

### Solutions to Limits Worksheet (1)

| Question | Answer         |
|----------|----------------|
| 1        | $-\infty$      |
| 2        | $\infty$       |
| 3        | $\infty$       |
| 4        | $-\infty$      |
| 5        | 2              |
| 6        | $\frac{1}{2}$  |
| 7        | Does not exist |
| 8        | 4              |
| 9        | 0              |

### Solutions to Limits Worksheet (2)

| Question | Answer | Question | Answer |
|----------|--------|----------|--------|
| 1        | B      | 11       | A      |
| 2        | A      | 12       | B      |
| 3        | C      | 13       | C      |
| 4        | C      | 14       | A      |
| 5        | D      | 15       | D      |
| 6        | C      | 16       | A      |
| 7        | C      | 17       | B      |
| 8        | B      | 18       | B      |
| 9        | A      | 19       | C      |
| 10       | C      | 20       | A      |

### Solutions to Continuity Worksheet (1)

| Question | Answer           |
|----------|------------------|
| 1        | Discontinuous    |
| 2        | Continuous       |
| 3        | Discontinuous    |
| 4        | Right continuous |
| 5        | Continuous       |
| 6        | Discontinuous    |
| 7        | Left continuous  |
| 8        | Continuous       |
| 9        | Continuous       |
| 10       | Discontinuous    |
| 11       | Discontinuous    |
| 12       | Continuous       |

### Solutions to Continuity Worksheet (2)

| Question | Answer |
|----------|--------|
| 1        | B      |
| 2        | D      |
| 3        | C      |
| 4        | C      |
| 5        | B      |
| 6        | C      |
| 7        | A      |

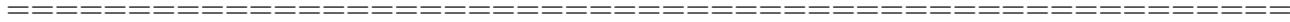
**MATH 110**  
**Continuity**  
**Worksheet (2)**

Choose the correct answer:

|   |                                 |
|---|---------------------------------|
| 1. The function $f(x) = \begin{cases} 3x - 1 & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$ is ..... at $x = 1$ . |                                 |
| (a) continuous  | (c) right continuous            |
| (b) left continuous   | (d) discontinuous               |
| 2. The function $f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ is ..... at $x = 0$ .        |                                 |
| (a) continuous  | (c) right continuous            |
| (b) left continuous   | (d) discontinuous               |
| 3. The function $f(x) = \sqrt{3x - 5}$ is continuous on the interval.....   |                                 |
| (a) $(-\infty, \frac{5}{3}]$  | (c) $[\frac{5}{3}, \infty)$     |
| (b) $(-\infty, -\frac{5}{3}]$   | (d) $[-\frac{5}{3}, \infty)$    |
| 4. The function $f(x) = [x]$ is continuous on ..... where $n$ is an integer.  |                                 |
| (a) every interval $(n, n + 1]$   | (c) every interval $[n, n + 1)$ |
| (b) every interval $(n - 1, n]$   | (d) every interval $[n - 1, n)$ |

