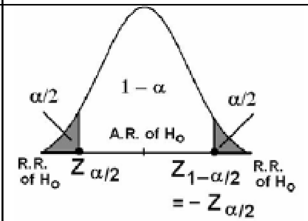
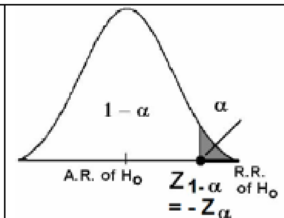
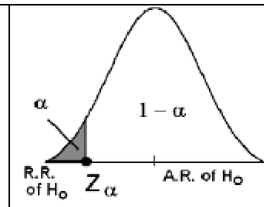
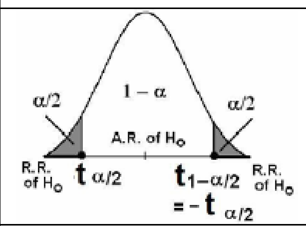
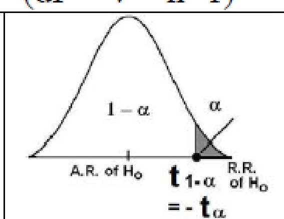
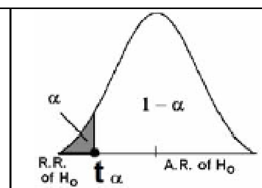
Hypotheses Testing

1-Single Mean

(*if σ known*):

Hypotheses	H_o: $\mu = \mu_o$ H_A: $\mu \neq \mu_o$	H_o: $\mu \leq \mu_o$ H_A: $\mu > \mu_o$	H_o: $\mu \geq \mu_o$ H_A: $\mu < \mu_o$
Test Statistic (T.S.)	Calculate the value of: $Z = \dfrac{\overline{X} - \mu_o}{\sigma / \sqrt{n}} \sim N(0,1)$		
R.R. & A.R. of H_o			
Critical value (s)	$Z_{\alpha/2}$ and $-Z_{\alpha/2}$	$Z_{1-\alpha} = -Z_{\alpha}$	Z_{α}
Decision:	We reject H_o (and accept H_A) at the significance level α if:		
	$Z < Z_{\alpha/2}$ or $Z > Z_{1-\alpha/2} = -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{1-\alpha} = -Z_{\alpha}$ One-Sided Test	$Z < Z_{\alpha}$ One-Sided Test

(*if σ unknown*):

Hypotheses	H_o: $\mu = \mu_o$ H_A $\mu \neq \mu_o$	H_o: $\mu \leq \mu_o$ H_A: $\mu > \mu_o$	H_o: $\mu \geq \mu_o$ H_A: $\mu < \mu_o$
Test Statistic (T.S.)	Calculate the value of: $t = \dfrac{\overline{X} - \mu_o}{S / \sqrt{n}} \sim t(n-1)$ $(df = \nu = n-1)$		
R.R. & A.R. of H_o			
Critical value (s)	$t_{\alpha/2}$ and $-t_{\alpha/2}$	$t_{1-\alpha} = -t_{\alpha}$	t_{α}
Decision:	We reject H_o (and accept H_A) at the significance level α if:		
	$t < t_{\alpha/2}$ or $t > t_{1-\alpha/2} = -t_{\alpha/2}$ Two-Sided Test	$t > t_{1-\alpha} = -t_{\alpha}$ One-Sided Test	$t < t_{\alpha}$ One-Sided Test

Question 1:

Suppose that we are interested in estimating the true average time in seconds it takes an adult to open a new type of tamper-resistant aspirin bottle. It is known that the population standard deviation is $\sigma = 5.71$seconds. A random sample of 40 adults gave a mean of 20.6 seconds. Let μ be the population mean, then, to test if the meanμ is 21 seconds at level of significant 0.05 ($H_0: \mu = 21$ vs $H_A: \mu \neq 21$) then:

