

المملكة العربية السعودية

وزارة التعليم

MINISTRY OF EDUCATION



لكل المهتمين و المهتمات
بدروس و مراجع الجامعية

هام

مدونة المناهج السعودية eduschool40.blog

1.3 - New Functions From Old Functions

Suppose $f(x)$ is odd function with $D_{f(x)} = [a, b]$; $R_{f(x)} = [e, d]$
and $c > 0$

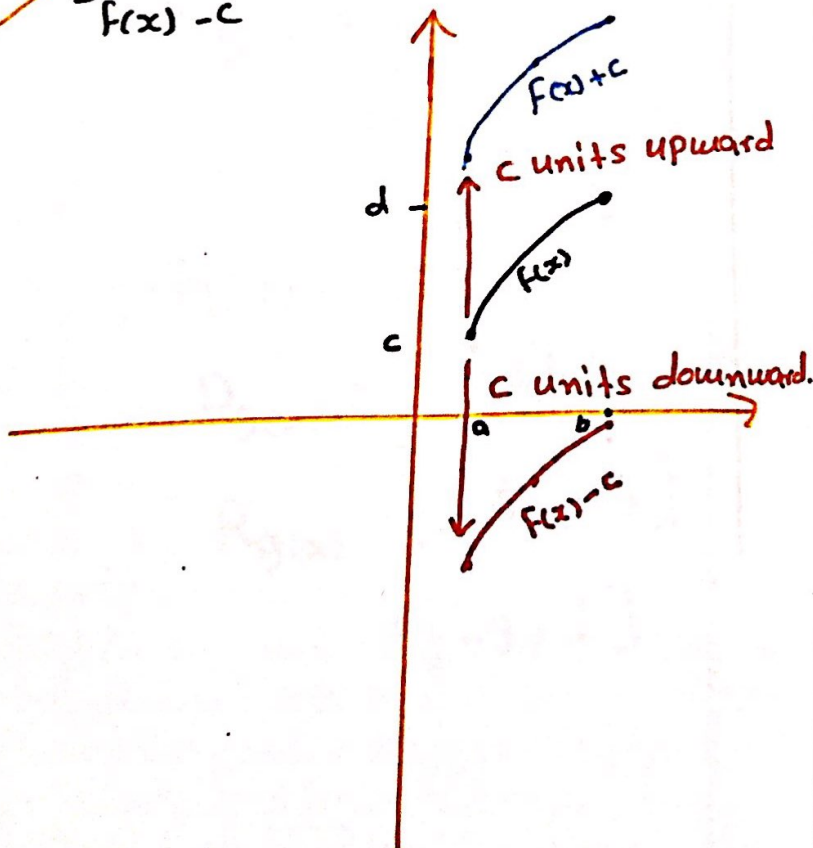
Vertical Shift

✓ $y = f(x) + c$, Shift the graph of $y = f(x)$ a distance c units upward

✓ $D_{f(x)+c} = [a, b]$; $R_{f(x)+c} = [e+c, d+c]$

✓ $y = f(x) - c$; Shift the graph of $y = f(x)$ a distance c units downward.

✓ $D_{f(x)-c} = [a, b]$; $R_{f(x)-c} = [e-c, d-c]$



Horizontal Shift

✓ $y = f(x+c)$, Shift the graph of $y = f(x)$ a distance c units to left

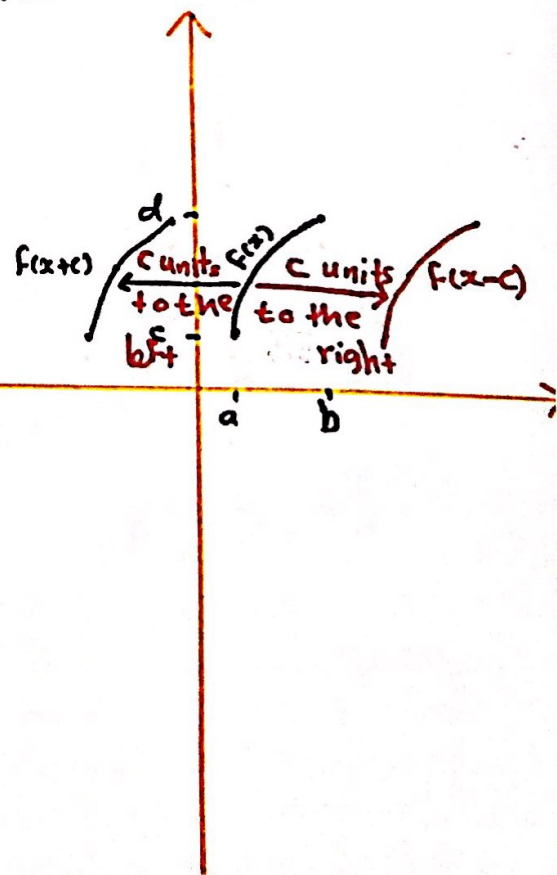
✓ $D_{f(x+c)} = [a-c, b-c]$

$R_{f(x+c)} = [e, d]$

✓ $y = f(x-c)$, Shift the graph of $y = f(x)$ a distance c units to right

✓ $D_{f(x-c)} = [a+c, b+c]$

$R_{f(x-c)} = [e, d]$

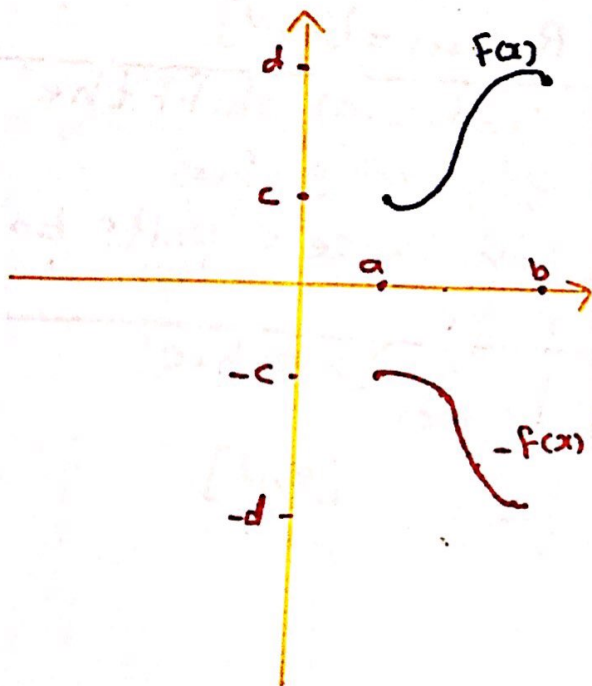


Vertical Reflecting

✓ $y = -f(x)$, reflect the graph of $y = f(x)$ about the x -axis

✓ $D_{-f(x)} = [a, b]$

✓ $R_{-f(x)} = [-d, -c]$

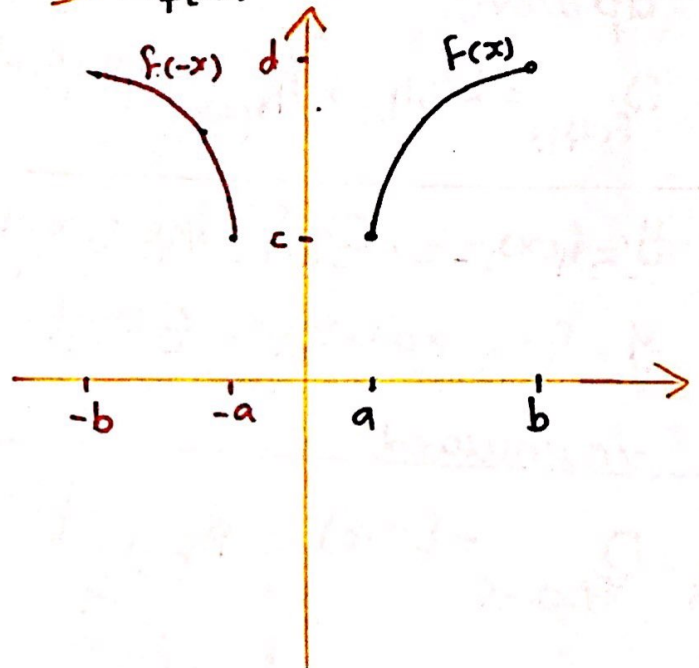


Horizontal Reflecting

✓ $y = f(-x)$, reflect the graph of $y = f(x)$ about the y -axis

✓ $D_{f(-x)} = [-b, -a]$

✓ $R_{f(-x)} = [e, d]$



Example (1)

If $f(x)$ is a function with Domain: $[-6, 5]$ and Range: $[-1, 3]$
then find the Domain and Range of the following.

a) $g(x) = f(x) + 2$

$$D_{g(x)} = [-6, 5]$$

$$R_{g(x)} = [-1+2, 3+2] \\ = [1, 5]$$

b) $g(x) = f(x) - 3$

$$D_{g(x)} = [-6, 5]$$

$$R_{g(x)} = [-1-3, 3-3] \\ = [-4, 0]$$

c) $g(x) = -f(x)$

$$D_{g(x)} = [-6, 5]$$

$$R_{g(x)} = [-(1), -3] \\ = [-3, 1]$$

d) $g(x) = f(x+2)$

$$R_{g(x)} = [-1, 3]$$

$$D_{g(x)} = [-6-2, 5-2] \\ = [-8, 3]$$

e) $g(x) = f(x-3)$

$$R_{g(x)} = [-1, 3]$$

$$D_{g(x)} = [-6+3, 5+3] \\ = [-3, 8]$$

f) $g(x) = f(-x)$

$$R_{g(x)} = [-1, 3]$$

$$D_{g(x)} = [-(6), -5] \\ = [-5, 6]$$

$$h) g(x) = 3 - f(x)$$

$$D_{g(x)} = [-6, 5]$$

$$R_{g(x)} : \text{i) } R_{-f(x)} = [-(-1), -3] \\ = [-3, 1]$$

$$\text{ii) } R_{2-f(x)} = [-3+3, 1+3] \\ = [0, 4]$$

$$\Rightarrow R_{g(x)} = [0, 4] \#$$

$$I) g(x) = f(x-1) - 6$$

$$D_{g(x)} = [-6+1, 5+1] = [-5, 6]$$

$$R_{g(x)} = [-1-6, 3-6] = [-7, -3]$$

H.W

if $D_{f(x)} = [0, 6]$ and $R_{f(x)} = [4, 2]$ then

find ① $D_{1-f(x)}$ and $R_{1-f(x)}$

② $D_{f(x+2)+1}$ and $R_{f(x+2)+1}$

③ $D_{f(-x)-2}$ and $R_{f(-x)-2}$

④ $D_{f(x)+4}$ and $R_{f(x)+4}$

⑤ $R_{f(x+4)}$ and $D_{f(x+4)}$

$$\textcircled{1} D_{1-f(x)} = [0, 6)$$

$$R_{1-f(x)} : \text{i) } R_{-f(x)} = [-(-4), -2) \\ = (-2, 4]$$

$$\text{ii) } R_{1-f(x)} = (-2+1, 4+1) \\ = (-1, 5]$$

$$\textcircled{2} D_{f(x+2)+1} = [0-2, 6-2) \\ = [-2, 4)$$

$$R_{f(x+2)+1} = [-4+1, 2+1) \\ = [-3, 3)$$

$$\textcircled{3} D_{f(-x)-2} = [-0, -6) \\ = (-6, 0]$$

$$R_{f(-x)-2} = [-4-2, 2-2) \\ = [-6, 0)$$

④

$$D_{f(x)+4} = [0, 6)$$

$$R_{f(x)+4} = [-4+4, 2+4)$$
$$= [0, 6)$$

⑤

$$D_{f(x+4)} = [0-4, 6-4)$$
$$= [-4, 2)$$

$$R_{f(x+4)} = [-4, 2)$$

Example (1)

Find the Domain and Range of the following:

$$1 - y = \sqrt{x} + 2$$

The old function: $f(x) = \sqrt{x}$

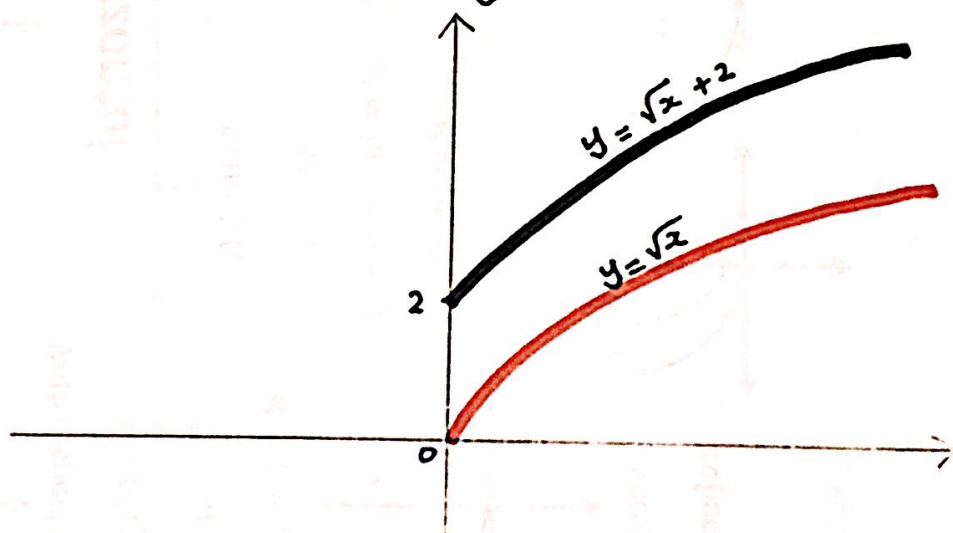
$$D_{f(x)} = [0, \infty)$$

$$R_{f(x)} = [0, \infty)$$

The New function: $y = \sqrt{x} + 2$

$$D_{y(x)} = [0, \infty)$$

$$R_{y(x)} = [0+2, \infty+2) = [2, \infty)$$



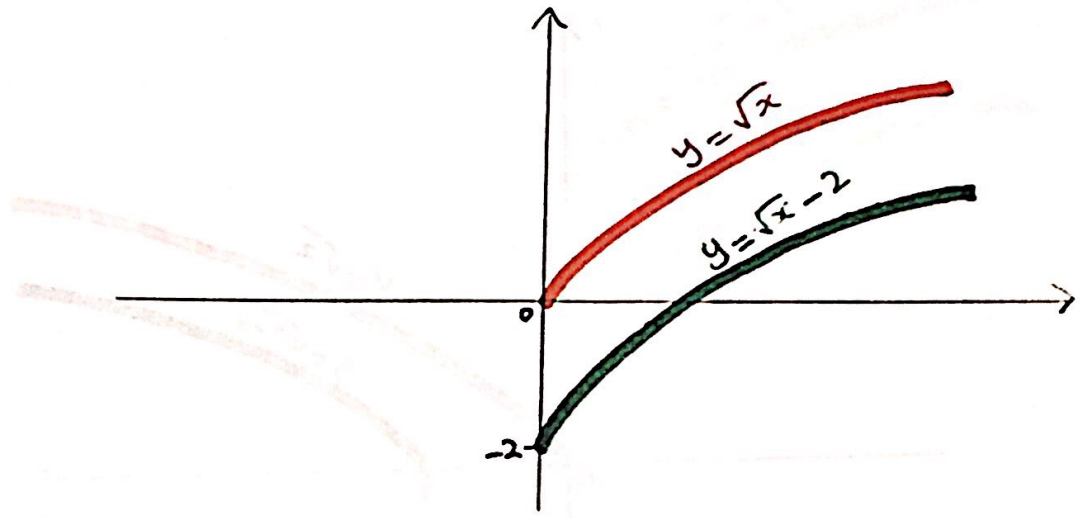
Type of the Shift: vertical shift

$y = \sqrt{x} + 2$, Shift the graph of $y = \sqrt{x}$ a distance 2 units upward.

2. $y = \sqrt{x} - 2$

The old function: $f(x) = \sqrt{x}$
 $D_{f(x)} = [0, \infty)$
 $R_{f(x)} = [0, \infty)$

The new function: $y = \sqrt{x} - 2$
 $D_{y(x)} = [0, \infty)$
 $R_{y(x)} = [0 - 2, \infty - 2)$
 $= [-2, \infty)$



Type of the Shift : Vertical shift.

$y = \sqrt{x} - 2$, shift the graph ^{of} $y = \sqrt{x}$ a distance 2 units downward.

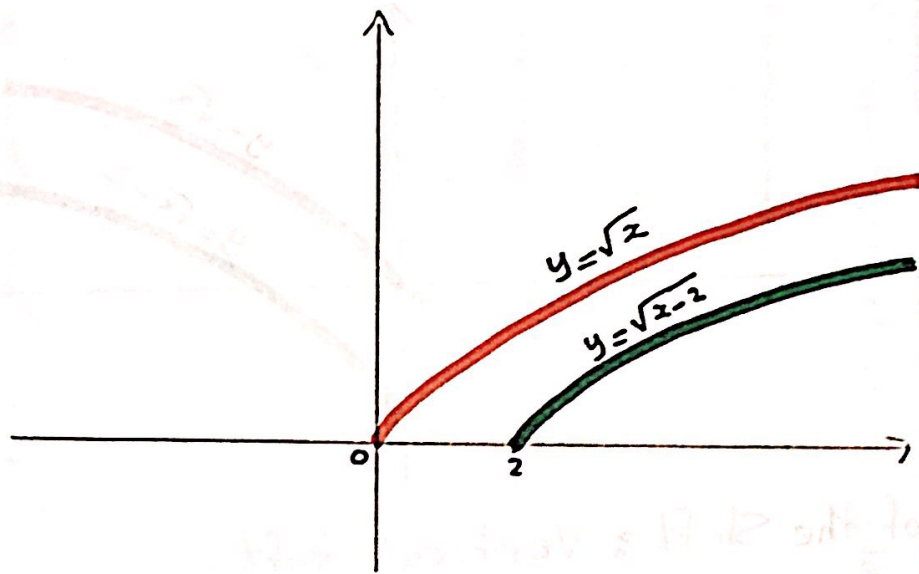
3) $y = \sqrt{x-2}$

The new function: $f(x) = \sqrt{x}$

- $D_{f(x)} = [0, \infty)$
- $R_{f(x)} = [0, \infty)$

The new function: $y = \sqrt{x-2}$

- $D_{y(x)} = [0+2, \infty+2) = [2, \infty)$
- $R_{y(x)} = [0, \infty)$



Type of the Shift: **Horizontal Shift**

$y = \sqrt{x-2}$, Shift the graph of $y = \sqrt{x}$ a distance 2 units to the right

$$4) y = \sqrt{x+2}$$

The old function : $f(x) = \sqrt{x}$

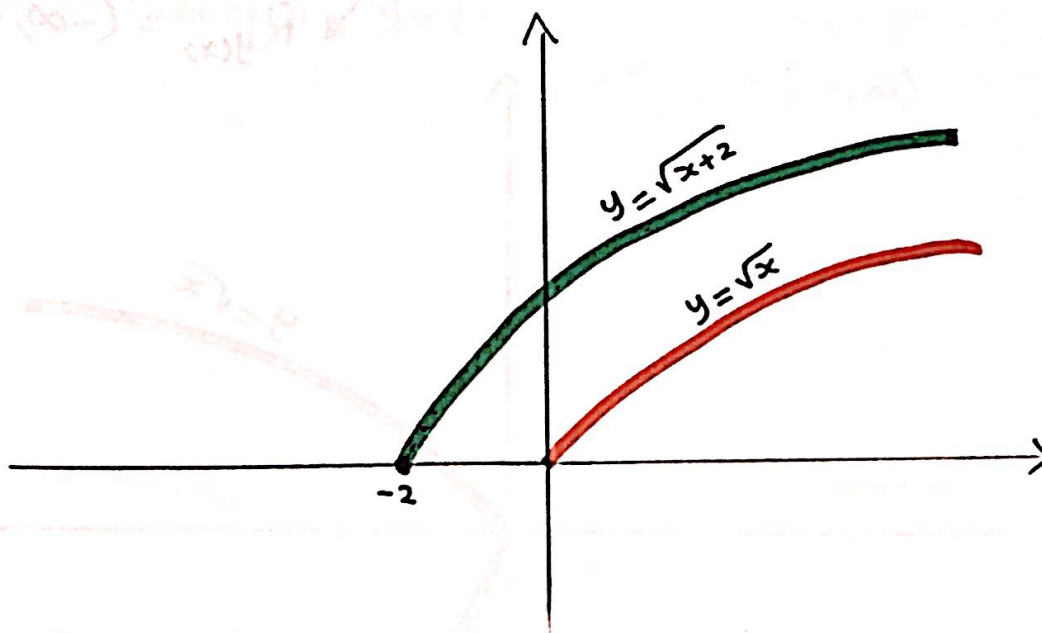
$$D_{f(x)} = [0, \infty)$$

$$R_{f(x)} = [0, \infty)$$

The new function : $y = \sqrt{x+2}$

$$D_{y(x)} = [0-2, \infty-2) = [-2, \infty)$$

$$R_{y(x)} = [0, \infty)$$



Type of the Shift : Horizontal Shift

$y = \sqrt{x+2}$, shift the graph of $y = \sqrt{x}$ a distance 2 units to the left.

5) $y = -\sqrt{x}$

The old function: $f(x) = \sqrt{x}$

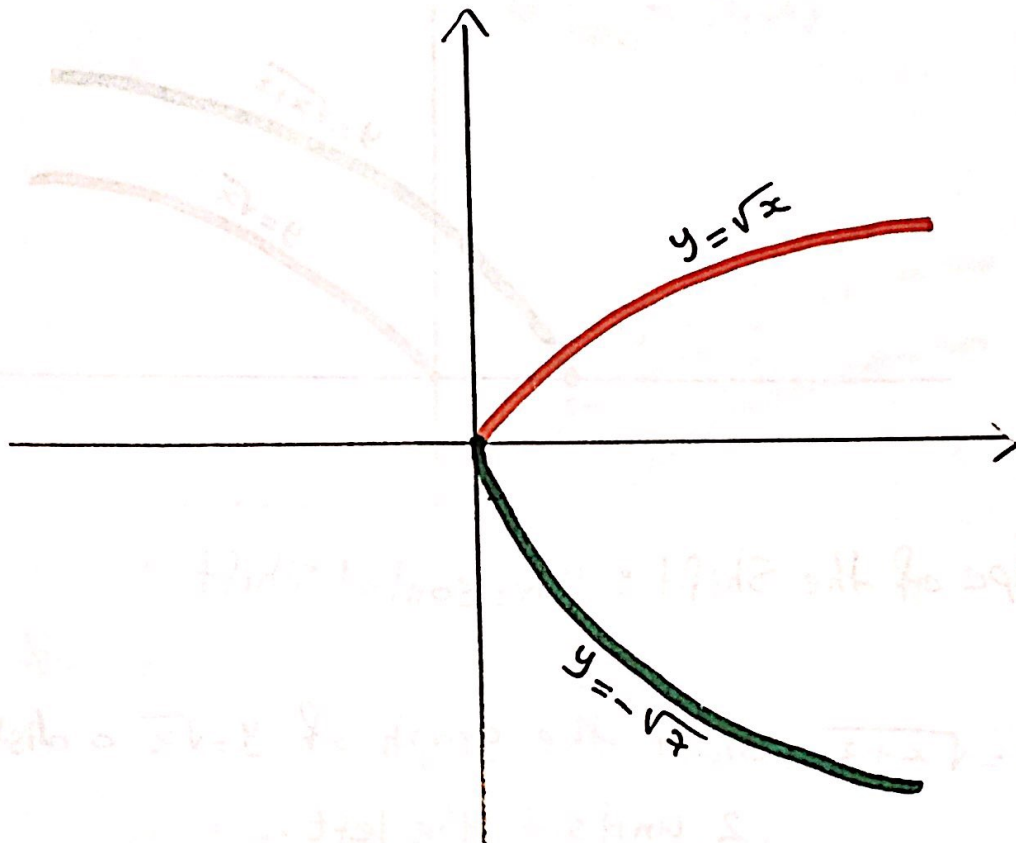
$D_{f(x)} = [0, \infty)$

$R_{f(x)} = [0, \infty)$

The new function: $y = -\sqrt{x}$

$D_{y(x)} = [0, \infty)$

$R_{y(x)} = (-\infty, 0]$



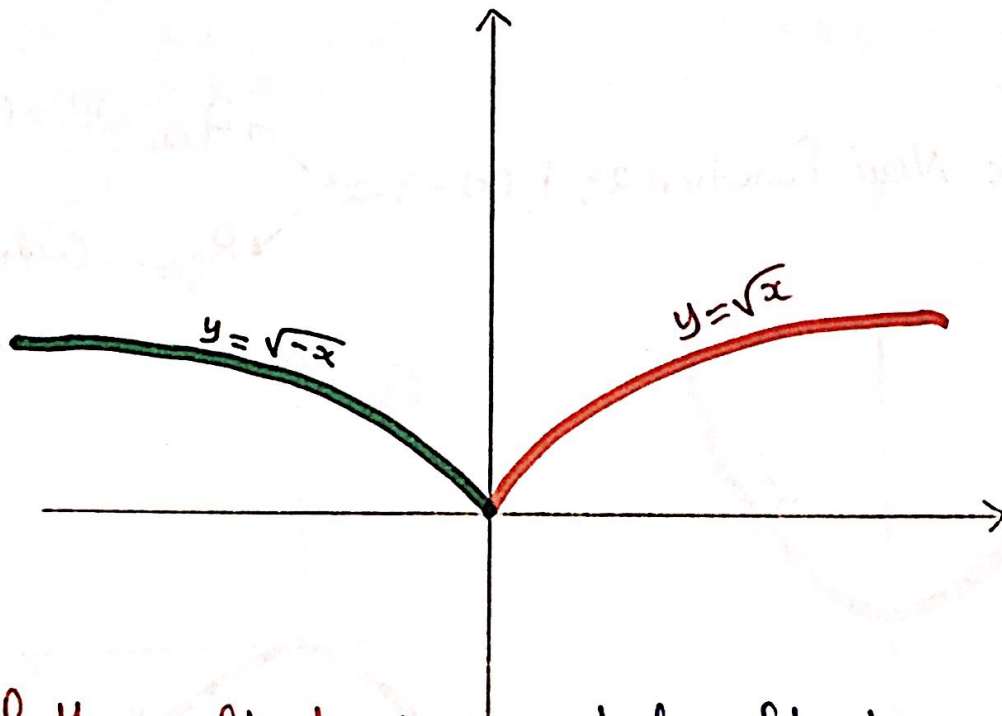
Type of the reflect : Vertical reflecting

$y = -\sqrt{x}$, reflect the graph of $y = \sqrt{x}$ about
x-axis " $y = 0$ "

$$6) y = \sqrt{-x}$$

The old function: $f(x) = \sqrt{x}$ $\rightarrow D_{f(x)} = [0, \infty)$
 $\rightarrow R_{f(x)} = [0, \infty)$

The new function: $y = \sqrt{-x}$ $\rightarrow D_{y(x)} = (-\infty, 0]$
 $\rightarrow R_{y(x)} = [0, \infty)$

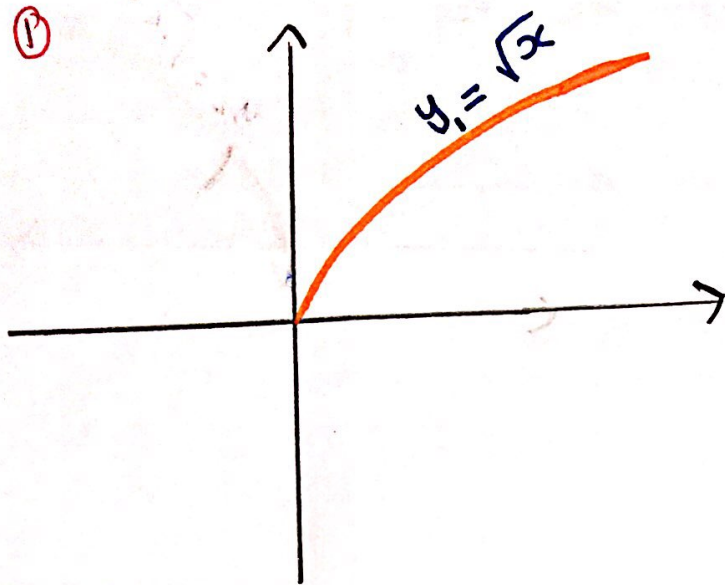


Type of the reflect: Horizontal reflecting.

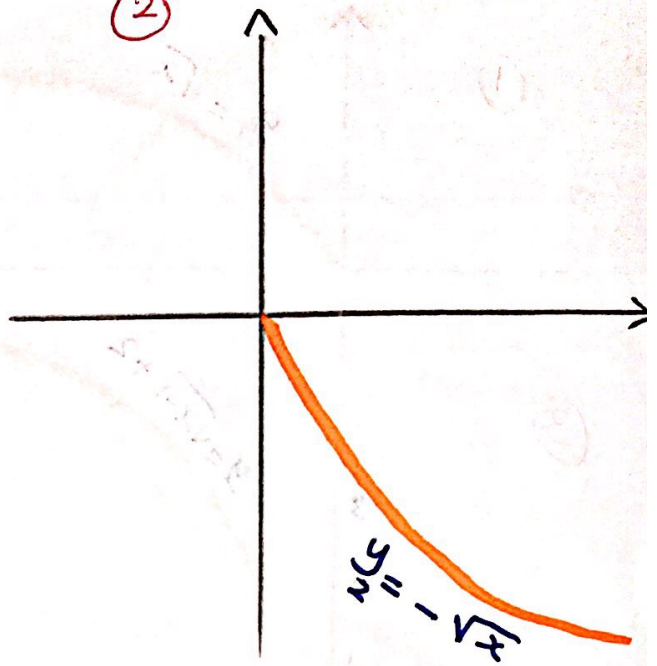
$y = \sqrt{-x}$, reflect the graph of $y = \sqrt{x}$ about
y-axis "x=0"

7) $y = 2 - \sqrt{x}$

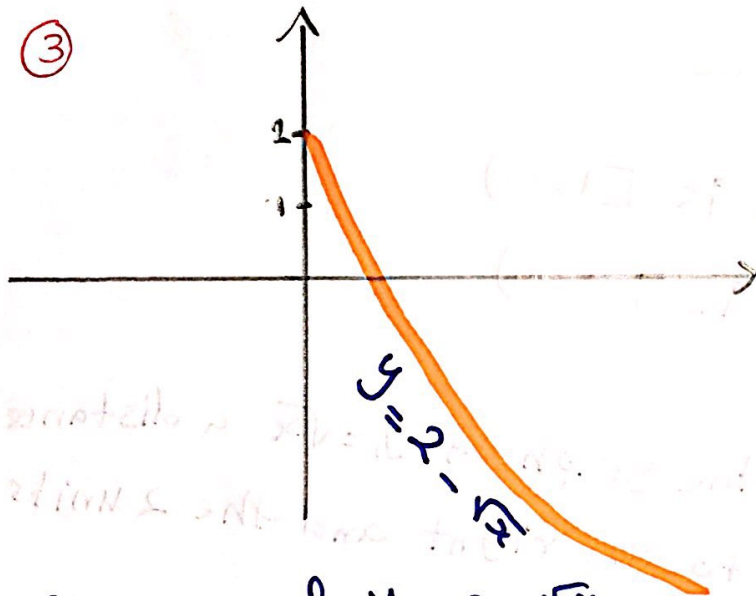
①



②



③



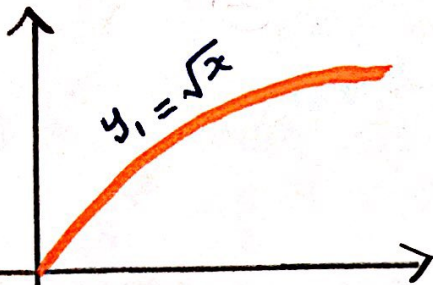
Domain of $y = 2 - \sqrt{x}$
is $[0, \infty)$

Range of $y = 2 - \sqrt{x}$ is $(-\infty, 2]$

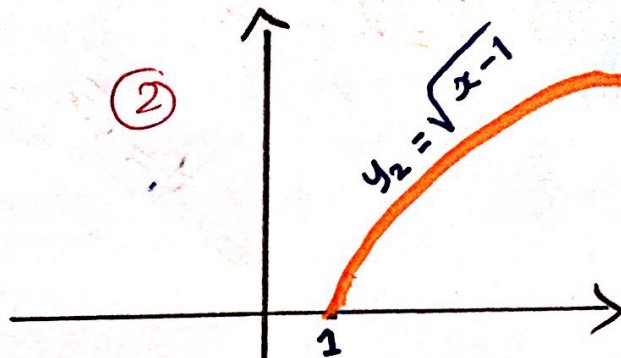
$y = 2 - \sqrt{x}$, reflect the graph of $f_1(x) = \sqrt{x}$ about x-axis and shift the graph of $f_2(x) = -\sqrt{x}$ a distance 2 units upward.

8) $y = \sqrt{x-1} + 2$

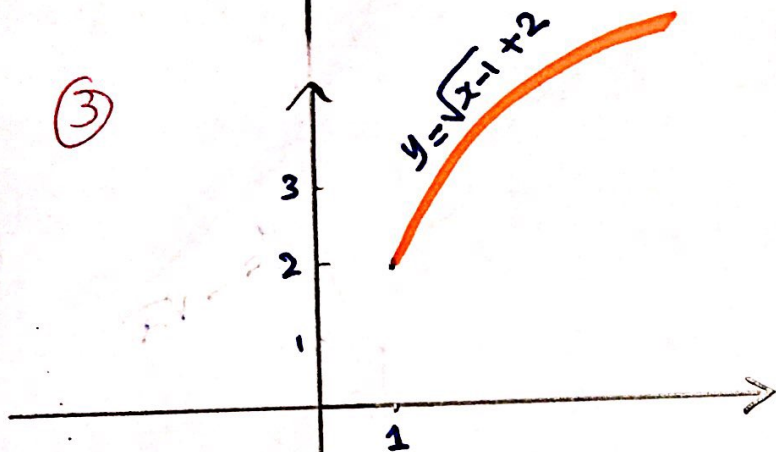
①



②



③



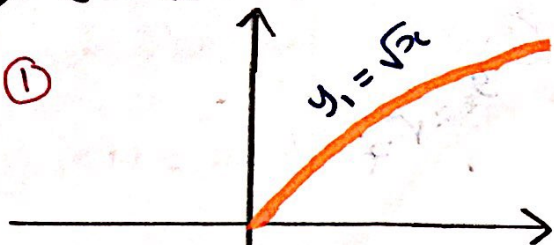
Domain of $y = \sqrt{x-1} + 2$ is $[1, \infty)$

Range of $y = \sqrt{x-1} + 2$ is $[2, \infty)$

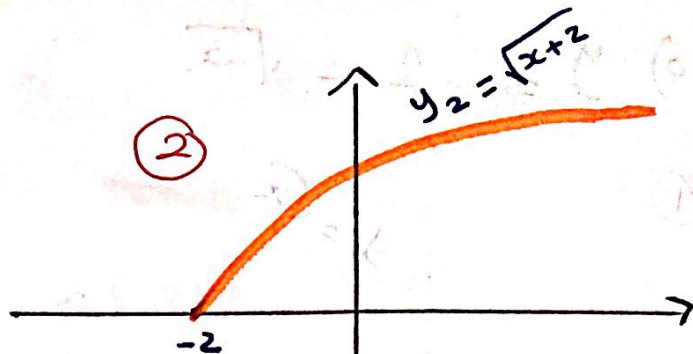
$y = \sqrt{x-1} + 2$, Shift the graph of $y_1 = \sqrt{x}$ a distance 1 unit to the right and the 2 units upward.

⑨ $y = 3 - \sqrt{x+2}$

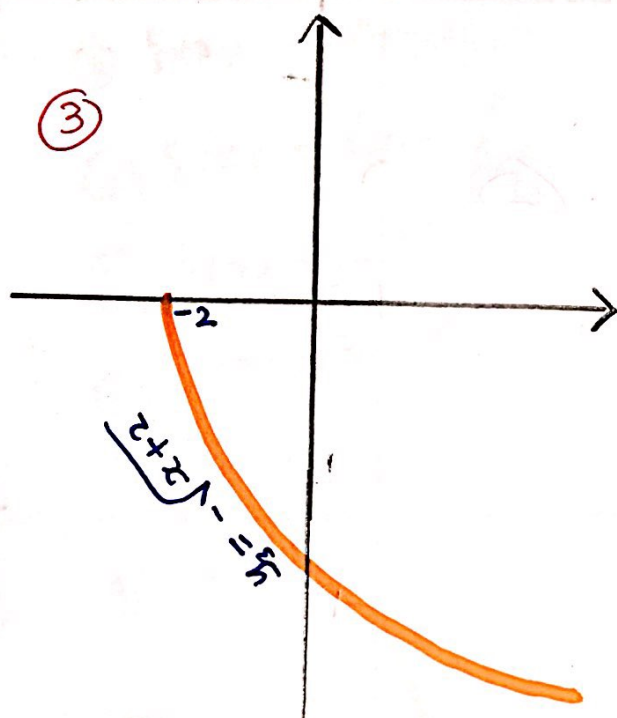
①



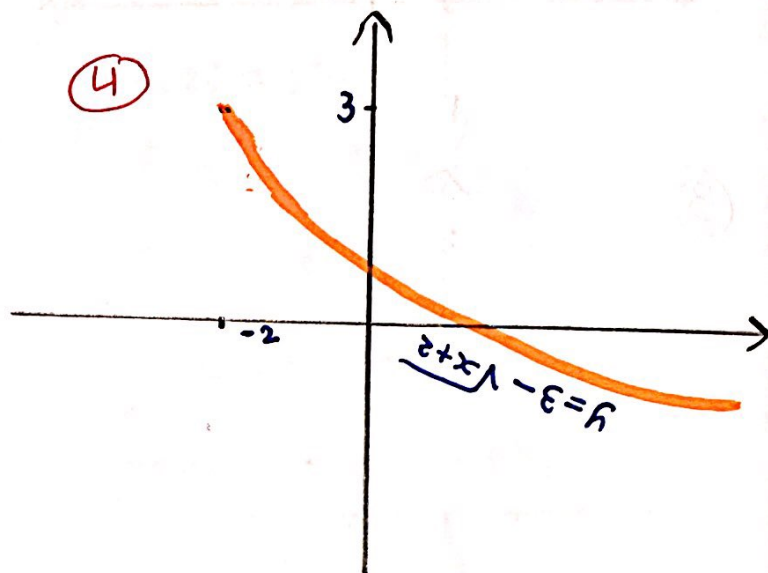
②



③



④



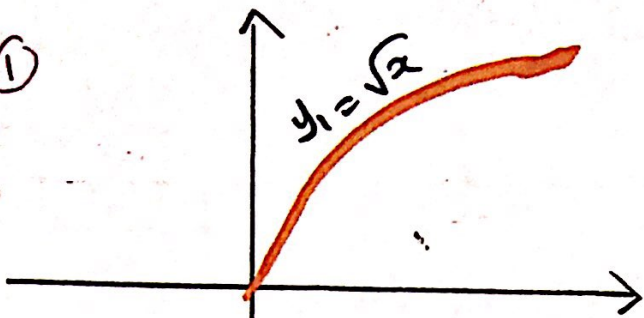
Domain of $y = 3 - \sqrt{x+2} = [-2, \infty)$

Range of $y = 3 - \sqrt{x+2} = (-\infty, 3]$

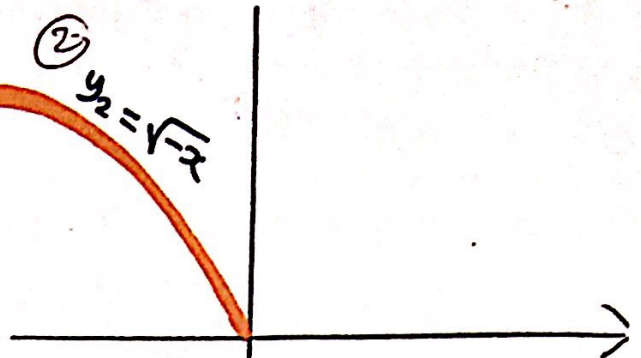
$y = 3 - \sqrt{x+2}$, Shift the graph $y_1 = \sqrt{x}$ a distance 2 units to the left and reflect the graph $y_2 = \sqrt{x+2}$ about x -axis and Shift the graph $y = -\sqrt{x+2}$ a distance 3 units upward.

$$10) y = -1 - \sqrt{-x}$$

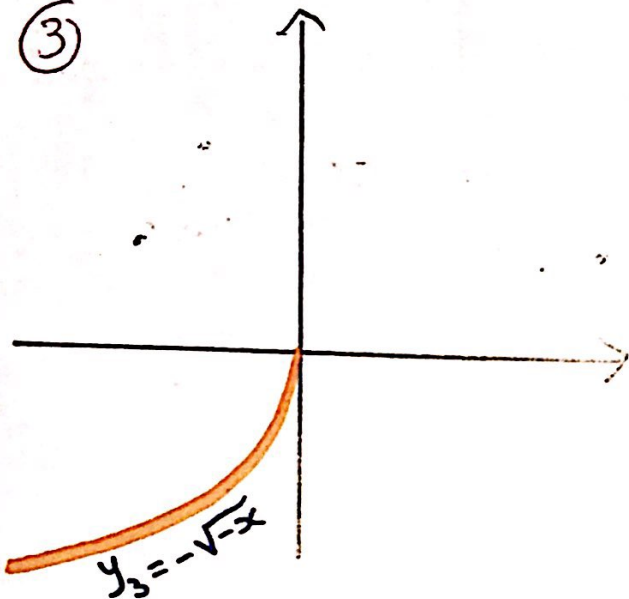
①



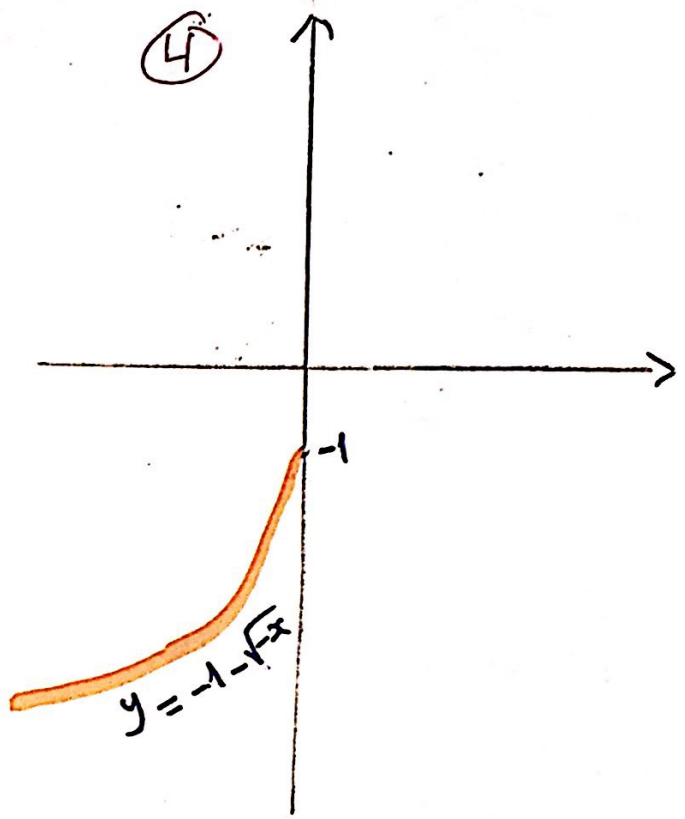
② $y_2 = \sqrt{-x}$



③



④



Domain of $y = -1 - \sqrt{-x}$ is $(-\infty, 0]$

Range of $y = -1 - \sqrt{-x}$ is $(-\infty, -1]$

$y = -1 - \sqrt{-x}$, reflect the graph of $y_1 = \sqrt{x}$ about y-axis and x-axis and shift the graph $y_3 = -\sqrt{-x}$ a distance 1 unit downward.