|   |   |
|---|---|
| 5. If $f(x) = \begin{cases} x^2 - k & \text{if } x \leq 6 \\ x & \text{if } x > 6 \end{cases}$ is a continuous function, then $k = \dots\dots\dots$ |   |
| (a) 36  | (c) 6   |
| (b) 30  | (d) 3   |
| 6. The continuous extension of the function $f(x) = \frac{x^2 - 2x - 3}{x^2 - 9}$ at $x = 3$ is .....   |   |
| (a) $F(x) = \begin{cases} \frac{x^2 - 2x - 3}{x^2 - 9} & \text{if } x \neq 3 \\ 3 & \text{if } x = 3 \end{cases}$                                   | (c) $F(x) = \begin{cases} \frac{x^2 - 2x - 3}{x^2 - 9} & \text{if } x \neq 3 \\ \frac{2}{3} & \text{if } x = 3 \end{cases}$ |
| (b) $F(x) = \begin{cases} \frac{x^2 - 2x - 3}{x^2 - 9} & \text{if } x \neq 3 \\ 0 & \text{if } x = 3 \end{cases}$                                   | (d) $F(x) = \begin{cases} \frac{x^2 - 2x - 3}{x^2 - 9} & \text{if } x \neq 3 \\ -3 & \text{if } x = 3 \end{cases}$          |
| 7. Which of the following functions has a removable discontinuity at $x = 1$ ?  |   |
| (a) $g(x) = \begin{cases} x & \text{if } x \neq 1 \\ -1 & \text{if } x = 1 \end{cases}$   | (c) $g(x) = \begin{cases} x & \text{if } x < 1 \\ -1 & \text{if } x > 1 \end{cases}$  |
| (b) $g(x) = \begin{cases} x & \text{if } x < 1 \\ -1 & \text{if } x \geq 1 \end{cases}$   | (d) $g(x) = \begin{cases} x & \text{if } x > 1 \\ -1 & \text{if } x \leq 1 \end{cases}$                                     |

**MATH 110**  
**Differentiation**  
**Worksheet (1)**



Choose the correct answer:

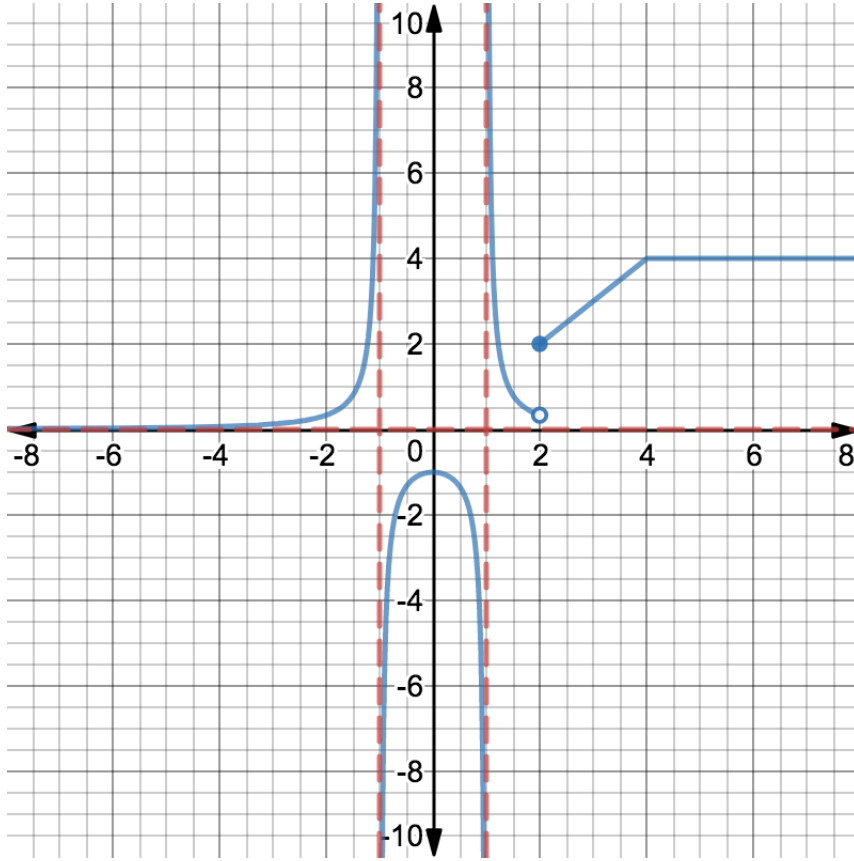
|   |  |
|---|--|
| 1. The function $f(x) =  x $ has a ..... at $x = 0$ .                                   |  |
| (a) vertical tangent line   | (c) nonvertical tangent line                                       |
| (b) horizontal tangent line   | (d) no tangent line  |
| 2. The function $f(x) = x^3 + 5$ has a ..... at the point $(1, 6)$ .                    |  |
| (a) vertical tangent line   | (c) nonvertical tangent line                                       |
| (b) horizontal tangent line   | (d) no tangent line  |
| 3. The derivative formula of the function $f(x) = x^2 + 1$ is $f'(x) = \dots\dots\dots$ |  |
| (a) $\lim_{h \rightarrow 0} \frac{(x^2 + h + 1) - (x^2 + 1)}{h}$                        | (c) $\lim_{h \rightarrow 0} \frac{(x + h)^2 + 1 - (x^2 + 1)}{h}$   |
| (b) $\lim_{h \rightarrow 0} \frac{(x + h + 1)^2 - (x^2 + 1)}{h}$                        | (d) $\lim_{h \rightarrow 0} \frac{(x^2 + h)^2 + 1 - (x^2 + 1)}{h}$ |
| 4. The derivative of the function $f(x) = 5x^6 + x^3 + 1$ is .....                      |  |
| (a) $f'(x) = 6x^5 + 3x^2$   | (c) $f'(x) = 30x^5 + 3x + 1$                                       |
| (b) $f'(x) = 11x^5 + 3x^2$  | (d) $f'(x) = 30x^5 + 3x^2$   |