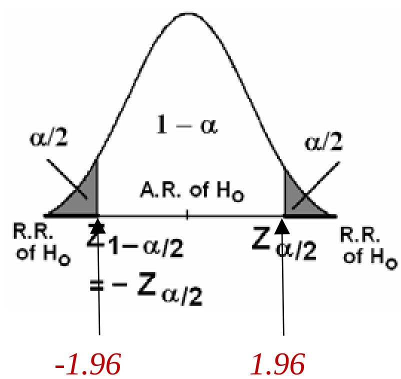
(1) The value of the test statistic is:

$$\sigma = 5.71 \quad n = 40 \quad \bar{X} = 20.6$$

$$Z = \frac{\bar{X} - \mu_o}{\sigma/\sqrt{n}} = \frac{20.6 - 21}{5.71/\sqrt{40}} = -0.443$$

(A) 0.443 *(B) - 0.012* *<u>(C) -0.443</u>* *(D) 0.012*

(2) The acceptance area is:



$$Z_{\frac{\alpha}{2}} = Z_{\frac{0.05}{2}} = Z_{0.025} = 1.96$$

<u>(A) (-1.96, 1.96)</u> *(B) (1.96, ∞)* *(C) (-∞, 1.96)* *(D) (-∞, 1.645)*

(3) The decision is:

(A) Reject H_0 *<u>(B)</u> Accept H_0* *(C) no decision* *(D) None of these*

Question 2:

If the hemoglobin level of pregnant women (امرأه حامل) is normally distributed, and if the mean and standard deviation of a sample of 25 pregnant women were $\bar{X} = 13$ (g/dl), $s = 2$ (g/dl). Using $\alpha = 0.05$, to test if the average hemoglobin level for the pregnant women is greater than 10 (g/dl) [$H_0 : \mu \le 10$, $H_A: \mu > 10$].

(1) The test statistic is:

(A) $Z = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$ (B) $Z = \frac{\bar{X}-10}{S/\sqrt{n}}$ (C) $t = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$ (D) $t = \frac{\bar{X}-10}{S/\sqrt{n}}$

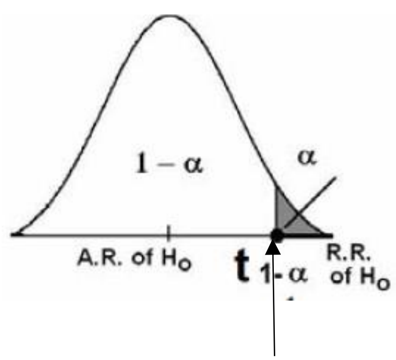
(2) The value of the test statistic is:

$$s = 2 \quad n = 25 \quad \bar{X} = 13$$

$$t = \frac{\bar{X} - \mu_o}{S/\sqrt{n}} = \frac{13 - 10}{2/\sqrt{25}} = 7.5$$

(A) 10 (B) 1.5 (C) 7.5 (D) 37.5

(3) The rejection of H_0 is :



1.711

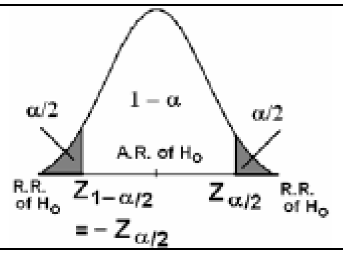
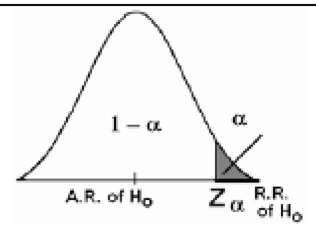
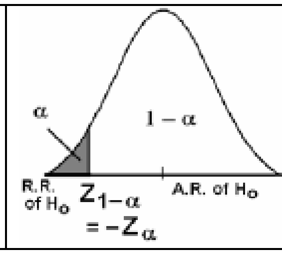
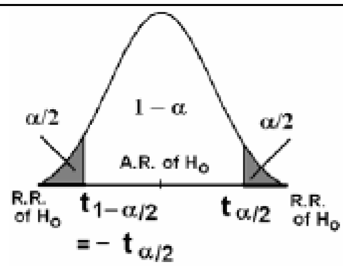
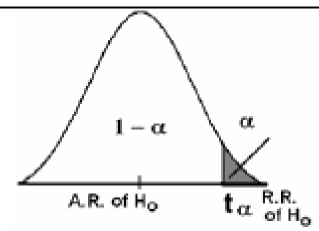
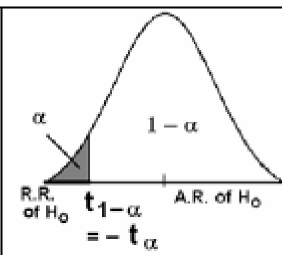
$$t_{1-\alpha,n-1} = t_{0.95,24} = 1.711$$