Example

Sketch and Find the Domain and Range

of $f(x) = x^2 + 6x + 10$

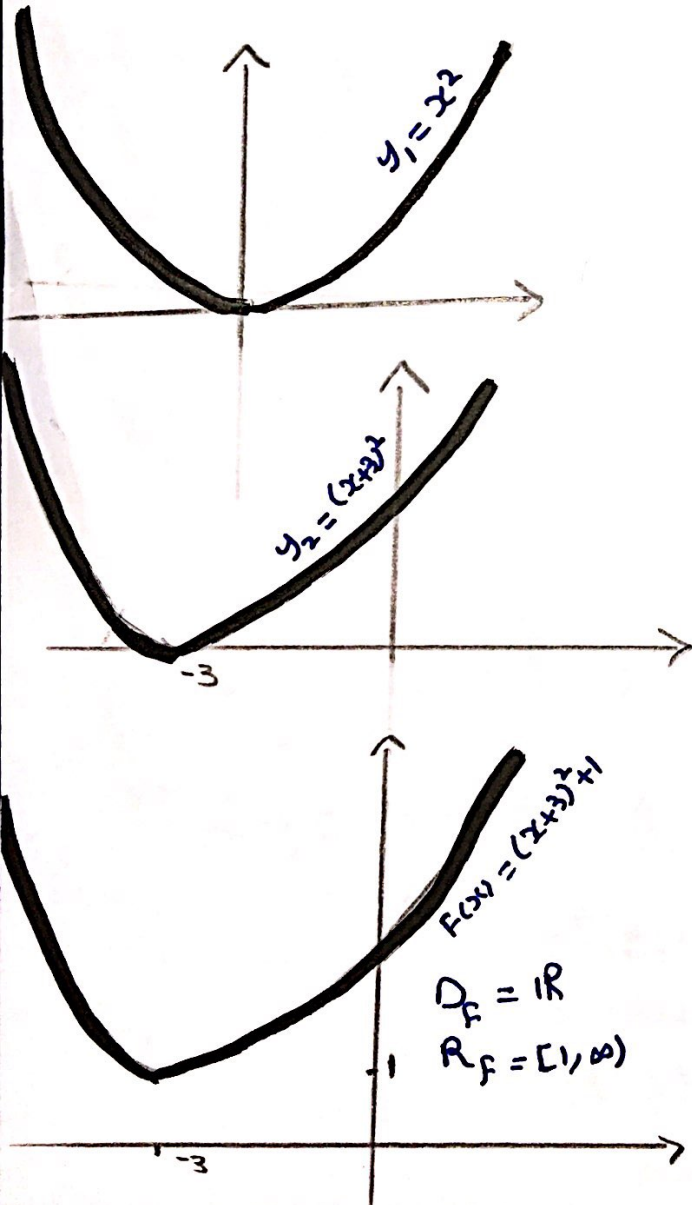
$f(x) = 5 - 4x - x^2$

$f(x) = x^2 + 6x + 10$

$f(x) = x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 10$

$= x^2 + 6x + 9 - 9 + 10$

$= (x + 3)^2 + 1$



$f(x) = 5 - 4x - x^2$

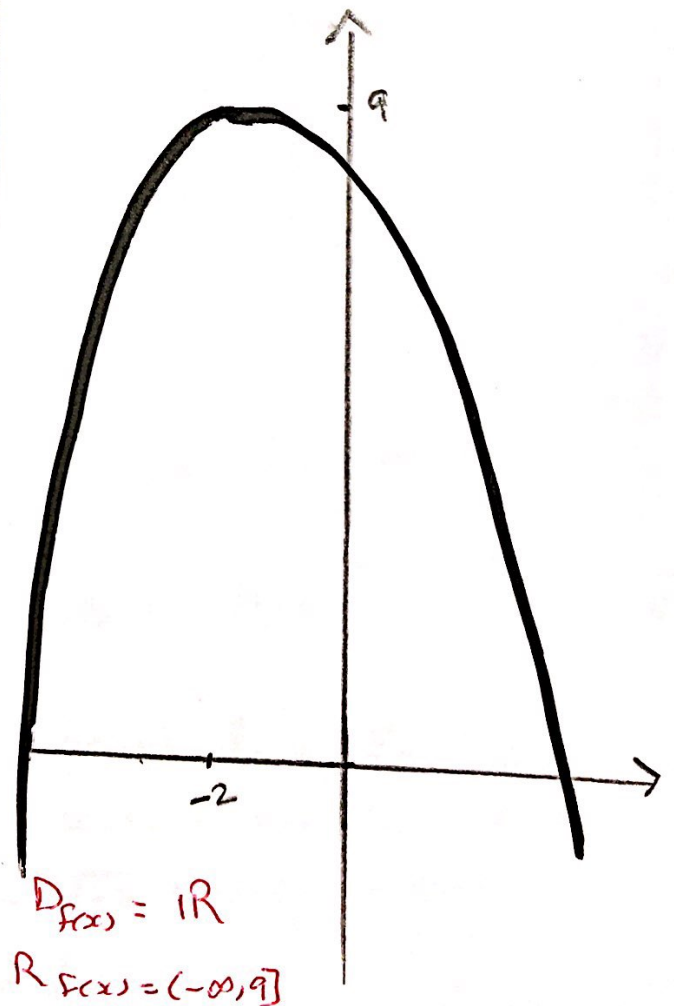
$f(x) = -[x^2 + 4x - 5]$

$= -[x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 5]$

$= -[x^2 + 4x + 4 - 4 - 5]$

$= -[(x+2)^2 - 9]$

$= 9 - (x+2)^2$



H.W

Find the Domain and Range of the following

① $f(x) = x^2 + 2$

② $f(x) = x^2 - 2$

③ $f(x) = (x-3)^2$

④ $f(x) = (x+3)^2$

⑤ $f(x) = -x^2$

⑥ $f(x) = 3 - x^2$

⑦ $f(x) = (x-1)^2 - 3$

⑧ $f(x) = 4 - (x+1)^2$

$f(x) = |x| + 3$

$f(x) = |x| - 3$

$f(x) = |x-2|$

$f(x) = |x+2|$

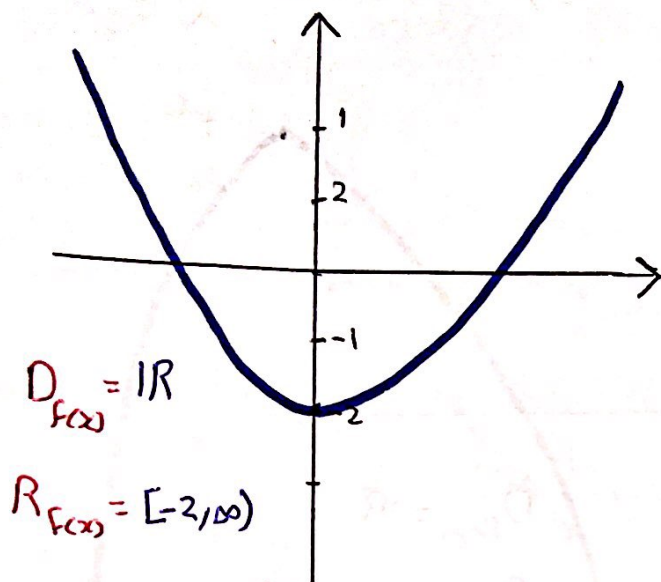
$f(x) = -|x|$

$f(x) = 5 - |x|$

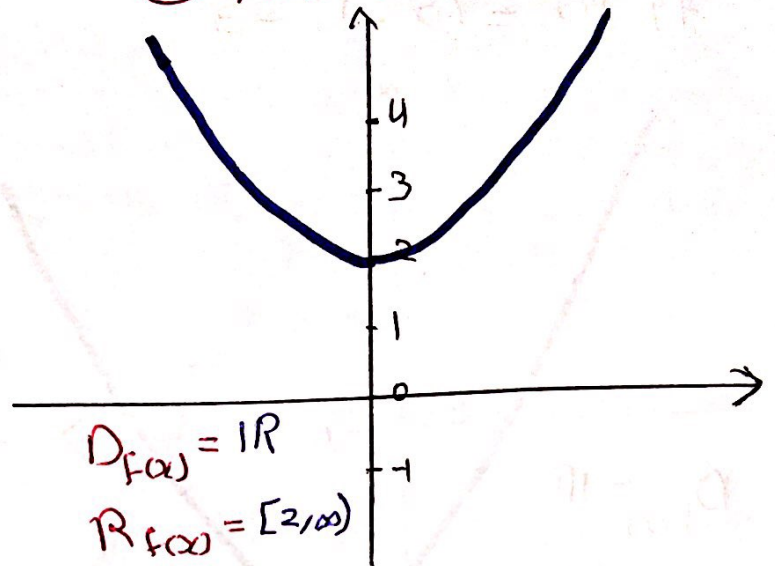
$f(x) = |x-3| + 1$

$f(x) = 2 - |x+4|$

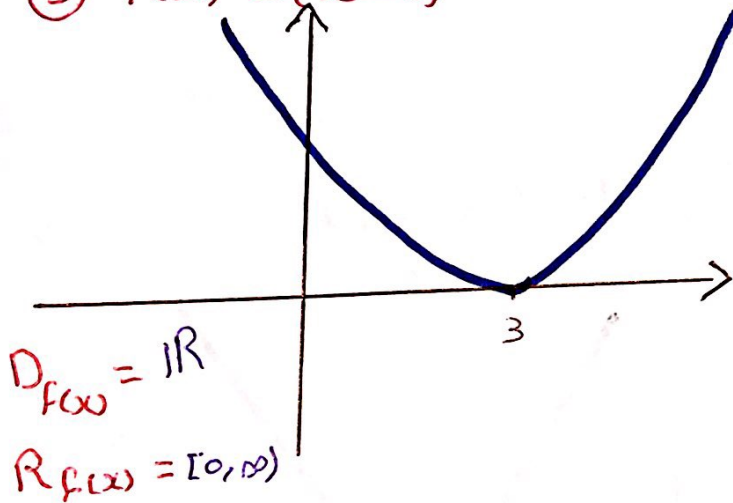
① $f(x) = x^2 - 2$



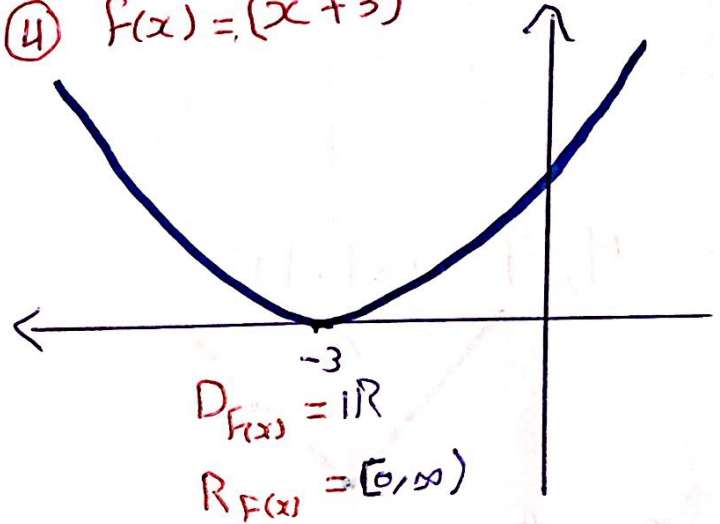
② $f(x) = x^2 + 2$



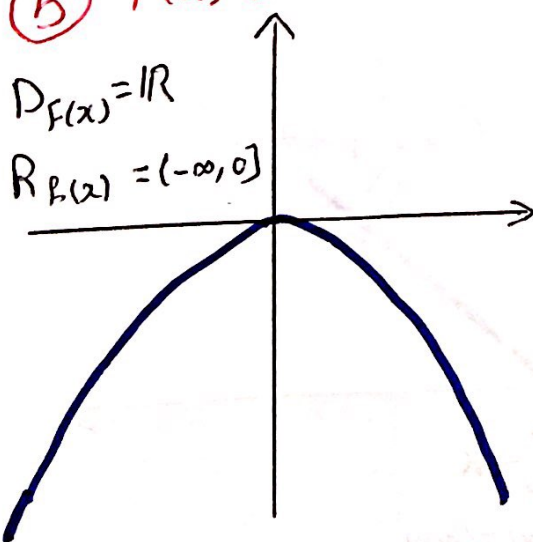
③ $f(x) = (x-3)^2$



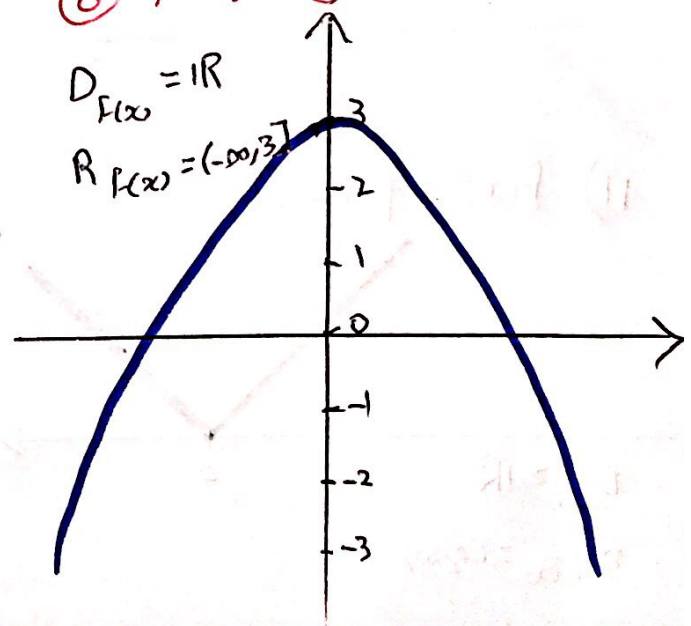
④ $f(x) = (x+3)^2$



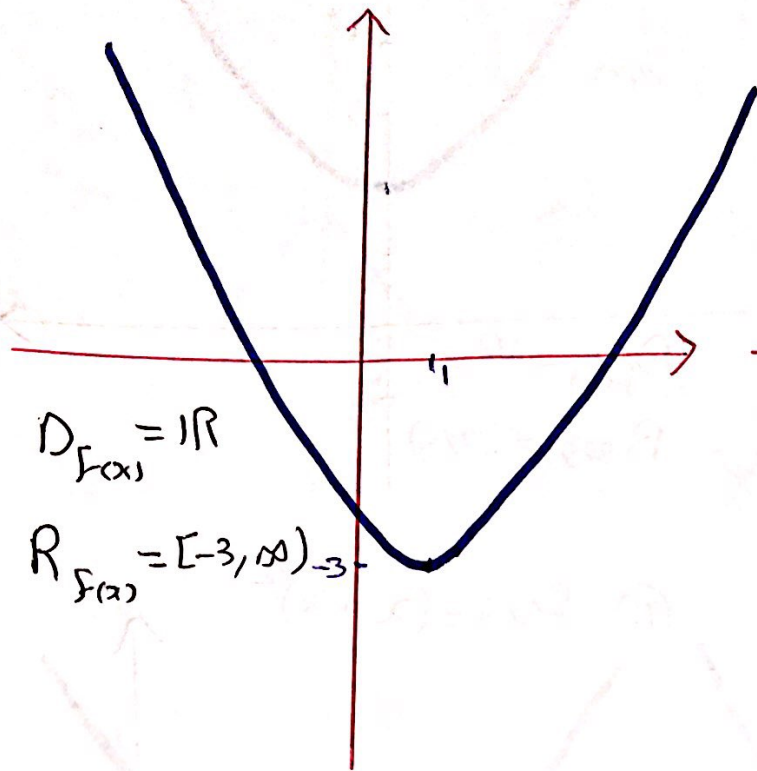
⑤ $f(x) = -x^2$



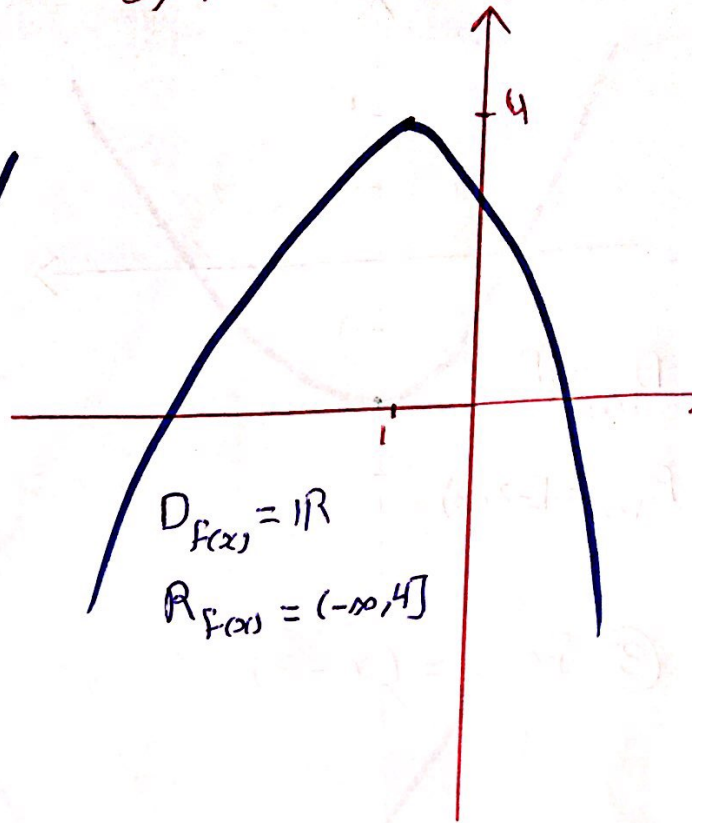
⑥ $f(x) = 3 - x^2$



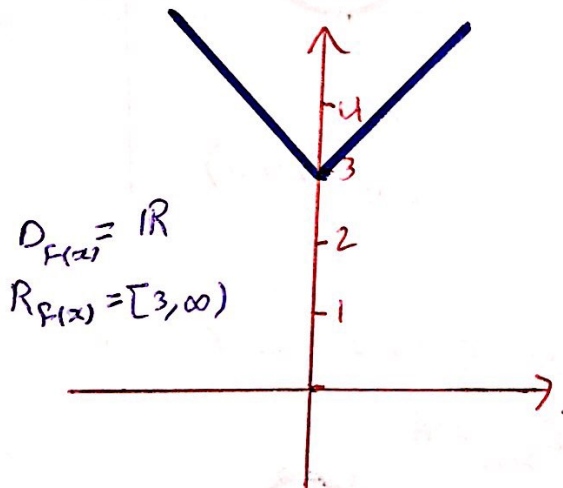
7) $f(x) = (x-1)^2 - 3$



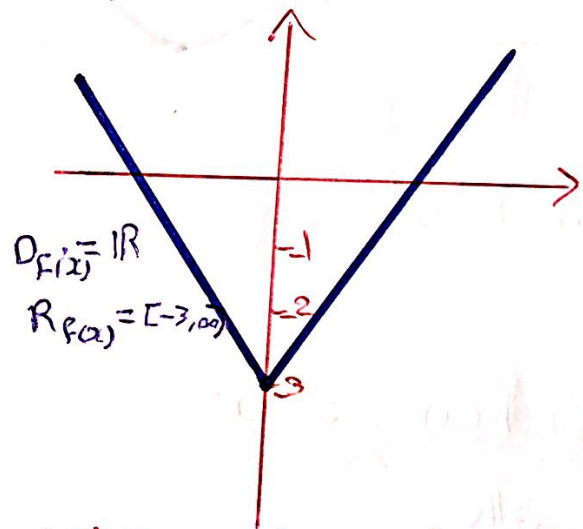
8) $f(x) = 4 - (x+1)^2$



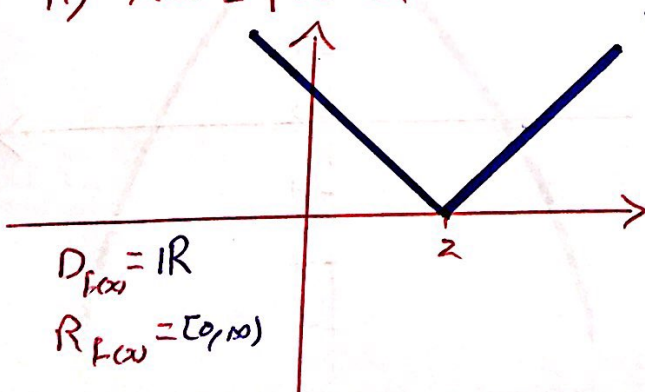
9) $f(x) = |x| + 3$



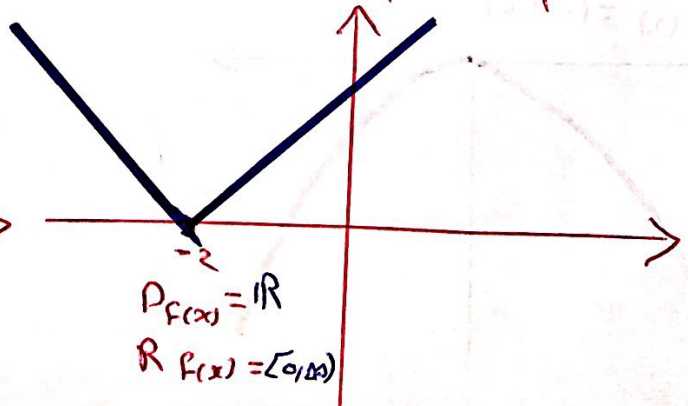
10) $f(x) = |x| - 3$



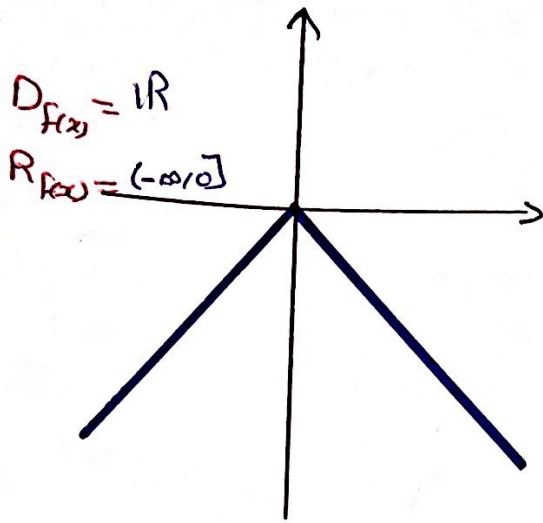
11) $f(x) = |x-2|$



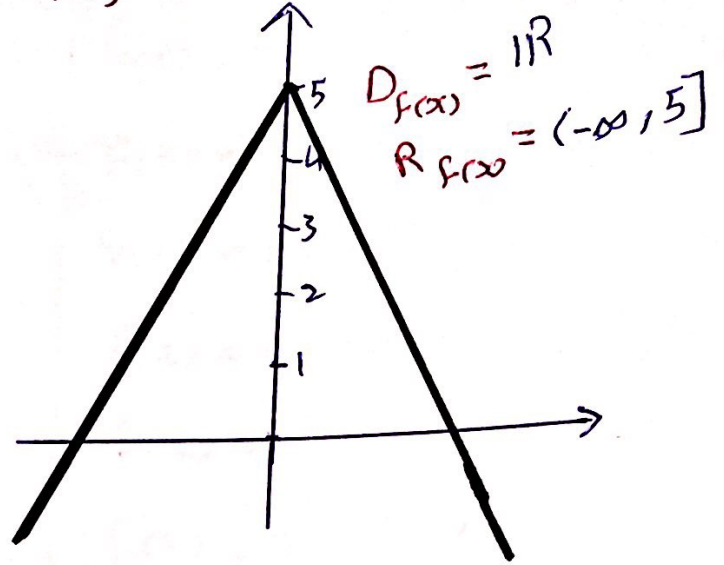
12) $f(x) = |x+2|$



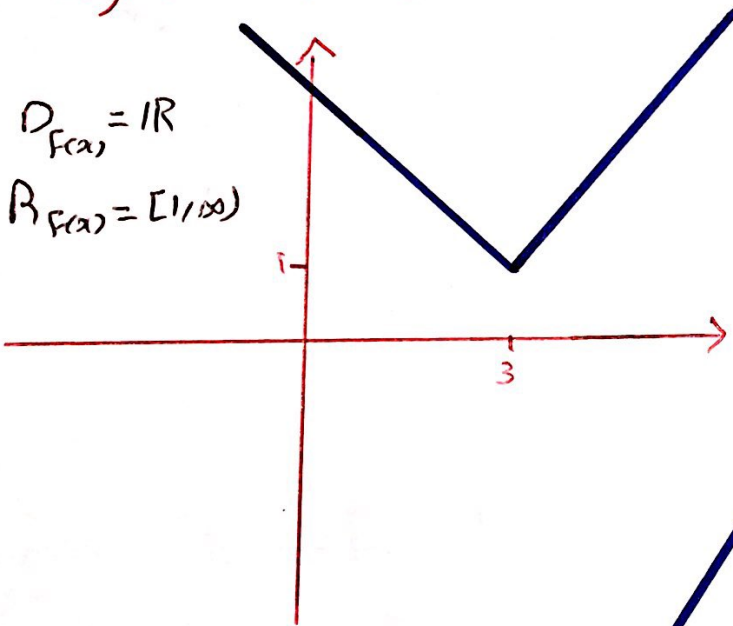
13) $f(x) = -|x|$



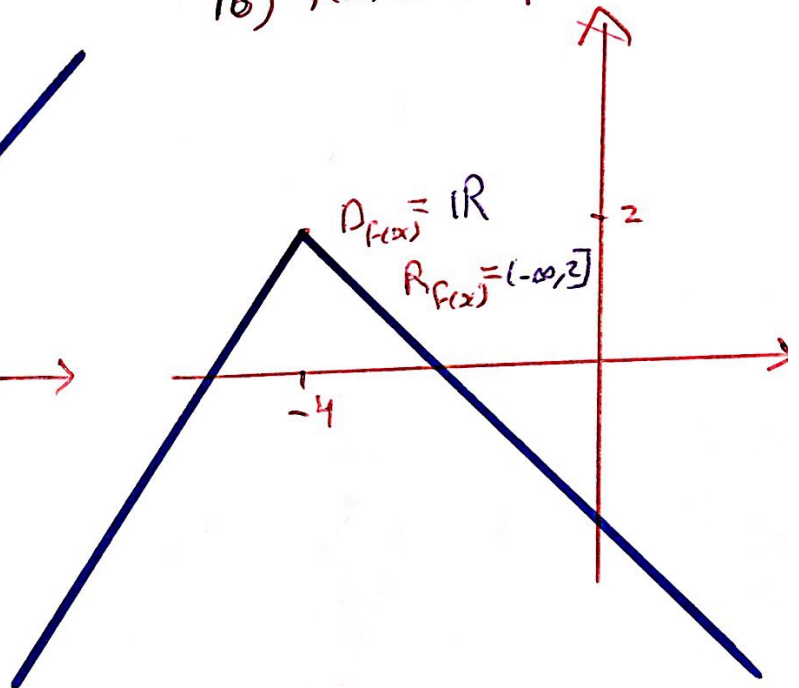
14) $f(x) = 5 - |x|$



15) $f(x) = |x-3| + 1$



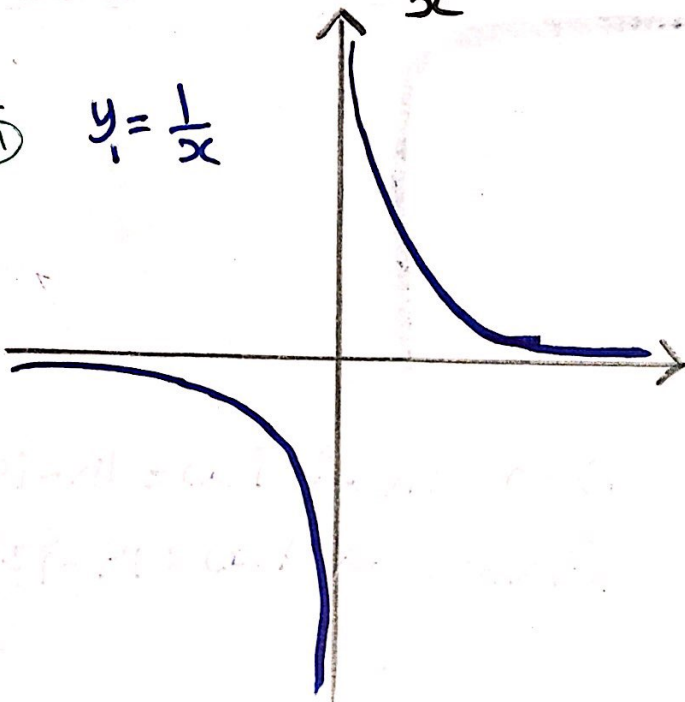
16) $f(x) = 2 - |x+4|$



Example (3) : Sketch the graph and find the Domain and Range.

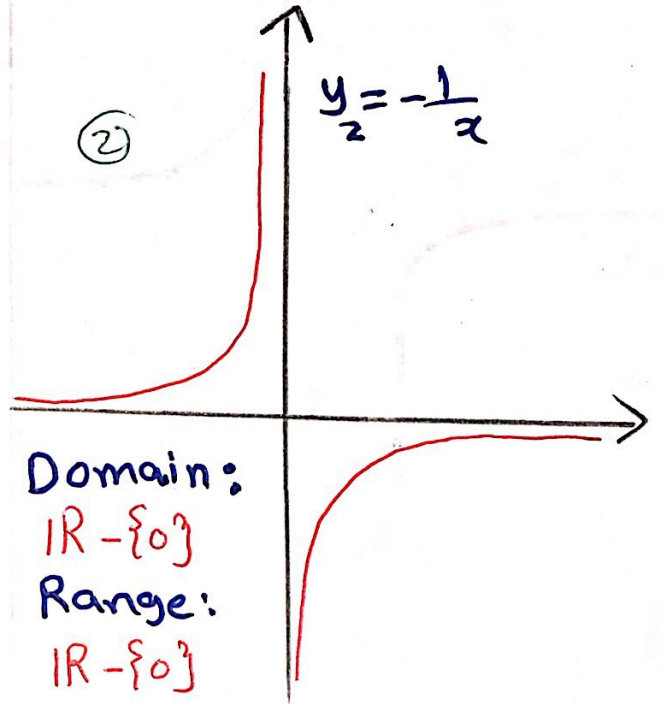
1) $f(x) = -\frac{1}{x}$

① $y_1 = \frac{1}{x}$



②

$y_2 = -\frac{1}{x}$



Domain:

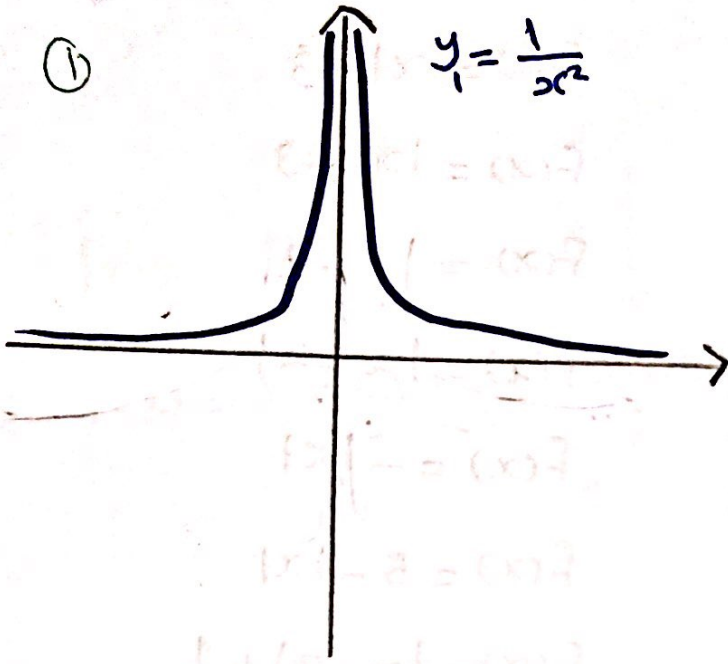
$\mathbb{R} - \{0\}$

Range:

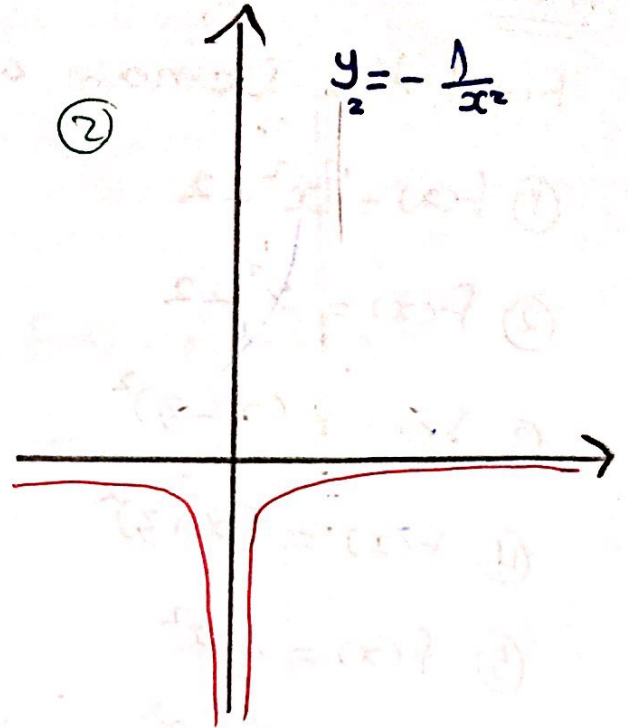
$\mathbb{R} - \{0\}$

② $f(x) = -\frac{1}{x^2}$

①



②

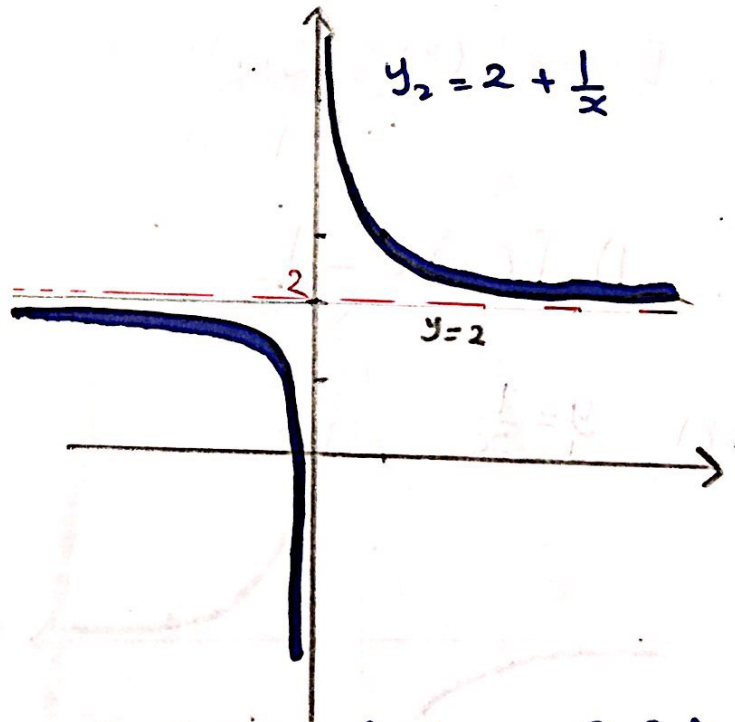
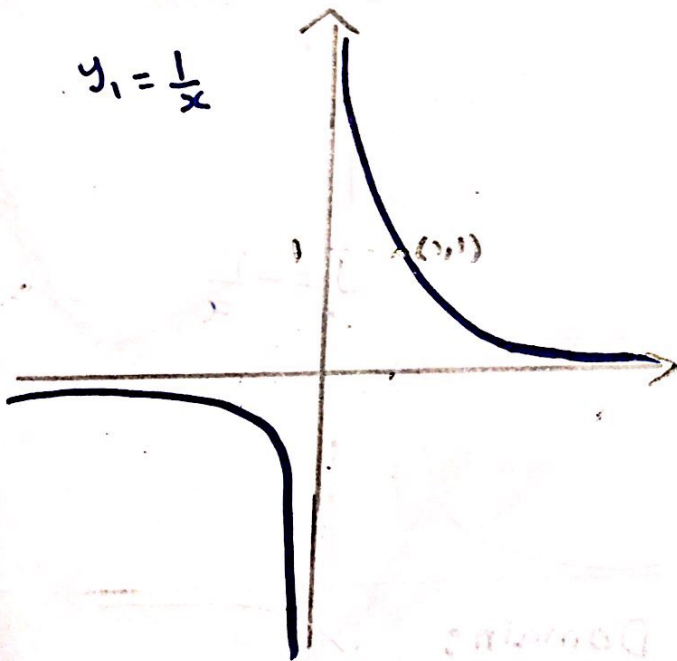


Domain: $\mathbb{R} - \{0\}$

Range: $(-\infty, 0)$

③ $f(x) = 2 + \frac{1}{x}$

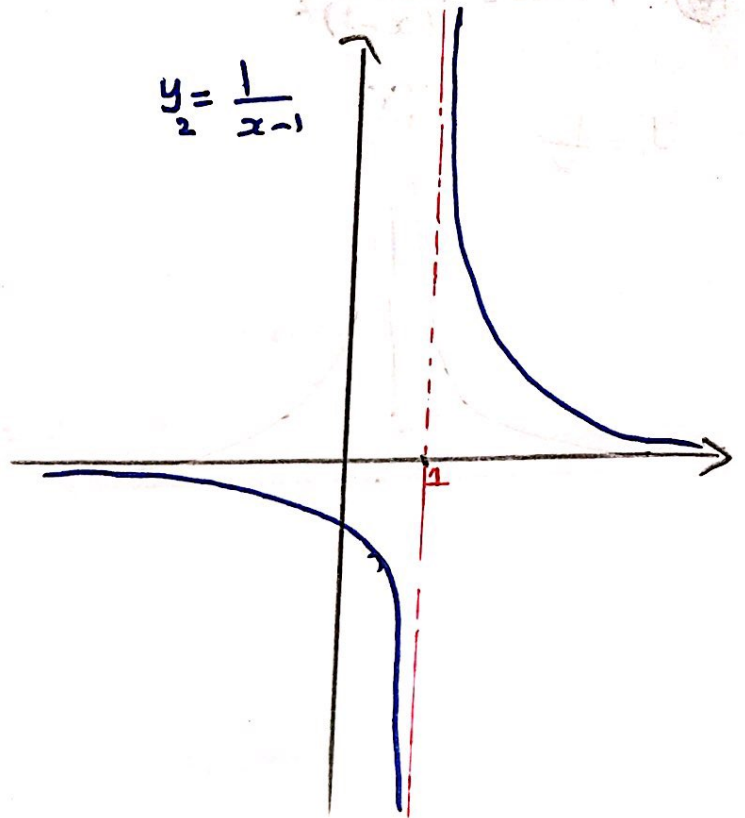
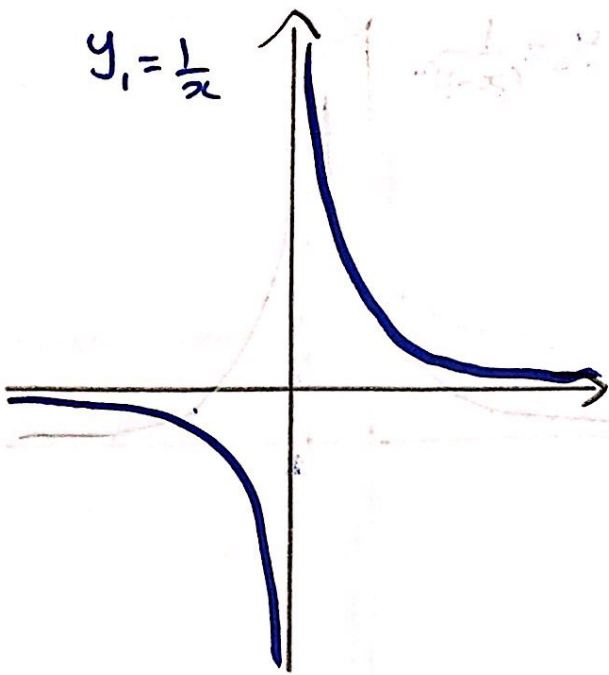
$y_1 = \frac{1}{x}$



Domain of $f(x) = \mathbb{R} - \{0\}$

Range of $f(x) = \mathbb{R} - \{2\}$

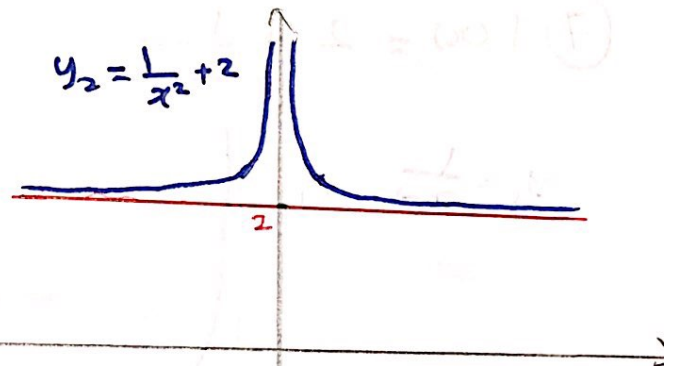
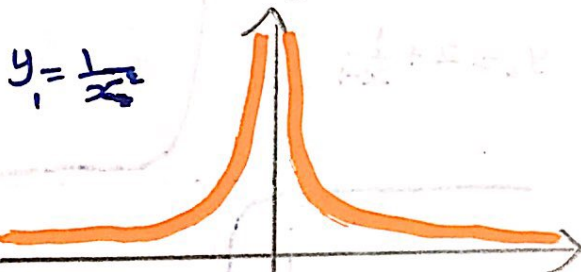
$$4) f(x) = \frac{1}{x-1}$$



Domain: $\mathbb{R} - \{1\}$

Range: $\mathbb{R} - \{0\}$

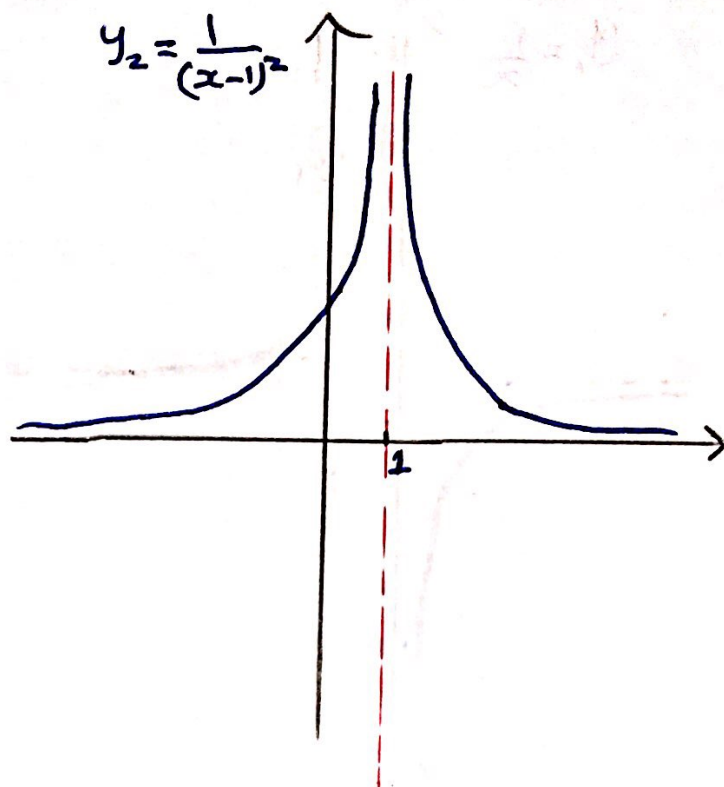
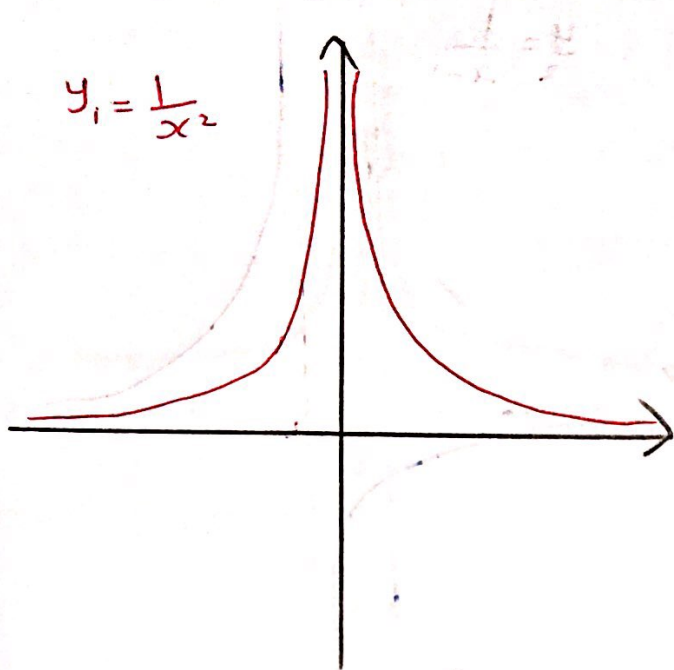
$$5) f(x) = \frac{1}{x^2} + 2$$



