Workshop Solutions to Sections 2.1 and 2.2 (1.1 & 1.2)

1) Find the domain of the function $f(x) = 9 - x^2$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$ **Note:** The domain of any polynomial is $\mathbb{R}$.	2) Find the range of the function $f(x) = 9 - x^2$. Solution: $R_f = (-\infty, 9]$
3) Find the domain of the function $f(x) = 6 - 2x$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	4) Find the range of the function $f(x) = 6 - 2x$. Solution: Since $f(x)$ is a polynomial of degree one ($i.e.$ is of an odd degree), then $R_f = \mathbb{R} = (-\infty, \infty)$
5) Find the domain of the function $f(x) = x^2 - 2x - 3$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
7) Find the domain of the function $f(x) = 5$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	8) Find the range of the function $f(x) = 5$. Solution: $R_f = \{5\}$
9) Find the domain of the function $f(x) = \|x - 1\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$ **Note:** The domain of an absolute value of any polynomial is $\mathbb{R}$.	10) Find the domain of the function $f(x) = \|x + 5\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
11) Find the domain of the function $f(x) = \|x\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	12) Find the range of the function $f(x) = \|x\|$. Solution: $R_f = [0, \infty)$ **Note:** The range of an absolute value of any polynomial is always $[0, \infty)$.
13) Find the domain of the function $f(x) = \|3x - 6\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	14) Find the domain of the function $f(x) = \|9 - 3x\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
15) Find the domain of the function $f(x) = \frac{x+2}{x-3}$ Solution: $f(x)$ is defined when $x - 3 \neq 0 \implies x \neq 3$. So, $D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$	16) Find the domain of the function $f(x) = \frac{x-2}{x+3}$ Solution: $f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So, $D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$

17) Find the domain of the function $$f(x) = \frac{x+2}{x^2-9}$$ Solution: $f(x)$ is defined when $x^2 - 9 \neq 0 \Longrightarrow x^2 \neq 9 \Longrightarrow x \neq \pm 3$. So, $$D_f = \mathbb{R} \setminus \{-3,3\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$$	18) Find the domain of the function $$f(x) = \frac{x+2}{x^2-5x+6}$$ Solution: $f(x)$ is defined when $x^2 - 5x + 6 \neq 0$ $\Longrightarrow (x-2)(x-3) \neq 0 \Longrightarrow x \neq 2$ or $x \neq 3$. So, $$D_f = \mathbb{R} \setminus \{2,3\} = (-\infty,2) \cup (2,3) \cup (3,\infty)$$
19) Find the domain of the function $$f(x) = \frac{x+2}{x^2-x-6}$$ Solution: $f(x)$ is defined when $x^2 - x - 6 \neq 0$ $\Longrightarrow (x+2)(x-3) \neq 0 \Longrightarrow x \neq -2$ or $x \neq 3$. So, $$D_f = \mathbb{R} \setminus \{-2,3\} = (-\infty,-2) \cup (-2,3) \cup (3,\infty)$$	20) Find the domain of the function $$f(x) = \frac{x+2}{x^2+9}$$ Solution: $f(x)$ is defined when $x^2 + 9 \neq 0$ but for any value x the denominator $x^2 + 9$ cannot be 0. So, $$D_f = \mathbb{R} = (-\infty,\infty)$$
21) Find the domain of the function $$f(x) = \sqrt[3]{x-3}$$ Solution: $$D_f = \mathbb{R} = (-\infty,\infty)$$ **Note:** The domain of an odd root of any polynomial is $\mathbb{R}$.	22) Find the domain of the function $$f(x) = \sqrt{x-3}$$ Solution: $f(x)$ is defined when $x - 3 \geq 0 \Longrightarrow x \geq 3$ because $f(x)$ is an even root. So, $$D_f = [3,\infty)$$
23) Find the domain of the function $$f(x) = \sqrt{3-x}$$ Solution: $f(x)$ is defined when $3 - x \geq 0 \Longrightarrow -x \geq -3 \Longrightarrow x \leq 3$ because $f(x)$ is an even root. So, $$D_f = (-\infty,3]$$	24) Find the domain of the function $$f(x) = \sqrt{x+3}$$ Solution: $f(x)$ is defined when $x + 3 \geq 0 \Longrightarrow x \geq -3$ because $f(x)$ is an even root. So, $$D_f = [-3,\infty)$$
25) Find the domain of the function $$f(x) = \sqrt{-x}$$ Solution: $f(x)$ is defined when $-x \geq 0 \Longrightarrow x \leq 0$ because $f(x)$ is an even root. So, $$D_f = (-\infty,0]$$	26) Find the range of the function $$f(x) = \sqrt{-x}$$ Solution: $$R_f = [0,\infty)$$ **Note:** The range of an even root is always ≥ 0.
27) Find the domain of the function $$f(x) = \sqrt{9-x^2}$$ Solution: $f(x)$ is defined when $9 - x^2 \geq 0 \Longrightarrow -x^2 \geq -9 \Longrightarrow$ $x^2 \leq 9 \Longrightarrow \sqrt{x^2} \leq \sqrt{9} \Longrightarrow \|x\| \leq 3 \Longrightarrow -3 \leq x \leq 3$. So, $$D_f = [-3,3]$$	28) Find the domain of the function $$f(x) = \frac{x+2}{\sqrt{x-3}}$$ Solution: $f(x)$ is defined when $x - 3 > 0 \Longrightarrow x > 3$. So, $$D_f = (3,\infty)$$
29) Find the domain of the function $$f(x) = \frac{x+2}{\sqrt{9-x^2}}$$ Solution: $f(x)$ is defined when $9 - x^2 > 0 \Longrightarrow -x^2 > -9$ $\Longrightarrow x^2 < 9 \Longrightarrow \sqrt{x^2} < \sqrt{9} \Longrightarrow \|x\| < 3 \Longrightarrow -3 < x < 3$. So, $$D_f = (-3,3)$$	30) Find the domain of the function $$f(x) = \sqrt{x^2-9}$$ Solution: $f(x)$ is defined when $x^2 - 9 \geq 0 \Longrightarrow x^2 \geq 9$ $\Longrightarrow \sqrt{x^2} \geq \sqrt{9} \Longrightarrow \|x\| \geq 3 \Longrightarrow x \geq 3$ or $x \leq -3$. So, $$D_f = (-\infty,-3] \cup [3,\infty)$$

31) Find the range of the function

$$f(x) = \sqrt{x^2 - 9}$$

Solution:

$$R_f = [0, \infty)$$

32) Find the domain of the function

$$f(x) = \frac{x + 2}{\sqrt{x^2 - 9}}$$

Solution:

$f(x)$ is defined when $x^2 - 9 > 0 \implies x^2 > 9$

$\implies \sqrt{x^2} > \sqrt{9} \implies |x| > 3 \implies x > 3 \text{ or } x < -3$.

So,

$$D_f = (-\infty, -3) \cup (3, \infty)$$

33) Find the domain of the function

$$f(x) = \sqrt{9 + x^2}$$

Solution:

$f(x)$ is defined when $9 + x^2 \geq 0$ but it is always true for any value x . So,

$$D_f = \mathbb{R}$$

34) Find the domain of the function

$$f(x) = \sqrt[4]{x^2 - 25}$$

Solution:

$f(x)$ is defined when $x^2 - 25 \geq 0 \implies x^2 \geq 25$

$\implies \sqrt{x^2} \geq \sqrt{25} \implies |x| \geq 5 \implies x \geq 5 \text{ or } x \leq -5$.

So,

$$D_f = (-\infty, -5] \cup [5, \infty)$$

35) Find the domain of the function

$$f(x) = \sqrt[6]{16 - x^2}$$

Solution:

$f(x)$ is defined when $16 - x^2 \geq 0 \implies -x^2 \geq -16 \implies$

$x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16} \implies |x| \leq 4 \implies -4 \leq x \leq 4$.

So,

$$D_f = [-4,4]$$

36) Find the range of the function

$$f(x) = \sqrt{16 - x^2}$$

Solution:

We know that $f(x)$ is defined when $16 - x^2 \geq 0$

$\implies -x^2 \geq -16 \implies x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16}$

$\implies |x| \leq 4 \implies -4 \leq x \leq 4$. So,

$$D_f = [-4,4]$$

Using D_f we find the outputs vary from 0 to 4 . Hence,

$$R_f = [0,4]$$

37) Find the domain of the function

$$f(x) = \frac{x + |x|}{x}$$

Solution:

$f(x)$ is defined when $x \neq 0$. So,

$$D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$$

38) Find the domain of the function

$$f(x) = \begin{cases} -\dfrac{1}{x} , & x < 0 \\ x , & x \geq 0 \end{cases}$$

Solution:

It is clear from the definition of the function $f(x)$ that

$$D_f = \mathbb{R} = (-\infty, \infty)$$

39) Find the domain of the function

$$f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$$

Solution:

$f(x)$ is defined when

1- $x \geq 0 \quad \implies \quad D_{\sqrt{x}} = [0, \infty)$

2- $x^2 + 1 > 0$ but this is always true for all x

$\implies D_{\sqrt{x^2+1}} = \mathbb{R}$.

Hence,

$$D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2+1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$$

40) Find the domain of the function

$$f(x) = \sqrt{x - 1} + \sqrt{x + 3}$$

Solution:

$f(x)$ is defined when

1- $x - 1 \geq 0 \implies x \geq 1 \implies D_{\sqrt{x-1}} = [1, \infty)$

2- $x + 3 \geq 0 \implies x \geq -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$

Hence,

$$D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$$

41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function.

42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function.

43) The function $f(x) = -3x^2 + 7$ is a quadratic function.

44) The function $f(x) = 2x + 3$ is a linear function.

45) The function $f(x) = x^7$ is a power function.

46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function.

47) The function $f(x) = \frac{x-3}{x+2}$ is a rational function and we can say it is an algebraic function as well.

48) The function $f(x) = \sin x$ is a trigonometric function.

49) The function $f(x) = e^x$ is a natural exponential function.

50) The function $f(x) = 3^x$ is a general exponential function.

51) The function $f(x) = x^2 + \sqrt{x-2}$ is an algebraic function.

52) The function $f(x) = -3$ is a constant function.

53) The function $f(x) = \log_3 x$ is a general logarithmic function.

54) The function $f(x) = \ln x$ is a natural logarithmic function.

55) The function $f(x) = 3x^4 + x^2 + 1$ is

<u>Solution:</u>

$$f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$$

Hence,

$f(x)$ is an even function.

56) The function $f(x) = 9 - x^2$ is

<u>Solution:</u>

$$f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$$

Hence,

$f(x)$ is an even function.

57) The function $f(x) = x^5 - x$ is

<u>Solution:</u>

$$f(-x) = (-x)^5 - (-x) = -x^5 + x$$
$$= -(x^5 - x) = -f(x)$$

Hence,

$f(x)$ is an odd function.

58) The function $f(x) = 2 - \sqrt[5]{x}$ is

<u>Solution:</u>

$$f(-x) = 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} = 2 + \sqrt[5]{x}$$
$$= -\left(-2 - \sqrt[5]{x}\right)$$

Hence,

$f(x)$ is neither even nor odd.

59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is

<u>Solution:</u>

$$f(-x) = 3(-x) + \frac{2}{\sqrt{(-x)^2 + 9}} = -3x + \frac{2}{\sqrt{x^2+9}}$$
$$= -\left(3x - \frac{2}{\sqrt{x^2+9}}\right)$$

Hence,

$f(x)$ is neither even nor odd.

60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is

<u>Solution:</u>

$$f(-x) = \frac{3}{\sqrt{(-x)^2+9}} = \frac{3}{\sqrt{x^2+9}} = f(x)$$

Hence,

$f(x)$ is an even function.

61) The function $f(x) = \sqrt{4 + x^2}$ is

<u>Solution:</u>

$$f(-x) = \sqrt{4 + (-x)^2} = \sqrt{4 + x^2} = f(x)$$

Hence,

$f(x)$ is an even function.

62) The function $f(x) = 3$ is

<u>Solution:</u>

Since the graph of the constant function 3 is symmetric about the $y - axis$, then

$f(x)$ is an even function.

63) The function $f(x) = \frac{9-x^2}{x-2}$ is

<u>Solution:</u>

$$f(-x) = \frac{9 - (-x)^2}{(-x) - 2} = \frac{9 - x^2}{-x - 2}$$
$$= -\left(\frac{9 - x^2}{x + 2}\right)$$

Hence,

$f(x)$ is neither even nor odd.

64) The function $f(x) = \frac{x^2-4}{x^2+1}$ is

<u>Solution:</u>

$$f(-x) = \frac{(-x)^2 - 4}{(-x)^2 + 1} = \frac{x^2 - 4}{x^2 + 1} = f(x)$$

Hence,

$f(x)$ is an even function.

65) The function $f(x) = 3|x|$ is

<u>Solution:</u>

$$f(-x) = 3|(-x)| = 3|x| = f(x)$$

Hence,

$f(x)$ is an even function.

66) The function $f(x) = x^{-2}$ is

<u>Solution:</u>

$$f(x) = x^{-2} = \frac{1}{x^2}$$
$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$$

Hence, $f(x)$ is an even function.

67) The function $f(x) = x^3 - 2x + 5$ is <u>Solution:</u> $f(-x) = (-x)^3 - 2(-x) + 5 \; = -x^3 + 2x + 5$ $= -(x^3 - 2x - 5)$ Hence, $f(x)$ is neither even nor odd.	68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is <u>Solution:</u> $f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) \; = -\sqrt[3]{x^5} + x^3 - x$ $= -\left(\sqrt[3]{x^5} - x^3 + x\right) = -f(x)$ Hence, $f(x)$ is an odd function.
69) The function $f(x) = 7$ is <u>Solution:</u> Since the graph of the constant function 7 is symmetric about the $y - axis$, then $f(x)$ is an even function.	70) The function $f(x) = \frac{x^3-4}{x^3+1}$ is <u>Solution:</u> $f(-x) = \dfrac{(-x)^3 - 4}{(-x)^3 + 1} = \dfrac{-x^3 - 4}{-x^3 + 1} = -\dfrac{x^3 + 4}{-x^3 + 1}$ Hence, $f(x)$ is neither even nor odd.
71) The function $f(x) = \frac{x^2-1}{x^3+3}$ is <u>Solution:</u> $f(-x) = \dfrac{(-x)^2 - 1}{(-x)^3 + 3} = \dfrac{x^2 - 1}{-x^3 + 3} = -\dfrac{x^2 - 1}{x^3 - 3}$ Hence, $f(x)$ is neither even nor odd.	72) The function $f(x) = x^6 - 4x^2 + 1$ is <u>Solution:</u> $f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.
73) The function $f(x) = x^2$ is increasing on $(0, \infty)$.	74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$.
75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$.	76) The function $f(x) = x^3$ is not decreasing at all.
77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$.	78) The function $f(x) = \sqrt{x}$ is not decreasing at all.
79) The function $f(x) = \frac{1}{x}$ is not increasing at all.	80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty) \setminus \{0\}$

Workshop Solutions to Sections 2.3 and 2.4 (1.3 & app D)

1) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f+g)(x) =$ Solution: $(f+g)(x) = x^2 + \sqrt{4-x}$	2) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f+g} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \Leftrightarrow x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{f+g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
3) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f-g)(x) =$ Solution: $(f-g)(x) = x^2 - \sqrt{4-x}$	4) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f-g} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \Leftrightarrow x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{f-g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
5) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(fg)(x) =$ Solution: $(fg)(x) = x^2\sqrt{4-x}$	6) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{fg} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \Leftrightarrow x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{fg} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
7) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f \circ g)(x) =$ Solution: $(f \circ g)(x) = f(g(x))$ $= f(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4 - x$	8) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f \circ g} =$ Solution: $(f \circ g)(x) = f(g(x))$ $= f(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4 - x$ $D_g = (-\infty, 4]$ $D_{f(g(x))} = \mathbb{R}$ $D_{f \circ g} = D_g \cap D_{f(g(x))} = (-\infty, 4] \cap \mathbb{R} = (-\infty, 4]$
9) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{4-x^2}$	10) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{g \circ f} =$ Solution: $(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{4-x^2}$ $D_f = \mathbb{R}$ $D_{g(f(x))} = [-2,2]$ $D_{g \circ f} = D_f \cap D_{g(f(x))} = \mathbb{R} \cap [-2,2] = [-2,2]$
11) If $f(x) = x^2$, then $(f \circ f)(x) =$ Solution: $(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$	12) If $f(x) = x^2$, then $D_{f \circ f} =$ Solution: $(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$ $D_f = \mathbb{R}$ $D_{f(f(x))} = \mathbb{R}$ $D_{f \circ f} = D_f \cap D_{f(f(x))} = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

13) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{f}{g}\right)(x) =$

Solution:

$$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$$

14) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{f}{g}} =$

Solution:

$$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$$

$D_f = \mathbb{R}$

$g(x)$ is defined when $4 - x \geq 0 \Leftrightarrow x \leq 4$. Thus,

$D_g = (-\infty, 4]$

$$D_{\frac{f}{g}} = \{x \in D_f \cap D_g | g(x) \neq 0\}$$

$$= \mathbb{R} \cap (-\infty, 4) = (-\infty, 4)$$

15) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{g}{f}\right)(x) =$

Solution:

$$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$$

16) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{g}{f}} =$

Solution:

$$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$$

$D_f = \mathbb{R}$

$g(x)$ is defined when $4 - x \geq 0 \Leftrightarrow x \leq 4$. Thus,

$D_g = (-\infty, 4]$

$$D_{\frac{g}{f}} = \{x \in D_f \cap D_g | f(x) \neq 0\}$$

$$= \mathbb{R} \setminus \{0\} \cap (-\infty, 4] = (-\infty, 0) \cup (0, 4]$$

17) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f+g)(x) =$

Solution:

$$(f+g)(x) = (9 - x^2) + (10) = 9 - x^2 + 10$$
$$= 19 - x^2$$

18) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f-g)(x) =$

Solution:

$$(f-g)(x) = (9 - x^2) - (10) = 9 - x^2 - 10$$
$$= -x^2 - 1$$

19) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g-f)(x) =$

Solution:

$$(g-f)(x) = (10) - (9 - x^2) = 10 - 9 + x^2$$
$$= 1 + x^2$$

20) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(fg)(x) =$

Solution:

$$(fg)(x) = (9 - x^2)(10) = 90 - 10x^2$$

21) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f \circ g)(x) =$

Solution:

$$(f \circ g)(x) = f(g(x)) = f(10)$$
$$= 9 - 10^2 = 9 - 100 = -91$$

22) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g \circ f)(x) =$

Solution:

$$(g \circ f)(x) = g(f(x)) = g(9 - x^2) = 10$$

23) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f \circ f)(x) =$

Solution:

$$(f \circ f)(x) = f(f(x)) = f(9 - x^2)$$
$$= 9 - (9 - x^2)^2$$

24) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g \circ g)(x) =$

Solution:

$$(g \circ g)(x) = g(g(x)) = g(10) = 10$$

25) If $f(x) = 9 - x^2$, $g(x) = \sin x$ and $h(x) = 3x + 2$, then $(f \circ g \circ h)(x) =$

Solution:

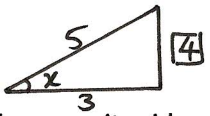
$$(f \circ g \circ h)(x) = f\big(g(h(x))\big)$$
$$= f\big(g(3x+2)\big)$$
$$= f(\sin(3x+2))$$
$$= 9 - (\sin(3x+2))^2$$
$$= 9 - sin^2(3x+2)$$

26) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $(f+g)(x) =$

Solution:

$$(f+g)(x) = \sqrt{25 + x^2} + x^3$$

27) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $(f-g)(x)=$ Solution: $(f-g)(x)=\sqrt{25+x^2}-x^3$	28) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $(fg)(x)=$ Solution: $(fg)(x)=x^3\sqrt{25+x^2}$
29) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $\left(\frac{f}{g}\right)(x)=$ Solution: $\left(\frac{f}{g}\right)(x)=\frac{\sqrt{25+x^2}}{x^3}$	30) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))=f(x^3)=\sqrt{25+(x^3)^2}$ $=\sqrt{25+x^6}$
31) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g\left(\sqrt{25+x^2}\right)=\left(\sqrt{25+x^2}\right)^3$ $=\sqrt{(25+x^2)^3}$	32) If $f(x)=\sqrt{x}$ and $g(x)=x-2$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))=f(x-2)=\sqrt{x-2}$
33) If $f(x)=\sqrt{x}$ and $g(x)=x-2$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g(\sqrt{x})=\sqrt{x}-2$	34) If $f(x)=\sqrt{x}$ and $g(x)=x-2$, then $(g\circ g)(x)=$ Solution: $(g\circ g)(x)=g(g(x))=g(x-2)=(x-2)-2$ $=x-2-2=x-4$
35) If $f(x)=\sqrt{x}$ and $g(x)=x-2$, then $(fg)(x)=$ Solution: $(fg)(x)=(\sqrt{x})(x-2)=(x-2)\sqrt{x}$	36) If $f(x)=\sin 5x$ and $g(x)=x^2+3$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))=f(x^2+3)=\sin 5(x^2+3)$
37) If $f(x)=\sin 5x$ and $g(x)=x^2+3$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g(\sin 5x)=(\sin 5x)^2+3$ $=sin^2 5x+3$	38) If $f(x)=\sin 5x$ and $g(x)=x^2+3$, then $(fg)(x)=$ Solution: $(fg)(x)=(\sin 5x)(x^2+3)=(x^2+3)\sin 5x$
39) If $f(x)=\sqrt{x}$ and $g(x)=\cos x$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g(\sqrt{x})=\cos\sqrt{x}$	40) If $f(x)=x+\frac{1}{x}$ and $g(x)=1-x^2$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))=f(1-x^2)=(1-x^2)+\frac{1}{1-x^2}$
41) If $f(x)=x+\frac{1}{x}$ and $g(x)=1-x^2$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g\left(x+\frac{1}{x}\right)=1-\left(x+\frac{1}{x}\right)^2$	42) If $f(x)=x+\frac{1}{x}$ and $g(x)=1-x^2$, then $(fg)(x)=$ Solution: $(fg)(x)=\left(x+\frac{1}{x}\right)(1-x^2)$
43) If the graph of the function $f(x)=x^2$ is shifted a distance 2 units upwards, then the new graph represented the graph of the function is Solution: x^2+2	44) If the graph of the function $f(x)=x^2$ is shifted a distance 2 units downwards, then the new graph represented the graph of the function is Solution: x^2-2
45) If the graph of the function $f(x)=x^2$ is shifted a distance 2 units to the right, then the new graph represented the graph of the function is Solution: $(x-2)^2=x^2-4x+4$	46) If the graph of the function $f(x)=x^2$ is shifted a distance 2 units to the left, then the new graph represented the graph of the function is Solution: $(x+2)^2=x^2+4x+4$

47) If the graph of the function $f(x) = \cos x$ is stretched vertically by a factor of 2 , then the new graph represented the graph of the function is Solution: $2\cos x$	48) If the graph of the function $f(x) = \cos x$ is compressed vertically by a factor of $\frac{1}{2}$, then the new graph represented the graph of the function is Solution: $\frac{1}{2}\cos x$
49) If the graph of the function $f(x) = \cos x$ is compressed horizontally by a factor of 2 , then the new graph represented the graph of the function is Solution: $\cos 2x$	50) If the graph of the function $f(x) = \cos x$ is stretched horizontally by a factor of $\frac{1}{2}$, then the new graph represented the graph of the function is Solution: $\cos\frac{x}{2}$
51) The graph of the function $f(x) = \sqrt{x}$ is reflected about the $x - axis$ if Solution: $f(x) = -\sqrt{x}$	52) The graph of the function $f(x) = \sqrt{x}$ is reflected about the $y - axis$ if Solution: $f(x) = \sqrt{-x}$
53) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units upwards , then the new graph represented the graph of the function is Solution: $e^x + 2$	54) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units downwards , then the new graph represented the graph of the function is Solution: $e^x - 2$
55) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units to the right , then the new graph represented the graph of the function is Solution: e^{x-2}	56) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units to the left , then the new graph represented the graph of the function is Solution: e^{x+2}
57) $\frac{2\pi}{3}$ rad $= \frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ$	58) $\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$
59) $\frac{7\pi}{6}$ rad $= \frac{7\pi}{6} \times \frac{180^\circ}{\pi} = 210^\circ$	60) $\frac{3\pi}{2}$ rad $= \frac{3\pi}{2} \times \frac{180^\circ}{\pi} = 270^\circ$
61) $120^\circ = 120 \times \frac{\pi}{180^\circ} = \frac{2\pi}{3}$ rad	62) $270^\circ = 270 \times \frac{\pi}{180^\circ} = \frac{3\pi}{2}$ rad
63) $\frac{5\pi}{12}$ rad $= \frac{5\pi}{12} \times \frac{180^\circ}{\pi} = 75^\circ$	64) $\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$ (Repeated)
65) $150^\circ = 150 \times \frac{\pi}{180^\circ} = \frac{5\pi}{6}$ rad	66) $210^\circ = 210 \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$ rad
67) $\frac{1}{\sec x} = \cos x$	68) $\frac{1}{\csc x} = \sin x$
69) $\frac{1}{\cot x} = \tan x$	70) $\frac{\sin x}{\cos x} = \tan x$
71) $\frac{\cos x}{\sin x} = \cot x$	
72) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\cot x =$ Solution:  $\cos x = \frac{3}{5} = \frac{adj}{hyp}$ Now, we should find the length of the opposite side using the Pythagorean Theorem, so $\lvert\text{opposite}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ $\therefore \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = \frac{3}{4}$	73) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\tan x =$ Solution: $\cos x = \frac{3}{5} = \frac{adj}{hyp}$ Now, we should find the length of the opposite side using the Pythagorean Theorem, so $\lvert\text{opposite}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ $\therefore \tan x = \frac{1}{\cot x} = \frac{opp}{adj} = \frac{4}{3}$

74) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\sin x =$

Solution:

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$$\therefore \sin x = \frac{opp}{hyp} = \frac{4}{5}$$

75) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\csc x =$

Solution:

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$$\therefore \csc x = \frac{1}{\sin x} = \frac{hyp}{opp} = \frac{5}{4}$$

76) $\sin\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\sin\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$\sin\left(\frac{5\pi}{6}\right) = \sin(150^\circ) = \sin(180^\circ - 30^\circ) = \sin(30^\circ) =$

$\sin\pi6=12$

77) $\cos\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\cos\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\cos\left(\frac{5\pi}{6}\right) = \cos(150^\circ) = \cos(180^\circ - 30^\circ)$$

$$= -\cos(30^\circ) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

78) $\tan\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\tan\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\tan\left(\frac{5\pi}{6}\right) = \tan(150^\circ) = \tan(180^\circ - 30^\circ)$$

$$= -\tan(30^\circ) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

79) $\cot\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

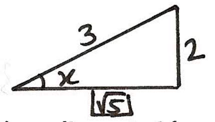
So, we deduce now that $\cot\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\cot\left(\frac{5\pi}{6}\right) = \cot(150^\circ) = \cot(180^\circ - 30^\circ)$$

$$= -\cot(30^\circ) = -\cot\left(\frac{\pi}{6}\right) = -\sqrt{3}$$

80) If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $\sec x =$

Solution:



$$\sin x = \frac{2}{3} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - 2^2} = \sqrt{9 - 4} = \sqrt{5}$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{hyp}{adj} = \frac{3}{\sqrt{5}}$$

81) If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $\csc x =$

Solution:

$$\sin x = \frac{2}{3} = \frac{opp}{hyp}$$

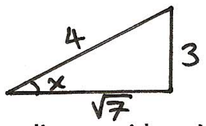
Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - 2^2} = \sqrt{9 - 4} = \sqrt{5}$

$$\therefore \csc x = \frac{1}{\sin x} = \frac{hyp}{opp} = \frac{3}{2}$$

82) If $\sin x = \frac{3}{4}$ and $0 < x < \frac{\pi}{2}$, then $\cos x =$

Solution:



$$\sin x = \frac{3}{4} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$

$$\therefore \cos x = \frac{adj}{hyp} = \frac{\sqrt{7}}{4}$$

83) If $\sin x = \frac{3}{4}$ and $0 < x < \frac{\pi}{2}$, then $\cot x =$

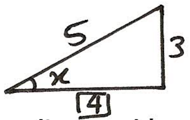
Solution:

$$\sin x = \frac{3}{4} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$

$$\therefore \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = \frac{\sqrt{7}}{3}$$

84) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cos x =$ <u>Solution:</u>  $\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$ Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $\lvert\text{adjacent}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$ $\therefore\ \cos x = \frac{adj}{hyp} = \frac{4}{5}$	85) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\sec x =$ <u>Solution:</u> $\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$ Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $\lvert\text{adjacent}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$ $\therefore\ \sec x = \frac{1}{\cos x} = \frac{hyp}{adj} = \frac{5}{4}$
86) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cot x =$ <u>Solution:</u> $\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$ Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $\lvert\text{adjacent}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$ $\therefore\ \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = -\frac{4}{3}$	87) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\tan x =$ <u>Solution:</u> $\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$ Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $\lvert\text{adjacent}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$ $\therefore\ \tan x = \frac{1}{\cot x} = \frac{opp}{adj} = -\frac{3}{4}$
88) If $f(x) = \sin x$, then $D_f = \mathbb{R}$	89) If $f(x) = \cos x$, then $D_f = \mathbb{R}$
88) If $f(x) = \sin x$, then $R_f = [-1,1]$	88) If $f(x) = \sin x$, then $R_f = [-1,1]$

Workshop Solutions to Section 2.5 (1.5)

How to find the domain and range of the exponential function $f(x) = a^x$?