(A) Z< -1.645 (B) z> 1.645 (C) t < -1.711 (D) t > 1.711

(4) *The decision is:*

(A) Reject H_0 (B) Do not reject (Accept) H_0
(C) Accept both H_0 and H_A (D) Reject both H_0 and H_A

2-Two Means:

Hypotheses	$H_o: \mu_1 - \mu_2 = 0$ $H_A: \mu_1 - \mu_2 \neq 0$	$H_o: \mu_1 - \mu_2 \leq 0$ $H_A: \mu_1 - \mu_2 > 0$	$H_o: \mu_1 - \mu_2 \geq 0$ $H_A: \mu_1 - \mu_2 < 0$
Test Statistic For the First Case:	$Z = \dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$	{if σ_1^2 and σ_2^2 are known}	
R.R. and A.R. of H_o (For the First Case)			
Test Statistic For the Second Case:	$T = \dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{\dfrac{S_p^2}{n_1} + \dfrac{S_p^2}{n_2}}} \sim t(n_1+n_2-2)$	{if $\sigma_1^2 = \sigma_2^2 = \sigma^2$ is unknown}	
R.R. and A.R. of H_o (For the Second Case)			
Decision:	Reject H_o (and accept H_A) at the significance level α if:		
	T.S. ∈ R.R. Two-Sided Test	T.S. ∈ R.R. One-Sided Test	T.S. ∈ R.R. One-Sided Test

Question 1:

A standardized chemistry test was given to 50 girls and 75 boys. The girls made an average of 84, while the boys made an average grade of 82. Assume the population standard deviations are 6 and 8 for girls and boys respectively. To test the null hypothesis $H_0: \mu_1 - \mu_2 \le 0$ against the alternative hypothesis $H_A: \mu_1 - \mu_2 > 0$ at 0.05 level of significance:

(1) The standard error of $(\bar{X}_1 - \bar{X}_2)$ is:

$$\begin{array}{ll} girls: & n_1 = 50,\ \bar{X}_1 = 84,\ \sigma_1 = 6 \\ boys: & n_2 = 75,\ \bar{X}_2 = 82,\ \sigma_2 = 8 \end{array}$$

$$S.E(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{6^2}{50} + \frac{8^2}{75}} = 1.2543$$

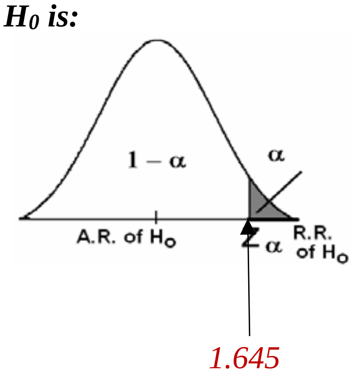
(A) 0.2266 (B) 2 (C) 1.5733 (D) 1.2543

(2) The value of the test statistic is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(84 - 82)}{\sqrt{\frac{6^2}{50} + \frac{8^2}{75}}} = \frac{2}{1.2543} = 1.5945$$

(A) -1.59 (B) 1.59 (C) 1.25 (D) 4.21

(3) The rejection region (RR) of H_0 is:



$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$

(A) $(1.645, \infty)$ (B) $(-\infty, -1.645)$
(C) $(1.96, \infty)$ (D) $(-\infty, -1.96)$

(4) The decision is:

(A) Reject H_0 (B) Do not reject (Accept) H_0
(C) Accept both H_0 and H_A (D) Reject both H_0 and H_A

Question 2:

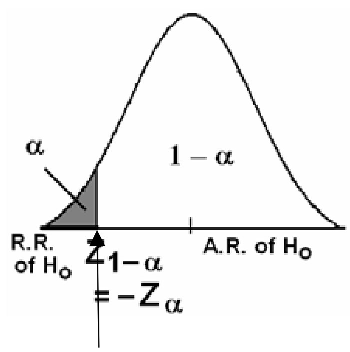
Cortisol level determinations were made on two samples of women at childbirth. Group 1 subjects underwent emergency cesarean section following induced labor. Group 2 subjects natural childbirth route following spontaneous labor. The sample sizes, mean cortisol levels, and standard deviations were $(n_1 = 40, \bar{x}_1 = 575, \sigma_1 = 70), (n_2 = 44, \bar{x}_2 = 610, \sigma_2 = 80)$
If we are interested to test if the mean Cortisol level of group 1 (μ_1) *is less than that of group 2* (μ_2) *at level 0.05 (or* $H_0: \mu_1 \geq \mu_2$ *vs* $H_1: \mu_1 < \mu_2$*), then:*

(1) *The value of the test statistic is:*

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(575 - 610)}{\sqrt{\frac{70^2}{40} + \frac{80^2}{44}}} = -2.138$$

(A) -1.326 *(B) -2.138* *(C) -2.576* *(D) -1.432*

(2) *Reject* H_0 ***if :***



-1.645

$$Z_{1-\alpha} = Z_{0.95} = 1.645$$

(A) Z >1.645 *(B) T >1.98* *(C) Z< -1.645* *(D) T < -1.98*

(3) *The decision is:*

(A) Reject H_0 *(B) Accept* H_0 *(C) no decision* *(D) none of these*

Question 3:

An experiment was conducted to compare time length (duration time in minutes) of two types of surgeries (A) and (B). 10 surgeries of type (A) and 8 surgeries of type (B) were performed. The data for both samples is shown below.

Surgery type	A	B
Sample size	10	8
Sample mean	14.2	12.8
Sample standard deviation	1.6	2.5

Assume that the two random samples were independently selected from two normal populations with equal variances. If μ_A and μ_B are the population means of the time length of surgeries of type (A) and type (B), then, to test if μ_A is greater than μ_B at level of significant 0.05 ($H_0: \mu_A \leq \mu_B$ vs $H_A: \mu_A > \mu_B$) then:

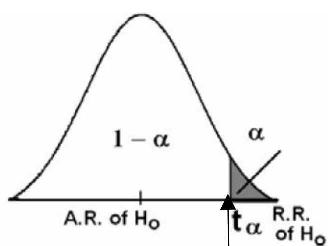
(4) The value of the test statistic is:

$$Sp^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2} = \frac{1.6^2(10 - 1) + 2.5^2(8 - 1)}{10 + 8 - 2} = 4.174$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(14.2 - 12.8)}{\sqrt{4.174}\sqrt{\frac{1}{10} + \frac{1}{8}}} = 1.44$$

(A) -1.44 *(B) 1.44* *(C) - 0.685* *(D) 0.685*

(5) Reject H_0 if :



1.746

$$t_{\alpha, n_1 + n_2 - 2} = t_{0.05, 10+8-2} = t_{0.05, 16} = 1.746$$

(A) Z >1.645 *(B) Z<-1.645* *(C) T >1.746* *(D) T< -1.746*

(6) The decision is:

(A) Reject H_0 *(B) Accept H_0* *(C) no decision* *(D) none of these*

Question 4:

A researcher was interested in comparing the mean score of female students μ_1, with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:

	Female	male
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\bar{x}_1 = 82.63$	$\bar{x}_2 = 80.04$
Variance	$s_1^2 = 15.05$	$s_2^2 = 20.79$

Test that is there is a difference between the mean score of female students and the mean score of male students.

