(2x-3)(x-1)}{x-1} = \lim_$
$(2^{1}-3^{1}-3^{2}-3) = 5$ $2 - \lim_{x \to 1^{+}} x^{1} = \lim_{x \to 2} (2^{2}+1) = 3$ $3 - \lim_{x \to 2} x^{1} - 2 = 6$ $(2^{1}-3^{1}-3^{2}-3) = 6$ $(2^{1}-3^{1}-3^{2}-3) = 6$ $(2^{1}-3^{1}-3^{2}-3) = 6$ $(2^{1}-3^{1}-3^{1}-3) = 6$ $(2^{1}-3^{1}-3$
$2 - \lim_{x \to 3} \frac{1}{x^2 - 9} = \lim_{x \to 1} \frac{1}{(2)^2 - 9} = \frac{1}{-5} = -\frac{1}{5}$ $3 - \lim_{x \to 2} \frac{x + 1}{x^2 - 9} = f(2)$ OR We know that $D_f = \mathbb{R} \setminus \{\pm 3\}$, so $\{2\} \in D_f$. Note: Any function is continuous on its domain. 34) The function $f(x) = \frac{x+1}{x^2 - 9}$ is continuous on its domain. 34) The function $f(x) = \frac{x+1}{x^2 - 9}$ is continuous on its domain. 34) The function $f(x) = \frac{x+1}{x^2 - 9}$ is continuous on its domain. 34) The function $f(x) = \frac{x+1}{x^2 - 9}$ is continuous on its domain. 35) The function $f(x) = \frac{x+1}{x^2 - 9}$ is continuous at $a = 0$ because $1 \cdot f(0) = 3$ $2 \cdot \lim_{x \to 0} \frac{\sin 3x}{x} = 3(1) = 3$ $3 \cdot \lim_{x \to 0} f(x) = f(0)$ 36) The function $f(x) = \left\{ \frac{\sin 3x}{x}, x \neq 0 \\ 5, x = 0 \text{ is discontinuous} \\ 1 \cdot f(0) = 5$ $1 \cdot f(0) = 5$ $1 \cdot f(0) = 5$ $2 \cdot \lim_{x \to 0} \frac{\sin 3x}{x} = 3(1) = 3$ $3 \cdot \lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \left\{ \frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \\ 1, x = 1 \\ continuous at a = 1 because 1 \cdot f(1) = 7 2 \cdot \lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1 3 \cdot \lim_{x \to 1} f(x) = f(1) 4 \circ The function f(x) = \left\{ \frac{2x^2 - 3x + 1}{x + 1}, x \neq 2 \\ 1, x = 1 \\ continuous at a = 1 because 1 \cdot f(1) = 7 2 \cdot \lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1 3 \cdot \lim_{x \to 1} f(x) = f(1) 4 \circ The function f(x) = \left\{ \frac{2x + 3}{x + 1}, x \geq 2 \\ x = 1 \\ x = $
$3 - \lim_{x \to 2} \frac{x^{2} - 9}{x^{2} - 9} = f(2)$ OR We know that $D_{f} = \mathbb{R} \setminus \{\pm 3\}$, so $\{2\} \in D_{f}$. Note: Any function is continuous on its domain. $34 \text{ The function } f(x) = \frac{x+1}{x^{2} - 9} \text{ is continuous on its domain.}$ $34 \text{ The function } f(x) = \frac{x+1}{x^{2} - 9} \text{ is continuous on its domain.}$ $34 \text{ The function } f(x) = \frac{x+1}{x^{2} - 9} \text{ is continuous on its domain.}$ $35 \text{ The function } f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 3, x = 0 \end{cases}$ $35 \text{ The function } f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 3, x = 0 \end{cases}$ $35 \text{ The function } f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 3, x = 0 \end{cases}$ $36 \text{ The function } f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 5, x = 0 \end{cases}$ $36 \text{ The function } f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 5, x = 0 \end{cases}$ $37 \text{ The function } f(x) = \begin{cases} \frac{2x^{2} - 3x + 1}{x - 1}, x \neq 1 \\ 1, x = 1 \end{cases}$ $38 \text{ The function } f(x) = \begin{cases} \frac{2x^{2} - 3x + 1}{x - 1}, x \neq 1 \\ 1, x = 1 \end{cases}$ $39 \text{ The function } f(x) = \frac{x^{2} - x - 2}{x - 2} \text{ is discontinuous at } a = 2 \text{ because } 1 \cdot f(1) = 1 \\ 2 \cdot \lim_{x \to 1} \frac{2x^{2} - 3x + 1}{x - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x - 1} = \lim_{x \to 1} (2x - 1) = 1 \\ 3 \cdot \lim_{x \to 1} f(x) \neq f(1)$ $38 \text{ The function } f(x) = \begin{cases} \frac{2x^{2} - 3x + 1}{x - 1}, x \neq 1 \\ 1, x = 1 \\ x = 2 \text{ because } (2) \notin D_{f} \text{ .} \end{cases}$ $41 \text{ The function } f(x) = \frac{x^{2} - 3x - 2}{x^{2} - 3} \text{ is ontinuous on its domain where } f(x) \text{ is defined, we mean that } x^{2} - 4 > 0 \Rightarrow x^{2} > 4 \Rightarrow \sqrt{x^{2}} > \sqrt{4} \Rightarrow x > 2 \Leftrightarrow x > 2 \text{ or } x < -2 \\ \text{Hence,} \\ D_{f} = (-\infty, -2) \cup (2,\infty) \text{ .} \end{cases}$ $42 \text{ The function } f(x) = \sqrt{x^{2} - 4} \text{ is continuous on its domain where } f(x) \text{ is defined, we mean that } x^{2} - 4 \ge 0 \Rightarrow x^{2} > 4 \Rightarrow \sqrt{x^{2}} > \sqrt{4} \Rightarrow x > 2 \Leftrightarrow x > 2 \text{ or } x < -2 \\ \text{Hence,} \\ D_{f} = (-\infty, -2) \cup (2,\infty) \text{ .} \end{cases}$
$x + 2x^{2} - 9 f < f < f \\ x + 2x^{2} - 9 f < f < f \\ x + 2x^{2} - 9 f < f < f \\ x + 2x^{2} - 9 f < f < f \\ x + 2x^{2} - 9 f < f < f \\ x + 2x^{2} - 9 f < f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 9 f < f \\ x + 2x^{2} - 3x^{2} - 3x^$
We know that $D_f = \mathbb{R} \setminus \{\pm 3\}$, so $\{2\} \in D_f$. Note: Any function is continuous on its domain. 34) The function $f(x) = \frac{x+1}{x^{2}-y}$ is continuous on its domain. 35) The function $f(x) = \begin{cases} \frac{\sin 3x}{x^2}, x \neq 0 \\ 3, x = 0 \end{cases}$ is continuous at $a = 0$ because 1 $f(0) = 3$ 2 $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 1} \frac{\sin 3x}{3x} = 3(1) = 3$ 3 $\lim_{x \to 0} f(x) \neq f(0)$ 36) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 5, x = 0 \end{cases}$ is discontinuous at $a = 0$ because 1 $f(0) = 5$ 2 $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 1} \frac{\sin 3x}{3x} = 3(1) = 3$ 3 $\lim_{x \to 0} f(x) \neq f(0)$ 37) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x}, x \neq 1 \\ 1, x = 1 \end{cases}$ is discontinuous at $a = 1$ because 1 $f(1) = 1$ 38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x}, x \neq 1 \\ \frac{1}{x-1}, x = 1 \end{cases}$ is continuous at $a = 1$ because 1 $f(1) = 1$ 39) The function $f(x) = \frac{x^{2} - 3x + 1}{x^{2} - 1}$ is discontinuous at $a = 2$ because $\{2\} \notin D_f$. 30) The function $f(x) = \frac{x^{2} - 3x + 1}{x^{2} - 1}$ is discontinuous at $a = 2$ because $\{2\} \notin D_f$. 41) The function $f(x) = \frac{x^{2} - 3x}{\sqrt{x^{2}} - \sqrt{x}}$ is defined, we mean that $x^{2} - 4 > 0 \Rightarrow x^{2} > 4 \Rightarrow \sqrt{x^{2}} > \sqrt{4}$ $\Rightarrow x > 2 \Leftrightarrow x > 2$ or $x < -2$ Hence, $D_f = (-\infty, -2) \cup (2, \infty)$. 42) The function $f(x) = \sqrt{x^{2} - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^{2} - 4 \ge 0 \Rightarrow x^{2} \ge 4 \Rightarrow \sqrt{x^{2}} \le \sqrt{4}$ $\Rightarrow x > 2 \Leftrightarrow x > 2$ or $x < -2$ Hence, $D_f = (-\infty, -2) \cup (2, \infty)$. 43) The function $f(x) = \sqrt{4 - x^{2}}$ is continuous on its domain where $f(x)$ is defined, we mean that $4 - x^{2} \ge 0 \Rightarrow -x^{2} \ge 4 \Rightarrow x^{2} \le 4$
Note: Any function is continuous on its domain.34) The function $f(x) = \frac{x+1}{x^2-y}$ is continuous on its domain which is $D_f = \mathbb{R} \setminus \{\pm 3\}$.35) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 3, x = 0 \end{cases}$ is continuous at $a = 0$ because36) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 5, x = 0 \end{cases}$ is discontinuous $5, x = 0$ 37) The function $f(x) = \begin{cases} \frac{2x^2-3x+1}{x-1}, x \neq 1 \\ 1, x = 1 \end{cases}$ is $\frac{2x+1}{x-1} = \lim_{x \to 1} \frac{2x-1}{x-1} = \lim_{x \to 1} (2x-1) = 1$ $3 \lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^2-3x+1}{x-1}, x \neq 1 \\ 1, x = 1 \end{cases}$ is $\frac{2x+1}{x-1} = \lim_{x \to 1} \frac{2x-1}{x-1} = \lim_{x \to 1} (2x-1) = 1$ $\frac{2x+1}{x-1} = \lim_{x \to 1} \frac{2x-1}{x-1} = \lim_{x \to 1} (2x-1) = 1$ $3 \lim_{x \to 0} f(x) \neq f(1)$ 38) The function $f(x) = \begin{cases} \frac{2x^2-3x+1}{x-1}, x \neq 1 \\ 1, x = 1 \end{cases}$ $x = 1$ $x = 1$ $x = 1$ 39) The function $f(x) = \frac{2x^2-3x+1}{x-1} = \lim_{x \to 1} (2x-1) = 1$ $\lim_{x \to 1} f(x) = f(1)$ 40) The function $f(x) = \begin{pmatrix} 2x+3, x > 2 \\ 3x + 1, x \leq 2 \\ 3x + 1, x \leq 2 \\ 1x = 2 \end{pmatrix}$ $\lim_{x \to 2} f(x) = 7$ 41) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 > 0 \Rightarrow x^2 > 4 \Rightarrow \sqrt{x^2} > \sqrt{4}$ $\Rightarrow x > 2 \Leftrightarrow x > 2$ or $x < -2$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \Rightarrow x^2 \ge 4 \Rightarrow \sqrt{x^2} \ge \sqrt{4}$ 43) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $4 - x^2 \ge 0 \Rightarrow -x^2 \ge 4 \Rightarrow \sqrt{x^2} \le 4$
Note: Any function is continuous on its domain. 34) The function $f(x) = \frac{x+1}{x^{2}-9}$ is continuous on its domain which is $D_{f} = \mathbb{R} \setminus \{\pm 3\}$. 35) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 3, x = 0 \end{cases}$ is continuous at a = 0 because 1. $f(0) = 3$ 2. $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3. $\lim_{x \to 0} f(x) = f(0)$ 36) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 5, x = 0 \end{cases}$ is discontinuous at $a = 0$ because 1. $f(0) = 5$ 2. $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3. $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^{2}-3x+1}{x-1}, x \neq 1 \\ 1, x = 1 \end{cases}$ is continuous at $a = 1$ because 1. $f(1) = 7$ 2. $\lim_{x \to 1} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^{2}-3x+1}{x-1}, x \neq 1 \\ 1, x = 1 \end{cases}$ is continuous at $a = 1$ because 1. $f(1) = 7$ 3. $\lim_{x \to 1} f(x) \neq f(0)$ 39) The function $f(x) = \frac{x^{2}-x-2}{x-2}$ is discontinuous at $a = 2$ because $\{2\} \notin D_{f}$. 41) The function $f(x) = \frac{x^{2}-x-2}{\sqrt{x^{2}} \sqrt{4}}$ $\Rightarrow x > 2 \Leftrightarrow x > 2 \text{ or } x < -2$ Hence, $\lim_{x \to 1} (x) = f(2)$ 42) The function $f(x) = \sqrt{x^{2}-4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^{2}-4 \ge 0 \Rightarrow x^{2} \ge 4 \Rightarrow \sqrt{x^{2}} \ge \sqrt{4}$ $\Rightarrow x > 2 \Leftrightarrow x > 2 \text{ or } x^{2} \le 4 \Rightarrow \sqrt{x^{2}} \le 4$ $\Rightarrow x > 2 \Leftrightarrow x > 2 \text{ or } x^{2} \le 4 \Rightarrow \sqrt{x^{2}} \le 4$ $\Rightarrow \sqrt{x^{2}} \le \sqrt{4} \Rightarrow \sqrt{x^{2}} \le 4$
34) The function $f(x) = \frac{1}{x-9}$ is continuous on its domain which is $D_f = \mathbb{R} \setminus \{\pm 3\}$. 35) The function $f(x) = \begin{cases} \frac{1}{x-1}, x \neq 0 \\ 3, x = 0 \end{cases}$ is continuous at $a = 0$ because 1 $f(0) = 3$ 2 $\lim_{x \to 0} \frac{\sin 3x}{x} = 3(1) = 3$ 3 $\lim_{x \to 0} f(x) = f(0)$ 36) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ 5, x = 0 \end{cases}$ is discontinuous at $a = 0$ because 1 $f(0) = 5$ 2 $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3 $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x-1}, x \neq 1 \\ 1, x = 1 \end{cases}$ is discontinuous at $a = 1$ because 1 $f(1) = 7$ 2 $\lim_{x \to 0} \frac{2x^2 - 3x + 1}{x-1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x-1} = \lim_{x \to 1} (2x - 1) = 1$ 38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x-1}, x \neq 1 \\ 1, x = 1 \end{cases}$ is continuous at $a = 1$ because 1 $f(1) = 7$ 2 $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x-1} = \lim_{x \to 1} (2x - 1) = 1$ 39) The function $f(x) = \frac{x^2 - 2x + 1}{x-1} = \lim_{x \to 1} (2x - 1) = 1$ 31) The function $f(x) = \frac{x^2 - 3x + 1}{x-1} = \lim_{x \to 1} (2x - 1) = 1$ 32) $\lim_{x \to 1} f(x) = f(1)$ 40) The function $f(x) = \begin{cases} 2x^2 - 3x + 1 \\ x - 1 \end{cases}$ is continuous at $a = 2$ because 1 $f(2) = 3(2) + 1 = 7$ $\lim_{x \to 2} f(x) = f(2)$ 41) The function $f(x) = \sqrt{x^2} > 4 \implies \sqrt{x^2} > \sqrt{4} \implies x > 2 \iff x > 2 \text{ or } x < -2$ Hence, $D_f = (-\infty, -2) \cup (2, \infty)$. 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \le 4$ $\Rightarrow x > 2 \iff x \ge 2 \text{ or } x^2 \ge 4 \implies \sqrt{x^2} \le 4$ $\Rightarrow x > 2 \iff x \ge 2 \text{ or } x^2 \le 4 \implies \sqrt{x^2} \le 4$
domain which is $D_f = \mathbb{R} \setminus \{\pm 3\}$. $a = 0 \text{ because}$ $1 - f(0) = 3$ $2 - \lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ $3 - \lim_{x \to 0} f(x) = f(0)$ $36) \text{ The function } f(x) = \left\{\frac{\sin 3x}{5}, x \neq 0 \\ 5, x = 0\right\} \text{ discontinuous}}{5, x = 0}$ $37) \text{ The function } f(x) = \left\{\frac{2x^2 - 3x + 1}{x}, x \neq 1 \\ 1, x = 1\right\} \text{ discontinuous at } a = 1 \text{ because}$ $1 - f(0) = 5$ $2 - \lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ $3 - \lim_{x \to 0} f(x) \neq f(0)$ $38) \text{ The function } f(x) = \left\{\frac{2x^2 - 3x + 1}{x}, x \neq 1 \\ 1, x = 1\right\} \text{ discontinuous at } a = 1 \text{ because}$ $1 - f(1) = 7$ $2 - \lim_{x \to 1} \frac{2x^{2} - 3x + 1}{x^{2} - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x^{2} - 1} = \lim_{x \to 1} (2x - 1) = 1$ $38) \text{ The function } f(x) = \left\{\frac{2x^{2} - 3x + 1}{x - 1}, x \neq 1 \\ 1, x = 1 \\ \text{ continuous at } a = 1 \text{ because}$ $1 - f(1) = 1$ $2 - \lim_{x \to 1} \frac{2x^{2} - 3x + 1}{x^{2} - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x^{2} - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x^{2} - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x^{2} - 1} = \lim_{x \to 1} (2x - 1) = 1$ $39) \text{ The function } f(x) = \frac{x^{2} - 3x + 1}{x^{2} - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x^{2} - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x^{2} - 1} = \lim_{x \to 1} (2x - 1) = 1$ $31) \text{ The function } f(x) = \frac{x^{2} - 3x + 2}{x^{2} - 4} \text{ is continuous at } a = 2 \text{ because} \{2\} \notin D_{f}.$ $39) \text{ The function } f(x) = \frac{x^{2} + 3}{x^{2} - 4} \text{ is continuous on its}$ $31) \text{ The function } f(x) = \frac{x^{2} + 3}{x^{2} + 4} \text{ is continuous on its}$ $32 \text{ lim}_{x \to 2} (2x + 3) = 2(2) + 3 = 7$ $3 - \lim_{x \to 2} f(x) = f(2)$ $32) \text{ The function } f(x) = \sqrt{x^{2} - 4} \text{ is continuous on its}$ $33) \text{ The function } f(x) = \sqrt{x^{2} - 4} \text{ is continuous on its}$ $34) \text{ The function } f(x) = \sqrt{x^{2} - 4} \text{ is continuous on its}$ $34) \text{ The function } f(x) = \sqrt{x^{2} - 4} \text{ is continuous on its}$ $34) \text{ The function } f(x) = \sqrt{x^{2} - 4} \text{ is continuous on its}$ $34) \text{ The function } f(x) = \sqrt{x^{2} - 4} \text{ is } \sqrt{x^{2}} \leq 4$ $34) \text{ The function } f(x) = \sqrt{x^{2} - 4} \text{ is } \sqrt{x^{2}} \leq 4$ $35) $
$1 \cdot f(0) = 3$ $2 \cdot \lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{x} = 3(1) = 3$ $3 \cdot \lim_{x \to 0} f(x) = f(0)$ $36) \text{ The function } f(x) = \left\{ \frac{\sin 3x}{x}, x \neq 0 \text{ is discontinuous}}{x, x \neq 0} \text{ is discontinuous} \right\}$ $37) \text{ The function } f(x) = \left\{ \frac{2x^2 - 3x + 1}{x}, x \neq 1 \text{ is}}{7, x = 1} \text{ is} \right\}$ $37) \text{ The function } f(x) = \left\{ \frac{2x^2 - 3x + 1}{x}, x \neq 1 \text{ is}}{7, x = 1} \text{ is} \right\}$ $37) \text{ The function } f(x) = \left\{ \frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \text{ is}}{1, x = 1} \text{ continuous at } a = 1 \text{ because}$ $1 \cdot f(1) = 1$ $2 \cdot \lim_{x \to 0} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ $38) \text{ The function } f(x) = \left\{ \frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \text{ is}}{1, x = 1} \text{ continuous at } a = 1 \text{ because}$ $1 \cdot f(1) = 1$ $2 \cdot \lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ $3 \cdot \lim_{x \to 1} f(x) = f(1)$ $40) \text{ The function } f(x) = \left\{ \frac{2x + 3, x > 2}{3x + 1, x \leq 2} \text{ is} \text{ continuous at } a = 2 \text{ because } \{2\} \notin D_f.$ $41) \text{ The function } f(x) = \frac{x + 3}{\sqrt{x^2} - 4} \text{ is continuous on its} \text{ domain where } f(x) \text{ is defined, we mean that} x^2 - 4 > 0 \implies x^2 > 4 \implies \sqrt{x^2} > \sqrt{4} \implies x > 2 \iff x > 2 \text{ or } x < -2 \text{ Hence,} $ $D_f = (-\infty, -2) \cup (2, \infty).$ $42) \text{ The function } f(x) = \sqrt{x^2 - 4} \text{ is continuous on its} \text{ domain where } f(x) \text{ is defined, we mean that} $ $x^2 - 4 > 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4} \implies \sqrt{x^2} \ge \sqrt{4} \implies \sqrt{x^2} \ge \sqrt{4} \implies \sqrt{x^2} \le \sqrt{4} \implies$
$\begin{array}{lll} 2 & \lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3\\ 3 & \lim_{x \to 0} f(x) = f(0) \end{array}$ $\begin{array}{lll} 36) \text{ The function } f(x) = \left\{\frac{\sin 3x}{x}, x \neq 0 \\ 5, x = 0 \end{array} \text{ is discontinuous} \\ at a = 0 \text{ because} \\ 1 & f(0) = 5\\ 2 & \lim_{x \to 0} \frac{\sin 3x}{3x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3\\ 3 & \lim_{x \to 0} f(x) \neq f(0) \end{array}$ $\begin{array}{lll} 37) \text{ The function } f(x) = \left\{\frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \\ 1 & x = 1 \end{array} \right. \text{ is } \\ 38) \text{ The function } f(x) = \left\{\frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \\ 1 & x = 1 \end{array} \right. \text{ is } \\ 39) \text{ The function } f(x) = \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1\\ 3 & \lim_{x \to 1} f(x) \neq f(1) \end{array}$ $\begin{array}{lll} 39) \text{ The function } f(x) = \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1\\ 3 & \lim_{x \to 1} f(x) = f(1) \end{array}$ $\begin{array}{ll} 39) \text{ The function } f(x) = \frac{x^2 - 3x - 2}{x - 2} \text{ is discontinuous at } a = 2 \text{ because } \left\{2\right\} \notin D_f. \end{array}$ $\begin{array}{ll} 41) \text{ The function } f(x) = \frac{x + 3}{\sqrt{x^2 - 4}} \text{ is continuous on its } \\ \text{ domain where } f(x) \text{ is defined, we mean that } \\ x^2 - 4 \ge 0 \Rightarrow x^2 \ge 4 \Rightarrow \sqrt{x^2} \ge \sqrt{4} \\ \Rightarrow x \ge 2 \Rightarrow x \ge 2 \text{ or } x < -2 \end{array}$ $\begin{array}{ll} 43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its } \\ \text{ domain where } f(x) \text{ is defined, we mean that } \\ x^2 - 4 \ge 0 \Rightarrow x^2 \ge 4 \Rightarrow \sqrt{x^2} \ge \sqrt{4} \\ \Rightarrow \sqrt{x^2} \le \sqrt{4} \Rightarrow x \le 2 \Rightarrow -2 \le x \le 2 \end{array}$
$3 \cdot \lim_{x \to 0} f(x) = f(x) = \int_{x \to 0}^{x \to 0} f(x) = \int_{x \to 1}^{x \to 0} f(x) = \int_{x \to 1}^{x \to 1} f(x) = f(x) = \int_{x \to 1}^{x \to 1} f(x) = \int_{x \to 1}$
36) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ \overline{s}, & x = 0 \end{cases}$ is discontinuous at $a = 0$ because 1- $f(0) = 5$ 2- $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3- $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x}, & x \neq 1 \\ \overline{x} = -1 \\ \overline{x} = -1 \end{cases}$, $x \neq 1$ is $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x}, & x \neq 1 \\ 1, & x = 1 \end{cases}$ 39) The function $f(x) = \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1 \\ 3 = \lim_{x \to 1} f(x) \neq f(1)$ 39) The function $f(x) = \frac{x^2 - x - 2}{x - 2}$ is discontinuous at $a = 2$ because $\{2\} \notin D_f$. 39) The function $f(x) = \frac{x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1 \\ 3 = \lim_{x \to 1} f(x) = f(1)$ 40) The function $f(x) = \frac{2x + 3}{x + 1} = \lim_{x \to 1} (2x - 1) = 1 \\ 3 = \lim_{x \to 1} f(x) = f(1)$ 41) The function $f(x) = \frac{2x + 3}{x + 1} = \frac{2x + 3}{x + 2}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 > 0 \implies x^2 > 4 \implies \sqrt{x^2} > \sqrt{4} \implies x > 2 \implies x^2 > 0 \implies x^2 > 4 \implies \sqrt{x^2} > \sqrt{4} \implies x > 2 \implies x^2 > 0 \implies x^2 > 4 \implies \sqrt{x^2} > \sqrt{4} \implies x > 2 \implies x^2 > 0 \implies x^2 - 4 \implies \sqrt{x^2} > 4$
36) The function $f(x) = \begin{cases} x & y, x \neq 0 \text{ is discontinuous} \\ 5 & , x = 0 \end{cases}$ at $a = 0$ because 1 $f(0) = 5$ 2 $\lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3 $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \text{ is} \\ 1, x = 1 \text{ continuous at } a = 1 \text{ because} \end{cases}$ 1 $f(1) = 1$ 2 $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 39) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \text{ is} \\ 1, x = 1 \text{ continuous at } a = 1 \text{ because} \end{cases}$ 1 $f(1) = 1$ 2 $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 30) The function $f(x) = \frac{2x + 3, x > 2}{3x + 1, x \leq 2}$ is continuous at $a = 2$ because $\{2\} \notin D_f$. 41) The function $f(x) = \frac{x + 3}{\sqrt{x^2 - 4}}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 > 0 \Rightarrow x^2 > 4 \Rightarrow \sqrt{x^2} > \sqrt{4}$ 31) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 > 0 \Rightarrow x^2 \geq 4 \Rightarrow \sqrt{x^2} > \sqrt{4}$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \geq 0 \Rightarrow x^2 \geq 4 \Rightarrow \sqrt{x^2} \geq \sqrt{4}$ 32) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \geq 0 \Rightarrow x^2 \geq 4 \Rightarrow \sqrt{x^2} \geq \sqrt{4}$ 33) The function $f(x) = \sqrt{4 - x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $4 - x^2 \geq 0 \Rightarrow -x^2 \geq 4 \Rightarrow x^2 \leq 4$ $4 - x^2 \geq 0 \Rightarrow -x^2 \geq 4 \Rightarrow x^2 \leq 4$
at $a = 0$ because 1. $f(0) = 5$ 2. $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3. $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \\ 1, x = 1 \end{cases}$ continuous at $a = 1$ because 1. $f(1) = 7$ 2. $\lim_{x \to 1} f(x) \neq f(1)$ 39) The function $f(x) = \frac{x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 3. $\lim_{x \to 1} f(x) \neq f(1)$ 40) The function $f(x) = \begin{cases} 2x + 3, x > 2 \\ 3x + 1, x \leq 2 \end{cases}$ is continuous at $a = 2$ because 1. $f(2) = 3(2) + 1 = 7$ 2. $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 31) $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 32) The function $f(x) = \begin{cases} 2x + 3, x > 2 \\ 3x + 1, x \leq 2 \end{cases}$ is continuous at $a = 2$ because 1. $f(2) = 3(2) + 1 = 7$ 2. $\lim_{x \to 2^{-1}} (3x + 1) = 3(2) + 1 = 7$ $\lim_{x \to 2^{-1}} (3x + 1) = 3(2) + 1 = 7$ $\lim_{x \to 2^{-1}} f(x) = f(2)$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\lim_{x \to 2^{-1}} f(x) = f(2)$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\lim_{x \to 2^{-1}} (x - 1) = 7$ $\lim_{x \to 2^{-1}} (x - 1) = 7$ $\lim_{x \to 2^{-1}} f(x) = 7$ $\lim_{x \to 2^{-1}} f(x) = 7$ $\lim_{x \to 2^{-1}} f(x) = 7$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\lim_{x \to 2^{-1}} (x - 4) \implies x^2 \le 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\lim_{x \to 1^{-1}} (x - 4) \implies x^2 \le 4 \implies 2 $ or $x < -2$ $\lim_{x \to 2^{-1}} (x - 2) \implies x^2 \le 4 \implies 2 $ or $x < -2$ $\lim_{x \to 2^{-1}} (x - 2) \implies x^2 \le 4 \implies 2 $ or $x < -2$ $\lim_{x \to 2^{-1}} (x - 2) \implies x^2 \le 4 \implies 2 $ or $x^2 \ge 2 $ or $x^2 \ge 4 \implies $
1- $f(0) = 5$ 2- $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3- $\lim_{x \to 0} f(x) \neq f(0)$ 38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \\ 1, x = 1 \end{cases}$ continuous at $a = 1$ because 1- $f(1) = 1$ 2- $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 3. $\lim_{x \to 1} f(x) = f(1)$ 40) The function $f(x) = \begin{cases} \frac{2x + 3}{x - 1}, x \geq 2 \\ 3x + 1, x \leq 2 \end{cases}$ is continuous at $a = 2$ because 1- $f(2) = 3(2) + 1 = 7$ 2- $\lim_{x \to 2} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 3- $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 3- $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 3- $\lim_{x \to 1} f(x) = f(1)$ 40) The function $f(x) = \begin{cases} 2x + 3, x > 2 \\ 3x + 1, x \leq 2 \end{cases}$ is continuous at $a = 2$ because 1- $f(2) = 3(2) + 1 = 7$ 2- $\lim_{x \to 2} (2x + 3) = 2(2) + 3 = 7$ $\lim_{x \to 2} f(x) = f(2)$ 41) The function $f(x) = \frac{x + 3}{\sqrt{x^2} - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 > 0 \implies x^2 > 4 \implies \sqrt{x^2} > \sqrt{4}$ $\implies x > 2 \iff x > 2$ or $x < -2$ Hence, $D_f = (-\infty, -2) \cup (2, \infty)$. 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x > 2 \iff x > 2$ or $x < -2$ $= x \ge 2 \iff x > 2$ or $x < -2$ $= x \ge 2 \iff x > 2$ or $x < -2$ $= x \ge 2 \iff x > 2$ or $x < -2$ $= x \ge 2 \iff x > 2$ or $x < -2$ $= x \ge 2 \iff x > 2$ or $x < -2$ $= x \ge 2 \iff x > 2$ or $x < -2$
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38) The function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{x - 1}, x \neq 1 \\ 1, x = 1 \end{cases}$ 39) The function $f(x) = \frac{x^2 - x - 2}{x - 2}$ is discontinuous at $a = 2$ because $\{2\} \notin D_f$. 39) The function $f(x) = \frac{x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 3 $\lim_{x \to 1} f(x) = f(1)$ 40) The function $f(x) = \begin{cases} 2x + 3, x > 2 \\ 3x + 1, x \leq 2 \end{cases}$ is continuous at $a = 2$ because $\{2\} \notin D_f$. 41) The function $f(x) = \frac{x + 3}{\sqrt{x^2 - 4}}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \le \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \le \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \le \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \le \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \le \sqrt{4}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \le \sqrt{4}$
$a = 2 \text{ because } \{2\} \notin D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $b = 1 \text{ because } \{1\} \text{ because } \{1\} \text{ because } \{1\} \text{ because } \{1\} \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $b = 1 \text{ because } \{2\} \# D_f$ $a = 2 \text{ because } \{2\} \# D_f$ $b = 1 \text{ because } \{1\} becau$
continuous at $a = 1$ because 1- $f(1) = 1$ 2- $\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x - 1} = \lim_{x \to 1} (2x - 1) = 1$ 3- $\lim_{x \to 1} f(x) = f(1)$ 40) The function $f(x) = \begin{cases} 2x + 3, x > 2 \\ 3x + 1, x \le 2 \end{cases}$ is continuous at $a = 2$ because 1- $f(2) = 3(2) + 1 = 7$ 2- $\lim_{x \to 2^+} (2x + 3) = 2(2) + 3 = 7$ $\lim_{x \to 2^+} (3x + 1) = 3(2) + 1 = 7$ $\therefore \lim_{x \to 2^+} f(x) = f(2)$ 41) The function $f(x) = \frac{x + 3}{\sqrt{x^2 - 4}}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ Hence, $D_f = (-\infty, -2) \cup (2, \infty)$. 43) The function $f(x) = \sqrt{4 - x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \implies x \ge 2 \text{ or } x < -2$ Hence, $D_f = (-\infty, -2) \cup (2, \infty)$. 43) The function $f(x) = \sqrt{4 - x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \implies x \ge 2 \text{ or } x^2 \ge 4$ $\implies \sqrt{x^2} \le \sqrt{4} \implies x \le 2 \implies -2 \le x \le 2$
$\begin{array}{l} 2 \int (x)^{-2} \\ \lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} \\ = \lim_{x \to 1} \frac{(2x - 1)(x - 1)}{x - 1} \\ = \lim_{x \to 1} (2x - 1) \\ = 1 \\ \end{array}$ $\begin{array}{l} 2 \lim_{x \to 1} \frac{2x^2 - 3x + 1}{x - 1} \\ = \lim_{x \to 1} \frac{1}{x - 1} \\ = \lim_{x \to 1} \frac{1}{x - 1} \\ = \lim_{x \to 1} \frac{1}{x - 1} \\ \end{array}$ $\begin{array}{l} 41 \text{The function } f(x) = \frac{x + 3}{\sqrt{x^2 - 4}} \text{ is continuous on its} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x - 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x - 2} \\ \text{function } f(x) = \sqrt{x^2 - 4} \text{ is continuous on its} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x - 2} \\ \text{function } f(x) = \sqrt{x^2 - 4} \text{ is continuous on its} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x - 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x + 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x + 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x + 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x + 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x + 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x + 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{x^2 - 4 \ge 0}{x + 2} \\ \frac{x + 2}{x + 2} \\ \text{domain where } f(x) \text{ is defined, we mean that} \\ \frac{43}{x - 4} \\ \frac{4 - x^2 \ge 0}{x + 2} \\ \frac{5 - 4}{x + 2} \\ \frac{5 - 4}{$
$2 - \lim_{x \to 1} x - 1 - \lim_{x \to 1} x - 1 - \lim_{x \to 1} (2x - 1) - 1$ $3 - \lim_{x \to 1} f(x) = f(1)$ 40) The function $f(x) = \begin{cases} 2x + 3, x > 2 \\ 3x + 1, x \le 2 \end{cases}$ is continuous at $a = 2$ because $1 - f(2) = 3(2) + 1 = 7$ $2 - \lim_{x \to 2^{+}} (2x + 3) = 2(2) + 3 = 7$ $\lim_{x \to 2^{+}} (3x + 1) = 3(2) + 1 = 7$ $\therefore \lim_{x \to 2^{-}} f(x) = f(2)$ 42) The function $f(x) = \sqrt{x^{2} - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^{2} - 4 \ge 0 \implies x^{2} \ge 4 \implies \sqrt{x^{2}} \ge \sqrt{4}$ $x \ge 2 \text{ or } x < -2$ Hence, $D_{f} = (-\infty, -2) \cup (2, \infty) .$ 43) The function $f(x) = \sqrt{4 - x^{2}}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^{2} - 4 \ge 0 \implies x^{2} \ge 4 \implies \sqrt{x^{2}} \ge \sqrt{4}$ $\Rightarrow x \ge 2 \iff x \ge 2 \text{ or } x < -2$ $43) \text{ The function } f(x) = \sqrt{4 - x^{2}} \text{ is continuous on its domain where } f(x) \text{ is defined, we mean that}$ $4 - x^{2} \ge 0 \implies -x^{2} \ge -4 \implies x^{2} \le 4$ $\Rightarrow \sqrt{x^{2}} \le \sqrt{4} \implies x \le 2 \iff -2 \le x \le 2$
40) The function $f(x) = \begin{cases} 2x + 3, x > 2 \\ 3x + 1, x \le 2 \end{cases}$ is continuous at $a = 2$ because 1- $f(2) = 3(2) + 1 = 7$ 2- $\lim_{x \to 2^+} (2x + 3) = 2(2) + 3 = 7$ $\lim_{x \to 2^+} (3x + 1) = 3(2) + 1 = 7$ $\therefore \lim_{x \to 2^+} f(x) = 7$ 3- $\lim_{x \to 2} f(x) = f(2)$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\Rightarrow x \ge 2 \iff x \ge 2 \text{ or } x < -2$ Hence, $D_f = (-\infty, -2) \cup (2, \infty)$. 43) The function $f(x) = \sqrt{4 - x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\Rightarrow x \ge 2 \iff x \ge 2 \text{ or } x < -2$ $x \ge 2 \text{ or } x < -2$
40) The function $f(x) = \begin{cases} 2x + 3, x \neq 2 \\ 3x + 1, x \leq 2 \end{cases}$ continuous at $a = 2$ because 1- $f(2) = 3(2) + 1 = 7$ 2- $\lim_{x \to 2^+} (2x + 3) = 2(2) + 3 = 7$ $\lim_{x \to 2^-} (3x + 1) = 3(2) + 1 = 7$ $\therefore \lim_{x \to 2^-} f(x) = f(2)$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \geq 0 \implies x^2 \geq 4 \implies \sqrt{x^2} \geq \sqrt{4}$ $\Rightarrow x \geq 2 \iff x \geq 2 \text{ or } x < -2$ Hence, $D_f = (-\infty, -2) \cup (2, \infty)$. 43) The function $f(x) = \sqrt{4 - x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \geq 0 \implies x^2 \geq 4 \implies \sqrt{x^2} \geq \sqrt{4}$ $\Rightarrow x \geq 2 \iff x \geq 2 \text{ or } x < -2$
continuous at $a = 2$ because 1- $f(2) = 3(2) + 1 = 7$ 2- $\lim_{x \to 2^+} (2x + 3) = 2(2) + 3 = 7$ $\lim_{x \to 2^-} (3x + 1) = 3(2) + 1 = 7$ $\therefore \lim_{x \to 2} f(x) = 7$ 3- $\lim_{x \to 2} f(x) = f(2)$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \implies x \ge 2 \text{ or } x < -2$ 43) The function $f(x) = \sqrt{4 - x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \implies x \ge 2 \text{ or } x < -2$
1- $f(2) = 3(2) + 1 = 7$ 2- $\lim_{x \to 2^+} (2x + 3) = 2(2) + 3 = 7$ $\lim_{x \to 2^-} (3x + 1) = 3(2) + 1 = 7$ $\therefore \lim_{x \to 2} f(x) = 7$ 3- $\lim_{x \to 2} f(x) = f(2)$ 42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \iff x \ge 2$ or $x < -2$ 43) The function $f(x) = \sqrt{4 - x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \iff x \ge 2$ or $x < -2$
$\frac{1}{x \to 2^{-1}} (2x + 3) = 2(2) + 3 = 7$ $\lim_{x \to 2^{-1}} (3x + 1) = 3(2) + 1 = 7$ $\therefore \lim_{x \to 2} f(x) = 7$ $3 - \lim_{x \to 2} f(x) = f(2)$ $42) \text{ The function } f(x) = \sqrt{x^2 - 4} \text{ is continuous on its}$ $domain \text{ where } f(x) \text{ is defined, we mean that}$ $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \iff x \ge 2 \text{ or } x \le -2$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its}$ $43) \text{ The function } f(x) = 4 - x^2$
$\lim_{x \to 2^{-}} f(x) = f(2)$ $D_f = (-\infty, -2) \cup (2, \infty).$ $D_f = (-\infty, -2) \cup ($
$3- \lim_{x \to 2} f(x) = f(2)$ $42) \text{ The function } f(x) = \sqrt{x^2 - 4} \text{ is continuous on its} \\ \text{ domain where } f(x) \text{ is defined, we mean that} \\ x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4} \\ \implies x \ge 2 \iff x \ge 2 \text{ or } x \le -2$ $43) \text{ The function } f(x) = \sqrt{4 - x^2} \text{ is continuous on its} \\ \text{ domain where } f(x) \text{ is defined, we mean that} \\ 4-x^2 \ge 0 \implies -x^2 \ge -4 \implies x^2 \le 4 \\ \implies \sqrt{x^2} \le \sqrt{4} \implies x \le 2 \iff -2 \le x \le 2$
42) The function $f(x) = \sqrt{x^2 - 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \iff x \ge 2$ or $x \le -2$ 43) The function $f(x) = \sqrt{4 - x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $4 - x^2 \ge 0 \implies -x^2 \ge -4 \implies x^2 \le 4$ $\implies \sqrt{x^2} \le \sqrt{4} \implies x \le 2 \iff -2 \le x \le 2$
domain where $f(x)$ is defined, we mean that $x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$ $\implies x \ge 2 \iff x \ge 2 \text{ or } x \le -2$ domain where $f(x)$ is defined, we mean that $4 - x^2 \ge 0 \implies -x^2 \ge -4 \implies x^2 \le 4$ $\implies \sqrt{x^2} \le \sqrt{4} \implies \sqrt{x^2} \le \sqrt{4}$
$\begin{array}{ll} x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4} \\ \implies x \ge 2 \iff x \ge 2 \text{ or } x \le -2 \end{array} \qquad \begin{array}{ll} 4 - x^2 \ge 0 \implies -x^2 \ge -4 \implies x^2 \le 4 \\ \implies \sqrt{x^2} \le \sqrt{4} \implies x \le 2 \iff -2 \le x \le 2 \end{array}$
\Rightarrow $ x > 2 \Leftrightarrow x > 2$ or $x < -2$ $\Rightarrow \sqrt{x^2} \le \sqrt{4} \Rightarrow x \le 2 \Leftrightarrow -2 \le x \le 2$
Hence, $D_c = (-\infty - 2) \cup [2, \infty)$ $D_c = [-2, 2]$
44) The function $f(x) = \frac{x+3}{x+3}$ is continuous on its 45) The function $f(x) = \frac{x+1}{x+3}$ is continuous on its
domain where $f(x)$ is defined, we mean that domain where $f(x)$ is defined, we mean that
$4 - x^2 > 0 \implies -x^2 > -4 \implies x^2 < 4 \qquad \qquad x^2 - 4 \neq 0 \implies x^2 \neq 4 \implies x \neq \pm 2$
$\Rightarrow \sqrt{x^2} < \sqrt{4} \Rightarrow x < 2 \iff -2 < x < 2$ Hence, $D = \mathbb{D} \setminus (+2)$
Hence, $\nu_f = \mathbb{E} \setminus \{ \perp 4 \}$