|  |   |
|--|---|
| 5. The derivative of the function $f(x) = \frac{2x + 1}{x + 2}$ is .....           |   |
| (a) $f'(x) = \frac{2}{(x + 2)^2}$  | (c) $f'(x) = \frac{5}{(x + 2)^2}$             |
| (b) $f'(x) = \frac{3}{(x + 2)^2}$  | (d) $f'(x) = \frac{3x + 2}{(x + 2)^2}$        |
| 6. The derivative of the function $f(x) = \frac{1}{4x}$ is .....                   |   |
| (a) $f'(x) = -\frac{1}{4x^2}$  | (c) $f'(x) = \frac{1}{4}$                     |
| (b) $f'(x) = -\frac{1}{16x^2}$   | (d) $f'(x) = 4$                               |
| 7. $\frac{d}{dx} \left( \frac{x^5 + 2x^4 + 3x^2 + 1}{x} \right) = \dots\dots\dots$ |   |
| (a) $4x^3 + 6x^2 + 3 + \frac{1}{x^2}$  | (c) $4x^3 + 6x^2 + 3 - \frac{1}{x^2}$         |
| (b) $5x^2 + 8x + \frac{6}{x}$  | (d) $4x^3 + 6x^2 + 4$                         |
| 8. If $f(x) = x^{5/2} + 2x^{3/2} + 4$ , then $f'(x) = \dots\dots\dots$             |   |
| (a) $\frac{5}{2}x^2 + 3x$  | (c) $\frac{5}{2}x^{3/2} + 3x^{1/2}$           |
| (b) $\frac{5}{2}x^3 + 3x$  | (d) $\frac{5}{2}x^{3/2} + \frac{3}{2}x^{1/2}$ |
| 9. $\frac{d}{dx} \left( \frac{4x}{1 + x^2} \right) \Big _{x=2} = \dots\dots\dots$  |   |
| (a) $\frac{12}{25}$  | (c) $\frac{12}{5}$                            |
| (b) $-\frac{12}{25}$   | (d) $-\frac{12}{5}$                           |

|   |                              |
|---|------------------------------|
| 10. The equation of the tangent line to $f(x) = x^2 + 2x$ at $x = 4$ is ..... |                              |
| (a) $y = 10x + 64$  | (c) $y = 10x + 16$           |
| (b) $y = 10x - 64$  | (d) $y = 10x - 16$           |
| 11. The equation of the normal line to $f(x) = \sqrt{x}$ at $x = 9$ is .....  |                              |
| (a) $y = -6x + 57$  | (c) $y = 6x + 57$            |
| (b) $y = \frac{1}{6}x + 57$   | (d) $y = -\frac{1}{6}x + 57$ |