Domain: $\mathbb{R} - \{0\}$

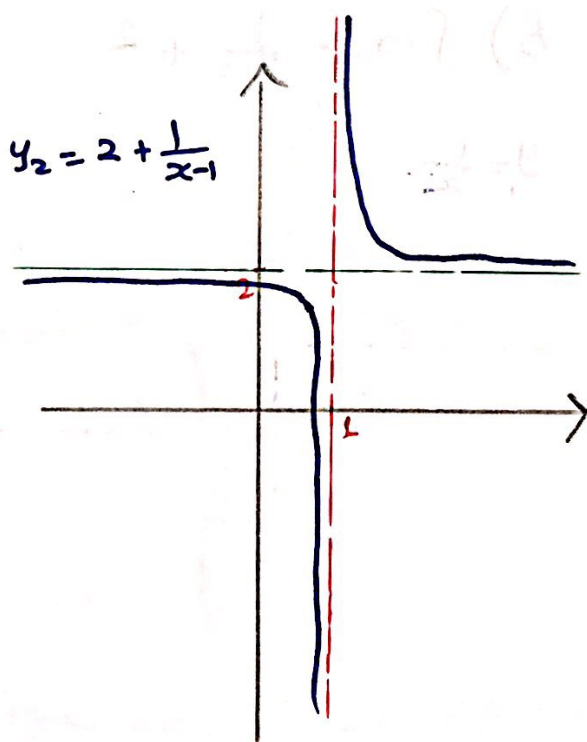
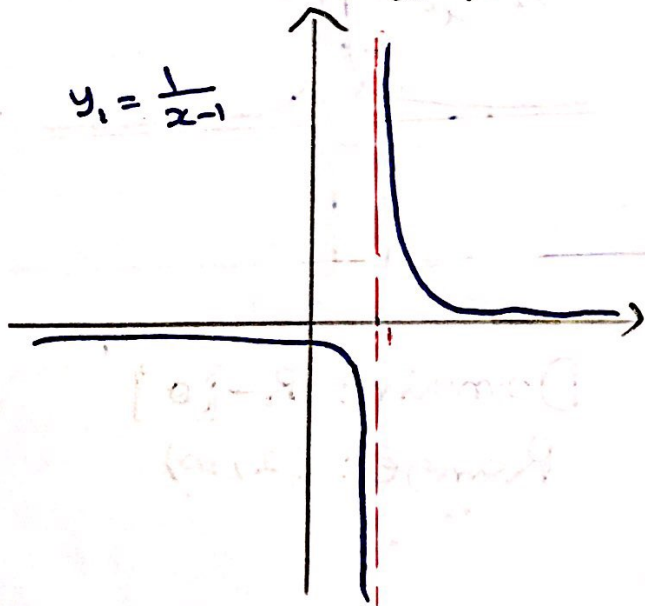
Range: $(2, \infty)$

⑥ $f(x) = \frac{1}{(x-1)^2}$



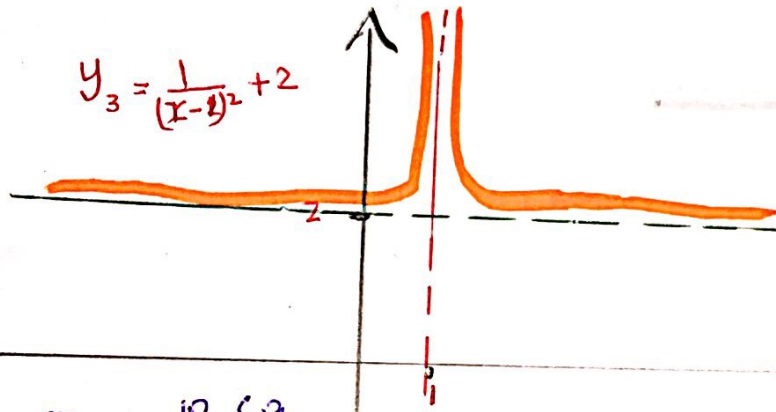
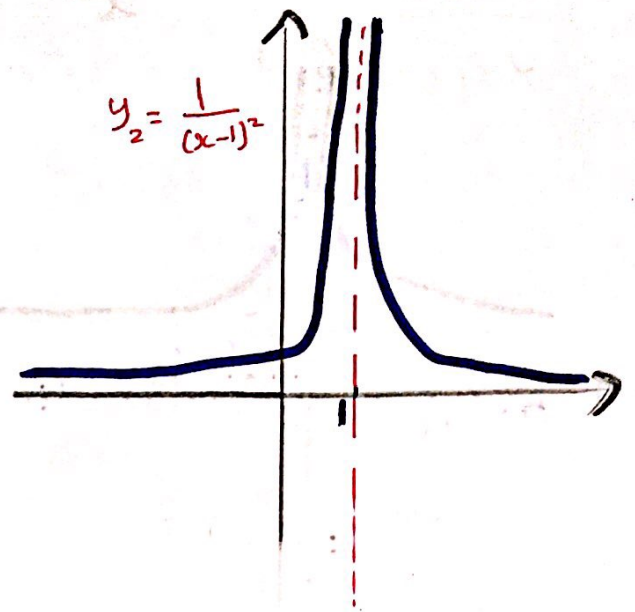
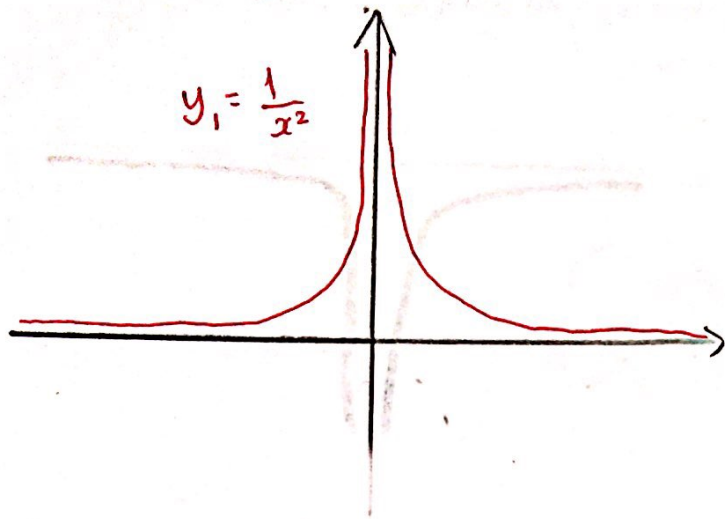
Domain: $\mathbb{R} - \{1\}$
 Range: $(0, \infty)$

⑦ $f(x) = 2 + \frac{1}{x-1}$



Domain: $\mathbb{R} - \{1\}$
 Range: $\mathbb{R} - \{2\}$

⑧ $f(x) = \frac{1}{(x-1)^2} + 2$ H.W

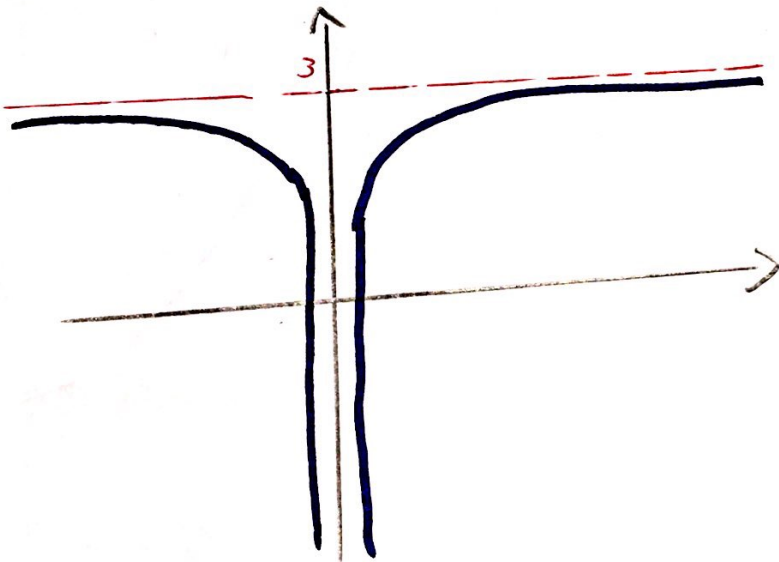
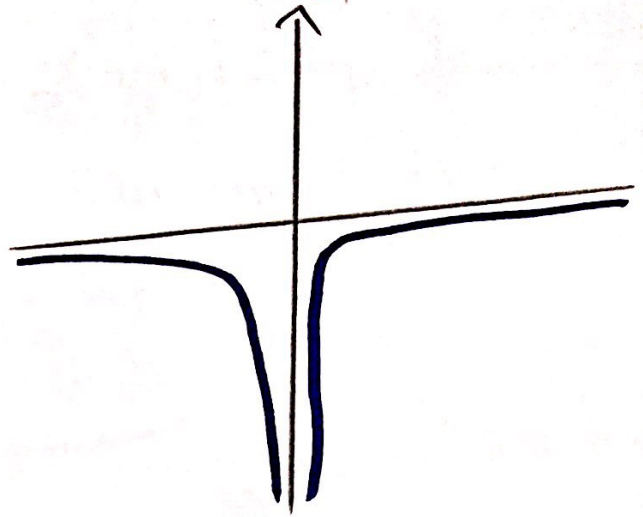
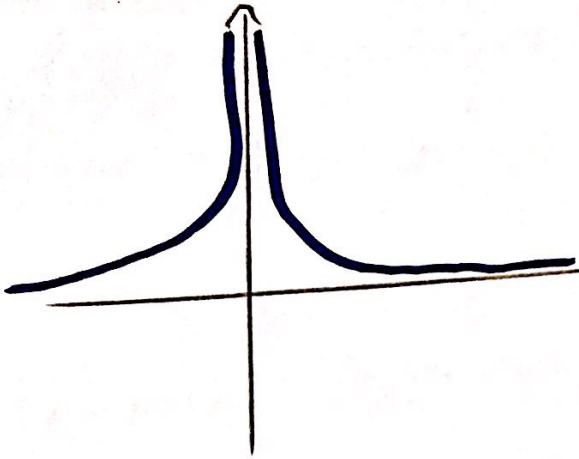


$D_{f(x)} = \mathbb{R} - \{1\}$

$R_{f(x)} = (2, \infty)$

⑨ $f(x) = 3 - \frac{1}{x^2}$

W.H. $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$ (1)

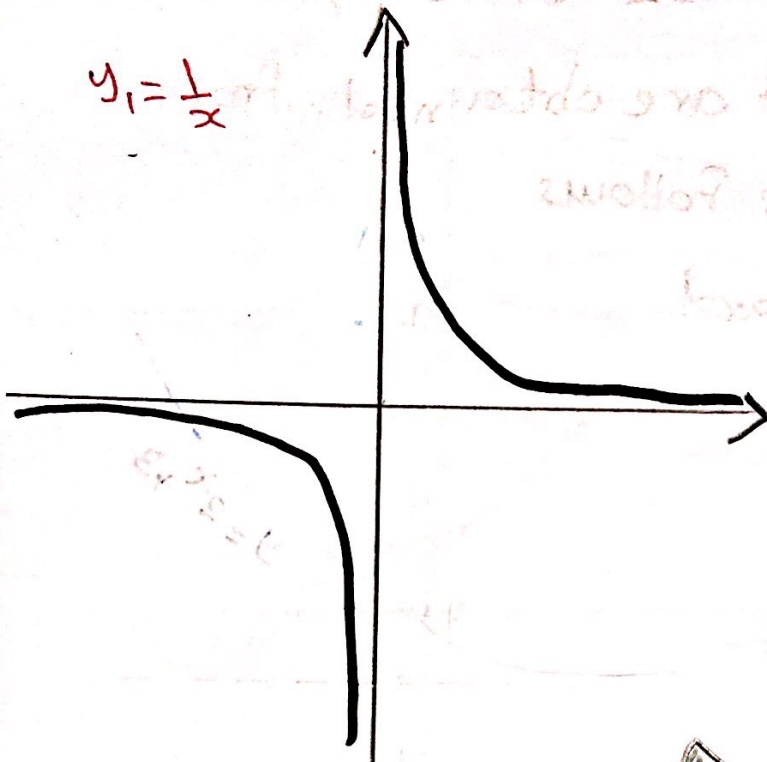


Domain: $\mathbb{R} - \{0\}$

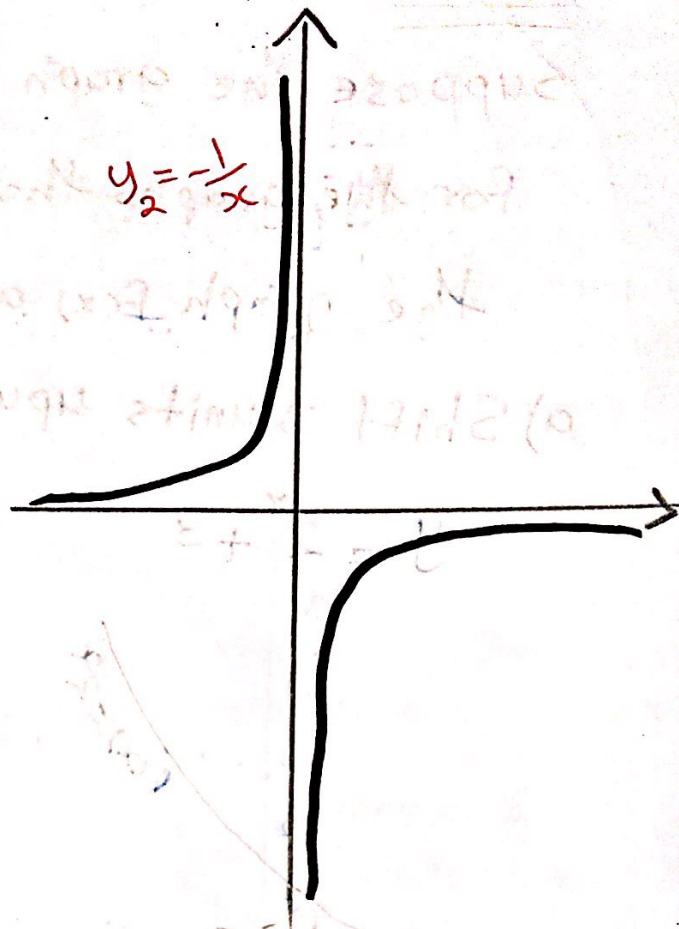
Range: $(-\infty, 3)$

⑩ $f(x) = 3 - \frac{1}{x}$ H.W

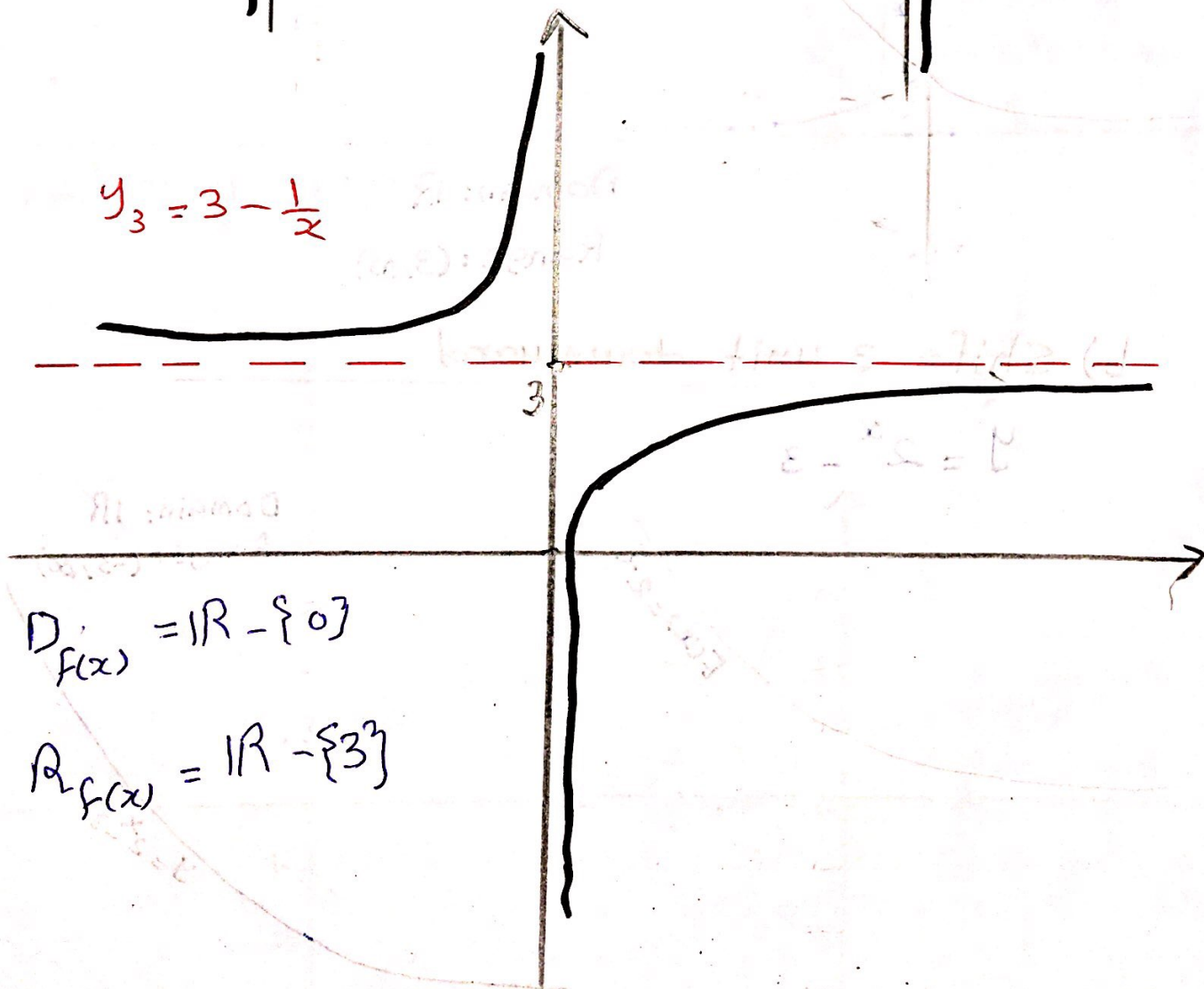
$y_1 = \frac{1}{x}$



$y_2 = -\frac{1}{x}$



$y_3 = 3 - \frac{1}{x}$



$D_{f(x)} = \mathbb{R} - \{0\}$

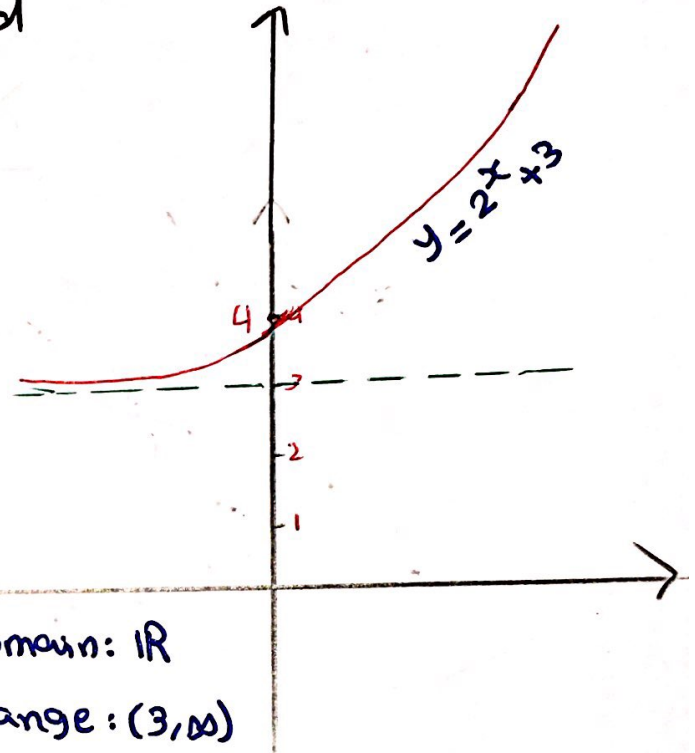
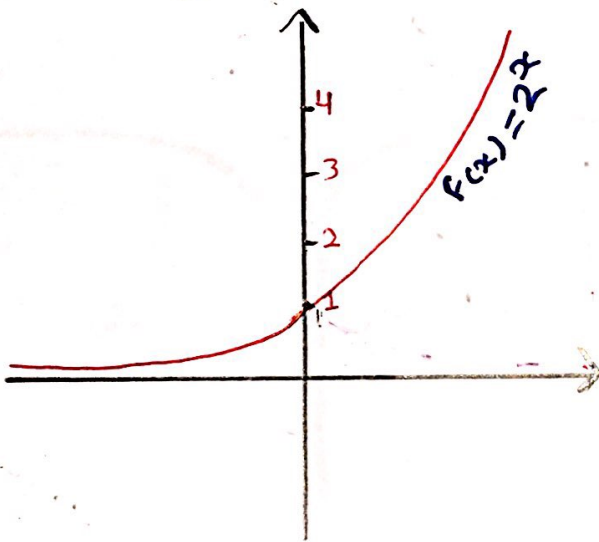
$R_{f(x)} = \mathbb{R} - \{3\}$

Example

Suppose the graph $f(x) = 2^x$ write equation for the graphs that are obtained from the graph $f(x)$ as follows

a) Shift 3 units upward

$$y = 2^x + 3$$

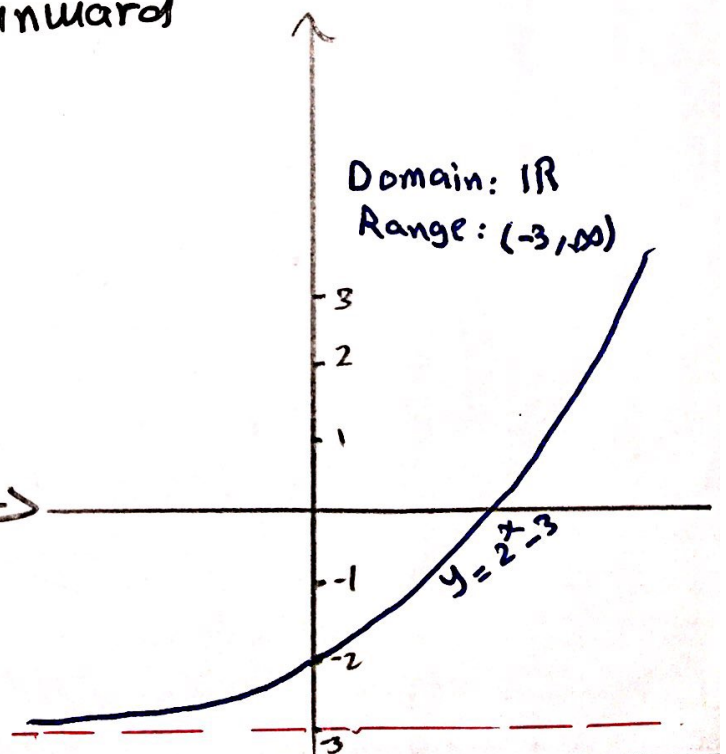
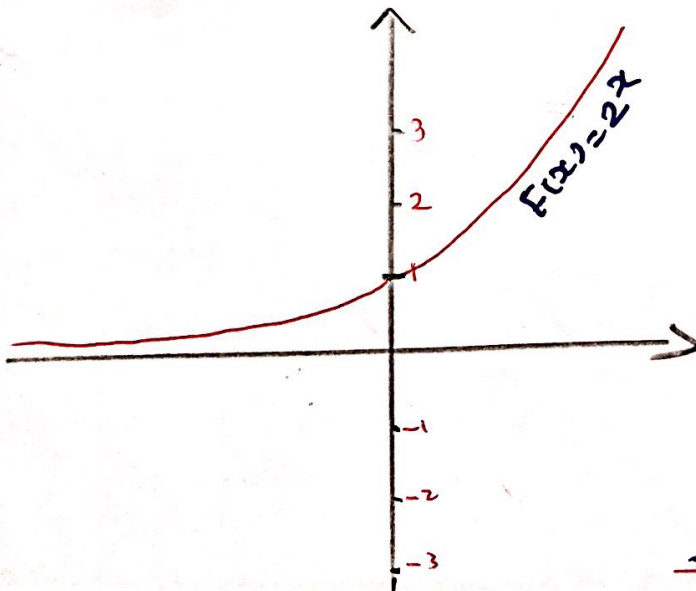


Domain: \mathbb{R}

Range: $(3, \infty)$

b) Shift 3 unit downward

$$y = 2^x - 3$$

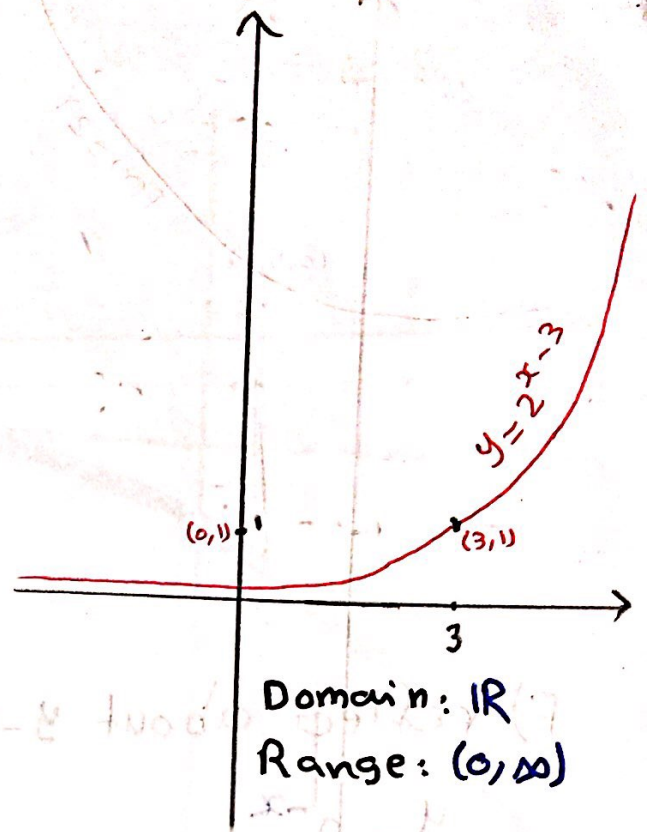
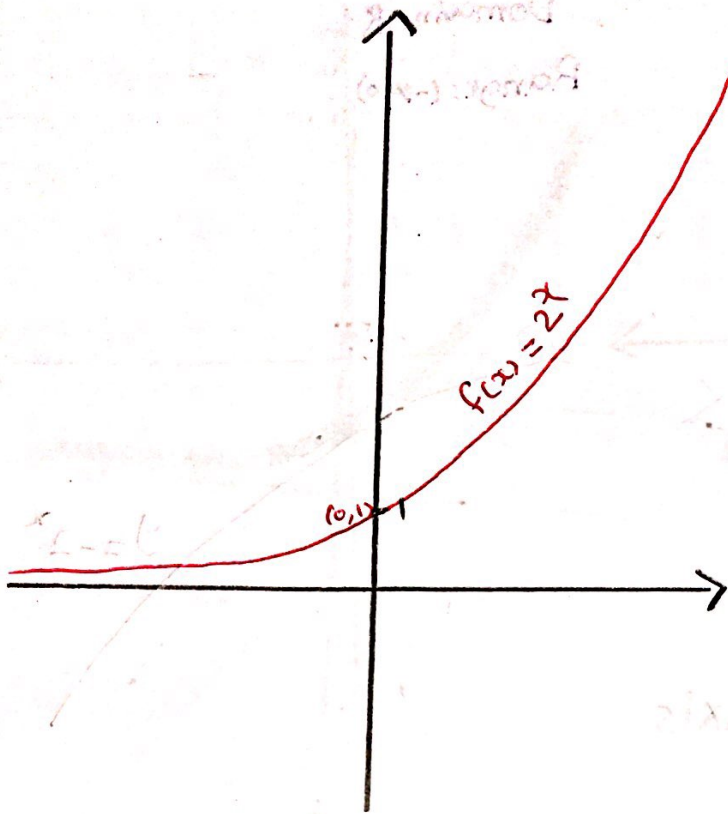


Domain: \mathbb{R}

Range: $(-3, \infty)$

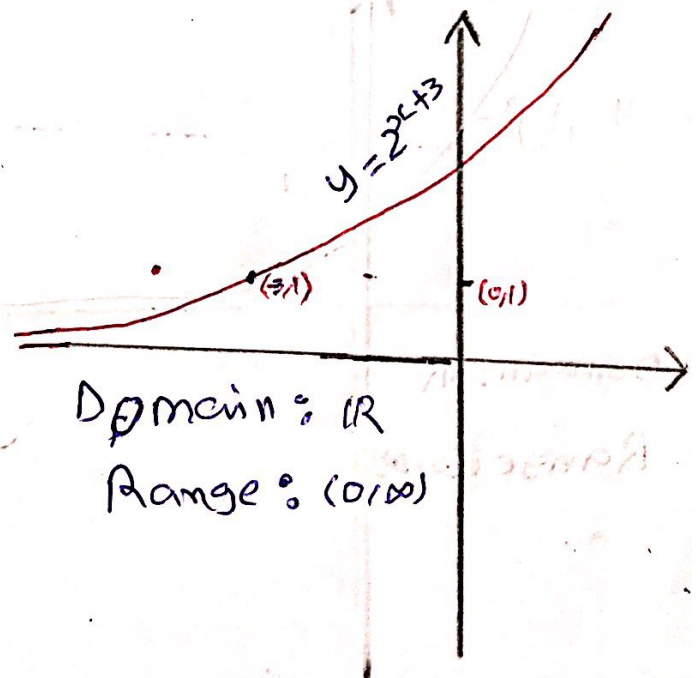
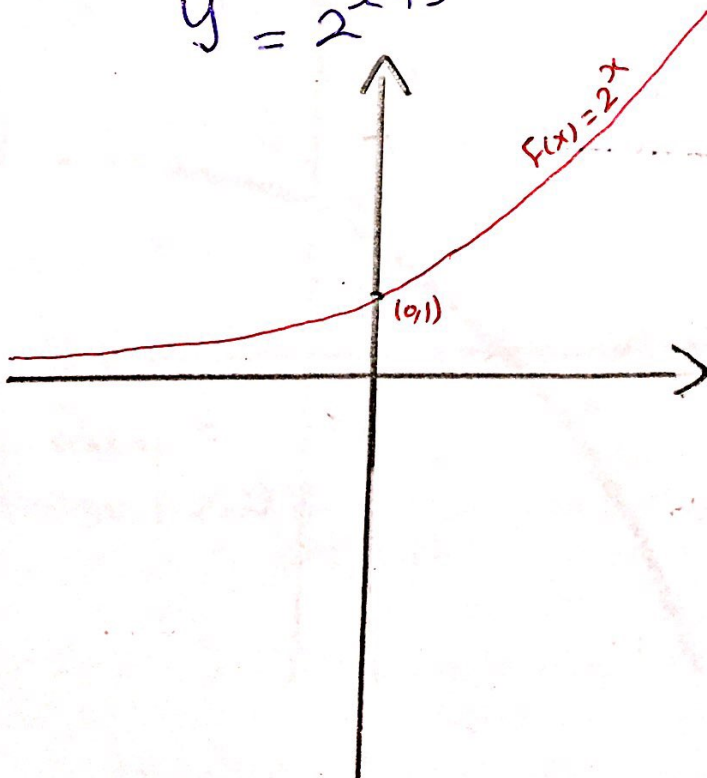
c) Shift 3 units to the right

$$y = 2^{x-3}$$



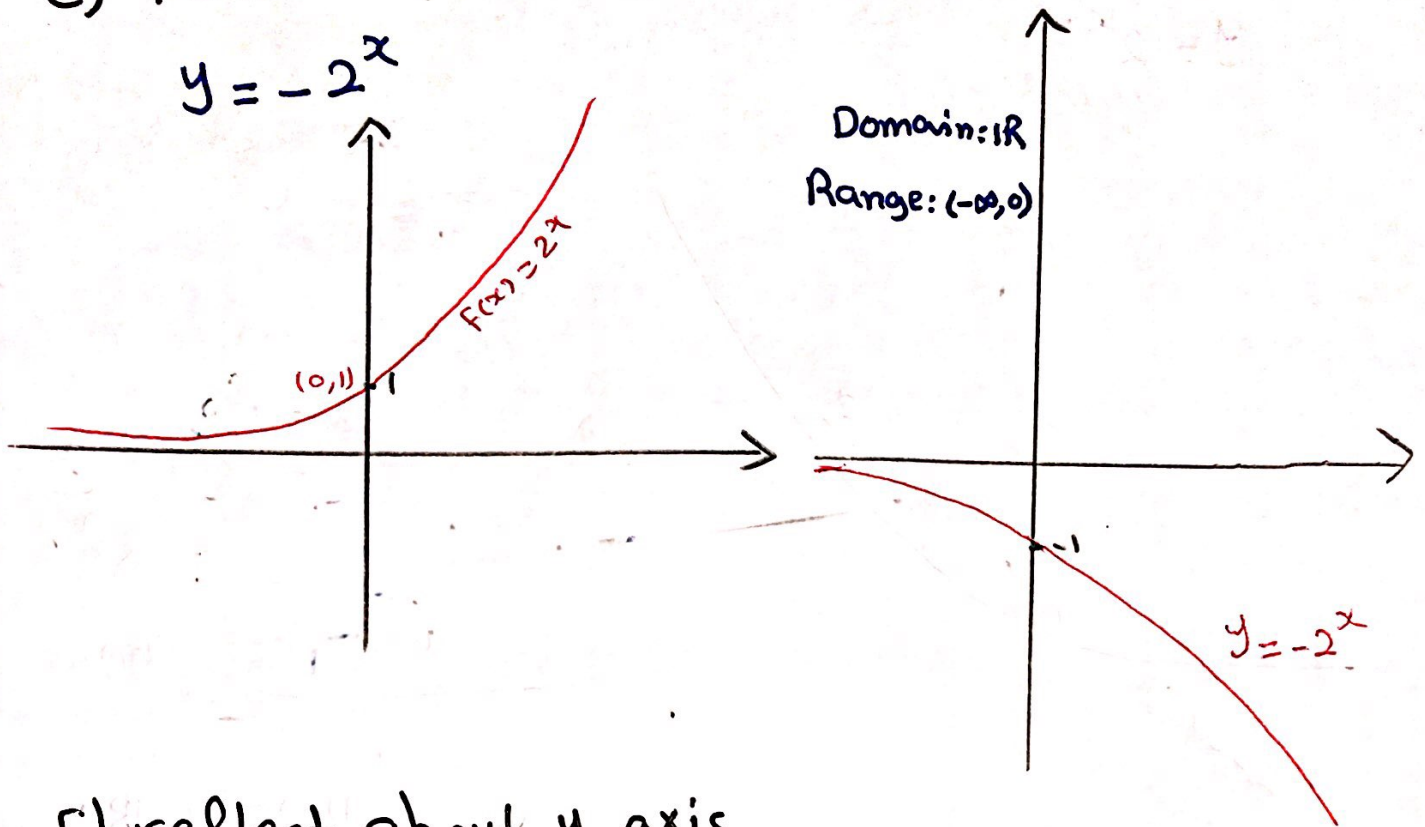
d) Shift 3 unit to the left. H.W

$$y = 2^{x+3}$$



e) reflect about x-axis

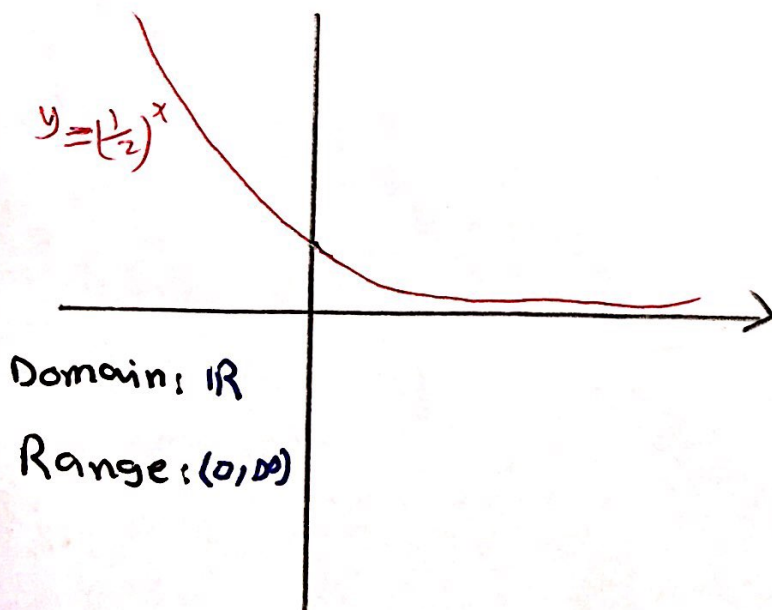
$$y = -2^x$$



f) reflect about y-axis

$$y = 2^{-x}$$

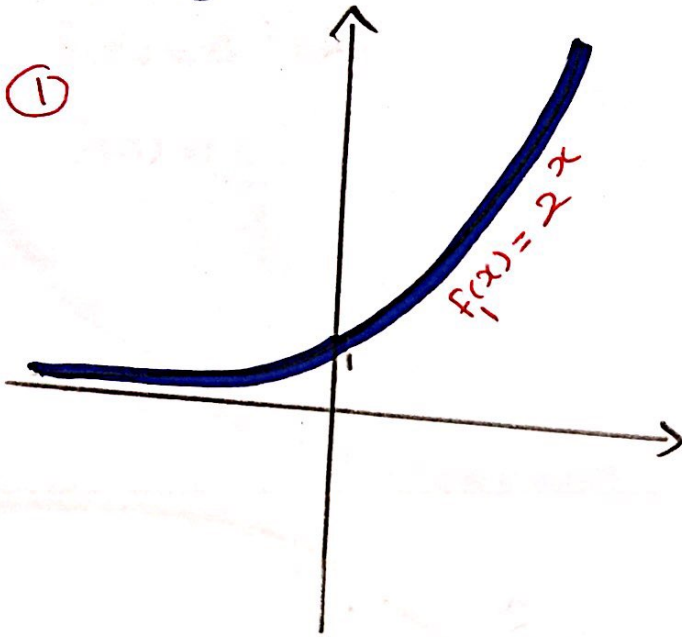
$$= \left(\frac{1}{2}\right)^x$$



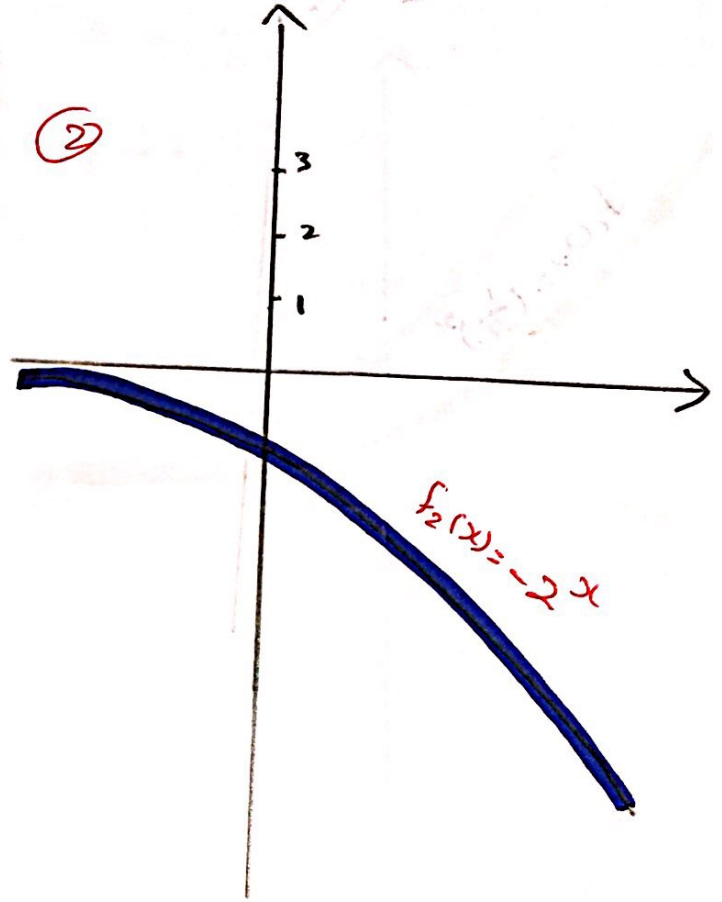
9) reflect about x-axis and Shift 3 units upward.

$$y = 3 - 2^x$$

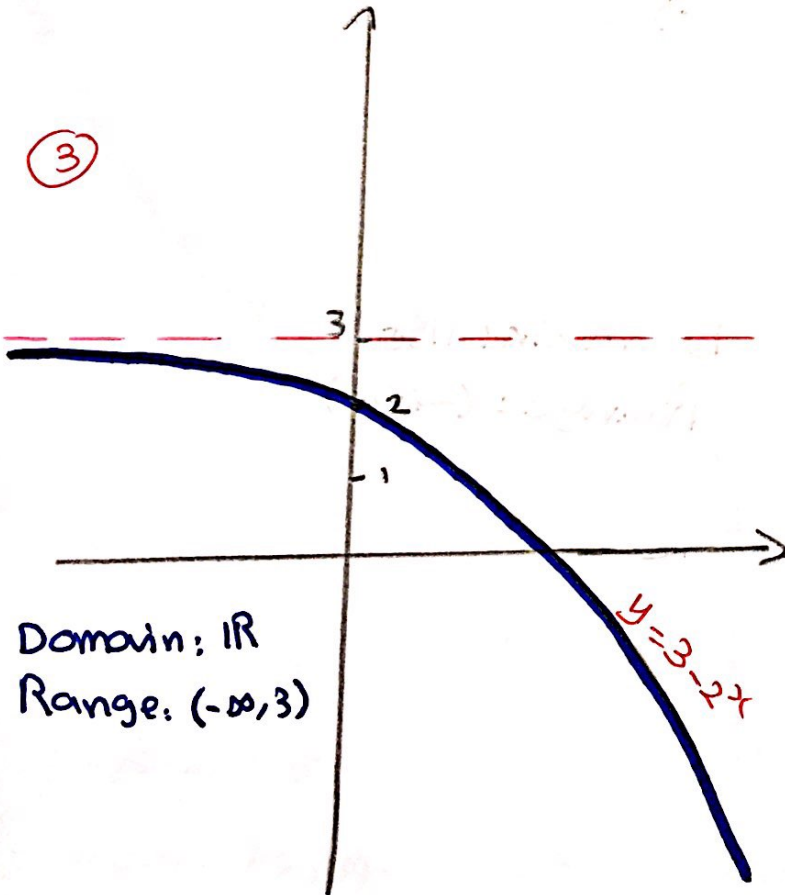
①



②



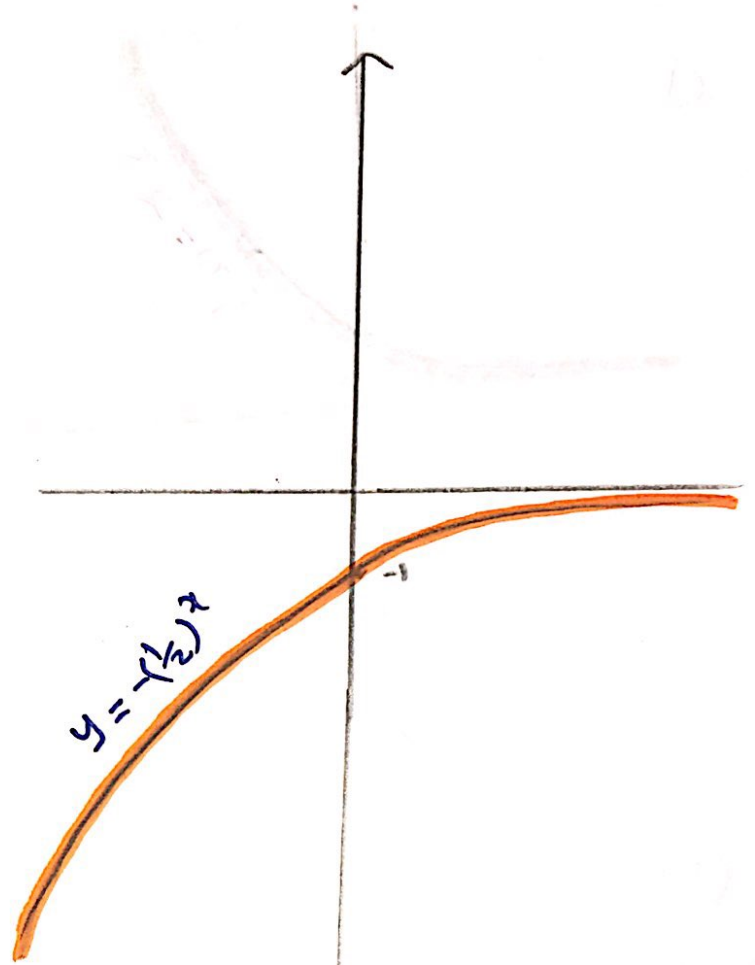
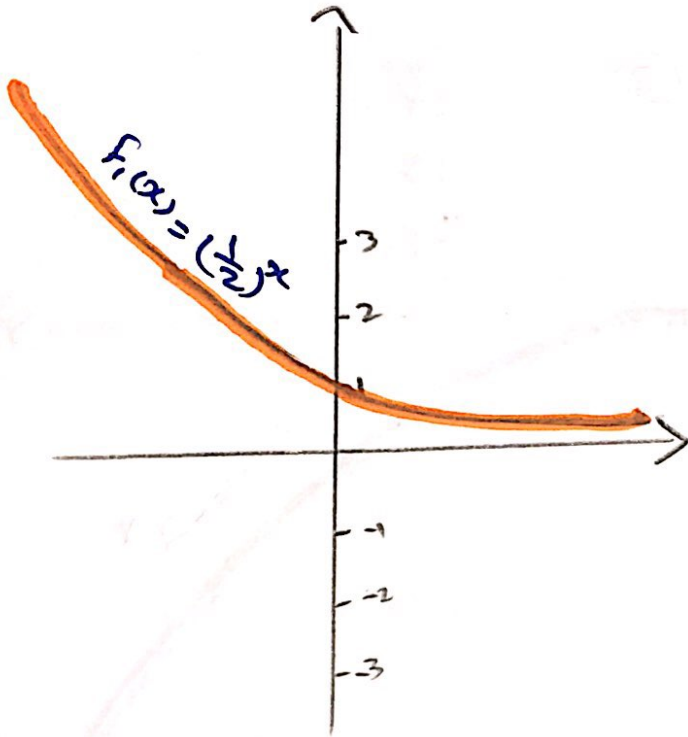
③