46) The function $f(x) = \log_2(x+2)$ is continuous on its domain where $f(x)$ is defined we mean that	47) The function $f(x) = \sqrt{x-1} + \sqrt{x+4}$ is continuous
$r + 2 > 0 \implies r > -2$	on its domain where $f(x)$ is defined, we mean that
$x + 2 \ge 0 \implies x \ge 2$	$x - 1 \ge 0$ and $x + 4 \ge 0 \implies x \ge 1$ if $x \ge -4$ Hence
$D_f = (-2, \infty) .$	$D_f = [1, \infty)$.
48) The function $f(x) = 5^x$ is continuous	49) The function $f(x) = e^x$ is continuous
on its domain .	on its domain .
Hence,	Hence,
$D_f = \mathbb{R} = (-\infty, \infty)$.	$D_f = \mathbb{R} = (-\infty, \infty)$.
50) The function $f(x) = \sin^{-1}(3x - 5)$ is continuous on its domain where $f(x)$ is defined, we mean that	51) The function $f(x) = \cos^{-1}(3x + 5)$ is continuous on its domain where $f(x)$ is defined, we mean that
$-1 \le 3x - 5 \le 1 \iff 4 \le 3x \le 6 \iff \frac{4}{3} \le x \le 2$.	$-1 \le 3x + 5 \le 1 \iff -6 \le 3x \le -4 \iff -2 \le x \le -\frac{4}{3}.$
Hence,	Hence,
$D_f = \left[\frac{4}{2}, 2\right].$	$D_f = \left[-2, -\frac{4}{2}\right].$
52) The number c that makes $f(x) = \int c + x$, $x > 2$	53) The number <i>c</i> that makes
S2) The number c that makes $f(x) = \{2x - c, x \le 2\}$	$f(x) = \{cx^2 - 2x + 1, x \le -1 \ \text{is continuous at } -1 \ \text{is} \}$
is continuous at $x = 2$ is	$\int (x)^{-1} (3x+2), x > -1$
$\frac{\text{Solution:}}{\text{lim } f(x) \text{ exists if}}$	Solution: $\lim_{x \to \infty} f(x)$ exists if
$\lim_{x \to 2} f(x) = \lim_{x \to 2} f(x)$	$\lim_{x \to -1} f(x) \in A(x)$
$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$	$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$
$\lim_{x \to 2^+} (c + x) = \lim_{x \to 2^-} (2x - c)$	$\lim_{x \to -1^+} (3x+2) = \lim_{x \to -1^-} (cx^2 - 2x + 1)$
c + 2 = 4 - c	$3(-1) + 2 = c(-1)^2 - 2(-1) + 1$
c + c = 4 - 2	-1 = c + 3
c = 1	c = -1 - 3
54) The number c that makes	c = 1
$\int \frac{\sin cx}{\cos x} + 2x - 1, x < 0$	55) The value c that makes $f(x) = \begin{cases} x^3 - cx & x > 2 \end{cases}$
$f'(x) = \begin{cases} x + 2x + 4 & x > 0 \\ 3x + 4 & x > 0 \end{cases}$ is continuous at 0 is	is continuous at 2 is
Solution:	Solution:
$\lim_{x \to \infty} f(x)$ exists if	$\lim_{x \to 2} f(x)$ exists if
$\lim_{x \to 0} f(x) = \lim_{x \to 0} f(x)$	$ \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) $
$x \to 0^+$ $x \to 0^ x \to 0^ x \to 0^-$	$\lim_{x \to 2^+} (x^3 - cx) = \lim_{x \to 2^-} (cx^2 + 2x)$
$\lim_{x \to 0^+} (3x + 4) = \lim_{x \to 0^-} \left(\frac{1}{x} + 2x - 1 \right)$	$(2)^3 - c(2) = c(2)^2 + 2(2)$
3(0) + 4 = c(1) + 2(0) - 1	8 - 2c = 4c + 4
4 = c - 1	-2c - 4c = 4 - 8
c = 4 + 1	-6c = -4
c = 5	$c = \frac{-4}{-6}$
	$c=\frac{2}{2}$
$\int c^{2} x^{2} - 1, x \leq 3$	57) The number a that makes $f(x) = (x-2, x > 5)$
So the number c that makes $f(x) = \{x+5, x>3\}$	S7) The number c that makes $f(x) = \{cx - 3, x \le 5\}$
is continuous at 3 is	is continuous at 5 is
Solution:	Solution:
$\lim_{x \to 3} f(x)$ exists if	$\lim_{x \to 5} f(x)$ exists if
$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$	$ \lim_{x \to 5^{+}} f(x) = \lim_{x \to 5^{-}} f(x) $
$\lim_{x \to 1} (x+5) = \lim_{x \to 1} (c^2 x^2 - 1)$	$\lim_{x \to 5^{-1}} (x - 2) = \lim_{x \to 5^{-1}} (cx - 3)$
$(3) + 5 = c^2(3)^2 - 1$	(5) - 2 = c(5) - 3
$8 = 9c^2 - 1$	3 = 5c - 3
$9c^2 = 8 + 1$	5c = 3 + 3
$c^{2} = 1$	5c = 6
$c = \pm 1$	$c = \frac{o}{r}$
	5

58) The number <i>c</i> that makes $f(x) = \begin{cases} x+3, \ x > -1 \\ 2x-c, \ x < -1 \end{cases}$	
is continuous at -1 is	
Solution:	
$\lim_{x \to -1} f(x)$ exists if	
$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$	
$\lim_{x \to -1^+} (x+3) = \lim_{x \to -1^-} (2x-c)$	
(-1) + 3 = 2(-1) - c	
2 = -2 - c	
c = -2 - 2	
c = -4	

Workshop Solutions to Section 3.3 ^(2.6 & page 192,193)