MATH 110  
Limits of Functions  
Worksheet (1)

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Use the above graph of the function  $f(x)$  to find the following limits:

1.  $\lim_{x \rightarrow -1^+} f(x) = \dots\dots\dots$
2.  $\lim_{x \rightarrow -1^-} f(x) = \dots\dots\dots$
3.  $\lim_{x \rightarrow 1^+} f(x) = \dots\dots\dots$
4.  $\lim_{x \rightarrow 1^-} f(x) = \dots\dots\dots$
5.  $\lim_{x \rightarrow 2^+} f(x) = \dots\dots\dots$
6.  $\lim_{x \rightarrow 2^-} f(x) = \dots\dots\dots$
7.  $\lim_{x \rightarrow 2} f(x) = \dots\dots\dots$
8.  $\lim_{x \rightarrow \infty} f(x) = \dots\dots\dots$
9.  $\lim_{x \rightarrow -\infty} f(x) = \dots\dots\dots$



**MATH 110**  
**Limits of Functions**  
**Worksheet (2)**

=====

Choose the correct answer:

|  |       |              |               |
|--|-------|--------------|---------------|
| 1. $\lim_{x \rightarrow 5} 6 = \dots\dots\dots$                                |       |              |               |
| (a) 5  | (b) 6 | (c) $\infty$ | (d) $-\infty$ |
| 2. $\lim_{x \rightarrow 0} x^2 - 1 = \dots\dots\dots$                          |       |              |               |
| (a) -1   | (b) 0 | (c) 1        | (d) $\infty$  |
| 3. $\lim_{x \rightarrow 0} \frac{1}{x^2} = \dots\dots\dots$                    |       |              |               |
| (a) 0  | (b) 2 | (c) $\infty$ | (d) $-\infty$ |
| 4. $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x - 3} = \dots\dots\dots$       |       |              |               |
| (a) 0  | (b) 1 | (c) 2        | (d) $\infty$  |
| 5. $\lim_{x \rightarrow \infty} \frac{3x^3 - 2x + 5}{x - 3} = \dots\dots\dots$ |       |              |               |
| (a) 0  | (b) 1 | (c) 2        | (d) $\infty$  |
| 6. $\lim_{x \rightarrow \infty} \frac{2x + 5}{x - 3} = \dots\dots\dots$        |       |              |               |
| (a) 0  | (b) 1 | (c) 2        | (d) $\infty$  |
| 7. $\lim_{x \rightarrow 3^+} \frac{x + 2}{x^2 - 9} = \dots\dots\dots$          |       |              |               |
| (a) 0  | (b) 1 | (c) $\infty$ | (d) $-\infty$ |

|  |                    |                    |                    |
|--|--------------------|--------------------|--------------------|
| 8. $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x^2 - 9x} = \dots\dots\dots$  |                    |                    |                    |
| (a) 0  | (b) $\frac{1}{54}$ | (c) $\frac{1}{81}$ | (d) $\infty$       |
| 9. $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{9-x} - \sqrt{9+x}} = \dots\dots\dots$   |                    |                    |                    |
| (a) -6   | (b) -3             | (c) 3              | (d) 6              |
| 10. $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2 + 2}}{12x} = \dots\dots\dots$   |                    |                    |                    |
| (a) $\frac{1}{4}$  | (b) $\frac{3}{4}$  | (c) $-\frac{1}{4}$ | (d) $-\frac{3}{4}$ |
| 11. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2x} - x}{2} = \dots\dots\dots$  |                    |                    |                    |
| (a) $\frac{1}{2}$  | (b) 1              | (c) 2              | (d) $\infty$       |
| 12. $\lim_{x \rightarrow 6^+} \frac{ x-6 }{x^2 - 4x - 12} = \dots\dots\dots$   |                    |                    |                    |
| (a) 0  | (b) $\frac{1}{8}$  | (c) $-\frac{1}{8}$ | (d) $\infty$       |
| 13. $\lim_{x \rightarrow 2^-} \frac{ x-2 }{x^2 - 4} = \dots\dots\dots$   |                    |                    |                    |
| (a) 0  | (b) $\frac{1}{4}$  | (c) $-\frac{1}{4}$ | (d) $-\infty$      |
| 14. If $2x \leq f(x) \leq x^4 - x^2 + 2$ , then $\lim_{x \rightarrow 1} f(x) = \dots\dots\dots$  |                    |                    |                    |
| (a) 2  | (b) 3              | (c) $\infty$       | (d) $-\infty$      |
| 15. Let $f(x) = \begin{cases} x-1 & \text{if } x \leq 1 \\ 3x^2 & \text{if } x > 1 \end{cases}$ be a function. Then, $\lim_{x \rightarrow 1} f(x) = \dots\dots\dots$ |                    |                    |                    |
| (a) 0  | (b) 1              | (c) 3              | (d) dose not exist |