1- If $f(x) = c.a^{\pm x} \pm k$ where c and k are positive constants, then
$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$
2- If $f(x) = -c.a^{\pm x} \pm k$ where c and k are positive constants, then
$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$
3- If $f(x) = c.e^{\pm x} \pm k$ where c and k are positive constants, then
$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$
4- If $f(x) = -c.e^{\pm x} \pm k$ where c and k are positive constants, then
$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$

1) Find the domain of the function $f(x) = 4^x$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	2) Find the range of the function $f(x) = 4^x$. Solution: From Step (1) above, we deduce that $R_f = (0, \infty)$
3) Find the domain of the function $f(x) = 4^x - 3$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	4) Find the range of the function $f(x) = 4^x - 3$. Solution: From Step (1) above, we deduce that $R_f = (-3, \infty)$
5) Find the domain of the function $f(x) = 5 - 3^x$. Solution: From Step (2) above, we deduce that $D_f = \mathbb{R}$	6) Find the range of the function $f(x) = 5 - 3^x$. Solution: From Step (2) above, we deduce that $R_f = (-\infty, 5)$
7) Find the domain of the function $f(x) = 3^{-x} + 1$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	8) Find the range of the function $f(x) = 3^{-x} + 1$. Solution: From Step (1) above, we deduce that $R_f = (1, \infty)$
9) Find the domain of the function $f(x) = e^x$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	10) Find the range of the function $f(x) = e^x$. Solution: From Step (3) above, we deduce that $R_f = (0, \infty)$
11) Find the domain of the function $f(x) = e^x - 3$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	12) Find the range of the function $f(x) = e^x - 3$. Solution: From Step (3) above, we deduce that $R_f = (-3, \infty)$
13) Find the domain of the function $f(x) = e^x + 1$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	14) Find the domain of the function $f(x) = \frac{1}{1-e^x}$. Solution: $f(x)$ is defined when $1 - e^x \neq 0$ $\Leftrightarrow e^x \neq 1 \Leftrightarrow \ln e^x \neq \ln 1$ $\Leftrightarrow x \neq 0$ $\therefore D_f = \mathbb{R} \setminus \{0\}$

<table>
<tr>
<td>15) Find the domain of the function $f(x) = \frac{1}{1+e^x}$.
<u>Solution:</u>
$f(x)$ is defined when $1 + e^x \neq 0$.
But there is no value of x makes $1 + e^x = 0$. Therefore,
$D_f = \mathbb{R}$</td>
<td>16) Find the domain of the function $f(x) = \sqrt{1 + 3^x}$.
<u>Solution:</u>
$f(x)$ is defined when $1 + 3^x \geq 0$.
But $1 + 3^x > 0$ always. Therefore,
$D_f = \mathbb{R}$</td>
</tr>
<tr>
<td>17) If $4^{(x+1)} = 8$, then $x =$
<u>Solution:</u>
$4^{(x+1)} = 8$
$(2^2)^{(x+1)} = 2^3$
$2^{2(x+1)} = 2^3$
$2(x+1) = 3$
$2x + 2 = 3$
$2x = 3 - 2 = 1$
$\therefore \ x = \frac{1}{2}$</td>
<td>18) If $4^{(x-1)} = 8$, then $x =$
<u>Solution:</u>
$4^{(x-1)} = 8$
$(2^2)^{(x-1)} = 2^3$
$2^{2(x-1)} = 2^3$
$2(x-1) = 3$
$2x - 2 = 3$
$2x = 3 + 2 = 5$
$\therefore \ x = \frac{5}{2}$</td>
</tr>
<tr>
<td>19) If $9^{(x+1)} = 27$, then $x =$
<u>Solution:</u>
$9^{(x+1)} = 27$
$(3^2)^{(x+1)} = 3^3$
$3^{2(x+1)} = 3^3$
$2(x+1) = 3$
$2x + 2 = 3$
$2x = 3 - 2 = 1$
$\therefore \ x = \frac{1}{2}$</td>
<td>20) If $9^{(x-1)} = 27$, then $x =$
<u>Solution:</u>
$9^{(x-1)} = 27$
$(3^2)^{(x-1)} = 3^3$
$3^{2(x-1)} = 3^3$
$2(x-1) = 3$
$2x - 2 = 3$
$2x = 3 + 2 = 5$
$\therefore \ x = \frac{5}{2}$</td>
</tr>
<tr>
<td>21) If $5^{2(x-1)} = 125$, then $x =$
<u>Solution:</u>
$5^{2(x-1)} = 125$
$5^{2(x-1)} = 5^3$
$2(x-1) = 3$
$2x - 2 = 3$
$2x = 3 + 2 = 5$
$\therefore \ x = \frac{5}{2}$</td>
<td>22) If $5^{2(x+1)} = 125$, then $x =$
<u>Solution:</u>
$5^{2(x+1)} = 125$
$5^{2(x+1)} = 5^3$
$2(x+1) = 3$
$2x + 2 = 3$
$2x = 3 - 2 = 1$
$\therefore \ x = \frac{1}{2}$</td>
</tr>
</table>

Workshop Solutions to Section 2.6 (1.6)

1) The inverse of the function
$f = \{(0,3), (-2,1), (3,4), (5,-2), (1,7)\}$ is
$f^{-1} = \{(3,0), (1,-2), (4,3), (-2,5), (7,1)\}$

2) Find the inverse of the function $f(x) = 2x + 3$.
Solution:
Let $y = 2x + 3$
$2x = y - 3$
$x = \frac{y-3}{2}$
Now, change x with y $(x \Leftrightarrow y)$
$y = \frac{x-3}{2}$
$\therefore\ f^{-1}(x) = \frac{x-3}{2}$

3) Find the inverse of the function $f(x) = 3 - 2x$.
Solution:
Let $y = 3 - 2x$
$2x = 3 - y$
$x = \frac{3-y}{2}$
Now, change x with y $(x \Leftrightarrow y)$
$y = \frac{3-x}{2}$
$\therefore\ f^{-1}(x) = \frac{3-x}{2}$

4) Find the inverse of the function $f(x) = 3 - \frac{x}{2}$.
Solution:
Let $y = 3 - \frac{x}{2}$
$2y = 6 - x$
$x = 6 - 2y$
Now, change x with y $(x \Leftrightarrow y)$
$y = 6 - 2x$
$\therefore\ f^{-1}(x) = 6 - 2x$

5) Find the inverse of the function $f(x) = \sqrt{2x-3}$.
Solution:
Let $y = \sqrt{2x-3}$ by squaring both sides
$y^2 = 2x - 3$
$2x = y^2 + 3$
$x = \frac{y^2+3}{2}$
Now, change x with y $(x \Leftrightarrow y)$
$y = \frac{x^2+3}{2}$
$\therefore\ f^{-1}(x) = \frac{x^2+3}{2}$

6) Find the inverse of the function $f(x) = \sqrt[3]{3-2x}$.
Solution:
Let $y = \sqrt[3]{3-2x}$ by cubing both sides
$y^3 = 3 - 2x$
$2x = 3 - y^3$
$x = \frac{3-y^3}{2}$
Now, change x with y $(x \Leftrightarrow y)$
$y = \frac{3-x^3}{2}$
$\therefore\ f^{-1}(x) = \frac{3-x^3}{2}$

7) Find the inverse of the function
$f(x) = (2x+3)^2, x \in [0, \infty)$.
Solution:
Let $y = (2x+3)^2$
Take the square root for both sides
$\sqrt{y} = 2x + 3$
$2x = \sqrt{y} - 3$
$x = \frac{\sqrt{y}-3}{2}$
Now, change x with y $(x \Leftrightarrow y)$
$y = \frac{\sqrt{x}-3}{2}$
$\therefore\ f^{-1}(x) = \frac{\sqrt{x}-3}{2}$

8) Find the inverse of the function $f(x) = -(x-3)^3$.
Solution:
Let $y = -(x-3)^3$
$-y = (x-3)^3$
Take the cubic root for both sides
$\sqrt[3]{-y} = x - 3$
$x = \sqrt[3]{-y} + 3$
Now, change x with y $(x \Leftrightarrow y)$
$y = \sqrt[3]{-x} + 3$
$\therefore\ f^{-1}(x) = \sqrt[3]{-x} + 3$

9) Find the inverse of the function $f(x) = \frac{x}{x-3}$.
Solution:
Let $y = \frac{x}{x-3}$
$y(x-3) = x$
$xy - 3y = x$
$xy - x = 3y$
$x(y-1) = 3y$
$x = \frac{3y}{y-1}$
Now, change x with y $(x \Leftrightarrow y)$
$y = \frac{3x}{x-1}$
$\therefore\ f^{-1}(x) = \frac{3x}{x-1}$

10) Find the inverse of the function $f(x) = \frac{x-3}{x}$.
Solution:
Let $y = \frac{x-3}{x}$
$xy = x - 3$
$xy - x = -3$
$x(y-1) = -3$
$x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$
Now, change x with y $(x \Leftrightarrow y)$
$y = \frac{3}{1-x}$
$\therefore\ f^{-1}(x) = \frac{3}{1-x}$

11) Find the inverse of the function $f(x) = \frac{x+2}{x-3}$.

Solution:

Let $y = \frac{x+2}{x-3}$

$y(x-3) = x+2$

$xy - 3y = x + 2$

$xy - x = 3y + 2$

$x(y-1) = 3y + 2$

$x = \frac{3y+2}{y-1}$

Now, change x with y ($x \Leftrightarrow y$)

$y = \frac{3x+2}{x-1}$

$\therefore\ f^{-1}(x) = \frac{3x+2}{x-1}$

12) Find the inverse of the function $f(x) = \sqrt{x} + 5$.

Solution:

Let $y = \sqrt{x} + 5$

$\sqrt{x} = y - 5$ by squaring both sides

$x = (y-5)^2$

Now, change x with y ($x \Leftrightarrow y$)

$y = (x-5)^2$

$\therefore\ f^{-1}(x) = (x-5)^2$

13) Find the inverse of the function $f(x) = \sqrt[3]{x^5}$.

Solution:

Let $y = \sqrt[3]{x^5}$

$y = x^{\frac{5}{3}}$

$y^{\frac{3}{5}} = (x^{\frac{5}{3}})^{\frac{3}{5}}$

$x = \sqrt[5]{y^3}$

Now, change x with y ($x \Leftrightarrow y$)

$y = \sqrt[5]{x^3}$

$\therefore\ f^{-1}(x) = \sqrt[5]{x^3}$

14) Find the inverse of the function $f(x) = 2x^3 - 5$.

Solution:

Let $y = 2x^3 - 5$

$2x^3 = y + 5$

$x^3 = \frac{y+5}{2}$ take the cubic root for both sides

$x = \sqrt[3]{\frac{y+5}{2}}$

Now, change x with y ($x \Leftrightarrow y$)

$y = \sqrt[3]{\frac{x+5}{2}}$

$\therefore\ f^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$

15) Find the inverse of the function $f(x) = \sqrt[3]{\frac{x+2}{5}}$.

Solution:

Let $y = \sqrt[3]{\frac{x+2}{5}}$ by cubing both sides

$y^3 = \frac{x+2}{5}$

$5y^3 = x + 2$

$x = 5y^3 - 2$

Now, change x with y ($x \Leftrightarrow y$)

$y = 5x^3 - 2$

$\therefore\ f^{-1}(x) = 5x^3 - 2$

16) Evaluate

$2^{log_2(5x+3)}$.

Solution:

$2^{log_2(5x+3)} = 5x + 3$

17) Evaluate

$log_2 2^{(5x+3)}$.

Solution:

$log_2 2^{(5x+3)} = 5x + 3$

18) $log_2 64 - log_2 32 + log_2 2 = log_2 \frac{64\times 2}{32}$

$= log_2 4 = log_2 2^2$

$= 2log_2 2$

$= 2 \times 1 = 2$

OR

$log_2 64 - log_2 32 + log_2 2 = log_2 2^6 - log_2 2^5 + log_2 2$

$= 6 - 5 + 1 = 2$

19) $log_3 27 - log_3 81 + 5\, log_3 3 = log_3 \frac{27\times 3^5}{81}$

$= log_3 81 = log_3 3^4$

$= 4log_3 3$

$= 4 \times 1 = 4$

OR

$log_3 27 - log_3 81 + 5\, log_3 = log_3 3^3 - log_3 3^4 + 5 \times 1$

$= 3 - 4 + 5 = 4$

20) $log_3 54 - log_3 2 = log_3 \frac{54}{2}$

$= log_3 27 = log_3 3^3 = 3$

21) If $log_2(6 + 2x) = 1$, then $x =$

Solution:

$log_2(6 + 2x) = 1$

$2^{log_2(6+2x)} = 2^1$

$6 + 2x = 2$

$2x = 2 - 6 = -4$

$x = -2$

22) If $ln(x+3) = 5$, then $x =$

Solution:

$ln(x+3) = 5$

$e^{ln(x+3)} = e^5$

$x + 3 = e^5$

$x = e^5 - 3$

23) If $ln(x) = 5$, then $x =$

Solution:

$ln(x) = 5$

$e^{ln(x)} = e^5$

$x = e^5$

24) If $e^{(2x-3)} = 5$, then $x =$

Solution:

$$e^{(2x-3)} = 5$$
$$lne^{(2x-3)} = ln5$$
$$2x - 3 = ln5$$
$$2x = ln5 + 3$$
$$x = \frac{ln5+3}{2}$$

25) $log_3 2 = \frac{ln2}{ln3}$

26) $\log\ 25 + \log\ 4 = \log(25 \times 4)$

$$= \log 100 = \log 10^2$$
$$= 2$$

27) $log_3 18 - log_3 6 = log_3 \frac{18}{6}$

$$= log_3 3$$
$$= 1$$

28) $log_2 6 - log_2 15 + log_2 20 = log_2 \frac{6\times 20}{15}$

$$= log_2 8 = log_2 2^3$$
$$= 3$$

29) $e^{3ln2} = e^{ln2^3} = 2^3 = 8$

30) If $3^{2-x} = 6$, then $x =$

Solution:

$$3^{2-x} = 6$$
$$log_3 3^{2-x} = log_3 6$$
$$2 - x = log_3 6$$
$$x = 2 - log_3 6 = 2 - log_3(3 \times 2)$$
$$= 2 - (log_3 3 + log_3 2) = 2 - (1 + log_3 2)$$
$$= 2 - 1 - log_3 2$$
$$= 1 - log_3 2$$

31) Find the inverse of the function $f(x) = 5 + lnx$.

Solution:

Let $y = 5 + lnx$

$$lnx = y - 5$$
$$e^{lnx} = e^{y-5}$$
$$x = e^{y-5}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = e^{x-5}$$
$$\therefore\ f^{-1}(x) = e^{x-5}$$

32) Find the domain of the function

$$f(x) = \sin^{-1}(3x + 5)\ .$$

Solution:

We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$. So,

$$-1 \le 3x + 5 \le 1$$
$$-6 \le 3x \le -4$$
$$-2 \le x \le -\frac{4}{3}$$
$$\therefore\ D_f = \left[-2, -\frac{4}{3}\right]$$

33) Find the domain of the function

$$f(x) = \cos^{-1}(3x - 5)\ .$$

Solution:

We know that the domain of $\cos^{-1}(x)$ is $[-1,1]$. So,

$$-1 \le 3x - 5 \le 1$$
$$4 \le 3x \le 6$$
$$\frac{4}{3} \le x \le 2$$
$$\therefore\ D_f = \left[\frac{4}{3}, 2\right]$$

34) Find the domain of the function

$$f(x) = 2\sin^{-1}(x) + 1\ .$$

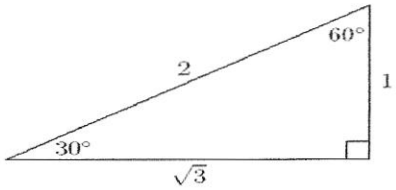
Solution:

We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$. So,

$$\therefore\ D_f = [-1,1]$$

Before proceeding to the questions 35-55, we should be aware of the following well-known right triangles:

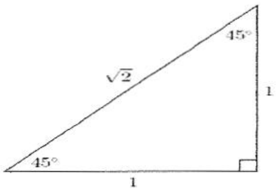
$30^o - 60^o$ Right Triangle



We know that $30^o = \frac{\pi}{6}$ and $60^o = \frac{\pi}{3}$, so

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$
$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \qquad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$
$$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \qquad \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$
$$\cot\left(\frac{\pi}{6}\right) = \sqrt{3} \qquad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$
$$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \qquad \sec\left(\frac{\pi}{3}\right) = 2$$
$$\csc\left(\frac{\pi}{6}\right) = 2 \qquad \csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$$

$30^o - 60^o$ Right Triangle



We know that $45^o = \frac{\pi}{4}$, so

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$\tan\left(\frac{\pi}{4}\right) = 1$$
$$\cot\left(\frac{\pi}{4}\right) = 1$$
$$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$
$$\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$$

35) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin\theta = \frac{\sqrt{3}}{2}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{3}$$

36) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin\theta = \frac{\sqrt{3}}{2}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{3}$$

37) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$

Solution:

Let $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\tan\theta = \frac{1}{\sqrt{3}}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{6}$$

38) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\sin\theta = \frac{1}{\sqrt{2}}$

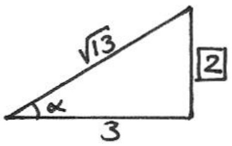
Use the $45^o - 45^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{4}$$

39) If $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$, then $\tan\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$



$\cos\alpha = \frac{3}{\sqrt{13}} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{\left(\sqrt{13}\right)^2 - 3^2} = \sqrt{13-9} = \sqrt{4} = 2$$

$$\therefore \tan\alpha = \frac{opp}{adj} = \frac{2}{3}$$

40) If $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$, then $\csc\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$

$\cos\alpha = \frac{3}{\sqrt{13}} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{\left(\sqrt{13}\right)^2 - 3^2} = \sqrt{13-9} = \sqrt{4} = 2$$

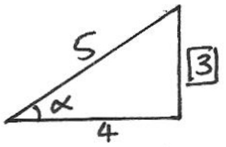
$$\therefore \csc\alpha = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{\sqrt{13}}{2}$$

41) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\csc\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$



Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \csc\alpha = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{5}{3}$$

42) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\cot\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \cot\alpha = \frac{1}{\tan\alpha} = \frac{adj}{opp} = \frac{4}{3}$$

43) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\tan\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \tan\alpha = \frac{1}{\cot\alpha} = \frac{opp}{adj} = \frac{3}{4}$$

44) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\sin\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \sin\alpha = \frac{opp}{hyp} = \frac{3}{5}$$

45) $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) =$

Solution:

Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \sin(\alpha) = \frac{opp}{hyp} = \frac{3}{5}$$

46) $\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) =$

Solution:

Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

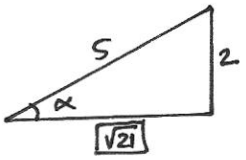
$$\therefore \tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \tan(\alpha) = \frac{opp}{adj} = \frac{3}{4}$$

47) $\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) =$

Solution:

Let $\alpha = \sin^{-1}\left(\frac{2}{5}\right)$

$\sin\alpha = \frac{2}{5} = \frac{opp}{hyp}$



Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{5^2 - 2^2} = \sqrt{25-4} = \sqrt{21}$

$$\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha)$$

Now, use the identity $\sin(2x) = 2\sin x.\cos x$. Thus,

$$\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

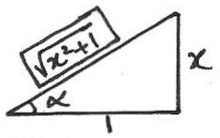
$$= 2 \times \frac{2}{5} \times \frac{\sqrt{21}}{5} = \frac{4\sqrt{21}}{25}$$

48) $\cos(\tan^{-1}x) =$

Solution:

Let $\alpha = \tan^{-1}x$

$\tan\alpha = x = \frac{opp}{adj}$



Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\cos(\tan^{-1}x) = \cos(\alpha) = \frac{adj}{hyp} = \frac{1}{\sqrt{x^2+1}}$$

49) $\sin(\tan^{-1}x) =$

Solution:

Let $\alpha = \tan^{-1}x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\sin(\tan^{-1}x) = \sin(\alpha) = \frac{opp}{hyp} = \frac{x}{\sqrt{x^2+1}}$$

50) $\csc(\tan^{-1}x) =$

Solution:

Let $\alpha = \tan^{-1}x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\csc(\tan^{-1}x) = \csc(\alpha) = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{\sqrt{x^2+1}}{x}$$

51) $\sec(\tan^{-1} x) =$

Solution:

Let $\alpha = \tan^{-1} x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

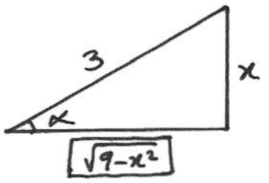
$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$

$$\sec(\tan^{-1} x) = \sec(\alpha) = \frac{1}{\cos\alpha} = \frac{hyp}{adj} = \frac{\sqrt{x^2+1}}{1} = \sqrt{x^2+1}$$

52) $\sec\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$



$\sin\alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

$$\sec\left(\sin^{-1}\frac{x}{3}\right) = \sec(\alpha) = \frac{1}{\cos\alpha} = \frac{hyp}{adj} = \frac{3}{\sqrt{9-x^2}}$$

53) $\cot\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin\alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

$$\cot\left(\sin^{-1}\frac{x}{3}\right) = \cot(\alpha) = \frac{1}{\tan\alpha} = \frac{adj}{opp} = \frac{\sqrt{9-x^2}}{x}$$

54) $\tan\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin\alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

$$\tan\left(\sin^{-1}\frac{x}{3}\right) = \tan(\alpha) = \frac{1}{\cot\alpha} = \frac{opp}{adj} = \frac{x}{\sqrt{9-x^2}}$$

55) $\cos\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin\alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

$$\cos\left(\sin^{-1}\frac{x}{3}\right) = \cos(\alpha) = \frac{adj}{hyp} = \frac{\sqrt{9-x^2}}{3}$$

Workshop Solutions to Sections 3.4 and 3.5 (2.2 & 2.5)

1) $\lim\limits_{x\to 3^{+}} \dfrac{2}{x-3} =$

Solution:

If $x \to 3^{+}$, then $x > 3 \implies x-3>0$

$$\therefore \lim_{x\to 3^{+}} \frac{2}{x-3} = \infty$$

2) $\lim\limits_{x\to 3^{-}} \dfrac{2}{x-3} =$

Solution:

If $x \to 3^{-}$, then $x < 3 \implies x-3<0$

$$\therefore \lim_{x\to 3^{-}} \frac{2}{x-3} = -\infty$$

3) $\lim\limits_{x\to 3^{+}} \dfrac{-2}{x-3} =$

Solution:

If $x \to 3^{+}$, then $x > 3 \implies x-3>0$

$$\therefore \lim_{x\to 3^{+}} \frac{-2}{x-3} = -\infty$$

4) $\lim\limits_{x\to 3^{-}} \dfrac{-2}{x-3} =$

Solution:

If $x \to 3^{-}$, then $x < 3 \implies x-3<0$

$$\therefore \lim_{x\to 3^{-}} \frac{2}{x-3} = \infty$$

5) $\lim\limits_{x\to -3^{+}} \dfrac{2}{x+3} =$

Solution:

If $x \to -3^{+}$, then $x > -3 \implies x+3>0$

$$\therefore \lim_{x\to -3^{+}} \frac{2}{x+3} = \infty$$

6) $\lim\limits_{x\to -3^{-}} \dfrac{2}{x+3} =$

Solution:

If $x \to -3^{-}$, then $x < -3 \implies x+3<0$

$$\therefore \lim_{x\to -3^{-}} \frac{2}{x+3} = -\infty$$

7) $\lim\limits_{x\to 2^{+}} \dfrac{3x-1}{x-2} =$

Solution:

If $x \to 2^{+}$, then $x > 2 \implies x-2>0$ and $3x-1>0$

$$\therefore \lim_{x\to 2^{+}} \frac{3x-1}{x-2} = \infty$$

8) $\lim\limits_{x\to 2^{-}} \dfrac{3x-1}{x-2} =$

Solution:

If $x \to 2^{-}$, then $x < 2 \implies x-2<0$ and $3x-1>0$

$$\therefore \lim_{x\to 2^{-}} \frac{3x-1}{x-2} = -\infty$$

9) $\lim\limits_{x\to -2^{+}} \dfrac{1-x}{(x+2)^2} =$

Solution:

If $x \to -2^{+}$, then $x > -2$

$\implies 1-x>0$ and $(x+2)^2>0$

$$\therefore \lim_{x\to -2^{+}} \frac{1-x}{(x+2)^2} = \infty$$

10) $\lim\limits_{x\to -2^{-}} \dfrac{1-x}{(x+2)^2} =$

Solution:

If $x \to -2^{-}$, then $x < -2$

$\implies 1-x>0$ and $(x+2)^2>0$

$$\therefore \lim_{x\to -2^{+}} \frac{1-x}{(x+2)^2} = \infty$$

11) $\lim\limits_{x\to -2^{+}} \dfrac{x-1}{(x+2)^2} =$

Solution:

If $x \to -2^{+}$, then $x > -2$

$\implies x-1<0$ and $(x+2)^2>0$

$$\therefore \lim_{x\to -2^{+}} \frac{x-1}{(x+2)^2} = -\infty$$

12) $\lim\limits_{x\to -2^{-}} \dfrac{x-1}{(x+2)^2} =$

Solution:

If $x \to -2^{-}$, then $x < -2$

$\implies x-1<0$ and $(x+2)^2>0$

$$\therefore \lim_{x\to -2^{-}} \frac{x-1}{(x+2)^2} = -\infty$$

13) $\lim\limits_{x\to 2^{+}} \dfrac{6x-1}{x^2-4} =$

Solution:

If $x \to 2^{+}$, then $x^2 > 4$

$\implies x^2-4>0$ and $6x-1>0$

$$\therefore \lim_{x\to 2^{+}} \frac{6x-1}{x^2-4} = \infty$$

14) $\lim\limits_{x\to 2^{-}} \dfrac{6x-1}{x^2-4} =$

Solution:

If $x \to 2^{-}$, then $x^2 < 4$

$\implies x^2-4<0$ and $6x-1>0$

$$\therefore \lim_{x\to 2^{+}} \frac{6x-1}{x^2-4} = -\infty$$

15) $\lim_{x\to-2^+}\dfrac{6x-1}{x^2-4}=$

Solution:

If $x\to-2^+$, then $x^2<4$

$$\Longrightarrow\ x^2-4<0 \text{ and } 6x-1<0$$

$$\therefore\ \lim_{x\to2^+}\frac{6x-1}{x^2-4}=\infty$$

16) $\lim_{x\to-2^-}\dfrac{6x-1}{x^2-4}=$

Solution:

If $x\to-2^-$, then $x^2>4$

$$\Longrightarrow\ x^2-4>0 \text{ and } 6x-1<0$$

$$\therefore\ \lim_{x\to2^+}\frac{6x-1}{x^2-4}=-\infty$$

17) $\lim_{x\to-2^-}\dfrac{6x-1}{x^2-x-6}=$

Solution:

$$f(x)=\frac{6x-1}{x^2-x-6}=\frac{6x-1}{(x-3)(x+2)}$$

If $x\to-2^-$, then $x<-2$

$$\Longrightarrow\ x-3<0\,,\ x+2<0 \text{ and } 6x-1<0$$

$$\therefore\ \lim_{x\to-2^-}\frac{6x-1}{x^2-x-6}=-\infty$$

18) $\lim_{x\to-2^+}\dfrac{6x-1}{x^2-x-6}=$

Solution:

$$f(x)=\frac{6x-1}{x^2-x-6}=\frac{6x-1}{(x-3)(x+2)}$$

If $x\to-2^+$, then $x>-2$

$$\Longrightarrow\ x-3<0\,,\ x+2>0 \text{ and } 6x-1<0$$

$$\therefore\ \lim_{x\to-2^+}\frac{6x-1}{x^2-x-6}=\infty$$

19) $\lim_{x\to3^+}\dfrac{-1}{x^2-x-6}=$

Solution:

$$f(x)=\frac{-1}{x^2-x-6}=\frac{-1}{(x-3)(x+2)}$$

If $x\to3^+$, then $x>3$

$$\Longrightarrow\ x-3>0\,,\ x+2>0 \text{ and } -1<0$$

$$\therefore\ \lim_{x\to3^+}\frac{-1}{x^2-x-6}=-\infty$$

20) $\lim_{x\to3^-}\dfrac{-1}{x^2-x-6}=$

Solution:

$$f(x)=\frac{-1}{x^2-x-6}=\frac{-1}{(x-3)(x+2)}$$

If $x\to3^-$, then $x<3$

$$\Longrightarrow\ x-3<0\,,\ x+2>0 \text{ and } -1<0$$

$$\therefore\ \lim_{x\to3^-}\frac{-1}{x^2-x-6}=\infty$$

21) $\lim_{x\to(\pi/2)^+}\tan x=$

Solution:

$$\lim_{x\to(\pi/2)^+}\tan x=-\infty$$

22) $\lim_{x\to(\pi/2)^-}\tan x=$

Solution:

$$\lim_{x\to(\pi/2)^-}\tan x=\infty$$

23) The vertical asymptote of $f(x)=\frac{1-x}{2x+1}$ is

Solution:

We see that the function $f(x)$ is not defined when $2x+1=0\ \Longrightarrow\ x=-\frac{1}{2}$. Since

$$\lim_{x\to\left(-\frac{1}{2}\right)^+}\frac{1-x}{2x+1}=\infty$$

and

$$\lim_{x\to\left(-\frac{1}{2}\right)^-}\frac{1-x}{2x+1}=-\infty$$

then, $x=-\frac{1}{2}$ is a vertical asymptote.