Domain: \mathbb{R}
Range: $(-\infty, 3)$

h) reflect about x-axis and y-axis

$$y = -2^{-x}$$
$$= -\left(\frac{1}{2}\right)^x$$



Domain: \mathbb{R}
Range: $(-\infty, 0)$

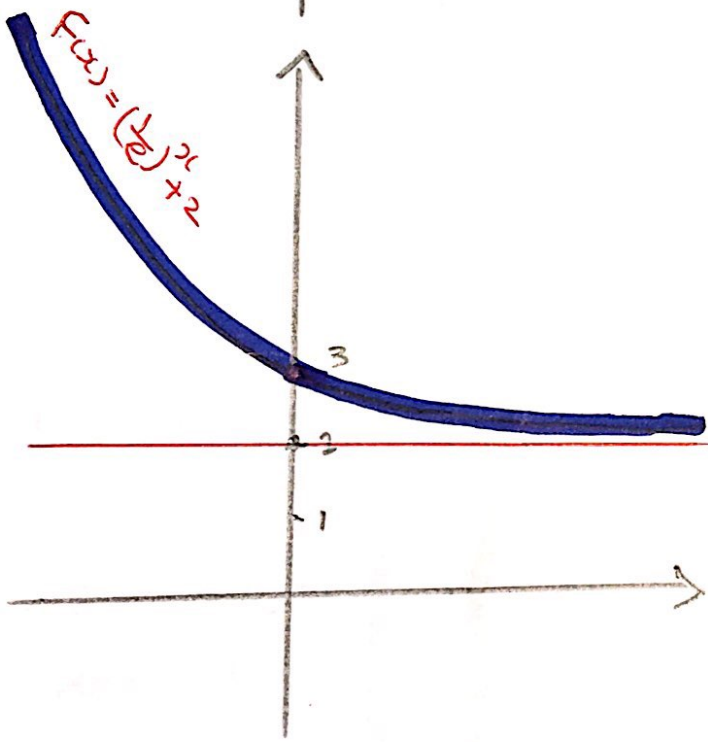
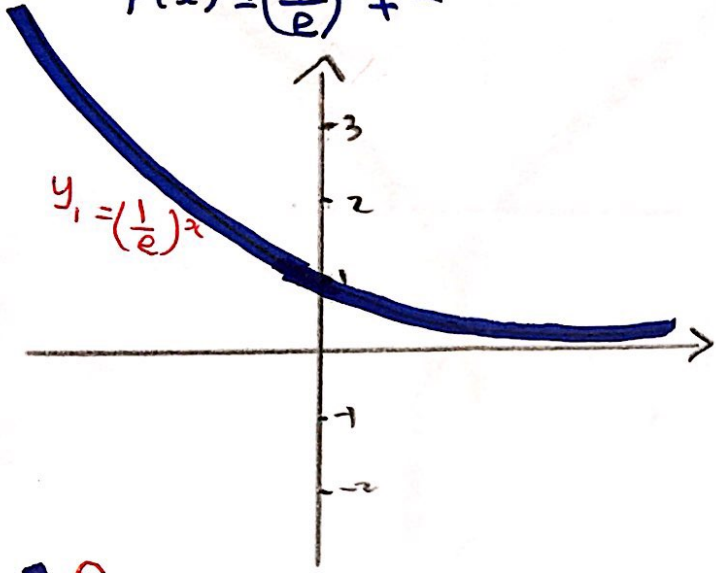
Example find the Domain and Range of

① $f(x) = e^{-x} + 2$

② $f(x) = \left(\frac{1}{\pi}\right)^{-x} - 2$

$f(x) = e^{-x} + 2$

$f(x) = \left(\frac{1}{e}\right)^x + 2$

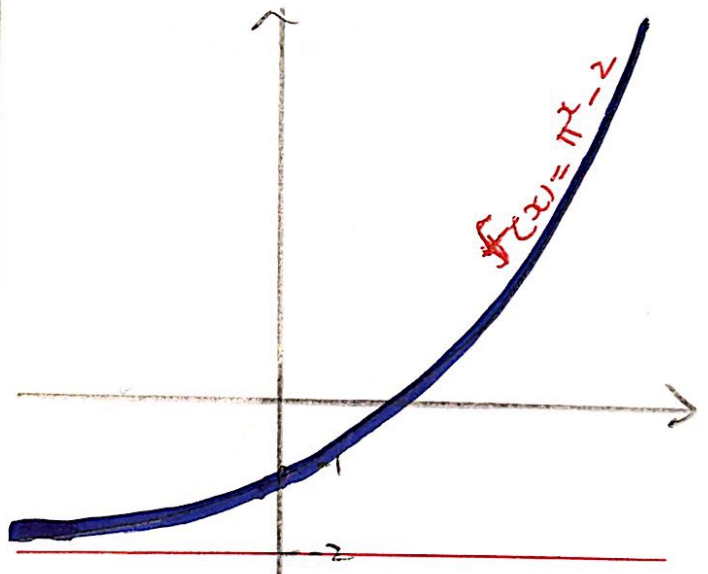
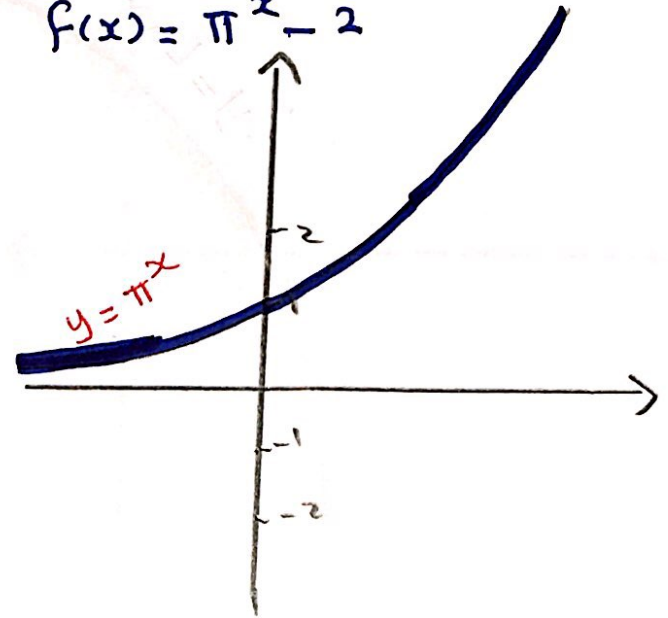


Domain: \mathbb{R}

Range: $(2, \infty)$

$f(x) = \left(\frac{1}{\pi}\right)^{-x} - 2$

$f(x) = \pi^x - 2$



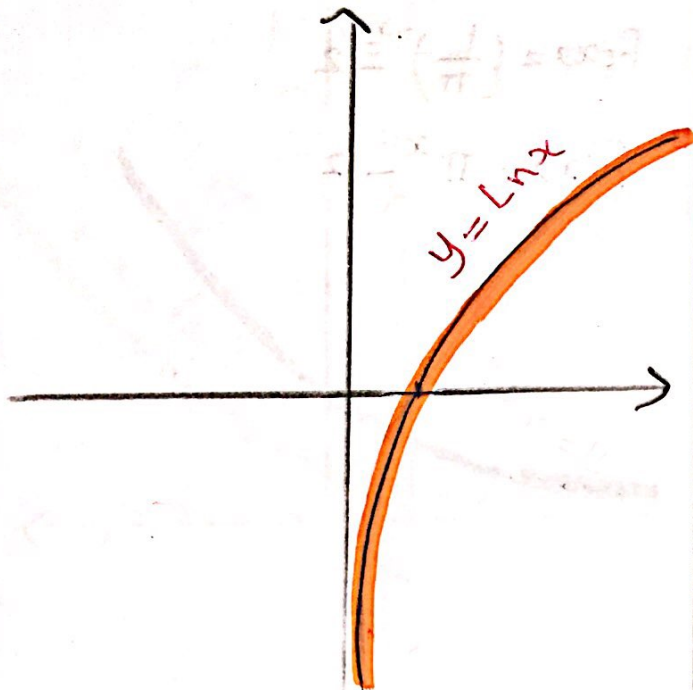
Domain: \mathbb{R}

Range: $(-2, \infty)$

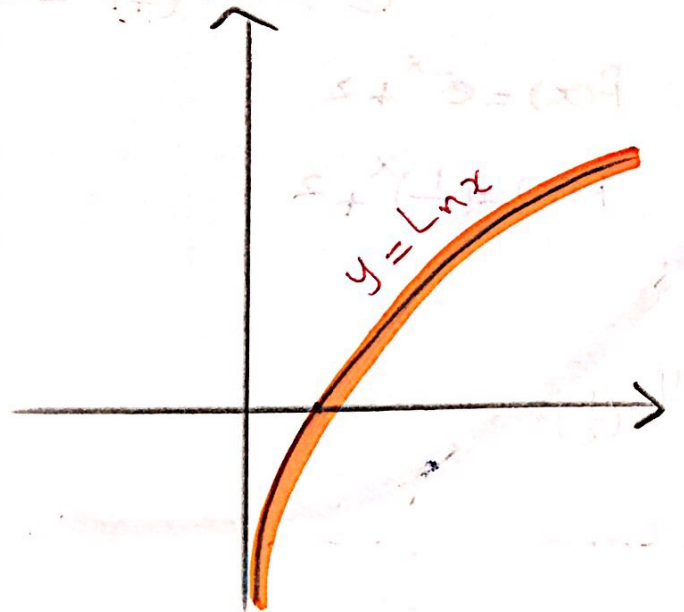
③ $f(x) = -\ln x$

④ $f(x) = \ln(-x)$

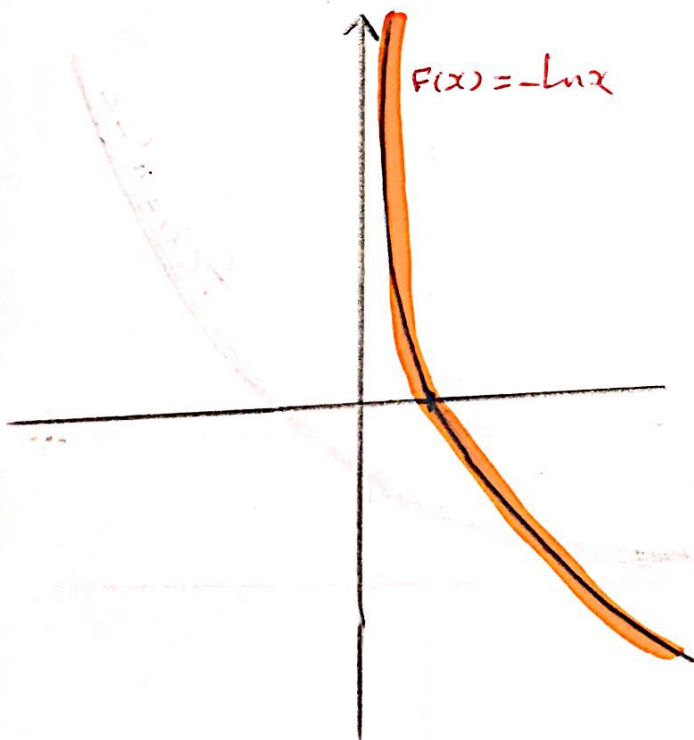
$f(x) = -\ln x$



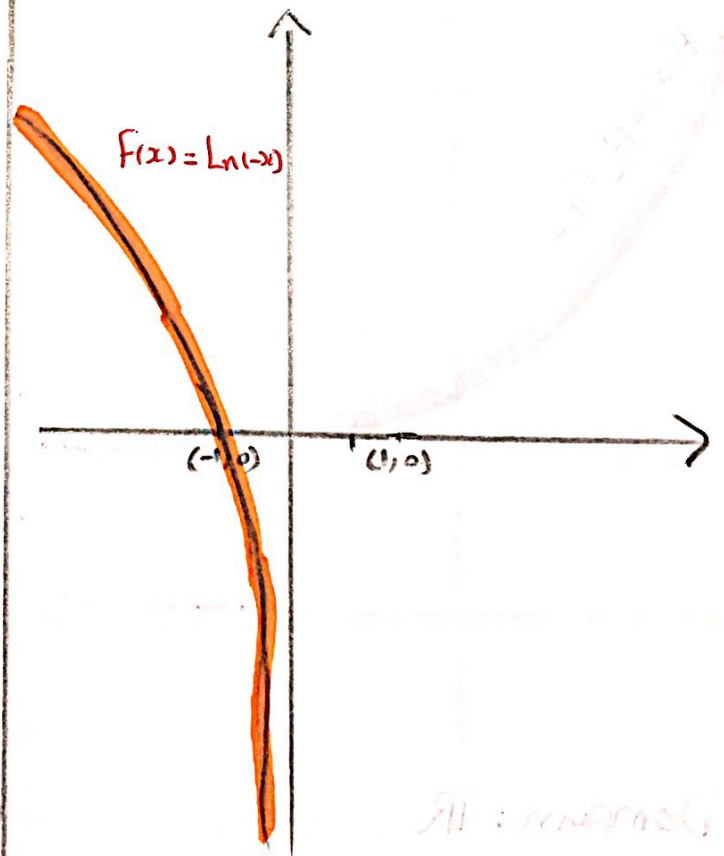
$f(x) = \ln(-x)$



$f(x) = -\ln x$



$f(x) = \ln(-x)$



Domain: $(0, \infty)$

Range: \mathbb{R}

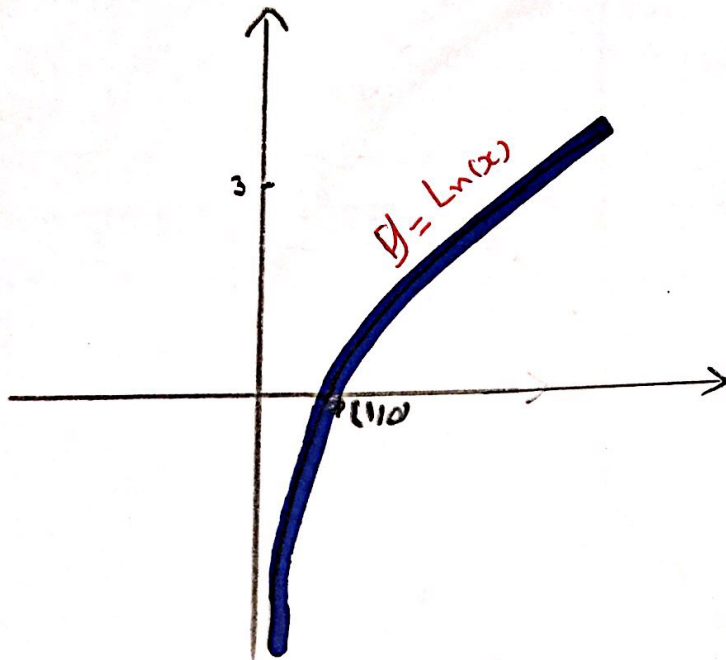
Domain: $(-\infty, 0)$

Range: \mathbb{R}

⑤ $f(x) = \ln(x) + 3$

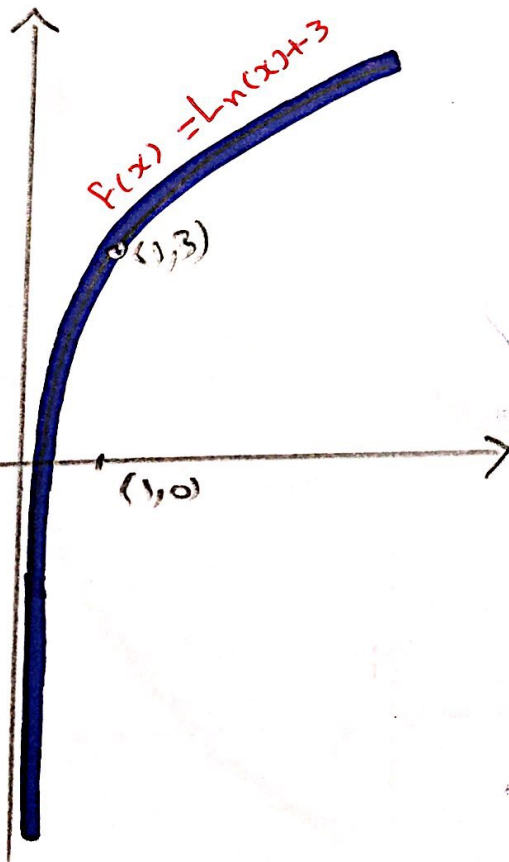
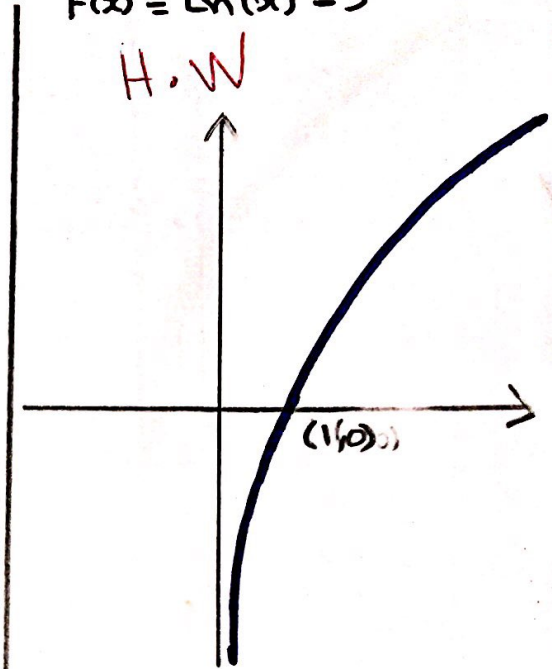
⑥ $f(x) = \ln(x) - 3$

$f(x) = \ln(x) + 3$



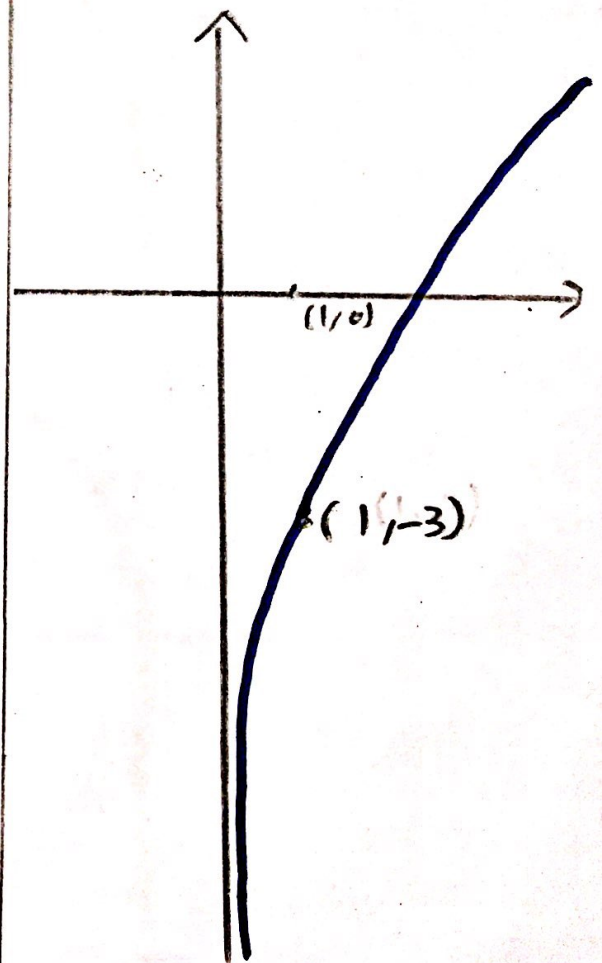
$f(x) = \ln(x) - 3$

H.W



Domain: $(0, \infty)$

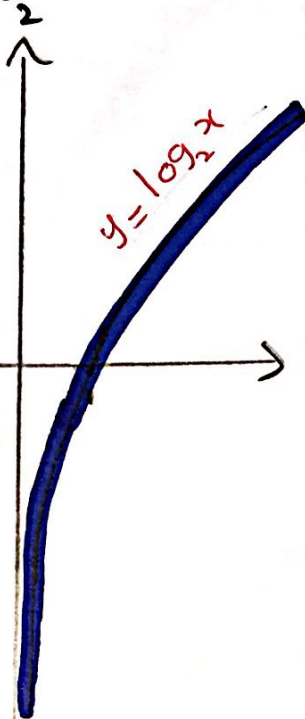
Range: \mathbb{R}



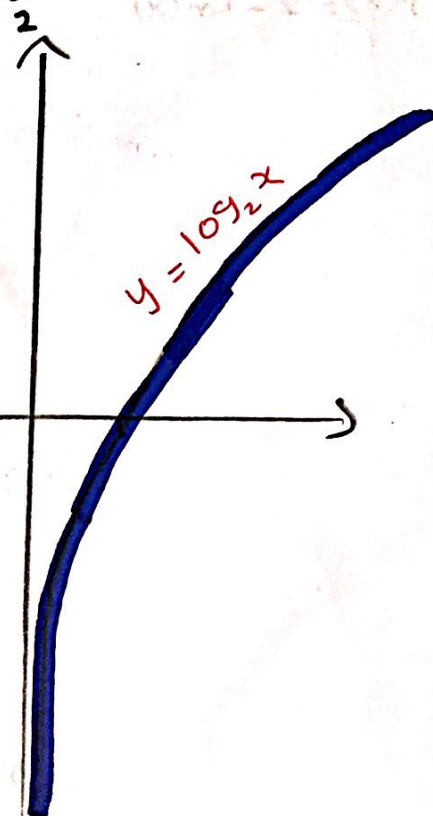
⑦ $f(x) = \log_2(x-1)$

⑧ $f(x) = \log_2(x+1)$

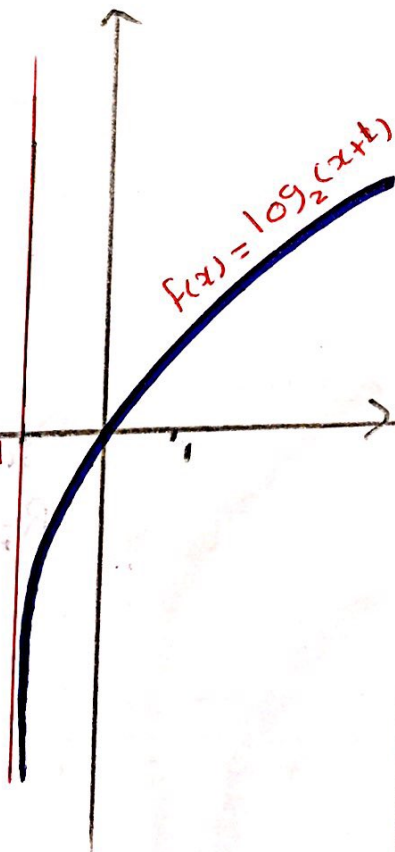
$f(x) = \log_2(x+1)$



$f(x) = \log_2(x+1)$



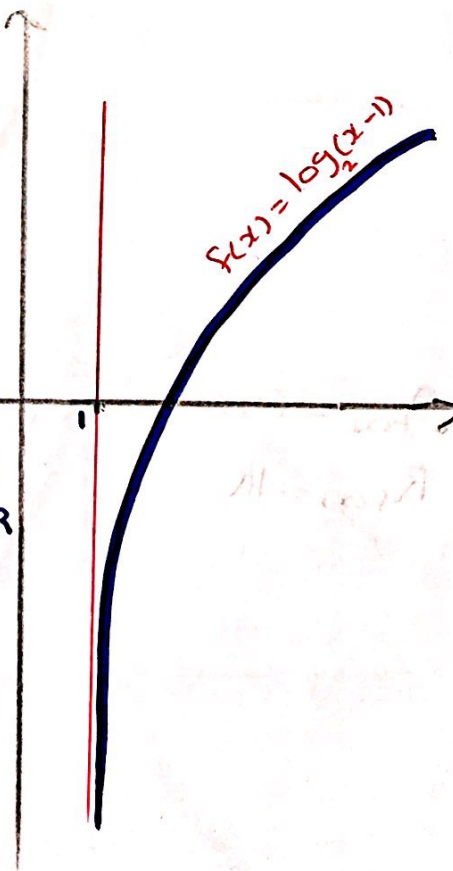
$f(x) = \log_2(x+1)$



Domain: $(-1, \infty)$

Range: \mathbb{R}

$f(x) = \log_2(x-1)$



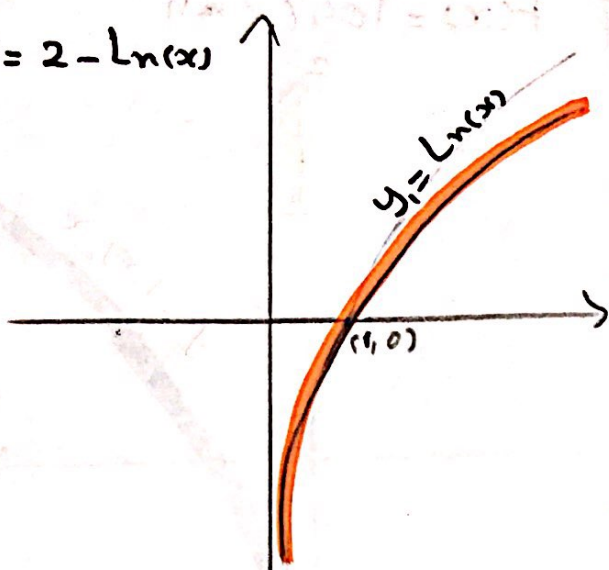
Domain: $(1, \infty)$

Range: \mathbb{R}

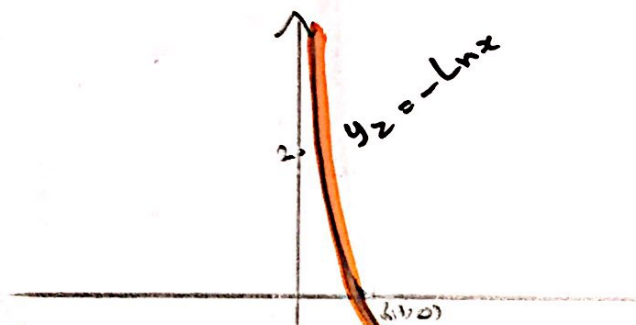
9) $f(x) = 2 - \ln(x)$

10) $f(x) = 2 + \ln(-x)$

$f(x) = 2 - \ln(x)$

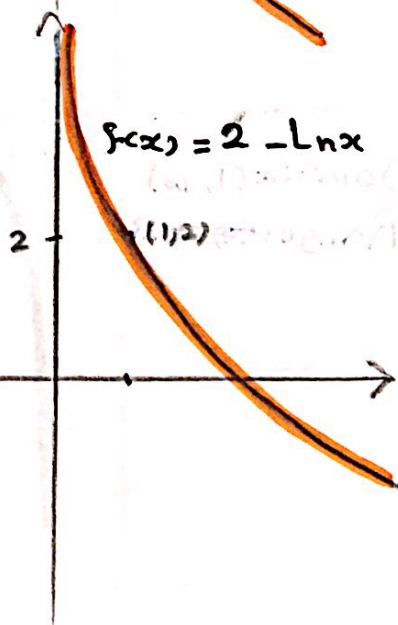


$y_2 = -\ln(x)$

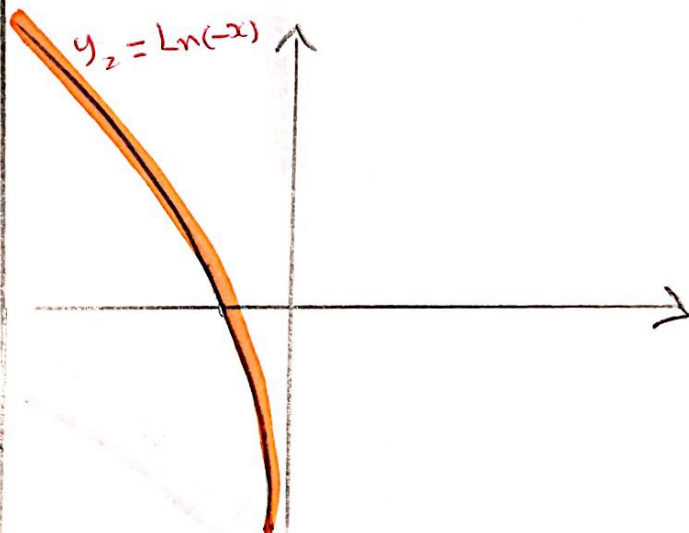
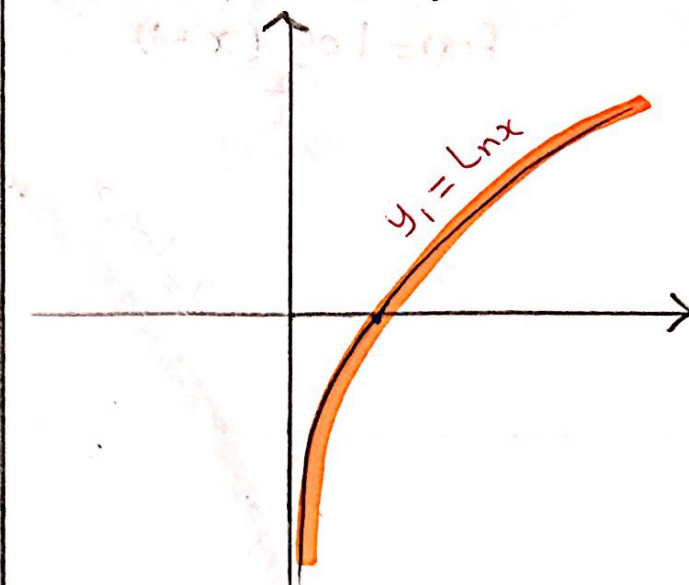


$D_{f(x)} = (0, \infty)$

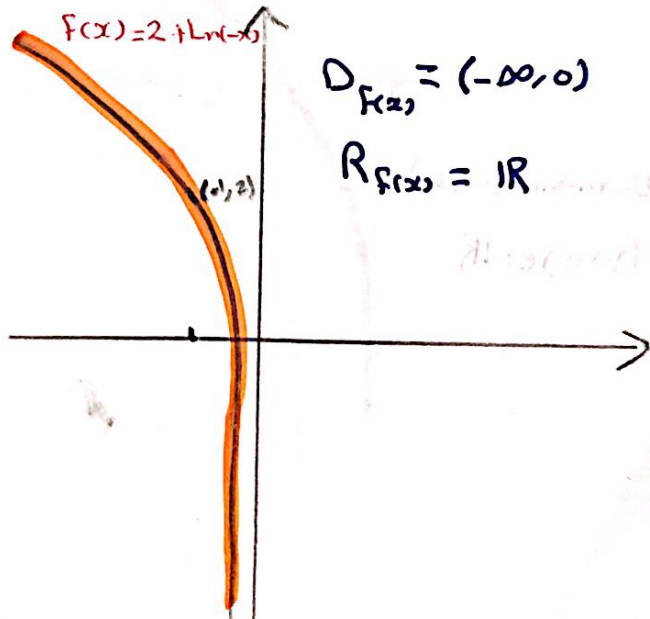
$R_{f(x)} = \mathbb{R}$



$f(x) = 2 + \ln(-x)$



$f(x) = 2 + \ln(-x)$

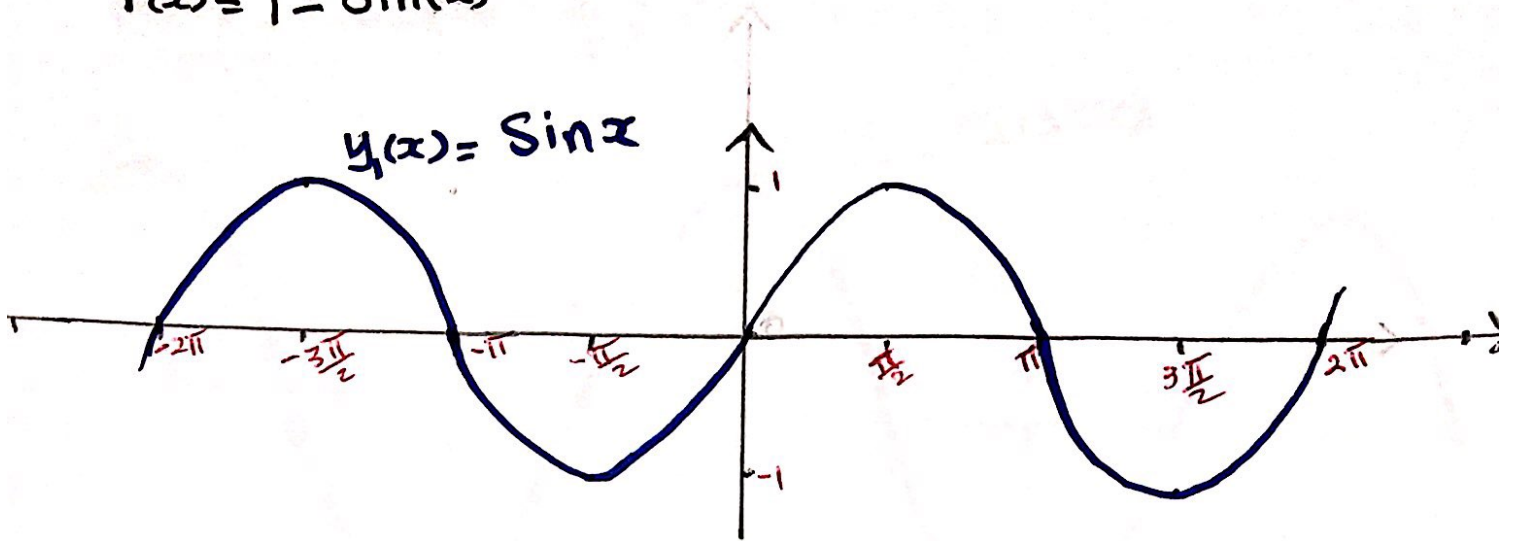


$D_{f(x)} = (-\infty, 0)$

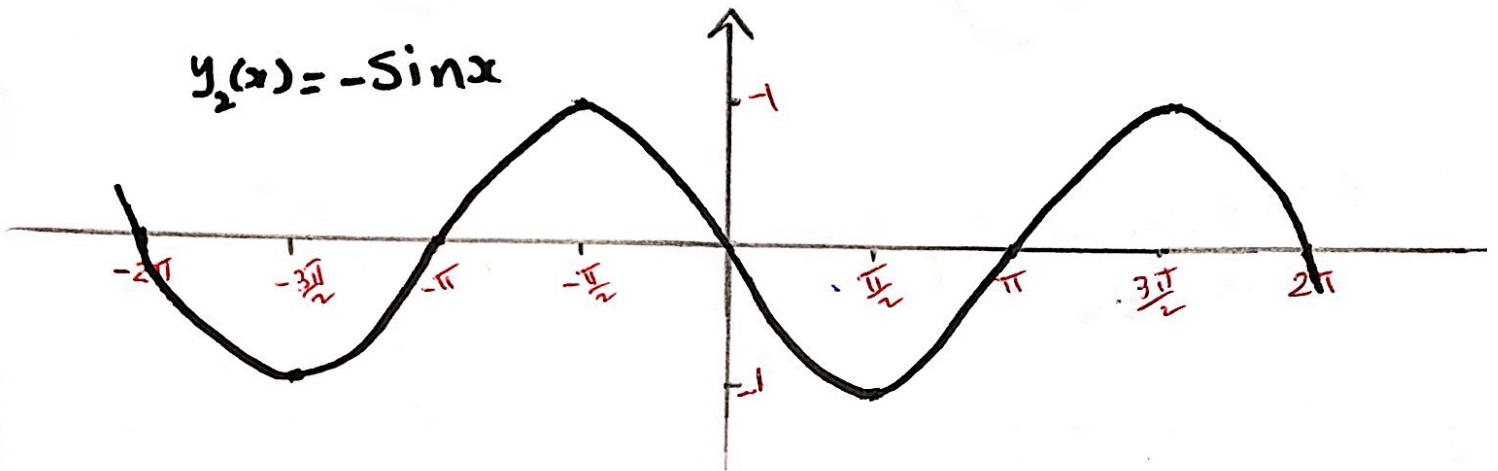
$R_{f(x)} = \mathbb{R}$

$$f(x) = 1 - \sin(x)$$

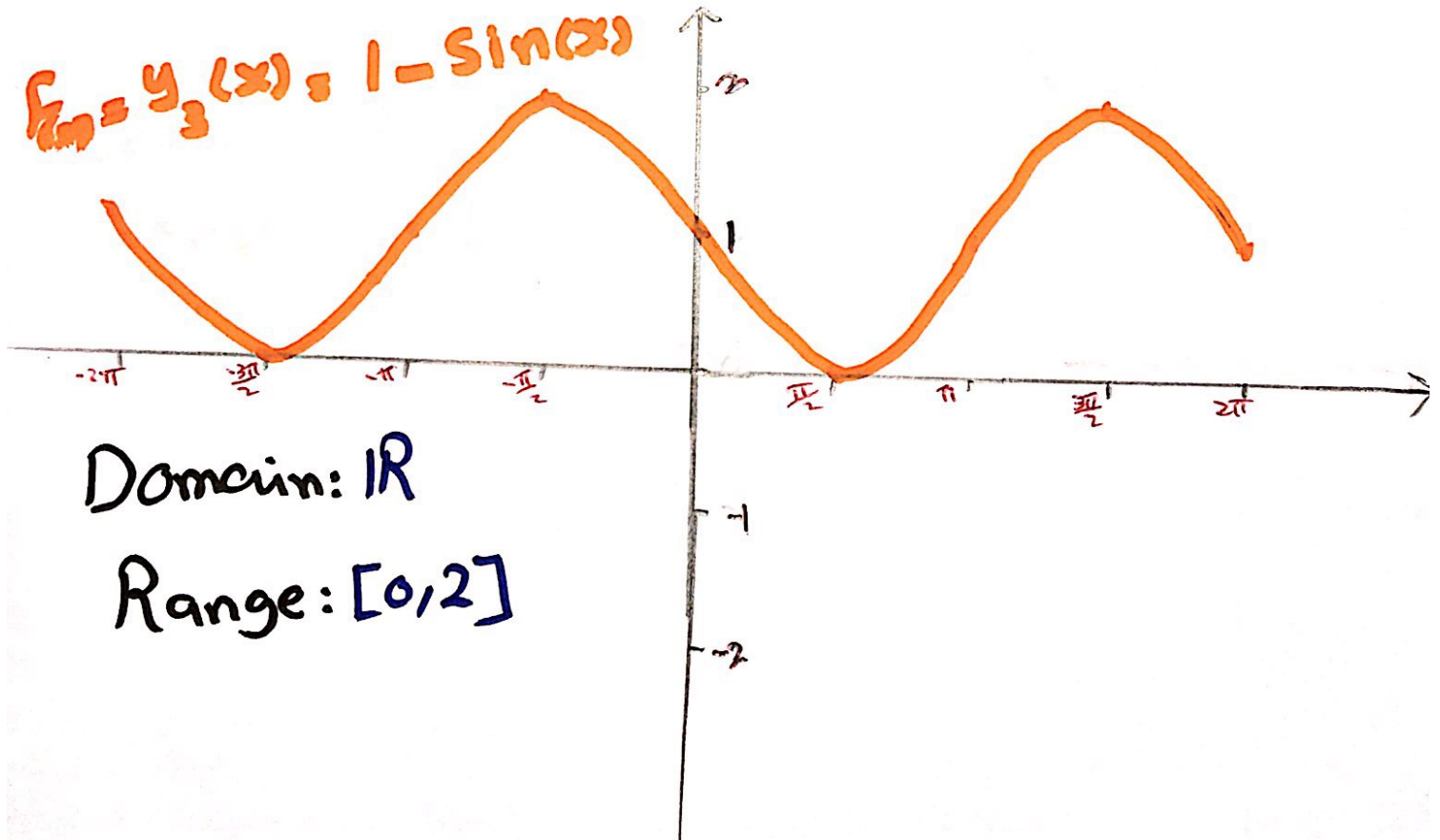
$$y_1(x) = \sin x$$



$$y_2(x) = -\sin x$$



$$f(x) = y_3(x) = 1 - \sin(x)$$



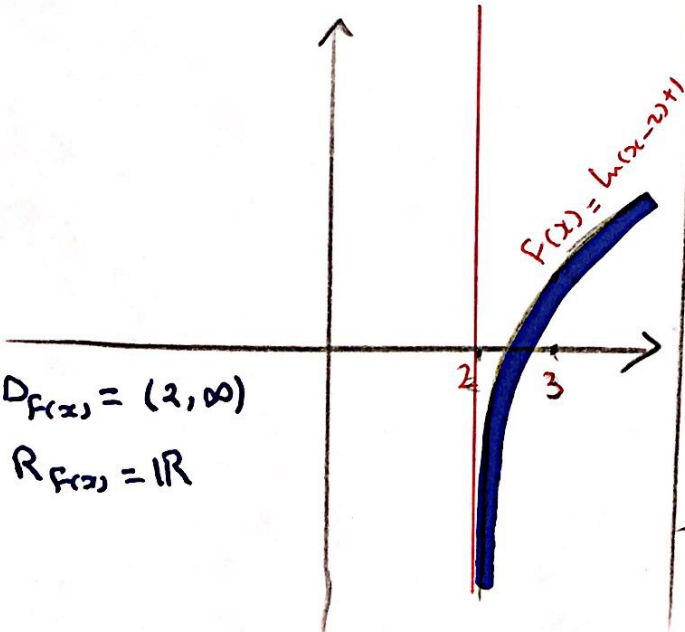
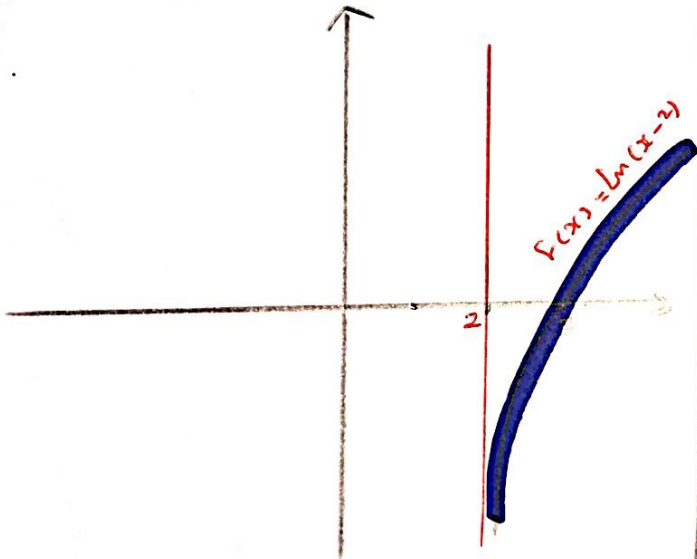
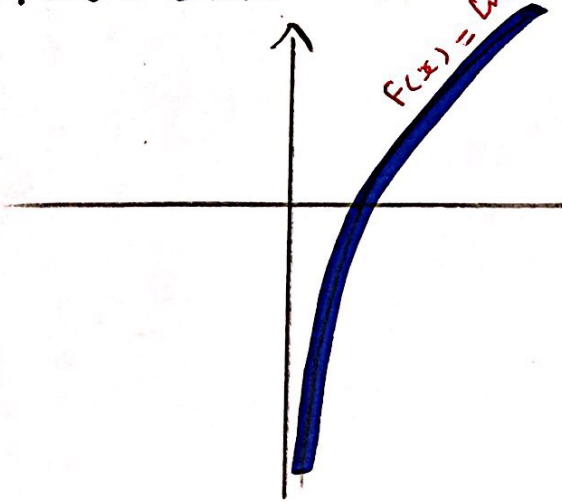
Domain: \mathbb{R}