|  |        |                    |                     |
|--|--------|--------------------|---------------------|
| 16. Let $f(x) = \begin{cases} x - 1 & \text{if } x \leq 1 \\ 3x^2 & \text{if } x > 1 \end{cases}$ be a function. Then, $\lim_{x \rightarrow 1^-} f(x) = \dots\dots\dots$ |        |                    |                     |
| (a) 0  | (b) 1  | (c) 3              | (d) dose not exist  |
| 17. Let $f(x) = \begin{cases} x - 1 & \text{if } x \neq 3 \\ -2 & \text{if } x = 3 \end{cases}$ be a function. Then, $\lim_{x \rightarrow 3} f(x) = \dots\dots\dots$     |        |                    |                     |
| (a) -2   | (b) 2  | (c) 3              | (d) dose not exist  |
| 18. If $\lim_{x \rightarrow 2} f(x) = -4$ and $\lim_{x \rightarrow 2} g(x) = 3$ , then $\lim_{x \rightarrow 2} \frac{2f(x) + 5}{g(x)} = \dots\dots\dots$                 |        |                    |                     |
| (a) 1  | (b) -1 | (c) $\frac{13}{3}$ | (d) $-\frac{13}{3}$ |
| 19. $\lim_{x \rightarrow -\infty} (5x^6 - 9x^2 + 2) = \dots\dots\dots$   |        |                    |                     |
| (a) 5  | (b) 6  | (c) $\infty$       | (d) $-\infty$       |
| 20. $\lim_{x \rightarrow 5^-} [x] = \dots\dots\dots$   |        |                    |                     |
| (a) 4  | (b) 5  | (c) 6              | (d) does not exist  |



(1) The absolute maximum point of  $f(x) = x^2 - 2x$  on  $[-1,2]$  is

نوجد المشتقة الاولى ثم نساويها بالصفر لايجاد قيمة  $x$

$$f'(x) = 2x - 2$$

$$2x - 2 = 0$$

$$2x = 2$$

$$x = 1$$

نعوض عن قيمة  $x$  واطراف الفترة لتحديد القيمة العظمى المطلقة (ذات الناتج الاكبر) والقيمة الصغرى المطلقة (ذات الناتج الاصغر)

$$f(1) = (1)^2 - 2(1) = 1 - 2 = -1 \rightarrow \text{AbsMinimum}$$

$$f(-1) = (-1)^2 - 2(-1) = 1 + 2 = 3 \rightarrow \text{AbsMaximum}$$

$$f(2) = (2)^2 - 2(2) = 4 - 4 = 0$$

Absolute Maximum at  $(-1, f(-1)) = (-1, 3)$

Absolute Minimum at  $(1, f(1)) = (1, -1)$

(2) The absolute minimum point of  $f(x) = x^2 - 2x$  on  $[-1,2]$  is

Absolute Minimum at  $(1, f(1)) = (1, -1)$

(3) The critical numbers of  $f(x) = 2x^3 - 3x^2 - 12x + 3$  are  $-1, 2$

نوجد المشتقة الاولى ثم نساويها بالصفر لايجاد قيمة  $x$

$$f'(x) = 6x^2 - 6x - 12$$

$$6x^2 - 6x - 12 = 0$$

$$\frac{6}{6}x^2 - \frac{6}{6}x - \frac{12}{6} = \frac{0}{6}$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