(1) The hypotheses are:

(A) $H_o: \mu_1 = \mu_2$, $H_A: \mu_1 \neq \mu_2$ (B) $H_o: \mu_1 = \mu_2$, $H_A: \mu_1 < \mu_2$ (C) $H_o: \mu_1 < \mu_2$, $H_A: \mu_1 > \mu_2$ (D) $H_o: \mu_1 \leq \mu_2$, $H_A: \mu_1 > \mu_2$

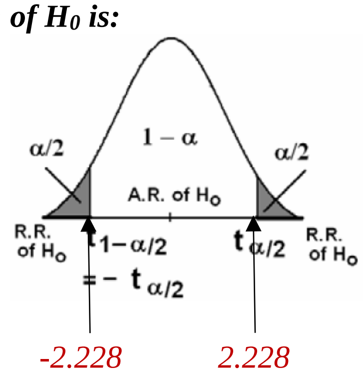
(2) The value of the test statistic is:

$$Sp^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{15.05(4)+20.79(6)}{5+7-2} = 18.494$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{82.63-80.04}{\sqrt{18.494}\sqrt{\frac{1}{5}+\frac{1}{7}}} = 1.029$$

(A) 1.3 (B) 1.029 (C) 0.46 (D) 0.93

(4) The acceptance region (AR) of H_0 is:



$$t_{\frac{\alpha}{2}, n_1+n_2-2} = t_{\frac{0.05}{2}, 5+7-2} = t_{0.025,10} = 2.228$$

(A) $(2.2281, \infty)$ (B) $(-\infty, -2.2281)$
(C) (-2.228, 2.228) (D) (-1.96, 1.96)

Question 5:

A nurse researcher wished to know if graduates of baccalaureate nursing program and graduate of associate degree nursing program differ with respect to mean scores on personality inventory at α = 0.02. A sample of 50 associate degree graduates (sample A) and a sample of 60 baccalaureate graduates (sample B) yielded the following means and standard deviations:

$$\bar{X}_A = 88.12,\ \ S_A = 10.5,\ \ n_A = 50$$
$$\bar{X}_B = 83.25,\ \ S_B = 11.2,\ \ n_B = 60$$

1) *The hypothesis is:*

A) $H_0\colon \mu_1 \le \mu_2\ \ vs\ \ H_1\colon \mu_1 > \mu_2$ *B)* $H_0\colon \mu_1 \ge \mu_2\ \ vs\ \ H_1\colon \mu_1 < \mu_2$

C) $H_0\colon \mu_1 = \mu_2\ \ vs\ \ H_1\colon \mu_1 \ne \mu_2$ *D) None of the above.*

2) *The test statistic is:*

A) Z *B) t* *C) F* *D) None of the above.*

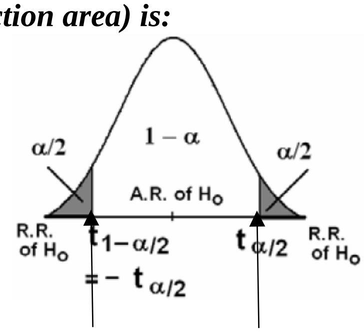
3) *The computed value of the test statistic is:*

$$Sp^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{10.5^2(50-1)+11.2^2(60-1)}{50+60-2} = 118.55$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{88.12-83.25}{\sqrt{118.55}\sqrt{\frac{1}{50}+\frac{1}{60}}} = 48.19$$

A) 2.72 *B) 1.50* *C) 1.86* *D) 2.35*

4) *The critical region (rejection area) is:*



-2.33 2.33

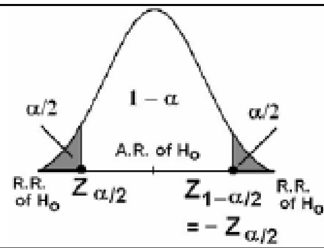
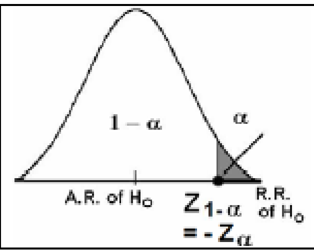
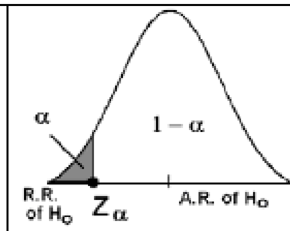
$$t_{\frac{\alpha}{2},n_1+n_2-2} = t_{\frac{0.02}{2},\ 50+60-2} = t_{0.01,108} = 2.33$$