Range: $[0, 2]$

⑪ $f(x) = \ln(x-2) + 1$

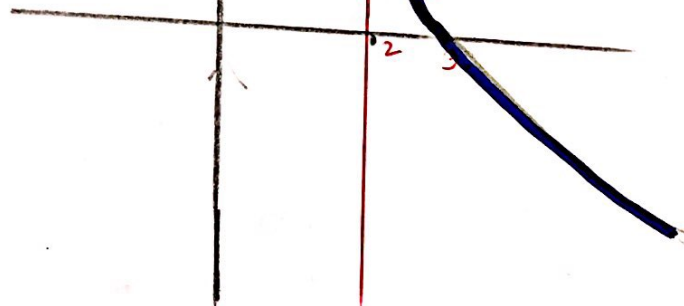
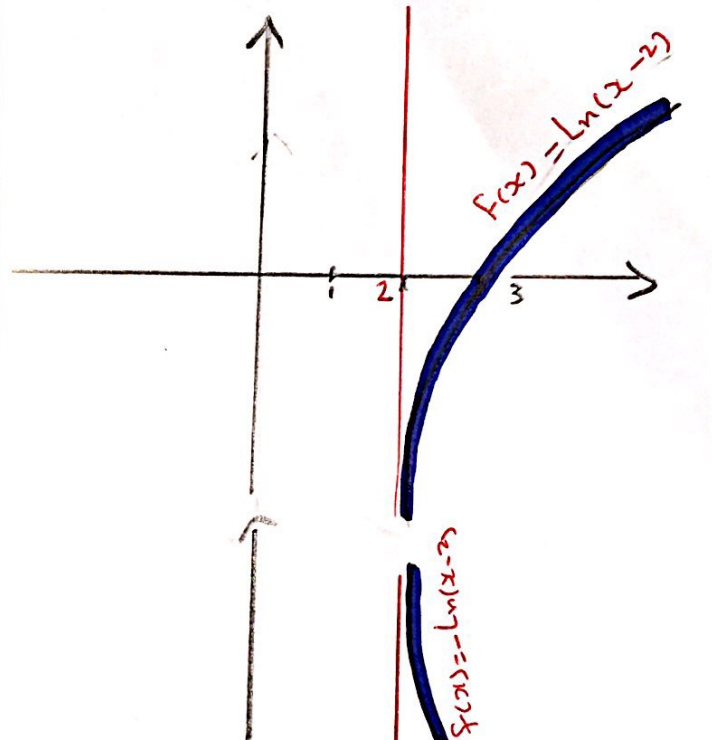
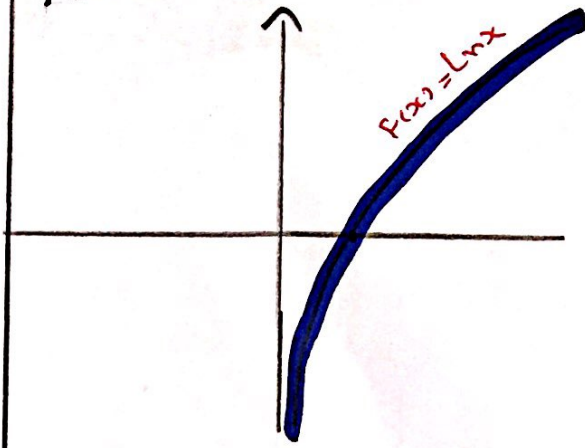
⑫ $f(x) = 1 - \ln(x-2)$

$f(x) = \ln(x-2) + 1$

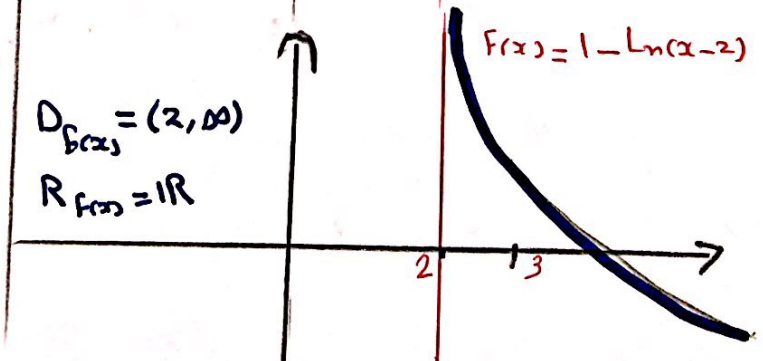


$D_{f(x)} = (2, \infty)$
 $R_{f(x)} = \mathbb{R}$

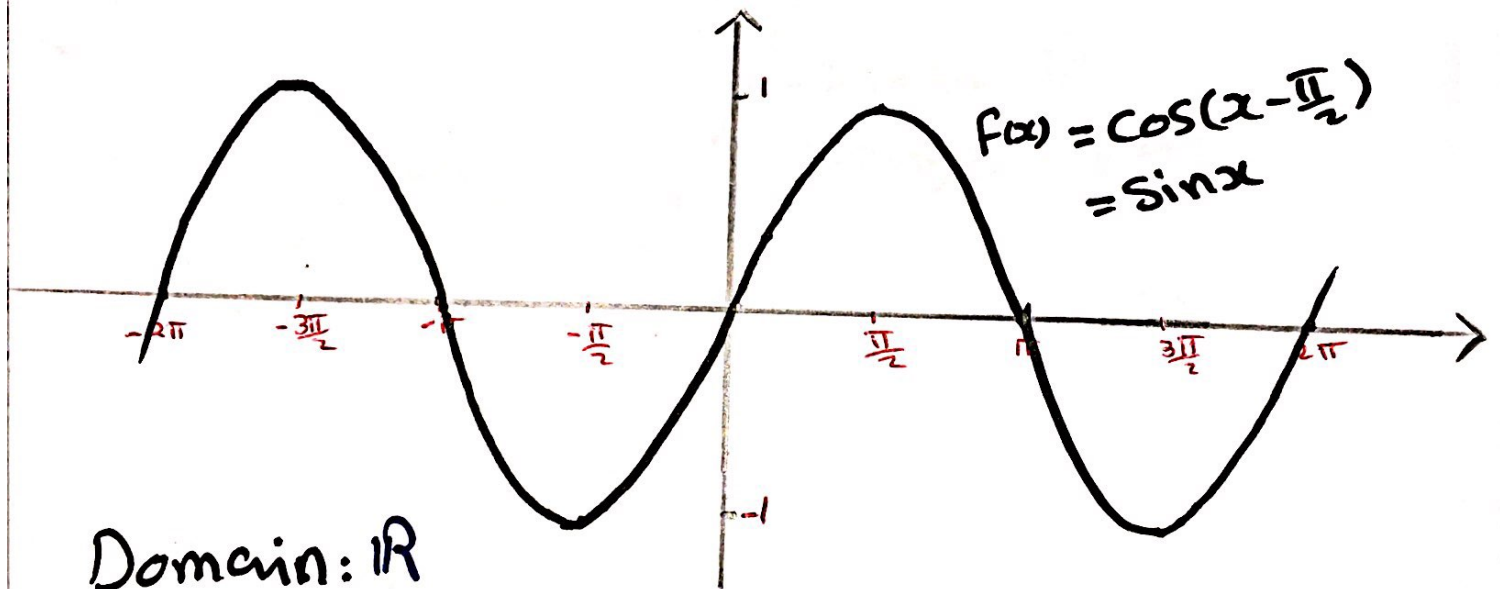
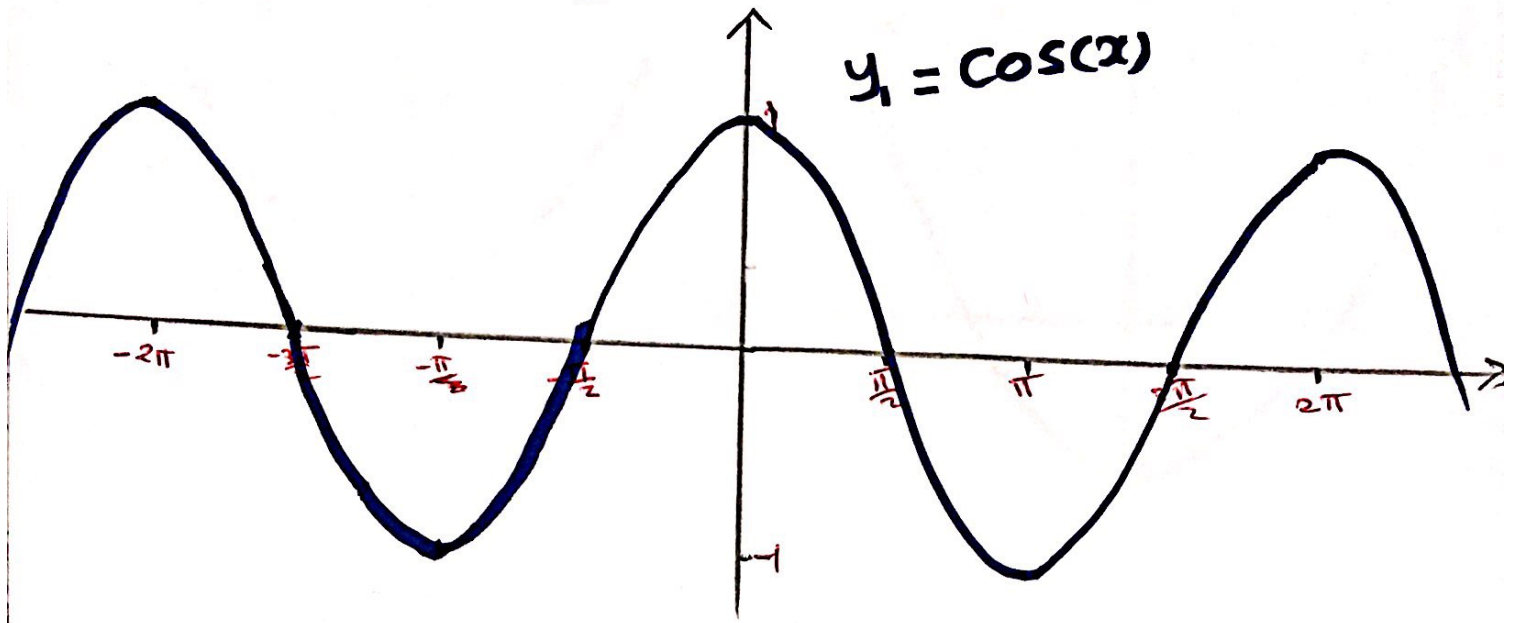
$f(x) = 1 - \ln(x-2)$



$D_{f(x)} = (2, \infty)$
 $R_{f(x)} = \mathbb{R}$



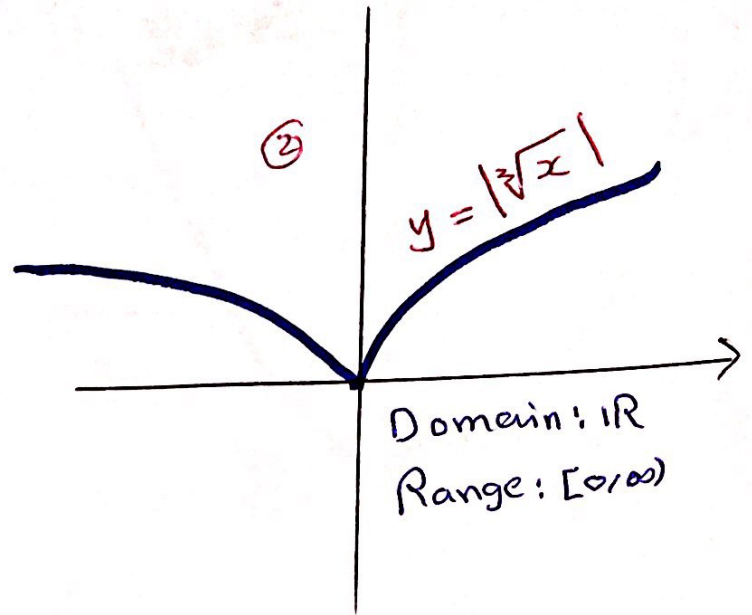
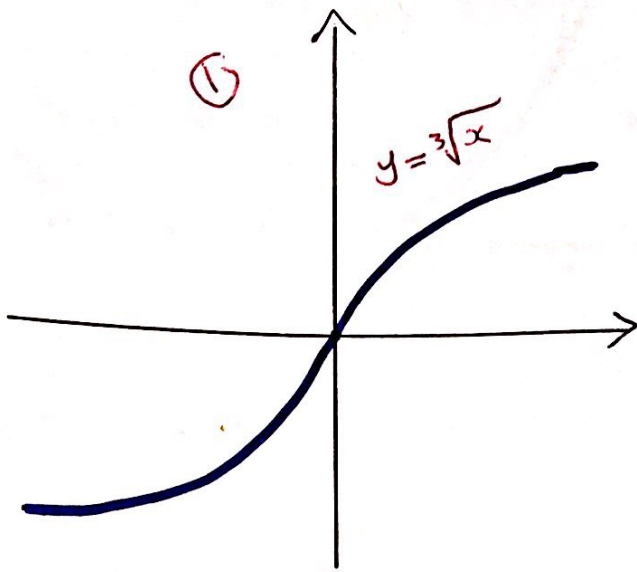
$$f(x) = \cos\left(x - \frac{\pi}{2}\right)$$



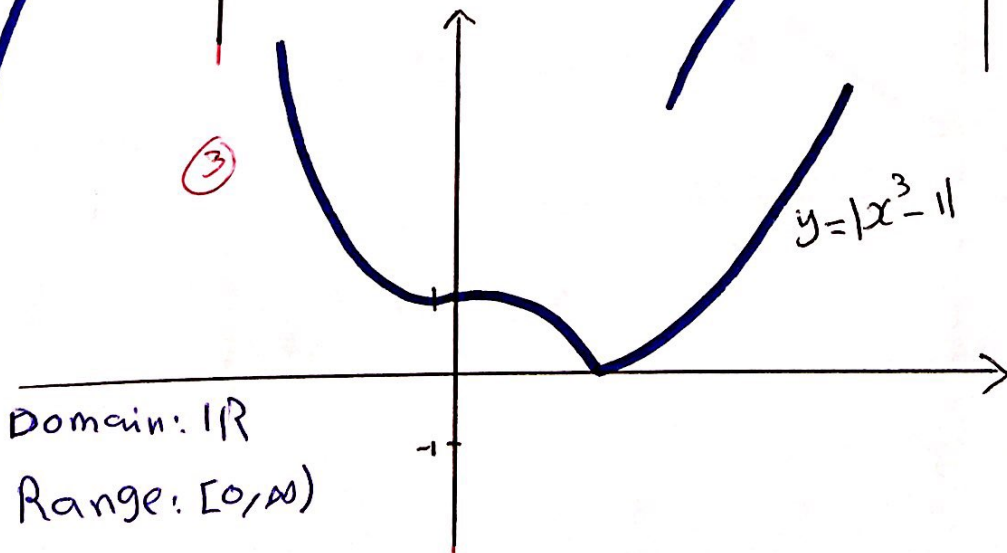
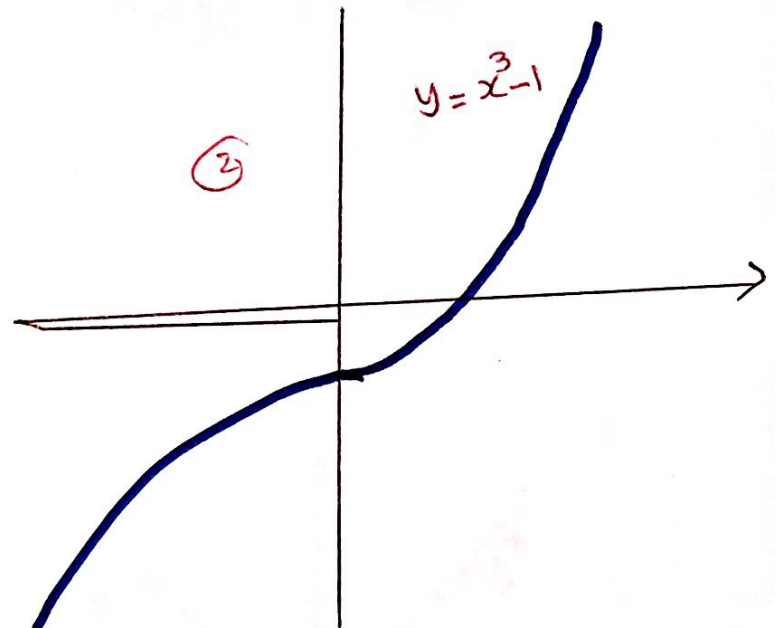
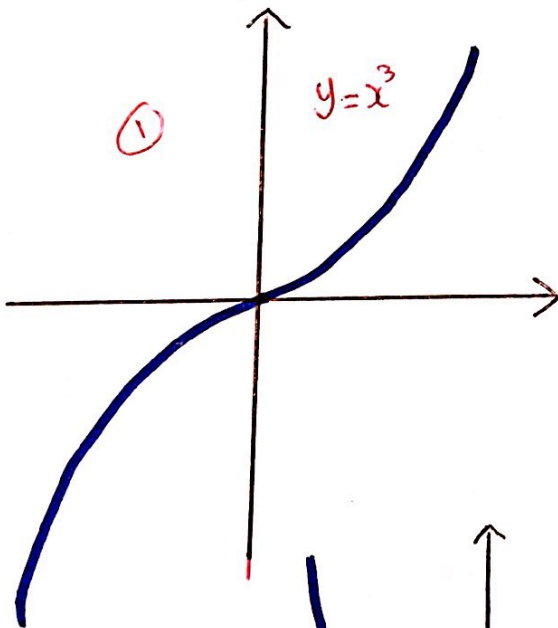
Domain: \mathbb{R}

Range: $[-1, 1]$

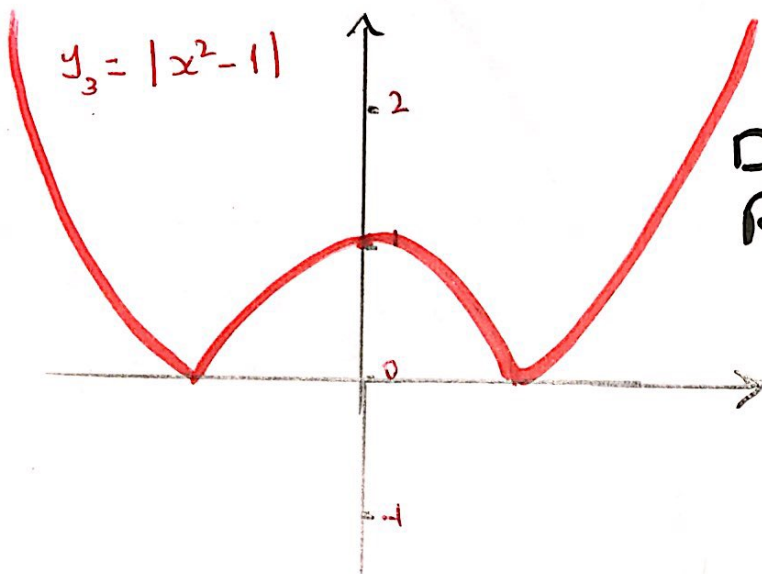
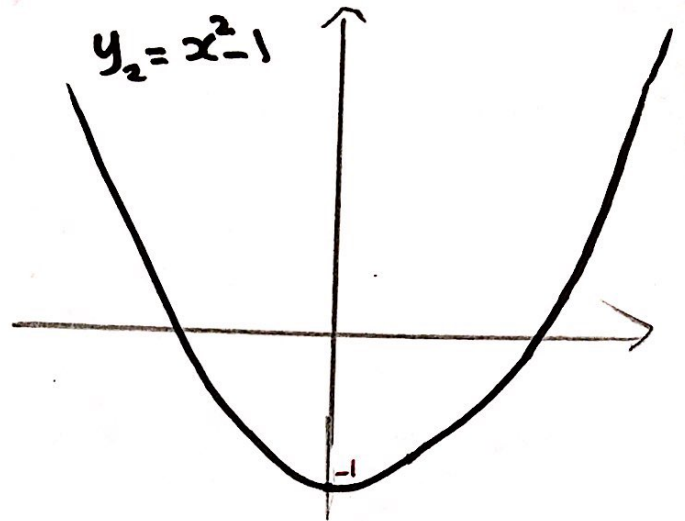
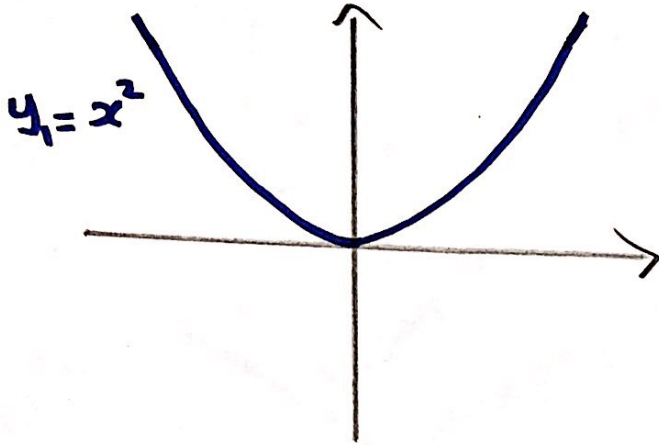
$$f(x) = |\sqrt[3]{x}|$$



$$f(x) = |x^3 - 1|$$



$$f(x) = |x^2 - 1|$$

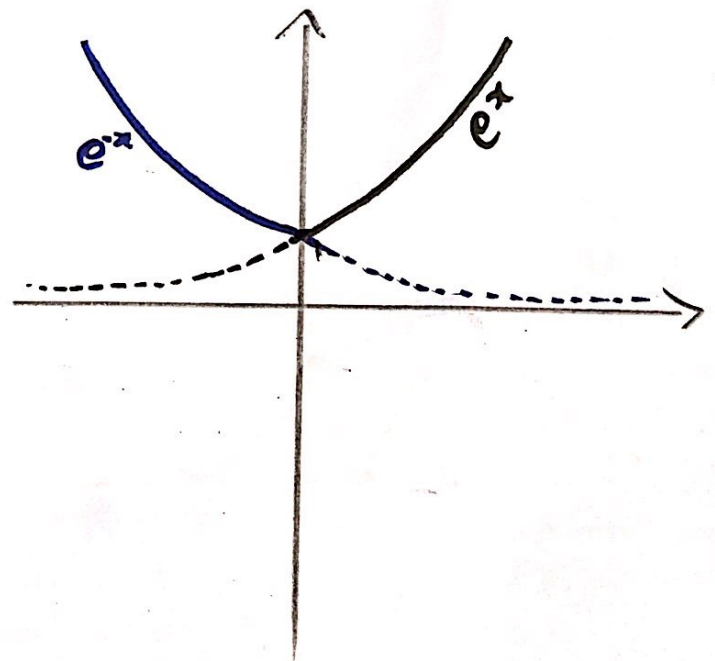


Domain: \mathbb{R}
Range: $[0, \infty)$

$$f(x) = e^{|x|}$$

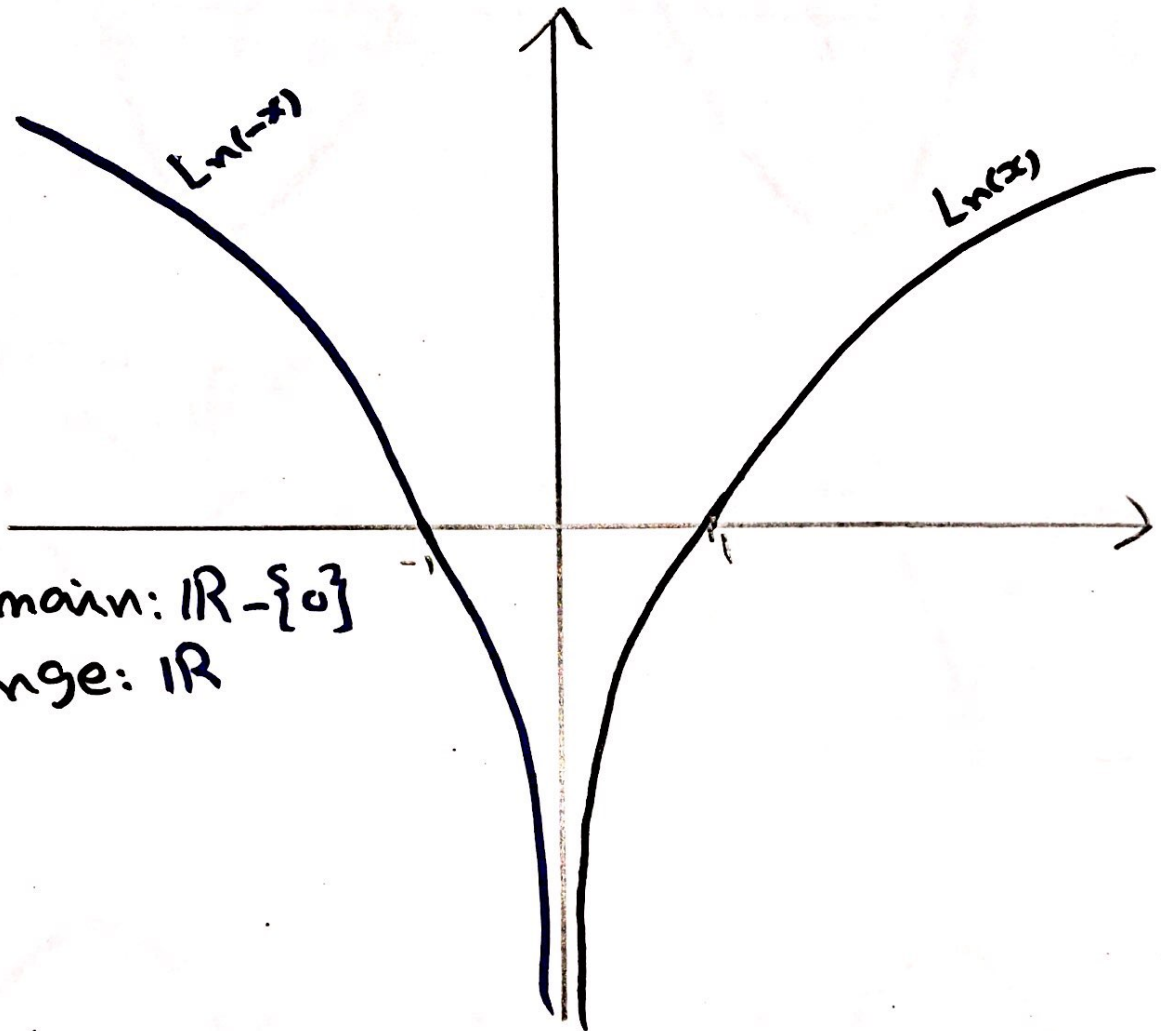
$$f(x) = \begin{cases} e^x & \text{if } x \geq 0 \\ e^{-x} & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} e^x & \text{if } x \geq 0 \\ \left(\frac{1}{e}\right)^x & \text{if } x < 0 \end{cases}$$



$$f(x) = \ln|x|$$

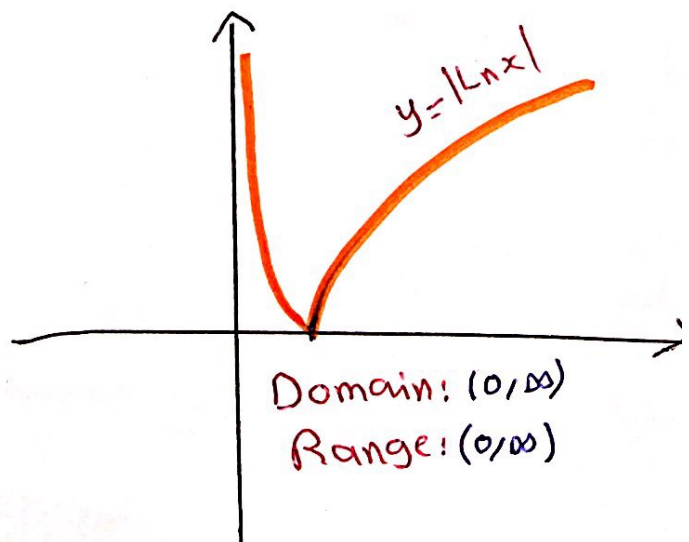
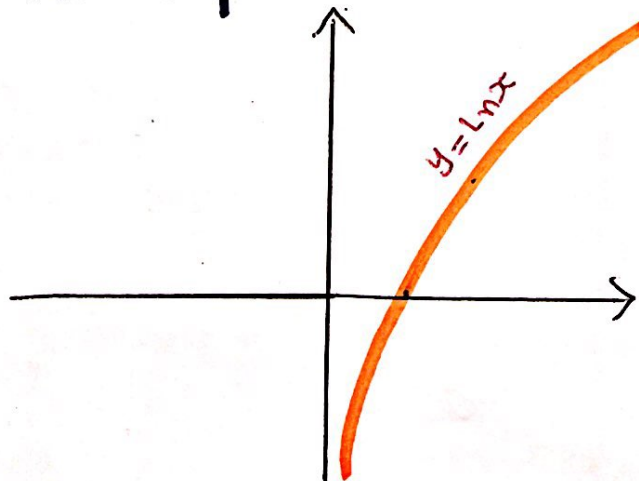
$$= \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}$$



Domain: $\mathbb{R} - \{0\}$
Range: \mathbb{R}

H.W

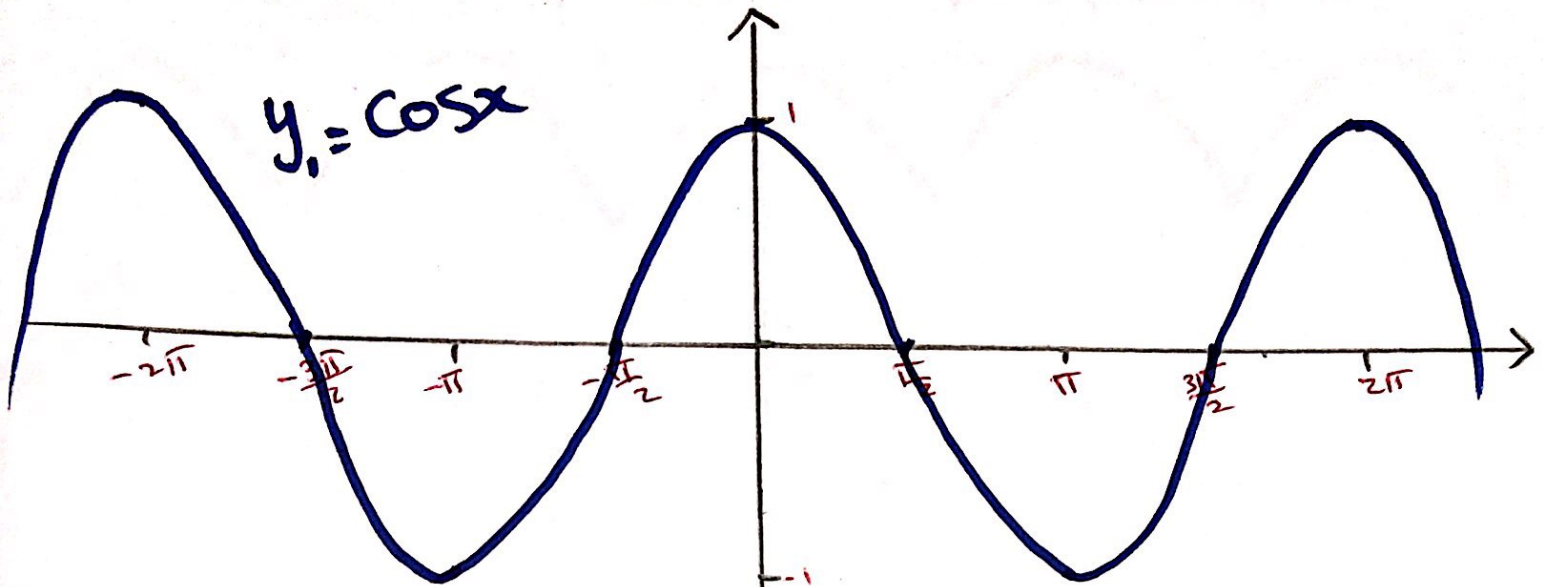
$$f(x) = |\ln(x)|$$



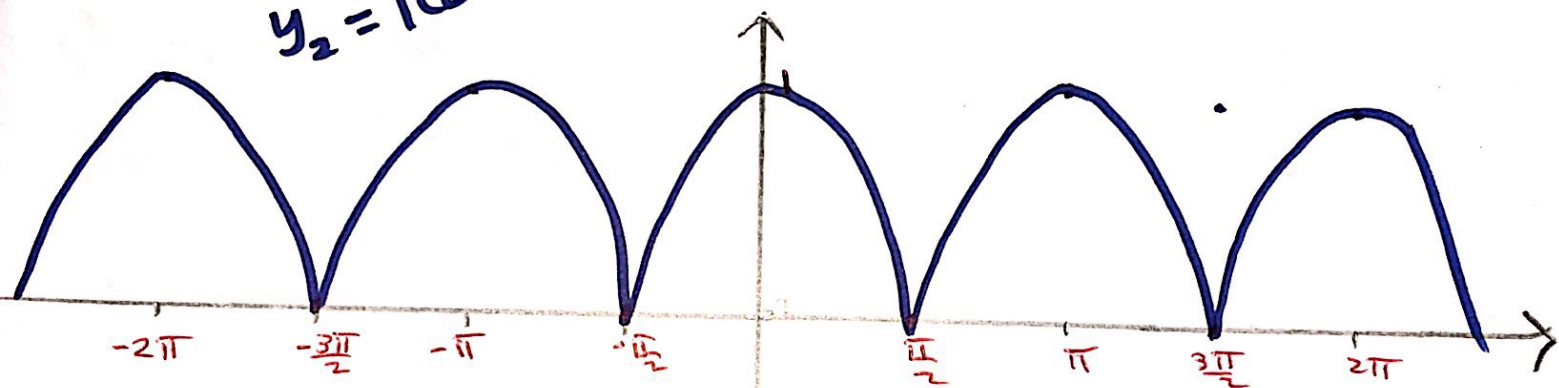
Domain: $(0, \infty)$
Range: $(0, \infty)$