24) The vertical asymptote of $f(x)=\frac{3-x}{x^2-4}$ is

Solution:

We see that the function $f(x)$ is not defined when $x^2-4=0\ \Longrightarrow\ x=\pm2$. Since

$$\lim_{x\to2^+}\frac{3-x}{x^2-4}=\infty\,,\qquad \lim_{x\to2^-}\frac{3-x}{x^2-4}=-\infty$$

and

$$\lim_{x\to-2^+}\frac{3-x}{x^2-4}=-\infty\,,\qquad \lim_{x\to-2^-}\frac{3-x}{x^2-4}=\infty$$

then, $x=\pm2$ are vertical asymptotes.

25) The vertical asymptote of $f(x)=\frac{3-x}{x^2-x-6}$ is

Solution:

$$f(x)=\frac{3-x}{x^2-x-6}=\frac{3-x}{(x-3)(x+2)}=\frac{-(x-3)}{(x-3)(x+2)}=-\frac{1}{x+2}$$

We see that the function $f(x)$ is not defined when $x^2-x-6=0 \implies (x-3)(x+2)=0 \implies x=3$ or $x=-2$. Since

$$\lim_{x\to3}\frac{3-x}{x^2-x-6}=\lim_{x\to3}\frac{3-x}{(x-3)(x+2)}=\lim_{x\to3}\frac{-(x-3)}{(x-3)(x+2)}=\lim_{x\to3}\frac{-1}{x+2}=-\frac{1}{5}$$

then, $x=3$ is a removable discontinuity.

$$\lim_{x\to-2^+}\frac{3-x}{x^2-x-6}=\lim_{x\to-2^+}\frac{3-x}{(x-3)(x+2)}=\infty$$

and

$$\lim_{x\to-2^-}\frac{3-x}{x^2-x-6}=\lim_{x\to-2^-}\frac{3-x}{(x-3)(x+2)}=-\infty$$

then, $x=-2$ is a vertical asymptote only.

26) The vertical asymptote of $f(x)=\frac{7-x}{x^2-5x+6}$ is

Solution:

$$f(x)=\frac{7-x}{x^2-5x+6}=\frac{7-x}{(x-3)(x-2)}$$

We see that the function $f(x)$ is not defined when $x-3=0$ or $x-2=0 \implies x=3$ or $x=2$. Since

$$\lim_{x\to3^+}\frac{7-x}{x^2-5x+6}=\lim_{x\to3^+}\frac{7-x}{(x-3)(x-2)}=\infty$$
$$\lim_{x\to3^-}\frac{7-x}{x^2-5x+6}=\lim_{x\to3^-}\frac{7-x}{(x-3)(x-2)}=-\infty$$

and

$$\lim_{x\to2^+}\frac{7-x}{x^2-5x+6}=\lim_{x\to2^+}\frac{7-x}{(x-3)(x-2)}=-\infty$$
$$\lim_{x\to2^-}\frac{7-x}{x^2-5x+6}=\lim_{x\to2^-}\frac{7-x}{(x-3)(x-2)}=\infty$$

then, $x=3$ and $x=2$ are vertical asymptotes.

27) The vertical asymptote of $f(x)=\frac{x-7}{x^2+5x+6}$ is

Solution:

$$f(x)=\frac{x-7}{x^2+5x+6}=\frac{x-7}{(x+3)(x+2)}$$

We see that the function $f(x)$ is not defined when $x+3=0$ or $x+2=0 \implies x=-3$ or $x=-2$. Since

$$\lim_{x\to-3^+}\frac{x-7}{x^2+5x+6}=\lim_{x\to-3^+}\frac{x-7}{(x+3)(x+2)}=\infty$$
$$\lim_{x\to-3^-}\frac{x-7}{x^2+5x+6}=\lim_{x\to-3^-}\frac{x-7}{(x+3)(x+2)}=-\infty$$

and

$$\lim_{x\to-2^+}\frac{x-7}{x^2+5x+6}=\lim_{x\to-2^+}\frac{x-7}{(x+3)(x+2)}=-\infty$$
$$\lim_{x\to-2^-}\frac{x-7}{x^2+5x+6}=\lim_{x\to-2^-}\frac{x-7}{(x+3)(x+2)}=\infty$$

then, $x=-3$ and $x=-2$ are vertical asymptotes.

28) The vertical asymptote of $f(x)=\frac{x-7}{x^2+3x}$ is

Solution:

$$f(x)=\frac{x-7}{x^2+3x}=\frac{x-7}{x(x+3)}$$

We see that the function $f(x)$ is not defined when $x=0$ or $x+3=0 \implies x=0$ or $x=-3$. Since

$$\lim_{x\to-3^+}\frac{x-7}{x^2+3x}=\lim_{x\to-3^+}\frac{x-7}{x(x+3)}=\infty$$
$$\lim_{x\to-3^-}\frac{x-7}{x^2+3x}=\lim_{x\to-3^-}\frac{x-7}{x(x+3)}=-\infty$$

and

$$\lim_{x\to0^+}\frac{x-7}{x^2+3x}=\lim_{x\to0^+}\frac{x-7}{x(x+3)}=-\infty$$
$$\lim_{x\to0^-}\frac{x-7}{x^2+3x}=\lim_{x\to0^-}\frac{x-7}{x(x+3)}=\infty$$

then, $x=-3$ and $x=0$ are vertical asymptotes.

29) The vertical asymptote of $f(x)=\frac{x-7}{x^2-3x}$ is

Solution:

$$f(x)=\frac{x-7}{x^2-3x}=\frac{x-7}{x(x-3)}$$

We see that the function $f(x)$ is not defined when $x=0$ or $x-3=0 \implies x=0$ or $x=3$. Since

$$\lim_{x\to3^+}\frac{x-7}{x^2-3x}=\lim_{x\to3^+}\frac{x-7}{x(x-3)}=-\infty$$
$$\lim_{x\to3^-}\frac{x-7}{x^2-3x}=\lim_{x\to3^-}\frac{x-7}{x(x-3)}=\infty$$

and

$$\lim_{x\to0^+}\frac{x-7}{x^2-3x}=\lim_{x\to0^+}\frac{x-7}{x(x-3)}=\infty$$
$$\lim_{x\to0^-}\frac{x-7}{x^2-3x}=\lim_{x\to0^-}\frac{x-7}{x(x-3)}=-\infty$$

then, $x=3$ and $x=0$ are vertical asymptotes.

30) The vertical asymptotes of $f(x)=\frac{2x^2+1}{x^2-9}$ are

Solution:

$$f(x)=\frac{2x^2+1}{x^2-9}=\frac{2x^2+1}{(x+3)(x-3)}$$

We see that the function $f(x)$ is not defined when $x^2-9=0 \implies x=\pm3$. Since

$$\lim_{x\to3^+}\frac{2x^2+1}{x^2-9}=\lim_{x\to3^+}\frac{2x^2+1}{(x+3)(x-3)}=\infty$$
$$\lim_{x\to3^-}\frac{2x^2+1}{x^2-9}=\lim_{x\to3^-}\frac{2x^2+1}{(x+3)(x-3)}=-\infty$$

and

$$\lim_{x\to-3^+}\frac{2x^2+1}{x^2-9}=\lim_{x\to-3^+}\frac{2x^2+1}{(x+3)(x-3)}=-\infty$$
$$\lim_{x\to-3^-}\frac{2x^2+1}{x^2-9}=\lim_{x\to-3^-}\frac{2x^2+1}{(x+3)(x-3)}=\infty$$

then, $x=\pm3$ are vertical asymptotes.

31) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous at $a=2$ because

$1-\quad f(2)=\dfrac{(2)+1}{(2)^2-9}=\dfrac{3}{-5}=-\dfrac{3}{5}$

$2-\quad \lim\limits_{x\to 3^-}\dfrac{x+1}{x^2-9}=\lim\limits_{x\to 2}\dfrac{(2)+1}{(2)^2-9}=\dfrac{3}{-5}=-\dfrac{3}{5}$

$3-\quad \lim\limits_{x\to 2}\dfrac{x+1}{x^2-9}=f(2)$

OR

We know that $D_f=\mathbb{R}\setminus\{\pm3\}$, so $\{2\}\in D_f$.

Note: Any function is continuous on its domain.

32) The function $f(x)=\frac{x+1}{x^2-9}$ is discontinuous at $a=\pm3$ because we know that $D_f=\mathbb{R}\setminus\{\pm3\}$, so $\{\pm3\}\notin D_f$.

33) The function $f(x)=\frac{x+1}{x^2-9}$ is discontinuous at ±3 because $\{\pm3\}\notin D_f$.

34) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous on its domain which is $D_f=\mathbb{R}\setminus\{\pm3\}$.

35) The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ 3, & x=0\end{cases}$ is continuous at $a=0$ because

1- $f(0)=3$

2- $\lim\limits_{x\to0}\frac{\sin 3x}{x}=3\lim\limits_{x\to0}\frac{\sin 3x}{3x}=3(1)=3$

3- $\lim\limits_{x\to0}f(x)=f(0)$

36) The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ 5, & x=0\end{cases}$ is discontinuous at $a=0$ because

1- $f(0)=5$

2- $\lim\limits_{x\to0}\frac{\sin 3x}{x}=3\lim\limits_{x\to0}\frac{\sin 3x}{3x}=3(1)=3$

3- $\lim\limits_{x\to0}f(x)\neq f(0)$

37) The function $f(x)=\begin{cases}\frac{2x^2-3x+1}{x-1}, & x\neq 1\\ 7, & x=1\end{cases}$ is discontinuous at $a=1$ because

1- $f(1)=7$

2- $\lim\limits_{x\to1}\frac{2x^2-3x+1}{x-1}=\lim\limits_{x\to1}\frac{(2x-1)(x-1)}{x-1}=\lim\limits_{x\to1}(2x-1)=1$

3- $\lim\limits_{x\to1}f(x)\neq f(1)$

38) The function $f(x)=\begin{cases}\frac{2x^2-3x+1}{x-1}, & x\neq 1\\ 1, & x=1\end{cases}$ is continuous at $a=1$ because

1- $f(1)=1$

2- $\lim\limits_{x\to1}\frac{2x^2-3x+1}{x-1}=\lim\limits_{x\to1}\frac{(2x-1)(x-1)}{x-1}=\lim\limits_{x\to1}(2x-1)=1$

3- $\lim\limits_{x\to1}f(x)=f(1)$

39) The function $f(x)=\frac{x^2-x-2}{x-2}$ is discontinuous at $a=2$ because $\{2\}\notin D_f$.

40) The function $f(x)=\begin{cases}2x+3, & x>2\\ 3x+1, & x\leq 2\end{cases}$ is continuous at $a=2$ because

1- $f(2)=3(2)+1=7$

2- $\lim\limits_{x\to2^+}(2x+3)=2(2)+3=7$

$\lim\limits_{x\to2^-}(3x+1)=3(2)+1=7$

$\therefore\ \lim\limits_{x\to2}f(x)=7$

3- $\lim\limits_{x\to2}f(x)=f(2)$

41) The function $f(x)=\frac{x+3}{\sqrt{x^2-4}}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4>0\ \Longrightarrow\ x^2>4\ \Longrightarrow\ \sqrt{x^2}>\sqrt{4}$$
$$\Longrightarrow\ |x|>2\ \Leftrightarrow\ x>2\ \text{or}\ x<-2$$

Hence,

$D_f=(-\infty,-2)\cup(2,\infty)$.

42) The function $f(x)=\sqrt{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4\geq0\ \Longrightarrow\ x^2\geq4\ \Longrightarrow\ \sqrt{x^2}\geq\sqrt{4}$$
$$\Longrightarrow\ |x|\geq2\ \Leftrightarrow\ x\geq2\ \text{or}\ x\leq-2$$

Hence,

$D_f=(-\infty,-2]\cup[2,\infty)$.

43) The function $f(x)=\sqrt{4-x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$4-x^2\geq0\ \Longrightarrow\ -x^2\geq-4\ \Longrightarrow\ x^2\leq4$$
$$\Longrightarrow\sqrt{x^2}\leq\sqrt{4}\ \Longrightarrow\ |x|\leq2\ \Leftrightarrow\ -2\leq x\leq2$$

Hence,

$D_f=[-2,2]$.

44) The function $f(x)=\frac{x+3}{\sqrt{4-x^2}}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$4-x^2>0\ \Longrightarrow\ -x^2>-4\ \Longrightarrow\ x^2<4$$
$$\Longrightarrow\ \sqrt{x^2}<\sqrt{4}\ \Longrightarrow\ |x|<2\ \Leftrightarrow\ -2<x<2$$

Hence,

$D_f=(-2,2)$.

45) The function $f(x)=\frac{x+1}{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4\neq0\ \Longrightarrow\ x^2\neq4\ \Longrightarrow\ x\neq\pm2$$

Hence,

$D_f=\mathbb{R}\setminus\{\pm2\}$

$=(-\infty,-2)\cup(-2,2)\cup(2,\infty)=\{x\in\mathbb{R}:x\neq\pm2\}$.

46) The function $f(x) = \log_2(x+2)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x+2>0 \implies x>-2$$

Hence,

$D_f = (-2, \infty)$.

47) The function $f(x) = \sqrt{x-1} + \sqrt{x+4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x-1 \geq 0 \text{ and } x+4 \geq 0 \implies x \geq 1 \ \cap \ x \geq -4$$

Hence,

$D_f = [1, \infty)$.

48) The function $f(x) = 5^x$ is continuous on its domain .

Hence,

$D_f = \mathbb{R} = (-\infty, \infty)$.

49) The function $f(x) = e^x$ is continuous on its domain .

Hence,

$D_f = \mathbb{R} = (-\infty, \infty)$.

50) The function $f(x) = \sin^{-1}(3x-5)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$-1 \leq 3x-5 \leq 1 \iff 4 \leq 3x \leq 6 \iff \frac{4}{3} \leq x \leq 2 \,.$$

Hence,

$D_f = \left[\frac{4}{3}, 2\right]$.

51) The function $f(x) = \cos^{-1}(3x+5)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$-1 \leq 3x+5 \leq 1 \iff -6 \leq 3x \leq -4 \iff -2 \leq x \leq -\frac{4}{3} \,.$$

Hence,

$D_f = \left[-2, -\frac{4}{3}\right]$.

52) The number c that makes $f(x) = \begin{cases} c+x\,, & x>2 \\ 2x-c, & x \leq 2 \end{cases}$ is continuous at $x=2$ is

<u>Solution:</u>

$\lim\limits_{x\to 2} f(x)$ exists if

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$$
$$\lim_{x\to 2^+} (c+x) = \lim_{x\to 2^-} (2x-c)$$
$$c+2 = 4-c$$
$$c+c = 4-2$$
$$2c = 2$$
$$c = 1$$

53) The number c that makes $f(x) = \begin{cases} cx^2-2x+1, & x \leq -1 \\ 3x+2 \quad , & x>-1 \end{cases}$ is continuous at -1 is

<u>Solution:</u>

$\lim\limits_{x\to -1} f(x)$ exists if

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x)$$
$$\lim_{x\to -1^+} (3x+2\,) = \lim_{x\to -1^-} (cx^2-2x+1)$$
$$3(-1)+2 = c(-1)^2-2(-1)+1$$
$$-1 = c+3$$
$$c = -1-3$$
$$c = -4$$

54) The number c that makes $f(x) = \begin{cases} \frac{\sin cx}{x} + 2x - 1, & x<0 \\ 3x+4 \quad , & x \geq 0 \end{cases}$ is continuous at 0 is

<u>Solution:</u>

$\lim\limits_{x\to 0} f(x)$ exists if

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$$
$$\lim_{x\to 0^+} (3x+4) = \lim_{x\to 0^-} \left(\frac{\sin cx}{x} + 2x - 1\right)$$
$$3(0)+4 = c(1)+2(0)-1$$
$$4 = c-1$$
$$c = 4+1$$
$$c = 5$$

55) The value c that makes $f(x) = \begin{cases} cx^2+2x, & x \leq 2 \\ x^3-cx \,, & x>2 \end{cases}$ is continuous at 2 is

<u>Solution:</u>

$\lim\limits_{x\to 2} f(x)$ exists if

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$$
$$\lim_{x\to 2^+} (x^3-cx\ ) = \lim_{x\to 2^-} (cx^2+2x)$$
$$(2)^3-c(2) = c(2)^2+2(2)$$
$$8-2c = 4c+4$$
$$-2c-4c = 4-8$$
$$-6c = -4$$
$$c = \frac{-4}{-6}$$
$$c = \frac{2}{3}$$

56) The number c that makes $f(x) = \begin{cases} c^2x^2-1, & x \leq 3 \\ x+5 \quad , & x>3 \end{cases}$ is continuous at 3 is

<u>Solution:</u>

$\lim\limits_{x\to 3} f(x)$ exists if

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x)$$
$$\lim_{x\to 3^+} (x+5) = \lim_{x\to 3^-} (c^2x^2-1)$$
$$(3)+5 = c^2(3)^2-1$$
$$8 = 9c^2-1$$
$$9c^2 = 8+1$$
$$c^2 = 1$$
$$c = \pm 1$$

57) The number c that makes $f(x) = \begin{cases} x-2\,, & x>5 \\ cx-3, & x \leq 5 \end{cases}$ is continuous at 5 is

<u>Solution:</u>

$\lim\limits_{x\to 5} f(x)$ exists if

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^-} f(x)$$
$$\lim_{x\to 5^+} (x-2) = \lim_{x\to 5^-} (cx-3)$$
$$(5)-2 = c(5)-3$$
$$3 = 5c-3$$
$$5c = 3+3$$
$$5c = 6$$
$$c = \frac{6}{5}$$

58) The number c that makes $f(x) = \begin{cases} x+3\,, & x > -1 \\ 2x - c, & x \leq -1 \end{cases}$ is continuous at -1 is Solution: $\lim_{x\to -1} f(x)$ exists if $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x)$ $\lim_{x\to -1^+} (x+3) = \lim_{x\to -1^-} (2x - c)$ $(-1) + 3 = 2(-1) - c$ $2 = -2 - c$ $c = -2 - 2$ $c = -4$	

Workshop Solutions to Section 3.3 (2.6 & page 192,193)

1) If $f(x)=\begin{cases}2x+3; & x\ge -2\\ 2x+5; & x<-2\end{cases}$ then $\lim\limits_{x\to(-2)^-} f(x)=$ Solution: $\lim\limits_{x\to(-2)^-} f(x)=\lim\limits_{x\to(-2)^-}(2x+5)=2(-2)+5=-4+5=1$	2) If $f(x)=\begin{cases}2x+3; & x\ge -2\\ 2x+5; & x<-2\end{cases}$ then $\lim\limits_{x\to(-2)^+} f(x)=$ Solution: $\lim\limits_{x\to(-2)^+} f(x)=\lim\limits_{x\to(-2)^+}(2x+3)=2(-2)+3=-4+3=-1$
3) If $f(x)=\begin{cases}2x+3; & x\ge -2\\ 2x+5; & x<-2\end{cases}$ then $\lim\limits_{x\to -2} f(x)=$ Solution: $\lim\limits_{x\to -2} f(x)$ does not exist because $\lim\limits_{x\to(-2)^-} f(x)\neq \lim\limits_{x\to(-2)^+} f(x)$	4) If $f(x)=\begin{cases}x^2-2x+3; & x\ge 3\\ x^3-3x-12; & x<3\end{cases}$ then $\lim\limits_{x\to 3} f(x)=$ Solution: $\lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^-}(x^3-3x-12)=(3)^3-3(3)-12=27-9-12=6$ $\lim\limits_{x\to 3^+} f(x)=\lim\limits_{x\to 3^+}(x^2-2x+3)=(3)^2-2(3)+3=9-6+3=6$ $\therefore \lim\limits_{x\to 3} f(x)=6$
5) If $f(x)=\begin{cases}x^2-7x; & x<1\\ 5; & 1\le x\le 3\\ 3x+1; & x>3\end{cases}$ then $\lim\limits_{x\to 1^-} f(x)=$ Solution: $\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-}(x^2-7x)=(1)^2-7(1)=1-7=-6$	6) If $f(x)=\begin{cases}x^2-7x; & x<1\\ 5; & 1\le x\le 3\\ 3x+1; & x>3\end{cases}$ then $\lim\limits_{x\to 1^+} f(x)=$ Solution: $\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}(5)=5$
7) If $f(x)=\begin{cases}x^2-7x; & x<1\\ 5; & 1\le x\le 3\\ 3x+1; & x>3\end{cases}$ then $\lim\limits_{x\to 3^-} f(x)=$ Solution: $\lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^-}(5)=5$	8) If $f(x)=\begin{cases}x^2-7x; & x<1\\ 5; & 1\le x\le 3\\ 3x+1; & x>3\end{cases}$ then $\lim\limits_{x\to 3^+} f(x)=$ Solution: $\lim\limits_{x\to 3^+} f(x)=\lim\limits_{x\to 3^+}(3x+1)=3(3)+1=9+1=10$
9) If $f(x)=\begin{cases}\frac{x^2+x-6}{x^2-4}; & x^2-4>0\\ \frac{x^2+x-6}{4-x^2}; & x^2-4<0\end{cases}$ then $\lim\limits_{x\to 2^+} f(x)=$ Solution: $f(x)=\begin{cases}\frac{x^2+x-6}{x^2-4}; & x^2-4>0\\ \frac{x^2+x-6}{4-x^2}; & x^2-4<0\end{cases}$ $=\begin{cases}\frac{x^2+x-6}{x^2-4}; & x^2>4\\ \frac{x^2+x-6}{-(x^2-4)}; & x^2<4\end{cases}$ $=\begin{cases}\frac{(x+3)(x-2)}{(x-2)(x+2)}; & \lvert x\rvert>4\\ \frac{(x+3)(x-2)}{-(x-2)(x+2)}; & \lvert x\rvert<4\end{cases}$ $=\begin{cases}\frac{x+3}{x+2}; & x>2 \text{ or } x<-2\\ -\frac{x+3}{x+2}; & -2<x<2\end{cases}$ then $\therefore \lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2^+}\left(\frac{x+3}{x+2}\right)=\frac{(2)+3}{(2)+2}=\frac{5}{4}$	10) If $f(x)=\begin{cases}\frac{x^2+x-6}{x^2-4}; & x^2-4>0\\ \frac{x^2+x-6}{4-x^2}; & x^2-4<0\end{cases}$ then $\lim\limits_{x\to 2^-} f(x)=$ Solution: $f(x)=\begin{cases}\frac{x^2+x-6}{x^2-4}; & x^2-4>0\\ \frac{x^2+x-6}{4-x^2}; & x^2-4<0\end{cases}$ $=\begin{cases}\frac{x^2+x-6}{x^2-4}; & x^2>4\\ \frac{x^2+x-6}{-(x^2-4)}; & x^2<4\end{cases}$ $=\begin{cases}\frac{(x+3)(x-2)}{(x-2)(x+2)}; & \lvert x\rvert>4\\ \frac{(x+3)(x-2)}{-(x-2)(x+2)}; & \lvert x\rvert<4\end{cases}$ $=\begin{cases}\frac{x+3}{x+2}; & x>2 \text{ or } x<-2\\ -\frac{x+3}{x+2}; & -2<x<2\end{cases}$ then $\therefore \lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^-}\left(-\frac{x+3}{x+2}\right)=-\frac{(2)+3}{(2)+2}=-\frac{5}{4}$

11)

$$\lim_{x\to a^-}\frac{|x-a|}{x-a}=$$

Solution:

$$f(x)=\frac{|x-a|}{x-a}=\begin{cases}\dfrac{x-a}{x-a} & ;\ x-a>0\\ \dfrac{-(x-a)}{x-a} & ;\ x-a<0\end{cases}=\begin{cases}1; & x>a\\ -1; & x<a\end{cases}$$

$$\therefore\quad \lim_{x\to a^-}\frac{|x-a|}{x-a}=\lim_{x\to a^-}\frac{-(x-a)}{x-a}=\lim_{x\to a^-}(-1)=-1$$

12)

$$\lim_{x\to a^+}\frac{|x-a|}{x-a}=$$

Solution:

$$f(x)=\frac{|x-a|}{x-a}=\begin{cases}\dfrac{x-a}{x-a} & ;\ x-a>0\\ \dfrac{-(x-a)}{x-a} & ;\ x-a<0\end{cases}=\begin{cases}1; & x>a\\ -1; & x<a\end{cases}$$

$$\therefore\quad \lim_{x\to a^+}\frac{|x-a|}{x-a}=\lim_{x\to a^+}\frac{(x-a)}{x-a}=\lim_{x\to a^+}(1)=1$$

13)

$$\lim_{x\to a}\frac{|x-a|}{x-a}=$$

Solution:

$\lim_{x\to a}\frac{|x-a|}{x-a}$ does not exist because

$$\lim_{x\to a^-}\frac{|x-a|}{x-a}\neq\lim_{x\to a^+}\frac{|x-a|}{x-a}$$

It is clearly obvious from questions (11) and (12) above.