A) 2.60 Or -2.60 *B) 2.06 Or -2.06* *C) 2.33 Or - 2.33* *D) 2.58*

5) *Your decision is:*

A) accept & reject H_0 *B) accept* H_0 *C) reject* H_0 *D) no decision.*

Single proportion:

Hypotheses	H_o: $p = p_o$ H_A: $p \neq p_o$	H_o: $p \leq p_o$ H_A: $p > p_o$	H_o: $p \geq p_o$ H_A: $p < p_o$
Test Statistic (T.S.)	$Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$ ~N(0,1)		
R.R. & A.R. of H_o			
Decision:	Reject H_o (and accept H_A) at the significance level α if:		
	$Z < Z_{\alpha/2}$ or $Z > Z_{1-\alpha/2} = -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{1-\alpha} = -Z_{\alpha}$ One-Sided Test	$Z < Z_{\alpha}$ One-Sided Test

Question 1:

Toothpaste (معجون الأسنان) company claims thatmorethan75% of the dentists recommend their product to the patients. Suppose that 161 out of 200 dental patients reported receiving a recommendation for this toothpaste from their dentist. Do you suspect that the proportion is actually morethan75%. If we use 0.05 level of significance to test $H_0: P \leq 0.75$, $H_A: P > 0.75$, then:

(1) The sample proportion $\hat{p}$ is:

$$n = 200,\ \hat{p} = \frac{161}{200} = 0.8050$$

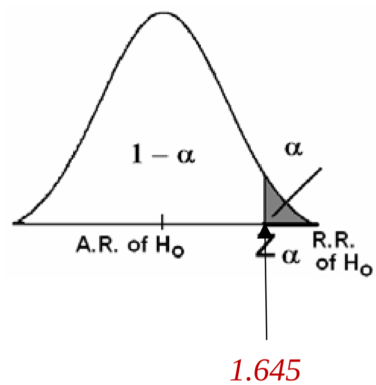
(A) 0.75 *(B) 0.195* *(C) 0.805* *(D) 0.25*

(2) The value of the test statistic is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.805 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{200}}} = 1.7963$$

(A) 1.963 *(B) 1.796* *(C) -1.796* *(D) -1.963*

(3)The decision is:



$$\alpha = 0.05 \rightarrow Z_{1-\alpha} = Z_{0.95} = 1.645$$

(A) Reject H_0 *(B) Do not reject (Accept) H_o*
(C) Accept both H_o and H_A *(D) Reject both H_o and H_A*

Question 2:

A researcher was interested in studying the obesity (السمنة) disease in a certain population. A random sample of 400 people was taken from this population. It was found that 152 people in this sample have the obesity disease. If p is the population proportion of people who are obese. Then, to test if p is greater than 0.34 at level 0.05 ($H_0: p \leq 0.34$ vs $H_A: p > 0.34$) then:

(1) The value of the test statistic is:

$$n = 400, \quad \hat{p} = \frac{152}{400} = 0.38$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.38 - 0.34}{\sqrt{\frac{0.34 \times 0.66}{400}}} = 1.69$$