1) If $f(x) = \begin{cases} 2x + 3; & x \ge -2 \\ 2x + 5; & x \ge -2 \end{cases}$ then	2) If $f(x) = \begin{cases} 2x + 3; & x \ge -2 \\ 2x + 5; & x \ge -2 \end{cases}$ then
$f(x) = \frac{1}{2x+5}; x < -2$ $\lim_{x \to -2} f(x) = 1$	(2x + 5; x < -2) lim $f(x) =$
$x \rightarrow (-2)^{-y}$ Solution:	$x \rightarrow (-2)^+$ Solution:
$\lim_{x \to 1} f(x) = \lim_{x \to 1} (2x+5) = 2(-2) + 5 = -4 + 5$	$\lim_{x \to 0} f(x) = \lim_{x \to 0} (2x+3) = 2(-2) + 3 = -4 + 3$
$\begin{array}{ccc} x \rightarrow (-2) & x \rightarrow (-2) \\ & = 1 \end{array}$	$\begin{array}{ccc} x \rightarrow (-2)^{+} & x \rightarrow (-2)^{+} \\ & = -1 \end{array}$
3) If $f(x) = \begin{cases} 2x+3; \ x \ge -2 \\ 2x+5; \ x < -2 \end{cases}$ then	4) If $f(x) = \begin{cases} x^2 - 2x + 3; x \ge 3 \\ 2 & \text{then} \end{cases}$
f(2x + 5); x < -2 $\lim_{x \to -2} f(x) = 0$	$(x^3 - 3x - 12; x < 3)$ $\lim_{x \to \infty} f(x) =$
Solution: $x \rightarrow -2$	Solution:
$\lim_{x \to -2} f(x)$ does not exist because	$\lim_{x \to 2^{-}} f(x) = \lim_{x \to 2^{-}} (x^3 - 3x - 12) = (3)^3 - 3(3) - 12$
$\lim_{x \to (-2)^{-}} f(x) \neq \lim_{x \to (-2)^{+}} f(x)$	$x^{7/3} = 27 - 9 - 12 = 6$
	$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x^2 - 2x + 3) = (3)^2 - 2(3) + 3$
	= 9 - 6 + 3 = 6 $\therefore \lim_{x \to 0} f(x) = 6$
$(x^2 - 7x) \cdot x < 1$	$(x^2 - 7x; x < 1)$
5) If $f(x) = \begin{cases} x & x \\ 5 & x \\ 5 & x \\ 1 & \leq x \\ 2 & x \\ 3 & x \\ 5 & x \\ 5 & x \\ 1 & \leq x \\ 2 & x \\ 5 & x \\ 5 & x \\ 1 & \leq x \\ 2 & x \\ 5 & x \\ 1 & x \\ 2 & x \\ 2 & x \\ 1 & x \\ 2 & x \\ 2 & x \\ 1 & x \\ 2 & x \\$	6) If $f(x) = \begin{cases} x & y \\ 5 & y \\ 1 & \leq x \\ 2 & \leq 3 \end{cases}$ then
(3x+1; x > 3) $\lim_{x \to 0} f(x) = 0$	(3x + 1; x > 3) $\lim_{x \to 1} f(x) = 0$
Solution:	Solution:
$\lim_{x \to 1^{-}} f(x) = \lim_{x \to 1^{-}} (x^2 - 7x) = (1)^2 - 7(1) = 1 - 7 = -6$	$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (5) = 5$
$(x^2 - 7x; x < 1)$	$(x^2 - 7x; x < 1)$
7) If $f(x) = \begin{cases} 5 ; & 1 \le x \le 3 \\ 3x + 1 ; & x > 3 \end{cases}$ then	8) If $f(x) = \begin{cases} 5 ; & 1 \le x \le 3 \\ 3x + 1 ; & x > 3 \end{cases}$ then
$\lim_{x \to 2^-} f(x) =$	$\lim_{x \to 0^+} f(x) =$
Solution:	Solution:
$\lim_{x \to 3^{-}} f(x) = \lim_{x \to 3^{-}} (5) = 5$	$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (3x+1) = 3(3) + 1 = 9 + 1 = 10$
9) If $f(x) = \int \frac{x^2 + x - 6}{x^2 - 4}$; $x^2 - 4 > 0$ then	$\int_{100}^{100} \int_{100}^{100} \frac{f(x) - 5}{x^2 - 4}; \ x^2 - 4 > 0$
$\int \frac{x^2 + x - 6}{4 - x^2}; \ x^2 - 4 < 0$	$\int \frac{107}{4} \frac{x^2 + x - 6}{4 - x^2}; \ x^2 - 4 < 0$
$\lim_{x \to 2^+} f(x) =$	$\lim_{x \to 2^-} f(x) =$
Solution:	Solution:
$\left(\frac{x^2 + x - 6}{x^2 - 4}; \ x^2 - 4 > 0\right)$	$\left(\frac{x^2 + x - 6}{x^2 - 4}; \ x^2 - 4 > 0\right)$
$f(x) = \begin{cases} x^{2} + x - 6 \\ x^{2} + x - 6 \end{cases}, x^{2} - 4 < 0$	$f(x) = \begin{cases} x^2 + x - 6 \\ x^2 + x - 6 \end{cases}; \ x^2 - 4 < 0$
$\begin{pmatrix} 4-x^2 & x^2-4 < 0 \\ (x^2+x-6) \end{pmatrix}$	$\begin{pmatrix} 4 - x^2 & x & -4 \\ x^2 + x & 6 \end{pmatrix}$
$\int \frac{x^2 + x - 6}{x^2 - 4}; \ x^2 > 4$	$\int \frac{x + x - 6}{x^2 - 4}; \ x^2 > 4$
$=$ $\frac{x^2 + x - 6}{x^2 + x^2}$; $x^2 < 4$	$=$ $\frac{x^2 + x - 6}{(x^2 - 1)^2}$; $x^2 < 4$
$(-(x^2-4)^2)$ ((x+3)(x-2))	$(-(x^2-4)^2)$ ((x+3)(x-2))
$\int \frac{d(x-y)(x-y)}{(x-2)(x+2)}; x > 4$	$\int \frac{d(x-y)(x-y)}{(x-2)(x+2)}; x > 4$
$\frac{1}{(x+3)(x-2)}; x < 4$	$\frac{(x+3)(x-2)}{(x-2)(x+2)}; x < 4$
(-(x-2)(x+2)) (x+3) $(x+3)$ $(x+2)$ $(x+3)$	$\begin{pmatrix} -(x-2)(x+2) \\ x+3 \\ x+3 \\ x+2 \\ x+3 \\ x+2 \\ $
$=\begin{cases} \frac{1}{x+2}; & x > 2 \text{ of } x < -2 \\ x+3 & \text{then} \end{cases}$	$= \begin{cases} \frac{1}{x+2}; & x > 2 \text{ of } x < -2 \\ x+3 & \text{then} \end{cases}$
$\left(-\frac{x+3}{x+2}; -2 < x < 2\right)$	$\left(-\frac{x+3}{x+2}; -2 < x < 2\right)$
$\therefore \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(\frac{x+3}{x+2}\right) = \frac{(2)+3}{(2)+2} = \frac{5}{4}$	$\therefore \lim_{x \to 2^{-}} f(x) = \lim_{x \to 2^{-}} \left(-\frac{x+3}{x+2} \right) = -\frac{(2)+3}{(2)+2} = -\frac{5}{4}$
$\frac{x \rightarrow 2}{x \rightarrow 2} \frac{x \rightarrow 2}{x \rightarrow 2} \frac{x + 2}{x + 2} \frac{2}{x \rightarrow 2} + 2 \frac{4}{x \rightarrow 2}$	$x - 2 \qquad x - 2 \qquad x + 2 \qquad (2) + 2 \qquad 4$

$$\begin{array}{c} 11) \\ \begin{array}{c} \lim_{x \to w} \frac{|x-a|}{x-a} = \\ \\ \hline \text{Solution:} \\ f(x) = \left[\frac{x-a}{x-a}\right] = \left\{ \frac{x-a}{x-a} : x-a > 0 \\ \frac{x-a}{x-a} : x-a < 0 \\ \frac{x-a}{x-$$

30) If $m \neq 0$, then	31) If $m \neq 0$, then
$\lim_{x \to 0} \frac{\sin(nx)}{\sin(mx)} =$	$\lim_{x \to 0} \frac{\sin(nx)}{\tan(mx)} =$
$\lim_{x \to 0} \frac{\sin(nx)}{\sin(mx)} = \frac{n}{m} \left(\lim_{x \to 0} \frac{\sin(nx)}{nx} \right) \left(\lim_{x \to 0} \frac{mx}{\sin(mx)} \right)$ $= \frac{n}{m} (1)(1) = \frac{n}{m}$	$\lim_{x \to 0} \frac{\sin(nx)}{\tan(mx)} = \frac{n}{m} \left(\lim_{x \to 0} \frac{\sin(nx)}{nx} \right) \left(\lim_{x \to 0} \frac{mx}{\tan(mx)} \right)$ $= \frac{n}{m} (1)(1) = \frac{n}{m}$
32) If $m \neq 0$, then	33) If $m \neq 0$, then
$\lim_{x \to 0} \frac{\tan(nx)}{\tan(mx)} =$ $\lim_{x \to 0} \frac{\tan(nx)}{\tan(mx)} = \frac{n}{m} \left(\lim_{x \to 0} \frac{\tan(nx)}{nx} \right) \left(\lim_{x \to 0} \frac{mx}{\tan(mx)} \right)$ $= \frac{n}{m} (1)(1) = \frac{n}{m}$	$\lim_{x \to 0} \frac{\tan(nx)}{\sin(mx)} =$ $\frac{\text{Solution:}}{\lim_{x \to 0} \frac{\tan(nx)}{\sin(mx)}} = \frac{n}{m} \left(\lim_{x \to 0} \frac{\tan(nx)}{nx} \right) \left(\lim_{x \to 0} \frac{mx}{\sin(mx)} \right)$ $= \frac{n}{m} (1)(1) = \frac{n}{m}$
$\sin(1-\cos x)$	sin(sin(2x))
$\lim_{x \to 0} \frac{1 - \cos x}{1 - \cos x} =$	$\lim_{x \to 0} \frac{1}{\sin(2x)} =$
Solution: $\lim_{x \to 0} \frac{\sin(1 - \cos x)}{1 - \cos x} = 1$	Solution: $\lim_{x \to 0} \frac{\sin(\sin(2x))}{\sin(2x)} = 1$
36)	37)
$\lim_{x \to 0} \frac{1 - \cos(2x)}{x^2} =$	$\lim_{n \to \infty} \frac{1}{2} - \frac{3}{2} + 4 =$
Solution:	$x \to \infty \sqrt{x^2 - x}$
$\lim_{x \to 0} \frac{1 - \cos(2x)}{x^2} = \lim_{x \to 0} \frac{2 \sin^2 x}{x^2} = 2\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2$ $= 2\left(\lim_{x \to 0} \frac{\sin x}{x}\right)^2 = 2(1)^2 = 2$	$\frac{\text{Solution:}}{\lim_{x \to \infty} \sqrt{\frac{1}{x^2} - \frac{3}{x} + 4}} = \sqrt{\lim_{x \to \infty} \left(\frac{1}{x^2} - \frac{3}{x} + 4\right)} = \sqrt{0 - 0 + 4}$ $= 2$
38)	39)
$\lim_{x \to \infty} \left(\frac{1}{\chi^{2}/5} + 2 \right) =$ Solution:	$\lim_{x \to \infty} \frac{3x + 13}{9x^2 + 4x - 13} =$ <u>Solution:</u>
$\lim_{x \to -\infty} \left(\frac{1}{\chi^{2/5}} + 2 \right) = 0 + 2 = 2$	$\lim_{x \to \infty} \frac{3x + 15}{9x^2 + 4x - 13} = \lim_{x \to \infty} \frac{\frac{3x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}}$
	$= \lim_{x \to \infty} \frac{\frac{3}{x} + \frac{15}{x^2}}{9 + \frac{4}{x} - \frac{13}{x^2}} = \frac{0+0}{9+0+0} = 0$
40)	41)
$\lim_{x \to \infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} =$	$\lim_{x \to -\infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} =$
Solution:	Solution:
$\lim \frac{3x^2 - 8x + 15}{x^2 - 15} = \lim \frac{\frac{5x^2}{x^2} - \frac{6x}{x^2} + \frac{15}{x^2}}{\frac{15}{x^2} - \frac{6x}{x^2} + \frac{15}{x^2}}$	$\lim \frac{3x^2 - 8x + 15}{-x^2} = \lim \frac{\frac{5x^2}{-x^2} - \frac{6x}{-x^2} + \frac{15}{-x^2}}{-x^2}$
$ x \to \infty 9x^2 + 4x - 13 \qquad x \to \infty \frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2} $	$x \to -\infty 9x^{2} + 4x - 13 \qquad x \to -\infty \frac{9x^{2}}{-x^{2}} + \frac{4x}{-x^{2}} - \frac{13}{-x^{2}}$
$3 - \frac{8}{x} + \frac{15}{x^2} 3 - 0 + 0 1$	$-3 + \frac{8}{x} - \frac{15}{x^2} - 3 + 0 - 0 = 1$
$= \lim_{x \to \infty} \frac{x}{9 + \frac{4}{x} - \frac{13}{x^2}} = \frac{1}{9 + 0 + 0} = \frac{1}{3}$	$= \lim_{x \to -\infty} \frac{\pi}{-9 - \frac{4}{x} + \frac{13}{x^2}} = \frac{-9 - 0 + 0}{-9 - 0 + 0} = \frac{1}{3}$

$$\begin{array}{l} 42) \\ \lim_{x \to \infty} \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \lim_{x \to \infty} \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \lim_{x \to \infty} \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{3x^{5} - 8x + 15}{9x^{2} + 4x - 13} = \\ \underline{Solution:} \\ \frac{1}{1x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{1x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} - x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} + x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} + x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} + x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} + x \right) = \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x + 7} + x \right) = \\ \underline{Solution:} \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1}} \left(\sqrt{x^{2} - 3x^{-1} + x^{-1}} + x \right) = \\ \underline{Solution:} \\ \underline{Solution:} \\ \frac{1}{x^{-1}x^{-1$$

$$\begin{array}{l} \text{48} \\ \text{49} \\ \text{Solution:} \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \\ \frac{1}{y_{x \to \infty}} \frac{1}{y_$$

$$\begin{array}{l} 53) \text{ The horizontal asymptote of} \\ f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 - 2x - 3}}{2x + 7} \\ \hline f(x) = \frac{\sqrt{x^2 - 3}}{2x^2 + 7x - 1} \\ \hline f(x) = \frac{\sqrt{x^2 - 3}}{2x^2$$

Workshop Solutions to Chapter 4_(chapter 3)

1) If $f(x)$ is a differentiable function then $f'(x) =$	2) If $f(x) = 4x^2$ then $f'(x) =$
(x) = (x) is a differentiable function, then $f(x) = (x)$	$(x) = 4x^2$, then $f(x) = 4x^2$
Solution:	Solution:
$f'(x) = \lim_{h \to \infty} f(x+h) - f(x)$	$f(x+h) - f(x) = 4(x+h)^2 - 4x^2$
$f(x) = \lim_{h \to 0} \frac{h}{h}$	$f'(x) = \lim_{h \to 0} \frac{1}{h} = \lim_{h \to 0} \frac{1}{h}$
2) If $f(x) = x^2 + 2$, then $f'(x)$	
3) If $f(x) = x^2 - 3$, then $f'(x) =$	4) If $f(x) = \sqrt{x}$, $x \ge 0$, then $f'(x) =$
Solution:	Solution:
f(x+h) - f(x)	$f(r+h) - f(r)$ $\sqrt{r+h} - \sqrt{r}$
$f'(x) = \lim_{h \to 0} \frac{f'(x)}{h}$	$f'(x) = \lim_{x \to \infty} \frac{f(x+n) - f(x)}{n} = \lim_{x \to \infty} \frac{f(x+n) - f(x)}{n}$
$\begin{bmatrix} n \neq 0 & n \\ [(r + h)^2 - 3] - [r^2 - 3] \end{bmatrix}$	$h \rightarrow 0$ h $h \rightarrow 0$ h
$= \lim \frac{[(x+n) - 3] - [x - 3]}{[x - 3]}$	
$h \rightarrow 0$ h	
5) If f is a differentiable function at a, then f is	6) If f is a continuous function at a, then f is
a continuous function at a	a differentiable function at a
	Solution:
	False
7) If $y = x^4 + 5x^2 + 3$, then $y' =$	8) If $y = x^4 - 5x^2 + 3$, then $y' =$
Solution	Solution
$y' = 4x^3 + 10x$	$y' = 4x^3 - 10x$
9) If $y = x^{-5/2}$, then $y' =$	10) If $y = \frac{1}{x^{-3}} + 2\sqrt{x} = \frac{1}{x^{-3}} + 2x^{1/2}$, then $y' = \frac{1}{x^{-3}} + \frac{1}{x^{$
Solution:	$3x^3$
	Solution:
$y' = -\frac{5}{2}r^{-\frac{5}{2}-1} = -\frac{5}{2}r^{-\frac{7}{2}}$	$u' = (-2) \begin{pmatrix} 1 \\ -3^{-1} \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ -3^{-1} \\ -1 \end{pmatrix} (2) u^{\frac{1}{2}-1}$
$y = 2^{x} = 2^{x}$	$y = (-3)(\frac{-3}{3})x^{-3} + (\frac{-2}{2})(2)x^{2}$
	$= -x^{-4} + x^{-1/2} = -\frac{1}{1/2} = -\frac{1}{$
	$x^{-1}x^{-1/2}$ $x^{-1}\sqrt{x}$
11) If $y = (x - 3)(x - 2)$, then $y' =$	12) If $y = (x^3 + 3)(x^2 - 1)$, then $y' =$
Solution:	Solution:
$y = (x - 3)(x - 2) = x^2 - 5x + 6$	$\frac{1}{y} = (r^3 + 3)(r^2 - 1) = r^5 - r^3 + 3r^2 - 3$
y' = (x + 3)(x + 2) = x + 3x + 6	y = (x + 3)(x + 1) = x + 3x
y = 2x = 3	y = 5x - 5x + 6x
13) If $y = \sqrt{x(2x+1)}$, then $y' =$	14) If $y = \frac{x+3}{x-2}$, then $y' =$
Solution:	Solution:
	5000000000000000000000000000000000000
$y = \sqrt{x(2x+1)} = 2x\sqrt{x} + \sqrt{x} = 2x^2 + x^2$	Use the rule $\left(\frac{f}{r}\right) = \frac{f g - f g}{r^2}$
$u' = \binom{3}{2} \binom{3}{2} \binom{3}{2} - \frac{3}{2} + \binom{1}{2} \binom{1}{2} \binom{1}{2} - \frac{1}{2} - \frac{1}{2} \binom{1}{2} + \frac{1}{2} \binom{1}{2}$	$(g) \qquad g^2$
$y = (\frac{1}{2})^{(2)x^2} + (\frac{1}{2})^{x^2} = 3x^2 + \frac{1}{2}x^2$	
1	(1)(x-2) - (x+3)(1) - x - 2 - x - 3 - 5
$=3\sqrt{x}+\frac{1}{2\sqrt{x}}$	$y = \frac{(x-2)^2}{(x-2)^2} = \frac{(x-2)^2}{(x-2)^2} = \frac{(x-2)^2}{(x-2)^2}$
$2\sqrt{x}$	5
OR	$=-\frac{3}{(2-3)^2}$
Use the rule $(f a)' = f'a + fa'$	$(x-2)^2$
(1) $2m+1$	
$y' = (2)(\sqrt{x}) + (\frac{1}{\sqrt{x}})(2x+1) = 2\sqrt{x} + \frac{2x+1}{\sqrt{x}}$	
$y = (2)(\sqrt{x}) + (2\sqrt{x})(2x + 1) = 2\sqrt{x} + 2\sqrt{x}$	
(15) If $y = \frac{x+3}{x+3}$ then $y' = \frac{1}{x+3}$	16) If $y = \frac{x-1}{x-1}$ then $y' = \frac{x-1}{x-1}$
15) If $y = \frac{1}{x-2}$, then $y _{x=4}$	$10) \text{ If } y = \frac{1}{x+2}$, then $y = \frac{1}{x+2}$
Solution:	Solution:
(1)(x-2) - (x+3)(1) $x-2-x-3$	(f)' f'g-fg'
$y' = \frac{(y' - y')}{(y - 2)^2} = \frac{(y - 2)^2}{(y - 2)^2}$	Use the rule $\left(\frac{-}{g}\right) = \frac{-}{g^2}$
$(x-2)^{-}$ $(x-2)^{-}$	
== =	(1)(r+2) - (r-1)(1) $r+2 - r+1$ 3
$(x-2)^2$ $(x-2)^2$	$y' = \frac{(1)(x+2)}{(x-1)(1)} = \frac{x+2}{(x-1)(1)} = \frac{3}{(x-1)(1)} = \frac{3}{(x$
, 5 5	$(x+2)^2$ $(x+2)^2$ $(x+2)^2$
$ y' _{x=4} = -\frac{1}{(4-2)^2} = -\frac{1}{4}$	

17) If $y = \sqrt{3x^2 + 6x}$, then $y' =$	18) If $y = \sqrt{3x^2 + 6x}$, then $y' _{x=1} =$
Use the rule $(\sqrt{u})' = \frac{u'}{u}$	$\frac{3000001}{6x+6} \qquad 6(x+1) \qquad 3(x+1)$
$2\sqrt{u}$	$y = \frac{1}{2\sqrt{3x^2 + 6x}} = \frac{1}{2\sqrt{3x^2 + 6x}} = \frac{1}{\sqrt{3x^2 + 6x}}$
$y' = \frac{6x+6}{3(x+1)} = \frac{6(x+1)}{3(x+1)} = \frac{3(x+1)}{3(x+1)}$	3((1)+1) 6 6
$y = 2\sqrt{3x^2 + 6x} = 2\sqrt{3x^2 + 6x} = \sqrt{3x^2 + 6x}$	$y' _{x=1} = \frac{(x-y-y)}{\sqrt{3(1)^2 + 6(1)}} = \frac{1}{\sqrt{9}} = \frac{1}{3} = 2$
19) The tangent line equation to the curve $y = x^2 + 2$	20) The tangent line equation to the curve $y = \frac{2x}{x+1}$
Solution:	at the point $(0,0)$ is
First, we have to find the slope of the curve which is	Solution:
y' = 2x	First, we have to find the slope of the curve which is (2)(r+1) - (2r)(1) - 2r + 2 - 2r = 2
Thus, the slope at $x = 1$ is	$y' = \frac{(2)(x+1)}{(x+1)^2} = \frac{2x+2}{(x+1)^2} = \frac{2}{(x+1)^2}$
$y' _{x=1} = 2(1) = 2$	Thus, the slope at $x = 0$ is
Hence, the tangent line equation passing through the point (1.3) with slope $m = 2$ is	$y'_{1} = \frac{2}{2} = 2$
v - 3 = 2(x - 1)	$y_{1x=0} - \frac{1}{(0+1)^2} - 2$
y - 3 = 2x - 2	Hence, the tangent line equation passing through the point
y = 2x - 2 + 3	(0,0) with slope $m = 2$ is
y = 2x + 1	y - 0 = (2)(x - 0) y = 2r
21) The tangent line equation to the curve $y = 3x^2 - 13$	22) The tangent line equation to the curve
at the point $(2, -1)$ is	$y = 3x^{2} + 2x + 5$ at the point (0,5) is
Solution:	Solution:
First, we have to find the slope of the curve which is	First, we have to find the slope of the curve which is
y' = 6x	y' = 6x + 2
y' = 6(2) = 12	$v'_{1} = 6(0) + 2 = 2$
Hence, the tangent line equation passing through the	Hence, the tangent line equation passing through the point
point $(2, -1)$ with slope $m = 12$ is	(0,5) with slope $m = 2$ is
y - (-1) = 12(x - 2)	y-5=2(x-0)
y + 1 = 12x - 24	y-5=2x
y = 12x - 24 - 1 y = 12r - 25	$y \equiv 2x + 5$
23) If $y = xe^x$, then $y' =$	24) If $y = x - e^x$, then $y'' =$
Solution:	Solution:
Use the rules $(f \cdot g)' = f'g + fg'$ and $(e^u) = e^u \cdot u'$	Use the rules $(f - g)' = f' - g'$ and $(e^u) = e^u \cdot u'$
$y' = (1)(e^x) + (x)(e^x) = e^x + xe^x = e^x(1+x)$	$y' = 1 - e^x$ $y'' = -e^x$
25) If $x^2 - y^2 = 4$, then $y' =$	26) If $x^2 + y^2 = 4$, then $y' =$
$\frac{\text{Solution:}}{2r - 2wv' = 0}$	$\frac{\text{Solution:}}{2r+2w'=0}$
2x 2yy = 0 $-2yy' = -2x$	2x + 2yy = 0 $2yy' = -2x$
$\frac{-2x}{x'}$	$x' = \frac{-2x}{x}$
$y = \frac{1}{-2y}$	$y = \frac{1}{2y}$
$y' = \frac{x}{y}$	$y' = -\frac{x}{y}$
27) If $y = \frac{x+1}{x}$, then $y' =$	28) If $y = \frac{1}{2\sqrt{-1}} + \sec x$, then $y' = \frac{1}{2\sqrt{-1}} + \sec x$
Solution:	Solution:
Use the rule $\left(\frac{f}{f}\right)' = \frac{f'g - fg'}{f}$	Use the rules
$(g) = g^2$	$(f+g)' = f' + g'$ and $(\sec u)' = \sec u \tan u \cdot u'$
$y' = \frac{(1)(x+2) - (x+1)(1)}{(x+2)^2} = \frac{x+2-x-1}{(x+2)^2}$	$y = \frac{1}{1} + \sec x = x^{-\frac{5}{2}} + \sec x$
$(x+2)^2$ $(x+2)^2$	$y = \frac{1}{\sqrt{x^5}} + \sec x = x^2 + \sec x$
$=\frac{1}{(x+2)^2}$	$y' = \left(-\frac{5}{2}\right)x^{-\frac{5}{2}-1} + \sec x \tan x = -\frac{5}{2}x^{-7/2} + \sec x \tan x$