14)

$$\lim_{x\to a^+}\frac{|a-x|}{x-a}=$$

Solution:

$$f(x)=\frac{|a-x|}{x-a}=\begin{cases}\dfrac{a-x}{x-a} & ;\ a-x>0\\ \dfrac{-(a-x)}{x-a} & ;\ a-x<0\end{cases}=\begin{cases}\dfrac{-(x-a)}{x-a} & ;\ a>x\\ \dfrac{(x-a)}{x-a} & ;\ a<x\end{cases}=\begin{cases}-1; & x<a\\ 1; & x>a\end{cases}$$

$$\therefore\quad \lim_{x\to a^+}\frac{|a-x|}{x-a}=\lim_{x\to a^+}(1)=1$$

15)

$$\lim_{x\to a^-}\frac{|a-x|}{x-a}=$$

Solution:

$$f(x)=\frac{|a-x|}{x-a}=\begin{cases}\dfrac{a-x}{x-a} & ;\ a-x>0\\ \dfrac{-(a-x)}{x-a} & ;\ a-x<0\end{cases}=\begin{cases}\dfrac{-(x-a)}{x-a} & ;\ a>x\\ \dfrac{(x-a)}{x-a} & ;\ a<x\end{cases}=\begin{cases}-1; & x<a\\ 1; & x>a\end{cases}$$

$$\therefore\quad \lim_{x\to a^-}\frac{|a-x|}{x-a}=\lim_{x\to a^-}(-1)=-1$$

16)

$$\lim_{x\to a}\frac{|a-x|}{x-a}=$$

Solution:

$\lim_{x\to a}\frac{|a-x|}{x-a}$ does not exist because

$$\lim_{x\to a^-}\frac{|a-x|}{x-a}\neq\lim_{x\to a^+}\frac{|a-x|}{x-a}$$

It is clearly obvious from questions (14) and (15) above.

17)

$$\lim_{x\to(-a)^-}\frac{|x+a|}{x+a}=$$

Solution:

$$f(x)=\frac{|x+a|}{x+a}=\begin{cases}\dfrac{x+a}{x+a} & ;\ x+a>0\\ \dfrac{-(x+a)}{x+a} & ;\ x+a<0\end{cases}=\begin{cases}1; & x>-a\\ -1; & x<-a\end{cases}$$

$$\therefore\quad \lim_{x\to(-a)^-}\frac{|x+a|}{x+a}=\lim_{x\to(-a)^-}(-1)=-1$$

18)

$$\lim_{x\to(-a)^+}\frac{|x+a|}{x+a}=$$

Solution:

$$f(x)=\frac{|x+a|}{x+a}=\begin{cases}\dfrac{x+a}{x+a} & ;\ x+a>0\\ \dfrac{-(x+a)}{x+a} & ;\ x+a<0\end{cases}=\begin{cases}1; & x>-a\\ -1; & x<-a\end{cases}$$

$$\therefore\quad \lim_{x\to(-a)^+}\frac{|x+a|}{x+a}=\lim_{x\to(-a)^+}(1)=1$$

19)

$$\lim_{x\to-a}\frac{|x+a|}{x+a}=$$

Solution:

$\lim_{x\to-a}\frac{|x+a|}{x+a}$ does not exist because

$$\lim_{x\to(-a)^-}\frac{|x+a|}{x+a}\neq\lim_{x\to(-a)^+}\frac{|x+a|}{x+a}$$

It is clearly obvious from questions (17) and (18) above.

20)

$$\lim_{x\to 0^+}\frac{2x-|x|}{x^2+|x|}=$$

Solution:

$$f(x)=\frac{2x-|x|}{x^2+|x|}=\begin{cases}\dfrac{2x-(x)}{x^2+(x)} & ;\ x>0\\ \dfrac{2x-(-x)}{x^2+(-x)} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{2x-x}{x^2+x} & ;\ x>0\\ \dfrac{2x+x}{x^2-x} & ;\ x<0\end{cases}=\begin{cases}\dfrac{x}{x^2+x} & ;\ x>0\\ \dfrac{3x}{x^2-x} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{x}{x(x+1)} & ;\ x>0\\ \dfrac{3x}{x(x-1)} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{1}{x+1} & ;\ x>0\\ \dfrac{3}{x-1} & ;\ x<0\end{cases}$$

$$\therefore\quad \lim_{x\to 0^+}\frac{2x-|x|}{x^2+|x|}=\lim_{x\to 0^+}\frac{1}{x+1}=\frac{1}{0+1}=1$$

21)

$$\lim_{x\to 0^-}\frac{2x-|x|}{x^2+|x|}=$$

Solution:

$$f(x)=\frac{2x-|x|}{x^2+|x|}=\begin{cases}\dfrac{2x-(x)}{x^2+(x)} & ;\ x>0\\ \dfrac{2x-(-x)}{x^2+(-x)} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{2x-x}{x^2+x} & ;\ x>0\\ \dfrac{2x+x}{x^2-x} & ;\ x<0\end{cases}=\begin{cases}\dfrac{x}{x^2+x} & ;\ x>0\\ \dfrac{3x}{x^2-x} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{x}{x(x+1)} & ;\ x>0\\ \dfrac{3x}{x(x-1)} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{1}{x+1} & ;\ x>0\\ \dfrac{3}{x-1} & ;\ x<0\end{cases}$$

$$\therefore\quad \lim_{x\to 0^-}\frac{2x-|x|}{x^2+|x|}=\lim_{x\to 0^-}\frac{3}{x-1}=\frac{3}{0-1}=-3$$

22)

$$\lim_{x\to 0}\frac{2x-|x|}{x^2+|x|}=$$

Solution:

$\lim_{x\to 0}\frac{2x-|x|}{x^2+|x|}$ does not exist because

$$\lim_{x\to 0^-}\frac{2x-|x|}{x^2+|x|}\neq\lim_{x\to 0^+}\frac{2x-|x|}{x^2+|x|}$$

It is clearly obvious from questions (20) and (21) above.

23)

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos^2 x-\sin^2 x}=$$

Solution:

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos^2 x-\sin^2 x}=\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x+\sin x}=\frac{1}{\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)}$$

$$=\frac{1}{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}=\frac{1}{\frac{2}{\sqrt{2}}}=\frac{\sqrt{2}}{2}$$

24)

$$\lim_{x\to 0}\frac{\cos^2 x+2\cos x-3}{2\cos^2 x-\cos x-1}=$$

Solution:

$$\lim_{x\to 0}\frac{\cos^2 x+2\cos x-3}{2\cos^2 x-\cos x-1}=\lim_{x\to 0}\frac{(\cos x+3)(\cos x-1)}{(2\cos x+1)(\cos x-1)}$$

$$=\lim_{x\to 0}\frac{\cos x+3}{2\cos x+1}=\frac{\cos(0)+3}{2\cos(0)+1}$$

$$=\frac{1+3}{2(1)+1}=\frac{4}{3}$$

25)

$$\lim_{x\to 0}(\sin^2 x+3\tan x-4)=$$

Solution:

$$\lim_{x\to 0}(\sin^2 x+3\tan x-4)=\sin^2(0)+3\tan(0)-4$$

$$=0+3(0)-4=-4$$

26) If $m\neq 0$, then

$$\lim_{x\to 0}\frac{\sin(nx)}{mx}=$$

Solution:

$$\lim_{x\to 0}\frac{\sin(nx)}{mx}=\frac{n}{m}\lim_{x\to 0}\frac{\sin(nx)}{nx}=\frac{n}{m}(1)=\frac{n}{m}$$

27) If $m\neq 0$, then

$$\lim_{x\to 0}\frac{\tan(nx)}{mx}=$$

Solution:

$$\lim_{x\to 0}\frac{\tan(nx)}{mx}=\frac{n}{m}\lim_{x\to 0}\frac{\tan(nx)}{nx}=\frac{n}{m}(1)=\frac{n}{m}$$

28) If $m\neq 0$, then

$$\lim_{x\to 0}\frac{nx}{\sin(mx)}=$$

Solution:

$$\lim_{x\to 0}\frac{nx}{\sin(mx)}=\frac{n}{m}\lim_{x\to 0}\frac{mx}{\sin(mx)}=\frac{n}{m}(1)=\frac{n}{m}$$

29) If $m\neq 0$, then

$$\lim_{x\to 0}\frac{nx}{\tan(mx)}=$$

Solution:

$$\lim_{x\to 0}\frac{nx}{\tan(mx)}=\frac{n}{m}\lim_{x\to 0}\frac{mx}{\tan(mx)}=\frac{n}{m}(1)=\frac{n}{m}$$

30) If $m \neq 0$, then

$$\lim_{x\to 0}\frac{\sin(nx)}{\sin(mx)} =$$

Solution:

$$\lim_{x\to 0}\frac{\sin(nx)}{\sin(mx)} = \frac{n}{m}\left(\lim_{x\to 0}\frac{\sin(nx)}{nx}\right)\left(\lim_{x\to 0}\frac{mx}{\sin(mx)}\right)$$

$$= \frac{n}{m}(1)(1) = \frac{n}{m}$$

31) If $m \neq 0$, then

$$\lim_{x\to 0}\frac{\sin(nx)}{\tan(mx)} =$$

Solution:

$$\lim_{x\to 0}\frac{\sin(nx)}{\tan(mx)} = \frac{n}{m}\left(\lim_{x\to 0}\frac{\sin(nx)}{nx}\right)\left(\lim_{x\to 0}\frac{mx}{\tan(mx)}\right)$$

$$= \frac{n}{m}(1)(1) = \frac{n}{m}$$

32) If $m \neq 0$, then

$$\lim_{x\to 0}\frac{\tan(nx)}{\tan(mx)} =$$

Solution:

$$\lim_{x\to 0}\frac{\tan(nx)}{\tan(mx)} = \frac{n}{m}\left(\lim_{x\to 0}\frac{\tan(nx)}{nx}\right)\left(\lim_{x\to 0}\frac{mx}{\tan(mx)}\right)$$

$$= \frac{n}{m}(1)(1) = \frac{n}{m}$$

33) If $m \neq 0$, then

$$\lim_{x\to 0}\frac{\tan(nx)}{\sin(mx)} =$$

Solution:

$$\lim_{x\to 0}\frac{\tan(nx)}{\sin(mx)} = \frac{n}{m}\left(\lim_{x\to 0}\frac{\tan(nx)}{nx}\right)\left(\lim_{x\to 0}\frac{mx}{\sin(mx)}\right)$$

$$= \frac{n}{m}(1)(1) = \frac{n}{m}$$

34)

$$\lim_{x\to 0}\frac{\sin(1-\cos x)}{1-\cos x} =$$

Solution:

$$\lim_{x\to 0}\frac{\sin(1-\cos x)}{1-\cos x} = 1$$

35)

$$\lim_{x\to 0}\frac{\sin(\sin(2x))}{\sin(2x)} =$$

Solution:

$$\lim_{x\to 0}\frac{\sin(\sin(2x))}{\sin(2x)} = 1$$

36)

$$\lim_{x\to 0}\frac{1-\cos(2x)}{x^2} =$$

Solution:

$$\lim_{x\to 0}\frac{1-\cos(2x)}{x^2} = \lim_{x\to 0}\frac{2\sin^2 x}{x^2} = 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2$$

$$= 2\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2 = 2(1)^2 = 2$$

37)

$$\lim_{x\to\infty}\sqrt{\frac{1}{x^2}-\frac{3}{x}+4} =$$

Solution:

$$\lim_{x\to\infty}\sqrt{\frac{1}{x^2}-\frac{3}{x}+4} = \sqrt{\lim_{x\to\infty}\left(\frac{1}{x^2}-\frac{3}{x}+4\right)} = \sqrt{0-0+4}$$

$$= 2$$

38)

$$\lim_{x\to\infty}\left(\frac{1}{x^{2/5}}+2\right) =$$

Solution:

$$\lim_{x\to-\infty}\left(\frac{1}{x^{2/5}}+2\right) = 0+2 = 2$$

39)

$$\lim_{x\to\infty}\frac{3x+15}{9x^2+4x-13} =$$

Solution:

$$\lim_{x\to\infty}\frac{3x+15}{9x^2+4x-13} = \lim_{x\to\infty}\frac{\frac{3x}{x^2}+\frac{15}{x^2}}{\frac{9x^2}{x^2}+\frac{4x}{x^2}-\frac{13}{x^2}}$$

$$= \lim_{x\to\infty}\frac{\frac{3}{x}+\frac{15}{x^2}}{9+\frac{4}{x}-\frac{13}{x^2}} = \frac{0+0}{9+0+0} = 0$$

40)

$$\lim_{x\to\infty}\frac{3x^2-8x+15}{9x^2+4x-13} =$$

Solution:

$$\lim_{x\to\infty}\frac{3x^2-8x+15}{9x^2+4x-13} = \lim_{x\to\infty}\frac{\frac{3x^2}{x^2}-\frac{8x}{x^2}+\frac{15}{x^2}}{\frac{9x^2}{x^2}+\frac{4x}{x^2}-\frac{13}{x^2}}$$

$$= \lim_{x\to\infty}\frac{3-\frac{8}{x}+\frac{15}{x^2}}{9+\frac{4}{x}-\frac{13}{x^2}} = \frac{3-0+0}{9+0+0} = \frac{1}{3}$$

41)

$$\lim_{x\to-\infty}\frac{3x^2-8x+15}{9x^2+4x-13} =$$

Solution:

$$\lim_{x\to-\infty}\frac{3x^2-8x+15}{9x^2+4x-13} = \lim_{x\to-\infty}\frac{\frac{3x^2}{-x^2}-\frac{8x}{-x^2}+\frac{15}{-x^2}}{\frac{9x^2}{-x^2}+\frac{4x}{-x^2}-\frac{13}{-x^2}}$$

$$= \lim_{x\to-\infty}\frac{-3+\frac{8}{x}-\frac{15}{x^2}}{-9-\frac{4}{x}+\frac{13}{x^2}} = \frac{-3+0-0}{-9-0+0} = \frac{1}{3}$$

42)

$$\lim_{x\to\infty}\frac{3x^5-8x+15}{9x^2+4x-13}=$$

Solution:

$$\lim_{x\to\infty}\frac{3x^5-8x+15}{9x^2+4x-13}=\lim_{x\to\infty}\frac{\frac{3x^5}{x^2}-\frac{8x}{x^2}+\frac{15}{x^2}}{\frac{9x^2}{x^2}+\frac{4x}{x^2}-\frac{13}{x^2}}$$

$$=\lim_{x\to\infty}\frac{3x^3-\frac{8}{x}+\frac{15}{x^2}}{9+\frac{4}{x}-\frac{13}{x^2}}=\frac{3(\infty)-0+0}{9+0+0}=\infty$$

43)

$$\lim_{x\to-\infty}\frac{3x^5-8x+15}{9x^2+4x-13}=$$

Solution:

$$\lim_{x\to-\infty}\frac{3x^5-8x+15}{9x^2+4x-13}=\lim_{x\to-\infty}\frac{\frac{3x^5}{-x^2}-\frac{8x}{-x^2}+\frac{15}{-x^2}}{\frac{9x^2}{-x^2}+\frac{4x}{-x^2}-\frac{13}{-x^2}}$$

$$=\lim_{x\to-\infty}\frac{-3x^3+\frac{8}{x}-\frac{15}{x^2}}{-9-\frac{4}{x}+\frac{13}{x^2}}=\frac{-3(-\infty)+0-0}{-9-0+0}=-\infty$$

44)

$$\lim_{x\to\infty}\left(\sqrt{x^2-3x+7}-x\right)=$$

Solution:

$$\lim_{x\to\infty}\left(\sqrt{x^2-3x+7}-x\right)$$

$$=\lim_{x\to\infty}\left[\left(\sqrt{x^2-3x+7}-x\right)\times\frac{\left(\sqrt{x^2-3x+7}+x\right)}{\left(\sqrt{x^2-3x+7}+x\right)}\right]$$

$$=\lim_{x\to\infty}\left(\frac{(x^2-3x+7)-x^2}{\sqrt{x^2-3x+7}+x}\right)=\lim_{x\to\infty}\left(\frac{-3x+7}{\sqrt{x^2-3x+7}+x}\right)$$

$$=\lim_{x\to\infty}\frac{\frac{-3x}{x}+\frac{7}{x}}{\frac{\sqrt{x^2-3x+7}}{x}+\frac{x}{x}}$$

$$=\lim_{x\to\infty}\frac{-3+\frac{7}{x}}{\sqrt{\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{7}{x^2}}+1}$$

$$=\lim_{x\to\infty}\frac{-3+\frac{7}{x}}{\sqrt{1-\frac{3}{x}+\frac{7}{x^2}}+1}$$

$$=\frac{-3+0}{\sqrt{1-0+0}+1}=\frac{-3}{1+1}=-\frac{3}{2}$$

45)

$$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)=$$

Solution:

$$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$$

$$=\lim_{x\to\infty}\left[\left(\sqrt{x^2+x}-x\right)\times\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}\right]$$

$$=\lim_{x\to\infty}\left(\frac{(x^2+x)-x^2}{\sqrt{x^2+x}+x}\right)$$

$$=\lim_{x\to\infty}\left(\frac{x}{\sqrt{x^2+x}+x}\right)$$

$$=\lim_{x\to\infty}\frac{\frac{x}{x}}{\frac{\sqrt{x^2+x}}{x}+\frac{x}{x}}=\lim_{x\to\infty}\frac{1}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+1}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\frac{1}{\sqrt{1+0}+1}=\frac{1}{1+1}$$

$$=\frac{1}{2}$$

46)

$$\lim_{x\to\infty}(x^2-5x+4)=$$

Solution:

$$\lim_{x\to\infty}(x^2-5x+4)=\lim_{x\to\infty}x^2\left(\frac{x^2}{x^2}-\frac{5x}{x^2}+\frac{4}{x^2}\right)$$

$$=\lim_{x\to\infty}x^2\left(1-\frac{5}{x}+\frac{4}{x^2}\right)=(\infty)^2(1-0+0)=\infty$$

OR

$$\lim_{x\to\infty}(x^2-5x+4)=\lim_{x\to\infty}(x^2)=\infty$$

47)

$$\lim_{x\to-\infty}(x^4-2x^3+9)=$$

Solution:

$$\lim_{x\to-\infty}(x^4-2x^3+9)=\lim_{x\to-\infty}x^4\left(\frac{x^4}{x^4}-\frac{2x^3}{x^4}+\frac{9}{x^4}\right)$$

$$=\lim_{x\to-\infty}x^4\left(1-\frac{2}{x}+\frac{9}{x^4}\right)=(-\infty)^4(1-0+0)=\infty$$

OR

$$\lim_{x\to-\infty}(x^4-2x^3+9)=\lim_{x\to-\infty}(x^4)=\infty$$

48)

$$\lim_{x\to-\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=$$

Solution:

$$\lim_{x\to-\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=\lim_{x\to-\infty}\frac{\frac{\sqrt{3x^2-8}}{-x}+\frac{2}{-x}}{\frac{x}{-x}+\frac{5}{-x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{3x^2-8}{x^2}}-\frac{2}{x}}{-1-\frac{5}{x}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{3x^2}{x^2}-\frac{8}{x^2}}-\frac{2}{x}}{-1-\frac{5}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{3-\frac{8}{x^2}}-\frac{2}{x}}{-1-\frac{5}{x}}=\frac{\sqrt{3-0}-0}{-1-0}=-\sqrt{3}$$

49)

$$\lim_{x\to\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=$$

Solution:

$$\lim_{x\to\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=\lim_{x\to\infty}\frac{\frac{\sqrt{3x^2-8}}{x}+\frac{2}{x}}{\frac{x}{x}+\frac{5}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{\frac{3x^2-8}{x^2}}+\frac{2}{x}}{1+\frac{5}{x}}=\lim_{x\to\infty}\frac{\sqrt{\frac{3x^2}{x^2}-\frac{8}{x^2}}+\frac{2}{x}}{1+\frac{5}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{3-\frac{8}{x^2}}+\frac{2}{x}}{1+\frac{5}{x}}=\frac{\sqrt{3-0}+0}{1+0}=\sqrt{3}$$

50) The horizontal asymptotes of

$$f(x)=\frac{\sqrt{3x^2-8}+2}{x+5}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{\sqrt{3x^2-8}+2}{x+5}$$

It is clear from the previous questions (48) and (49) that

$$\lim_{x\to\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=\sqrt{3}$$

and

$$\lim_{x\to-\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=-\sqrt{3}$$

Thus, the horizontal asymptotes are

$$y=\pm\sqrt{3}$$

51) The horizontal asymptote of

$$f(x)=\frac{1-x}{2x+1}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{1-x}{2x+1}$$

$$\lim_{x\to\infty}\frac{1-x}{2x+1}=\lim_{x\to\infty}\frac{\frac{1}{x}-\frac{x}{x}}{\frac{2x}{x}+\frac{1}{x}}=\lim_{x\to\infty}\frac{\frac{1}{x}-1}{2+\frac{1}{x}}=\frac{0-1}{2+0}=-\frac{1}{2}$$

$$\lim_{x\to-\infty}\frac{1-x}{2x+1}=\lim_{x\to-\infty}\frac{\frac{1}{-x}-\frac{x}{-x}}{\frac{2x}{-x}+\frac{1}{-x}}=\lim_{x\to-\infty}\frac{\frac{1}{-x}+1}{-2-\frac{1}{x}}=\frac{0+1}{-2-0}$$

$$=-\frac{1}{2}$$

Thus, the horizontal asymptote is

$$y=-\frac{1}{2}$$

52) The horizontal asymptote of

$$f(x)=\frac{7x^2+5}{3x^2+2}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{7x^2+5}{3x^2+2}$$

$$\lim_{x\to\infty}\frac{7x^2+5}{3x^2+2}=\lim_{x\to\infty}\frac{\frac{7x^2}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{2}{x^2}}=\lim_{x\to\infty}\frac{7+\frac{5}{x^2}}{3+\frac{2}{x^2}}=\frac{7+0}{3+0}=\frac{7}{3}$$

$$\lim_{x\to-\infty}\frac{7x^2+5}{3x^2+2}=\lim_{x\to-\infty}\frac{\frac{7x^2}{-x^2}+\frac{5}{-x^2}}{\frac{3x^2}{-x^2}+\frac{2}{-x^2}}$$

$$=\lim_{x\to-\infty}\frac{-7-\frac{5}{x^2}}{-3-\frac{2}{x^2}}=\frac{-7-0}{-3-0}=\frac{7}{3}$$

Thus, the horizontal asymptote is

$$y=\frac{7}{3}$$

53) The horizontal asymptote of

$$f(x)=\frac{\sqrt{x^2+2x-3}}{2x+7}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{\sqrt{x^2+2x-3}}{2x+7}$$

$$\lim_{x\to\infty}\frac{\sqrt{x^2+2x-3}}{2x+7}=\lim_{x\to\infty}\frac{\frac{\sqrt{x^2+2x-3}}{x}}{\frac{2x}{x}+\frac{7}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{\frac{x^2+2x-3}{x^2}}}{2+\frac{7}{x}}=\lim_{x\to\infty}\frac{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}}{2+\frac{7}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}}{2+\frac{7}{x}}=\frac{\sqrt{1+0-0}}{2+0}=\frac{1}{2}$$

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+2x-3}}{2x+7}=\lim_{x\to-\infty}\frac{\frac{\sqrt{x^2+2x-3}}{-x}}{\frac{2x}{-x}+\frac{7}{-x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{x^2+2x-3}{x^2}}}{-2-\frac{7}{x}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}}{-2-\frac{7}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}}{-2-\frac{7}{x}}=\frac{\sqrt{1+0-0}}{-2-0}=-\frac{1}{2}$$

Thus, the horizontal asymptotes are

$$y=\pm\frac{1}{2}$$

54) The horizontal asymptote of

$$f(x)=\frac{\sqrt{2x-3}}{2x^2+7x-1}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{\sqrt{2x-3}}{2x^2+7x-1}$$

$$\lim_{x\to\infty}\frac{\sqrt{2x-3}}{2x^2+7x-1}=\lim_{x\to\infty}\frac{\frac{\sqrt{2x-3}}{x^2}}{\frac{2x^2}{x^2}+\frac{7x}{x^2}-\frac{1}{x^2}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{\frac{2x-3}{x^4}}}{2+\frac{7}{x}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\sqrt{\frac{2x}{x^4}-\frac{3}{x^4}}}{2+\frac{7}{x}-\frac{1}{x^2}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{\frac{2}{x^3}-\frac{3}{x^4}}}{2+\frac{7}{x}-\frac{1}{x^2}}=\frac{\sqrt{0-0}}{2+0-0}=\frac{0}{2}=0$$

$$\lim_{x\to-\infty}\frac{\sqrt{2x-3}}{2x^2+7x-1}=\lim_{x\to-\infty}\frac{\frac{\sqrt{2x-3}}{-x^2}}{\frac{2x^2}{-x^2}+\frac{7x}{-x^2}-\frac{1}{-x^2}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{2x-3}{x^4}}}{-2-\frac{7}{x}+\frac{1}{x^2}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{2x}{x^4}-\frac{3}{x^4}}}{-2-\frac{7}{x}+\frac{1}{x^2}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{2}{x^3}-\frac{3}{x^4}}}{-2-\frac{7}{x}+\frac{1}{x^2}}=\frac{\sqrt{0-0}}{-2-0+0}=\frac{0}{-2}=0$$

Thus, the horizontal asymptote is

$$y=0$$

55)

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2-8}+3}{x+1}=$$

Solution:

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2-8}+3}{x+1}=\lim_{x\to-\infty}\frac{\frac{\sqrt{4x^2-8}}{-x}+\frac{3}{-x}}{\frac{x}{-x}+\frac{1}{-x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{4x^2-8}{x^2}}-\frac{3}{x}}{-1-\frac{1}{x}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{4x^2}{x^2}-\frac{8}{x^2}}-\frac{3}{x}}{-1-\frac{1}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{4-\frac{8}{x^2}}-\frac{3}{x}}{-1-\frac{1}{x}}=\frac{\sqrt{4-0}-0}{-1-0}=-2$$

56)

$$\lim_{x\to\infty}\frac{\sqrt{4x^2-8}+3}{x+1}=$$

Solution:

$$\lim_{x\to\infty}\frac{\sqrt{4x^2-8}+3}{x+1}=\lim_{x\to\infty}\frac{\frac{\sqrt{4x^2-8}}{x}+\frac{3}{x}}{\frac{x}{x}+\frac{1}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{4x^2-8}{x^2}}+\frac{3}{x}}{1+\frac{1}{x}}=\lim_{x\to\infty}\frac{\sqrt{\frac{4x^2}{x^2}-\frac{8}{x^2}}+\frac{3}{x}}{1+\frac{1}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{4-\frac{8}{x^2}}+\frac{3}{x}}{1+\frac{1}{x}}=\frac{\sqrt{4-0}+0}{1+0}=2$$

Workshop Solutions to Chapter 4 (chapter 3)

1) If $f(x)$ is a differentiable function, then $f'(x) =$

Solution:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

2) If $f(x) = 4x^2$, then $f'(x) =$

Solution:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{4(x+h)^2 - 4x^2}{h}$$

3) If $f(x) = x^2 - 3$, then $f'(x) =$

Solution:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^2 - 3] - [x^2 - 3]}{h}$$

4) If $f(x) = \sqrt{x}$, $x \geq 0$, then $f'(x) =$

Solution:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

5) If f is a differentiable function at a, then f is a continuous function at a.