(A) 0.023 *(B) 1.96* *(C) 2.50* *(<u>D</u>) 1.69*

(2) The P-value is

$$P - value = P(Z > 1.96) = 1 - P(Z < 1.96) = 1 - 0.9545 = 0.0455$$

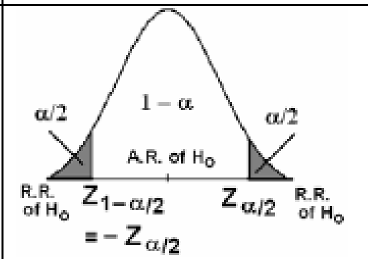
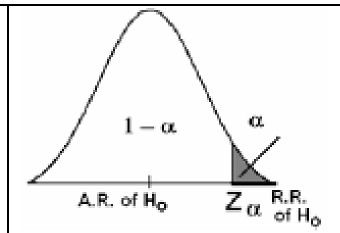
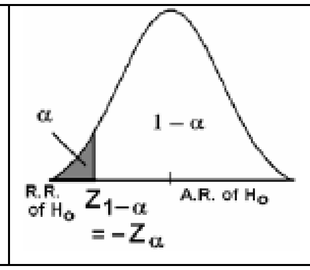
(A) 0.9545 *(B) 0.0910* *(<u>C</u>) 0.0455* *(D)1.909*

(3) The decision is:

$$P - value = 0.0455 < 0.05$$

(<u>A</u>) Reject H_0 *(B) Accept H_0* *(C) no decision* *(D) none of these*

Two proportions:

Hypotheses	$H_o: p_1 - p_2 = 0$ $H_A: p_1 - p_2 \neq 0$	$H_o: p_1 - p_2 \leq 0$ $H_A: p_1 - p_2 > 0$	$H_o: p_1 - p_2 \geq 0$ $H_A: p_1 - p_2 < 0$
Test Statistic (T.S.)	$Z = \dfrac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\dfrac{\overline{p}(1-\overline{p})}{n_1} + \dfrac{\overline{p}(1-\overline{p})}{n_2}}} \sim \mathrm{N}(0,1)$		
R.R. and A.R. of H_o			
Decision:	Reject H_o (and accept H_1) at the significance level α if $Z \in$ R.R.:		
Critical Values	$Z > Z_{\alpha/2}$ or $Z < -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{\alpha}$ One-Sided Test	$Z < -Z_{\alpha}$ One-Sided Test

Question 1:

In a first sample of 200 men, 130 said they used seat belts and a second sample of 300 women, 150 said they used seat belts. To test the claim that men are more safety-conscious than women ($H_0: p_1 - p_2 \leq 0, H_1: p_1 - p_2 > 0$)*, at 0.05 level of significant:*

(1) The value of the test statistic is:

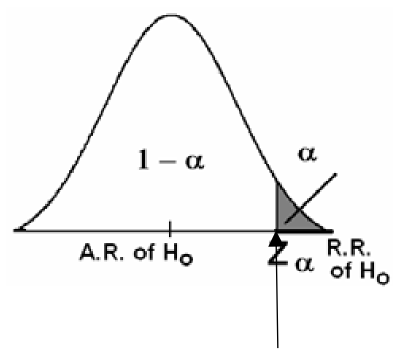
$$n_1 = 200, \;\; \hat{p}_1 = \frac{130}{200} = 0.65 \qquad n_2 = 300, \;\; \hat{p}_2 = \frac{150}{300} = 0.5$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{130 + 150}{200 + 300} = 0.56$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.65 - 0.5)}{\sqrt{(0.56)(0.44)\left(\frac{1}{200} + \frac{1}{300}\right)}} = 3.31$$

(A) -3.31 *(B)5.96* *(C)1.15* *(D) 3.31*

(2) The decision is:



1.645

$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$

(A) Reject H_0 *(B) Do not reject (Accept)* H_0
(C) Accept both H_0 *and* H_A *(D) Reject both* H_0 *and* H_A

(3) We can conclude that from confidence interval that

(A) The diabetes proportions may be equal for both proportion.
(B) The diabetes proportions may not be equal for both proportion.