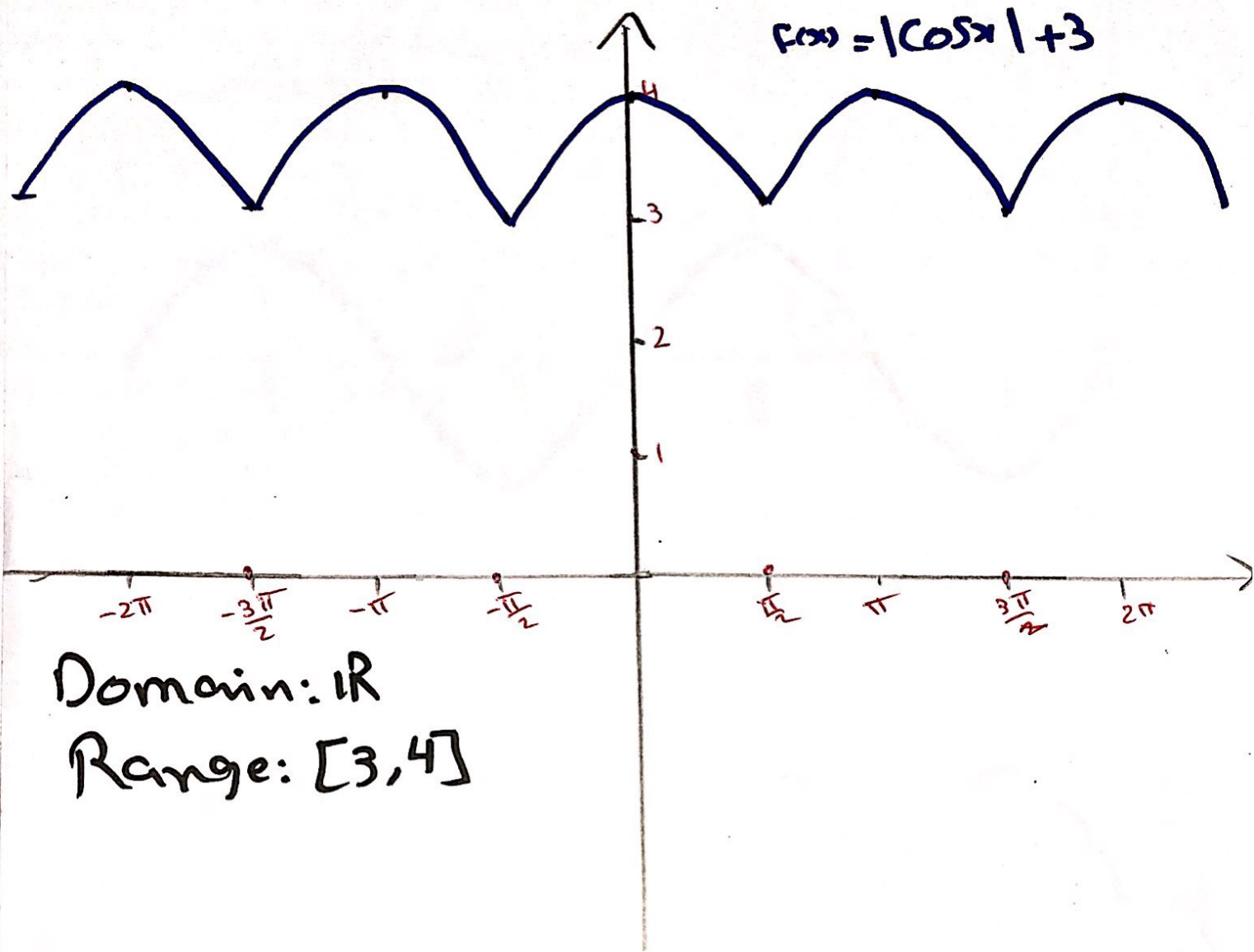
$$f(x) = |\cos x| + 3$$

$$y_1 = \cos x$$



$$y_2 = |\cos x|$$





Domain: \mathbb{R}

Range: $[3, 4]$

H.W

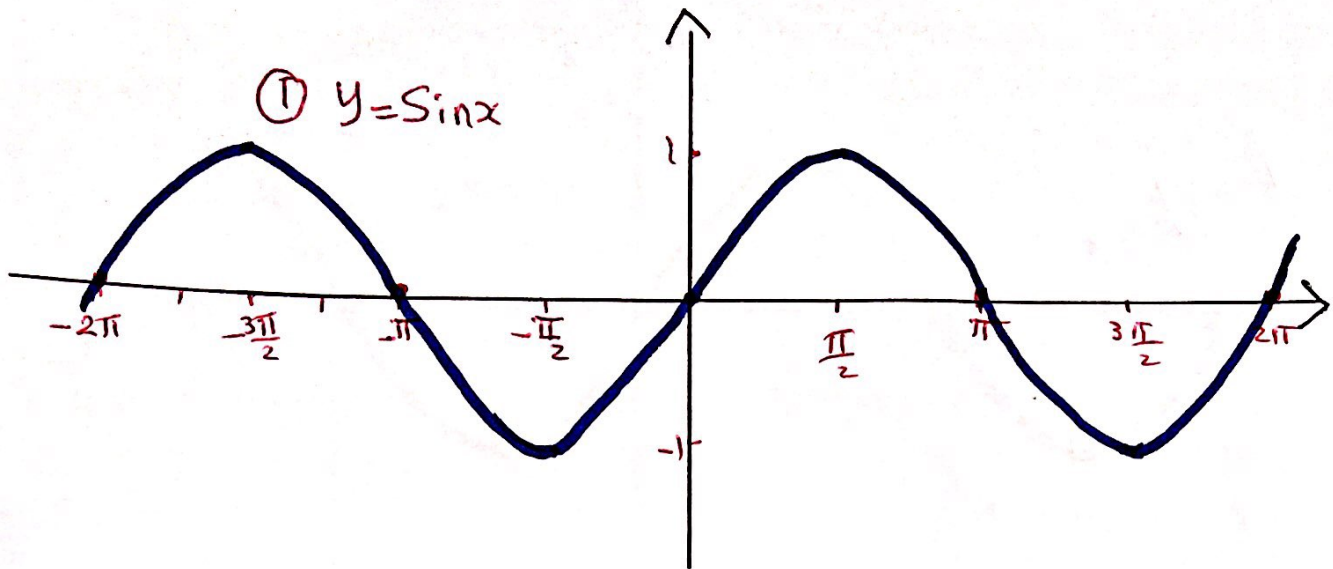
$$f(x) = |\sin x| + 2$$

$$f(x) = \sin\left(x - \frac{\pi}{2}\right)$$

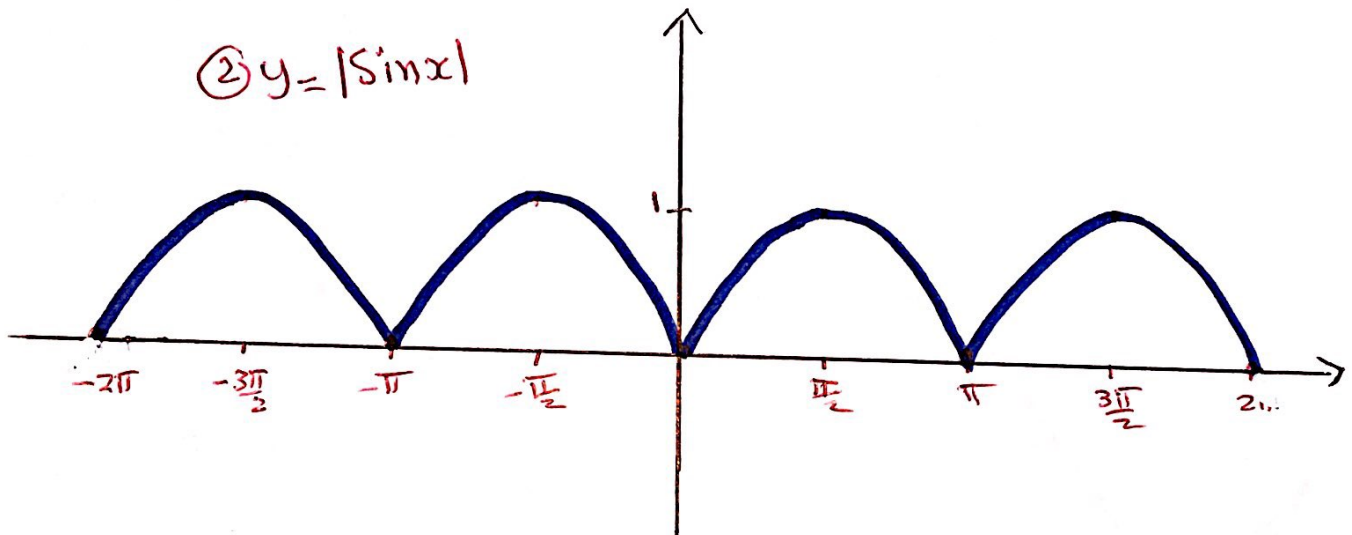
$$f(x) = 3 + \cos x$$

$$f(x) = |\sin x| + 2$$

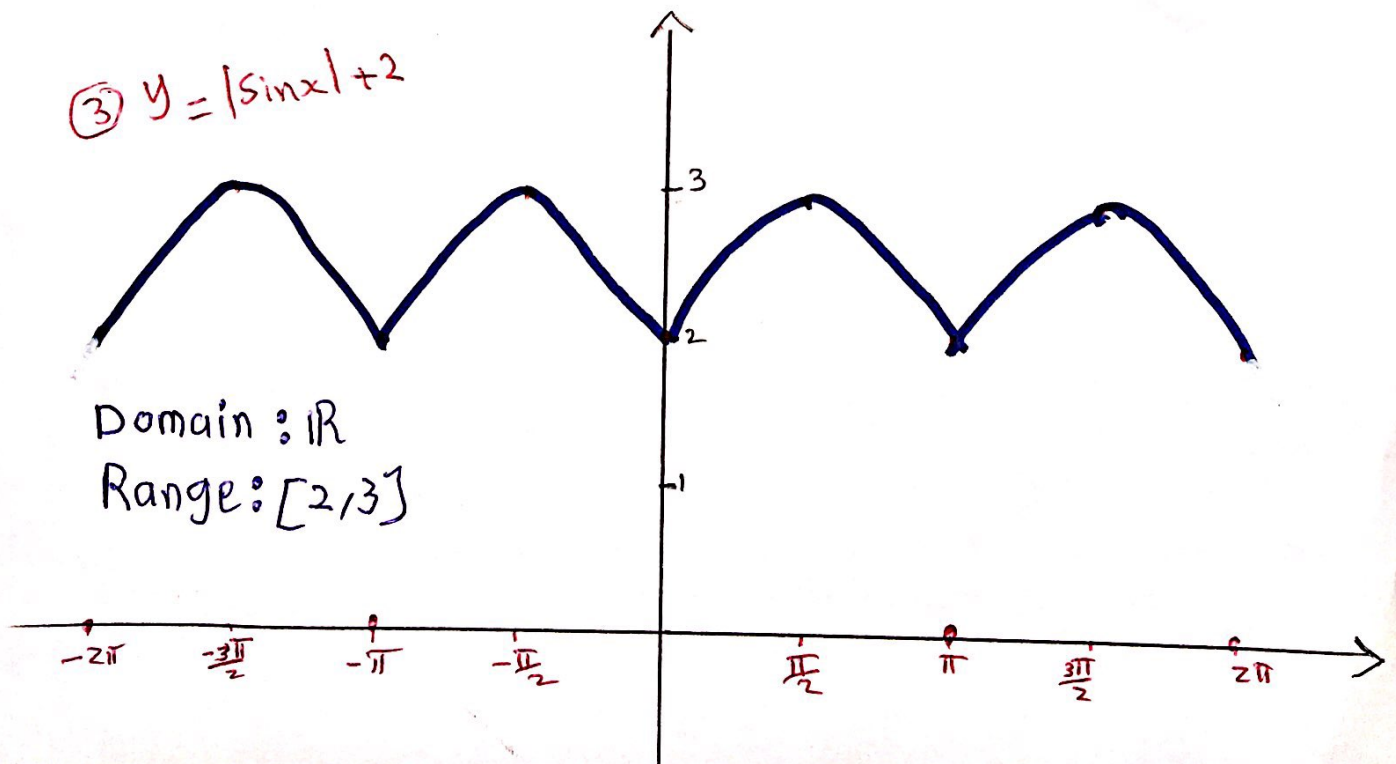
① $y = \sin x$



② $y = |\sin x|$

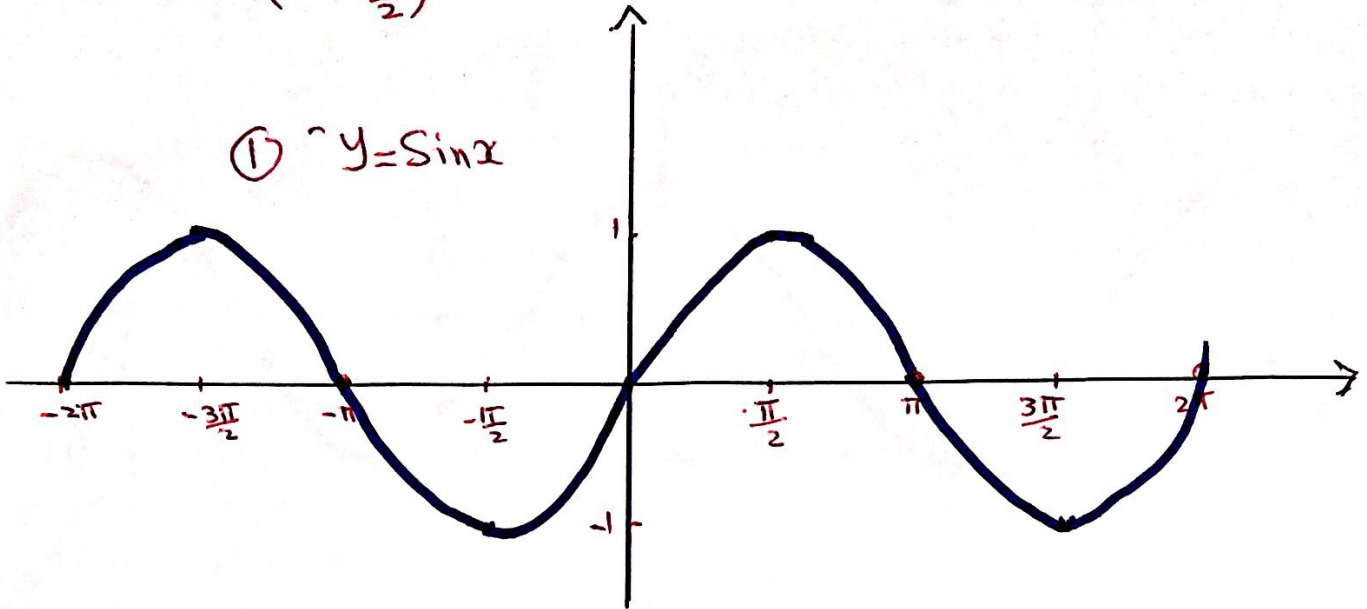


③ $y = |\sin x| + 2$

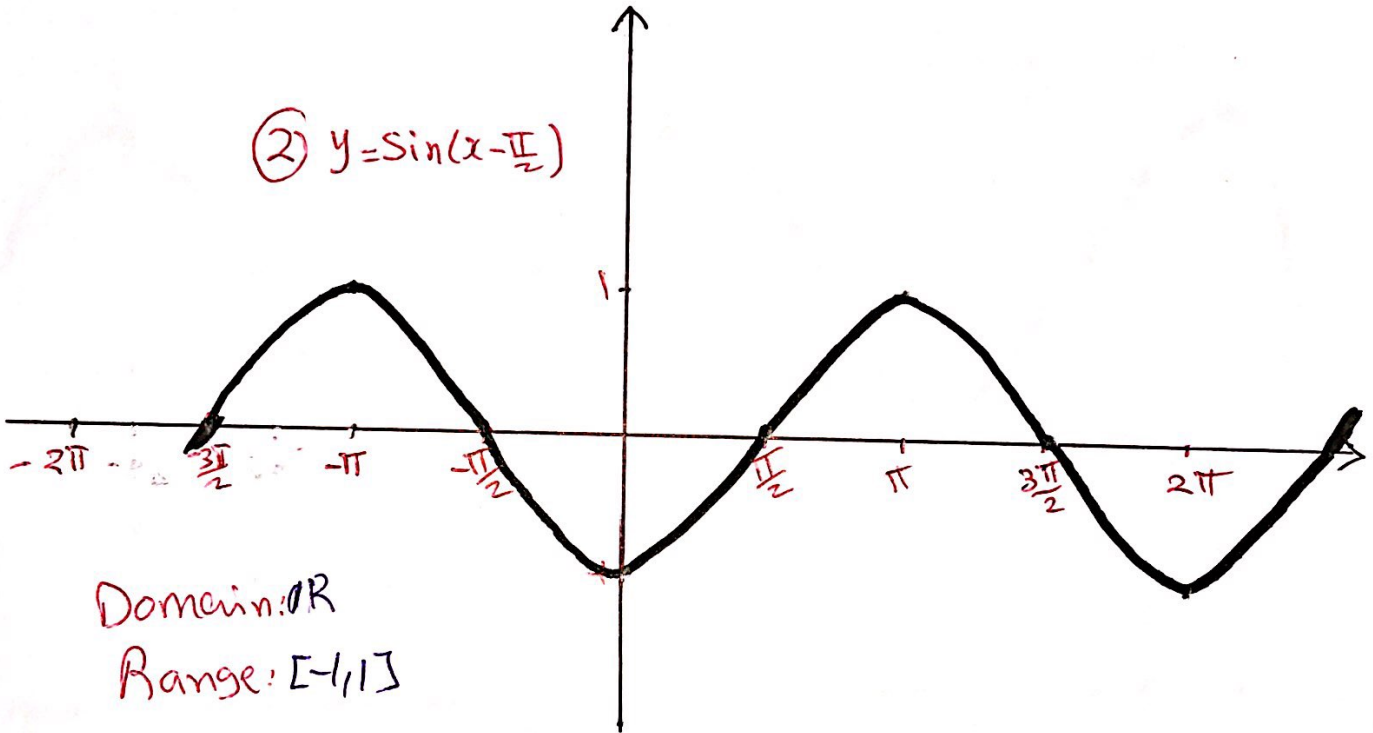


$$f(x) = \sin(x - \frac{\pi}{2})$$

① $y = \sin x$



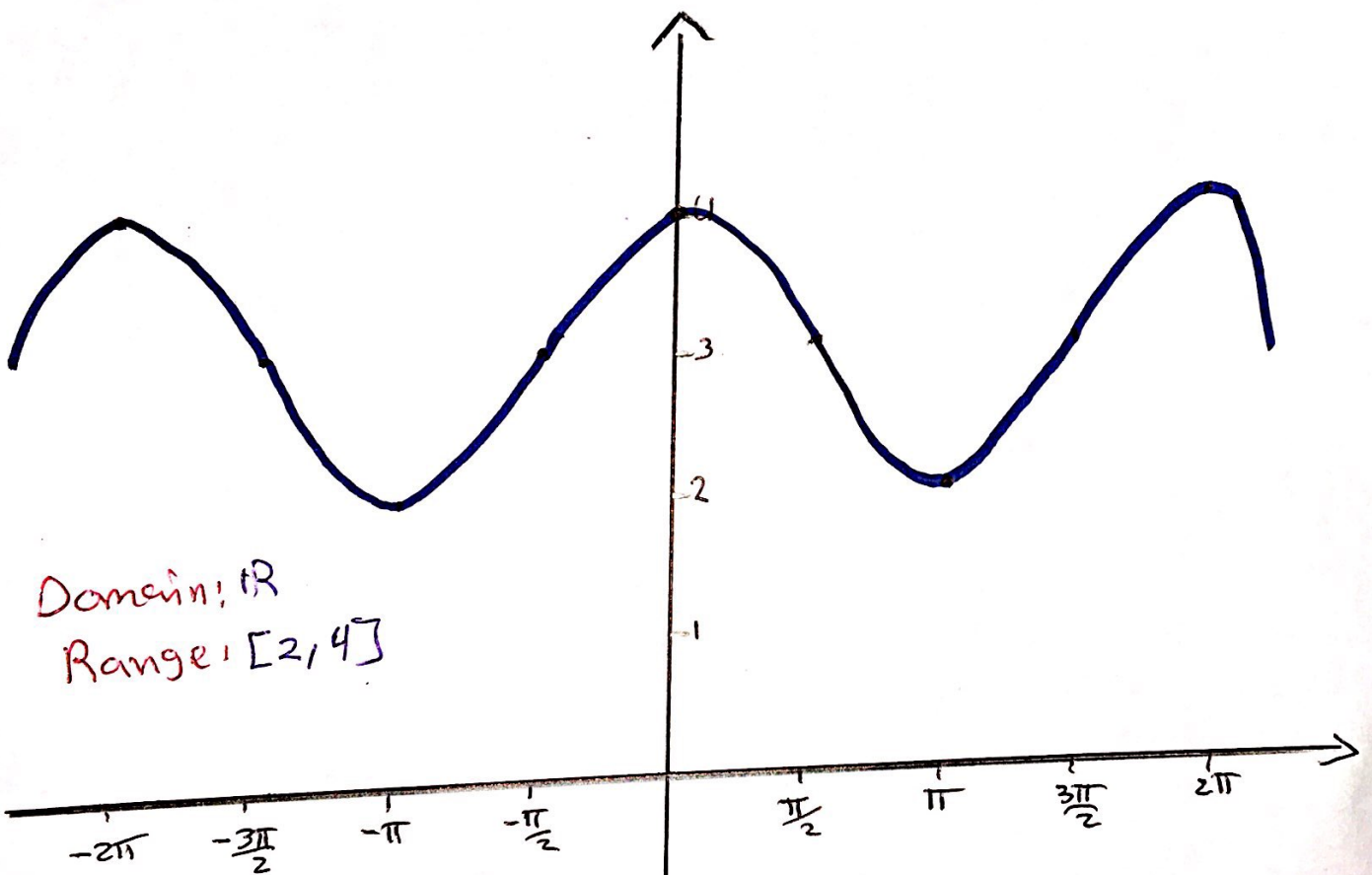
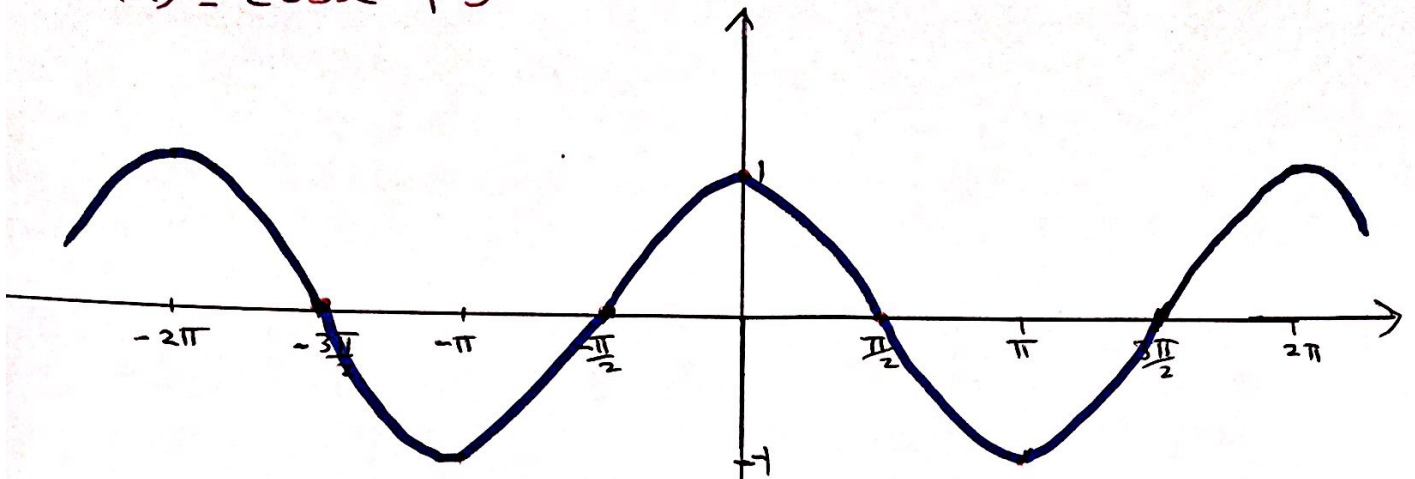
② $y = \sin(x - \frac{\pi}{2})$



Domain: \mathbb{R}

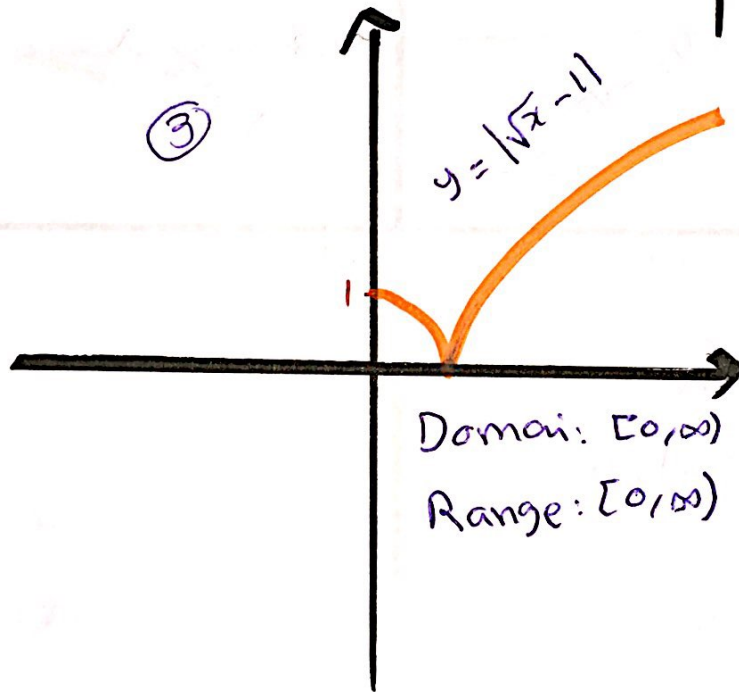
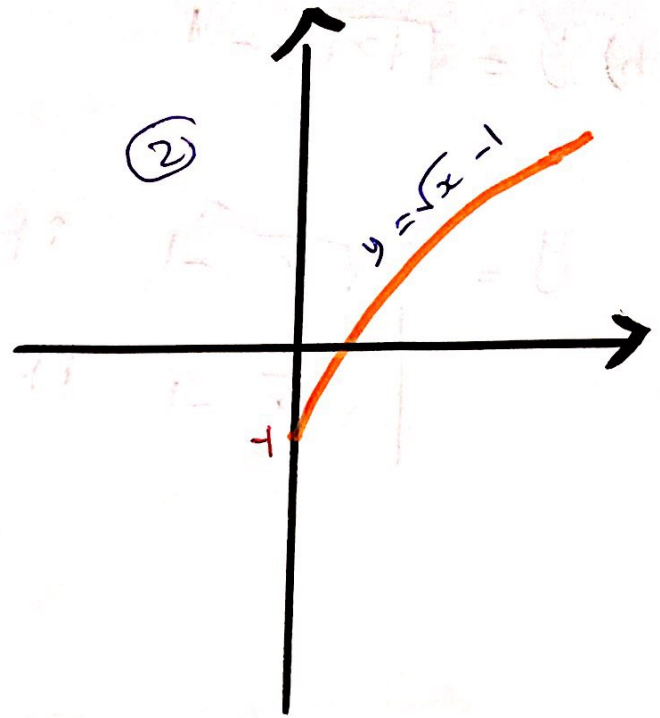
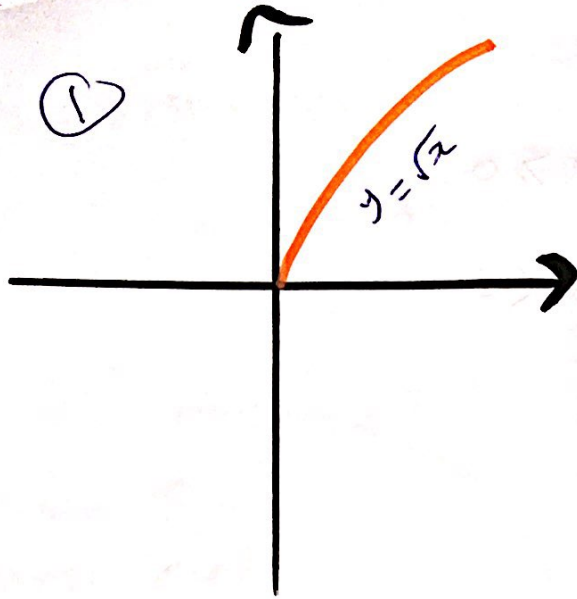
Range: $[-1, 1]$

$$f(x) = \cos x + 3$$



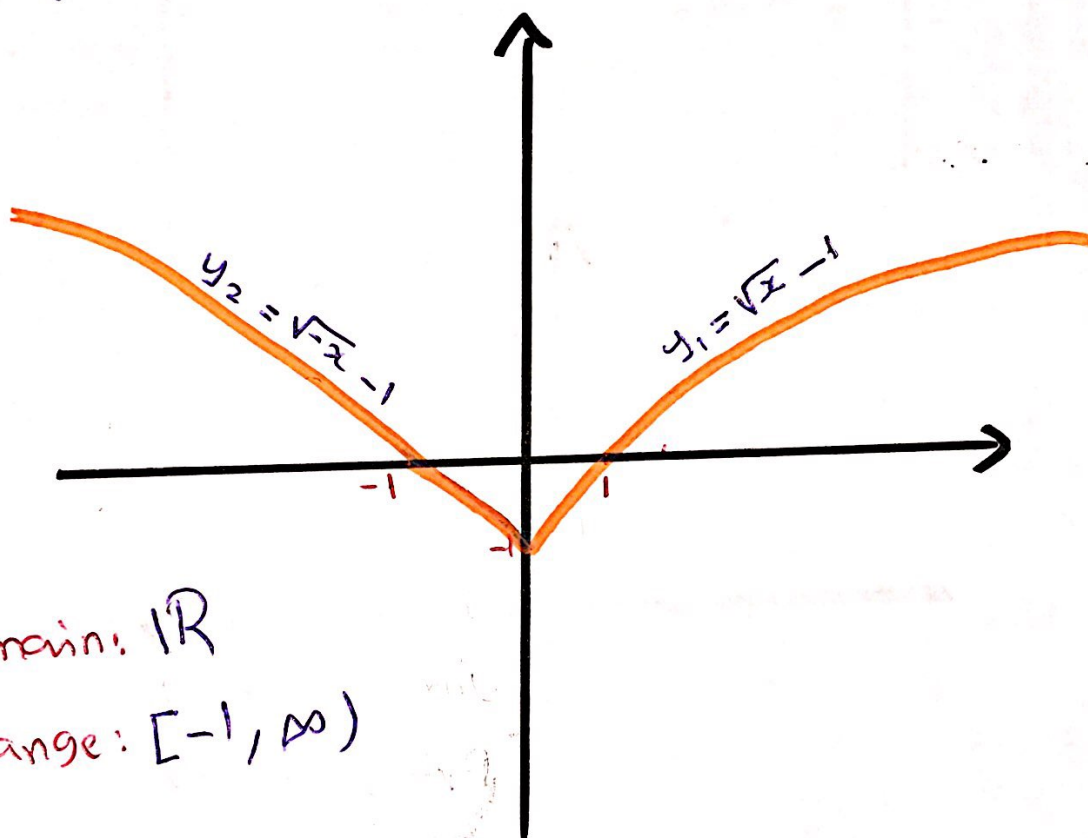
Find the Domain and Range of the following:

a) $y = |\sqrt{x} - 1|$



$$b) y = \sqrt{|x|} - 1$$

$$y = \begin{cases} \sqrt{x} - 1 & \text{if } x \geq 0 \\ \sqrt{-x} - 1 & \text{if } x < 0 \end{cases}$$



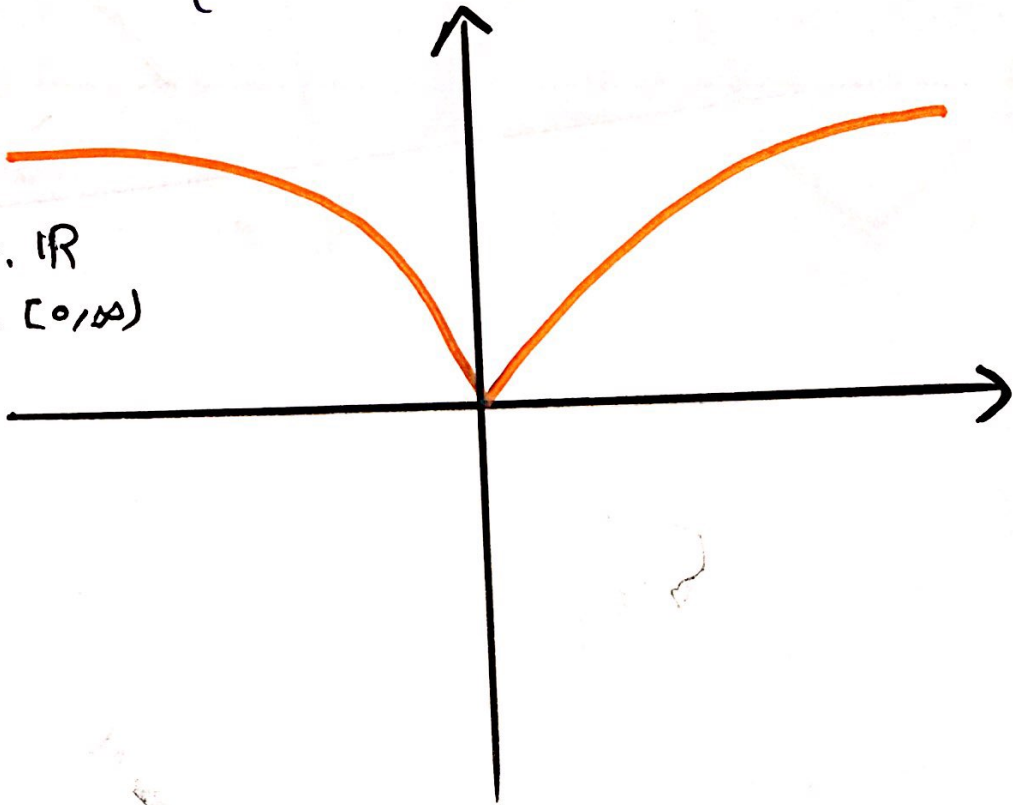
Domain: \mathbb{R}

Range: $[-1, \infty)$

c) $y = \sqrt{|x|}$

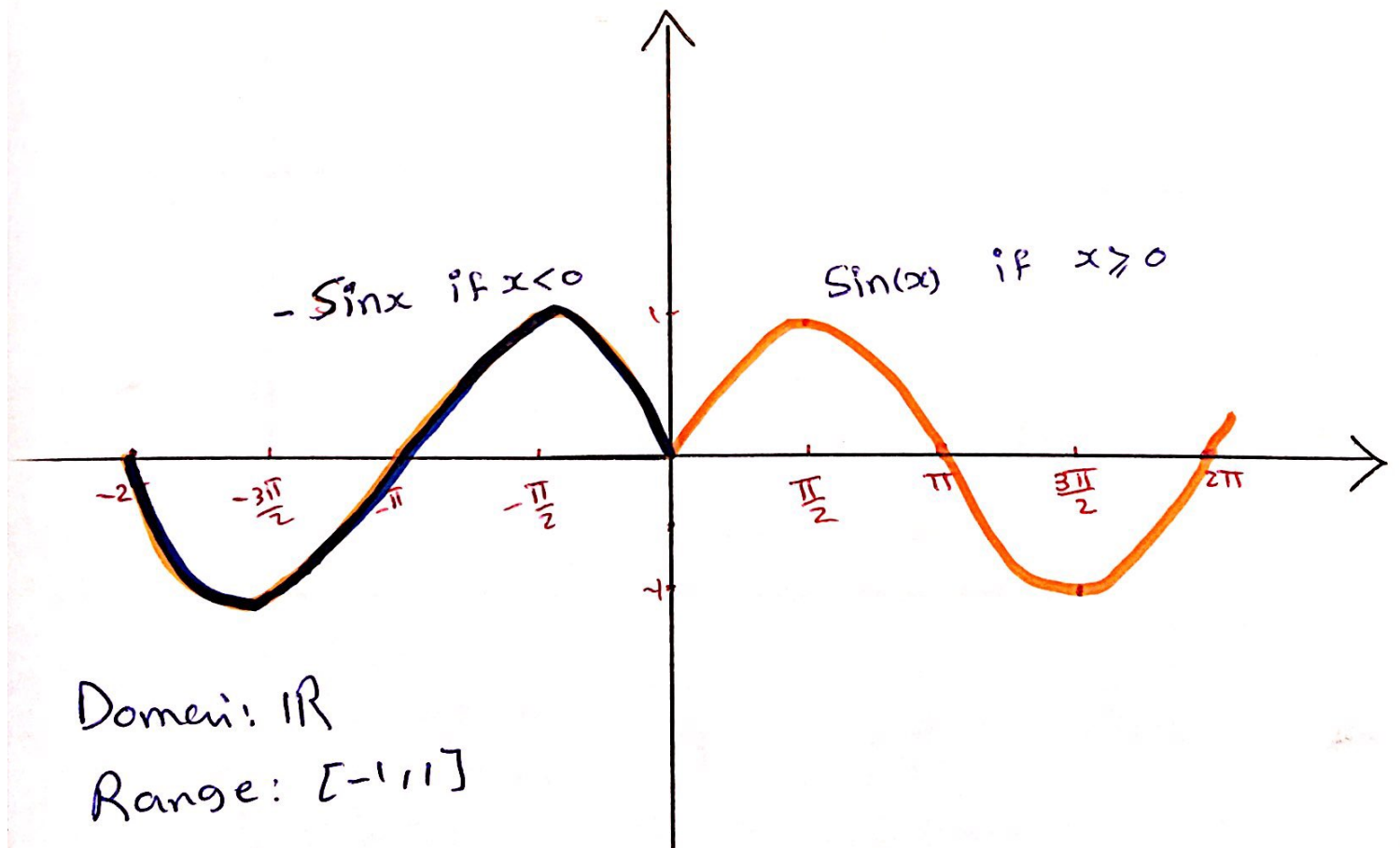
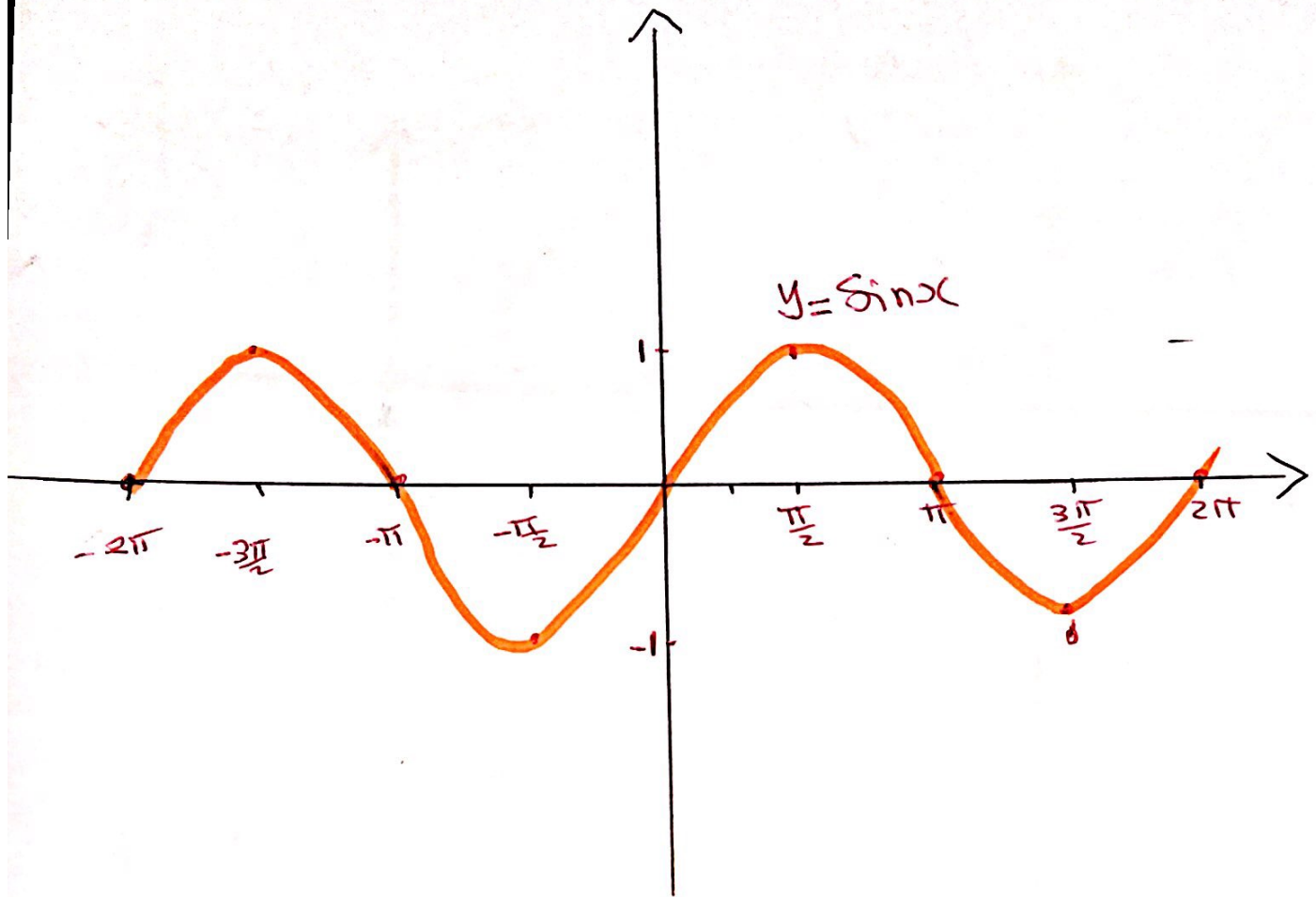
$$y = \sqrt{|x|} = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ \sqrt{-x} & \text{if } x < 0 \end{cases}$$

Domain: \mathbb{R}
Range: $[0, \infty)$



d) $y = \sin|x|$

$$y = \sin|x| = \begin{cases} \sin x & \text{if } x \geq 0 \\ \sin(-x) & \text{if } x < 0 \end{cases}$$
$$= \begin{cases} \sin x & \text{if } x \geq 0 \\ -\sin x & \text{if } x < 0 \end{cases}$$

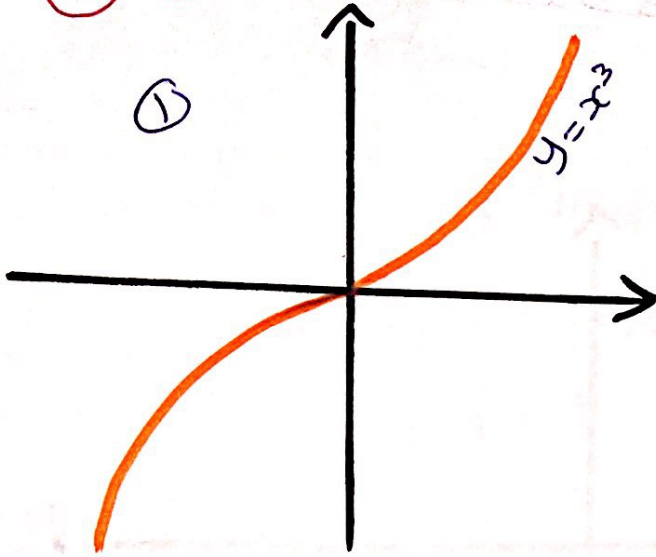


Domain: \mathbb{R}

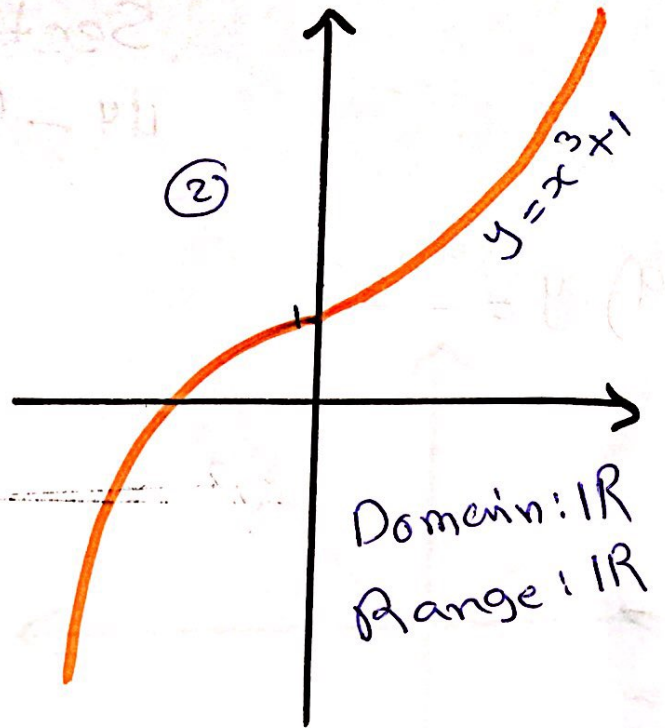
Range: $[-1, 1]$

Q) $y = x^3 + 1$

①

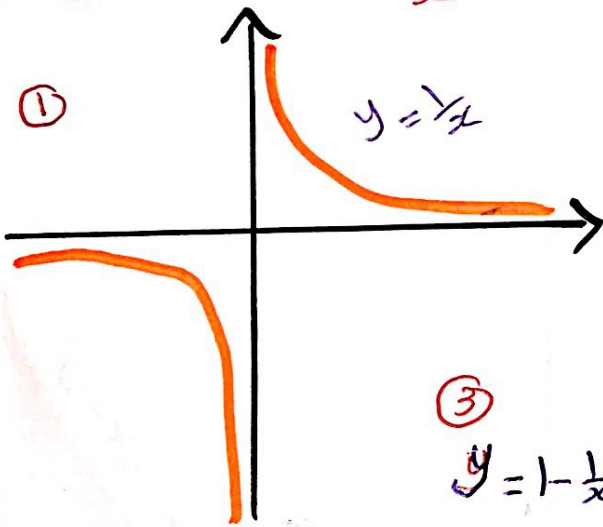


②

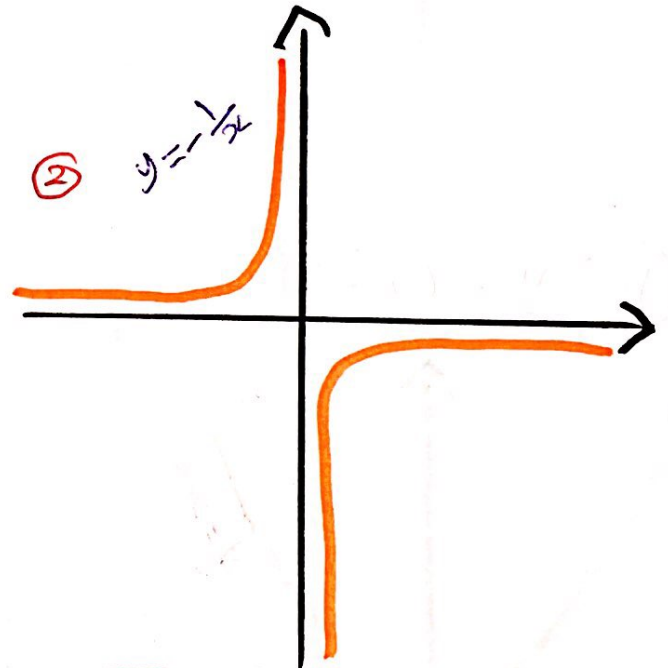


Q) $y = 1 - \frac{1}{x}$

①

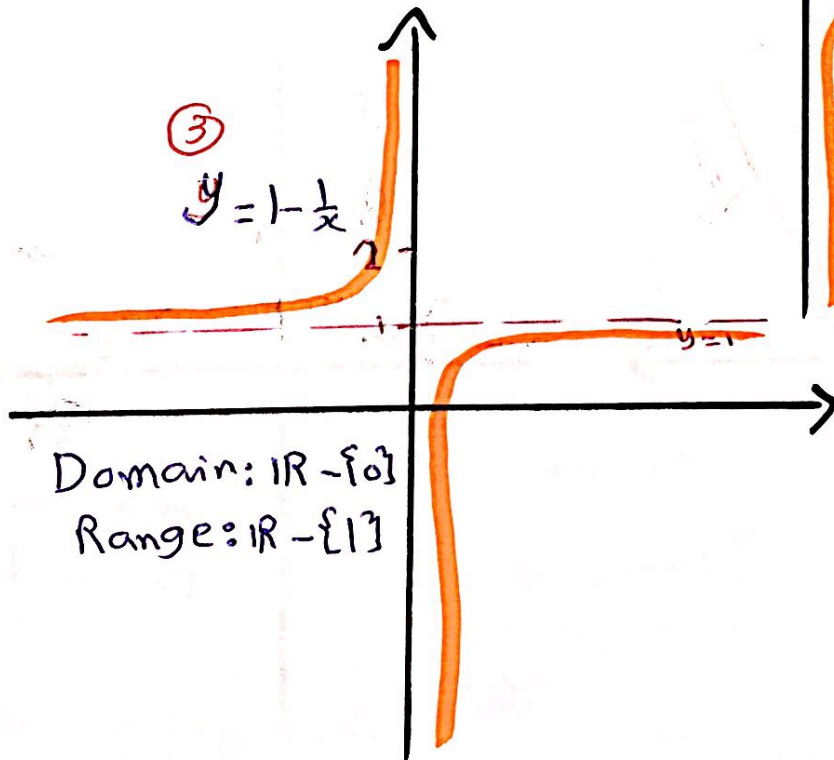


②



③

$y = 1 - \frac{1}{x}$

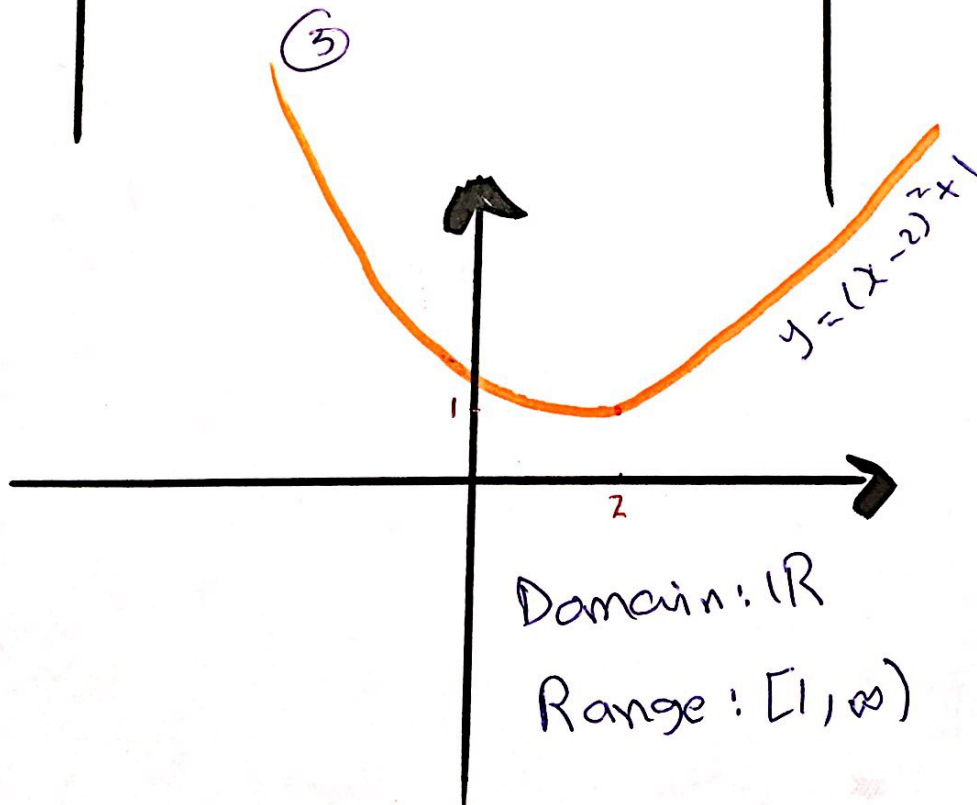
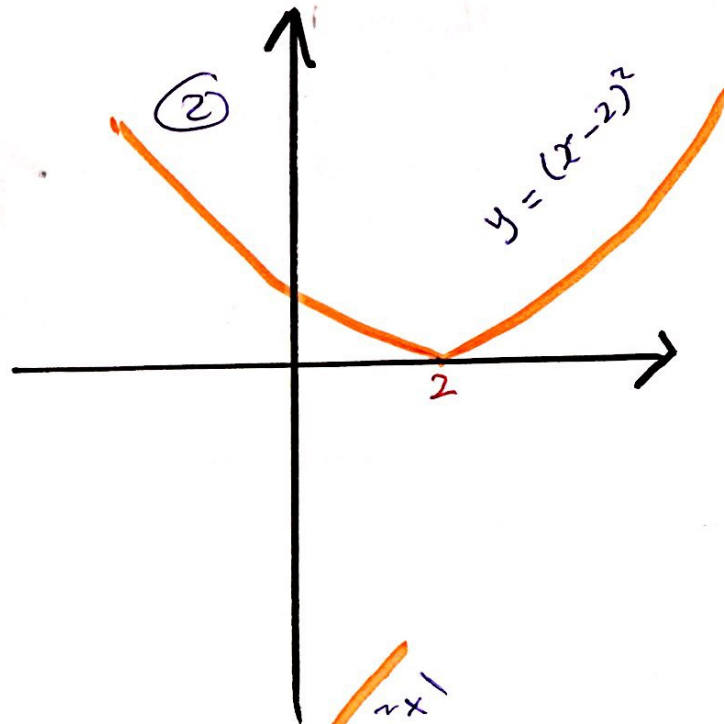
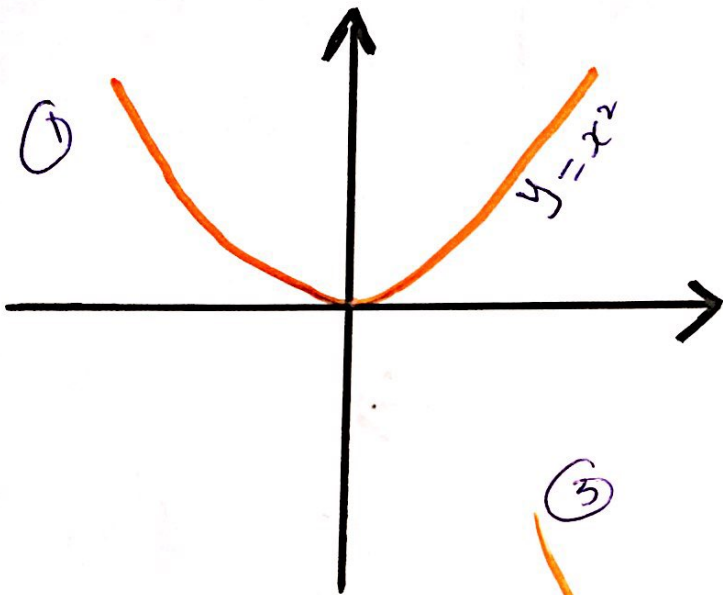


⑤ $y = x^2 - 4x + 5$
 نقوم بإكمال مربع

$$y = x^2 - 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 + 5$$

$$= x^2 - 4x + 4 - 4 + 5$$

$$= (x - 2)^2 + 1$$



Example

سؤال 1 من 42

Suppose the graph of f is given

Write equation for the graphs that

are obtained from the graph of f as follows

a) Shift 3 units up

$$y = f(x) + 3$$

b) Shift 3 units down

$$y = f(x) - 3$$

c) Shift 3 units to the right

$$y = f(x - 3)$$

d) Shift 3 units to the left

$$y = f(x + 3)$$

e) reflect about the x -axis

$$y = -f(x)$$

f) reflect about the y -axis

$$y = f(-x)$$

Example 2

سوال 2 ص 92

Explain how each graph is obtained from the graph $y = f(x)$

$y = f(x) + 8$, Shift the graph of $y = f(x)$ a distance 8 units up

$y = f(x+8)$, Shift the graph of $y = f(x)$ a distance 8 units to the left

$y = -f(x) - 1$
reflect of the graph of $y = f(x)$ about x -axis and shift the graph of $y = -f(x)$ a distance 1 unit down.