6) If f is a continuous function at a, then f is a differentiable function at a.

Solution:

False

7) If $y = x^4 + 5x^2 + 3$, then $y' =$

Solution:

$$y' = 4x^3 + 10x$$

8) If $y = x^4 - 5x^2 + 3$, then $y' =$

Solution:

$$y' = 4x^3 - 10x$$

9) If $y = x^{-5/2}$, then $y' =$

Solution:

$$y' = -\frac{5}{2} x^{-\frac{5}{2}-1} = -\frac{5}{2} x^{-7/2}$$

10) If $y = \frac{1}{3x^3} + 2\sqrt{x} = \frac{1}{3}x^{-3} + 2x^{1/2}$, then $y' =$

Solution:

$$y' = (-3)\left(\frac{1}{3}\right) x^{-3-1} + \left(\frac{1}{2}\right)(2)x^{\frac{1}{2}-1}$$

$$= -x^{-4} + x^{-1/2} = -\frac{1}{x^4} + \frac{1}{x^{1/2}} = -\frac{1}{x^4} + \frac{1}{\sqrt{x}}$$

11) If $y = (x-3)(x-2)$, then $y' =$

Solution:

$$y = (x-3)(x-2) = x^2 - 5x + 6$$

$$y' = 2x - 5$$

12) If $y = (x^3+3)(x^2-1)$, then $y' =$

Solution:

$$y = (x^3+3)(x^2-1) = x^5 - x^3 + 3x^2 - 3$$

$$y' = 5x^4 - 3x^2 + 6x$$

13) If $y = \sqrt{x}(2x+1)$, then $y' =$

Solution:

$$y = \sqrt{x}(2x+1) = 2x\sqrt{x} + \sqrt{x} = 2x^{\frac{3}{2}} + x^{\frac{1}{2}}$$

$$y' = \left(\frac{3}{2}\right)(2)x^{\frac{3}{2}-1} + \left(\frac{1}{2}\right)x^{\frac{1}{2}-1} = 3x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$= 3\sqrt{x} + \frac{1}{2\sqrt{x}}$$

OR

Use the rule $(f.g)' = f'g + fg'$

$$y' = (2)(\sqrt{x}) + \left(\frac{1}{2\sqrt{x}}\right)(2x+1) = 2\sqrt{x} + \frac{2x+1}{2\sqrt{x}}$$

14) If $y = \frac{x+3}{x-2}$, then $y' =$

Solution:

Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$y' = \frac{(1)(x-2) - (x+3)(1)}{(x-2)^2} = \frac{x-2-x-3}{(x-2)^2} = \frac{-5}{(x-2)^2}$$

$$= -\frac{5}{(x-2)^2}$$

15) If $y = \frac{x+3}{x-2}$, then $y'|_{x=4} =$

Solution:

$$y' = \frac{(1)(x-2) - (x+3)(1)}{(x-2)^2} = \frac{x-2-x-3}{(x-2)^2}$$

$$= \frac{-5}{(x-2)^2} = -\frac{5}{(x-2)^2}$$

$$y'|_{x=4} = -\frac{5}{(4-2)^2} = -\frac{5}{4}$$

16) If $y = \frac{x-1}{x+2}$, then $y' =$

Solution:

Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$y' = \frac{(1)(x+2) - (x-1)(1)}{(x+2)^2} = \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$

17) If $y = \sqrt{3x^2 + 6x}$, then $y' =$

Solution:

Use the rule $\left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}}$

$$y' = \frac{6x + 6}{2\sqrt{3x^2 + 6x}} = \frac{6(x+1)}{2\sqrt{3x^2 + 6x}} = \frac{3(x+1)}{\sqrt{3x^2 + 6x}}$$

18) If $y = \sqrt{3x^2 + 6x}$, then $y'|_{x=1} =$

Solution:

$$y' = \frac{6x + 6}{2\sqrt{3x^2 + 6x}} = \frac{6(x+1)}{2\sqrt{3x^2 + 6x}} = \frac{3(x+1)}{\sqrt{3x^2 + 6x}}$$

$$y'|_{x=1} = \frac{3((1)+1)}{\sqrt{3(1)^2 + 6(1)}} = \frac{6}{\sqrt{9}} = \frac{6}{3} = 2$$

19) The tangent line equation to the curve $y = x^2 + 2$ at the point $(1,3)$ is

Solution:

First, we have to find the slope of the curve which is

$$y' = 2x$$

Thus, the slope at $x = 1$ is

$$y'|_{x=1} = 2(1) = 2$$

Hence, the tangent line equation passing through the point $(1,3)$ with slope $m = 2$ is

$$y - 3 = 2(x - 1)$$
$$y - 3 = 2x - 2$$
$$y = 2x - 2 + 3$$
$$y = 2x + 1$$

20) The tangent line equation to the curve $y = \frac{2x}{x+1}$ at the point $(0,0)$ is

Solution:

First, we have to find the slope of the curve which is

$$y' = \frac{(2)(x+1) - (2x)(1)}{(x+1)^2} = \frac{2x + 2 - 2x}{(x+1)^2} = \frac{2}{(x+1)^2}$$

Thus, the slope at $x = 0$ is

$$y'|_{x=0} = \frac{2}{(0+1)^2} = 2$$

Hence, the tangent line equation passing through the point $(0,0)$ with slope $m = 2$ is

$$y - 0 = (2)(x - 0)$$
$$y = 2x$$

21) The tangent line equation to the curve $y = 3x^2 - 13$ at the point $(2, -1)$ is

Solution:

First, we have to find the slope of the curve which is

$$y' = 6x$$

Thus, the slope at $x = 2$ is

$$y'|_{x=2} = 6(2) = 12$$

Hence, the tangent line equation passing through the point $(2, -1)$ with slope $m = 12$ is

$$y - (-1) = 12(x - 2)$$
$$y + 1 = 12x - 24$$
$$y = 12x - 24 - 1$$
$$y = 12x - 25$$

22) The tangent line equation to the curve $y = 3x^2 + 2x + 5$ at the point $(0,5)$ is

Solution:

First, we have to find the slope of the curve which is

$$y' = 6x + 2$$

Thus, the slope at $x = 2$ is

$$y'|_{x=0} = 6(0) + 2 = 2$$

Hence, the tangent line equation passing through the point $(0,5)$ with slope $m = 2$ is

$$y - 5 = 2(x - 0)$$
$$y - 5 = 2x$$
$$y = 2x + 5$$

23) If $y = xe^x$, then $y' =$

Solution:

Use the rules $(f.g)' = f'g + fg'$ and $(e^u) = e^u.u'$

$$y' = (1)(e^x) + (x)(e^x) = e^x + xe^x = e^x(1 + x)$$

24) If $y = x - e^x$, then $y'' =$

Solution:

Use the rules $(f - g)' = f' - g'$ and $(e^u) = e^u.u'$

$$y' = 1 - e^x$$
$$y'' = -e^x$$

25) If $x^2 - y^2 = 4$, then $y' =$

Solution:

$$2x - 2yy' = 0$$
$$-2yy' = -2x$$
$$y' = \frac{-2x}{-2y}$$
$$y' = \frac{x}{y}$$

26) If $x^2 + y^2 = 4$, then $y' =$

Solution:

$$2x + 2yy' = 0$$
$$2yy' = -2x$$
$$y' = \frac{-2x}{2y}$$
$$y' = -\frac{x}{y}$$

27) If $y = \frac{x+1}{x+2}$, then $y' =$

Solution:

Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$y' = \frac{(1)(x+2) - (x+1)(1)}{(x+2)^2} = \frac{x + 2 - x - 1}{(x+2)^2}$$
$$= \frac{1}{(x+2)^2}$$

28) If $y = \frac{1}{\sqrt[2]{x^5}} + \sec x$, then $y' =$

Solution:

Use the rules

$(f + g)' = f' + g'$ and $(\sec u)' = \sec u \tan u \,. u'$

$$y = \frac{1}{\sqrt[2]{x^5}} + \sec x = x^{-\frac{5}{2}} + \sec x$$
$$y' = \left(-\frac{5}{2}\right)x^{-\frac{5}{2}-1} + \sec x \tan x = -\frac{5}{2}x^{-7/2} + \sec x \tan x$$

29) If $y = \tan^{-1}(x^3)$, then $y' =$

Solution:

Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$

$$y' = \frac{1}{1+(x^3)^2} . (3x^2) = \frac{3x^2}{1+x^6}$$

30) If $y = \tan x - x$, then $y' =$

Solution:

Use the rules

$(f-g)' = f' - g'$ and $(\tan u)' = \sec^2 u \; .u'$

$$y' = \sec^2 x - 1$$

31) If $y = \sec^2 x - 1$, then $y' =$

Solution:

Use the rules $(f-g)' = f' - g'$, $(u)^n = n(u)^{n-1}.u'$ and $(\sec u)' = \sec u \tan u \, . u'$

$$y' = 2\sec x \, . \, \sec x \tan x = 2\sec^2 x \tan x$$

32) If $y = x^{\sin x}$, then $y' =$

Solution:

Use the rule $(\sin u)' = \cos u \; .u'$

$$y = x^{\sin x}$$
$$\ln y = \ln x^{\sin x}$$
$$\ln y = \sin x \, . \ln x$$
$$\frac{y'}{y} = \cos x \, . \, \ln x + \sin x \, . \frac{1}{x} = \cos x \, . \, \ln x + \frac{\sin x}{x}$$
$$y' = y\left(\cos x \, . \, \ln x + \frac{\sin x}{x}\right) = x^{\sin x}\left(\cos x \, . \, \ln x + \frac{\sin x}{x}\right)$$

33) If $y = x^{\cos x}$, then $y' =$

Solution:

Use the rule $(\cos u)' = -\sin u \; .u'$

$$y = x^{\cos x}$$
$$\ln y = \ln x^{\cos x}$$
$$\ln y = \cos x \, . \ln x$$
$$\frac{y'}{y} = -\sin x \, . \, \ln x + \cos x \, . \frac{1}{x} = -\sin x \, . \, \ln x + \frac{\cos x}{x}$$
$$y' = y\left(-\sin x \, . \, \ln x + \frac{\cos x}{x}\right)$$
$$= x^{\cos x}\left(\frac{\cos x}{x} - \sin x \, . \, \ln x\right)$$

34) If $y = (2x^2 + \csc x)^9$, then $y' =$

Solution:

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\csc u)' = -\csc u \cot u \, . u'$

$$y' = 9(2x^2 + \csc x)^8 . (4x - \csc x \cot x)$$

35) If $y = \frac{5^x}{\cot x}$, then $y' =$

Solution:

Use the rules

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$, $(a^u)' = a^u . \ln a \, . u'$

and $(\csc u)' = -\csc u \cot u \, . u'$

$$y' = \frac{(5^x \ln 5)(\cot x) - (5^x)(-\csc^2 x)}{(\cot x)^2}$$
$$= \frac{5^x(\ln 5 \cot x + \csc^2 x)}{\cot^2 x}$$

36) If $y = e^{2x}$, then $y^{(6)} =$

Solution:

Use the rule $(e^u)' = e^u . u'$

$$y' = 2e^{2x}$$
$$y'' = 4e^{2x}$$
$$y''' = 8e^{2x}$$
$$y^{(4)} = 16e^{2x}$$
$$y^{(5)} = 32e^{2x}$$
$$y^{(6)} = 64e^{2x}$$

37) If $y = x^{-2}e^{\sin x}$, then $y' =$

Solution:

Use the rules $(f.g)' = f'g + fg'$, $(e^u) = e^u . u'$ and $(\sin u)' = \cos u \; .u'$

$$y' = (-2x^{-3})(e^{\sin x}) + (x^{-2})(e^{\sin x} . \cos x)$$
$$= -2x^{-3}e^{\sin x} + x^{-2}\cos x \, e^{\sin x}$$
$$= x^{-3}e^{\sin x}(-2 + x\cos x)$$
$$= x^{-3}e^{\sin x}(x\cos x - 2)$$

38) If $y = 5^{\tan x}$, then $y' =$

Solution:

Use the rules

$(a^u)' = a^u . \ln a \, . u'$ and $(\tan u)' = \sec^2 u \, . u'$

$$y' = 5^{\tan x} . \ln 5 \, . \sec^2 x$$

39) If $x^2 + y^2 = 3xy + 7$, then $y' =$

Solution:

$$2x + 2yy' = 3y + 3xy'$$
$$2yy' - 3xy' = 3y - 2x$$
$$y'(2y - 3x) = 3y - 2x$$
$$y' = \frac{3y - 2x}{2y - 3x}$$

40) If $y = \sin^3(4x)$, then $y^{(6)} =$ y'=

Solution:

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\sin u)' = \cos u \, . u'$

$$y' = 3\sin^2(4x) . \cos(4x) . (4)$$
$$= 12\sin^2(4x) . \cos(4x)$$

41) If $y = 3^x \cot x$, then $y' =$

<u>Solution:</u>

Use the rules $(f.g)' = f'g + fg'$, $(a^u)' = a^u . \ln a . u'$ and $(\cot u)' = -\csc^2 u\ .u'$

$$y' = (3^x . \ln 3)(\cot x) + (3^x)(-\csc^2 x)$$
$$= 3^x \ln 3\ \cot x - 3^x \csc^2 x$$
$$= 3^x(\ln 3\ \cot x - \csc^2 x)$$

42) If $y = (2x^2 + \sec x)^7$, then $y' =$

<u>Solution:</u>

Use the rules $(u)^n = n(u)^{n-1}.u'$ and $(\sec u)' = \sec u \tan u . u'$

$$y' = 7(2x^2 + \sec x)^6 . (4x + \sec x \tan x)$$

43) If $f(x) = \cos x$, then $f^{(45)}(x) =$

<u>Solution:</u>

$$f'(x) = -\sin x$$
$$f''(x) = -\cos x$$
$$f'''(x) = \sin x$$
$$f^{(4)}(x) = \cos x$$

Note: $f^{(n)}(x) = \cos x$ whenever n is a multiple of 4.

Hence,

$$f^{(44)}(x) = \cos x$$
$$f^{(45)}(x) = -\sin x$$

44) If $D^{47}(\sin x) =$

<u>Solution:</u>

$$D(\sin x) = \cos x$$
$$D^2(\sin x) = -\sin x$$
$$D^3(\sin x) = -\cos x$$
$$D^4(\sin x) = \sin x$$

Note: $D^n(\sin x) = \sin x$ whenever n is a multiple of 4.

Hence,

$$D^{44}(\sin x) = \sin x$$
$$D^{45}(\sin x) = \cos x$$
$$D^{46}(\sin x) = -\sin x$$
$$D^{47}(\sin x) = -\cos x$$

45) If $y = x^x$, then $y' =$

<u>Solution:</u>

Use the rule $(\ln u)' = \frac{u'}{u}$

$$y = x^x$$
$$\ln y = \ln x^x$$
$$\ln y = x \ln x$$
$$\frac{y'}{y} = (1)(\ln x) + (x)\left(\frac{1}{x}\right)$$
$$\frac{y'}{y} = \ln x + 1$$
$$y' = y(1 + \ln x) = x^x(1 + \ln x)$$

46) If $f(x) = \frac{\ln x}{x^2}$, then $f'(1) =$

<u>Solution:</u>

Use the rules $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ and $(\ln u)' = \frac{u'}{u}$

$$f'(x) = \frac{\left(\frac{1}{x}\right)(x^2) - (\ln x)(2x)}{(x^2)^2} = \frac{x - 2x \ln x}{x^4}$$
$$= \frac{x(1 - 2\ln x)}{x^4} = \frac{1 - 2\ln x}{x^3}$$

$$\therefore \quad f'(1) = \frac{1 - 2\ln(1)}{(1)^3} = \frac{1 - 2(0)}{1} = 1$$

47) If $y = \cot^{-1}(e^x)$, then $y' =$

<u>Solution:</u>

Use the rules $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ and $(e^u) = e^u.u'$

$$y' = -\frac{1}{1 + (e^x)^2} . e^x = -\frac{e^x}{1 + e^{2x}}$$

48) If $y = \tan^{-1}(e^x)$, then $y' =$

<u>Solution:</u>

Use the rules $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ and $(e^u) = e^u.u'$

$$y' = \frac{1}{1 + (e^x)^2} . e^x = \frac{e^x}{1 + e^{2x}}$$

49) If $y = \sin^{-1}(e^x)$, then $y' =$

<u>Solution:</u>

Use the rules $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ and $(e^u) = e^u.u'$

$$y' = \frac{1}{\sqrt{1 - (e^x)^2}} . e^x = \frac{e^x}{\sqrt{1 - e^{2x}}}$$

50) If $y = \cos^{-1}(e^x)$, then $y' =$

<u>Solution:</u>

Use the rules $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ and $(e^u) = e^u.u'$

$$y' = -\frac{1}{\sqrt{1 - (e^x)^2}} . e^x = -\frac{e^x}{\sqrt{1 - e^{2x}}}$$

51) If $y = \cos(2x^3)$, then $y' =$

<u>Solution:</u>

Use the rule $(\cos u)' = -\sin u\ .u'$

$$y' = -\sin(2x^3) . (6x^2) = -6x^2 \sin(2x^3)$$

52) If $y = \csc x \cot x$, then $y' =$

<u>Solution:</u>

Use the rules $(f.g)' = f'g + fg'$, $(\csc u)' = -\csc u \cot u . u'$ and $(\cot u)' = -\csc^2 u\ .u'$

$$y' = (-\csc x \cot x)(\cot x) + (\csc x)(-\csc^2 x)$$
$$= -\csc x \cot^2 x - \csc^3 x = -\csc x(\cot^2 x + \csc^2 x)$$

53) If $y = \sqrt{x^2 - 2\sec x}$, then $y' =$

Solution:

Use the rules

$$\left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}} \quad \text{and} \quad (\sec u)' = \sec u \tan u\,.\,u'$$

$$y' = \frac{2x - 2\sec x \tan x}{2\sqrt{x^2 - 2\sec x}} = \frac{2(x - \sec x \tan x)}{2\sqrt{x^2 - 2\sec x}} = \frac{x - \sec x \tan x}{\sqrt{x^2 - 2\sec x}}$$

54) If $y = (3x^2 + 1)^6$, then $y' =$

Solution:

Use the rule $\quad (u)^n = n(u)^{n-1}.u'$

$$y' = 6(3x^2 + 1)^5.(6x) = 36x(3x^2 + 1)^5$$

55) If $xy + \tan x = 2x^3 + \sin y$, then $y' =$

Solution:

$$[(1)(y) + (x)(y')] + \sec^2 x = 6x^2 + \cos y\,.\,y'$$
$$y + xy' + \sec^2 x = 6x^2 + y' \cos y$$
$$xy' - y' \cos y = 6x^2 - y - \sec^2 x$$
$$y'(x - \cos y) = 6x^2 - y - \sec^2 x$$
$$y' = \frac{6x^2 - y - \sec^2 x}{x - \cos y}$$

56) If $y = x^{-1} \sec x$, then $y' =$

Solution:

Use the rules

$(f.g)' = f'g + fg'$ and $(\sec u)' = \sec u \tan u\,.\,u'$

$$y' = (-x^{-2})(\sec x) + (x^{-1})(\sec x \tan x)$$
$$= x^{-1} \sec x \tan x - x^{-2} \sec x$$
$$= x^{-2} \sec x\,(x \tan x - 1)$$

57) If $y = \sin^{-1}(x^3)$, then $y' =$

Solution:

Use the rule $\quad (\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$

$$y' = \frac{1}{\sqrt{1 - (x^3)^2}}\,.3x^2 = \frac{3x^2}{\sqrt{1 - x^6}}$$

58) If $y = \cos^{-1}(x^3)$, then $y' =$

Solution:

Use the rule $\quad (\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$

$$y' = -\frac{1}{\sqrt{1 - (x^3)^2}}\,.3x^2 = -\frac{3x^2}{\sqrt{1 - x^6}}$$

59) If $y = \sec^{-1}(x^3)$, then $y' =$

Solution:

Use the rule $\quad (\sec^{-1} u)' = \frac{u'}{|u|\sqrt{u^2-1}}$

$$y' = \frac{1}{x^3\sqrt{(x^3)^2 - 1}}\,.3x^2 = \frac{3x^2}{x^3\sqrt{x^6 - 1}} = \frac{3}{x\sqrt{x^6 - 1}}$$

60) If $y = \csc^{-1}(x^3)$, then $y' =$

Solution:

Use the rule $\quad (\csc^{-1} u)' = -\frac{u'}{|u|\sqrt{u^2-1}}$

$$y' = -\frac{1}{x^3\sqrt{(x^3)^2 - 1}}\,.3x^2 = -\frac{3x^2}{x^3\sqrt{x^6 - 1}} = -\frac{3}{x\sqrt{x^6 - 1}}$$

61) If $y = \ln(x^3 - 2\sec x)$, then $y' =$

Solution:

Use the rules

$$(\ln u)' = \frac{u'}{u} \quad \text{and} \quad (\sec u)' = \sec u \tan u\,.\,u'$$

$$y' = \frac{1}{x^3 - 2\sec x}\,.\,(3x^2 - 2\sec x \tan x) = \frac{3x^2 - 2\sec x \tan x}{x^3 - 2\sec x}$$

62) If $y = \ln(\cos x)$, then $y' =$

Solution:

Use the rules

$$(\ln u)' = \frac{u'}{u} \quad \text{and} \quad (\cos u)' = -\sin u\,.\,u'$$

$$y' = \frac{1}{\cos x}\,.\,(-\sin x) = -\frac{\sin x}{\cos x} = -\tan x$$

63) If $y = \ln(\sin x)$, then $y' =$

Solution:

Use the rules

$$(\ln u)' = \frac{u'}{u} \quad \text{and} \quad (\sin u)' = \cos u\,.\,u'$$

$$y' = \frac{1}{\sin x}\,.\,(\cos x) = \frac{\cos x}{\sin x} = \cot x$$

64) If $y = \ln\sqrt{3x^2 + 5x}$, then $y' =$

Solution:

Use the rules $\quad (\ln u)' = \frac{u'}{u} \quad$ and $\quad \left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}}$

$$y' = \frac{1}{\sqrt{3x^2 + 5x}}\,.\left(\frac{6x + 5}{2\sqrt{3x^2 + 5x}}\right) = \frac{6x + 5}{2(3x^2 + 5x)}$$

65) If $y=\log_5(x^3-2\csc x)$, then $y'=$

Solution:

Use the rules

$$(\log_a u)'=\frac{u'}{u\ln a} \quad \text{and} \quad (\csc u)'=-\csc u\cot u\,.\,u'$$

$$y'=\frac{1}{(x^3-2\csc x)(\ln 5)}\,.\,[3x^2-2\,(-\csc x\cot x)]$$
$$=\frac{3x^2+2\csc x\cot x}{(x^3-2\csc x)(\ln 5)}$$

66) If $y=\ln\frac{x-1}{\sqrt{x+2}}$, then $y'=$

Solution:

Use the rules

$$(\ln u)'=\frac{u'}{u}\ ,\ \left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2} \quad \text{and} \quad \left(\sqrt{u}\right)'=\frac{u'}{2\sqrt{u}}$$

$$y'=\frac{1}{\frac{x-1}{\sqrt{x+2}}}\cdot\left(\frac{(1)\left(\sqrt{x+2}\right)-(x-1)\left(\frac{1}{2\sqrt{x+2}}\right)}{\left(\sqrt{x+2}\right)^2}\right)$$
$$=\frac{\sqrt{x+2}}{x-1}\cdot\left(\frac{\sqrt{x+2}-\frac{x-1}{2\sqrt{x+2}}}{x+2}\right)$$
$$=\frac{\sqrt{x+2}}{x-1}\cdot\left(\frac{\frac{2(x+2)-(x-1)}{2\sqrt{x+2}}}{x+2}\right)$$
$$=\frac{\sqrt{x+2}}{x-1}\cdot\left(\frac{\frac{x+5}{2\sqrt{x+2}}}{x+2}\right)$$
$$=\frac{\sqrt{x+2}}{x-1}\left(\frac{x+5}{2(x+2)\sqrt{x+2}}\right)$$
$$=\frac{x+5}{2(x-1)(x+2)}$$

67) If $y=2x^3-\sin x$, then $y'=$

Solution:

Use the rule $\quad (\sin u)'=\cos u\,.\,u'$

$$y'=6x^2-\cos x$$

68) If $y=x^3\cos x$, then $y'=$

Solution:

Use the rules

$$(f.g)'=f'g+fg' \quad \text{and} \quad (\cos u)'=-\sin u\,.\,u'$$

$$y'=(3x^2)(\cos x)+(x^3)(-\sin x)$$
$$=3x^2\cos x-x^3\sin x$$

69) If $y=x^{\sqrt{x}}$, then $y'=$

Solution:

Use the rule $\quad \left(\sqrt{u}\right)'=\frac{u'}{2\sqrt{u}}$

$$y=x^{\sqrt{x}}$$
$$\ln y=\ln x^{\sqrt{x}}$$
$$\ln y=\sqrt{x}\ln x$$
$$\frac{y'}{y}=\left(\frac{1}{2\sqrt{x}}\right)(\ln x)+\left(\sqrt{x}\right)\left(\frac{1}{x}\right)$$
$$\frac{y'}{y}=\frac{\ln x}{2\sqrt{x}}+\frac{\sqrt{x}}{x}=\frac{x\ln x+2x}{2x\sqrt{x}}=\frac{x(\ln x+2)}{2x\sqrt{x}}$$
$$=\frac{\ln x+2}{2\sqrt{x}}$$
$$y'=y\left(\frac{\ln x+2}{2\sqrt{x}}\right)=x^{\sqrt{x}}\left(\frac{\ln x+2}{2\sqrt{x}}\right)$$

70) If $y=(\sin x)^x$, then $y'=$

Solution:

Use the rule $\quad (\sin u)'=\cos u\,.\,u'$

$$y=(\sin x)^x$$
$$\ln y=\ln(\sin x)^x$$
$$\ln y=x\ln(\sin x)$$
$$\frac{y'}{y}=(1)(\ln(\sin x))+(x)\left(\frac{\cos x}{\sin x}\right)$$
$$\frac{y'}{y}=\ln(\sin x)+\frac{x\cos x}{\sin x}=\ln(\sin x)+x\cot x$$
$$y'=y(\ln(\sin x)+x\cot x)$$
$$=(\sin x)^x(\ln(\sin x)+x\cot x)$$

71) If $y=\log_7(x^3-2)$, then $y'=$

Solution:

Use the rule $\quad (\log_a u)'=\frac{u'}{u\ln a}$

$$y'=\frac{1}{(x^3-2)(\ln 7)}\,.\,(3x^2)=\frac{3x^2}{(x^3-2)(\ln 7)}$$

72) If $y=\cos(x^5)$, then $y'=$

Solution:

Use the rule $\quad (\cos u)'=-\sin u\,.\,u'$

$$y'=-\sin(x^5)\,.\,(5x^4)=-5x^4\sin(x^5)$$

73) If $y = \sec x \tan x$, then $y' =$

Solution:

$(f.g)' = f'g + fg'$, $(\sec u)' = \sec u \tan u \,.\, u'$ and $(\tan u)' = \sec^2 u \,.\, u'$

$$y' = (\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x)$$
$$= \sec x \tan^2 x + \sec^3 x = \sec x(\tan^2 x + \sec^2 x)$$

74) If $D^{99}(\cos x) =$

Solution:

$$D(\cos x) = -\sin x$$
$$D^2(\cos x) = -\cos x$$
$$D^3(\cos x) = \sin x$$
$$D^4(\cos x) = \cos x$$

Note: $D^n(\cos x) = \cos x$ whenever n is a multiple of 4.
Hence,

$$D^{96}(\cos x) = \cos x$$
$$D^{97}(\cos x) = -\sin x$$
$$D^{98}(\cos x) = -\cos x$$
$$D^{99}(\cos x) = \sin x$$

75) If $y = (x + \sec x)^3$, then $y' =$

Solution:

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\sec u)' = \sec u \tan u \,.\, u'$

$$y' = 3(x + \sec x)^2.(1 + \sec x \tan x)$$

76) If $x^2 = 5y^2 + \sin y$, then $y' =$

Solution:

$$2x = 10yy' + \cos y \,.\, y'$$
$$y'(10y + \cos y) = 2x$$
$$y' = \frac{2x}{10y + \cos y}$$

77) If $x^2 - 5y^2 + \sin y = 0$, then $y' =$

Solution:

$$2x - 10yy' + \cos y \,.\, y' = 0$$
$$y'(-10y + \cos y) = -2x$$
$$y' = \frac{-2x}{-10y + \cos y} = \frac{2x}{10y - \cos y}$$

78) If $y = \sin x \sec x$, then $y' =$

Solution:

$(f.g)' = f'g + fg'$, $(\sin u)' = \cos u \,.\, u'$ and $(\sec u)' = \sec u \tan u \,.\, u'$

$$y' = (\cos x)(\sec x) + (\sin x)(\sec x \tan x)$$
$$= 1 + \sin x \,.\, \frac{1}{\cos x} \,.\, \frac{\sin x}{\cos x} = 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$$
$$= \sec^2 x$$

79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$

Solution:

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\sin u)' = \cos u \,.\, u'$

$$f'(x) = 2\sin(x^3 + 1) \,.\, (\cos(x^3 + 1)) \,.\, (3x^2)$$
$$= 6x^2 \sin(x^3 + 1)\cos(x^3 + 1)$$

80) If $y = (x + \cot x)^3$, then $y' =$

Solution:

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\cot u)' = -\csc^2 u \,.\, u'$

$$y' = 3(x + \cot x)^2.(1 - \csc^2 x)$$

81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' =$

Solution:

Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$

$$y' = \frac{1}{1 + \left(\frac{x}{2}\right)^2} \,.\, \frac{1}{2} = \frac{1}{2\left(1 + \frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4 + x^2}{4}\right)} = \frac{2}{4 + x^2}$$

82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' =$

Solution:

Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$

$$y' = -\frac{1}{1 + \left(\frac{x}{2}\right)^2} \,.\, \frac{1}{2} = -\frac{1}{2\left(1 + \frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4 + x^2}{4}\right)}$$
$$= -\frac{2}{4 + x^2}$$

83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' =$

Solution:

Use the rule $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \,.\, \frac{1}{3} = \frac{1}{3\sqrt{1 - \frac{x^2}{9}}} = \frac{1}{3\sqrt{\frac{9 - x^2}{9}}}$$
$$= \frac{1}{\sqrt{9 - x^2}}$$

84) If $y = \cos^{-1}\left(\frac{x}{3}\right)$, then $y' =$

Solution:

Use the rule $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$

$$y' = -\frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \,.\, \frac{1}{3} = -\frac{1}{3\sqrt{1 - \frac{x^2}{9}}} = -\frac{1}{3\sqrt{\frac{9 - x^2}{9}}}$$
$$= -\frac{1}{\sqrt{9 - x^2}}$$

85) If $D^{99}(\sin x) =$ Solution: $D(\sin x) = \cos x$ $D^2(\sin x) = -\sin x$ $D^3(\sin x) = -\cos x$ $D^4(\sin x) = \sin x$ **Note:** $D^n(\sin x) = \sin x$ whenever n is a multiple of 4. Hence, $D^{96}(\sin x) = \sin x$ $D^{97}(\sin x) = \cos x$ $D^{98}(\sin x) = -\sin x$ $D^{99}(\sin x) = -\cos x$	

Workshop Solutions to Sections 5.1 and 5.2 (chapter 4)

1) The absolute maximum value of $f(x) = x^3 - 2x^2$ in $[-1,2]$ is at $x =$

Solution:

Since $f(x)$ is a continuous on $[-1,2]$, we can use the Closed Interval Method,

$$f(x) = x^3 - 2x^2$$
$$f'(x) = 3x^2 - 4x$$

Now, we find the critical numbers of $f(x)$ when

$$f'(x) = 0 \implies 3x^2 - 4x = 0 \implies x(3x-4) = 0$$
$$\implies x = 0 \text{ or } x = \frac{4}{3}$$

Thus,

$f(-1) = (-1)^3 - 2(-1)^2 = -1 - 2 = -3$

$f(2) = (2)^3 - 2(2)^2 = 8 - 8 = 0$

$f(0) = (0)^3 - 2(0)^2 = 0 - 0 = 0$

$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 = \frac{64}{27} - \frac{32}{9} = -\frac{32}{27}$

Hence, we see that the absolute maximum value is 0 at $x = 0$ and $x = 2$

2) The absolute minimum value of $f(x) = x^3 - 3x^2 + 1$ in $\left[-\frac{1}{2}, 4\right]$ is

Solution:

Since $f(x)$ is a continuous on $\left[-\frac{1}{2}, 4\right]$, we can use the Closed Interval Method,

$$f(x) = x^3 - 3x^2 + 1$$
$$f'(x) = 3x^2 - 6x$$

Now, we find the critical numbers of $f(x)$ when

$$f'(x) = 0 \implies 3x^2 - 6x = 0 \implies 3x(x-2) = 0$$
$$\implies x = 0 \text{ or } x = 2$$

Thus,

$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 1 = -\frac{1}{8} - \frac{3}{4} + 1 = \frac{1}{8}$

$f(4) = (4)^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 17$

$f(0) = (0)^3 - 3(0)^2 + 1 = 0 - 0 + 1 = 1$

$f(2) = (2)^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$

Hence, we see that the absolute minimum value is -3 at $x = 2$

3) The absolute maximum point of $f(x) = 3x^2 - 12x + 1$ in $[0,3]$ is

Solution:

Since $f(x)$ is a continuous on $[0,3]$, we can use the Closed Interval Method,

$$f(x) = 3x^2 - 12x + 1$$
$$f'(x) = 6x - 12$$

Now, we find the critical numbers of $f(x)$ when

$$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$$
$$\implies x = 2$$

Thus,

$f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$

$f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$

$f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$

Hence, we see that the absolute maximum point is $(0,1)$.

4) The absolute minimum point of $f(x) = 3x^2 - 12x + 1$ in $[0,3]$ is

Solution:

Since $f(x)$ is a continuous on $[0,3]$, we can use the Closed Interval Method,

$$f(x) = 3x^2 - 12x + 1$$
$$f'(x) = 6x - 12$$

Now, we find the critical numbers of $f(x)$ when

$$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$$
$$\implies x = 2$$

Thus,

$f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$

$f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$

$f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$

Hence, we see that the absolute minimum point is $(2,-11)$.

5) The absolute minimum point of $f(x) = 3x^2 - 12x + 2$ in $[0,3]$ is

Solution:

Since $f(x)$ is a continuous on $[0,3]$, we can use the Closed Interval Method,

$$f(x) = 3x^2 - 12x + 2$$
$$f'(x) = 6x - 12$$

Now, we find the critical numbers of $f(x)$ when

$$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$$
$$\implies x = 2$$

Thus,

$f(0) = 3(0)^2 - 12(0) + 2 = 0 - 0 + 2 = 2$

$f(3) = 3(3)^2 - 12(3) + 2 = 27 - 36 + 2 = -7$

$f(2) = 3(2)^2 - 12(2) + 2 = 12 - 24 + 2 = -10$

Hence, we see that the absolute minimum point is $(2,-10)$.

6) The values in $(-3,3)$ which make $f(x) = x^3 - 9x$ satisfy Rolle's Theorem on $[-3,3]$ are deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[-3,3]$.

2- $f(x)$ is differentiable on $(-3,3)$,

$f'(x) = 3x^2 - 9$

3- $f(-3) = (-3)^3 - 9(-3) = -27 + 27 = 0 = f(3)$

Then there is a number $c \in (-3,3)$ such that

$$f'(c) = 0 \implies 3c^2 - 9 = 0 \implies 3c^2 = 9$$
$$\implies c^2 = 3 \implies c = \pm\sqrt{3}$$

Hence, the values are $\pm\sqrt{3} \in (-3,3)$.

7) The values in $(0,2)$ which make $f(x) = x^3 - 3x^2 + 2x + 5$ satisfy Rolle's Theorem on $[0,2]$ are deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,2]$.

2- $f(x)$ is differentiable on $(0,2)$,

$f'(x) = 3x^2 - 6x + 2$

3- $f(0) = (0)^3 - 3(0)^2 + 2(0) + 5 = 5 = f(2)$

Then there is a number $c \in (0,2)$ such that

$$f'(c) = 0 \implies 3c^2 - 6c + 2 = 0$$

$$\implies c = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)} = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$= \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm \sqrt{3 \times 4}}{6} = \frac{6 \pm 2\sqrt{3}}{6}$$

$$= \frac{2(3 \pm \sqrt{3})}{6} = \frac{3 \pm \sqrt{3}}{3} = \frac{3}{3} \pm \frac{\sqrt{3}}{3}$$

$$= 1 \pm \frac{\sqrt{3}}{3}$$

Hence, the values are $1 \pm \frac{\sqrt{3}}{3} \in (0,2)$.

8) The value c in $(0,5)$ which makes $f(x) = x^2 - x - 6$ satisfy the Mean Value Theorem on $[0,5]$ is deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,5]$.

2- $f(x)$ is differentiable on $(0,5)$,

$f'(x) = 2x - 1$

Then there is a number $c \in (0,5)$ such that

$$f'(c) = \frac{f(5) - f(0)}{5 - 0}$$

$$\implies 2c - 1 = \frac{[(5)^2 - (5) - 6] - [(0)^2 - (0) - 6]}{5}$$

$$\implies 2c - 1 = \frac{(14) - (-6)}{5}$$

$$\implies 2c - 1 = \frac{14 + 6}{5}$$

$$\implies 2c - 1 = 4$$

$$\implies 2c = 4 + 1$$

$$\implies c = \frac{5}{2}$$

Hence, the value c is $\frac{5}{2} \in (0,5)$.

9) The value c in $(0,2)$ makes $f(x) = x^3 - x$ satisfied the Mean Value Theorem on $[0,2]$ are deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,2]$.

2- $f(x)$ is differentiable on $(0,2)$,

$f'(x) = 3x^2 - 1$

Then there is a number $c \in (0,3)$ such that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

$$\implies 3c^2 - 1 = \frac{[(2)^3 - (2)] - [(0)^3 - (0)]}{2}$$

$$\implies 3c^2 - 1 = \frac{(6) - (0)}{2}$$

$$\implies 3c^2 - 1 = \frac{6}{2}$$

$$\implies 3c^2 - 1 = 3$$

$$\implies 3c^2 = 3 + 1$$

$$\implies c^2 = \frac{4}{3}$$

$$\implies c = \pm\sqrt{\frac{4}{3}}$$

$$\implies c = \pm\frac{2}{\sqrt{3}}$$

Hence, the value c is $\frac{2}{\sqrt{3}} \in (0,2)$ but $-\frac{2}{\sqrt{3}} \notin (0,2)$.

10) The value in $(0,1)$ which makes $f(x) = 3x^2 + 2x + 5$ satisfy the Mean Value Theorem on $[0,1]$ is deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,1]$.

2- $f(x)$ is differentiable on $(0,1)$,

$f'(x) = 6x + 2$

Then there is a number $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\implies 6c + 2 = \frac{[3(1)^2 + 2(1) + 5] - [3(0)^2 + 2(0) + 5]}{1}$$

$$\implies 6c + 2 = (3 + 2 + 5) - (0 + 0 + 5)$$

$$\implies 6c + 2 = 10 - 5$$

$$\implies 6c + 2 = 5$$

$$\implies 6c = 5 - 2$$

$$\implies 6c = 3$$

$$\implies c = \frac{3}{6}$$

$$\implies c = \frac{1}{2}$$

Hence, the values are $\frac{1}{2} \in (0,1)$.

11) The critical numbers of the function $f(x) = x^3 + 3x^2 - 9x + 1$ are

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$$

$$\implies 3(x^2 + 2x - 3) = 0$$

$$\implies x^2 + 2x - 3 = 0$$

$$\implies (x + 3)(x - 1) = 0$$

$$\implies x = -3 \quad \text{or} \quad x = 1$$

12) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is decreasing on

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 + 6x - 9 = 0 \\ &\Longrightarrow \quad 3(x^2 + 2x - 3) = 0 \\ &\Longrightarrow \quad x^2 + 2x - 3 = 0 \\ &\Longrightarrow \quad (x+3)(x-1) = 0 \\ &\Longrightarrow \quad x = -3 \quad \text{or} \quad x = 1 \end{aligned}$$

-3 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-3,1)$

13) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is increasing on

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 + 6x - 9 = 0 \\ &\Longrightarrow \quad 3(x^2 + 2x - 3) = 0 \\ &\Longrightarrow \quad x^2 + 2x - 3 = 0 \\ &\Longrightarrow \quad (x+3)(x-1) = 0 \\ &\Longrightarrow \quad x = -3 \quad \text{or} \quad x = 1 \end{aligned}$$

-3 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty,-3) \cup (1,\infty)$

14) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative maximum value at the point

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 + 6x - 9 = 0 \\ &\Longrightarrow \quad 3(x^2 + 2x - 3) = 0 \\ &\Longrightarrow \quad x^2 + 2x - 3 = 0 \\ &\Longrightarrow \quad (x+3)(x-1) = 0 \\ &\Longrightarrow \quad x = -3 \quad \text{or} \quad x = 1 \end{aligned}$$

-3 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-3,28)$.

$$\begin{aligned} f(-3) &= (-3)^3 + 3(-3)^2 - 9(-3) + 1 \\ &= -27 + 27 + 27 + 1 = 28 \end{aligned}$$

15) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative minimum value at the point

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 + 6x - 9 = 0 \\ &\Longrightarrow \quad 3(x^2 + 2x - 3) = 0 \\ &\Longrightarrow \quad x^2 + 2x - 3 = 0 \\ &\Longrightarrow \quad (x+3)(x-1) = 0 \\ &\Longrightarrow \quad x = -3 \quad \text{or} \quad x = 1 \end{aligned}$$

-3 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(1,-4)$.

$$\begin{aligned} f(1) &= (1)^3 + 3(1)^2 - 9(1) + 1 \\ &= 1 + 3 - 9 + 1 = -4 \end{aligned}$$

16) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave upward on

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Longrightarrow \quad 6x + 6 = 0 \\ &\Longrightarrow \quad 6x = -6 \\ &\Longrightarrow \quad x = -\frac{6}{6} \\ &\Longrightarrow \quad x = -1 \end{aligned}$$

-1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-1,\infty)$

17) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave downward on

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Longrightarrow \quad 6x + 6 = 0 \\ &\Longrightarrow \quad 6x = -6 \\ &\Longrightarrow \quad x = -\frac{6}{6} \\ &\Longrightarrow \quad x = -1 \end{aligned}$$

-1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty,-1)$

18) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has an inflection point at

Solution:

$$f'(x) = 3x^2 + 6x - 9$$
$$f''(x) = 6x + 6$$
$$f''(x) = 0 \quad \Rightarrow \quad 6x + 6 = 0$$
$$\Rightarrow \quad 6x = -6$$
$$\Rightarrow \quad x = -\frac{6}{6}$$
$$\Rightarrow \quad x = -1$$

-1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(-1,12)$.

$$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 1$$
$$= -1 + 3 + 9 + 1 = 12$$

19) The critical numbers of the function $f(x) = x^3 - 3x^2 - 9x + 1$ are

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x - 9 = 0$$
$$\Rightarrow \quad 3(x^2 - 2x - 3) = 0$$
$$\Rightarrow \quad x^2 - 2x - 3 = 0$$
$$\Rightarrow \quad (x+1)(x-3) = 0$$
$$\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3$$

20) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x - 9 = 0$$
$$\Rightarrow \quad 3(x^2 - 2x - 3) = 0$$
$$\Rightarrow \quad x^2 - 2x - 3 = 0$$
$$\Rightarrow \quad (x+1)(x-3) = 0$$
$$\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3$$

-1 $\qquad$ 3

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1,3)$

21) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x - 9 = 0$$
$$\Rightarrow \quad 3(x^2 - 2x - 3) = 0$$
$$\Rightarrow \quad x^2 - 2x - 3 = 0$$
$$\Rightarrow \quad (x+1)(x-3) = 0$$
$$\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3$$

-1 $\qquad$ 3

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty,-1) \cup (3,\infty)$

22) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x - 9 = 0$$
$$\Rightarrow \quad 3(x^2 - 2x - 3) = 0$$
$$\Rightarrow \quad x^2 - 2x - 3 = 0$$
$$\Rightarrow \quad (x+1)(x-3) = 0$$
$$\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3$$

-1 $\qquad$ 3

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1,6)$.

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 1$$
$$= -1 - 3 + 9 + 1 = 6.$$

23) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x - 9 = 0$$
$$\Rightarrow \quad 3(x^2 - 2x - 3) = 0$$
$$\Rightarrow \quad x^2 - 2x - 3 = 0$$
$$\Rightarrow \quad (x+1)(x-3) = 0$$
$$\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3$$

-1 $\qquad$ 3

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(3,-26)$.

$$f(3) = (3)^3 - 3(3)^2 - 9(3) + 1$$
$$= 27 - 27 - 27 + 1 = -26.$$

24) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f''(x) = 6x - 6$$
$$f''(x) = 0 \Rightarrow 6x - 6 = 0 \Rightarrow 6x = 6 \Rightarrow x = \frac{6}{6} \Rightarrow x = 1$$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$

25) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f''(x) = 6x - 6$$
$$f''(x) = 0 \Rightarrow 6x - 6 = 0 \Rightarrow 6x = 6 \Rightarrow x = \frac{6}{6} \Rightarrow x = 1$$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$

26) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f''(x) = 6x - 6$$
$$f''(x) = 0 \Rightarrow 6x - 6 = 0 \Rightarrow 6x = 6 \Rightarrow x = \frac{6}{6} \Rightarrow x = 1$$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -10)$.

$$f(1) = (1)^3 - 3(1)^2 - 9(1) + 1$$
$$= 1 - 3 - 9 + 1 = -10$$

27) The critical numbers of the function $f(x) = x^3 + 3x^2 - 9x + 5$ are

Solution:

$$f'(x) = 3x^2 + 6x - 9$$
$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0 \Rightarrow 3(x^2 + 2x - 3) = 0 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0 \Rightarrow x = -3 \text{ or } x = 1$$

28) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is decreasing on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$
$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0 \Rightarrow 3(x^2 + 2x - 3) = 0 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0 \Rightarrow x = -3 \text{ or } x = 1$$

-3 $\quad$ 1

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-3,1)$.

29) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is increasing on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$
$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0 \Rightarrow 3(x^2 + 2x - 3) = 0 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0 \Rightarrow x = -3 \text{ or } x = 1$$

-3 $\quad$ 1

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -3) \cup (1, \infty)$.

30) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative minimum value at the point

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$$
$$\implies 3(x^2 + 2x - 3) = 0$$
$$\implies x^2 + 2x - 3 = 0$$
$$\implies (x+3)(x-1) = 0$$
$$\implies x = -3 \quad \text{or} \quad x = 1$$

-3 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point (1,0).

$$f(1) = (1)^3 + 3(1)^2 - 9(1) + 5$$
$$= 1 + 3 - 9 + 5 = 0$$

31) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative maximum value at the point

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$$
$$\implies 3(x^2 + 2x - 3) = 0$$
$$\implies x^2 + 2x - 3 = 0$$
$$\implies (x+3)(x-1) = 0$$
$$\implies x = -3 \quad \text{or} \quad x = 1$$

-3 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-3,32)$.

$$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5$$
$$= -27 + 27 + 27 + 5 = 32$$

32) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has an inflection point at

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$
$$f''(x) = 6x + 6$$

$$f''(x) = 0 \implies 6x + 6 = 0$$
$$\implies 6x = -6$$
$$\implies x = -\frac{6}{6}$$
$$\implies x = -1$$

-1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(-1,16)$.

$$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 5$$
$$= -1 + 3 + 9 + 5 = 16$$

33) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave downward on

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$
$$f''(x) = 6x + 6$$

$$f''(x) = 0 \implies 6x + 6 = 0$$
$$\implies 6x = -6$$
$$\implies x = -\frac{6}{6}$$
$$\implies x = -1$$

-1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, -1)$.

34) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave upward on

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$
$$f''(x) = 6x + 6$$

$$f''(x) = 0 \implies 6x + 6 = 0$$
$$\implies 6x = -6$$
$$\implies x = -\frac{6}{6}$$
$$\implies x = -1$$

-1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-1, \infty)$.

35) The critical numbers of the function $f(x) = x^3 - 3x^2 - 9x + 5$ are

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$$
$$\implies 3(x^2 - 2x - 3) = 0$$
$$\implies x^2 - 2x - 3 = 0$$
$$\implies (x+1)(x-3) = 0$$
$$\implies x = -1 \quad \text{or} \quad x = 3$$

36) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing on

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 - 6x - 9 = 0 \\ &\Longrightarrow \quad 3(x^2 - 2x - 3) = 0 \\ &\Longrightarrow \quad x^2 - 2x - 3 = 0 \\ &\Longrightarrow \quad (x+1)(x-3) = 0 \\ &\Longrightarrow \quad x = -1 \quad \text{or} \quad x = 3 \end{aligned}$$

-1 $\qquad$ 3

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -1) \cup (3, \infty)$.

37) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is decreasing on

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 - 6x - 9 = 0 \\ &\Longrightarrow \quad 3(x^2 - 2x - 3) = 0 \\ &\Longrightarrow \quad x^2 - 2x - 3 = 0 \\ &\Longrightarrow \quad (x+1)(x-3) = 0 \\ &\Longrightarrow \quad x = -1 \quad \text{or} \quad x = 3 \end{aligned}$$

-1 $\qquad$ 3

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1,3)$.

38) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative maximum value at the point

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 - 6x - 9 = 0 \\ &\Longrightarrow \quad 3(x^2 - 2x - 3) = 0 \\ &\Longrightarrow \quad x^2 - 2x - 3 = 0 \\ &\Longrightarrow \quad (x+1)(x-3) = 0 \\ &\Longrightarrow \quad x = -1 \quad \text{or} \quad x = 3 \end{aligned}$$

-1 $\qquad$ 3

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1,10)$.

$$\begin{aligned} f(-1) &= (-1)^3 - 3(-1)^2 - 9(-1) + 5 \\ &= -1 - 3 + 9 + 5 = 10. \end{aligned}$$

39) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative minimum value at the point

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 - 6x - 9 = 0 \\ &\Longrightarrow \quad 3(x^2 - 2x - 3) = 0 \\ &\Longrightarrow \quad x^2 - 2x - 3 = 0 \\ &\Longrightarrow \quad (x+1)(x-3) = 0 \\ &\Longrightarrow \quad x = -1 \quad \text{or} \quad x = 3 \end{aligned}$$

-1 $\qquad$ 3

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(3,-22)$.

$$\begin{aligned} f(3) &= (3)^3 - 3(3)^2 - 9(3) + 5 \\ &= 27 - 27 - 27 + 5 = -22. \end{aligned}$$

40) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave upward on

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Longrightarrow \quad 6x - 6 = 0 \\ &\Longrightarrow \quad 6x = 6 \\ &\Longrightarrow \quad x = \frac{6}{6} \\ &\Longrightarrow \quad x = 1 \end{aligned}$$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

41) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave downward on

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Longrightarrow \quad 6x - 6 = 0 \\ &\Longrightarrow \quad 6x = 6 \\ &\Longrightarrow \quad x = \frac{6}{6} \\ &\Longrightarrow \quad x = 1 \end{aligned}$$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

42) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f''(x) = 6x - 6$$

$f''(x) = 0 \implies 6x - 6 = 0$
$\implies 6x = 6$
$\implies x = \frac{6}{6}$
$\implies x = 1$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -6)$.

$$f(1) = (1)^3 - 3(1)^2 - 9(1) + 5$$
$$= 1 - 3 - 9 + 5 = -6$$

43) The critical numbers of the function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ are

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

44) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is increasing on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

-1 $\qquad$ 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -1) \cup (2, \infty)$.

45) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is decreasing on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

-1 $\qquad$ 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1, 2)$.

46) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ has a relative maximum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

-1 $\qquad$ 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $\left(-1, \frac{13}{6}\right)$.

$$f(-1) = \frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2 - 2(-1) + 1$$
$$= -\frac{1}{3} - \frac{1}{2} + 2 + 1 = \frac{13}{6}$$

47) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ has a relative minimum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

-1 $\qquad$ 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $\left(2, -\frac{7}{3}\right)$.

$$f(2) = \frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 - 2(2) + 1$$
$$= \frac{8}{3} - \frac{4}{2} - 4 + 1 = -\frac{7}{3}$$

48) The function $f(x)=\frac{1}{3}x^3-\frac{1}{2}x^2-2x+1$ concave upward on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2-2\left(\frac{1}{2}\right)x-2=x^2-x-2$$

$$f''(x)=2x-1$$

$f''(x)=0 \quad \Longrightarrow \quad 2x-1=0$

$\Longrightarrow \quad 2x=1$

$\Longrightarrow \quad x=\frac{1}{2}$

$\frac{1}{2}$

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave upward on $\left(\frac{1}{2},\infty\right)$.

49) The function $f(x)=\frac{1}{3}x^3-\frac{1}{2}x^2-2x+1$ concave downward on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2-2\left(\frac{1}{2}\right)x-2=x^2-x-2$$

$$f''(x)=2x-1$$

$f''(x)=0 \quad \Longrightarrow \quad 2x-1=0$

$\Longrightarrow \quad 2x=1$

$\Longrightarrow \quad x=\frac{1}{2}$

$\frac{1}{2}$

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave downward on $\left(-\infty,\frac{1}{2}\right)$.