Question 2:

In a study of diabetes, the following results were obtained from samples of males and females between the ages of 20 and 75. Male sample size is 300 of whom 129 are diabetes patients, and female sample size is 200 of whom 50 are diabetes patients. If P_M, P_F are the diabetes proportions in both populations and $\hat{p}_M, \hat{p}_F$ are the sample proportions, then:
A researcher claims that the Proportion of diabetes patients is found to be more in males than in female $(H_0: P_M - P_F \leq 0$ vs $H_A: P_M - P_F > 0)$. Do you agree with his claim, take α = 0.10

(1) The pooled proportion is:

$$\hat{p} = \frac{x_m + x_f}{n_m + n_f} = \frac{129 + 50}{300 + 200} = 0.358$$

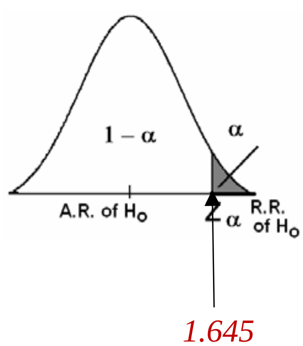
(A) 0.43 *(B) 0.18* *(C) 0.358* *(D) 0.68*

(2) The value of the test statistic is:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.43 - 0.25)}{\sqrt{(0.358)(1 - 0.358)\left(\frac{1}{300} + \frac{1}{200}\right)}} = 0.411$$

(A) -4.74 *(B) 4.74* *(C) 4.11* *(D) - 4.11*

(3) The decision is:

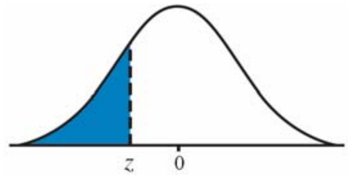


$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$

(A) Agree with the claim *(B) do not agree with the claim* *(C) Can't say*

Standard Normal Table

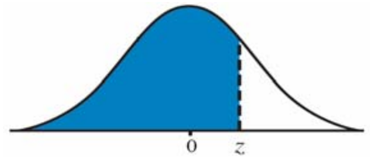
Areas Under the Standard Normal Curve



z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	**-3.50**
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	**-3.40**
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	**-3.30**
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	**-3.20**
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	**-3.10**
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	**-3.00**
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	**-2.90**
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	**-2.80**
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	**-2.70**
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	**-2.60**
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	**-2.50**
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	**-2.40**
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	**-2.30**
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	**-2.20**
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	**-2.10**
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	**-2.00**
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	**-1.90**
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	**-1.80**
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	**-1.70**
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	**-1.60**
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	**-1.50**
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	**-1.40**
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	**-1.30**
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	**-1.20**
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	**-1.10**
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	**-1.00**
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	**-0.90**
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	**-0.80**
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	**-0.70**
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	**-0.60**
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	**-0.50**
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	**-0.40**
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	**-0.30**
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	**-0.20**
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	**-0.10**
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	**-0.00**

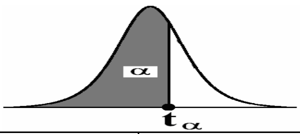
Standard Normal Table (continued)

Areas Under the Standard Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	**0.00**
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	**0.10**
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	**0.20**
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	**0.30**
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	**0.40**
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	**0.50**
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	**0.60**
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	**0.70**
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	**0.80**
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	**0.90**
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	**1.00**
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	**1.10**
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	**1.20**
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	**1.30**
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	**1.40**
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	**1.50**
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	**1.60**
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	**1.70**
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	**1.80**
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	**1.90**
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	**2.00**
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	**2.10**
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	**2.20**
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	**2.30**
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	**2.40**
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	**2.50**
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	**2.60**
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	**2.70**
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	**2.80**
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	**2.90**
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.9990	**3.00**
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	**3.10**
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	**3.20**
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	**3.30**
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	**3.40**
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	**3.50**

Critical Values of the t-distribution (t_α)



ν=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.2901	1.6602	1.9840	2.3642	2.6259
120	1.2886	1.6577	1.9799	2.3578	2.6174
140	1.2876	1.6558	1.9771	2.3533	2.6114
160	1.2869	1.6544	1.9749	2.3499	2.6069
180	1.2863	1.6534	1.9732	2.3472	2.6034
200	1.2858	1.6525	1.9719	2.3451	2.6006
∞	1.282	1.645	1.960	2.326	2.576