29) If $y = \tan^{-1}(x^3)$, then $y' =$	30) If $y = \tan x - x$, then $y' =$
Solution:	Solution:
$\frac{1}{(\tan t \ln t)} (\tan^{-1} u)' - \frac{u'}{1}$	Use the rules
Ose the fulle $(\tan u) = \frac{1}{1+u^2}$	$(f-g)' = f' - g'$ and $(\tan u)' = \sec^2 u \cdot u'$
$y' = \frac{1}{(3x^2)} = \frac{3x^2}{(3x^2)}$	
$y = 1 + (x^3)^2 + (5x^2) = 1 + x^6$	$y' = \sec^2 x - 1$
31) If $y = \sec^2 x - 1$, then $y' =$	32) If $y = x^{\sin x}$, then $y' =$
Solution:	Solution:
Use the rules $(f - g)' = f' - g'$, $(u)^n = n(u)^{n-1} \cdot u'$	Use the rule $(\sin u)' = \cos u \cdot u'$
and $(\sec u)' = \sec u \tan u \cdot u'$	
	$y = x^{\sin x}$
$y' = 2 \sec x \cdot \sec x \tan x = 2 \sec^2 x \tan x$	$\ln y = \ln x^{\sin x}$
	$\ln y = \sin x \cdot \ln x$
	$\frac{y'}{y} = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x} = \cos x \cdot \ln x + \frac{\sin x}{x}$
	$y' = y\left(\cos x \cdot \ln x + \frac{\sin x}{x}\right) = x^{\sin x}\left(\cos x \cdot \ln x + \frac{\sin x}{x}\right)$
33) If $v = x^{\cos x}$, then $v' =$	34) If $y = (2x^2 + \csc x)^9$, then $y' =$
Solution:	Solution:
Use the rule $(\cos u)' = -\sin u \cdot u'$	Use the rules
	$(u)^n = n(u)^{n-1} u'$ and $(\csc u)' = -\csc u \cot u u'$
$v = x^{\cos x}$	
$\ln y = \ln x^{\cos x}$	$y' = 9(2x^2 + \csc x)^8 \cdot (4x - \csc x \cot x)$
$\ln y = \cos x \cdot \ln x$	
y' 1 $\cos x$	
$\frac{1}{y} = -\sin x \cdot \ln x + \cos x \cdot - = -\sin x \cdot \ln x + \frac{1}{x}$	
$y' = y(\sin x + \ln x + \cos x)$	
$y = y\left(-\sin x + \frac{1}{x}\right)$	
$=x^{\cos x}\left(\frac{\cos x}{x}-\sin x\cdot\ln x\right)$	
$\frac{x}{25}$ if $x = \frac{5^x}{25}$ then $x' = \frac{5^x}{25}$	36) If $y = e^{2x}$, then $y^{(6)} =$
$\frac{33}{11} \frac{y}{y} = \frac{1}{\cot x}$, then $y = \frac{1}{3}$	Solution:
Solution:	Use the rule $(e^u)' = e^u \cdot u'$
Use the rules	
$\left(\frac{f}{d}\right) = \frac{f'g - fg'}{d}, (a^{u})' = a^{u} \ln a, u'$	$v' = 2e^{2x}$
(g) g^2 , (a) a matrix	$v^{\prime\prime} = 4e^{2x}$
and $(\csc u)' = -\csc u \cot u \cdot u'$	$y^{\prime\prime\prime} = 8e^{2x}$
	$y^{(4)} = 16e^{2x}$
$v' = \frac{(5^x \ln 5)(\cot x) - (5^x)(-\csc^2 x)}{(5^x \ln 5)(-\csc^2 x)}$	$y^{(5)} = 32e^{2x}$
$(\cot x)^2$	$y^{(6)} = 64e^{2x}$
$=\frac{5^{x}(\ln 5\cot x + \csc^{2} x)}{5^{x}(\ln 5\cot x + \csc^{2} x)}$	
$\cot^2 x$	
37) If $y = x^{-2}e^{\sin x}$, then $y' =$	38) If $y = 5^{4012}$, then $y' = 5^{4012}$
Solution:	Solution:
Use the rules $(f \cdot g)' = f'g + fg'$, $(e^u) = e^u \cdot u'$	Use the rules
and $(\sin u)^r = \cos u \cdot u^r$	$(a^{\alpha})^{\alpha} = a^{\alpha} . \ln a . u^{\alpha}$ and $(\tan u)^{\alpha} = \sec^{\alpha} u . u^{\alpha}$
$y' = (-2x^{-3})(a^{\sin x}) + (x^{-2})(a^{\sin x} \cos x)$	$y' = 5^{\tan x} \ln 5 \sec^2 x$
$y = (-2x)(e^{-3}e^{\sin x} + e^{-2}e^{\sin x})$	y 5 . mo.see x
$= -2x \cdot e^{-1} + x - \cos x e^{-1}$	
$= x^{-3} e^{\sin x} (-2 + x \cos x)$	
$= x \cdot e^{-1} (x \cos x - 2)$ 20) If $x^2 + y^2 - 2xy + 7$ then $x' - 1$	40) If $y = \sin^3(4x)$, then $x^{(6)} =$
Solution: $y = 3xy + 7$, then $y = 3xy + 7$	$\begin{array}{c} 40 & \text{if } y - 511 \\ \text{Solution:} \end{array} (4x), \text{then} y < y = \\ y' = $
$\frac{301000011}{284} = 284 \pm 2869'$	
$2x \pm 2yy - 3y \pm 3xy$ $2yy' - 3yy' - 3y = 2x$	$\frac{ 0 }{ 0 } = n(a_1)^{n-1} a_1' \text{and} (a = a_1)' = a_2 a_2 a_1 a_1'$
$\frac{2yy - 3xy - 3y - 2x}{y'(2y - 3y) - 3y - 2y}$	$(u) = n(u)$ $.u$ and $(\sin u) = \cos u . u$
3v - 2x	$u' = 2 \sin^2(4x) \cos(4x)$ (4)
$y' = \frac{y}{2y - 3x}$	$y = -3 \sin^2(4r) \cos(4r)$ = 12 sin ² (4r) cos(4r)

41) If $y = 3^x \cot x$, then $y' =$	42) If $y = (2x^2 + \sec x)^7$, then $y' =$
Solution:	Solution:
Use the rules $(f.g)' = f'g + fg'$, $(a^u)' = a^u \cdot \ln a \cdot u'$	Use the rules
and $(\cot u)' = -\csc^2 u \cdot u'$	$(u)^n = n(u)^{n-1} \cdot u'$ and $(\sec u)' = \sec u \tan u \cdot u'$
$y' = (3^{x} . \ln 3)(\cot x) + (3^{x})(-\csc^{2} x)$	$y' = 7(2x^2 + \sec x)^6 \cdot (4x + \sec x \tan x)$
$= 3^{x} \ln 3 \cot x - 3^{x} \csc^{2} x$	
$= 3^{x} (\ln 3 \cot x - \csc^{2} x)$	$AA \rightarrow A = D^{47} (\cdot \cdot \cdot)$
43) If $f(x) = \cos x$, then $f^{(43)}(x) =$	44) If $D^{(r)}(\sin x) =$
<u>Solution:</u>	$\frac{\text{Solution:}}{D(\sin x) = \cos x}$
$\int f'(x) = -\sin x$	$D(\sin x) = \cos x$ $D^2(\sin x) = -\sin x$
$f'''(x) = -\cos x$	$D^{3}(\sin x) = -\cos x$
$\int (x) - \sin x$ $f^{(4)}(x) - \cos x$	$D^{4}(\sin x) = \cos x$ $D^{4}(\sin x) = \sin x$
Note: $f^{(n)}(x) = \cos x$ whenever <i>n</i> is a multiple of 4	Note: $D^n(\sin x) = \sin x$ whenever <i>n</i> is a multiple of 4
Hence $f = cos x$ whenever <i>n</i> is a multiple of 4.	Hence $D^{-1}(\sin x) = \sin x$ whenever <i>n</i> is a matriple of 1.
$f^{(44)}(x) = \cos x$	$D^{44}(\sin x) = \sin x$
$f^{(45)}(x) = -\sin x$	$D^{45}(\sin x) = \cos x$
$\int f(x) = -\sin x$	$D^{46}(\sin x) = -\sin x$
	$D^{47}(\sin x) = -\cos x$
45) If $y = x^{x}$, then $y' =$	46) If $f(x) = \frac{\ln x}{1 + 1}$ then $f'(1) = \frac{1}{2}$
Solution:	$x^{2}, \text{ then } f(1) = \frac{1}{x^{2}}$
Use the rule $(\ln u)' = \frac{u'}{u}$	$\frac{\text{Solution:}}{(f)' + f'a - fa'} \qquad \qquad$
u = u	Use the rules $\left(\frac{f}{g}\right) = \frac{f(g)f(g)}{g^2}$ and $(\ln u)' = \frac{u}{u}$
$y = x^{\chi}$	
$y = x$ $\ln y = \ln x^{x}$	$\left(\frac{1}{2}\right)(r^2) - (\ln r)(2r)$ $r = 2r \ln r$
$\ln y = x \ln x$	$f'(x) = \frac{(x)^{(x-y)}(x-y)(2x)}{(x-y)^2} = \frac{x-2x \ln x}{4}$
y' (1)	$(x^2)^2$ x^4 $r(1-2\ln r)$ $1-2\ln r$
$\frac{1}{y} = (1)(\ln x) + (x)(\frac{1}{x})$	$=\frac{x(1-2 \ln x)}{x^4}=\frac{1-2 \ln x}{x^3}$
y' ,	
$\frac{1}{y} = \ln x + 1$	$1 - 2\ln(1) 1 - 2(0)$
$y' = y(1 + \ln x) = x^x(1 + \ln x)$	$\therefore f'(1) = \frac{1}{(1)^3} = \frac{1}{1} = 1$
47) If $y = \cot^{-1}(e^x)$, then $y' =$	48) If $y = \tan^{-1}(e^x)$, then $y' =$
Solution:	Solution:
Use the rules $(\cot^{-1}u)' = -\frac{u'}{u}$ and $(e^u) = e^u \cdot u'$	Use the rules $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ and $(e^u) = e^u \cdot u'$
$1+u^2$	$1+u^2$
$1 \qquad e^x$	$1 \qquad e^x$
$y' = -\frac{1}{1+(e^x)^2} \cdot e^x = -\frac{1}{1+e^{2x}}$	$y' = \frac{1}{1 + (e^x)^2} \cdot e^x = \frac{1}{1 + e^{2x}}$
49) If $y = \sin^{-1}(e^x)$, then $y' =$	50) If $y = \cos^{-1}(e^x)$, then $y' =$
Solution:	Solution:
Use the rules $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ and $(e^u) = e^u . u'$	Use the rules $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ and $(e^u) = e^u . u'$
1 e ^x	1 o ^x
$y' = \frac{1}{\sqrt{1 - (e^x)^2}} \cdot e^x = \frac{e}{\sqrt{1 - e^{2x}}}$	$y' = -\frac{1}{\sqrt{1 - (e^x)^2}} \cdot e^x = -\frac{e}{\sqrt{1 - e^{2x}}}$
51) If $v = \cos(2x^3)$, then $v' =$	52) If $y = \csc x \cot x$, then $y' =$
Solution:	Solution:
Use the rule $(\cos u)' = -\sin u \cdot u'$	Use the rules $(f.g)' = f'g + fg'$,
	$(\csc u)' = -\csc u \cot u \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$
$y' = -\sin(2x^3) \cdot (6x^2) = -6x^2 \sin(2x^3)$	
	$y' = (-\csc x \cot x)(\cot x) + (\csc x)(-\csc^2 x)$
	$= -\csc x \cot^2 x - \csc^3 x = -\csc x (\cot^2 x + \csc^2 x)$
53) If $y = \sqrt{x^2 - 2 \sec x}$, then $y' =$	54) If $y = (3x^2 + 1)^6$, then $y' =$
--	--
Solution:	Solution: Use the rule $(u)^n = n(u)^{n-1} u'$
u'	a = h(a) = h(a)
$(\sqrt{u}) = \frac{1}{2\sqrt{u}}$ and $(\sec u)' = \sec u \tan u \cdot u'$	$y' = 6(3x^2 + 1)^5 \cdot (6x) = 36x(3x^2 + 1)^5$
$y' = \frac{2x - 2 \sec x \tan x}{2\sqrt{2} + 2 \sec x} = \frac{2(x - \sec x \tan x)}{2\sqrt{2} + 2 \sec x}$	
$2\sqrt{x^2 - 2 \sec x} \qquad 2\sqrt{x^2 - 2 \sec x}$ $x - \sec x \tan x$	
$=\frac{1}{\sqrt{x^2-2\sec x}}$	
55) If $xy + \tan x = 2x^3 + \sin y$, then $y' =$	56) If $y = x^{-1} \sec x$, then $y' =$
Solution: $\begin{bmatrix} (1)(x) + (x)(x') \end{bmatrix} + \cos^2 x = 6x^2 + \cos x + x'$	Solution:
$[(1)(y) + (x)(y)] + \sec x = 6x^{2} + \cos y \cdot y$ $y + xy' + \sec^{2} x = 6x^{2} + y' \cos y$	$(f, q)' = f'q + fq'$ and $(\sec u)' = \sec u \tan u \cdot u'$
$xy' - y' \cos y = 6x^2 - y - \sec^2 x$	
$y'(x - \cos y) = 6x^2 - y - \sec^2 x$	$y' = (-x^{-2})(\sec x) + (x^{-1})(\sec x \tan x)$
$y' = \frac{6x^2 - y - \sec^2 x}{\cos^2 x}$	$= x^{-1} \sec x \tan x - x^{-2} \sec x$
$x - \cos y$	= x sec x (x tall x - 1)
57) If $y = \sin^{-1}(x^3)$, then $y' =$	58) If $y = \cos^{-1}(x^3)$, then $y' =$
Solution:	Solution:
Use the rule $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$	Use the rule $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$
$y' = \frac{1}{3x^2} = \frac{3x^2}{3x^2}$	
$y' = \sqrt{1 - (x^3)^2} \cdot 5x' = \sqrt{1 - x^6}$	$y' = -\frac{1}{\sqrt{3x^2}} \cdot 3x^2 = -\frac{3x^2}{\sqrt{3x^2}}$
	$\sqrt{1-(x^3)^2}$ $\sqrt{1-x^6}$
Solution: $y = \sec^{-1}(x^3)$, then $y^2 = \frac{1}{2}$	60) If $y = \csc^{-1}(x^3)$, then $y^2 = $
Use the rule $(\sec^{-1}u)' = \frac{u'}{u}$	Use the rule $(\csc^{-1}u)' = -\frac{u'}{2}$
Use the rule $(see - u) = \frac{1}{ u \sqrt{u^2-1}}$	Use the full $(Use u) = u \sqrt{u^2-1}$
$1 - 3x^2 - 3x^2$	$1 - 3 - 3x^2 - 3$
$y' = \frac{1}{x^3 \sqrt{(x^3)^2 - 1}} \cdot 3x^2 = \frac{1}{x^3 \sqrt{x^6 - 1}} = \frac{1}{x \sqrt{x^6 - 1}}$	$y' = -\frac{1}{x^3\sqrt{(x^3)^2 - 1}} \cdot 3x^2 = -\frac{1}{x^3\sqrt{x^6 - 1}} = -\frac{1}{x\sqrt{x^6 - 1}}$
61) If $y = \ln(x^3 - 2 \sec x)$, then $y' =$	62) If $y = \ln(\cos x)$, then $y' =$
Solution:	Solution:
Use the rules	Use the rules u'
$(\ln u)' = \frac{u}{u}$ and $(\sec u)' = \sec u \tan u \cdot u'$	$(\ln u)' = \frac{u}{u}$ and $(\cos u)' = -\sin u \cdot u'$
$y' = \frac{1}{x^3 - 2xxx^3} \cdot (3x^2 - 2\sec x \tan x)$	$y' = \frac{1}{x - x} \cdot (-\sin x) = -\frac{\sin x}{x - x} = -\tan x$
$3x^{2} - 2 \sec x$ $3x^{2} - 2 \sec x \tan x$	$\cos x$ $\cos x$
$=$ $\frac{1}{x^3 - 2 \sec x}$	
$\begin{array}{c} \hline \\ \hline $	(4) If $a_1 = \frac{\ln \sqrt{2u^2 + \Gamma_1}}{2u^2 + \Gamma_2}$ there a_1'
Solution: $y = m(sm x)$, then $y = m(sm x)$	64) If $y = \ln \sqrt{3x^2 + 5x}$, then $y =$
Use the rules	Use the rules $(\ln u)' = \frac{u'}{2}$ and $(\sqrt{u})' = \frac{u'}{2}$
$(\ln u)' = \frac{u'}{u}$ and $(\sin u)' = \cos u \cdot u'$	u and $(\sqrt{u}) = \frac{1}{2\sqrt{u}}$
	1 (6x + 5) 6x + 5
$u' = \frac{1}{1} (\cos x) = \frac{\cos x}{1 - \cot x}$	$y = \frac{1}{\sqrt{3x^2 + 5x}} \cdot \left(\frac{1}{2\sqrt{3x^2 + 5x}}\right) = \frac{1}{2(3x^2 + 5x)}$
$y - \frac{y}{\sin x} \cdot (\cos x) = \frac{1}{\sin x} = \cot x$	

$$\begin{aligned} \begin{aligned} & \text{(5)} & \text{if } y = \log_{x}(x^{3} - 2 \csc x) \text{, then } y' = \\ & \text{Solution:} \\ & \text{(bis the rules)} \\ & (\log_{x} u)' = \frac{u'}{u \ln a} \text{ and } (\csc u)' = - \sec u \cot u. u' \\ & y' = \frac{1}{(x^{3} - 2 \csc x)(\ln 5)} \cdot (3x^{2} - 2(-\csc x \cot x)) \\ & = \frac{3x^{2} + 2 \csc x \cot x}{(x^{3} - 2 \csc x)(\ln 5)} \end{aligned}$$

$$\begin{aligned} & \text{(6)} & \text{if } y = \ln \frac{x^{-1}}{u}, \text{ the } y' = \\ & \text{Solution:} \\ & \text{(in } u)' = \frac{u'}{u}, \text{ (f)} & \int_{y}^{y} - f(y) - f(y) + \frac{1}{2\sqrt{x} + 2} \end{aligned}$$

$$\begin{aligned} & \text{(in } u)' = \frac{u'}{u}, \text{ (f)} & \int_{y}^{y} - f(y) + \frac{1}{2\sqrt{x} + 2} \end{aligned}$$

$$\begin{aligned} & \text{(in } u)' = \frac{u'}{u}, \text{ (f)} & \int_{y}^{y} - f(y) + \frac{1}{2\sqrt{x} + 2} \end{aligned}$$

$$\begin{aligned} & \text{(in } u)' = \frac{u'}{u}, \text{ (f)} & \int_{y}^{y} - f(y) + \frac{1}{2\sqrt{x} + 2} \end{aligned}$$

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$$\begin{aligned} & \text{(f)} & \text{(f)} & \text{(f)} & \text{(f)} & \text{(f)} & \text{(f)} & \frac{1}{2\sqrt{x} + 2} \end{aligned}$$