50) The function $f(x)=\frac{1}{3}x^3-\frac{1}{2}x^2-2x+1$ has an inflection point at

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2-2\left(\frac{1}{2}\right)x-2=x^2-x-2$$

$$f''(x)=2x-1$$

$f''(x)=0 \quad \Longrightarrow \quad 2x-1=0$

$\Longrightarrow \quad 2x=1$

$\Longrightarrow \quad x=\frac{1}{2}$

$\frac{1}{2}$

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $\left(\frac{1}{2},-\frac{1}{12}\right)$.

$$f\left(\frac{1}{2}\right)=\frac{1}{3}\left(\frac{1}{2}\right)^3-\frac{1}{2}\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)+1$$

$$=\frac{1}{24}-\frac{1}{8}-1+1=-\frac{1}{12}$$

51) The critical numbers of the function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ are

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$f'(x)=0 \quad \Longrightarrow \quad x^2+x-2=0$

$\Longrightarrow \quad (x+2)(x-1)=0$

$\Longrightarrow \quad x=-2 \quad \text{or} \quad x=1$

52) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ is increasing on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$f'(x)=0 \quad \Longrightarrow \quad x^2+x-2=0$

$\Longrightarrow \quad (x+2)(x-1)=0$

$\Longrightarrow \quad x=-2 \quad \text{or} \quad x=1$

$-2 \qquad 1$

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty,-2)\cup(1,\infty)$.

53) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ is decreasing on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$f'(x)=0 \quad \Longrightarrow \quad x^2+x-2=0$

$\Longrightarrow \quad (x+2)(x-1)=0$

$\Longrightarrow \quad x=-2 \quad \text{or} \quad x=1$

$-2 \qquad 1$

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-2,1)$.

54) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$
has a relative maximum point

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$f'(x)=0 \quad\Longrightarrow\quad x^2+x-2=0$

$\Longrightarrow\quad (x+2)(x-1)=0$

$\Longrightarrow\quad x=-2 \quad\text{or}\quad x=1$

-2 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $\left(-2,\frac{13}{3}\right)$.

$$f(-2)=\frac{1}{3}(-2)^3+\frac{1}{2}(-2)^2-2(-2)+1$$

$$=-\frac{8}{3}+\frac{4}{2}+4+1=\frac{13}{3}$$

55) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$
has a relative minimum point

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$f'(x)=0 \quad\Longrightarrow\quad x^2+x-2=0$

$\Longrightarrow\quad (x+2)(x-1)=0$

$\Longrightarrow\quad x=-2 \quad\text{or}\quad x=1$

-2 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $\left(1,-\frac{1}{6}\right)$.

$$f(1)=\frac{1}{3}(1)^3+\frac{1}{2}(1)^2-2(1)+1$$

$$=\frac{1}{3}+\frac{1}{2}-2+1=-\frac{1}{6}$$

56) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ concave
upward on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$$f''(x)=2x+1$$

$f''(x)=0 \quad\Longrightarrow\quad 2x+1=0$

$\Longrightarrow\quad 2x=-1$

$\Longrightarrow\quad x=-\frac{1}{2}$

$-\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $\left(-\frac{1}{2},\infty\right)$.

57) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ concave
downward on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$$f''(x)=2x+1$$

$f''(x)=0 \quad\Longrightarrow\quad 2x+1=0$

$\Longrightarrow\quad 2x=-1$

$\Longrightarrow\quad x=-\frac{1}{2}$

$-\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $\left(-\infty,-\frac{1}{2}\right)$.

58) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ has an
inflection point at

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$$f''(x)=2x+1$$

$f''(x)=0 \quad\Longrightarrow\quad 2x+1=0$

$\Longrightarrow\quad 2x=-1 \quad\Longrightarrow\quad x=-\frac{1}{2}$

$-\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $\left(-\frac{1}{2},\frac{25}{12}\right)$.

$$f\left(-\frac{1}{2}\right)=\frac{1}{3}\left(-\frac{1}{2}\right)^3+\frac{1}{2}\left(-\frac{1}{2}\right)^2-2\left(-\frac{1}{2}\right)+1$$

$$=-\frac{1}{24}+\frac{1}{8}+1+1=\frac{25}{12}$$

59) The critical numbers of the function
$f(x)=x^3-12x+3$ are

Solution:

$$f'(x)=3x^2-12$$

$f'(x)=0 \quad\Longrightarrow\quad 3x^2-12=0$

$\Longrightarrow\quad 3(x^2-4)=0$

$\Longrightarrow\quad x^2-4=0$

$\Longrightarrow\quad x^2=4$

$\Longrightarrow\quad x=\pm 2$

60) The function $f(x) = x^3 - 12x + 3$ is increasing on

Solution:

$$f'(x) = 3x^2 - 12$$

$f'(x) = 0 \implies 3x^2 - 12 = 0$
$\implies 3(x^2 - 4) = 0$
$\implies x^2 - 4 = 0$
$\implies x^2 = 4$
$\implies x = \pm 2$

-2 $\quad$ 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -2) \cup (2, \infty)$.

61) The function $f(x) = x^3 - 12x + 3$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 12$$

$f'(x) = 0 \implies 3x^2 - 12 = 0$
$\implies 3(x^2 - 4) = 0$
$\implies x^2 - 4 = 0$
$\implies x^2 = 4$
$\implies x = \pm 2$

-2 $\quad$ 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-2,2)$.

62) The function $f(x) = x^3 - 12x + 3$ has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 12$$

$f'(x) = 0 \implies 3x^2 - 12 = 0$
$\implies 3(x^2 - 4) = 0$
$\implies x^2 - 4 = 0$
$\implies x^2 = 4$
$\implies x = \pm 2$

-2 $\quad$ 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(-2,19)$.

$f(-2) = (-2)^3 - 12(-2) + 3$
$= -8 + 24 + 3 = 19.$

63) The function $f(x) = x^3 - 12x + 3$ has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 12$$

$f'(x) = 0 \implies 3x^2 - 12 = 0$
$\implies 3(x^2 - 4) = 0$
$\implies x^2 - 4 = 0$
$\implies x^2 = 4$
$\implies x = \pm 2$

-2 $\quad$ 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2,-13)$.

$f(2) = (2)^3 - 12(2) + 3$
$= 8 - 24 + 3 = -13$

64) The function $f(x) = x^3 - 12x + 3$ concave upward on

Solution:

$$f'(x) = 3x^2 - 12$$
$$f''(x) = 6x$$

$f''(x) = 0 \implies 6x = 0$
$\implies x = \frac{0}{6}$
$\implies x = 0$

0

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(0, \infty)$.

65) The function $f(x) = x^3 - 12x + 3$ concave downward on

Solution:

$$f'(x) = 3x^2 - 12$$
$$f''(x) = 6x$$

$f''(x) = 0 \implies 6x = 0$
$\implies x = \frac{0}{6}$
$\implies x = 0$

0

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 0)$.

66) The function $f(x) = x^3 - 12x + 3$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 12$$
$$f''(x) = 6x$$
$$f''(x) = 0 \quad \Rightarrow \quad 6x = 0$$
$$\Rightarrow \quad x = \frac{0}{6}$$
$$\Rightarrow \quad x = 0$$

0

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(0,3)$.

$f(0) = (0)^3 - 12(0)^2 + 3$
$= 0 - 0 + 3 = 3$

67) The critical numbers of the function $f(x) = x^3 - 3x^2 + 1$ are

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

68) The function $f(x) = x^3 - 3x^2 + 1$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

0 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$.

69) The function $f(x) = x^3 - 3x^2 + 1$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

0 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(0,2)$.

70) The function $f(x) = x^3 - 3x^2 + 1$ has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

0 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(0,1)$.

$f(0) = (0)^3 - 3(0)^2 + 1$
$= 0 - 0 + 1 = 1.$

71) The function $f(x) = x^3 - 3x^2 + 1$ has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

0 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2,-3)$.

$f(2) = (2)^3 - 3(2)^2 + 1$
$= 8 - 12 + 1 = -3.$

72) The function $f(x) = x^3 - 3x^2 + 1$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$f''(x) = 0 \quad \Rightarrow \quad 6x - 6 = 0$

$\Rightarrow \quad 6x = 6$

$\Rightarrow \quad x = \frac{6}{6}$

$\Rightarrow \quad x = 1$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

73) The function $f(x) = x^3 - 3x^2 + 1$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$f''(x) = 0 \quad \Rightarrow \quad 6x - 6 = 0$

$\Rightarrow \quad 6x = 6$

$\Rightarrow \quad x = \frac{6}{6}$

$\Rightarrow \quad x = 1$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

74) The function $f(x) = x^3 - 3x^2 + 1$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$f''(x) = 0 \quad \Rightarrow \quad 6x - 6 = 0$

$\Rightarrow \quad 6x = 6$

$\Rightarrow \quad x = \frac{6}{6}$

$\Rightarrow \quad x = 1$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -1)$.

$f(1) = (1)^3 - 3(1)^2 + 1$

$= 1 - 3 + 1 = -1$

75) The critical numbers of the function $f(x) = x^3 - 3x^2 + 2$ are

Solution:

$$f'(x) = 3x^2 - 6x$$

$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$

$\Rightarrow \quad 3(x^2 - 2x) = 0$

$\Rightarrow \quad x^2 - 2x = 0$

$\Rightarrow \quad x(x - 2) = 0$

$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$

76) The function $f(x) = x^3 - 3x^2 + 2$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$

$\Rightarrow \quad 3(x^2 - 2x) = 0$

$\Rightarrow \quad x^2 - 2x = 0$

$\Rightarrow \quad x(x - 2) = 0$

$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$.

77) The function $f(x) = x^3 - 3x^2 + 2$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$

$\Rightarrow \quad 3(x^2 - 2x) = 0$

$\Rightarrow \quad x^2 - 2x = 0$

$\Rightarrow \quad x(x - 2) = 0$

$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(0,2)$.

78) The function $f(x) = x^3 - 3x^2 + 2$ has a relative minimum point at

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 - 6x = 0 \\ &\Longrightarrow \quad 3(x^2 - 2x) = 0 \\ &\Longrightarrow \quad x^2 - 2x = 0 \\ &\Longrightarrow \quad x(x-2) = 0 \\ &\Longrightarrow \quad x = 0 \quad \text{or} \quad x = 2 \end{aligned}$$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2, -2)$.

$f(2) = (2)^3 - 3(2)^2 + 2$
$= 8 - 12 + 2 = -2.$

79) The function $f(x) = x^3 - 3x^2 + 2$ has a relative maximum point at

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 - 6x = 0 \\ &\Longrightarrow \quad 3(x^2 - 2x) = 0 \\ &\Longrightarrow \quad x^2 - 2x = 0 \\ &\Longrightarrow \quad x(x-2) = 0 \\ &\Longrightarrow \quad x = 0 \quad \text{or} \quad x = 2 \end{aligned}$$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(0,2)$.

$f(0) = (0)^3 - 3(0)^2 + 2$
$= 0 - 0 + 2 = 2.$

80) The function $f(x) = x^3 - 3x^2 + 2$ concave downward on

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Longrightarrow \quad 6x - 6 = 0 \\ &\Longrightarrow \quad 6x = 6 \\ &\Longrightarrow \quad x = \frac{6}{6} \\ &\Longrightarrow \quad x = 1 \end{aligned}$$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

81) The function $f(x) = x^3 - 3x^2 + 2$ concave upward on

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Longrightarrow \quad 6x - 6 = 0 \\ &\Longrightarrow \quad 6x = 6 \\ &\Longrightarrow \quad x = \frac{6}{6} \\ &\Longrightarrow \quad x = 1 \end{aligned}$$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

82) The function $f(x) = x^3 - 3x^2 + 2$ has an inflection point at

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Longrightarrow \quad 6x - 6 = 0 \\ &\Longrightarrow \quad 6x = 6 \\ &\Longrightarrow \quad x = \frac{6}{6} \\ &\Longrightarrow \quad x = 1 \end{aligned}$$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1,0)$.

$f(1) = (1)^3 - 3(1)^2 + 2$
$= 1 - 3 + 2 = 0$

83) The critical numbers of the function $f(x) = x^3 - 6x^2 - 36x$ are

<u>Solution:</u>

$$f'(x) = 3x^2 - 12x - 36$$

$$\begin{aligned} f'(x) = 0 \quad &\Longrightarrow \quad 3x^2 - 12x - 36 = 0 \\ &\Longrightarrow \quad 3(x^2 - 4x - 12) = 0 \\ &\Longrightarrow \quad x^2 - 4x - 12 = 0 \\ &\Longrightarrow \quad (x+2)(x-6) = 0 \\ &\Longrightarrow \quad x = -2 \quad \text{or} \quad x = 6 \end{aligned}$$

84) The function $f(x) = x^3 - 6x^2 - 36x$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$$
$$\implies 3(x^2 - 4x - 12) = 0$$
$$\implies x^2 - 4x - 12 = 0$$
$$\implies (x+2)(x-6) = 0$$
$$\implies x = -2 \quad \text{or} \quad x = 6$$

-2 $\quad$ 6

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-2,6)$.

85) The function $f(x) = x^3 - 6x^2 - 36x$ is increasing on

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$$
$$\implies 3(x^2 - 4x - 12) = 0$$
$$\implies x^2 - 4x - 12 = 0$$
$$\implies (x+2)(x-6) = 0$$
$$\implies x = -2 \quad \text{or} \quad x = 6$$

-2 $\quad$ 6

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty,-2) \cup (6,\infty)$.

86) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$$
$$\implies 3(x^2 - 4x - 12) = 0$$
$$\implies x^2 - 4x - 12 = 0$$
$$\implies (x+2)(x-6) = 0$$
$$\implies x = -2 \quad \text{or} \quad x = 6$$

-2 $\quad$ 6

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(6,-216)$.

$$f(6) = (6)^3 - 6(6)^2 - 36(6)$$
$$= 216 - 216 - 216 = -216$$

87) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$$
$$\implies 3(x^2 - 4x - 12) = 0$$
$$\implies x^2 - 4x - 12 = 0$$
$$\implies (x+2)(x-6) = 0$$
$$\implies x = -2 \quad \text{or} \quad x = 6$$

-2 $\quad$ 6

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-2,40)$.

$$f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$$
$$= -8 - 24 + 72 = 40$$

88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 12x - 36$$
$$f''(x) = 6x - 12$$

$$f''(x) = 0 \implies 6x - 12 = 0$$
$$\implies 6x = 12$$
$$\implies x = \frac{12}{6}$$
$$\implies x = 2$$

2

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(2,-88)$.

$$f(2) = (2)^3 - 6(2)^2 - 36(2)$$
$$= 8 - 24 - 72 = -88$$

89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on

Solution:

$$f'(x) = 3x^2 - 12x - 36$$
$$f''(x) = 6x - 12$$

$$f''(x) = 0 \implies 6x - 12 = 0$$
$$\implies 6x = 12$$
$$\implies x = \frac{12}{6}$$
$$\implies x = 2$$

2

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 2)$.

90) The function $f(x) = x^3 - 6x^2 - 36x$ concave upward on

Solution:

$$f'(x) = 3x^2 - 12x - 36$$
$$f''(x) = 6x - 12$$

$f''(x) = 0 \Rightarrow 6x - 12 = 0$
$\Rightarrow 6x = 12$
$\Rightarrow x = \frac{12}{6}$
$\Rightarrow x = 2$

2

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(2, \infty)$.

91) The critical numbers of the function $f(x) = -x^3 - 6x^2 - 9x + 1$ are

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$
$\Rightarrow -3(x^2 + 4x + 3) = 0$
$\Rightarrow x^2 + 4x + 3 = 0$
$\Rightarrow (x + 3)(x + 1) = 0$
$\Rightarrow x = -3$ or $x = -1$

92) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ is decreasing on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$
$\Rightarrow -3(x^2 + 4x + 3) = 0$
$\Rightarrow x^2 + 4x + 3 = 0$
$\Rightarrow (x + 3)(x + 1) = 0$
$\Rightarrow x = -3$ or $x = -1$

-3 -1

$-$	$+$	$-$	Sign of $f'(x)$
↘	↗	↘	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-\infty, -3) \cup (-1, \infty)$.

93) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ is increasing on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$
$\Rightarrow -3(x^2 + 4x + 3) = 0$
$\Rightarrow x^2 + 4x + 3 = 0$
$\Rightarrow (x + 3)(x + 1) = 0$
$\Rightarrow x = -3$ or $x = -1$

-3 -1

$-$	$+$	$-$	Sign of $f'(x)$
↘	↗	↘	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-3, -1)$.

94) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has a relative minimum value at the point

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$
$\Rightarrow -3(x^2 + 4x + 3) = 0$
$\Rightarrow x^2 + 4x + 3 = 0$
$\Rightarrow (x + 3)(x + 1) = 0$
$\Rightarrow x = -3$ or $x = -1$

-3 -1

$-$	$+$	$-$	Sign of $f'(x)$
↘	↗	↘	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(-3,1)$.

$f(-3) = -(-3)^3 - 6(-3)^2 - 9(-3) + 1$
$= 27 - 54 + 27 + 1 = 1.$

95) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has a relative maximum value at the point

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$
$\Rightarrow -3(x^2 + 4x + 3) = 0$
$\Rightarrow x^2 + 4x + 3 = 0$
$\Rightarrow (x + 3)(x + 1) = 0$
$\Rightarrow x = -3$ or $x = -1$

-3 -1

$-$	$+$	$-$	Sign of $f'(x)$
↘	↗	↘	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1,5)$.

$f(-1) = -(-1)^3 - 6(-1)^2 - 9(-1) + 1$
$= 1 - 6 + 9 + 1 = 5.$

96) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has an inflection point at

Solution:

$$f'(x) = -3x^2 - 12x - 9$$
$$f''(x) = -6x - 12$$

$f''(x) = 0 \implies -6x - 12 = 0$
$\implies -6x = 12$
$\implies x = -\frac{12}{6}$
$\implies x = -2$

−2

+	−	Sign of $f''(x)$
∪	∩	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(-2,3)$.

$f(-2) = -(-2)^3 - 6(-2)^2 - 9(-2) + 1$
$= 8 - 24 + 18 + 1 = 3$

97) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ concave downward on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$
$$f''(x) = -6x - 12$$

$f''(x) = 0 \implies -6x - 12 = 0$
$\implies -6x = 12$
$\implies x = -\frac{12}{6}$
$\implies x = -2$

−2

+	−	Sign of $f''(x)$
∪	∩	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-2, \infty)$.

98) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ concave upward on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$
$$f''(x) = -6x - 12$$

$f''(x) = 0 \implies -6x - 12 = 0$
$\implies -6x = 12$
$\implies x = -\frac{12}{6}$
$\implies x = -2$

−2

+	−	Sign of $f''(x)$
∪	∩	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-\infty, -2)$.

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1) If $f(x) = 2x - 9$, then $f^{-1}(x) =$

a $\frac{x-9}{2}$ b $\frac{x}{2} - 9$ c $\frac{x+9}{2}$ d $\frac{x}{2} + 9$

2) If $y = \sqrt{3x^2 + 6x}$, then $y' =$

a $\frac{6(x+1)}{\sqrt{3x^2+6x}}$ b $\frac{x+6}{\sqrt{3x^2+6x}}$ c $\frac{3(x+1)}{\sqrt{3x^2+6x}}$ d $\frac{x+1}{2\sqrt{3x^2+6x}}$

3) If $y = \log_5(x^3 - 2\csc x)$, then $y' =$

a $\frac{3x^2 + 2\csc x \cot x}{x^3 - 2\csc x}$ b $\frac{3x^2 + 2\csc x \cot x}{x^3 - 2\csc x \ln 5}$

c $\frac{3x^2 + 2\csc x \cot x}{(x^3 - 2\csc x)\ln 5}$ d $\frac{3x^2 - 2\csc x \cot x}{(x^3 - 2\csc x)\ln 5}$

4) If $-7 \le 2x + 3 < 5$, then $x =$

a $(-5,1)$ b $(-5,1]$ c $[-5,1)$ d $[-5,1]$

5) If $f(x) = x^2$, then $f'(x) =$

a $\lim_{x \to 0} \frac{(x+h)^2 - x^2}{h}$ b $\lim_{x \to 0} \frac{(x+h)^2 + x^2}{h}$

c $\lim_{h \to 0} \frac{(x+h)^2 + x^2}{h}$ d $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

6) The function $f(x) = \frac{x+1}{x^2 - 4}$ is continuous on

a $\{\pm 2\}$ b $[-2,2]$ c $\{x \in \mathbb{R} : x \ne \pm 2\}$ d $(-\infty,-2) \cup (2,\infty)$

7) The domain of $\frac{x+3}{\sqrt{x^2-4}}$ is

a $[-2,2]$ b $(-2,2)$ c $(-\infty,-2) \cup (2,\infty)$ d $(-\infty,-2] \cup [2,\infty)$

8) $\csc(\tan^{-1} x) =$

A $\frac{1}{\sqrt{x^2+1}}$ B $\frac{x}{\sqrt{x^2+1}}$ C $\sqrt{x^2+1}$ D $\frac{\sqrt{x^2+1}}{x}$

9) $\lim_{x\to 5^+} \frac{x+1}{x-5} =$ [a] -5 [b] 5 [c] $-\infty$ [d] ∞
10) $\lim_{x\to 1} \frac{x^2-1}{x-1} =$ [a] 0 [b] does not exist [c] 2 [d] $\frac{1}{2}$
11) The values in $(-1,3)$ which makes $f(x) = x^2 - 5x + 7$ satisfied Mean Value Theorem on $[-1,3]$ is [a] 1 [b] -4 [c] 0 [d] 2
12) $\lim_{x\to\infty} \left(\sqrt{x^2+x} - x\right) =$ [a] 1 [b] $-\frac{1}{2}$ [c] 0 [d] $\frac{1}{2}$
13) If $y = \ln(\cos x)$, then $y' =$ [a] $\tan x$ [b] $-\tan x$ [c] $\cot x$ [d] $-\cot x$
14) If $f(x) = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$ [a] $\tan^{-1} x \tan x$ [b] x [c] 1 [d] $\tan x$
15) The absolute maximum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on $[0,4]$ is [a] 2 [b] 6 [c] 7 [d] 12
16) The absolute minimum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on $[0,4]$ is [a] 2 [b] 6 [c] 0 [d] -3
17) If $y = x^x$, then $y' =$ [a] $x^x(1+\ln x)$ [b] $1+\ln x$ [c] x^x [d] $x^x \ln x$
18) If $y = \tan^{-1}\left(\frac{2x}{3}\right)$, then $y' =$ [a] $-\frac{6}{9+4x^2}$ [b] $\frac{9}{9+4x^2}$ [c] $-\frac{9}{9+4x^2}$ [d] $\frac{6}{9+4x^2}$
19) If $x^2 + y^2 = 3xy + 7$, then $y' =$ [a] $\frac{2x+y}{3x-2y}$ [b] $\frac{3y-2x}{2y-3x}$ [c] $\frac{2x}{3-2y}$ [d] $\frac{2x}{y}$

20) If $y = \sin x \sec x$, then $y' =$

a $\sin x \tan x + 1$ b $\sec^2 x$ c $\sin x \tan x - 1$ d $\sin x \sec x \tan x - 1$

21) If $y = \sin^3(4x)$, then $y' =$

a $4\cos^3(4x)$ b $3\sin^2(4x)\cos(4x)$

c $12\sin^2(4x)\cos(4x)$ d $4\sin^3(4x) + 12\sin^2 x \cos x$

22) The tangent line equation to the curve $y = \dfrac{2x}{x+1}$ at the point $(0,0)$ is

a $y = -2x$ b $y = -2x + 1$ c $y = 2x$ d $y = 2x - 1$

23) If $y = 3^x \cot x$, then $y' =$

a $3^x \ln 3 \cot x + 3^x \sec^2 x$ b $3^x \cot x + 3^x \sec^2 x$

c $3^x \cot x - 3^x \csc^2 x$ d $3^x \ln 3 \cot x - 3^x \csc^2 x$

24) If $y = \left(2x^2 + \sec x\right)^7$, then $y' =$

a $7\left(2x^2 + \sec x\right)^6$ b $7\left(2x^2 + \sec x\right)^6 (4x - \sec x \tan x)$

c $7\left(2x^2 + \sec x\right)^6 (4x + \sec x \tan x)$ d $28x\left(2x^2 + \sec x\right)^6$

25) The slope of the perpendicular line to the line $3y - 2x - 6 = 0$ is

a $\dfrac{2}{3}$ b $-\dfrac{2}{3}$ c $-\dfrac{3}{2}$ d $\dfrac{3}{2}$

26) If the graph of the function $f(x) = 3^x$ is shifted a distance 2 units upward, then the new graph represented the graph of the function

a 3^{x+2} b $3^x + 2$ c 3^{x-2} d $3^x - 2$

27) If $y = \ln \dfrac{x+1}{x-2}$, then $y' =$

a $-\dfrac{3}{(x+1)(x-2)}$ b $\dfrac{3}{(x+1)(x-2)}$

c $\dfrac{1}{(x+1)(x-2)}$ d $-\dfrac{1}{(x+1)(x-2)}$

28) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{5x} =$

a $\dfrac{3}{5}$ b $\dfrac{5}{3}$ c $\dfrac{1}{5}$ d 3

29) $D^{(125)}(\cos x) =$			
a $\sin x$	b $-\sin x$	c $\cos x$	d $-\cos x$
30) $\frac{5\pi}{6}$ rad $=$			
a 120^0	b 150^0	c 270^0	d 210^0
31) The distance between the points $(-1,2)$ and $(2,-1)$ is			
a $2\sqrt{3}$	b $3\sqrt{2}$	c 9	d 3
32) If $y = e^{2x}$, then $y^{(5)} =$			
a $128e^{2x}$	b $16e^{2x}$	c $64e^{2x}$	d $32e^{2x}$
33) The critical numbers of the function $f(x) = 2x^3 + 3x^2 - 12x + 15$ are			
a $1,-2$	b $-1,2$	c $1,2$	d $-1,-2$
34) The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ is increasing on			
a $(-\infty,-2)\cup(-1,\infty)$	b $(-\infty,-2)\cup(1,\infty)$	c $(-\infty,-1)\cup(2,\infty)$	d $(-\infty,1)\cup(2,\infty)$
35) The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ is decreasing on			
a $(-2,-1)$	b $(-2,1)$	c $(1,2)$	d $(-1,2)$
36) The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ has a relative maximum at			
a $(1,8)$	b $(-1,28)$	c $(2,19)$	d $(-2,35)$
37) The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ has a relative minimum at			
a $(1,8)$	b $(-1,28)$	c $(2,19)$	d $(-2,35)$
38) The graph of $f(x) = 2x^3 + 3x^2 - 12x + 15$ concave upward on			
a $(-\infty,\frac{1}{2})$	b $(-\infty,-\frac{1}{2})$	c $(-\frac{1}{2},\infty)$	d $(\frac{1}{2},\infty)$
39) The graph of $f(x) = 2x^3 + 3x^2 - 12x + 15$ concave downward on			
a $(-\infty,\frac{1}{2})$	b $(-\infty,-\frac{1}{2})$	c $(-\frac{1}{2},\infty)$	d $(\frac{1}{2},\infty)$
40) The function $f(x) = 2x^3 + 3x^2 - 12x + 15$ has an inflection at			
a $(\frac{1}{2},10)$	b $(-\frac{1}{2},10)$	c $(\frac{1}{2},\frac{43}{2})$	d $(-\frac{1}{2},\frac{43}{2})$

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1) If $y = \cos x \csc x$, then $y' =$

a $-\csc^2 x$ b $1-\cos x \cot x$ c $-1+\cos x \cot x$ d $1-\cos x \csc x \cot x$

2) If $f(x)=\cot^{-1}(x)$ and $g(x)=\cot(x)$ then $(f \circ g)(x)=$

a 1 b $\cot x \cot^{-1} x$ c x d $\cot x$

3) The function $f(x)=\dfrac{x+1}{x^2-49}$ is continuous on

a $\{x \in \mathbb{R} : x \neq \pm 7\}$ b $[-7,7]$ c $(-\infty,-7)\cup(7,\infty)$ d $\{\pm 7\}$