Example...

The graph of $y = f(x)$ is given

Match each equation with its graph

انتزري الرسومات من 42 سؤال 3

a) $y = f(x-4)$
graph (3)

b) $y = f(x) + 3$
graph (1)

d) $y = -f(x+4)$
graph (5)

Combinations of Functions

Let $f(x)$ and $g(x)$ are functions, $D_{f(x)} = A$ and $D_{g(x)} = B$ then.

$$1) (f \pm g)(x) = f(x) \pm g(x)$$

$$D_{(f \pm g)(x)} = D_{f(x)} \cap D_{g(x)} = A \cap B$$

$$2) (fg)(x) = f(x) \cdot g(x)$$

$$D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)} = A \cap B$$

$$3) \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$D_{\left(\frac{f}{g}\right)(x)} = \{x \in A \cap B \mid g(x) \neq 0\}$$

$$= A \cap B - \{ \text{صفار المقام} \}$$

$$4) \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$$

$$D_{\left(\frac{g}{f}\right)(x)} = \{x \in A \cap B \mid f(x) \neq 0\}$$

$$= A \cap B - \{ \text{صفار المقام} \}$$

Example

Find a) $f+g$; b) $f-g$; c) fg ; d) $\frac{f}{g}$; e) $\frac{g}{f}$

and state their domains.

$$1) f(x) = x^2 - 1 \quad ; \quad g(x) = 2x + 1$$

$$\begin{aligned} \text{a) } (f+g)(x) &= f(x) + g(x) \\ &= x^2 - 1 + 2x + 1 \\ &= x^2 + 2x \end{aligned}$$

$$\begin{aligned} \text{b) } (f-g)(x) &= f(x) - g(x) \\ &= x^2 - 1 - (2x + 1) \\ &= x^2 - 1 - 2x - 1 \\ &= x^2 - 2x - 2 \end{aligned}$$

$$\begin{aligned} \text{c) } (fg)(x) &= f(x)g(x) \\ &= (x^2 - 1)(2x + 1) \\ &= x^2(2x + 1) - 1(2x + 1) \\ &= 2x^3 + x^2 - 2x - 1 \end{aligned}$$

$$D_{f(x)} = \mathbb{R} \quad ; \quad D_{g(x)} = \mathbb{R}$$

$$\begin{aligned} D_{(f+g)(x)} &= D_{(f-g)(x)} = D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)} \\ &= \mathbb{R} \cap \mathbb{R} \\ &= \mathbb{R} \\ &= (-\infty, \infty) \end{aligned}$$

$$d) \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{x^2-1}{2x+1}$$

$$D_{\left(\frac{f}{g}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \left\{ \text{مقامات صفرا} \right\}$$

مقامات صفرا:

$$2x+1=0$$

$$2x=-1$$

$$\frac{2x}{2} = \frac{-1}{2}$$

$$\boxed{x = -\frac{1}{2}}$$

$$\therefore D_{\left(\frac{f}{g}\right)(x)} = \mathbb{R} \cap \mathbb{R} - \left\{ -\frac{1}{2} \right\}$$

$$= \mathbb{R} - \left\{ -\frac{1}{2} \right\}$$

$$\text{or } = \left\{ x \mid x \neq -\frac{1}{2} \right\}$$

$$\text{or } = (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$$

$$e) \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{2x+1}{x^2-1}$$

$$D_{\left(\frac{g}{f}\right)(x)} = D_{g(x)} \cap D_{f(x)} - \left\{ \text{مقامات صفرا} \right\}$$

مقامات صفرا:

$$x^2-1=0$$

$$x^2=1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$|x| = 1$$

$$\boxed{x = \pm 1}$$

$$\therefore D_{\left(\frac{g}{f}\right)(x)} = \mathbb{R} \cap \mathbb{R} - \{ \pm 1 \}$$

$$= \mathbb{R} - \{ 1, -1 \}$$

$$2) f(x) = x^3 + 2x^2 \quad ; \quad g(x) = 3x^2 - 1$$

$$\begin{aligned} a) (f+g)(x) &= f(x) + g(x) \\ &= x^3 + \underline{2x^2} + \underline{3x^2} - 1 \\ &= x^3 + \underline{5x^2} - 1 \end{aligned}$$

$$\begin{aligned} b) (f-g)(x) &= f(x) - g(x) \\ &= x^3 + 2x^2 - (3x^2 - 1) \\ &= x^3 + \underline{2x^2} - \underline{3x^2} + 1 \\ &= x^3 - \underline{x^2} + 1 \end{aligned}$$

$$\begin{aligned} c) (fg)(x) &= f(x)g(x) \\ &= (x^3 + 2x^2)(3x^2 - 1) \\ &= x^3(3x^2 - 1) + 2x^2(3x^2 - 1) \\ &= 3x^5 - x^3 + 6x^4 - 2x^2 \\ &= 3x^5 + 6x^4 - x^3 - 2x^2 \end{aligned}$$

$$D_{f(x)} = \mathbb{R} \quad D_{g(x)} = \mathbb{R}$$

$$\begin{aligned} D_{(f+g)(x)} &= D_{(f-g)(x)} = D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)} = \mathbb{R} \cap \mathbb{R} \\ &= \mathbb{R} \\ &= (-\infty, \infty) \end{aligned}$$

$$d) \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{x^3 + 2x^2}{3x^2 - 1}$$

$$D_{\left(\frac{f}{g}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \left\{ \text{أصفا المقام} \right\}$$

$$3x^2 - 1 = 0 \quad \text{: أصفا المقام}$$

$$3x^2 = 1$$

$$\frac{3x^2}{3} = \frac{1}{3}$$

$$x^2 = \frac{1}{3}$$

$$\sqrt{x^2} = \sqrt{\frac{1}{3}}$$

$$|x| = \frac{1}{\sqrt{3}} \Rightarrow x = \pm \frac{1}{\sqrt{3}}$$

$$\therefore D_{\left(\frac{f}{g}\right)(x)} = \mathbb{R} \cap \mathbb{R} - \left\{ \pm \frac{1}{\sqrt{3}} \right\} = \mathbb{R} - \left\{ \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right\} = \mathbb{R} - \left\{ \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3} \right\}$$

$$e) \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{3x^2 - 1}{x^3 + 2x^2}$$

$$D_{\left(\frac{g}{f}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \left\{ \text{أصفا المقام} \right\}$$

$$\therefore D_{\left(\frac{g}{f}\right)(x)} = \mathbb{R} \cap \mathbb{R} - \{0, -2\}$$

$$= \mathbb{R} - \{0, -2\}$$

$$\text{or} = \{x \mid x \neq 0, x \neq -2\}$$

$$\text{or} = (-\infty, -2) \cup (-2, 0) \cup (0, \infty)$$

$$x^3 + 2x^2 = 0 \quad \text{: أصفا المقام}$$

$$x^2(x+2) = 0 \quad \begin{cases} \nearrow x^2 = 0 \Rightarrow x = 0 \\ \text{or} \end{cases}$$

$$\searrow x+2 = 0 \Rightarrow x = -2$$

$$3) f(x) = \sqrt{3-x} \quad g(x) = \sqrt{x^2-1}$$

$$a) (f+g)(x) = \sqrt{3-x} + \sqrt{x^2-1}$$

$$b) (f-g)(x) = \sqrt{3-x} - \sqrt{x^2-1}$$

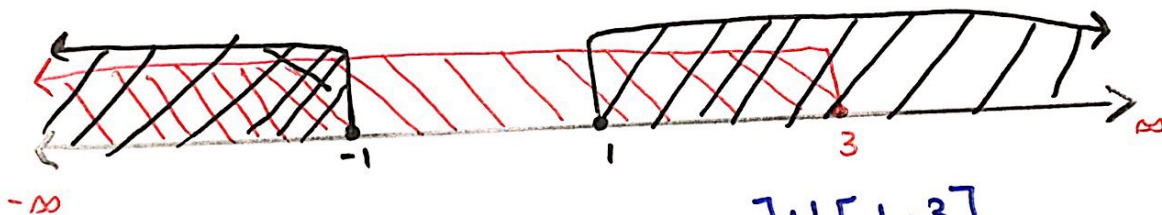
$$c) (fg)(x) = \sqrt{3-x} \sqrt{x^2-1}$$

$$D_{f(x)} = (-\infty, 3]$$

$$D_{g(x)} = (-\infty, -1] \cup [1, \infty)$$

"1.1 س، > ید"

$$D_{(f+g)(x)} = D_{(f-g)(x)} = D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)} \\ = (-\infty, 3] \cap [(-\infty, -1] \cup [1, \infty)]$$



$$= (-\infty, -1] \cup [1, 3]$$

Note

$$(fg)(x) = \sqrt{3-x} \sqrt{x^2-1} = \sqrt{(3-x)(x^2-1)} \\ = \sqrt{3x^2 - 3 - x^3 + x} \\ = \sqrt{-x^3 + 3x^2 + x - 3} \quad \text{on } D_{(fg)(x)}$$

$$= \sqrt{-x^3 + 3x^2 + x - 3} \quad \text{on } (-\infty, -1] \cup [1, 3].$$

$$d) \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{\sqrt{3-x}}{\sqrt{x^2-1}}$$

$$D_{\left(\frac{f}{g}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \left\{ \text{أصفار المقام} \right\}$$

$$\sqrt{x^2-1} = 0 \quad ; \quad \text{أصفار المقام}$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$|x| = 1$$

$$x = \pm 1$$

$$\therefore D_{\left(\frac{f}{g}\right)(x)} = (-\infty, -1] \cup [1, 3] - \{-1, 1\}$$

$$= (-\infty, -1) \cup (1, 3]$$

$$e) \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{x^2-1}}{\sqrt{3-x}}$$

$$D_{\left(\frac{g}{f}\right)(x)} = D_{g(x)} \cap D_{f(x)} - \left\{ \text{أصفار المقام} \right\}$$

$$\sqrt{3-x} = 0 \quad ; \quad \text{أصفار المقام}$$

$$3-x = 0$$

$$-x = -3$$

$$\frac{-x}{-1} = \frac{-3}{-1}$$

$$x = 3$$

$$\therefore D_{\left(\frac{g}{f}\right)(x)} = (-\infty, -1] \cup [1, 3] - \{3\}$$

$$= (-\infty, -1] \cup [1, 3)$$

Note : $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{3-x}}{\sqrt{x^2-1}} = \sqrt{\frac{3-x}{x^2-1}}$ on $(-\infty, -1) \cup (1, 3]$

$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{x^2-1}}{\sqrt{3-x}} = \sqrt{\frac{x^2-1}{3-x}}$ on $(-\infty, -1] \cup [1, 3)$

Example

Find the Domain of the following functions:

$$1) g(t) = \sqrt{3-t} - \sqrt{2+t}$$

$$\text{let } f_1(t) = \sqrt{3-t} \text{ and } f_2(t) = \sqrt{2+t}$$

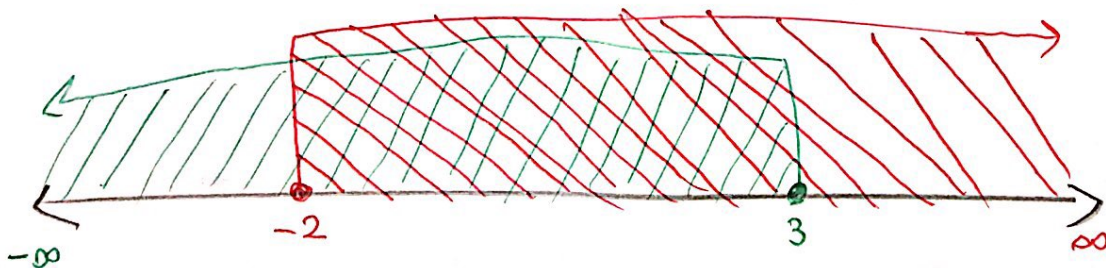
$$\checkmark D_{f_1(t)} : 3-t \geq 0 \Rightarrow -t \geq -3 \Rightarrow t \leq 3$$

$$\therefore D_{f_1(t)} = (-\infty, 3]$$

$$\checkmark D_{f_2(t)} : 2+t \geq 0 \Rightarrow t \geq -2$$

$$\therefore D_{f_2(t)} = [-2, \infty)$$

$$\begin{aligned} \checkmark D_{g(t)} &= D_{f_1(t)} \cap D_{f_2(t)} \\ &= (-\infty, 3] \cap [-2, \infty) \\ &= [-2, 3] \end{aligned}$$



$$2) f(x) = \frac{1}{\sqrt{2x^2+5}}$$

$$\text{let } f_1(x) = 1 \quad ; \quad f_2(x) = \sqrt{2x^2+5}$$

$$D_{f_1(x)} = \mathbb{R}$$

$$D_{f_2(x)} : \begin{cases} 2x^2 + 5 \neq 0 \\ 2x^2 \neq -5 \end{cases}$$

$$\underline{2x^2} \neq \underline{-5}$$

$$x^2 \neq -\frac{5}{2}$$

$$\sqrt{x^2} \neq \sqrt{-\frac{5}{2}}$$

$$|x| \neq \sqrt{-\frac{5}{2}}$$

$$x \neq \pm \sqrt{-\frac{5}{2}} \notin \mathbb{R}$$

$$\therefore D_{f_2(x)} = \mathbb{R}$$

$$\Rightarrow D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \left\{ \text{أصفار المقام} \right\}$$

$$= \mathbb{R} \cap \mathbb{R}$$

$$= \mathbb{R}$$

$$2x^2 + 5 = 0 \quad \text{أصفار المقام}$$

$$2x^2 = -5$$

$$x^2 = -\frac{5}{2}$$

$$\sqrt{x^2} = \sqrt{-\frac{5}{2}}$$

$$|x| = \sqrt{-\frac{5}{2}}$$

$$x = \pm \sqrt{-\frac{5}{2}} \notin \mathbb{R}$$

لا يوجد أصفار مقام

$$3) f(x) = \frac{\ln x}{\sqrt{5-x^2}}$$

$$\text{Let } f_1(x) = \ln(x) ; f_2(x) = \sqrt{5-x^2}$$

$$D_{f_1(x)} = (0, \infty)$$

$$D_{f_2(x)} = [-\sqrt{5}, \sqrt{5}]$$

$$\begin{aligned} \therefore D_{f(x)} &= D_{f_1(x)} \cap D_{f_2(x)} - \left\{ \begin{array}{l} \text{أصفار المقام} \\ \sqrt{5}, -\sqrt{5} \end{array} \right\} \\ &= (0, \infty) \cap [-\sqrt{5}, \sqrt{5}] - \left\{ \begin{array}{l} \text{أصفار المقام} \\ \sqrt{5}, -\sqrt{5} \end{array} \right\} \\ &= (0, \sqrt{5}] - \left\{ \sqrt{5}, -\sqrt{5} \right\} \\ &= (0, \sqrt{5}) \end{aligned}$$

$$5-x^2=0$$

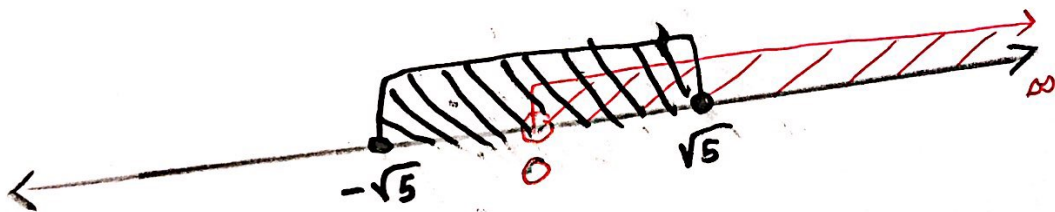
$$-x^2=-5$$

$$x^2=5$$

$$\sqrt{x^2}=\sqrt{5}$$

$$|x|=\sqrt{5}$$

$$x = \pm\sqrt{5}$$



$$4) f(x) = \frac{\sqrt{9-x^2}}{x-2}$$

$$\text{let } f_1(x) = \sqrt{9-x^2} \quad ; \quad f_2(x) = \underline{x-2}$$

$$D_{f_1(x)} = [-3, 3] \quad \text{and} \quad D_{f_2(x)} = \mathbb{R}$$

$$x-2=0 : \text{ مقام, لپه } \uparrow$$

$$x=2$$

$$D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{ \text{مقام, لپه } \uparrow \}$$

$$= [-3, 3] \cap \mathbb{R} - \{ 2 \}$$

$$= [-3, 3] - \{ 2 \}$$

$$= [-3, 2) \cup (2, 3]$$

$$5) f(x) = \frac{x-2}{\sqrt{9-x^2}} \quad \text{"H.W"}$$

$$\text{let } f_1(x) = x-2 \Rightarrow D_{f_1(x)} = \mathbb{R}$$

$$f_2(x) = \sqrt{9-x^2} \Rightarrow D_{f_2(x)} = [-3, 3]$$

$$9-x^2=0 : \text{ مقام, لپه } \uparrow$$

$$-x^2=9$$

$$x^2=9$$

$$\sqrt{x^2} = \sqrt{9} \Rightarrow |x|=3$$

$$\Rightarrow \boxed{x = \pm 3}$$

$$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{ \text{مقام, لپه } \uparrow \}$$

$$= \mathbb{R} \cap [-3, 3] - [-3, 3]$$

$$= [-3, 3] - [-3, 3]$$

$$= (-3, 3)$$

$$6) f(x) = \frac{x-1}{\sqrt[3]{x^2-6x+5}}$$

$$\text{let } f_1(x) = x-1 \quad ; \quad f_2(x) = \sqrt[3]{x^2-6x+5}$$

$$D_{f_1(x)} = \mathbb{R} \quad ; \quad D_{f_2(x)} = \mathbb{R}$$

$$x^2-6x+5=0: \text{ مقام, صفا } \uparrow$$

$$(x-1)(x-5)=0$$

$$\begin{array}{l} \downarrow \\ x-1=0 \\ \boxed{x=1} \end{array} \quad \begin{array}{l} \downarrow \\ x-5=0 \\ \boxed{x=5} \end{array}$$

$$D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{ \text{مقام, صفا} \}$$

$$= \mathbb{R} \cap \mathbb{R} - \{1, 5\}$$

$$= \mathbb{R} - \{1, 5\}$$

$$\textcircled{7} f(x) = \frac{\sqrt[3]{x^2-6x+5}}{x-1}$$

"H.W"

مسائل اضافی

$$* f(x) = \frac{1}{\sqrt[5]{6-2x}}$$

$$7) \text{ let } f_1(x) = \sqrt[3]{x^2-6x+5} \quad \text{and } f_2(x) = x-1$$

$$D_{f_1(x)} = \mathbb{R} \quad ; \quad D_{f_2(x)} = \mathbb{R}$$

$$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{ \text{مقام, صفا} \}$$

$$x-1=0: \text{ مقام, صفا } \uparrow \\ \boxed{x=1}$$

$$= \mathbb{R} \cap \mathbb{R} - \{1\}$$

$$= \mathbb{R} - \{1\} = (-\infty, 1) \cup (1, \infty)$$

$$f(x) = \frac{1}{\sqrt[5]{6-2x}}$$

حل المثال الإضافي :

let $f_1(x) = 1$ and $f_2(x) = \sqrt[5]{6-2x}$

$D_{f_1(x)} = \mathbb{R}$ and $D_{f_2(x)} = \mathbb{R}$

$6-2x=0$: المقام، لايف

$-2x = -6$

$2x = 6$

$x = \frac{6}{2}$

$x = 3$

$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{ \text{أصفاء المقام} \}$

$= \mathbb{R} \cap \mathbb{R} - \{3\}$

$= \mathbb{R} - \{3\}$

or $= \{x \mid x \neq 3\}$

or $= (-\infty, 3) \cup (3, \infty)$

$$(8) \quad g(x) = \frac{x}{e^x + 1}$$

$$\text{Let } g_1(x) = x \quad ; \quad g_2(x) = e^x + 1$$

$$D_{g_1(x)} = \mathbb{R}$$

$$D_{g_2(x)} : \text{ Let } g_3(x) = e^x \quad ; \quad g_4(x) = 1$$

$$D_{g_3(x)} = \mathbb{R} \quad ; \quad D_{g_4(x)} = \mathbb{R}$$

$$\therefore D_{g_2(x)} = D_{g_3(x)} \cap D_{g_4(x)} \\ = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

$$\Rightarrow D_{g_2(x)} = \mathbb{R}$$

$$\therefore D_{g(x)} = D_{g_1(x)} \cap D_{g_2(x)} - \left\{ \text{أصفاء المقام} \right\}$$
$$= \mathbb{R} \cap \mathbb{R}$$
$$= \mathbb{R}$$

أصفاء المقام :

$$e^x + 1 = 0$$

$$e^x = -1$$

لا يوجد أصفاء مقام
لأن $e^x > 0$

$$\textcircled{9} \quad G(x) = \frac{5^x}{2^x - 1}$$

$$\text{Let } G_1(x) = 5^x \quad ; \quad G_2(x) = 2^x - 1$$

$$D_{G_1(x)} = \mathbb{R}$$

$$D_{G_2(x)} = \mathbb{R}$$

$$\therefore D_{G(x)} = D_{G_1(x)} \cap D_{G_2(x)} - \left\{ \begin{array}{l} \text{القيم} \\ \text{المقام} \end{array} \right\}$$

$$D_{G(x)} = \mathbb{R} \cap \mathbb{R} - \{0\}$$

$$= \mathbb{R} - \{0\} \neq$$

∴ القيم، المقام

$$2^x - 1 = 0$$

$$2^x = 1$$

$$2^x = 2^0$$

$$\boxed{x=0}$$

Note

if $H(x) = e^{f(x)}$ then $D_{H(x)} = D_{e^x} \cap D_{f(x)}$

if $H(x) = a^{f(x)}$ then $D_{H(x)} = D_{a^x} \cap D_{f(x)}$

if $H(x) = \sin(f(x))$ then $D_{H(x)} = D_{\sin x} \cap D_{f(x)}$

if $H(x) = \cos(f(x))$ then $D_{H(x)} = D_{\cos x} \cap D_{f(x)}$

Example

Find the Domain of the following functions:

a) $f(x) = e^{\sqrt{x}}$

$$D_{f(x)} = D_{e^x} \cap D_{\sqrt{x}}$$

$$= \mathbb{R} \cap [0, \infty)$$

$$= [0, \infty)$$

b) $f(x) = \pi^{2x^2+1}$

$$D_{f(x)} = D_{\pi^x} \cap D_{(2x^2+1)}$$

$$= \mathbb{R} \cap \mathbb{R}$$

$$= \mathbb{R}$$

$$c) f(x) = \cos(x^{-3})$$

$$\begin{aligned} D_{f(x)} &= D_{\cos x} \cap D_{x^{-3}} \\ &= D_{\cos x} \cap D_{\frac{1}{x^3}} \\ &= \mathbb{R} \cap \mathbb{R} - \{0\} \\ &= \mathbb{R} - \{0\} \\ &= (-\infty, 0) \cup (0, \infty) \end{aligned}$$

$$d) \sin(e^t) = f(x)$$

$$\begin{aligned} D_{f(x)} &= D_{\sin t} \cap D_{e^{-t}} \\ &= \mathbb{R} \cap \mathbb{R} \\ &= \mathbb{R} \\ &= (-\infty, \infty) \end{aligned}$$

$$d) f(x) = \cos(\pi^x + \sqrt{x^2-1})$$

$$D_{\pi^x} = \mathbb{R} \quad ; \quad D_{\sqrt{x^2-1}} = (-\infty, -1] \cup [1, \infty)$$

$$\begin{aligned} D_{(\pi^x + \sqrt{x^2-1})} &= D_{\pi^x} \cap D_{\sqrt{x^2-1}} \\ &= \mathbb{R} \cap (-\infty, -1] \cup [1, \infty) \\ &= (-\infty, -1] \cup [1, \infty) \end{aligned}$$

$$e) f(x) = \frac{1 - e^{x^2}}{1 - e^{1-x^2}}$$

$$D_{(1-e^{x^2})} = \mathbb{R} \quad ; \quad D_{(1-e^{1-x^2})} = \mathbb{R}$$

$$\begin{aligned} D_{f(x)} &= D_{(1-e^{x^2})} \cap D_{(1-e^{1-x^2})} - \left\{ \frac{1}{1} \right\} \\ &= \mathbb{R} - \{\pm 1\} \end{aligned}$$

$1 - e^{1-x^2} = 0$: المقام، ليه إ
 $-e^{1-x^2} = -1$
 $e^{1-x^2} = 1$
 $e^{1-x^2} = e^0$
 $1-x^2 = 0 \Rightarrow x = \pm 1$

$$f) f(x) = \frac{1+x}{e^{\cos x}}$$

$$D_{(1+x)} = \mathbb{R} ; D_{e^{\cos x}} = \mathbb{R}$$

أصفر المقام $e^{\cos x} = 0$
لا يوجد أصفر المقام لأن $e^{\cos x} > 0$

$$\begin{aligned} \therefore D_{f(x)} &= D_{(1+x)} \cap D_{e^{\cos x}} - \{\text{أصفر المقام}\} \\ &= \mathbb{R} \cap \mathbb{R} \\ &= \mathbb{R} \end{aligned}$$

$$g) H(t) = \sqrt{10^t - 100}$$

$$10^t - 100 \geq 0 \Rightarrow 10^t \geq 100 \Rightarrow 10^t \geq 10^2$$

$$\Rightarrow t \geq 2$$

$$\therefore D_{H(t)} = [2, \infty)$$

$$h) g(x) = \sqrt[4]{1 - 2^x}$$

$$1 - 2^x \geq 0 \Rightarrow -2^x \geq -1 \Rightarrow 2^x \leq 1 \Rightarrow 2^x \leq 2^0$$

$$\Rightarrow x \leq 0 \Rightarrow D_{g(x)} = (-\infty, 0]$$

$$i) f(x) = \sqrt[4]{2^x + 1}$$

$$D_{f(x)} : 2^x + 1 \geq 0$$

$$2^x \geq -1$$

مستحيل لأنه دائماً
الدالة الأسية موجبة
أي $2^x > 0$

$$\therefore D_{f(x)} = \mathbb{R}$$

$$j) f(x) = \sqrt{|x| + 1}$$

$$D_{f(x)} : |x| + 1 \geq 0$$

$$|x| \geq -1$$

مستحيل لأنه دائماً
دالة القيمة المطلقة

موجبة أي $|x| \geq 0$

$$\therefore D_{f(x)} = \mathbb{R}$$

$$k) f(x) = \sqrt{|x| - 1}$$

$$D_{f(x)} : |x| - 1 \geq 0$$

$$|x| \geq 1$$

$$x \geq 1 \text{ or } x \leq -1$$

$$\therefore D_{f(x)} = (-\infty, -1] \cup [1, \infty) = \mathbb{R} - (-1, 1)$$

$$l) f(x) = \frac{1}{\sqrt{x+2}}$$

$$\text{let } f_1(x) = 1 \text{ and}$$

$$f_2(x) = \sqrt{x+2}$$

$$D_{f_1}(x) = \mathbb{R} \text{ and}$$

$$D_{f_2}(x) = [0, \infty)$$

أضف المقام:

$$\sqrt{x+2} = 0$$

$$\sqrt{x} = -2$$

مستحيل لأنه $\sqrt{x} \geq 0$
أي أنه دائماً
موجبة

لأنه لا يوجد أضف
مقام

$$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)}$$

$$= \mathbb{R} \cap [0, \infty)$$

$$= [0, \infty)$$

$$m) f(x) = \sqrt{1 - |x|}$$

$$D_{f(x)} : 1 - |x| \geq 0$$

$$-|x| \geq -1$$

$$|x| \leq 1$$

$$-1 \leq x \leq 1$$

$$\therefore D_{f(x)} = [-1, 1]$$

$$N) f(x) = \frac{1}{\sqrt{x} - 2}$$

$$\text{Let } f_1(x) = 1 \Rightarrow D_{f_1(x)} = \mathbb{R}$$

$$f_2(x) = \sqrt{x} - 2 \Rightarrow D_{f_2(x)} = [0, \infty)$$

$$\sqrt{x} - 2 = 0 : \text{نقطة التقاطع}$$

$$\sqrt{x} = 2$$

$$(\sqrt{x})^2 = (2)^2$$

$$\boxed{x = 4}$$

$$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{ \text{نقطة التقاطع} \}$$

$$= \mathbb{R} \cap [0, \infty) - \{4\}$$

$$= [0, \infty) - \{4\}$$

$$= [0, 4) \cup (4, \infty)$$

$$P) f(x) = \sqrt{\cos x + 3}$$

$$D_{f(x)}: \cos x + 3 \geq 0$$

$$\cos x \geq -3$$

مستحيل لأنه دائماً $-1 \leq \cos x \leq 1$

$$\therefore D_{f(x)} = \mathbb{R}$$

$$9) f(x) = \frac{1}{\sin x - 3}$$

$$\text{let } f_1(x) = 1 \text{ and } f_2(x) = \sin x - 3$$

$$D_{f_1}(x) = \mathbb{R} \quad ; \quad D_{f_2}(x) = \mathbb{R}$$

$$\sin x - 3 = 0 \quad \text{أضربا المقام}$$

$$\sin x = 3$$

مستحيل لأنه دائماً $-1 \leq \sin x \leq 1$

لا بد أن لا يوجد أضربا المقام

$$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)}$$

$$= \mathbb{R} \cap \mathbb{R}$$

$$= \mathbb{R}$$

Definition:-

$$(f \circ g)(x) = f(g(x))$$

$f \circ g$ is called the Composition of f and g .

Example.

If $f(x) = x^2$ and $g(x) = x - 3$

then find $f \circ g$ and $g \circ f$?

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f(x - 3) \\ &= (x - 3)^2 = x^2 - 6x + 9\end{aligned}$$

$$(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 3$$

Note $f \circ g \neq g \circ f$

Example: If $f(x) = \frac{x}{x+1}$; $g(x) = x^{10}$
and $h(x) = x+3$ then find $f \circ g \circ h$
and $h \circ g \circ f$

$$\begin{aligned}\checkmark (f \circ g \circ h)(x) &= f(g(h(x))) \\ &= f(g(x+3)) \\ &= f((x+3)^{10}) \\ &= \frac{(x+3)^{10}}{(x+3)^{10} + 1}\end{aligned}$$

$$\begin{aligned}\checkmark (h \circ g \circ f)(x) &= h(g(f(x))) \\ &= h\left(g\left(\frac{x}{x+1}\right)\right) \\ &= h\left(\left(\frac{x}{x+1}\right)^{10}\right) \\ &= \left(\frac{x}{x+1}\right)^{10} + 3\end{aligned}$$

Example :: Given $F(x) = \cos^2(x+9)$, find functions f , g and h such that $F = f \circ g \circ h$

$$F(x) = \cos^2(x+9)$$

$$(f \circ g \circ h)(x) = [\cos(x+9)]^2$$

$$\underline{f(\underline{g(\underline{h(x)})})} = \underline{[\underline{\cos(\underline{x+9})}]^2}$$

$$h(x) = x+9$$

$$g(x) = \cos(x)$$

$$f(x) = x^2$$

Example: $F(x) = \sqrt[3]{\frac{x}{1+x}}$ find f and g such that

$$F = F \circ g$$

$$F(x) = \sqrt[3]{\frac{x}{1+x}}$$

$$(F \circ g)(x) = \sqrt[3]{\frac{x}{1+x}}$$

$$f(g(x)) = \sqrt[3]{\frac{x}{1+x}}$$

$$g(x) = \frac{x}{x+1}; f(x) = \sqrt[3]{x}$$

Example $F(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$ Find f and g

Such that $F = F \circ g$

$$F(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$$

$$(F \circ g)(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$$

$$F(g(x)) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$$

$$g(x) = \sqrt[3]{x}$$

$$f(x) = \frac{x}{1+x}$$

Example $R(x) = \sqrt{\sqrt{x} - 1}$

$$R(x) = \sqrt{\sqrt{x} - 1}$$

$$(F \circ g \circ h)(x) = \sqrt{\sqrt{x} - 1}$$

$$F(g(h(x))) = \sqrt{\sqrt{x} - 1}$$

$$h(x) = \sqrt{x} \quad g(x) = x - 1 \quad f(x) = \sqrt{x}$$

الواجب في درس 1.3

$$39) f(x) = 3x - 2 \quad ; \quad g(x) = \sin x \quad ; \quad h(x) = x^2$$

$$\begin{aligned}(f \circ g \circ h)(x) &= f(g(h(x))) \\ &= f(g(x^2)) \\ &= f(\sin(x^2))\end{aligned}$$

$$= 3 \sin(x^2) - 2$$

$$40) f(x) = |x - 4| \quad ; \quad g(x) = 2^x \quad ; \quad h(x) = \sqrt{x}$$

$$\begin{aligned}(f \circ g \circ h)(x) &= f(g(h(x))) \\ &= f(g(\sqrt{x})) \\ &= f(2^{\sqrt{x}})\end{aligned}$$

$$= |2^{\sqrt{x}} - 4|$$

$$41) f(x) = \sqrt{x - 3} \quad , \quad g(x) = x^2 \quad ; \quad h(x) = x^3 + 2$$

$$\begin{aligned}(f \circ g \circ h)(x) &= f(g(h(x))) \\ &= f(g(x^3 + 2)) \\ &= f((x^3 + 2)^2) \\ &= \sqrt{(x^3 + 2)^2 - 3}\end{aligned}$$

$$(42) f(x) = \tan x \quad ; \quad g(x) = \frac{x}{x-1} \quad , \quad h(x) = \sqrt[3]{x}$$

$$\begin{aligned}(F \circ g \circ h)(x) &= F(g(h(x))) \\ &= F\left(g\left(\sqrt[3]{x}\right)\right) \\ &= F\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right) \\ &= \tan\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)\end{aligned}$$

Express the function in the form $f \circ g$

$$43) F(x) = (2x + x^2)^4$$

$$(F \circ g)(x) = (2x + x^2)^4$$

$$F(g(x)) = (2x + x^2)^4$$

$$g(x) = 2x + x^2 \quad ; \quad f(x) = x^4$$

$$45) F(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$$

$$(F \circ g)(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$$

$$F(g(x)) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$$

$$g(x) = \sqrt[3]{x} \quad ; \quad f(x) = \frac{x}{1+x}$$

$$47) V(t) = \sec(t^2) \tan(t^2)$$

$$(F \circ g)(t) = \sec(t^2) \tan(t^2)$$

$$F(g(t)) = \sec(t^2) \tan(t^2)$$

$$g(t) = t^2 \quad ; \quad F(t) = \sec(t) \tan(t)$$

$$(48) U(t) = \frac{\tan(t)}{1 + \tan(t)}$$

$$(F \circ g)(t) = \frac{\tan(t)}{1 + \tan(t)}$$

$$F(g(t)) = \frac{\tan(t)}{1 + \tan(t)}$$

$$g(t) = \tan(t) \quad ; \quad F(t) = \frac{t}{1+t}$$

$$(46) G(x) = \sqrt[3]{\frac{x}{1+x}}$$

$$(F \circ g)(x) = \sqrt[3]{\frac{x}{1+x}}$$

$$F(g(x)) = \sqrt[3]{\frac{x}{1+x}}$$

$$g(x) = \frac{x}{1+x} \quad ; \quad F(x) = \sqrt[3]{x}$$

$$(44) h(x) = \cos^2 x$$

$$(F \circ g)(x) = [\cos x]^2$$

$$F(g(x)) = [\cos x]^2$$

$$g(x) = \cos x \quad ; \quad F(x) = x^2$$

Express the function in the form fogoh

$$49) R(x) = \sqrt{\sqrt{x} - 1}$$

$$(F \circ g \circ h)(x) = \sqrt{\sqrt{x} - 1}$$

$$F(g(h(x))) = \sqrt{\sqrt{x} - 1}$$

$$h(x) = \sqrt{x} \quad ; \quad g(x) = \cancel{x-1} \quad ; \quad f(x) = \sqrt{x} \\ = x - 1$$

$$50) H(x) = \sqrt[8]{2 + |x|}$$

$$(F \circ g \circ h)(x) = \sqrt[8]{2 + |x|}$$

$$f(g(h(x))) = \sqrt[8]{2 + |x|}$$

$$h(x) = |x| \quad ; \quad g(x) = 2 + x \quad ; \quad f(x) = \sqrt[8]{x}$$

$$(51) S(t) = \sin^2(\cos t)$$

$$(F \circ g \circ h)(t) = \sin^2(\cos t)$$

$$f(g(h(t))) = [\sin(\cos t)]^2$$

$$h(t) = \cos t \quad ; \quad g(t) = \sin(t) \quad ; \quad f(t) = t^2$$

Example:

$$\text{if } f(x) = \frac{1}{x^2+5}, \quad g(x) = \frac{3}{2}$$

$$\text{then } (f \circ g)(2) = f(g(2))$$

$$= f\left(\frac{3}{2}\right)$$

$$= \frac{1}{\left(\frac{3}{2}\right)^2+5} = \frac{1}{\frac{9}{4}+5}$$

$$= \frac{1}{\left(\frac{9+20}{4}\right)}$$

$$= \frac{1}{\left(\frac{29}{4}\right)}$$

$$= \frac{4}{29} \neq$$

$$(g \circ f)(2) = g(f(2))$$

$$= g\left(\frac{1}{2^2+5}\right) = g\left(\frac{1}{4+5}\right) = g\left(\frac{1}{9}\right)$$

$$= \frac{3}{2} \neq$$

Example:

If $f(x) = \sqrt{3-x}$; $g(x) = \sqrt{x^2-1}$ then
find each function and its Domain

a) $f \circ f$

$$(f \circ f)(x) = f(f(x)) = f(\sqrt{3-x}) = \sqrt{3-\sqrt{3-x}}$$

New
Function

$$D_{f(x)} : 3-x \geq 0$$

$$-x \geq -3$$

$$\frac{-x}{-1} \leq \frac{-3}{-1}$$

$$x \leq 3$$

$$\therefore D_{f(x)} = \{x \mid x \leq 3\} = (-\infty, 3]$$

$$D_{\text{New}} : 3 - \sqrt{3-x} \geq 0$$

$$-\sqrt{3-x} \geq -3$$

$$\sqrt{3-x} \leq 3$$

$$(\sqrt{3-x})^2 \leq 3^2$$

$$3-x \leq 9$$

$$-x \leq 9-3$$

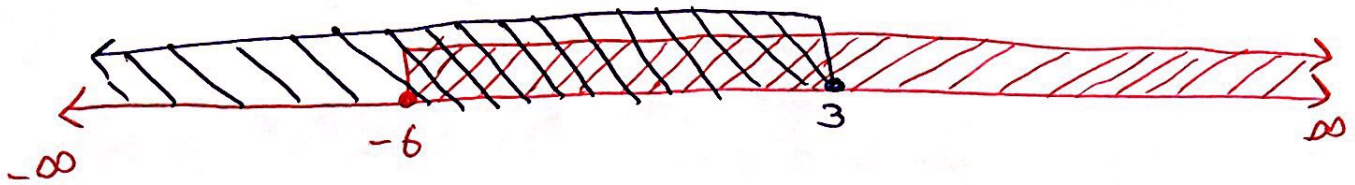
$$-x \leq 6$$

$$x \geq -6$$

$$\therefore D_{\text{New}} = [-6, \infty)$$

$$D_{(f \circ f)(x)} = D_{\text{New}} \cap D_{f(x)}$$

$$= [-6, \infty) \cap (-\infty, 3] = [-6, 3]$$



(b) $(g \circ f)(x)$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{3-x}) = \sqrt{(\sqrt{3-x})^2 - 1}$$

$$= \sqrt{3-x-1} = \sqrt{2-x} \text{ New function}$$

$$D_{f(x)}: 3-x > 0 \Rightarrow -x > -3 \Rightarrow x < 3$$

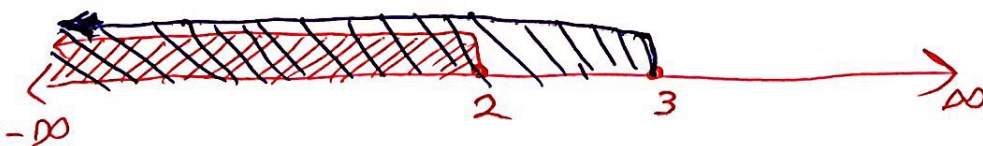
$$\therefore D_{f(x)} = (-\infty, 3]$$

$$D_{\text{New}}: 2-x > 0 \Rightarrow -x > -2 \Rightarrow x < 2$$

$$\therefore D_{\text{New}} = (-\infty, 2]$$

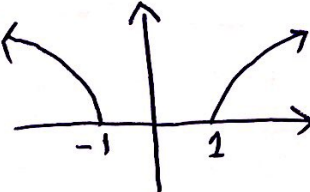
$$D_{(g \circ f)(x)} = D_{\text{New}} \cap D_{f(x)}$$

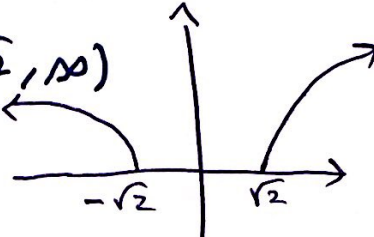
$$= (-\infty, 2] \cap (-\infty, 3] = (-\infty, 2]$$



c) $g \circ g$

$$(g \circ g)(x) = g(g(x)) = g(\sqrt{x^2 - 1}) = \sqrt{[\sqrt{x^2 - 1}]^2 - 1} \\ = \sqrt{x^2 - 1 - 1} = \sqrt{x^2 - 2} \text{ New function}$$

$$D_{g(x)} : (-\infty, -1] \cup [1, \infty)$$


$$D_{\text{New}} : (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$$


$$D_{(g \circ g)(x)} = D_{\text{New}} \cap D_{g(x)} \\ = [(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)] \cap [(-\infty, -1] \cup [1, \infty)] \\ = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$$



d) fog

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x^2-1})$$

$$= \sqrt{3 - \sqrt{x^2-1}} \quad \text{New function}$$

$$D_{g(x)}: (-\infty, -1] \cup [1, \infty)$$

$$D_{\text{New}}: 3 - \sqrt{x^2-1} \geq 0$$

$$-\sqrt{x^2-1} \geq -3$$

$$\sqrt{x^2-1} \leq 3$$

$$(\sqrt{x^2-1})^2 \leq (3)^2$$

$$x^2 - 1 \leq 9$$

$$x^2 \leq 9 + 1$$

$$x^2 \leq 10$$

$$\sqrt{x^2} \leq \sqrt{10}$$

$$|x| \leq \sqrt{10}$$

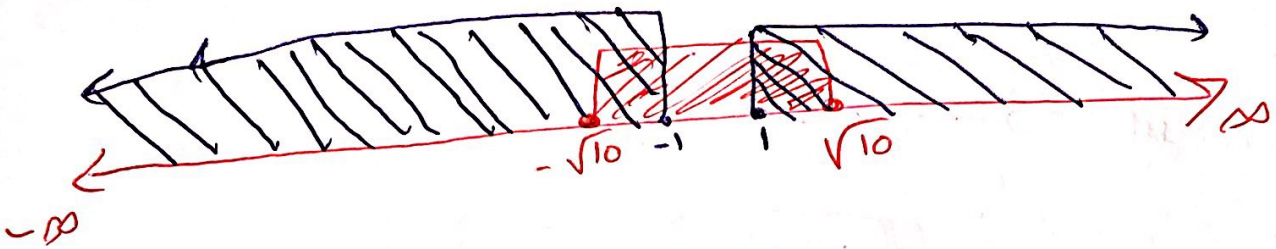
$$-\sqrt{10} \leq x \leq \sqrt{10}$$

$$\therefore D_{\text{New}} = [-\sqrt{10}, \sqrt{10}]$$

$$D_{(f \circ g)(x)} = D_{\text{New}} \cap D_{g(x)}$$

$$= [-\sqrt{10}, \sqrt{10}] \cap [(-\infty, -1] \cup [1, \infty)]$$

$$= [-\sqrt{10}, -1] \cup [1, \sqrt{10}]$$



Example

Find a) $f+g$, b) $f-g$; c) fg , d) $\frac{f}{g}$ and e) $\frac{g}{f}$ and state their domains.