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$$\end{aligned}$$

	= (p) (p)
73) If $y = \sec x \tan x$, then $y' =$	(74) If $D^{(3)}(\cos x) =$
Solution:	Solution:
$(f.g)' = f'g + fg'$, $(\sec u)' = \sec u \tan u \cdot u'$ and	$D(\cos x) = -\sin x$
$(\tan u)' = \sec^2 u \cdot u'$	$D^2(\cos x) = -\cos x$
	$D^{3}(\cos r) - \sin r$
	$D^{4}(\cos x) = \sin x$
	$D^{1}(\cos x) = \cos x$
$y' = (\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x)$	Note: $D^n(\cos x) = \cos x$ whenever <i>n</i> is a multiple of 4.
$= \sec x \tan^2 x + \sec^3 x = \sec x (\tan^2 x + \sec^2 x)$	Hence,
	$D^{96}(\cos x) = \cos x$
	$D^{97}(\cos x) = -\sin x$
	$\frac{D}{D} = \frac{1}{2} \frac{D}{D} $
	$D^{(c)}(\cos x) = -\cos x$
	$D^{99}(\cos x) = \sin x$
75) If $y = (x + \sec x)^3$, then $y' =$	76) If $x^2 = 5y^2 + \sin y$, then $y' =$
Solution:	Solution:
Lise the rules	$2r - 10yy' \pm \cos y y'$
Ose the rules	$2x = 10yy + \cos y \cdot y$
$(u)^n = n(u)^{n-1} \cdot u^n$ and $(\sec u)^n = \sec u \tan u \cdot u^n$	$y'(10y + \cos y) = 2x$
	$y' = \frac{Zx}{Zx}$
$y' = 3(x + \sec x)^2$. $(1 + \sec x \tan x)$	$\int \int \frac{y}{10y} + \cos y$
,, (, , , , , , , ,	
77) If $w^2 = Ew^2 + \sin w = 0$ then $w' =$	79) If $y = \sin x \sec x$, then $y' =$
$x^{-1} = -5y^{-1} + \sin y = 0$, then $y^{-1} = -5y^{-1} + \sin y = 0$	$y = \sin x \sec x$, then $y = \sin x \sec x$
Solution:	Solution:
$2x - 10yy' + \cos y \cdot y' = 0$	$(f.g)' = f'g + fg'$, $(\sin u)' = \cos u \cdot u'$ and
$v'(-10v + \cos v) = -2x$	$(\sec u)' = \sec u \tan u \cdot u'$
-2x $2x$	
$y' = \frac{10y}{10y} = \frac{10y}{10y} = \frac{10y}{10y}$	$u' = (\cos x)(\cos x) + (\sin x)(\cos x \tan x)$
$-10y + \cos y$ $10y - \cos y$	$y = (\cos x)(\sec x) + (\sin x)(\sec x \tan x)$
	$-1 + \sin x$ $\frac{1}{2}$ $\frac{\sin x}{2} - 1 + \frac{\sin^2 x}{2} - 1 + \tan^2 x$
	$-1 + \sin x \cdot \frac{1}{\cos x} \cdot \frac{1}{\cos x} - 1 + \frac{1}{\cos^2 x} - 1 + \tan^2 x$
	$= \sec^2 x$
70) If $f(x) = \sin^2(x^3 + 1)$ then $f'(x) =$	$(x + \cot x)^3 \text{then } x' =$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$	80) If $y = (x + \cot x)^3$, then $y' =$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = $ <u>Solution:</u>	80) If $y = (x + \cot x)^3$, then $y' = $ <u>Solution:</u>
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules	80) If $y = (x + \cot x)^3$, then $y' = \frac{\text{Solution:}}{\text{Use the rules}}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$	80) If $y = (x + \cot x)^3$, then $y' = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$	80) If $y = (x + \cot x)^3$, then $y' = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ <u>Solution:</u> Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (2x^2)$	80) If $y = (x + \cot x)^3$, then $y' = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$	80) If $y = (x + \cot x)^3$, then $y' = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$	80) If $y = (x + \cot x)^3$, then $y' = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$
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79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' =$	80) If $y = (x + \cot x)^3$, then $y' = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \cos^2 u \cdot u'$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' =$ Solution:	80) If $y = (x + \cot x)^3$, then $y' = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{\text{Solution:}}{2}$
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79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{\text{Solution:}}{\text{Use the rule}}$ Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Use the rule}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{\text{Solution:}}{\text{Use the rule}}$ Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1}.u'$ and $(\cot u)' = -\csc^2 u.u'$ $y' = 3(x + \cot x)^2.(1 - \csc^2 x)$ 82) If $y = \cot^{-1}(\frac{x}{2})$, then $y' = \frac{Solution:}{Use the rule}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}(\frac{x}{2})$, then $y' =$ Solution: Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $u' = \frac{1}{2} + $	80) If $y = (x + \cot x)^3$, then $y' = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1}.u'$ and $(\cot u)' = -\csc^2 u.u'$ $y' = 3(x + \cot x)^2.(1 - \csc^2 x)$ 82) If $y = \cot^{-1}(\frac{x}{2})$, then $y' = \frac{Solution:}{Use the rule}$ Use the rule $(\cot^{-1}u)' = -\frac{u'}{1+u^2}$ $u' = \frac{1}{2} + \frac{1}{2$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' =$ Solution: Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+u(x)^2} \cdot \frac{1}{2} = \frac{1}{2(1+x^2)} = \frac{1}{2(1+x^2)} = \frac{2}{1+x^2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Use the rule}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+u(x)^2} \cdot \frac{1}{2} = -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{2} \cdot \frac{1}{2} \cdot \frac$
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79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{1}{32}$ Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{50 \text{ lution:}}{1}$ Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{50 \text{ lution:}}{1}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1+\frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4+x^2}{4}\right)}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{1}{33}$ Solution: Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{50 \text{ lution:}}{1}$ Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{50 \text{ lution:}}{1}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1+\frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4+x^2}{4}\right)}$ $= -\frac{2}{4x^2}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ Solution: Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' =$ Solution: Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{50 \text{ lution:}}{100000000000000000000000000000000000$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2 \sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Use the rule}$ $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$ 83) If $y = \sin^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{1}{2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{50 \text{ lution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{50 \text{ lution:}}{\text{Use the rule}}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1+\frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4+x^2}{4}\right)}$ $= -\frac{2}{4+x^2}$ 84) If $y = \cos^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{1}{2}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{\text{Solution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{\text{Solution:}}{\text{Use the rule}}$ Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$ 83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{1}{2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{50 \text{ lution:}}{50 \text{ lution:}}$ Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{50 \text{ lution:}}{50 \text{ lution:}}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1+\frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4+x^2}{4}\right)}$ $= -\frac{2}{4+x^2}$ 84) If $y = \cos^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{50 \text{ lution:}}{50 \text{ lution:}}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$ 83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{Solution:}{Solution:}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{50 \text{lution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{50 \text{lution:}}{\text{Use the rule}}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1+\frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4+x^2}{4}\right)}$ $= -\frac{2}{4+x^2}$ 84) If $y = \cos^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{50 \text{lution:}}{2}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$ 83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\sin^{-1} u)' = \frac{u'}{(x-1)^2}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{50 \text{lution:}}{\text{Use the rules}}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{50 \text{lution:}}{\text{Use the rule}}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1+\frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4+x^2}{4}\right)}$ $= -\frac{2}{4+x^2}$ 84) If $y = \cos^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{50 \text{lution:}}{50 \text{lution:}}$ Use the rule $(\cos^{-1} u)' = -\frac{u'}{\sqrt{x+x^2}}$
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79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{Solution:}{Solution:}$ Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$ 83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ $y' = \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3} = \frac{1}{3\sqrt{1-\frac{x^2}{9}}} = \frac{1}{3\sqrt{\frac{9-x^2}{9}}} = \frac{1}{\sqrt{9-x^2}}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{50 \text{lution:}}{100 \text{ Use the rules}}$ (u) ^{n} = $n(u)^{n-1}.u'$ and ($\cot u$)' = $-\csc^2 u.u'$ $y' = 3(x + \cot x)^2.(1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{50 \text{lution:}}{1 + \left(\frac{x}{2}\right)^2}$, $\frac{1}{2} = -\frac{1}{2\left(1 + \frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4 + x^2}{4}\right)}$ $y' = -\frac{1}{1 + \left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1 + \frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4 + x^2}{4}\right)}$ $= -\frac{2}{4 + x^2}$ 84) If $y = \cos^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{50 \text{lution:}}{50 \text{lution:}}$ Use the rule ($\cos^{-1}u$)' = $-\frac{u'}{\sqrt{1 - u^2}}$ $y' = -\frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3} = -\frac{1}{3\sqrt{1 - \frac{x^2}{9}}} = -\frac{1}{3\sqrt{\frac{9 - x^2}{9}}}$ $= -\frac{1}{\sqrt{9 - x^2}}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{Solution:}{Solution:}$ Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2\sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$ 83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ $y' = \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3} = \frac{1}{3\sqrt{1-\frac{x^2}{9}}} = \frac{1}{3\sqrt{\frac{9-x^2}{9}}} = \frac{1}{\sqrt{9-x^2}}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}(\frac{x}{2})$, then $y' = \frac{Solution:}{Use the rule}$ $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+(\frac{x}{2})^2} \cdot \frac{1}{2} = -\frac{1}{2(1+\frac{x^2}{4})} = -\frac{1}{2(\frac{4+x^2}{4})}$ $= -\frac{2}{4+x^2}$ 84) If $y = \cos^{-1}(\frac{x}{3})$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ $y' = -\frac{1}{\sqrt{1-(\frac{x}{3})^2}} \cdot \frac{1}{3} = -\frac{1}{3\sqrt{1-\frac{x^2}{9}}} = -\frac{1}{3\sqrt{\frac{9-x^2}{9}}}$ $= -\frac{1}{\sqrt{9-x^2}}$
79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2 \sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$ 81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Use the rule} (\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$ 83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ $y' = \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3} = \frac{1}{3\sqrt{1-\frac{x^2}{9}}} = \frac{1}{3\sqrt{\frac{9-x^2}{9}}}$ $= \frac{1}{\sqrt{9-x^2}}$	80) If $y = (x + \cot x)^3$, then $y' = \frac{Solution:}{Use the rules}$ $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$ 82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1+\frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4+x^2}{4}\right)}$ $= -\frac{2}{4+x^2}$ 84) If $y = \cos^{-1}\left(\frac{x}{3}\right)$, then $y' = \frac{Solution:}{Solution:}$ Use the rule $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ $y' = -\frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3} = -\frac{1}{3\sqrt{1-\frac{x^2}{9}}} = -\frac{1}{3\sqrt{\frac{9-x^2}{9}}}$ $= -\frac{1}{\sqrt{9-x^2}}$

85) If $D^{99}(\sin x) =$
Solution:
$D(\sin x) = \cos x$
$D^2(\sin x) = -\sin x$
$D^3(\sin x) = -\cos x$
$D^4(\sin x) = \sin x$
Note: $D^n(\sin x) = \sin x$ whenever <i>n</i> is a multiple of 4.
Hence,
$D^{96}(\sin x) = \sin x$
$D^{97}(\sin x) = \cos x$
$D^{98}(\sin x) = -\sin x$
$D^{99}(\sin x) = -\cos x$

Workshop Solutions to Sections 5.1 and 5.2 (chapter 4)

1) The absolute maximum value of $f(x) = x^3 - 2x^2$ in	2) The absolute minimum value of $f(x) = x^3 - 3x^2 + 1$ in
[-1.2] is at $x =$	$\begin{bmatrix} -1 \\ -1 \end{bmatrix}$ is
Solution:	
Since $f(x)$ is a continuous on $[-1,2]$, we can use the Closed	Solution:
Interval Method,	Since $f(x)$ is a continuous on $\left[-\frac{1}{2}, 4\right]$, we can use the
$f(x) = x^3 - 2x^2$	Closed Interval Method,
$f'(x) = 3x^2 - 4x$	$f(x) = x^3 - 3x^2 + 1$
Now, we find the critical numbers of $f(x)$ when	$f'(x) = 3x^2 - 6x$
$f'(x) = 0 \implies 3x^2 - 4x = 0 \implies x(3x - 4) = 0$	Now, we find the critical numbers of $f(x)$ when
\Rightarrow $r = 0$ or $r = \frac{4}{2}$	$f'(x) = 0 \implies 3x^2 - 6x = 0 \implies 3x(x - 2) = 0$
\rightarrow $\chi = 0$ or $\chi = \frac{3}{3}$	$\Rightarrow x = 0 \text{ or } x = 2$
$[hus, (1)^3 - 2(1)^2 - 1 - 2 - 2]$	Thus,
$f(-1) = (-1)^{3} - 2(-1)^{2} = -1 - 2 = -3$ $f(2) = (2)^{3} - 2(2)^{2} - 0 = 0$	$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^{3} - 3\left(-\frac{1}{2}\right)^{2} + 1 = -\frac{1}{2} - \frac{3}{2} + 1 = \frac{1}{2}$
$f(2) = (2)^{2} - 2(2)^{2} = 0 - 0 = 0$ $f(0) = (0)^{3} - 2(0)^{2} = 0 - 0 = 0$	$\binom{1}{2}\binom{2}{2}\binom{2}{2}\binom{2}{2}$
$f(0) = (0)^{-2}(0)^{-2}(0)^{-0} = 0^{-0} = 0$	$f(4) = (4)^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 1/$
$f\left(\frac{4}{2}\right) = \left(\frac{4}{2}\right) - 2\left(\frac{4}{2}\right) = \frac{64}{27} - \frac{52}{2} = -\frac{52}{27}$	$f(0) = (0)^{2} - 3(0)^{2} + 1 = 0 - 0 + 1 = 1$ $f(2) = (2)^{3} - 2(2)^{2} + 1 = 0 - 12 + 1 = -2$
(3) (3) (3) (2) (3)	J(2) = (2) = 5(2) + 1 = 0 = 12 + 1 = -5
r = 0 and $r = 2$	r = 2
3) The absolute maximum point of $f(r) = 3r^2 - 12r + 1$	4) The absolute minimum point of $f(x) = 3x^2 - 12x + 1$
in [0.3] is	in [0,3] is
Solution:	Solution:
Since $f(x)$ is a continuous on [0.3], we can use the Closed	Since $f(x)$ is a continuous on [0.3], we can use the Closed
Interval Method,	Interval Method,
$f(x) = 3x^2 - 12x + 1$	$f(x) = 3x^2 - 12x + 1$
f'(x) = 6x - 12	f'(x) = 6x - 12
Now, we find the critical numbers of $f(x)$ when	Now, we find the critical numbers of $f(x)$ when
$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$	$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$
$\implies x = 2$	$\Rightarrow x = 2$
Thus,	Thus,
$f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$	$f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$
$f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$	$f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$
$f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$	$f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$
Hence, we see that the absolute maximum point is $(0,1)$.	Hence, we see that the absolute minimum point is $(2, -11)$.
5) The absolute minimum point of $f(x) = 3x^2 - 12x + 2$ in [0.2] is	b) The values in (-3,3) which make $f(x) = x^3 - 9x$
III [0,3] IS	Solution:
Since $f(x)$ is a continuous on $[0, 2]$ we can use the Closed	$\frac{501011011}{1000}$
Interval Method	f(x) is a polynomial, then 1 - f(x) is a continuous on $[-3, 3]$
$f(r) - 3r^2 - 12r \pm 2$	f(x) is a continuous on $[-3,3]$.
f'(x) = 5x - 12x + 2 f'(x) = 6x - 12	$f'(x) = 3x^2 - 9$
Now we find the critical numbers of $f(x)$ when	$f(x) = 3x^{-3}$ 3. $f(-3) = (-3)^{3} = 9(-3) = -27 + 27 = 0 = f(3)$
$f'(r) = 0 \implies 6r - 12 = 0 \implies 6r = 12$	Then there is a number $c \in (-3, 3)$ such that
$ \begin{array}{c} y(x) = 0 \Rightarrow 0x 12 = 0 \Rightarrow 0x = 12 \\ \Rightarrow x = 2 \end{array} $	$f'(c) = 0 \implies 3c^2 - 9 = 0 \implies 3c^2 - 9$
Thus.	$ \begin{array}{c} (c) = 0 \rightarrow 5c j = 0 \rightarrow 5c = j \\ \rightarrow c^2 = 2 \rightarrow c = \pm \sqrt{2} \end{array} $
$f(0) = 3(0)^2 - 12(0) + 2 = 0 - 0 + 2 = 2$	$\rightarrow t = 5 \Rightarrow t = 1\sqrt{5}$
$f(3) = 3(3)^2 - 12(3) + 2 = 27 - 36 + 2 = -7$	Hence the values are $\pm \sqrt{2} \in (-2,2)$
$f(2) = 3(2)^2 - 12(2) + 2 = 12 - 24 + 2 = -10$	Hence, the values are $\pm \sqrt{3} \in (-3, 5)$.
Hence, we see that the absolute minimum point is $(2, -10)$.	

8) The value c in (0,5) which makes $f(x) = x^2 - x - 6$ 7) The values in (0,2) which make $f(x) = x^3 - 3x^2 + 2x + 5$ satisfy Rolle's Theorem on satisfy the Mean Value Theorem on [0,5] is deleted [0,2] are deleted Solution: \therefore f(x) is a polynomial, then Solution: \therefore f(x) is a polynomial, then 1- f(x) is a continuous on [0,5]. 1- f(x) is a continuous on [0,2]. 2- f(x) is differentiable on (0,5), 2- f(x) is differentiable on (0,2), f'(x) = 2x - 1 $f'(x) = 3x^2 - 6x + 2$ Then there is a number $c \in (0,5)$ such that 3- $f(0) = (0)^3 - 3(0)^2 + 2(0) + 5 = 5 = f(2)$ $f'(c) = \frac{f(5) - f(0)}{5 - 0}$ $\begin{array}{l} 5 - 0 \\ \Rightarrow & 2c - 1 = \frac{\left[(5)^2 - (5) - 6 \right] - \left[(0)^2 - (0) - 6 \right]}{5} \\ \Rightarrow & 2c - 1 = \frac{(14) - (-6)}{5} \\ \Rightarrow & 2c - 1 = \frac{14 + 6}{5} \\ \Rightarrow & 2c - 1 = 4 \\ \Rightarrow & 2c = 4 + 1 \\ \Rightarrow & c = \frac{5}{2} \end{array}$ Then there is a number $c \in (0,2)$ such that $f'(c) = 0 \implies 3c^2 - 6c + 2 = 0$ $c = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)} = \frac{6 \pm \sqrt{36 - 24}}{6}$ $= \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm \sqrt{3 \times 4}}{6} = \frac{6 \pm 2\sqrt{3}}{6}$ $= \frac{2(3 \pm \sqrt{3})}{6} = \frac{3 \pm \sqrt{3}}{3} = \frac{3}{3} \pm \frac{\sqrt{3}}{3}$ $=1\pm\frac{\sqrt{3}}{2}$ Hence, the values are $1 \pm \frac{\sqrt{3}}{3} \in (0,2)$. Hence, the value *c* is $\frac{5}{2} \in (0,5)$. **10)** The value in (0,1) which makes $f(x) = 3x^2 + 2x + 5$ 9) The value c in (0,2) makes $f(x) = x^3 - x$ satisfied the Mean Value Theorem on [0,2] are deleted satisfy the Mean Value Theorem on [0,1] is deleted Solution: Solution: \therefore f(x) is a polynomial, then \therefore f(x) is a polynomial, then 1- f(x) is a continuous on [0,2]. 1- f(x) is a continuous on [0,1]. 2- f(x) is differentiable on (0,2), 2- f(x) is differentiable on (0,1), $f'(x) = 3x^2 - 1$ f'(x) = 6x + 2Then there is a number $c \in (0,3)$ such that Then there is a number $c \in (0,1)$ such that $f'(c) = \frac{f(1) - f(0)}{1 - 0}$ $f'(c) = \frac{f(2) - f(0)}{2 - 0}$ $\Rightarrow 3c^{2} - 1 = \frac{[(2)^{3} - (2)] - [(0)^{3} - (0)]}{2}$ $\Rightarrow 3c^{2} - 1 = \frac{(6) - (0)}{2}$ $\Rightarrow 6c + 2 = \frac{[3(1)^2 + 2(1) + 5] - [3(0)^2 + 2(0) + 5]}{1}$ $\implies 6c + 2 = (3 + 2 + 5) - (0 + 0 + 5)$ $\Rightarrow 6c + 2 = 10 - 5$ $\Rightarrow 3c^2 - 1 = \frac{6}{2}$ $\Rightarrow 3c^2 - 1 = 3$ $\Rightarrow 3c^2 = 3 + 1$ $\Rightarrow 6c + 2 = 5$ $\Rightarrow 6c = 5 - 2$ $\Rightarrow 6c = 3$ $\Rightarrow c = \frac{3}{6}$ $\Rightarrow c^2 = \frac{4}{3}$ $\Rightarrow c = \frac{1}{2}$ $\Rightarrow c = \pm \sqrt{\frac{4}{3}}$ Hence, the values are $\frac{1}{2} \in (0,1)$. $\Rightarrow c = \pm \frac{2}{\sqrt{3}}$ Hence, the value c is $\frac{2}{\sqrt{3}} \in (0,2)$ but $-\frac{2}{\sqrt{3}} \notin (0,2)$. 11) The critical numbers of the function $f(x) = x^3 + 3x^2 - 9x + 1$ are Solution: $f'(x) = 3x^2 + 6x - 9$ $f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$ $\implies 3(x^2 + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$ $\implies (x+3)(x-1) = 0$ \Rightarrow x = -3 or x = 1

12) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is decreasing	13) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is increasing
on	on
Solution:	Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$
$\Rightarrow 3(x^2 + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 2 = 0$	$\Rightarrow 3(x^2 + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 2 = 0$
$ \Rightarrow x + 2x - 3 = 0 \Rightarrow (r + 3)(r - 1) = 0 $	$ \Rightarrow x + 2x - 3 = 0 \Rightarrow (x + 3)(x - 1) = 0 $
$\Rightarrow x = -3 \text{ or } x = 1$	$\Rightarrow x = -3 \text{ or } x = 1$
-3 1	-3 1
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicit	monotonicit
γ	γ
Hence, the function $f(x)$ is decreasing on $(-3,1)$	Hence, the function $f(x)$ is increasing on
	$(-\infty, -3) \cup (1, \infty)$
14) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative	15) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative
maximum value at the point	minimum value at the point
Solution: $f'(x) = 2x^2 + (x = 0)$	Solution: $f'(x) = 2x^2 + (x = 0)$
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$ \begin{array}{cccc} (x) = 0 & \implies & 3x + 0x = 9 = 0 \\ \implies & 3(x^2 + 2x - 3) = 0 \end{array} $	$ \begin{array}{c} (x) = 0 \implies 3x + 0x = 9 = 0 \\ \implies 3(r^2 + 2r - 3) = 0 \end{array} $
$\Rightarrow x^2 + 2x - 3 = 0$	$\Rightarrow x^2 + 2x - 3 = 0$
$\Rightarrow (x+3)(x-1) = 0$	\Rightarrow $(x+3)(x-1) = 0$
\Rightarrow $x = -3$ or $x = 1$	\Rightarrow $x = -3$ or $x = 1$
-3 1	-3 1
+ $ +$ Sign of $f'(x)$	+ $ +$ Sign of $f'(x)$
Kind of	Kind of
monotonicit	monotonicit
y	Y
Hence, the function $f(x)$ has a relative maximum value at	Hence, the function $f(x)$ has a relative minimum value at
the point (-3,28). $f(-2) = (-2)^3 + 2(-2)^2 = 0(-2) + 1$	the point $(1, -4)$.
(-3) = (-3) + 3(-3) - 9(-3) + 1 = -27 + 27 + 27 + 1 = 28	f(1) = (1) + 3(1) = 9(1) + 1 = 1 + 3 - 9 + 1 = -4
16) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave	17) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave
upward on	downward on
Solution:	Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
f''(x) = 6x + 6	f''(x) = 6x + 6
$f''(x) = 0 \implies 6x + 6 = 0$	$f''(x) = 0 \implies 6x + 6 = 0$
$\Rightarrow bx = -b$	$\Rightarrow bx = -b$
$\Rightarrow x = -\frac{6}{6}$	$\Rightarrow x = -\frac{6}{6}$
$\Rightarrow x = -1$	$\Rightarrow x = -1$
-1	-1
- + Sign of $f''(x)$	- + Sign of $f''(x)$
Kind of	Kind of
II U concavity	II U concavity
Hence, the function $f(x)$ is concave upward on $(-1,\infty)$	Hence, the function $f(x)$ is concave downward on
	(−∞, −1)

18) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has an influencies point at	19) The critical numbers of the function $f(u) = u^3 - 2u^2 - 0u + 1$
Solution:	$f(x) = x^2 - 3x^2 - 9x + 1$ are Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
f''(x) = 6x + 6	$f'(x) = 0 \Longrightarrow 3x^2 - 6x - 9 = 0$
$f''(x) = 0 \implies 6x + 6 = 0$	$\implies 3(x^2 - 2x - 3) = 0$
$\Rightarrow 6x = -6$	$\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow x = -\frac{6}{6}$	$\Rightarrow (x+1)(x-3) = 0$ $\Rightarrow x = 1 \text{ or } x = 2$
$\Rightarrow x = -1$	$\Rightarrow x = -1$ or $x = 3$
-1	
- + Sign of $f''(x)$	
Kind of	
O U concavity	
Hence, the function $f(x)$ has an inflection point at	
(-1,12). $f(-1) = (-1)^3 + 3(-1)^2 = 9(-1) + 1$	
(-1) = (-1) + 3(-1) - 9(-1) + 1 = -1 + 3 + 9 + 1 = 12	
20) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is decreasing	21) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is increasing
on	on
Solution:	Solution:
$f'(x) = 3x^2 - 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$ $\implies 2(x^2 - 3x - 2) = 0$	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$ $\implies 2(x^2 - 2x - 3) = 0$
$\Rightarrow 3(x^2 - 2x - 3) = 0$ $\Rightarrow x^2 - 2x - 3 = 0$	$\Rightarrow 3(x^2 - 2x - 3) = 0$ $\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow x = 2x = 3 = 0$ $\Rightarrow (x+1)(x-3) = 0$	$\Rightarrow x = 2x = 3 = 0$ $\Rightarrow (x+1)(x-3) = 0$
$\Rightarrow x = -1 \text{ or } x = 3$	$\Rightarrow x = -1 \text{ or } x = 3$
-1 3	-1 3
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence the function $f(r)$ is decreasing on (-1.3)	Hence the function $f(x)$ is increasing on
	$(-\infty, -1) \cup (3, \infty)$
22) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative	23) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative
maximum value at the point	minimum value at the point
Solution: $f'(x) = 2x^2 - 6x = 0$	Solution: $f'(x) = 2x^2 - 6x = 0$
$\int (x) = 5x - 6x - 9$ $\int (x) = 3r^2 - 6r - 9 = 0$	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$
$ \begin{array}{cccc} y & (x) = 0 & \implies & 3x & 0x & y = 0 \\ & \implies & 3(x^2 - 2x - 3) = 0 \end{array} $	$ \begin{array}{c} y (x) = 0 \implies 3x 0x y = 0 \\ \implies 3(x^2 - 2x - 3) = 0 \end{array} $
$\Rightarrow x^2 - 2x - 3 = 0$	$\Rightarrow x^2 - 2x - 3 = 0$
$\implies (x+1)(x-3) = 0$	$\Rightarrow (x+1)(x-3) = 0$
\Rightarrow $x = -1$ or $x = 3$	\Rightarrow $x = -1$ or $x = 3$
+ $ +$ Sign of $f'(x)$	+ $ +$ Sign of $f'(x)$
Kind of monotonicity	
Hence, the function $f(x)$ has a relative maximum value at	Hence, the function $f(x)$ has a relative minimum value at
the point $(-1,6)$.	the point $(3, -26)$.
$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 1$	$f(3) = (3)^3 - 3(3)^2 - 9(3) + 1$
= -1 - 3 + 9 + 1 = 6.	= 27 - 27 - 27 + 1 = -26.

24) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave	25) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave
upward on	downward on
Solution: $(I(x) - 2x^2 - Cx - 0)$	Solution: $(I(x) - 2x^2 - Cx - 0)$
$\int (x) = 3x^2 - 6x - 9$ $f''(x) = 6x - 6$	$\int (x) = 3x^2 - 6x - 9$ $f''(x) = 6x - 6$
$f''(x) = 0 \implies 6x - 6 = 0$	$f''(x) = 0 \implies 6x - 6 = 0$
$\Rightarrow 6x = 6$	$\Rightarrow 6x = 6$
6	6
$\Rightarrow x = \frac{1}{6}$	$\Rightarrow x = \frac{1}{6}$
$\Rightarrow x = 1$	$\Rightarrow x = 1$
$\frac{1}{1}$	$\frac{1}{1}$
- $ -$	- $ -$
Kind of	Kind of
O U concavity	O U concavity
Hence, the function $f(x)$ is concave upward on $(1, \infty)$	Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$
26) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has an	27) The critical numbers of the function
inflection point at	$f(x) = x^3 + 3x^2 - 9x + 5$ are
Solution:	Solution:
$f'(x) = 3x^2 - 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$f''(x) = 0 \implies 6x - 6 = 0$	$\int (x) = 0 \implies 3x^2 + 6x - 9 = 0$ $\implies 2(x^2 + 2x - 2) = 0$
$ \begin{array}{c} y \\ (x) = 0 \end{array} \xrightarrow{\longrightarrow} 0 \\ (x) = 0 \end{array} \xrightarrow{\longrightarrow} 0 \\ (x) = $	$\Rightarrow 3(x + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$
6	$\Rightarrow (x+3)(x-1) = 0$
$\Rightarrow x = \frac{1}{6}$	\Rightarrow $x = -3$ or $x = 1$
$\Rightarrow x = 1$	
$\frac{1}{1}$	
- $+$ Sign of $f(x)$	
Kind of	
O U concavity	
Hence, the function $f(x)$ has an inflection point at	
(1, -10).	
$f(1) = (1)^3 - 3(1)^2 - 9(1) + 1$	
= 1 - 3 - 9 + 1 = -10	
28) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is decreasing	29) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is increasing
Off Solution:	Off Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$
$\Rightarrow 3(x^2 + 2x - 3) = 0$	$\Rightarrow 3(x^2 + 2x - 3) = 0$
$\implies x^2 + 2x - 3 = 0$	$\implies x^2 + 2x - 3 = 0$
$\Rightarrow (x+3)(x-1) = 0$	$\Rightarrow (x+3)(x-1) = 0$
$\Rightarrow x = -3 \text{ or } x = 1$	$\Rightarrow x = -3 \text{ or } x = 1$
f'(x)	f'(x)
Kind of	Kind of
monotonicit	monotonicit
γ	γ
Hence, the function $f(x)$ is decreasing on $(-3,1)$.	Hence, the function $f(x)$ is increasing on
	$(-\infty, -3) \cup (1, \infty).$

30) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative	31) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative
minimum value at the point	maximum value at the point
Solution:	Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$\int f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$ $\implies 2(x^2 + 2x - 2) = 0$	$\int f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$ $\implies 2(x^2 + 2x - 3) = 0$
$\Rightarrow 3(x^2 + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$	$\Rightarrow 3(x^2 + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 2 = 0$
$ \Rightarrow x + 2x - 5 = 0 \Rightarrow (r + 3)(r - 1) = 0 $	$ \Rightarrow x + 2x - 3 = 0 $ $ \Rightarrow (r + 3)(r - 1) = 0 $
$\Rightarrow (x+3)(x-1) = 0$ $\Rightarrow r = -3 \text{ or } r = 1$	$\Rightarrow (x+3)(x-1) = 0$ $\Rightarrow r = -3 \text{ or } r = 1$
-3 1	$-3 \qquad 1$
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicit	monotonicit
у	у
Hence, the function $f(x)$ has a relative minimum value at	Hence, the function $f(x)$ has a relative maximum value at
the point (1,0).	the point $(-3,32)$.
$f(1) = (1)^3 + 3(1)^2 - 9(1) + 5$	$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5$
= 1 + 3 - 9 + 5 = 0	= -27 + 27 + 27 + 5 = 32
32) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has an	33) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave
Inflection point at	downward on
<u>Solution:</u> $f'(x) = 2x^2 + 6x = 0$	<u>Solution:</u> $f'(x) = 2x^2 + 6x = 0$
f''(x) = 6x + 6	f''(x) = 6x + 6
$f''(x) = 0 \implies 6x + 6 = 0$	$f''(x) = 0 \implies 6x + 6 = 0$
$\Rightarrow 6x = -6$	$\Rightarrow 6x = -6$
6	6
$\Rightarrow x = -\frac{1}{6}$	$\Rightarrow x = -\frac{1}{6}$
$\Rightarrow x = -1$	$\Rightarrow x = -1$
-1 $-$ + Sign of $f''(x)$	-1 $-$ + Sign of $f''(x)$
O U concavity	O U concavity
Hence, the function $f(x)$ has an inflection point at	Hence, the function $f(x)$ is concave downward on
(-1,16).	(−∞, −1).
$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 5$	
= -1 + 3 + 9 + 5 = 16	
34) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave	35) The critical numbers of the function $f(u) = u^3 - 2u^2 = 0$
upward on Solution	$f(x) = x^3 - 3x^2 - 9x + 5$ are
$\frac{5010(1011)}{f'(x)} = 2x^2 + 6x = 0$	$\frac{501011011}{f'(x)} = 2x^2 - 6x = 0$
$\int (x) - 5x + 0x - 9$ $f''(x) - 6x + 6$	$f'(x) = 0 \implies 3x^2 = 6x = 9 = 0$
$f''(\mathbf{r}) = 0 \implies 6\mathbf{r} + 6 = 0$	$ \begin{array}{c} y (x) = 0 \implies 3x 0x y = 0 \\ \implies 3(x^2 - 2x - 3) = 0 \end{array} $
$ \begin{array}{c} f \\ f $	$\Rightarrow 3(x - 2x - 3) = 0$ $\Rightarrow x^2 - 2x - 3 = 0$
6	$\Rightarrow (x+1)(x-3) = 0$
$\Rightarrow x = -\frac{1}{6}$	\Rightarrow $x = -1$ or $x = 3$
$\Rightarrow x = -1$	
_1	
- + Sign of $f''(x)$	
$ \begin{array}{c c} - & + & \text{Sign of } f''(x) \\ \hline $	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
-+Sign of $f''(x)$ \bigcap UKind of concavityHence, the function $f(x)$ is concave upward on $(-1, \infty)$.	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	
-+Sign of $f''(x)$ \bigcap UKind of concavityHence, the function $f(x)$ is concave upward on $(-1, \infty)$.	
-+Sign of $f''(x)$ \bigcap UKind of concavityHence, the function $f(x)$ is concave upward on $(-1, \infty)$.	

36) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing	37) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is decreasing
on	on
Solution:	Solution:
$f'(x) = 3x^2 - 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$
$\Rightarrow 3(x^2 - 2x - 3) = 0$	$\Rightarrow 3(x^2 - 2x - 3) = 0$
$\Rightarrow r^2 - 2r - 3 = 0$	$\Rightarrow r^2 - 2r - 3 = 0$
$\Rightarrow x 2x 3 = 0$ $\Rightarrow (r+1)(r-3) = 0$	$\Rightarrow x = 2x = 3 = 0$ $\Rightarrow (r+1)(r-3) = 0$
$\Rightarrow (x + 1)(x - 3) = 0$ $\Rightarrow x = -1 \text{ or } x = 3$	$\Rightarrow (x + 1)(x - 3) = 0$ $\Rightarrow x = -1 \text{ or } x = 3$
\rightarrow $x = 1$ of $x = 3$	$ \xrightarrow{\longrightarrow} x = 1 \text{or} x = 3 $
$\frac{1}{1} = \frac{1}{1}$	$\begin{bmatrix} 1 \\ \vdots \end{bmatrix}$
+ $ +$ Sign of $f(x)$	$+$ $ +$ $\operatorname{Sign}(O) f(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ is increasing on	Hence, the function $f(x)$ is decreasing on $(-1,3)$.
$(-\infty, -1) \cup (3, \infty).$	
38) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative	39) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative
maximum value at the point	minimum value at the point
Solution:	Solution:
$f'(x) = 3x^2 - 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$	$f'(r) = 0 \implies 3r^2 - 6r - 9 = 0$
$ \Rightarrow 3(r^2 - 2r - 3) = 0 $	$ \Rightarrow 3(r^2 - 2r - 3) = 0 $
$ \rightarrow 3(x - 2x - 3) = 0 $	$ \rightarrow \qquad 3(x - 2x - 3) = 0 $
$\Rightarrow x - 2x - 5 = 0$ $\Rightarrow (x + 1)(x - 2) = 0$	$\Rightarrow x - 2x - 5 = 0$ $\Rightarrow (x + 1)(x - 2) = 0$
$\Rightarrow (x+1)(x-3) = 0$	$\Rightarrow (x+1)(x-3) = 0$
$\Rightarrow x = -1$ or $x = 3$	$\Rightarrow x = -1$ or $x = 3$
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum value at	Hence, the function $f(x)$ has a relative minimum value at
the point $(-1, 10)$.	the point $(3, -22)$.
$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 5$	$f(3) = (3)^3 - 3(3)^2 - 9(3) + 5$
= -1 - 3 + 9 + 5 = 10.	= 27 - 27 - 27 + 5 = -22.
40) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave	41) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave
$\frac{1}{1000}$	downward on
Solution	Solution
5000000000000000000000000000000000000	5000000000000000000000000000000000000
f''(x) = 5x - 6x - 9	f''(x) = 5x - 6x - 9
$\int (x) = 6x - 6$	$\int (x) = 6x - 6$
$f(x) = 0 \implies 6x - 6 = 0$	$f(x) = 0 \implies 6x - 6 = 0$
$\Rightarrow 6x = 6$	$\Rightarrow 6x = 6$
$\Rightarrow x = \frac{0}{1}$	$\Rightarrow x = \frac{0}{1}$
$ \stackrel{6}{\rightarrow} $	$ \rightarrow x = 1 $
$\Rightarrow x = 1$	$\Rightarrow x = 1$
- + Sign of $f''(x)$	- + Sign of $f''(x)$
Kind of	Kind of
II O concavity	II O concavity
Hence, the function $f(x)$ is concave upward on $(1, \infty)$.	Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

42) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has an	43) The critical numbers of the function
inflection point at	$f(r) = \frac{1}{r^3} - \frac{1}{r^2} - 2r + 1$ are
Solution:	$\int (x) = \frac{1}{3} \frac{1}{2} x + $
$f'(x) = 3x^2 - 6x - 9$	
$f^{\prime\prime}(x) = 6x - 6$	$f'(x) = 3\left(\frac{1}{2}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$
$f''(x) = 0 \Longrightarrow 6x - 6 = 0$	$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\Rightarrow 6x = 6$	$\Rightarrow (x+1)(x-2) = 0$
$\Rightarrow r = \frac{6}{7}$	\Rightarrow $x = -1$ or $x = 2$
\rightarrow $\lambda = \frac{6}{6}$	
$\Rightarrow x = 1$	
- + Sign of $f''(x)$	
Kind of	
O U concavity	
Hence, the function $f(x)$ has an inflection point at $(1, -6)$.	
$f(1) = (1)^3 - 3(1)^2 - 9(1) + 5$	
= 1 - 3 - 9 + 5 = -6	
44) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is	45) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is
increasing on	decreasing on
Solution:	Solution:
$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$	$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$
$f'(x) = 0 \implies x^2 - x - 2 = 0$	$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\Rightarrow (x+1)(x-2) = 0$	$\Rightarrow (x+1)(x-2) = 0$
\Rightarrow $x = -1$ or $x = 2$	\Rightarrow $x = -1$ or $x = 2$
-1 2	-1 2
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ is increasing on	Hence, the function $f(x)$ is decreasing on $(-1,2)$.
$(-\infty, -1) \cup (2, \infty).$	
46) The function $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 2x + 1$	47) The function $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 2x + 1$
has a relative maximum point	has a relative minimum point
Solution:	Solution:
$\frac{1}{2}$	$\frac{1}{1}$
$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$	$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$
$f'(x) = 0 \implies x^2 - x - 2 = 0$	$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$	$\implies (x+1)(x-2) = 0$
\implies $x = -1$ or $x = 2$	\implies $x = -1$ or $x = 2$
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum point at	Hence, the function $f(x)$ has a relative minimum point at
$\left(-1,\frac{13}{6}\right)$.	$\left(2,-\frac{7}{3}\right)$.
$f(-1) = \frac{1}{2}(-1)^3 = \frac{1}{2}(-1)^2 = 2(-1) + 1$	$f(2) = \frac{1}{2}(2)^3 = \frac{1}{2}(2)^2 = 2(2) \pm 1$
$\int (-1) - \frac{1}{3} (-1) - \frac{1}{2} (-1) - \frac{1}{2} (-1) + 1$	$\int (2) - \frac{1}{3} (2) - \frac{1}{2} (2) - \frac{1}{2} (2) - \frac{1}{2} (2) + 1$
$=-\frac{1}{2}-\frac{1}{2}+2+1=\frac{13}{2}$	$=\frac{8}{7}-\frac{4}{7}-4+1=-\frac{7}{7}$
3 2 6	3 2 3

48) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ concave	49) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ concave
upward on	downward on
Solution:	Solution:
$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$	$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$
f''(x) = 2x - 1	f''(x) = 2x - 1
$ \begin{array}{ccc} f & (x) = 0 & \Longrightarrow & 2x - 1 = 0 \\ & \implies & 2x = 1 \end{array} $	$ \begin{array}{ccc} f & (x) = 0 & \implies & 2x - 1 = 0 \\ & \implies & 2x = 1 \end{array} $
$\Rightarrow x = \overline{2}$	$\Rightarrow x = \overline{2}$
$\frac{1}{2}$	$\frac{1}{2}$
- + Sign of $f''(x)$	- + Sign of $f''(x)$
∩ U Kind of concavity	O U Kind of concevity
Hence, the function $f(x)$ is concave upward on $\left(\frac{1}{2}, \infty\right)$.	Hence, the function $f(x)$ is concave downward on $\left(-\infty, \frac{1}{2}\right)$.
50) The function $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 2x + 1$ has an	51) The critical numbers of the function
inflection point at	$f(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 - 2x + 1$ are
Solution:	Solution:
$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$	$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$
f''(x) = 2x - 1	$f'(x) = 0 \implies x^2 + x - 2 = 0$
$f''(x) = 0 \implies 2x - 1 = 0$	$\Rightarrow (x+2)(x-1) = 0$
$\Rightarrow 2x = 1$	$\Rightarrow x = -2 \text{ or } x = 1$
$\Rightarrow x = \frac{1}{2}$	
$\frac{1}{1}$	
2 + Sign of $f''(r)$	
∩ U Kind of concavity	
Hence, the function $f(x)$ has an inflection point at	
$\left(\frac{1}{2}, -\frac{1}{12}\right).$	
$f\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right)^3 - \frac{1}{2}\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1$	
$=\frac{1}{24}-\frac{1}{8}-1+1=-\frac{1}{12}$	
52) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ is	53) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ is
increasing on	decreasing on
Solution:	Solution:
$f'(x) = 3\left(\frac{1}{2}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$	$f'(x) = 3\left(\frac{1}{2}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$
$f'(x) = 0 \implies x^2 + x - 2 = 0$	$f'(x) = 0 \implies x^2 + x - 2 = 0$
$\Rightarrow (x+2)(x-1) = 0$	$\Rightarrow (x+2)(x-1) = 0$
$\Rightarrow x = -2 \text{ or } x = 1$	\Rightarrow $x = -2$ or $x = 1$
+ $ +$ Sign of $f'(x)$	+ - + Sign of $f'(x)$
Hence, the function $f(x)$ is increasing on	Hence, the function $f(x)$ is decreasing on $(-2,1)$.
$(-\infty, -2) \cup (1, \infty).$	

54) The function
$$f(x) = \frac{1}{4}x^{2} + \frac{1}{4}x^{2} - 2x + 1$$

has a relative maximum point
Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies x^{2} + x - 2 = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies x^{2} + x - 2 = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies x^{2} + x - 2 = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies x^{2} + x - 2 = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies x^{2} + x - 2 = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies x^{2} + x - 2 = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}$$

$$f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$$

$$f''(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 2x + 1$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f''(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x = -\frac{1}{2}$$

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\Rightarrow x^{2} - 1 \implies x^{2} = 4$$

$$\Rightarrow x^{2} = 4$$

$$\Rightarrow x = \pm 2$$



(c) The final $f(x) = x^3 + 12x + 2$ has a	(CT) The estimate subset of the frequine
66) The function $f(x) = x^{2} - 12x + 3$ has an	67) The critical numbers of the function
inflection point at	$f(x) = x^3 - 3x^2 + 1$ are
Solution:	Solution:
$f'(x) = 3x^2 - 12$	$f'(x) = 3x^2 - 6x$
$f^{\prime\prime}(x) = 6x$	$f'(x) = 0 \implies 3x^2 - 6x = 0$
$f''(x) = 0 \implies 6x = 0$	$\Rightarrow 3(x^2 - 2x) = 0$
	$\Rightarrow r^2 - 2r = 0$
$\Rightarrow x = \frac{1}{6}$	$ \xrightarrow{\longrightarrow} x \xrightarrow{2x} = 0 $
$\rightarrow r = 0$	$\implies x(x-2) = 0$
$\rightarrow x = 0$	$\Rightarrow x = 0 \text{ or } x = 2$
- $ $ + Sign of $f''(x)$	
Kind of	
Upped the function $f(u)$ has an inflaction point at $(0,2)$	
Hence, the function $f(x)$ has an inflection point at (0,3).	
$f(0) = (0)^3 - 12(0)^2 + 3$	
= 0 - 0 + 3 = 3	
68) The function $f(x) = x^3 - 3x^2 + 1$ is increasing	69) The function $f(x) = x^3 - 3x^2 + 1$ is decreasing
on	on
Solution	Solution
$\frac{30101011}{2}$	$\frac{30101011}{2}$
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 6x$
$f'(x) = 0 \implies 3x^2 - 6x = 0$	$f'(x) = 0 \implies 3x^2 - 6x = 0$
$\implies 3(x^2 - 2x) = 0$	$\implies 3(x^2 - 2x) = 0$
$\Rightarrow x^2 - 2x = 0$	$\Rightarrow x^2 - 2x = 0$
$\Rightarrow x(x-2) = 0$	$\Rightarrow x(x-2) = 0$
$ \rightarrow x = 0 \text{ or } x = 2 $	$ \rightarrow x = 0 \text{ or } x = 2 $
\rightarrow $x = 0$ of $x = 2$	$ \xrightarrow{\longrightarrow} x = 0 01 x = 2 $
+ $ +$ Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Using the function $f(x)$ is increasing on	Use the function $f(w)$ is decreasing on $(0,2)$
Hence, the function $f(x)$ is increasing on	Hence, the function $f(x)$ is decreasing on $(0,2)$.
$(-\infty,0) \cup (2,\infty).$	
70) The function $f(x) = x^3 - 3x^2 + 1$ has a relative	71) The function $f(x) = x^3 - 3x^2 + 1$ has a relative
maximum point at	minimum point at
Solution:	Solution:
$f'(x) - 2x^2 - 6x$	$f'(x) - 2x^2 - 6x$
$\int (x) - 5x = 0x$	$\int (x) - 5x = 0$
$\int (x) = 0 \implies 3x^2 - 6x = 0$	$\int (x) = 0 \implies 3x^2 - 6x = 0$
$\Rightarrow 3(x^2 - 2x) = 0$	$\Rightarrow 3(x^2 - 2x) = 0$
$\implies x^2 - 2x = 0$	$\Rightarrow x^2 - 2x = 0$
$\Rightarrow x(x-2) = 0$	$\Rightarrow x(x-2) = 0$
$\Rightarrow x = 0$ or $x = 2$	$\Rightarrow x = 0$ or $x = 2$
$ \begin{array}{ c c } \hline & & & \\ \hline \\ \hline$	$ \begin{array}{ c c } \hline \\ \hline $
\top $ \top$ Sign of $f(x)$	- + - + +
🖊 Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum point at	Hence, the function $f(x)$ has a relative minimum point at
(01)	(2 - 3)
(0,1).	(2, 3).
$\int (0)^{2} = (0)^{2} - 3(0)^{2} + 1$	$\int (2) = (2)^{2} - 3(2)^{2} + 1$
= 0 - 0 + 1 = 1.	= 8 - 12 + 1 = -3.

72) The function $f(x) = x^3 - 3x^2 + 1$ concave	73) The function $f(x) = x^3 - 3x^2 + 1$ concave
upward on	downward on
Solution:	Solution:
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 6x$
f''(x) = 6x - 6	f''(x) = 6x - 6
$\int^{\infty} (x) = 0 \implies 6x - 6 = 0$	$f^{(x)}(x) = 0 \implies 6x - 6 = 0$
$\Rightarrow 0x = 0$	$\Rightarrow 0x = 0$
$\Rightarrow x = \frac{3}{6}$	$\Rightarrow x = \frac{3}{6}$
$\Rightarrow x = 1$	$\Rightarrow x = 1$
1	1
- + Sign of $f''(x)$	- + Sign of $f''(x)$
Kind of	Kind of
Concavity	I U concavity
Hence, the function $f(x)$ is concave upward on $(1, \infty)$.	Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.
74) The function $f(x) = x^3 - 3x^2 + 1$ has an	75) The critical numbers of the function
inflection point at	$f(x) = x^3 - 3x^2 + 2$ are
Solution:	Solution:
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 6x$
f''(x) = 6x - 6	$f'(x) = 0 \implies 3x^2 - 6x = 0$
$f''(x) = 0 \implies 6x - 6 = 0$	$\implies 3(x^2 - 2x) = 0$
$\Rightarrow 0x = 0$	$\Rightarrow x^2 - 2x = 0$ $\Rightarrow x(x - 2) = 0$
$\Rightarrow x = \frac{3}{6}$	$ \Rightarrow x(x-2) = 0 \Rightarrow x = 0 \text{ or } x = 2 $
$\Rightarrow x = 1$	\rightarrow $\chi = 0$ of $\chi = 2$
1	
- + Sign of $f''(x)$	
Kind of	
$\bigcup_{x \in \mathcal{X}} Concavity \qquad \qquad$	
$f(1) = (1)^3 - 3(1)^2 + 1$ = 1 - 3 + 1 = -1	
76) The function $f(x) = x^3 - 3x^2 + 2$ is increasing on	77) The function $f(x) = x^3 - 3x^2 + 2$ is decreasing on
Solution:	Solution:
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 6x$
$f'(x) = 0 \implies 3x^2 - 6x = 0$	$f'(x) = 0 \implies 3x^2 - 6x = 0$
$\implies 3(x^2 - 2x) = 0$	$\implies 3(x^2 - 2x) = 0$
$\Rightarrow x^2 - 2x = 0$	$\Rightarrow x^2 - 2x = 0$
$\Rightarrow x(x-2) = 0$	$\Rightarrow x(x-2) = 0$
$\implies x = 0 \text{ or } x = 2$	$\Rightarrow x = 0 \text{ or } x = 2$
$\begin{array}{ c c c c c } \hline & & & & & \\ \hline & + & - & & + & & \\ \hline & + & & - & & + & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline & & & &$	\downarrow \downarrow \downarrow Sign of $f'(x)$
$- \frac{1}{1} = $	\checkmark \checkmark \checkmark Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ is increasing on	Hence, the function $f(x)$ is decreasing on (0,2).
$(-\infty,0) \cup (2,\infty).$	