4) If $x^2-4=3xy-y^2$, then $y'=$

a $\dfrac{3y-2x}{2y-3x}$ b $\dfrac{2x}{y}$ c $\dfrac{2x}{3-2y}$ d $\dfrac{2x+y}{3x-2y}$

5) If $y=3^x \tan x$, then $y'=$

a $3^x \ln 3 \tan x - 3^x \sec^2 x$ b $3^x \ln 3 \tan x + 3^x \sec^2 x$

c $3^x \tan x - 3^x \sec^2 x$ d $3^x \tan x + 3^x \sec^2 x$

6) If $y=\log_5(x^3-2\csc x)$, then $y'=$

a $\dfrac{3x^2+2\csc x \cot x}{(x^3-2\csc x)\ln 5}$ b $\dfrac{3x^2+2\csc x \cot x}{x^3-2\csc x \ln 5}$

c $\dfrac{3x^2+2\csc x \cot x}{x^3-2\csc x}$ d $\dfrac{3x^2-2\csc x \cot x}{(x^3-2\csc x)\ln 5}$

7) If $y=(2x^2+\csc x)^7$, then $y'=$

a $7(2x^2+\csc x)^6(4x-\csc x \cot x)$ b $7(2x^2+\csc x)^6$

c $7(2x^2+\csc x)^6(4x+\csc x \cot x)$ d $28x(2x^2+\csc x)^6$

8) The absolute minimum value of $f(x)=x^3-6x^2+9x+2$ on $[0,4]$ is

a 6   b 0   c 2   d -3

9) The absolute maximum value of $f(x)=x^3-6x^2+9x+2$ on $[0,4]$ is

a 6   b 2   c 7   d 12

10) If $y=\sqrt{3x^2-6x}$, then $y'=$ $\boxed{a}$ $\dfrac{x-6}{\sqrt{3x^2-6x}}$ $\boxed{b}$ $\dfrac{6(x-1)}{\sqrt{3x^2-6x}}$ $\boxed{c}$ $\dfrac{x-1}{2\sqrt{3x^2-6x}}$ $\boxed{d}$ $\dfrac{3(x-1)}{\sqrt{3x^2-6x}}$
11) The slope of the perpendicular line to the line $2y+3x-6=0$ is $\boxed{a}$ $\dfrac{2}{3}$ $\boxed{b}$ $-\dfrac{2}{3}$ $\boxed{c}$ $-\dfrac{3}{2}$ $\boxed{d}$ $\dfrac{3}{2}$
12) If $y=\ln\dfrac{x+1}{x-2}$, then $y'=$ $\boxed{a}$ $\dfrac{3}{(x+1)(x-2)}$ $\boxed{b}$ $-\dfrac{3}{(x+1)(x-2)}$ $\boxed{c}$ $\dfrac{1}{(x+1)(x-2)}$ $\boxed{d}$ $-\dfrac{1}{(x+1)(x-2)}$
13) $\sec\left(\tan^{-1}x\right)=$ $\boxed{A}$ $\dfrac{1}{\sqrt{x^2+1}}$ $\boxed{B}$ $\dfrac{x}{\sqrt{x^2+1}}$ $\boxed{C}$ $\sqrt{x^2+1}$ $\boxed{D}$ $\dfrac{\sqrt{x^2+1}}{x}$
14) $\lim\limits_{x\to 0}\dfrac{\tan 5x}{3x}=$ $\boxed{a}$ $\dfrac{1}{3}$ $\boxed{b}$ 5 $\boxed{c}$ $\dfrac{3}{5}$ $\boxed{d}$ $\dfrac{5}{3}$
15) If $f(x)=2x+7$, then $f^{-1}(x)=$ $\boxed{a}$ $\dfrac{x+7}{2}$ $\boxed{b}$ $\dfrac{x}{2}-7$ $\boxed{c}$ $\dfrac{x}{2}+7$ $\boxed{d}$ $\dfrac{x-7}{2}$
16) $D^{(127)}(\cos x)=$ $\boxed{a}$ $\sin x$ $\boxed{b}$ $-\sin x$ $\boxed{c}$ $\cos x$ $\boxed{d}$ $-\cos x$
17) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-x\right)=$ $\boxed{a}$ $\dfrac{1}{2}$ $\boxed{b}$ 1 $\boxed{c}$ 0 $\boxed{d}$ $-\dfrac{1}{2}$
18) If $y=\sin^4(3x)$, then $y'=$ $\boxed{a}$ $12\sin^3(3x)\cos(3x)$ $\boxed{b}$ $4\sin^3(3x)\cos(3x)$ $\boxed{c}$ $3\cos^2(3x)$ $\boxed{d}$ $3\sin^4(3x)+12\sin^3 x\cos x$

19) $\frac{2\pi}{3}$ rad $=$ a 120^0 b 150^0 c 270^0 d 210^0
20) If $f(x)=x^2$, then $f'(x)=$ a $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$ b $\lim_{x\to 0}\frac{(x+h)^2+x^2}{h}$ c $\lim_{h\to 0}\frac{(x+h)^2+x^2}{h}$ d $\lim_{x\to 0}\frac{(x+h)^2-x^2}{h}$
21) The tangent line equation to the curve $y=\frac{2x}{x-1}$ at the point $(0,0)$ is   a $y=-2x-1$ b $y=2x+1$ c $y=2x$ d $y=-2x$
22) If the graph of the function $f(x)=3^x$ is shifted a distance 2 units to the right, then the new graph represented the graph of the function   a 3^{x+2} b 3^x+2 c 3^{x-2} d 3^x-2
23) The distance between the points $(-1,2)$ and $(2,-1)$ is   a 3 b $2\sqrt{3}$ c 9 d $3\sqrt{2}$
24) If $y=\ln(\sin x)$, then $y'=$ a $\tan x$ b $-\tan x$ c $\cot x$ d $-\cot x$
25) If $-7\le 2x+3\le 5$, then $x=$ a $(-5,1)$ b $(-5,1]$ c $[-5,1)$ d $[-5,1]$
26) If $y=\cot^{-1}\left(\frac{2x}{3}\right)$, then $y'=$ a $-\frac{6}{9+4x^2}$ b $\frac{9}{9+4x^2}$ c $-\frac{9}{9+4x^2}$ d $\frac{6}{9+4x^2}$
27) If $y=e^{2x}$, then $y^{(4)}=$ a $128e^{2x}$ b $16e^{2x}$ c $64e^{2x}$ d $32e^{2x}$
28) $\lim_{x\to 3^-}\frac{x+1}{x-3}=$ a 3 b ∞ c -3 d $-\infty$
29) If $y=x^x$, then $y'=$ a $1+\ln x$ b x^x c $x^x(1+\ln x)$ d $x^x\ln x$

30) The critical numbers of the function $f(x)=2x^3-3x^2-12x+15$ are a $1,-2$ b $-1,2$ c $1,2$ d $-1,-2$
31) The function $f(x)=2x^3-3x^2-12x+15$ is increasing on a $(-\infty,-2)\cup(-1,\infty)$ b $(-\infty,-2)\cup(1,\infty)$ c $(-\infty,-1)\cup(2,\infty)$ d $(-\infty,1)\cup(2,\infty)$
32) The function $f(x)=2x^3-3x^2-12x+15$ is decreasing on a $(-2,-1)$ b $(-2,1)$ c $(1,2)$ d $(-1,2)$
33) The function $f(x)=2x^3-3x^2-12x+15$ has a relative maximum at a $(1,2)$ b $(-1,22)$ c $(2,-5)$ d $(-2,11)$
34) The function $f(x)=2x^3-3x^2-12x+15$ has a relative minimum at a $(1,2)$ b $(-1,22)$ c $(2,-5)$ d $(-2,11)$
35) The graph of $f(x)=2x^3-3x^2-12x+15$ concave upward on a $(-\infty,\frac{1}{2})$ b $(-\infty,-\frac{1}{2})$ c $(-\frac{1}{2},\infty)$ d $(\frac{1}{2},\infty)$
36) The graph of $f(x)=2x^3-3x^2-12x+15$ concave downward on a $(-\infty,\frac{1}{2})$ b $(-\infty,-\frac{1}{2})$ c $(-\frac{1}{2},\infty)$ d $(\frac{1}{2},\infty)$
37) The function $f(x)=2x^3-3x^2-12x+15$ has an inflection at a $(\frac{1}{2},20)$ b $(-\frac{1}{2},20)$ c $(\frac{1}{2},\frac{17}{2})$ d $(-\frac{1}{2},\frac{17}{2})$
38) $\lim_{x\to 1}\frac{x^2-1}{x-1}=$ a 0 b does not exist c 2 d $\frac{1}{2}$
39) The domain of $\frac{x+3}{\sqrt{4-x^2}}$ is a $[-2,2]$ b $(-\infty,-2)\cup(2,\infty)$ c $(-2,2)$ d $(-\infty,-2]\cup[2,\infty)$
40) The values in $(-1,3)$ which makes $f(x)=x^2-5x+7$ satisfied Mean Value Theorem on $[-1,3]$ is a -4 b 0 c 1 d 2

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1) $\lim_{x\to 5^+} \frac{x+1}{x-5} =$

a ∞ b $-\infty$ c 5 d -5

2) $\lim_{x\to\infty} \left(\sqrt{x^2+x} - x\right) =$

a 1 b $\frac{1}{2}$ c 0 d $-\frac{1}{2}$

3) $y = -\ln(\cos x)$, then $y' =$

a $\tan x$ b $-\tan x$ c $\cot x$ d $-\cot x$

4) The absolute maximum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on $[0,4]$ is

a 2 b 12 c 7 d 6

5) The absolute minimum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on $[0,4]$ is

a 6 b 2 c 0 d -3

6) If $f(x) = \tan^{-1}(x)$ and $g(x) = \tan(x)$ then $(f \circ g)(x) =$

a x b $\tan^{-1} x \tan x$ c 1 d $\tan x$

7) If $y = x^x$, then $y' =$

a $1 + \ln x$ b $x^x(1+\ln x)$ c x^x d $x^x \ln x$

8) If $x^2 + y^2 - 5 = 3xy$, then $y' =$

a $\frac{2x+y}{3x-2y}$ b $\frac{2x}{y}$ c $\frac{2x}{3-2y}$ d $\frac{3y-2x}{2y-3x}$

9) The tangent line equation to the curve $y = \frac{2x}{x+1}$ at the point $(0,0)$ is

a $y = 2x$ b $y = -2x + 1$ c $y = -2x$ d $y = 2x - 1$

10) If $y = 3^x \cot x$, then $y' =$

a $3^x \ln 3 \cot x - 3^x \csc^2 x$ b $3^x \cot x + 3^x \sec^2 x$

c $3^x \cot x - 3^x \csc^2 x$ d $3^x \ln 3 \cot x + 3^x \sec^2 x$

11) $D^{(126)}(\cos x) =$

a $\sin x$ b $-\sin x$ c $\cos x$ d $-\cos x$

12) If $f(x)=2x-5$, then $f^{-1}(x)=$ [a] $\frac{x+5}{2}$ [b] $\frac{x}{2}-5$ [c] $\frac{x-5}{2}$ [d] $\frac{x}{2}+5$
13) The slope of the perpendicular line to the line $3y+2x-6=0$ is   [a] $\frac{2}{3}$ [b] $-\frac{2}{3}$ [c] $-\frac{3}{2}$ [d] $\frac{3}{2}$
14) If the graph of the function $f(x)=3^x$ is shifted a distance 2 units downward, then the new graph represented the graph of the function   [a] 3^{x+2} [b] 3^x+2 [c] 3^{x-2} [d] 3^x-2
15) If $y=\ln\frac{x+1}{x-2}$, then $y'=$ [a] $\frac{1}{(x+1)(x-2)}$ [b] $-\frac{1}{(x+1)(x-2)}$ [c] $\frac{3}{(x+1)(x-2)}$ [d] $-\frac{3}{(x+1)(x-2)}$
16) $\lim_{x\to 0}\frac{\sin 5x}{3x}=$ [a] $\frac{3}{5}$ [b] $\frac{5}{3}$ [c] $\frac{1}{3}$ [d] 5
17) If $y=\left(2x^2+\sec x\right)^7$, then $y'=$ [a] $7\left(2x^2+\sec x\right)^6$ [b] $7\left(2x^2+\sec x\right)^6(4x+\sec x\tan x)$ [c] $7\left(2x^2+\sec x\right)^6(4x-\sec x\tan x)$ [d] $28x\left(2x^2+\sec x\right)^6$
18) If $f(x)=x^2$, then $f'(x)=$ [a] $\lim_{x\to 0}\frac{(x+h)^2+x^2}{h}$ [b] $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$ [c] $\lim_{h\to 0}\frac{(x+h)^2+x^2}{h}$ [d] $\lim_{x\to 0}\frac{(x+h)^2-x^2}{h}$
19) $\frac{7\pi}{6}$ rad $=$ [a] 120^0 [b] 150^0 [c] 270^0 [d] 210^0
20) If $y=\sin x\sec x$, then $y'=$ [a] $\sin x\tan x+1$ [b] $\sin x\sec x\tan x-1$ [c] $\sin x\tan x-1$ [d] $\sec^2 x$

21) The values in $(-1,3)$ which makes $f(x)=x^2-5x+7$ satisfied Mean Value Theorem on $[-1,3]$ is [a] -4 [b] 1 [c] 0 [d] 2	

22) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous on [a] $\{\pm 3\}$ [b] $[-3,3]$ [c] $(-\infty,-3)\cup(3,\infty)$ [d] $\{x\in\mathbb{R}: x\neq\pm 3\}$	

23) $\cos\left(\tan^{-1}x\right)=$

[A] $\frac{1}{\sqrt{x^2+1}}$ [B] $\frac{x}{\sqrt{x^2+1}}$ [C] $\sqrt{x^2+1}$ [D] $\frac{\sqrt{x^2+1}}{x}$

24) The distance between the points $(-1,2)$ and $(2,-1)$ is

[a] $3\sqrt{2}$ [b] $2\sqrt{3}$ [c] 9 [d] 3

25) If $-7<2x+3\leq 5$, then $x=$

[a] $(-5,1)$ [b] $(-5,1]$ [c] $[-5,1)$ [d] $[-5,1]$

26) If $y=e^{2x}$, then $y^{(6)}=$

[a] $128e^{2x}$ [b] $16e^{2x}$ [c] $64e^{2x}$ [d] $32e^{2x}$

27) If $y=\sin^3(4x)$, then $y'=$

[a] $4\cos^3(4x)$ [b] $3\sin^2(4x)\cos(4x)$

[c] $4\sin^3(4x)+12\sin^2 x\cos x$ [d] $12\sin^2(4x)\cos(4x)$

28) The domain of $\frac{x+3}{\sqrt{x^2-4}}$ is

[a] $[-2,2]$ [b] $(-\infty,-2)\cup(2,\infty)$ [c] $(-2,2)$ [d] $(-\infty,-2]\cup[2,\infty)$

29) $\lim_{x\to 3}\frac{x^2-9}{x-3}=$

[a] -6 [b] 6 [c] ∞ [d] 0

30) If $y=\sqrt{3x^2+6x}$, then $y'=$

[a] $\frac{x+6}{\sqrt{3x^2+6x}}$ [b] $\frac{6(x+1)}{\sqrt{3x^2+6x}}$

[c] $\frac{x+1}{2\sqrt{3x^2+6x}}$ [d] $\frac{3(x+1)}{\sqrt{3x^2+6x}}$

31) If $y = \tan^{-1}\left(\dfrac{3x}{2}\right)$, then $y' =$
[a] $-\dfrac{4}{4+9x^2}$ [b] $\dfrac{6}{4+9x^2}$ [c] $-\dfrac{6}{4+9x^2}$ [d] $\dfrac{4}{4+9x^2}$
32) The critical numbers of the function $f(x) = 2x^3 - 3x^2 - 12x + 16$ are
[a] $1, -2$ [b] $-1, 2$ [c] $1, 2$ [d] $-1, -2$
33) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ is increasing on
[a] $(-\infty,-2)\cup(-1,\infty)$ [b] $(-\infty,-2)\cup(1,\infty)$ [c] $(-\infty,-1)\cup(2,\infty)$ [d] $(-\infty,1)\cup(2,\infty)$
34) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ is decreasing on
[a] $(-2,-1)$ [b] $(-2,1)$ [c] $(1,2)$ [d] $(-1,2)$
35) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ has a relative maximum at
[a] $(1,3)$ [b] $(-1,-23)$ [c] $(2,-4)$ [d] $(-2,12)$
36) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ has a relative minimum at
[a] $(1,3)$ [b] $(-1,-23)$ [c] $(2,-4)$ [d] $(-2,12)$
37) The graph of $f(x) = 2x^3 - 3x^2 - 12x + 16$ concave upward on
[a] $(-\infty,\frac{1}{2})$ [b] $(-\infty,-\frac{1}{2})$ [c] $(-\frac{1}{2},\infty)$ [d] $(\frac{1}{2},\infty)$
38) The graph of $f(x) = 2x^3 - 3x^2 - 12x + 16$ concave downward on
[a] $(-\infty,\frac{1}{2})$ [b] $(-\infty,-\frac{1}{2})$ [c] $(-\frac{1}{2},\infty)$ [d] $(\frac{1}{2},\infty)$
39) The function $f(x) = 2x^3 - 3x^2 - 12x + 16$ has an inflection at
[a] $(\frac{1}{2},21)$ [b] $(-\frac{1}{2},21)$ [c] $(\frac{1}{2},\frac{19}{2})$ [d] $(-\frac{1}{2},\frac{19}{2})$
40) If $y = \log_5(x^3 - 2\csc x)$, then $y' =$
[a] $\dfrac{3x^2 + 2\csc x \cot x}{x^3 - 2\csc x \ln 5}$ [b] $\dfrac{3x^2 + 2\csc x \cot x}{\left(x^3 - 2\csc x\right)\ln 5}$
[c] $\dfrac{3x^2 + 2\csc x \cot x}{x^3 - 2\csc x}$ [d] $\dfrac{3x^2 - 2\csc x \cot x}{\left(x^3 - 2\csc x\right)\ln 5}$

King Abdul Aziz University	Faculty of Sciences	Mathematics Department
Math 110	Final Test Fall 2013 (40 Marks)	Time 120 m
Student Name:	Student Number:	D

1) If $y = -\ln(\sin x)$, then $y' =$

a $\tan x$ b $-\tan x$ c $\cot x$ d $-\cot x$

2) If $y = x^x$, then $y' =$

a $1+\ln x$ b x^x c $x^x \ln x$ d $x^x(1+\ln x)$

3) If $y = \cot^{-1}\left(\frac{3x}{2}\right)$, then $y' =$

a $-\frac{4}{4+9x^2}$ b $\frac{6}{4+9x^2}$ c $-\frac{6}{4+9x^2}$ d $\frac{4}{4+9x^2}$

4) If $y = \sin^4(3x)$, then $y' =$

a $4\sin^3(3x)\cos(3x)$ b $12\sin^3(3x)\cos(3x)$

c $3\cos^2(3x)$ d $3\sin^4(3x)+12\sin^3 x \cos x$

5) The tangent line equation to the curve $y = \frac{2x}{x-1}$ at the point $(0,0)$ is

a $y = -2x - 1$ b $y = -2x$

c $y = 2x$ d $y = 2x + 1$

6) If $y^2 - 2 = 3xy - x^2$, then $y' =$

a $\frac{2x}{3-2y}$ b $\frac{2x}{y}$ c $\frac{3y-2x}{2y-3x}$ d $\frac{2x+y}{3x-2y}$

7) If $y = 3^x \tan x$, then $y' =$

a $3^x \ln 3 \tan x - 3^x \sec^2 x$ b $3^x \tan x - 3^x \sec^2 x$

c $3^x \ln 3 \tan x + 3^x \sec^2 x$ d $3^x \tan x + 3^x \sec^2 x$

8) If $y = \left(2x^2 + \csc x\right)^7$, then $y' =$

a $28x\left(2x^2+\csc x\right)^6$ b $7\left(2x^2+\csc x\right)^6$

c $7\left(2x^2+\csc x\right)^6(4x + \csc x \cot x)$ d $7\left(2x^2+\csc x\right)^6(4x - \csc x \cot x)$

9) The slope of the perpendicular line to the line $2y - 3x - 6 = 0$ is

a $\frac{2}{3}$ b $-\frac{2}{3}$ c $-\frac{3}{2}$ d $\frac{3}{2}$

10) $D^{(128)}(\cos x) =$

a $\sin x$ b $-\sin x$ c $\cos x$ d $-\cos x$

11) If $y = \sqrt{3x^2 - 6x}$, then $y' =$

a $\dfrac{6(x-1)}{\sqrt{3x^2-6x}}$ b $\dfrac{x-6}{\sqrt{3x^2-6x}}$ c $\dfrac{3(x-1)}{\sqrt{3x^2-6x}}$ d $\dfrac{x-1}{2\sqrt{3x^2-6x}}$

12) If $y = \ln\dfrac{x+1}{x-2}$, then $y' =$

a $\dfrac{1}{(x+1)(x-2)}$ b $-\dfrac{1}{(x+1)(x-2)}$ c $\dfrac{3}{(x+1)(x-2)}$ d $-\dfrac{3}{(x+1)(x-2)}$

13) $\lim\limits_{x \to 3^-} \dfrac{x+1}{x-3} =$

a $-\infty$ b -3 c ∞ d 3

14) If the graph of the function $f(x) = 3^x$ is shifted a distance 2 units to the left, then the new graph represented the graph of the function

a 3^{x+2} b $3^x + 2$ c 3^{x-2} d $3^x - 2$

15) If $f(x) = x^2$, then $f'(x) =$

a $\lim\limits_{x \to 0} \dfrac{(x+h)^2 + x^2}{h}$ b $\lim\limits_{h \to 0} \dfrac{(x+h)^2 + x^2}{h}$

c $\lim\limits_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$ d $\lim\limits_{x \to 0} \dfrac{(x+h)^2 - x^2}{h}$

16) If $y = \log_5(x^3 - 2\csc x)$, then $y' =$

a $\dfrac{3x^2 + 2\csc x \cot x}{x^3 - 2\csc x \ln 5}$ b $\dfrac{3x^2 - 2\csc x \cot x}{\left(x^3 - 2\csc x\right)\ln 5}$

c $\dfrac{3x^2 + 2\csc x \cot x}{x^3 - 2\csc x}$ d $\dfrac{3x^2 + 2\csc x \cot x}{\left(x^3 - 2\csc x\right)\ln 5}$

17) $\lim\limits_{x \to 0} \dfrac{\tan 3x}{5x} =$

a $\dfrac{1}{5}$ b $\dfrac{5}{3}$ c $\dfrac{3}{5}$ d 3

18) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} =$

[a] ∞ [b] 0 [c] 4 [d] $\frac{1}{4}$

19) The distance between the points $(-1,2)$ and $(2,-1)$ is

[a] 3 [b] $2\sqrt{3}$ [c] $3\sqrt{2}$ [d] 9

20) $\frac{3\pi}{2}$ rad $=$

[a] 120^0 [b] 150^0 [c] 270^0 [d] 210^0

21) The absolute minimum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on $[0,4]$ is

[a] 6 [b] 0 [c] -3 [d] 2

22) The absolute maximum value of $f(x) = x^3 - 6x^2 + 9x + 2$ on $[0,4]$ is

[a] 7 [b] 2 [c] 6 [d] 12

23) The values in $(-1,3)$ which makes $f(x) = x^2 - 5x + 7$ satisfied Mean Value Theorem on $[-1,3]$ is

[a] -4 [b] 0 [c] 2 [d] 1

24) The domain of $\frac{x+3}{\sqrt{4 - x^2}}$ is

[a] $(-2,2)$ [b] $(-\infty,-2) \cup (2,\infty)$ [c] $[-2,2]$ [d] $(-\infty,-2] \cup [2,\infty)$

25) The function $f(x) = \frac{x+1}{x^2 - 25}$ is continuous on

[a] $[-5,5]$ [b] $\{x \in \mathbb{R} : x \neq \pm 5\}$ [c] $(-\infty,-5) \cup (5,\infty)$ [d] $\{\pm 5\}$

26) If $f(x) = \cot^{-1}(x)$ and $g(x) = \cot(x)$ then $(f \circ g)(x) =$

[a] 1 [b] $\cot x \cot^{-1} x$ [c] $\cot x$ [d] x

27) If $y = e^{2x}$, then $y^{(7)} =$

[a] $128e^{2x}$ [b] $16e^{2x}$ [c] $64e^{2x}$ [d] $32e^{2x}$

28) If $-7 < 2x + 3 < 5$, then $x =$

[a] $(-5,1)$ [b] $(-5,1]$ [c] $[-5,1)$ [d] $[-5,1]$

29) If $y = \cos x \csc x$, then $y' =$

[a] $-1 + \cos x \cot x$ [b] $1 - \cos x \cot x$ [c] $-\csc^2 x$ [d] $1 - \cos x \csc x \cot x$

30) The critical numbers of the function $f(x)=2x^3+3x^2-12x+16$ are				
	a $1,-2$	b $-1,2$	c $1,2$	d $-1,-2$
31) The function $f(x)=2x^3+3x^2-12x+16$ is increasing on				
	a $(-\infty,-2)\cup(-1,\infty)$	b $(-\infty,-2)\cup(1,\infty)$	c $(-\infty,-1)\cup(2,\infty)$	d $(-\infty,1)\cup(2,\infty)$
32) The function $f(x)=2x^3+3x^2-12x+16$ is decreasing on				
	$(-\infty,-1)\cup(2,\infty)$	b $(-\infty,-2)\cup(1,\infty)$	c $(-2,1)$	d $(-1,2)$
33) The function $f(x)=2x^3-3x^2-12x+15$ has a relative maximum at				
	a $(1,9)$	b $(-1,29)$	c $(2,20)$	d $(-2,36)$
34) The function $f(x)=2x^3+3x^2-12x+16$ has a relative minimum at				
	a $(1,9)$	b $(-1,29)$	c $(2,20)$	d $(-2,36)$
35) The graph of $f(x)=2x^3+3x^2-12x+16$ concave upward on				
	a $(-\infty,\frac{1}{2})$	b $(-\infty,-\frac{1}{2})$	c $(-\frac{1}{2},\infty)$	d $(\frac{1}{2},\infty)$
36) The graph of $f(x)=2x^3-3x^2-12x+15$ concave downward on				
	a $(-\infty,\frac{1}{2})$	b $(-\infty,-\frac{1}{2})$	c $(-\frac{1}{2},\infty)$	d $(\frac{1}{2},\infty)$
37) The function $f(x)=2x^3+3x^2-12x+16$ has an inflection at				
	a $(\frac{1}{2},11)$	b $(-\frac{1}{2},11)$	c $(\frac{1}{2},\frac{45}{2})$	d $(-\frac{1}{2},\frac{45}{2})$
38) $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)=$				
	a $\frac{1}{2}$	b 1	c 0	d $-\frac{1}{2}$
39) $\sin\left(\tan^{-1}x\right)=$				
	A $\frac{1}{\sqrt{x^2+1}}$	B $\frac{x}{\sqrt{x^2+1}}$	C $\sqrt{x^2+1}$	D $\frac{\sqrt{x^2+1}}{x}$
40) If $f(x)=2x+11$, then $f^{-1}(x)=$				
	a $\frac{x+11}{2}$	b $\frac{x-11}{2}$	c $\frac{x}{2}+11$	d $\frac{x}{2}-11$