*either*

$$x - 2 = 0 \rightarrow x = 2$$

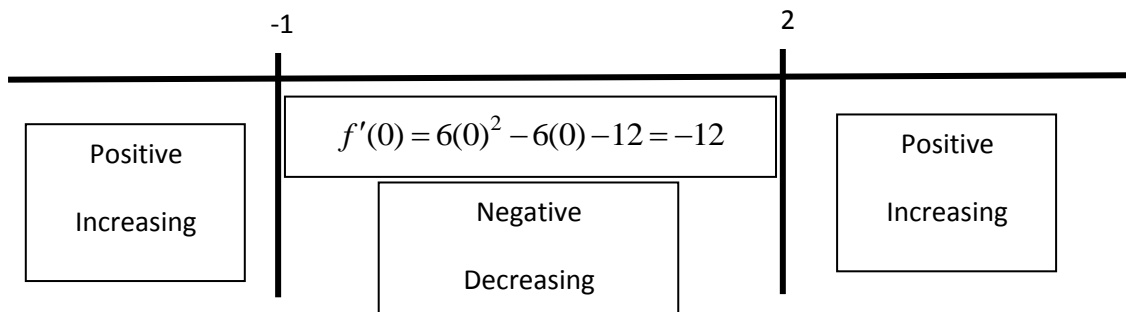
*or*

$$x + 1 = 0 \rightarrow x = -1$$

النقاط الحرجة هي -1,2

(4) The function  $f(x) = 2x^3 - 3x^2 - 12x + 3$  is increasing on

نمثل -1,2 على خط الاعداد ونختبر الاشارة بكل فترة بالتعويض عن القيم الاختيارية في المشتقة الاولى



F is increasing on  $(-\infty, -1) \cup (2, \infty)$

(5) The function  $f(x) = 2x^3 - 3x^2 - 12x + 3$  is decreasing on

F is decreasing on  $(-1, 2)$

(6) The function  $f(x) = 2x^3 - 3x^2 - 12x + 3$  has a local maximum at the point

التغيير من تزايدية الى تناقصية تعتبر نقطة عظمى محلية

Local maximum at  $(-1, f(-1)) = (-1, 10)$

(7) The function  $f(x) = 2x^3 - 3x^2 - 12x + 3$  has a local minimum at the point

التغيير من تناقصية الى تزايدية تعتبر نقطة صغرى محلية

Local minimum at  $(2, f(2)) = (2, -17)$

(8) The function  $f(x) = 2x^3 - 3x^2 - 12x + 3$  has an inflection point at

نوجد المشتقة الثانية ثم نساويها بالصفر لاجاد قيمة  $x$

$$f''(x) = 12x - 6$$

$$12x - 6 = 0$$

$$\frac{12}{12}x = \frac{6}{12}$$

$$x = \frac{1}{2}$$

Inflection point at  $(\frac{1}{2}, f(\frac{1}{2})) = (\frac{1}{2}, -\frac{7}{2})$

(9) The graph of the function  $f(x) = 2x^3 - 3x^2 - 12x + 3$  is concave up on

$$\frac{1}{2}$$

$$f''(0) = 12(0) - 6 = -6$$

Negative

Concave Down

$$f''(1) = 12(1) - 6 = 6$$

Positive

Concave Up

F is Concave up on  $(\frac{1}{2}, \infty)$

(10) The graph of the function  $f(x) = 2x^3 - 3x^2 - 12x + 3$  is concave down on

F is Concave down on  $(-\infty, \frac{1}{2})$