78) The function $f(x) = x^2 - 3x^2 + 2$ has a relative	79) The function $f(x) = x^3 - 3x^2 + 2$ has a relative		
minimum point at	maximum point at		
Solution:	Solution:		
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 6x$		
$\int f(x) = 0 \implies 3x^2 - 6x = 0$ $\implies 2(x^2 - 2x) = 0$	$f'(x) = 0 \implies 3x^2$	$x^2 - 6x = 0$	
$\implies 5(x - 2x) = 0$ $\implies x^2 - 2x = 0$	$\Rightarrow 3(x^2 - 2x) = 0$ $\Rightarrow x^2 - 2x = 0$		
$\Rightarrow x - 2x = 0$ $\Rightarrow r(r - 2) = 0$		(-2x - 0) - 0	
$\Rightarrow x(x-2) = 0$ $\Rightarrow x = 0 \text{ or } x = 2$	$ \Rightarrow x = x = x = x = x = x = x = x = x = x$	= 0 or r = 2	
$\overrightarrow{0}$ \overrightarrow{x} $\overrightarrow{0}$ \overrightarrow{x} $\overrightarrow{2}$	$\vec{0}$	2^{-0} $\frac{1}{2}$	
+ - + Sign of $f'(x)$	+ -	· +	Sign of $f'(x)$
			Kind of
monotonicity			monotonicity
			monotonicity
Hence, the function $f(x)$ has a relative minimum point at	Hence the function f	(r) has a relative m	aximum noint at
(2 - 2)	(0.2)		
$f(2) = (2)^3 - 3(2)^2 + 2$	$f(0) = (0)^3 = 3(0)^2$	+ 2	
= 8 - 12 + 2 = -2	= 0 - 0 + 2 = 2		
80) The function $f(x) = x^3 - 3x^2 + 2$ concave	81) The function $f(x)$	$x = x^3 - 3x^2 + 2$	concave
downward on	unward on)	
Solution:	Solution:		
$f'(r) = 3r^2 - 6r$	<u>5010(1011.</u> f	$f'(x) = 3x^2 - 6x$	
f''(x) = 6x - 6	J	f''(x) = 6x - 6	
$f''(x) = 0 \implies 6x - 6 = 0$	$f''(x) = 0 \implies 6x$	-6=0	
$\Rightarrow 6x = 6$	$\Rightarrow 6x$	= 6	
6		6	
$\Rightarrow x = \frac{1}{6}$	$\Rightarrow x =$	$=\frac{1}{6}$	
$\Rightarrow x = 1$	$\Rightarrow x =$	= 1	
1	1		
- + Sign of $f''(x)$	-	+	Sign of $f''(x)$
- + Sign of $f''(x)$	-	+	Sign of $f''(x)$
- + Sign of f''(x) $Kind of$	-	+	Sign of $f''(x)$ Kind of
$\begin{array}{c c} - & + & \text{Sign of } f''(x) \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	- <u> </u>	+ U	Sign of $f''(x)$ Kind of concavity
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Hence, the function <i>f</i>	+ U (x) is concave upwa	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Hence, the function <i>f</i>	+ U (x) is concave upwa	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Hence, the function <i>f</i> (83) The critical number	+ U (x) is concave upwa	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Hence, the function <i>f</i> ($f(x) = x^3 - 6x^2$)	+ U (x) is concave upwaters of the function -36x are	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Hence, the function $f(x) = x^3 - 6x^2$ Solution:	+ U (x) is concave upwaters of the function -36x are	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Hence, the function $f(x) = x^3 - 6x^2$ Solution:	+ U (x) is concave upware ers of the function - 36x are $3x^{2} - 12x - 3x^{2}$	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$-$ Hence, the function <i>f</i> (4) $f(x) = x^{3} - 6x^{2}$ Solution: $f'(x) = 0 \implies 3x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = 3x ² - 12x - 3 ² - 12x - 36 = 0	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	- Hence, the function <i>f</i> (83) The critical number $f(x) = x^3 - 6x^2$ Solution: $f'(x) = 0 \implies 3x^2$ $\Rightarrow 3(x)$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Hence, the function $f(x) = x^3 - 6x^2$ Solution: $f'(x) = 0 \implies 3x^2$ $\Rightarrow 3(x)$ $\Rightarrow x^2$	+ U (x) is concave upware ers of the function - 36x are) = 3x ² - 12x - 33 ² - 12x - 36 = 0 x ² - 4x - 12) = 0 - 4x - 12 = 0	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$f'(x) = 0 \implies f'(x) = x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 36$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$ - $4x - 12 = 0$ + 2)(x - 6) = 0	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$-$ Hence, the function $f(x) = x^3 - 6x^2$ Solution: $f'(x) = 0 \implies 3x^2$ $\Rightarrow 3(x)$ $\Rightarrow x^2$ $\Rightarrow (x + y)$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$f'(x) = 0 \implies 3(x) = x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$ -4x - 12 = 0 + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{ c c c c c }\hline & - & + & \text{Sign of } f''(x) \\ \hline & & & \\ \hline \\ \hline$	$f'(x) = 0 \implies f'(x)$ $f'(x) = 0 \implies 3x^{2}$ $f'(x) = 0 \implies 3x^{2}$ $f'(x) = 0 \implies x^{2}$ $f'(x) = 0 \implies x^{2}$ $f'(x) = 0 \implies x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 36$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$ - $4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{ c c c c c }\hline & - & + & \text{Sign of } f''(x) \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & \hline & & \\ \hline \hline & & \\ \hline & & \\ \hline & & \\ \hline \hline & & \\ \hline & & \\ \hline \hline \hline & & \\ \hline \hline \hline & & \\ \hline \hline \hline \hline$	$f'(x) = 0 \implies x^{2}$ $x = x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{ c c c c c c }\hline & - & + & \text{Sign of } f''(x) \\ \hline & & & & \\ \hline \hline & & \\ \hline \hline \hline \hline$	$f'(x) = 0 \implies x^{2}$ $\Rightarrow x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$ -4x - 12 = 0 + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$f'(x) = 0 \implies x^{2}$ $x = x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 36$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$ - $4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$f'(x) = 0 \implies x^{2}$ $x = x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 32^2 - 12x - 32^2 - 12x - 36 = 0$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{ c c c c c }\hline & - & + & \operatorname{Sign of} f''(x) \\ \hline & & & U & \operatorname{Kind of} \\ \operatorname{concavity} \\ \hline \\ \text{Hence, the function } f(x) &= x^3 - 3x^2 + 2 \text{ has an} \\ \text{inflection point at} \\ \hline \\ \hline \\ Solution: & f'(x) &= 3x^2 - 6x \\ f''(x) &= 6x - 6 \\ f''(x) &= 0 & \Rightarrow & 6x - 6 = 0 \\ & \Rightarrow & 6x = 6 \\ & \Rightarrow & x = 1 \\ \hline \\$	$f'(x) = 0 \implies x^{2}$ $\Rightarrow x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$ - $4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{ c c c c c }\hline & - & + & \operatorname{Sign of} f''(x) \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \hline & & \\ \hline & & \\ \hline & & \\ \hline \hline & & \\ \hline & & \\ \hline \hline \hline \hline$	$f'(x) = 0 \implies x^{2}$ $x = x^{2}$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 36^2 - 12x - 36^2 - 12x - 36 = 0^2 - 12x - 36 = 0^2 - 4x - 12 $	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{ c c c c c }\hline & - & + & \text{Sign of } f''(x) \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & \hline \\ \hline & & \\ \hline \hline & & \\ \hline \hline & & \\ \hline & & \\ \hline \hline \hline \hline$	- Hence, the function f 83) The critical number $f(x) = x^3 - 6x^2$ Solution: $f'(x) = 0 \implies 3x^2$ $\Rightarrow 3(x)$ $\Rightarrow x^2$ $\Rightarrow (x + x)$ $\Rightarrow x = x^2$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{ c c c c c }\hline & - & + & \text{Sign of } f''(x) \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & \hline & & \\ \hline \hline & & \\ \hline \hline & & \\ \hline & & \\ \hline & & \\ \hline \hline \hline & & \\ \hline \hline \hline & & \\ \hline \hline \hline \\ \hline \hline \hline \hline$	- Hence, the function f 83) The critical number $f(x) = x^3 - 6x^2$ Solution: $f'(x) = 0 \implies 3x^2$ $\Rightarrow 3(x)$ $\Rightarrow x^2$ $\Rightarrow (x + y)$ $\Rightarrow (x + y)$ $\Rightarrow (x + y)$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$ - $4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	- Hence, the function f 83) The critical number $f(x) = x^3 - 6x^2$ Solution: $f'(x) = 0 \implies 3x^2$ $\Rightarrow 3(x)$ $\Rightarrow x^2$ $\Rightarrow (x + y)$ $\Rightarrow x = 0$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 33$ $x^2 - 12x - 36 = 0$ $x^2 - 4x - 12) = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.
$\begin{array}{ c c c c c }\hline & - & + & \text{Sign of } f''(x) \\ \hline & & & & \text{U} & \text{Kind of concavity} \\ \hline & & \text{Hence, the function } f(x) \text{ is concave downward on } (-\infty, 1). \\ \hline & \text{82) The function } f(x) = x^3 - 3x^2 + 2 \text{ has an inflection point at} \\ \hline & \text{Solution:} & f'(x) = 3x^2 - 6x \\ & & f''(x) = 6x - 6 \\ f''(x) = 0 & \Rightarrow & 6x - 6 = 0 \\ & \Rightarrow & 6x = 6 \\ & \Rightarrow & x = 1 \\ \hline & & & & & \\ \hline & & & & & \\ \hline & & & &$	- Hence, the function f 83) The critical number $f(x) = x^3 - 6x^2$ Solution: $f'(x) = 0 \implies 3x^2$ $\Rightarrow 3(x)$ $\Rightarrow x^2$ $\Rightarrow (x + x)$ $\Rightarrow x = x^2$	+ U (x) is concave upware ers of the function - 36x are) = $3x^2 - 12x - 32^2 - 12x - 32^2 - 12x - 36 = 0$ $x^2 - 4x - 12 = 0$ + 2)(x - 6) = 0 = -2 or x = 6	Sign of $f''(x)$ Kind of concavity ard on $(1, \infty)$.

84) The function $f(x) = x^3 - 6x^2 - 36x$ is decreasing on	85) The function $f(x) = x^3 - 6x^2 - 36x$ is increasing on		
Solution: $f(x) = 2x^2 + 12x + 2C$	Solution: $f(x) = 2x^2 + 12x + 26$		
$f'(x) = 3x^2 - 12x - 36$	$f'(x) = 3x^2 - 12x - 36$ $f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccc} f'(x) = 0 & \implies & 3x^2 - 12x - 36 = 0 \\ & \implies & 3(x^2 - 4x - 12) = 0 \end{array} $		
$\Rightarrow x^2 - 4x - 12 = 0$	$\Rightarrow x^2 - 4x - 12 = 0$		
$\implies (x+2)(x-6) = 0$	$\Rightarrow (x+2)(x-6) = 0$		
\Rightarrow $x = -2$ or $x = 6$	\Rightarrow $x = -2$ or $x = 6$		
-2 6	-2 6		
+ $ +$ Sign of $f'(x)$	+ $ +$ Sign of $f'(x)$		
	Kind Of monotonicity		
Hence, the function $f(x)$ is decreasing on $(-2,6)$.	Hence, the function $f(x)$ is increasing on		
	$(-\infty, -2) \cup (6, \infty).$		
86) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative	87) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative		
minimum value at the point	maximum value at the point		
Solution:	Solution:		
$f'(x) = 3x^2 - 12x - 36$	$f'(x) = 3x^2 - 12x - 36$		
$\int f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$ $\implies 2(x^2 - 4x - 12) = 0$	$\int f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$ $\implies 2(x^2 - 4x - 12) = 0$		
$ \Rightarrow 3(x - 4x - 12) = 0 \Rightarrow x^2 - 4x - 12 = 0 $	$ \Rightarrow 3(x - 4x - 12) = 0 \Rightarrow x^2 - 4x - 12 = 0 $		
$\Rightarrow x + 12 = 0$ $\Rightarrow (x+2)(x-6) = 0$	$\Rightarrow x + 12 = 0$ $\Rightarrow (x + 2)(x - 6) = 0$		
\Rightarrow $x = -2$ or $x = 6$	\Rightarrow $x = -2$ or $x = 6$		
-2 6	-2 6		
+ - + Sign of $f'(x)$	+ – + Sign of $f'(x)$		
Kind of	Kind of		
monotonicity	monotonicity		
$\begin{bmatrix} \cdot & \cdot \\ \cdot $	Hence the function $f(x)$ has a relative maximum value at		
\square			
the point $(6 - 216)$	the point (-2.40)		
the point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$	the point $(-2,40)$. $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$		
the point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216	the point (-2,40). $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40		
The point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216 88) The function $f(x) = x^3 - 6x^2 - 36x$ has an	the point (-2,40). $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40 89) The function $f(x) = x^3 - 6x^2 - 36x$ concave		
The point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216 88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at	the point (-2,40). $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40 89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on		
The note, the function $f(x)$ has a relative minimum value at the point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216 88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at <u>Solution:</u>	the point (-2,40). $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40 89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on <u>Solution:</u>		
The note, the function $f(x)$ has a relative minimum value at the point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216 88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at <u>Solution:</u> $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12	the point (-2,40). $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40 89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on <u>Solution:</u> $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12		
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Hence, the function $f(x)$ has a relative minimum value at the point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216 88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at Solution: $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$ $\implies 6x = 12$ $\implies x = \frac{12}{6}$ $\implies x = 2$ 2 $\begin{array}{c c} & & \\ \hline \hline & & \\ \hline \hline & & \\ \hline & & \\ \hline & & \\ \hline & & \\ \hline \hline \hline \hline$	There is the function $f(x)$ has a relative maximum value at the point $(-2,40)$. $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40 89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on <u>Solution:</u> $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$ $\implies 6x = 12$ $\implies x = \frac{12}{6}$ $\implies x = 2$ 2 <u>Concavity</u> Hence, the function $f(x)$ is concave downward on $(-\infty, 2)$.		
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Thence, the function $f(x)$ has a relative minimum value at the point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216 88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at Solution: $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$ $\implies 6x = 12$ $\implies x = \frac{12}{6}$ $\implies x = 2$ 2 $\frac{-}{2}$ Hence, the function $f(x)$ has an inflection point at (2, -88). $f(2) = (2)^3 - 6(2)^2 - 36(2)$ = 8 - 24 - 72 = -88	Therefore, the function $f(x)$ has a relative maximum value at the point $(-2,40)$. $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40 89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on Solution: $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$ $\implies 6x = 12$ $\implies x = \frac{12}{6}$ $\implies x = 2$ 2 $\begin{array}{c c c c c c c c c c c c c c c c c c c $		
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Therefore, the function $f(x)$ has a relative minimum value at the point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216 88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at Solution: $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$ $\implies 6x = 12$ $\implies x = \frac{12}{6}$ $\implies x = 2$ 2 $\begin{array}{c c c c c c c c c c c c c c c c c c c $	the function $f(x)$ has a relative maximum value at the point $(-2,40)$. $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40 89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on <u>Solution:</u> $f''(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$ $\implies 6x = 12$ $\implies x = \frac{12}{6}$ $\implies x = 2$ 2 $\boxed{- + Sign of f''(x)}$ Hence, the function $f(x)$ is concave downward on $(-\infty, 2)$.		
Hence, the function $f(x)$ has a relative minimum value at the point $(6, -216)$. $f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216 88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at Solution: $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$ $\implies 6x = 12$ $\implies x = \frac{12}{6}$ $\implies x = 2$ 2 $\boxed{-}$ + Sign of $f''(x)$ $\boxed{-}$ U Kind of concavity Hence, the function $f(x)$ has an inflection point at (2, -88). $f(2) = (2)^3 - 6(2)^2 - 36(2)$ = 8 - 24 - 72 = -88	the point $(-2,40)$. $f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40 89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on <u>Solution:</u> $f'(x) = 3x^2 - 12x - 36$ f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$ $\implies 6x = 12$ $\implies x = \frac{12}{6}$ $\implies x = 2$ <u>2</u> <u>- + Sign of $f''(x)$ Hence, the function $f(x)$ is concave downward on $(-\infty, 2)$.</u>		

90) The function $f(x) = x^3 - 6x^2 - 36x$ concave	91) The critical numbers of the function
upward on	$f(x) = -x^3 - 6x^2 - 9x + 1$ are
Solution:	Solution:
$f'(x) = 3x^2 - 12x - 36$	$f'(x) = -3x^2 - 12x - 9$
f''(x) = 6x - 12	$f'(x) = 0 \implies -3x^2 - 12x - 9 = 0$
$f''(x) = 0 \implies 6x - 12 = 0$	$\Rightarrow -3(x^2 + 4x + 3) = 0$
$\Rightarrow 6r = 12$	$ \rightarrow 3(x + 1x + 3) = 0 $ $ \rightarrow x^2 + 4x + 3 = 0 $
\rightarrow $0x = 12$ 12	$ \xrightarrow{\longrightarrow} (r+3)(r+1) = 0 $
$\Rightarrow x = \frac{1}{6}$	$ \rightarrow (x+3)(x+1) = 0 $ $ \rightarrow x = -3 \text{ or } x = -1 $
$\Rightarrow x = 2$	$\rightarrow x = -3$ or $x = -1$
2	
- $+$ Sign of $f''(r)$	
Kind of	
Hence, the function $f(x)$ is concave upward on $(2, \infty)$.	
92) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ is	93) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ is
decreasing on	increasing on
Solution:	Solution:
$f'(x) = -3x^2 - 12x - 9$	$f'(x) = -3x^2 - 12x - 9$
$f'(x) = 0 \implies -3x^2 - 12x - 9 = 0$	$f'(x) = 0 \implies -3x^2 - 12x - 9 = 0$
$\Rightarrow -3(x^2 + 4x + 3) = 0$	$\Rightarrow -3(x^2 + 4x + 3) = 0$
$\Rightarrow r^2 + 4r + 3 = 0$	$\Rightarrow r^2 + 4r + 3 = 0$
$\Rightarrow (r+3)(r+1) = 0$	$\Rightarrow x + 1x + 5 = 0$ $\Rightarrow (r + 3)(r + 1) = 0$
$\Rightarrow (x + 5)(x + 1) = 0$ $\Rightarrow x = -3 \text{ or } x = -1$	$\Rightarrow (x+3)(x+1) = 0$ $\Rightarrow x = -3 \text{ or } x = -1$
\rightarrow $x = 5$ of $x = 1$ -3 -1	$ \xrightarrow{-3} x \xrightarrow{-1} x -$
- $+$ $-$ Sign of $f'(r)$	- $+$ $-$ Sign of $f'(r)$
- $ -$	- $ -$
monotonicity	monotonicity
Hence, the function $f(x)$ is decreasing on	Hence, the function $f(x)$ is increasing on $(-3, -1)$.
$(-\infty, -3) \cup (-1, \infty).$	
94) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has	95) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has
a relative minimum value at the point	a relative maximum value at the point
Solution:	Solution:
$f'(x) = -3x^2 - 12x - 9$	$f'(x) = -3x^2 - 12x - 9$
$f'(x) = 0 \Longrightarrow -3x^2 - 12x - 9 = 0$	$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$
$\implies -3(x^2 + 4x + 3) = 0$	$\implies -3(x^2 + 4x + 3) = 0$
$\Rightarrow x^2 + 4x + 3 = 0$	$\Rightarrow x^2 + 4x + 3 = 0$
\Rightarrow $(x+3)(x+1) = 0$	\Rightarrow $(x+3)(x+1) = 0$
\Rightarrow $x = -3$ or $x = -1$	\Rightarrow $x = -3$ or $x = -1$
-3 -1	-3 -1
- + $-$ Sign of $f'(x)$	- + $-$ Sign of $f'(x)$
Kind of	
	monotonicity
Hence the function $f(r)$ has a relative minimum value at	Hence the function $f(r)$ has a relative maximum value at
the point (-3.1)	the point (-15)
$f(-3) = -(-3)^3 = 6(-2)^2 = 0(-2) + 1$	$f(-1) = -(-1)^3 = 6(-1)^2 = 0(-1) + 1$
J(-3) = -(-3) = 0(-3) = 3(-3) + 1	f(-1) = -(-1) = 0(-1) = 5(-1) + 1
-2/-3++2/+1=1.	-1 - 0 + 7 + 1 - 3.

96) The function $f(x)$	$x) = -x^3 - 6x^2 - $	9x + 1 has an	9	7) The function $f(x)$	$x) = -x^3 - 6x^2 - $	9x + 1 concave
inflection point at				downward on			
Solution:			S	Solution:			
$f'(x) = -3x^2 - 12x - 9$				$f'(x) = -3x^2 - 12x - 9$			
f''(x) = -6x - 12				f''(x) = -6x - 12			
$f''(x) = 0 \Rightarrow -6x - 12 = 0$			f	$f''(x) = 0 \implies -6x - 12 = 0$			
$\Rightarrow -6x = 12$				$\Rightarrow -6x = 12$			
12				\rightarrow $x = \frac{12}{12}$			
	$\rightarrow \lambda$	6			$\rightarrow \lambda$	= - 6	
	$\Rightarrow x$	= -2			$\Rightarrow x$	= -2	
	+	-	Sign of $f''(x)$		+	-	Sign of $f''(x)$
	11	\cap	Kind of		11	\cap	Kind of
	0		concavity		0		concavity
He	ence, the function f	f(x) has an inflecti	on point at $(-2,3)$.	Н	lence, the function <i>f</i>	f(x) is concave dow	wnward on $(-2, \infty)$.
<i>f</i> ($(-2) = -(-2)^3 -$	$6(-2)^2 - 9(-2) +$	- 1				
	= 8 - 24 + 1	8 + 1 = 3					
98	b) The function $f(x)$	$x) = -x^3 - 6x^2 - 6x$	9x + 1 concave				
	upward on						
<u>So</u>	lution:	2					
	f'(z)	$x) = -3x^2 - 12x + x^2 - 12x$	- 9				
	<i>j</i>	f''(x) = -6x - 12					
f'	$f'(x) = 0 \implies -6$	6x - 12 = 0					
	$\Rightarrow -6$	5x = 12					
	$\Rightarrow x$	$=-\frac{12}{6}$					
$\Rightarrow x = -2$							
		-2					
	+	—	Sign of $f''(x)$				
	11		Kind of				
	U	[[]	concavity				
He	ence, the function f	f(x) is concave upv	vard on $(-\infty, -2)$.				
				1			

King Abdul Aziz University Faculty of Sciences Mathematics Department Math 110 Final Test Fall 2013 (40 Marks) Time 120 m Student Name: Student Number: |A|1) If f(x) = 2x - 9, then $f^{-1}(x) =$ $a \frac{x-9}{2}$ $b \frac{x}{2} - 9$ $c \frac{x+9}{2}$ $d \frac{x}{2} + 9$ 2) If $v = \sqrt{3x^2 + 6x}$, then y' = $\boxed{a} \frac{6(x+1)}{\sqrt{3x^2+6x}} \qquad \boxed{b} \frac{x+6}{\sqrt{3x^2+6x}} \qquad \boxed{c} \frac{3(x+1)}{\sqrt{3x^2+6x}} \qquad \boxed{d} \frac{x+1}{2\sqrt{3x^2+6x}}$ 3) If $y = \log_5(x^3 - 2\csc x)$, then y' = $a \frac{3x^{2} + 2\csc x \cot x}{x^{3} - 2\csc x} \qquad b \frac{3x^{2} + 2\csc x \cot x}{x^{3} - 2\csc x \ln 5}$ $\boxed{c} \frac{3x^{2} + 2\csc x \cot x}{(x^{3} - 2\csc x) \ln 5} \qquad \boxed{d} \frac{3x^{2} - 2\csc x \cot x}{(x^{3} - 2\csc x) \ln 5}$ 4) If $-7 \le 2x + 3 < 5$, then x = $b(-5,1] \qquad c[-5,1] \qquad d[-5,1]$ \boxed{a} (-5,1)5) If $f(x) = x^2$, then f'(x) = $a \lim_{x \to 0} \frac{(x+h)^2 - x^2}{h} \qquad b \lim_{x \to 0} \frac{(x+h)^2 + x^2}{h}$ $\boxed{c} \lim_{h \to 0} \frac{(x+h)^2 + x^2}{h} \qquad \boxed{d} \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ 6) The function $f(x) = \frac{x+1}{x^2 - 4}$ is continuous on $b[-2,2] \qquad c \{x \in \mathbb{R} : x \neq \pm 2\} \qquad d (-\infty,-2) \cup (2,\infty)$ $a \{\pm 2\}$ 7) The domain of $\frac{x+3}{\sqrt{x^2-4}}$ is $\frac{[a] [-2,2]}{[a] [-2,2]} \quad \boxed{b} (-2,2) \quad \boxed{c} (-\infty,-2) \cup (2,\infty) \quad \boxed{d} (-\infty,-2] \cup [2,\infty)$ 8) $\operatorname{csc}(\operatorname{tan}^{-1} x) =$ $\underline{A} \quad \frac{1}{\sqrt{x^2+1}} \qquad \qquad \underline{B} \quad \frac{x}{\sqrt{x^2+1}} \qquad \qquad \underline{C} \quad \sqrt{x^2+1} \qquad \qquad \underline{D} \quad \frac{\sqrt{x^2+1}}{x}$

9)
$$\lim_{x \to 5} \frac{x+1}{x-5} = \frac{1}{|x|-5|} = \frac{1}{|x|-5|} = \frac{1}{|x|-1|} = \frac{1}{|x$$

20) If
$$y = \sin x \sec x$$
, then $y' =$
a $\sin x \tan x + 1$ b $\sec^2 x$ c $\sin x \tan x - 1$ d $\sin x \sec x \tan x - 1$
21) If $y = \sin^3(4x)$, then $y' =$
a $4\cos^3(4x)$ b $3\sin^2(4x)\cos(4x)$
c $12\sin^2(4x)\cos(4x)$ d $4\sin^3(4x) + 12\sin^2 x \cos x$
22) The tangent line equation to the curve $y = \frac{2x}{x+1}$ at the point (0,0) is
a $y = -2x$ b $y = -2x + 1$ c $y = 2x$ d $y = 2x - 1$
23) If $y = 3^3 \cot x$, then $y' =$
a $3^3 \ln 3\cot x + 3^3 \sec^2 x$ b $3^3 \cot x + 3^3 \sec^2 x$
c $3^3 \cot x - 3^3 \csc^2 x$ d $3^3 \ln 3\cot x - 3^3 \csc^2 x$
24) If $y = (2x^2 + \sec x)^7$, then $y' =$
a $7(2x^2 + \sec x)^6$ b $7(2x^2 + \sec x)^6$ ($4x - \sec x \tan x$)
c $7(2x^2 + \sec x)^6$ d $4x + \sec x \tan x$ d $28x(2x^2 + \sec x)^6$
25) The slope of the perpendicular line to the line $3y - 2x - 6 = 0$ is
a $\frac{2}{3}$ b $-\frac{2}{3}$ c $-\frac{3}{2}$ d $\frac{3}{2}$
26) If the graph of the function $f(x) = 3^3$ is shifted a distance 2 units upward, then the new graph represented the graph of the function
a 3^{3+2} b $3^3 + 2$ c 3^{3-2} d $3^3 - 2$
27) If $y = \ln \frac{x+1}{x-2}$, then $y' =$
a $-\frac{3}{(x+1)(x-2)}$ b $\frac{3}{(x+1)(x-2)}$
c $\frac{1}{(x+1)(x-2)}$ d $-\frac{1}{(x+1)(x-2)}$
28) $\lim_{x\to 0} \frac{\sin 3x}{5x} =$
a $\frac{3}{5}$ b $\frac{5}{3}$ c $\frac{1}{5}$ d 3

29) D ⁽	$(125)(\cos x) =$
$a \sin x$	$b -\sin x$ $c \cos x$ $d -\cos x$
30)	$\frac{5\pi}{6}$ rad =
$a 120^{\circ}$	$b 150^{\circ}$ $c 270^{\circ}$ $d 210^{\circ}$
31) The	e distance between the points $(-1,2)$ and $(2,-1)$ is
$a 2\sqrt{3}$	$b 3\sqrt{2}$ $c 9$ $d 3$
32) If y	$y = e^{2x}$, then $y^{(5)} =$
a $128e^{2x}$	b $16e^{2x}$ c $64e^{2x}$ d $32e^{2x}$
33)	The critical numbers of the function $f(x) = 2x^3 + 3x^2 - 12x + 15$ are
<i>a</i> 1,-2	b - 1, 2 $c 1, 2$ $d - 1, -2$
34)	The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ is increasing on
$a (-\infty, -2)$	$\cup (-1,\infty) \underline{b} (-\infty,-2) \cup (1,\infty) \qquad \underline{c} (-\infty,-1) \cup (2,\infty) \qquad \underline{d} (-\infty,1) \cup (2,\infty)$
35)	The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ is decreasing on
a (-2,-1)	b (-2,1) c (1,2) d (-1,2)
36)	The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ has a relative maximum at
<i>a</i> (1,8)	b (-1,28) c (2,19) d (-2,35)
37)	The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ has a relative minimum at
<i>a</i> (1,8)	b (-1,28) c (2,19) d (-2,35)
38)	The graph of $f(x) = 2x^3 + 3x^2 - 12x + 15$ concave upward on
a $(-\infty,\frac{1}{2})$	b $(-\infty, -\frac{1}{2})$ c $(-\frac{1}{2}, \infty)$ d $(\frac{1}{2}, \infty)$
39)	The graph of $f(x) = 2x^3 + 3x^2 - 12x + 15$ concave downward on
a $(-\infty, \frac{1}{2})$	b $(-\infty, -\frac{1}{2})$ c $(-\frac{1}{2}, \infty)$ d $(\frac{1}{2}, \infty)$
40)	The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ has an inflection at
$a^{(\frac{1}{2},10)}$	b $(-\frac{1}{2}, 10)$ c $(\frac{1}{2}, \frac{43}{2})$ d $(-\frac{1}{2}, \frac{43}{2})$

King Abdul Aziz University Faculty of Sciences Mathematics Department Math 110 Final Test Fall 2013 (40 Marks) Time 120 m Student Name: Student Number: B 1) If $y = \cos x \csc x$, then $y' = \cos x \csc x$ $b 1 - \cos x \cot x \quad c \quad -1 + \cos x \cot x \quad d \quad 1 - \cos x \csc x \cot x$ $|a| - \csc^2 x$ 2) If $f(x) = \cot^{-1}(x)$ and $g(x) = \cot(x)$ then $(f \circ g)(x) =$ $|b| \cot x \cot^{-1} x$ |c| x|a|1 $|d| \cot x$ 3) The function $f(x) = \frac{x+1}{x^2 - 49}$ is continuous on $\begin{array}{c|c}
\hline a & \{x \in \mathbb{R} : x \neq \pm 7\} & b & [-7,7] & c & (-\infty,-7) \cup (7,\infty) & d & \{\pm 7\} \\
\hline 4) & \text{If } x^2 - 4 = 3xy - y^2, \text{ then } y' =
\end{array}$ $\boxed{a} \frac{3y - 2x}{2y - 3x} \qquad \boxed{b} \frac{2x}{y} \qquad \boxed{c} \frac{2x}{3 - 2y} \qquad \boxed{d} \frac{2x + y}{3x - 2y}$ 5) If $y = 3^x \tan x$, then y' =a $3^x \ln 3 \tan x - 3^x \sec^2 x$ b $3^x \ln 3 \tan x + 3^x \sec^2 x$ $\boxed{c} 3^{x} \tan x - 3^{x} \sec^{2} x \qquad \boxed{d} 3^{x} \tan x + 3^{x} \sec^{2} x$ 6) If $y = \log_5(x^3 - 2\csc x)$, then y' = $\boxed{a} \frac{3x^{2} + 2\csc x \cot x}{(x^{3} - 2\csc x)\ln 5} \qquad \boxed{b} \frac{3x^{2} + 2\csc x \cot x}{x^{3} - 2\csc x \ln 5}$ $\boxed{c} \frac{3x^{2} + 2\csc x \cot x}{x^{3} - 2\csc x} \qquad \boxed{d} \frac{3x^{2} - 2\csc x \cot x}{(x^{3} - 2\csc x) \ln 5}$ 7) If $y = (2x^2 + \csc x)^7$, then y' = $a 7(2x^{2} + \csc x)^{6}(4x - \csc x \cot x) \qquad b 7(2x^{2} + \csc x)^{6}$ $c 7(2x^{2} + \csc x)^{6}(4x + \csc x \cot x) \qquad d 28x(2x^{2} + \csc x)^{6}$ 8) The absolute minimum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on [0,4] is $b \mid 0$ $c \mid 2$ |a| 6|d| -39) The absolute maximum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on [0,4] is |b| | 2*c* 7 |d| 12 |a| 6

10) If
$$y = \sqrt{3x^2 - 6x}$$
, then $y' =$
 $a \frac{x - 6}{\sqrt{3x^2 - 6x}}$ $b \frac{6(x - 1)}{\sqrt{3x^2 - 6x}}$ $c \frac{x - 1}{2\sqrt{3x^2 - 6x}}$ $d \frac{3(x - 1)}{\sqrt{3x^2 - 6x}}$
11) The slope of the perpendicular line to the line $2y + 3x - 6 = 0$ is
 $a \frac{2}{3}$ $b - \frac{2}{3}$ $c - \frac{3}{2}$ $d \frac{3}{2}$
12) If $y = \ln \frac{x + 1}{x - 2}$, then $y' =$
 $a \frac{3}{(x + 1)(x - 2)}$ $b - \frac{3}{(x + 1)(x - 2)}$
 $c \frac{1}{(x + 1)(x - 2)}$ $d - \frac{1}{(x + 1)(x - 2)}$
13) $\sec(\tan^{-1}x) =$
 $a \frac{1}{\sqrt{x^2 + 1}}$ $b \frac{x}{\sqrt{x^2 + 1}}$ $c \sqrt{x^2 + 1}$ $b \frac{\sqrt{x^2 + 1}}{x}$
14) $\lim_{x \to 0} \frac{\tan 5x}{3x} =$
 $a \frac{1}{3}$ $b 5$ $c \frac{3}{5}$ $d \frac{5}{3}$
15) If $f(x) = 2x + 7$, then $f^{-1}(x) =$
 $a \frac{x + 7}{2}$ $b \frac{x}{2} - 7$ $c \frac{x}{2} + 7$ $d \frac{x - 7}{2}$
16) $D^{(127)}(\cos x) =$
 $a \frac{\sin x}{(\sqrt{x^2 + x} - x)} =$
 $a \frac{1}{2}$ $b 1$ $c 0$ $d - \frac{1}{2}$
18) If $y = \sin^4(3x)$, then $y' =$
 $a \frac{1}{2} 2 b 1$ $c 0$ $d - \frac{1}{2}$
18) If $y = \sin^4(3x)$, then $y' =$
 $a \frac{12\sin^3(3x)\cos(3x)}{(3x)\cos(3x)}$ $b 4\sin^3(3x)\cos(3x)$

30)	The critical numbers of the function $f(x) = 2x^3 - 3x^2 - 12x + 15$ are
<i>a</i> 1,-2	b - 1, 2 $c 1, 2$ $d - 1, -2$
31)	The function $f(x) = 2x^3 - 3x^2 - 12x + 15$ is increasing on
$\boxed{a} (-\infty, -2)$	$\cup (-1,\infty) \boxed{b} (-\infty,-2) \cup (1,\infty) \boxed{c} (-\infty,-1) \cup (2,\infty) \boxed{d} (-\infty,1) \cup (2,\infty)$
32)	The function $f(x) = 2x^3 - 3x^2 - 12x + 15$ is decreasing on
a (-2,-1)	b (-2,1) c (1,2) d (-1,2)
33)	The function $f(x) = 2x^3 - 3x^2 - 12x + 15$ has a relative maximum at
<i>a</i> (1, 2)	b (-1,22) c (2,-5) d (-2,11)
34)	The function $f(x) = 2x^3 - 3x^2 - 12x + 15$ has a relative minimum at
<i>a</i> (1, 2)	b (-1,22) c (2,-5) d (-2,11)
35)	The graph of $f(x) = 2x^3 - 3x^2 - 12x + 15$ concave upward on
a $(-\infty, \frac{1}{2})$	$b (-\infty, -\frac{1}{2}) \qquad c (-\frac{1}{2}, \infty) \qquad d (\frac{1}{2}, \infty)$
36)	The graph of $f(x) = 2x^3 - 3x^2 - 12x + 15$ concave downward on
a $(-\infty, \frac{1}{2})$	$b (-\infty, -\frac{1}{2}) \qquad c (-\frac{1}{2}, \infty) \qquad d (\frac{1}{2}, \infty)$
37)	The function $f(x) = 2x^3 - 3x^2 - 12x + 15$ has an inflection at
a ($\frac{1}{2}$, 20)	b $(-\frac{1}{2}, 20)$ c $(\frac{1}{2}, \frac{17}{2})$ d $(-\frac{1}{2}, \frac{17}{2})$
38)	$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} =$
<i>a</i> 0	b does not exist $c 2$ $d \frac{1}{2}$
39)	The domain of $\frac{x+3}{\sqrt{4-x^2}}$ is
<i>a</i> [-2,2]	$b(-\infty,-2)\cup(2,\infty) \qquad c(-2,2) \qquad d(-\infty,-2]\cup[2,\infty)$
40)	The values in (-1,3) which makes $f(x) = x^2 - 5x + 7$ satisfied Mean
Value	e Theorem on $[-1,3]$ is
<u>a</u> –4	b 0 c 1 d 2

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$$C$$

1) $\lim_{x \to 5^+} \frac{x+1}{x-5} =$
 $a \to b \to c$ $b \to c$ $5 d -5$
2) $\lim_{x \to \infty} (\sqrt{x^2 + x - x}) =$
 $a \to b \to c$ $b \to c$ $c = 5 d -5$
2) $\lim_{x \to \infty} (\sqrt{x^2 + x - x}) =$
 $a \to b \to c$ $c = 5 d -5$
3) $y = -\ln(\cos x)$, then $y' =$
 $a \tan x$ $b = -\tan x$ $c = \cot x$ $d = -\cot x$
4) The absolute maximum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on [0,4] is
 $a = 2 b + 12 c - 7 d - 6$
5) The absolute minimum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on [0,4] is
 $a = 6 b + 2 c - 7 d - 6$
6) If $f(x) = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$
 $a x b = x^{-1}(x)$ and $g(x) = \tan^{-1}(x)$ and $g(x) = -3\pi$ and $g(x$

12) If
$$f(x) = 2x - 5$$
, then $f^{-1}(x) =$
 $\boxed{a} \frac{x+5}{2} = \boxed{b} \frac{x}{2} - 5 = \boxed{c} \frac{x-5}{2} = \boxed{d} \frac{x}{2} + 5$
13) The slope of the perpendicular line to the line $3y + 2x - 6 = 0$ is
 $\boxed{a} \frac{2}{3} = \boxed{b} - \frac{2}{3} = \boxed{c} - \frac{3}{2} = \boxed{d} \frac{3}{2}$
14) If the graph of the function $f(x) = 3^{\circ}$ is shifted a distance 2 units downward, then the new graph represented the graph of the function
 $\boxed{a} 3^{x+2} = \boxed{b} 3^{x} + 2 = \boxed{c} 3^{x-2} = \boxed{d} 3^{x} - 2$
15) If $y = \ln \frac{x+1}{x-2}$, then $y' =$
 $\boxed{a} \frac{1}{(x+1)(x-2)} = \boxed{b} - \frac{1}{(x+1)(x-2)} = \boxed{c} \frac{3}{(x+1)(x-2)} = \boxed{d} - \frac{3}{(x+1)(x-2)}$
16) $\lim_{x\to 0} \frac{\sin 5x}{3x} =$
 $\boxed{a} \frac{3}{5} = \boxed{b} \frac{5}{3} = \boxed{c} \frac{1}{3} = \boxed{d} 5$
17) If $y = (2x^2 + \sec x)^7$, then $y' =$
 $\boxed{a} 7(2x^2 + \sec x)^6 = (4x - \sec x \tan x) = \boxed{d} 28x (2x^2 + \sec x)^6 (4x + \sec x \tan x)$
 $\boxed{c} 7(2x^2 + \sec x)^6 (4x - \sec x \tan x) = \boxed{d} 28x (2x^2 + \sec x)^6$
18) If $f(x) = x^2$, then $f'(x) =$
 $\boxed{a} \lim_{x\to 0} \frac{(x+h)^2 + x^2}{h} = \boxed{b} \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$
 $\boxed{c} \lim_{h\to 0} \frac{(x+h)^2 + x^2}{h} = \boxed{d} \lim_{x\to 0} \frac{(x+h)^2 - x^2}{h}$
 $\boxed{19} \frac{7\pi}{6} \operatorname{rad} =$
 $\boxed{a} 120^9 = \boxed{b} 150^9 = \boxed{c} 270^9 = \boxed{d} 210^9$

21) The values in (-1,3) which makes
$$f(x) = x^2 - 5x + 7$$
 satisfied Mean Value Theorem on [-1,3] is
 $\boxed{a} -4 \boxed{b} 1 \boxed{c} 0 \boxed{d} 2$
22) The function $f(x) = \frac{x+1}{x^2-9}$ is continuous on
 $\boxed{a} \left\{ \pm 3 \right\} \boxed{b} \begin{bmatrix} -3,3 \end{bmatrix} \boxed{c} (-\infty, -3) \cup (3, \infty) \boxed{d} \left\{ x \in \mathbb{R} : x \neq \pm 3 \right\}$
23) $\cos(\tan^{-1}x) =$
 $\boxed{A} \frac{1}{\sqrt{x^2+1}} \boxed{B} \frac{x}{\sqrt{x^2+1}} \boxed{C} \sqrt{x^2+1} \boxed{D} \frac{\sqrt{x^2+1}}{x}$
24) The distance between the points (-1,2) and (2,-1) is
 $\boxed{a} 3\sqrt{2} \qquad \boxed{b} 2\sqrt{3} \qquad \boxed{c} 9 \qquad \boxed{d} 3$
25) If $-7 < 2x + 3 \le 5$, then $x =$
 $\boxed{a} (-5,1) \qquad \boxed{b} (-5,1] \qquad \boxed{c} [-5,1] \qquad \boxed{d} [-5,1]$
26) If $y = e^{2x}$, then $y^{(6)} =$
 $\boxed{a} 128e^{2x} \qquad \boxed{b} 16e^{2x} \qquad \boxed{c} 64e^{2x} \qquad \boxed{d} 32e^{2x}$
27) If $y = \sin^3(4x)$, then $y' =$
 $\boxed{a} 4\cos^3(4x) \qquad \boxed{b} 3\sin^2(4x)\cos(4x)$
 $\boxed{c} 4\sin^3(4x) + 12\sin^2x \cos x \qquad \boxed{d} 12\sin^2(4x)\cos(4x)$
 $28)$ The domain of $\frac{x+3}{\sqrt{x^2-4}}$ is
 $\boxed{a} [-2,2] \qquad \boxed{b} (-\infty, -2) \cup (2, \infty) \qquad \boxed{c} (-2,2) \qquad \boxed{d} (-\infty, -2] \cup [2, \infty)$
29) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} =$
 $\boxed{a} -6 \qquad \boxed{b} 6 \qquad \boxed{c} \qquad \infty \qquad \boxed{d} 0$
30) If $y = \sqrt{3x^2 + 6x}$, then $y' =$
 $\boxed{a} \frac{x+6}{\sqrt{3x^2+6x}} \qquad \boxed{b} \frac{6(x+1)}{\sqrt{3x^2+6x}}$

31) If $y = \tan^{-1}\left(\frac{3x}{2}\right)$, then $y' =$	
$\boxed{a} - \frac{4}{4+9x^2} \qquad \boxed{b} \frac{6}{4+9x^2} \qquad \boxed{c} - \frac{6}{4+9x^2} \qquad \boxed{d} \frac{4}{4+9x^2}$	
32) The critical numbers of the function $f(x) = 2x^3 - 3x^2 - 12x + 16$ are	
a 1,-2 $b -1,2$ $c 1,2$ $d -1,-2$	
33) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ is increasing on	
$\boxed{a} (-\infty, -2) \cup (-1, \infty) \boxed{b} (-\infty, -2) \cup (1, \infty) \boxed{c} (-\infty, -1) \cup (2, \infty) \boxed{d} (-\infty, 1) \cup (2, \infty)$	
34) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ is decreasing on	
a (-2,-1) b (-2,1) c (1,2) d (-1,2)	
35) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ has a relative maximum at	
a (1,3) b (-1,-23) c (2,-4) d (-2,12)	
36) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ has a relative minimum at	
a (1,3) b (-1,-23) c (2,-4) d (-2,12)	
37) The graph of $f(x) = 2x^3 - 3x^2 - 12x + 16$ concave upward on	
a $(-\infty, \frac{1}{2})$ b $(-\infty, -\frac{1}{2})$ c $(-\frac{1}{2}, \infty)$ d $(\frac{1}{2}, \infty)$	
38) The graph of $f(x) = 2x^3 - 3x^2 - 12x + 16$ concave downward on	
$\boxed{a} (-\infty, \frac{1}{2}) \qquad \boxed{b} (-\infty, -\frac{1}{2}) \qquad \boxed{c} (-\frac{1}{2}, \infty) \qquad \boxed{d} (\frac{1}{2}, \infty)$	
39) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ has an inflection at	
$a (\frac{1}{2}, 21) \qquad b (-\frac{1}{2}, 21) \qquad c (\frac{1}{2}, \frac{19}{2}) \qquad d (-\frac{1}{2}, \frac{19}{2})$	
40) If $y = \log_5(x^3 - 2\csc x)$, then $y' =$	
$ = 3x^2 + 2\csc x \cot x $ $ = 3x^2 + 2\csc x \cot x $	
$\frac{a}{x^3 - 2\csc x \ln 5}$ $\frac{b}{(x^3 - 2\csc x) \ln 5}$	
$\int \frac{3x^2 + 2\csc x \cot x}{d} \qquad \qquad$	
$x^{3} - 2\csc x$ $(x^{3} - 2\csc x) \ln 5$	

King Abdul Aziz University Faculty of Sciences Mathematics Department
Math 110 Final Test Fall 2013 (40 Marks) Time 120 m
Student Name: ID
1) If
$$y = -\ln(\sin x)$$
, then $y' =$
 \boxed{a} tan x \boxed{b} -tan x \boxed{c} cot x \boxed{d} -cot x
2) If $y = x^{+}$, then $y' =$
 \boxed{a} 1+ $\ln x$ \boxed{b} x^{+} \boxed{c} $x^{+} \ln x$ \boxed{d} $x^{+}(1+\ln x)$
3) If $y = \cot^{-1}\left(\frac{3x}{2}\right)$, then $y' =$
 $\boxed{a} - \frac{4}{4+9x^{2}}$ \boxed{b} $\frac{6}{4+9x^{2}}$ $\boxed{c} - \frac{6}{4+9x^{2}}$ \boxed{d} $\frac{4}{4+9x^{2}}$
4) If $y = \sin^{4}(3x)$, then $y' =$
 \boxed{a} 4sin³(3x)cos(3x) \boxed{b} 12sin³(3x)cos(3x)
 \boxed{c} 3cos²(3x) \boxed{d} 3sin⁴(3x) + 12sin³ x cos x
5) The tangent line equation to the curve $y = \frac{2x}{x-1}$ at the point (0,0) is
 \boxed{a} $y = -2x - 1$ \boxed{b} $y = -2x$
 \boxed{c} $y = 2x$ \boxed{d} $y = 2x + 1$
6) If $y^{2} - 2 = 3xy - x^{2}$, then $y' =$
 \boxed{a} $\frac{3^{2}}{2xy}$ \boxed{b} $\frac{2x}{y}$ \boxed{c} $\frac{3y - 2x}{2y - 3x}$ \boxed{d} $\frac{2x + y}{3x - 2y}$
7) If $y = 3^{3}$ tan x , then $y' =$
 \boxed{a} $\frac{3^{5}}{\ln 3 \tan x - 3^{5} \sec^{2} x$ \boxed{b} $3^{5} \tan x - 3^{5} \sec^{2} x$
8) If $y = (2x^{2} + \csc x)^{7}$, then $y' =$
 \boxed{a} $28x (2x^{2} + \csc x)^{7}$, then $y' =$
 \boxed{a} $28x (2x^{2} + \csc x)^{7}$, then $y' =$
 \boxed{a} $28x (2x^{2} + \csc x)^{7}$, then $y' =$
 \boxed{a} $\frac{28x (2x^{2} + \csc x)^{7}}{(4x + \csc x \cot x)}$ \boxed{d} $7(2x^{2} + \csc x)^{6}(4x - \csc x \cot x)$
9) The slope of the perpendicular line to the line $2y - 3x - 6 = 0$ is
 \boxed{a} $\frac{2}{3}$ $\boxed{b} - \frac{2}{3}$ $\boxed{c} - \frac{3}{2}$ \boxed{d} $\frac{3}{2}$
30)
The critical numbers of the function
$$f(x) = 2x^3 + 3x^2 - 12x + 16$$
 are

a)
1, -2
b)
-1, 2
c)
1, 2
a)
-1, -2

31)
The function $f(x) = 2x^3 + 3x^2 - 12x + 16$ is increasing on
a)
(- ∞ , -2) \cup (-1, ∞)
b)
(- ∞ , -2) \cup (1, ∞)
c)
(- ∞ , -1) \cup (2, ∞)
d)
(- ∞ , 1) \cup (2, ∞)

32)
The function $f(x) = 2x^3 + 3x^2 - 12x + 16$ is decreasing on
(- ∞ , -1) \cup (2, ∞)
b)
(- ∞ , -1) \cup (2, ∞)
d)
(- ∞ , -1) \cup (2, ∞)

33)
The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ is a creative maximum at
a)
(1.9)
b)
(-1, 29)
c)
(-2, 20)
d)
(-2, 36)

34)
The function $f(x) = 2x^3 + 3x^2 - 12x + 16$ has a relative minimum at
a)
(1.9)
b)
(-1, 29)
c)
(2, 20)
d)
(-2, 36)

35)
The graph of $f(x) = 2x^3 + 3x^2 - 12x + 16$ has a relative minimum at
a)
(1.9)
b)
(-1, 29)
c)
(-2, 20)
d)
(-2, 36)

35)
The graph of $f(x) = 2x^3 + 3x^2 - 12x + 16$ has an inflection at
a)
(1, $(-\infty, \frac{1}{2}))$
(2, $(-\frac{1}{2}, \infty)$
(36)
The function $f(x) = 2x^3 +$