1) $f(x) = \sqrt{x}$; $g(x) = \sqrt{2-x}$

$$D_{f(x)} = [0, \infty)$$

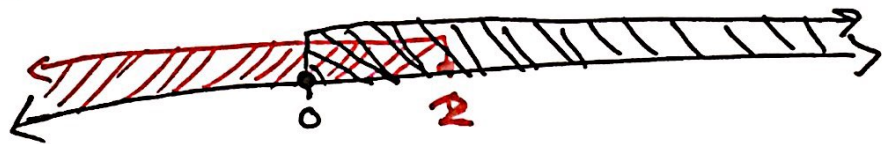
$$D_{g(x)}: 2-x \geq 0 \Rightarrow -x \geq -2 \Rightarrow x \leq 2$$

$$\therefore D_{g(x)} = (-\infty, 2]$$

$$\text{a) } (f+g)(x) = f(x) + g(x) \\ = \sqrt{x} + \sqrt{2-x}$$

$$\text{b) } (f-g)(x) = f(x) - g(x) \\ = \sqrt{x} - \sqrt{2-x}$$

$$D_{(f+g)(x)} = D_{f(x)} \cap D_{g(x)} = [0, \infty) \cap (-\infty, 2] = [0, 2]$$



$$(fg)(x) = f(x) \cdot g(x) = \sqrt{x} \cdot \sqrt{2-x}$$

$$D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)} = [0, 2]$$

Note $(fg)(x) = \sqrt{x} \cdot \sqrt{2-x} = \sqrt{x(2-x)}$
 $= \sqrt{2x-x^2}$ for all $x \in [0, 2]$

$$d) \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{2-x}}$$

✓ أصفار المقام: $2-x=0$

$$-x = -2$$

$$\boxed{x=2}$$

$$✓ D_{\left(\frac{f}{g}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \{ \text{أصفار المقام} \}$$

$$= [0, \infty) \cap (-\infty, 2] - \{2\}$$

$$= [0, 2] - \{2\}$$

$$= [0, 2)$$

Note

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{2-x}} = \sqrt{\frac{x}{2-x}} \text{ on } [0, 2)$$

$$e) \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{2-x}}{\sqrt{x}}$$

✓ أصفار المقام: $\boxed{x=0}$

$$✓ D_{\left(\frac{g}{f}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \{ \text{أصفار المقام} \}$$

$$= [0, \infty) \cap (-\infty, 2] - \{0\}$$

$$= [0, 2] - \{0\}$$

$$= (0, 2]$$

Note

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{2-x}}{\sqrt{x}}$$

$$= \sqrt{\frac{2-x}{x}} \quad \text{on } (0, 2]$$

$$= \sqrt{\frac{2}{x} - \frac{x}{x}} \quad \text{on } (0, 2]$$

$$= \sqrt{\frac{2}{x} - 1} \quad \text{on } (0, 2]$$

Example

Find the domine of $g(t) = \sqrt{3-t} - \sqrt{2+t}$

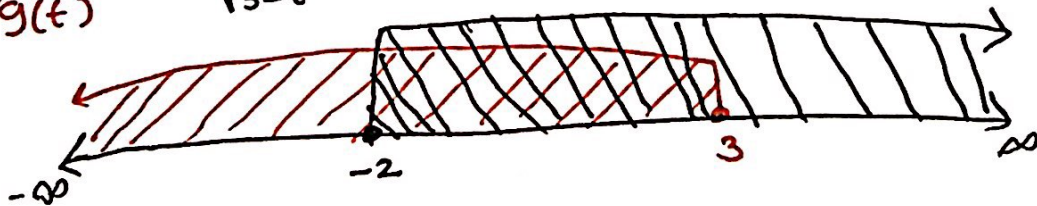
$$D_{\sqrt{3-t}} : 3-t \geq 0 \Rightarrow -t \geq -3 \Rightarrow t \leq 3$$

$$\therefore D_{\sqrt{3-t}} = (-\infty, 3]$$

$$D_{\sqrt{2+t}} : 2+t \geq 0 \Rightarrow t \geq -2$$

$$\therefore D_{\sqrt{2+t}} = [-2, \infty)$$

$$D_{g(t)} = D_{\sqrt{3-t}} \cap D_{\sqrt{2+t}} = (-\infty, 3] \cap [-2, \infty) = [-2, 3]$$



Example If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$

Find each function and its domain.

a) $f \circ g$

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}}$$
$$= \sqrt[4]{2-x} = \text{New Function.}$$

$$\checkmark D_{g(x)} : 2-x > 0 \Rightarrow -x > -2 \Rightarrow x \leq 2$$

$$\therefore D_{g(x)} = (-\infty, 2]$$

$$\checkmark D_{\text{New}} : 2-x > 0 \Rightarrow -x > -2 \Rightarrow x \leq 2$$

$$\therefore D_{\text{New}} = (-\infty, 2]$$

$$\checkmark D_{(f \circ g)(x)} = D_{g(x)} \cap D_{\text{New}} = (-\infty, 2] \cap (-\infty, 2]$$

$$D_{(f \circ g)(x)} = (-\infty, 2] \neq$$

⑥ $g \circ f$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) \\ = \sqrt{2 - \sqrt{x}} = \text{New function.}$$

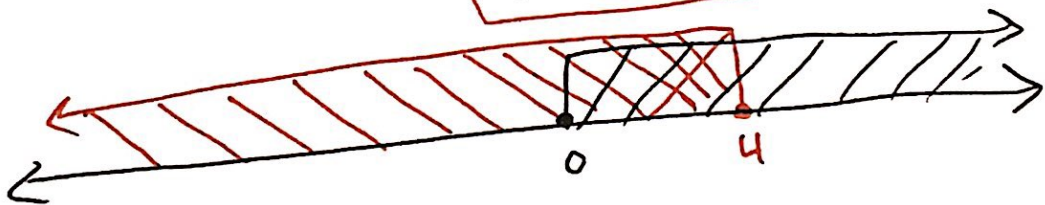
✓ $D_{f(x)} : x \geq 0$

∴ $D_{f(x)} = [0, \infty)$

✓ $D_{\text{New}} : 2 - \sqrt{x} \geq 0 \Rightarrow -\sqrt{x} \geq -2$
 $\sqrt{x} \leq 2 \Rightarrow (\sqrt{x})^2 \leq 2^2$
 $x \leq 4$

∴ $D_{\text{New}} = (-\infty, 4]$

✓ $D_{(g \circ f)(x)} = D_{\text{New}} \cap D_{f(x)} = (-\infty, 4] \cap [0, \infty)$
 $D_{(g \circ f)(x)} = [0, 4]$ #



c) f o f

$$(f \circ f)(x) = f(f(x)) = f(\sqrt{x})$$

$$= \sqrt{\sqrt{x}}$$

$$= \sqrt[4]{x} = \text{New Function}$$

✓ $D_{f(x)} : x \geq 0$

∴ $D_{f(x)} = [0, \infty)$

✓ $D_{\text{New}} : x \geq 0$

∴ $D_{\text{New}} = [0, \infty)$

✓ $D_{(f \circ f)(x)} = D_{\text{New}} \cap D_{f(x)} = [0, \infty) \cap [0, \infty)$

∴ $D_{(f \circ f)(x)} = [0, \infty)$

d) $g \circ g$

$$(g \circ g)(x) = g(g(x)) = g(\sqrt{2-x}) \\ = \sqrt{2 - \sqrt{2-x}} = \text{New Function}$$

$$\checkmark D_{g(x)}: 2-x \geq 0 \Rightarrow -x \geq -2 \Rightarrow x \leq 2$$

$$\therefore D_{g(x)} = (-\infty, 2]$$

$$\checkmark D_{\text{New}}: 2 - \sqrt{2-x} \geq 0 \Rightarrow -\sqrt{2-x} \geq -2 \\ \sqrt{2-x} \leq 2 \Rightarrow (\sqrt{2-x})^2 \leq 2^2$$

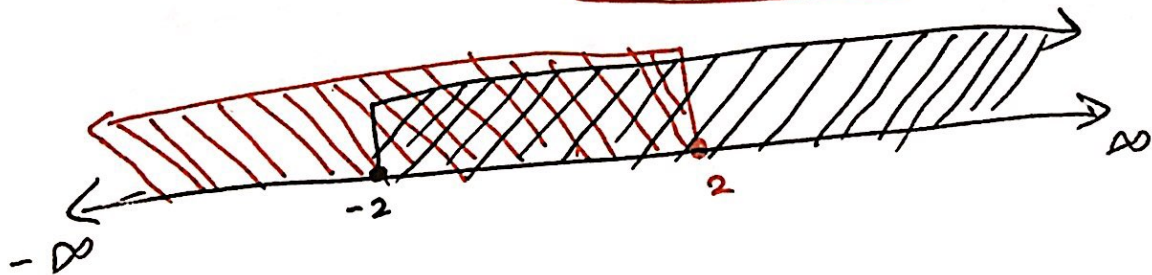
$$2-x \leq 4 \Rightarrow -x \leq 4-2$$

$$-x \leq 2 \Rightarrow x \geq -2$$

$$\therefore D_{\text{New}} = [-2, \infty)$$

$$\checkmark D_{(g \circ g)(x)} = D_{\text{New}} \cap D_{g(x)} = [-2, \infty) \cap (-\infty, 2]$$

$$\therefore D_{(g \circ g)(x)} = [-2, 2] \neq$$



33) الواجب منه > / < / س 1.3
 $f(x) = 3x + 5$; $g(x) = x^2 + x$

a) $(f \circ g)(x) = f(g(x)) = f(x^2 + x) = 3(x^2 + x) + 5$
 $= 3x^2 + 3x + 5$ New function

$$D_{(f \circ g)(x)} = D_{\text{New}} \cap D_g = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

b) $(g \circ f)(x) = g(f(x)) = g(3x + 5)$

$$= (3x + 5)^2 + (3x + 5)$$

$$= 9x^2 + \underline{30x} + \underline{25} + \underline{3x} + \underline{5}$$

$$= 9x^2 + 33x + 30$$
 New function

$$D_{(g \circ f)(x)} = D_{\text{New}} \cap D_f = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

c) $(f \circ f)(x) = f(f(x)) = f(3x + 5) = 3(3x + 5) + 5$

$$= 9x + 15 + 5$$

$$= 9x + 20$$
 New function

$$D_{(f \circ f)(x)} = D_{\text{New}} \cap D_f = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

d) $(g \circ g)(x) = g(g(x)) = g(x^2 + x) = (x^2 + x)^2 + (x^2 + x)$

$$= x^4 + 2x^3 + \underline{x^2} + \underline{x^2} + x$$

$$= x^4 + 2x^3 + 2x^2 + x$$
 New function

$$D_{(g \circ g)(x)} = D_{\text{New}} \cap D_g = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

$$35) f(x) = \sqrt{x+1} \quad g(x) = 4x - 3$$

$$D_{f(x)} : \begin{aligned} x+1 &\geq 0 \\ x &\geq -1 \end{aligned}$$

$$D_{g(x)} = \mathbb{R} = (-\infty, \infty)$$

$$\therefore D_{f(x)} = [-1, \infty)$$

$$a) (f \circ g)(x) = f(g(x)) = f(4x-3) = \sqrt{4x-3+1}$$

$$= \sqrt{4x-2} \quad \text{New Function}$$

$$D_{\text{New}} : 4x-2 \geq 0 \Rightarrow 4x \geq 2 \Rightarrow \frac{4x}{2} \geq \frac{2}{2} \Rightarrow x \geq \frac{1}{2}$$

$$\therefore D_{\text{New}} = [\frac{1}{2}, \infty)$$

$$D_{(f \circ g)(x)} = D_{\text{New}} \cap D_g = [\frac{1}{2}, \infty) \cap \mathbb{R} = [\frac{1}{2}, \infty)$$

$$b) (g \circ f)(x) = g(f(x)) = g(\sqrt{x+1}) = \frac{4\sqrt{x+1} - 3}{\text{New Function}}$$

$$\begin{aligned} \text{Domain of New Function} &= (\text{Domain of } 4\sqrt{x+1}) \cap (\text{Domain of } -3) \\ &= [-1, \infty) \cap \mathbb{R} \\ &= [-1, \infty) \end{aligned}$$

$$D_{(g \circ f)(x)} = D_{\text{New}} \cap D_f = [-1, \infty) \cap [-1, \infty) = [-1, \infty)$$

$$b) (f \circ f)(x) = f(f(x)) = f(\sqrt{x+1})$$

$$= \sqrt{\sqrt{x+1} + 1} \text{ New function}$$

$$D_{\text{New}} : \sqrt{x+1} + 1 \geq 0$$

$$\sqrt{x+1} \geq -1 \Rightarrow \sqrt{x+1} \text{ مستحيل لأن الجذر أكبر من 0}$$

$$\sqrt{x+1} \geq 0 \Rightarrow (\sqrt{x+1})^2 \geq 0^2$$

$$\therefore D_{\text{New}} = [-1, \infty) \Rightarrow \begin{matrix} x+1 \geq 0 \\ x \geq -1 \end{matrix}$$

$$D_{(f \circ f)(x)} = D_{\text{New}} \cap D_f = [-1, \infty) \cap [-1, \infty)$$

$$= [-1, \infty)$$

$$c) (g \circ g)(x) = g(g(x)) = g(4x-3)$$

$$= 4(4x-3) - 3$$

$$= 16x - 12 - 3$$

$$= 16x - 15 \text{ New function}$$

$$D_{\text{New}} = \mathbb{R}$$

$$D_{(g \circ g)(x)} = D_{\text{New}} \cap D_{g(x)} = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

$$38) f(x) = \sqrt{x} ; g(x) = \sqrt[3]{1-x}$$

$$D_{f(x)} : x \geq 0 \Rightarrow D_{f(x)} = [0, \infty)$$

$$D_{g(x)} : D_{g(x)} = \mathbb{R}$$

$$a) (f \circ g)(x) = f(g(x)) = f(\sqrt[3]{1-x}) = \sqrt[2]{\sqrt[3]{1-x}} = \sqrt[6]{1-x} \text{ New function}$$

$$D_{\text{New}} : 1-x \geq 0 \Rightarrow -x \geq -1 \Rightarrow x \leq 1$$

$$\therefore D_{\text{New}} = (-\infty, 1]$$

$$D_{(f \circ g)(x)} = D_{\text{New}} \cap D_{g(x)} \\ = (-\infty, 1] \cap \mathbb{R} = (-\infty, 1]$$

$$b) (g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt[3]{1-\sqrt{x}} \text{ New function}$$

$$D_{\text{New}} = D_{\sqrt[3]{x}} \cap D_{(1-\sqrt{x})} \\ = \mathbb{R} \cap [0, \infty) \\ = [0, \infty)$$

$$D_{(g \circ f)(x)} = D_{\text{New}} \cap D_{f(x)} \\ = [0, \infty) \cap [0, \infty) \\ = [0, \infty)$$

$$c) (f \circ f)(x) = f(f(x))$$

$$= f(\sqrt{x})$$

$$= \sqrt[2]{\sqrt{x}}$$

$$= \sqrt[4]{x} \quad \text{New function}$$

$$D_{\text{New}}: x \geq 0$$

$$\therefore D_{\text{New}} = [0, \infty)$$

$$D_{(f \circ f)(x)} = D_{\text{New}} \cap D_{f(x)}$$

$$= [0, \infty) \cap [0, \infty)$$

$$= [0, \infty)$$

$$d) (g \circ g)(x) = g(g(x)) = g(\sqrt[3]{1-x}) = \sqrt[3]{1 - \sqrt[3]{1-x}} \quad \text{New function}$$

$$D_{\text{New}} = D \cap D_{(1 - \sqrt[3]{1-x})}$$

$$= \mathbb{R} \cap \mathbb{R}$$

$$= \mathbb{R}$$

$$\therefore D_{(g \circ g)(x)} = D_{\text{New}} \cap D_{g(x)}$$

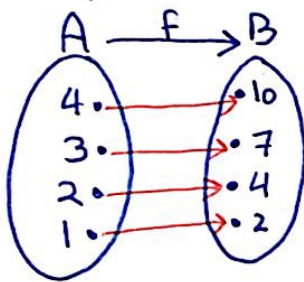
$$= \mathbb{R} \cap \mathbb{R}$$

$$= \mathbb{R}$$

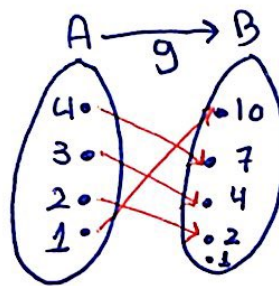
1.5 - Inverse Functions and Logarithms

Definition: A function f is called a **One-to-One function** if it never takes on the same value twice; that is $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$
 or $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$
 or $x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$

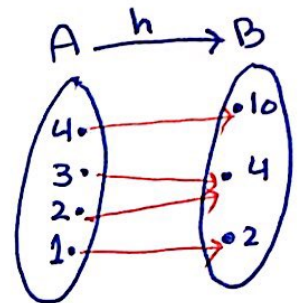
Example:



f is one-to-one function.



g is 1-1 function

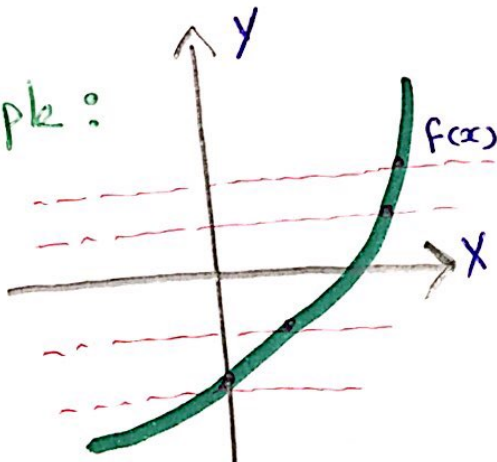


h is not one to one function
 since: $3 \rightarrow 4$
 $2 \rightarrow 4$
 or $h(3) = h(2) = 4$

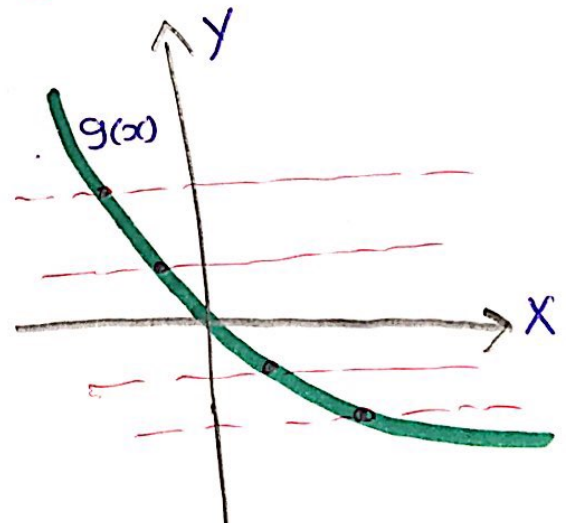
Horizontal Line Test

A function is one-to-one if and only if **no horizontal line intersects its graph more than once**

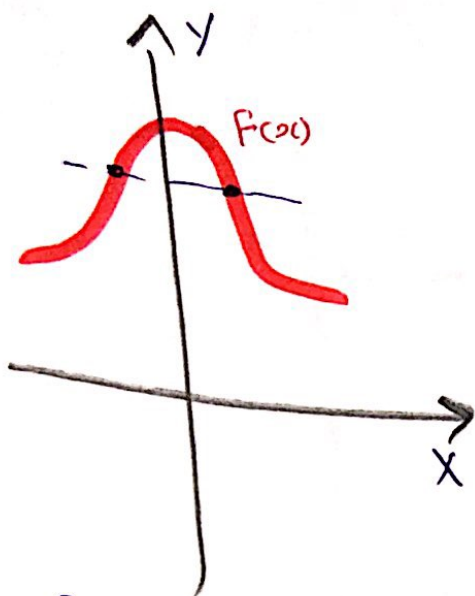
Example:



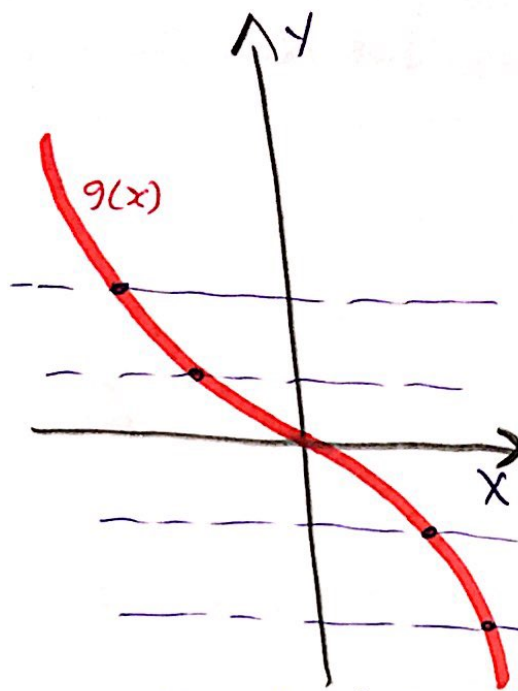
$f(x)$ is 1-1 function.



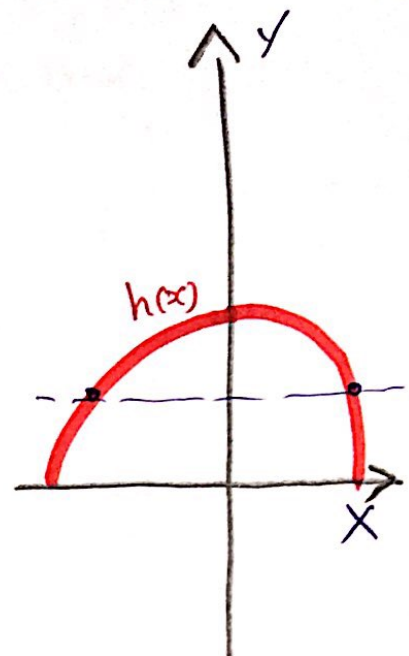
$g(x)$ is 1-1 function



$f(x)$ is not 1-1 function



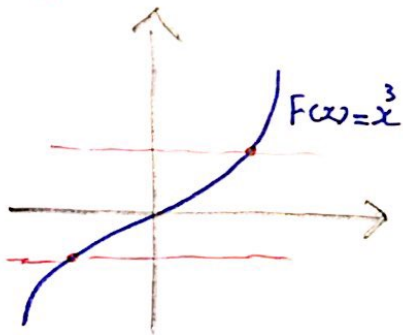
$g(x)$ is 1-1 function



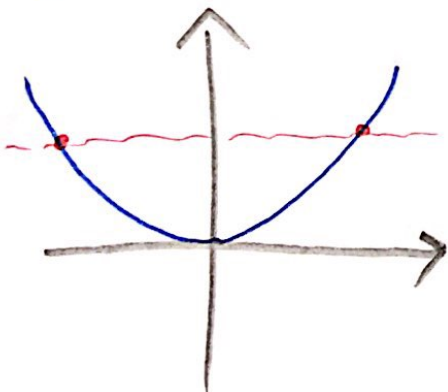
$h(x)$ is not 1-1 function

Example:- A function is given by a formula
Determine whether it is 1-1

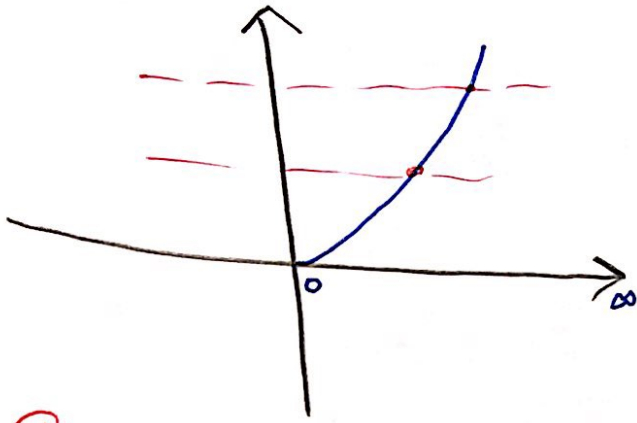
a) $f(x) = x^3$ is 1-1 function



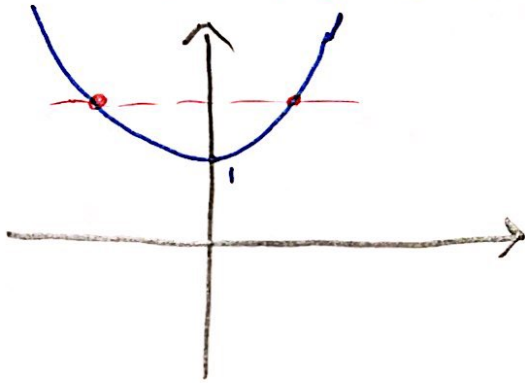
b) $f(x) = x^2$ is not 1-1 function



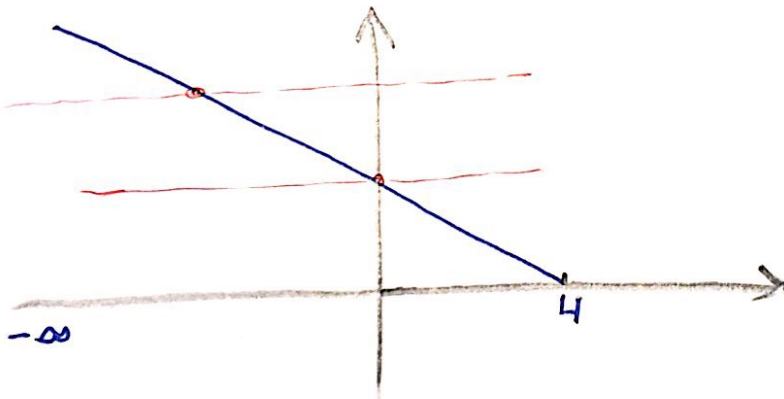
c) $f(x) = x^2$ on $[0, \infty)$ is 1-1 function



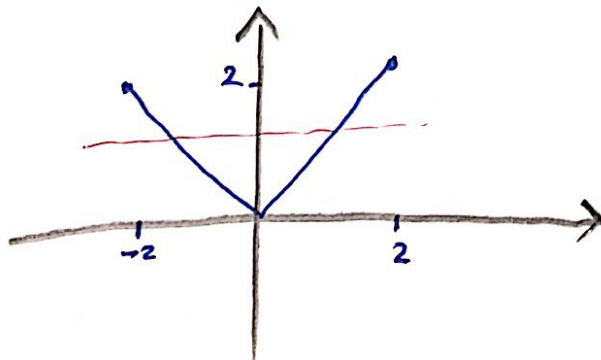
d) $f(x) = x^2 + 1$ is not 1-1 function



e) $f(x) = |x - 4|$ on $(-\infty, 4]$ is 1-1 function



f) $f(x) = |x|$ on $[-2, 2]$ is not 1-1 function



$$(g) f(x) = \sqrt[3]{1-x^5}$$

$$\text{let } x_1 \neq x_2$$

$$x_1^5 \neq x_2^5$$

$$-x_1^5 \neq -x_2^5$$

$$1-x_1^5 \neq 1-x_2^5$$

$$\sqrt[3]{1-x_1^5} \neq \sqrt[3]{1-x_2^5}$$

$$f(x_1) \neq f(x_2)$$

$$\therefore x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

$\therefore f(x) = \sqrt[3]{1-x^5}$ is 1-1 function

$$(h) f(x) = \frac{x-1}{x+3}$$

$$\text{let } f(x_1) = f(x_2)$$

$$\frac{x_1-1}{x_1+3} = \frac{x_2-1}{x_2+3}$$

$$(x_1-1)(x_2+3) = (x_1+3)(x_2-1)$$

$$\cancel{x_1}x_2 + 3x_1 - x_2 - \cancel{3} = \cancel{x_1}x_2 - x_1 + 3x_2 - \cancel{3}$$

$$3x_1 - x_2 = -x_1 + 3x_2$$

$$3x_1 + x_1 = 3x_2 + x_2$$

$$\underline{4x_1} = \underline{4x_2}$$

$$x_1 = x_2$$

$$\therefore f(x) = f(x_2) \Rightarrow x_1 = x_2$$

$\therefore f(x) = \frac{x-1}{x+3}$ is 1-1 function.

$$(i) f(x) = \sqrt{x}$$

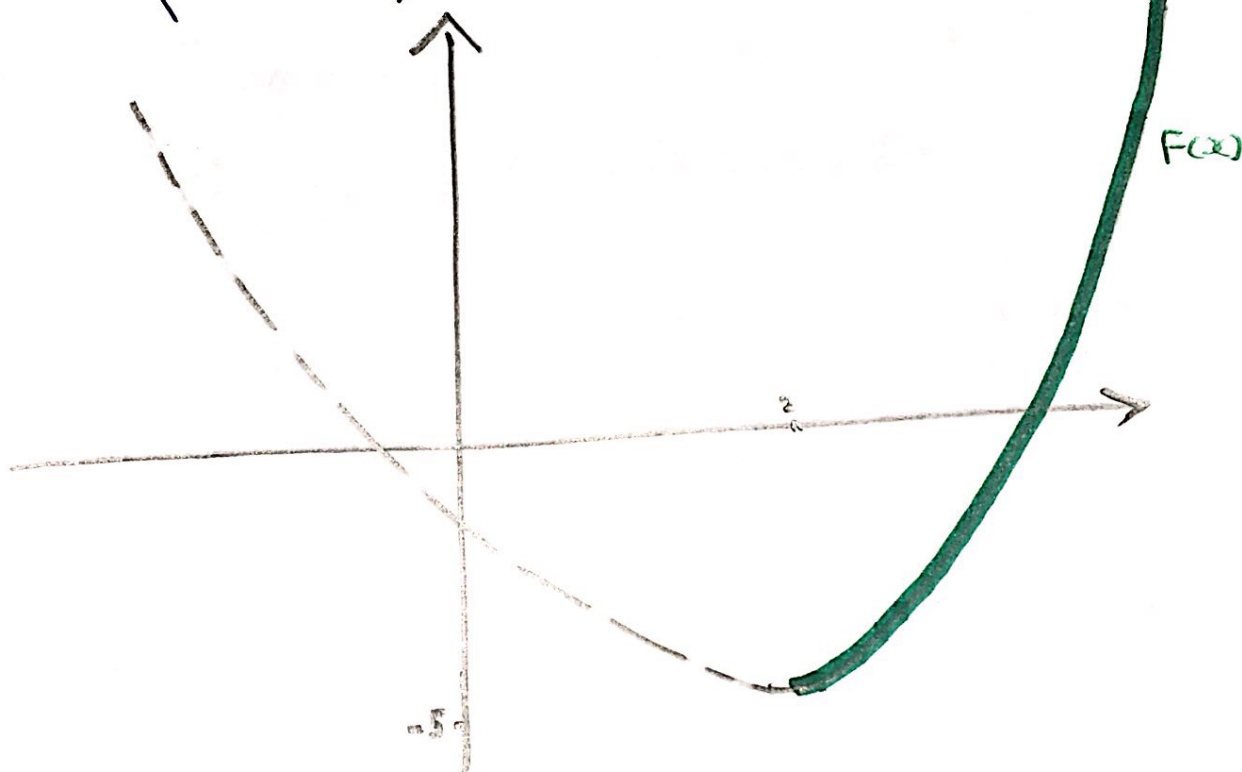
H.W

$$(j) f(x) = x^2 - 4x - 1 \text{ on } [2, \infty)$$

$$= x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 - 1$$

$$= x^2 - 4x + 4 - 4 - 1$$

$$= (x - 2)^2 - 5 \text{ is 1-1 function on } [2, \infty)$$



The inverse function

if $f(x) : A \rightarrow B$ is 1-1 function with

$$D_{f(x)} = A \quad \text{and} \quad R_{f(x)} = B \quad \text{"} R_{f(x)} = \text{Co-Domain"}$$

then $f^{-1}(x) : B \rightarrow A$ with $D_{f^{-1}(x)} = B$ and $R_{f^{-1}(x)} = A$

Note: ① Domain of $f^{-1} = \text{Range of } f$

Range of $f^{-1} = \text{Domain of } f$

$$\begin{aligned} \textcircled{2} \quad f(x) = y &\iff f^{-1}(y) = x & \textcircled{3} \quad f^{-1}(x) \neq \frac{1}{f(x)} \\ f(y) = x &\iff f^{-1}(x) = y \end{aligned}$$

Example:

① if $f(x) = \sqrt{3-x}$ then find Range of $f^{-1}(x)$

$$R_{f^{-1}(x)} = D_{f(x)} = (-\infty, 3] \quad \text{" 1.1 مع > من "}$$

② if $f(2) = 5$ then $f^{-1}(5) = 2 \dots$
if $f^{-1}(3) = 7$ then $f(7) = 3 \dots$

③ if $f(x) = \sqrt{x} + 10$ then find Domain of $f^{-1}(x)$

$$D_{f^{-1}(x)} = R_{f(x)} = [10, \infty) \quad \text{" 1.3 مع > من "}$$

③ if $f(x) = x^3 - 1$ then find $f^{-1}(7)$

$$\begin{aligned}f^{-1}(7) = x &\implies f(x) = 7 \\x^3 - 1 &= 7 \\x^3 &= 7 + 1 \\x^3 &= 8 \\\sqrt[3]{x^3} &= \sqrt[3]{8} \\x &= 2\end{aligned}$$

$$\therefore f^{-1}(7) = \underline{2}$$

Note

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x \quad \text{for every } x \in D_{f^{-1}} = B$$

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x \quad \text{for every } x \in D_{f^{-1}} = A$$

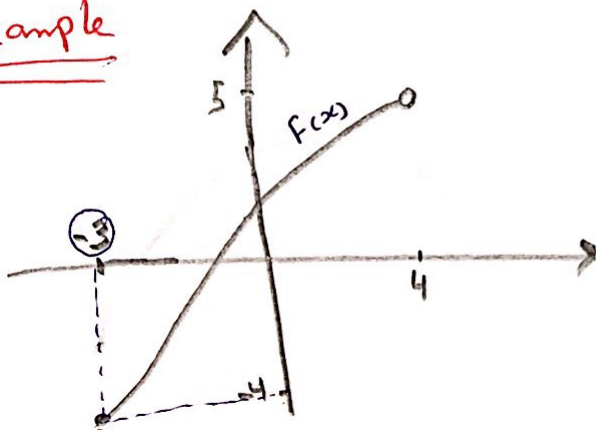
Example

if $f(x) = \sqrt[3]{3-2x}$ then find

$$a) (f \circ f^{-1})(2) = f(f^{-1}(2)) = 2 \in D_{f^{-1}} = \mathbb{R}$$

$$b) (f^{-1} \circ f)(2) = f^{-1}(f(2)) = 2 \in D_{f^{-1}} = \mathbb{R}$$

Example



$$① D_{f^{-1}} = R_{f(x)} = [-4, 5)$$

$$② R_{f^{-1}} = D_{f(x)} = [-3, 4)$$

$$③ f^{-1}(-4) = -3$$

How to find the inverse function of 1-1 function

- ① Write $y = f(x)$
- ② Solve this equation for x in terms of y
- ③ To express f^{-1} as a function of x , interchange x and y the resulting equation is $y = f^{-1}(x)$

Example

find the inverse function $f(x) = x^3 + 2$

$$\textcircled{1} y = x^3 + 2$$

$$\textcircled{2} y - 2 = x^3$$

$$\sqrt[3]{y-2} = \sqrt[3]{x^3}$$

$$\sqrt[3]{y-2} = x$$

$$\textcircled{3} \sqrt[3]{x-2} = y$$

$$\textcircled{4} f^{-1}(x) = \sqrt[3]{x-2}$$

Example

Find $f^{-1}(x)$ of the following functions:

1) $f(x) = 1 + \sqrt{2 + 3x}$

a) $y = 1 + \sqrt{2 + 3x}$

b) $y - 1 = \sqrt{2 + 3x}$

$$(y-1)^2 = (\sqrt{2+3x})^2$$

$$(y-1)^2 = 2 + 3x$$

$$(y-1)^2 - 2 = 3x$$

$$\frac{(y-1)^2 - 2}{3} = \frac{3x}{3}$$

$$\frac{(y-1)^2 - 2}{3} = x$$

c) $\frac{(\cancel{x}-1)^2 - 2}{3} = y$

d) $f^{-1}(x) = \frac{1}{3} [(x-1)^2 - 2]$

$$\textcircled{2} f(x) = \frac{4x-1}{2x+3}$$

$$y = \frac{4x-1}{2x+3}$$

$$y(2x+3) = 4x-1$$

$$2yx + 3y = 4x - 1$$

$$2yx - 4x = -1 - 3y$$

$$x(2y-4) = -1-3y$$

$$x = \frac{-1-3y}{2y-4}$$

$$y = \frac{-1-3x}{2x-4}$$

$$f^{-1}(x) = \frac{-1-3x}{2x-4} = \frac{-(3x+1)}{-(4-2x)} = \frac{3x+1}{4-2x}$$

$$\textcircled{3} f(x) = e^{2x-1}$$

$$y = e^{2x-1}$$

$$\ln(y) = \ln(e^{2x-1})$$

$$\ln(y) = 2x-1$$

$$\ln(y) + 1 = 2x$$

$$\frac{\ln(y) + 1}{2} = x$$

$$\frac{\ln(x) + 1}{2} = y \quad \Rightarrow \quad f^{-1}(x) = \frac{\ln(x) + 1}{2} = \frac{1}{2} [\ln(x) + 1]$$

$$\textcircled{4} \quad y = \ln(x+3)$$

$$y = \ln(x+3)$$

$$e^y = e^{\ln(x+3)}$$

$$e^y = x+3$$

$$e^y - 3 = x$$

$$e^x - 3 = y$$

$$f^{-1}(x) = e^x - 3$$

$$\textcircled{5} \quad y = \frac{e^x}{1+2e^x}$$

$$y = \frac{e^x}{1+2e^x}$$

$$y(1+2e^x) = e^x$$

$$y + 2ye^x = e^x$$

$$2ye^x - e^x = -y$$

$$e^x(2y-1) = -y$$

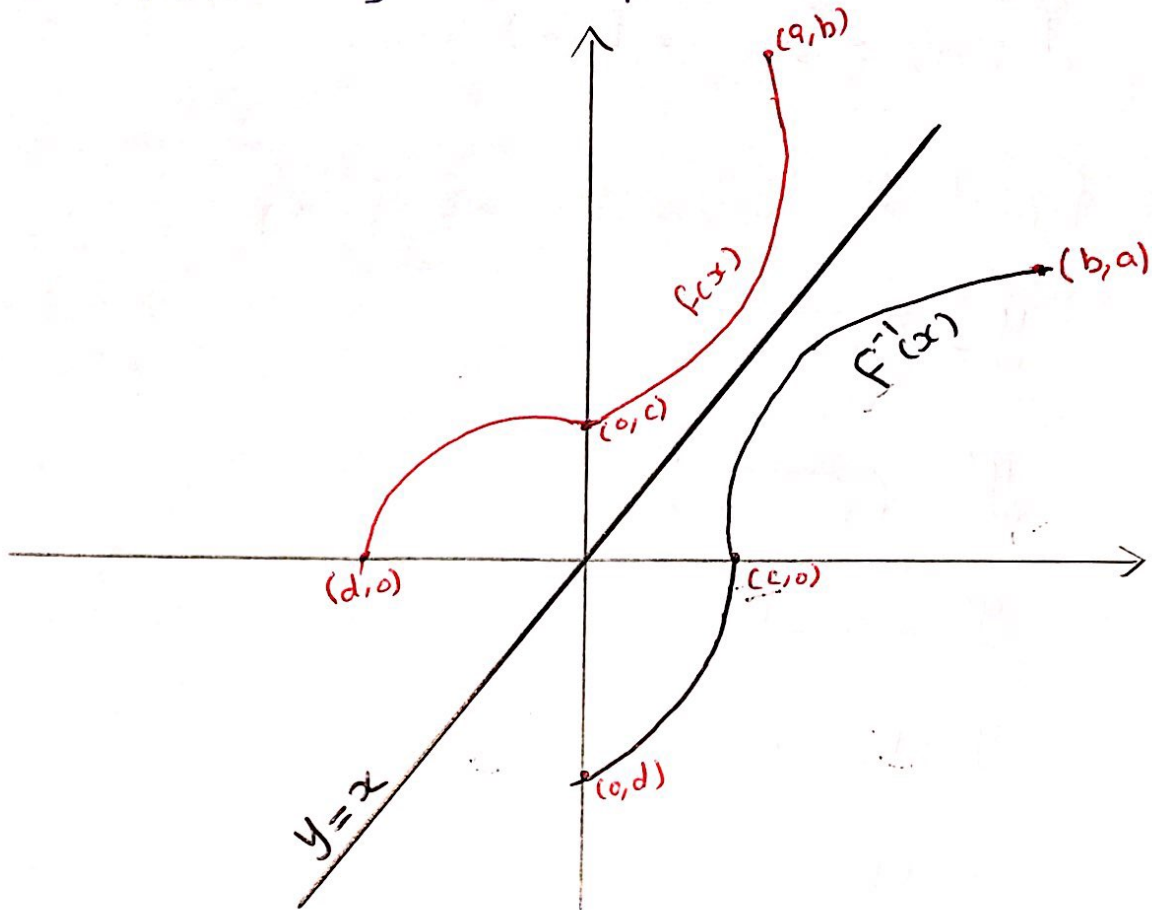
$$e^x = \frac{-y}{2y-1}$$

$$\ln e^x = \ln\left(\frac{-y}{2y-1}\right)$$

$$x = \ln\left(\frac{y}{1-2y}\right) \Rightarrow y = \ln\left(\frac{x}{1-2x}\right) = f^{-1}(x)$$

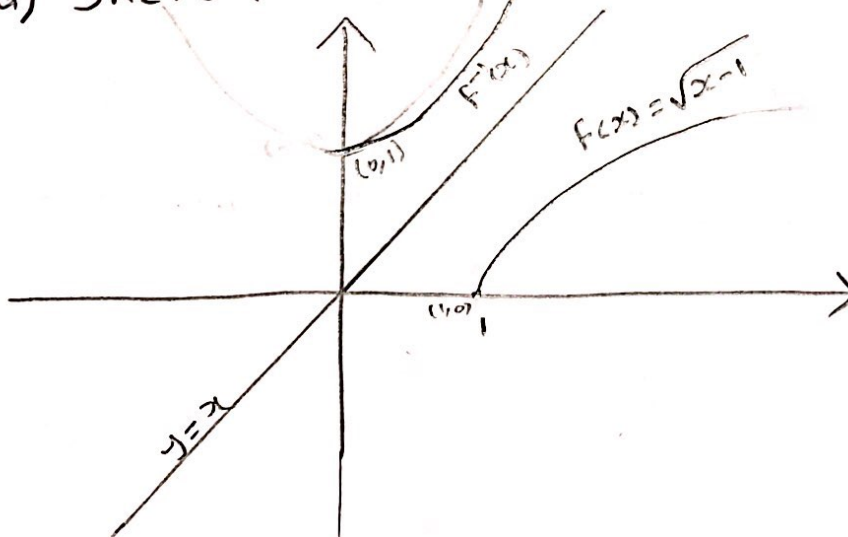
Note

The graph of f^{-1} is obtained by reflecting the graph f about Line $y=x$



Example

if $f(x) = \sqrt{x-1}$ then
a) sketch $f(x)$ and $f^{-1}(x)$



b) Find the Domain and Range of $f^{-1}(x)$

$$D_{f^{-1}(x)} = R_{f(x)} = [0, \infty)$$

$$R_{f^{-1}(x)} = D_{f(x)} = [1, \infty)$$

c) Find the formula of $f^{-1}(x)$

$$y = \sqrt{x-1}$$

$$y^2 = (\sqrt{x-1})^2$$

$$y^2 = x-1$$

$$y^2 + 1 = x$$

$$x^2 + 1 = y$$

$$f^{-1}(x) = x^2 + 1 \quad \text{on } [0, \infty) \text{ " } D_{f^{-1}(x)} \text{ "}$$

Logarithmic function

Rules of logarithms

$$1) \log_a 1 = 0 \quad ; \quad \ln(1) = 0$$

$$\text{Example: } \log_3 1 = 0$$

$$2) \log_a a = 1 \quad ; \quad \ln(e) = 1$$

$$\text{Example: } \log_7 7 = 1$$

$$3) \log_a b = \frac{\ln(b)}{\ln(a)} \quad ; \quad \ln(x) = \log_e x$$

$$\text{Example: } \log_2 3 = \frac{\ln 3}{\ln 2} \quad ; \quad \frac{\ln(5x)}{\ln(7)} = \log_7(5x)$$

$$4) \log_a x = y \Leftrightarrow a^y = x$$

$$\ln(x) = y \Leftrightarrow e^y = x$$

$$\text{Example: } \log_a (2b) = c \Leftrightarrow a^c = 2b$$

$$\ln(5d) = 3+d \Leftrightarrow e^{(3+d)} = 5d.$$

$$5) \log_a x^n = n \log_a x$$

$$\ln(x^n) = n \ln(x)$$

$$\text{Example: } \log_4 x^3 = 3 \log_4 x$$

$$\log_3 9 = \log_3 3^2$$

$$= 2 \log_3 3$$

$$= 2(1)$$

$$= 2$$

$$\ln(y^7) = 7 \ln(y)$$

$$\ln(9) = \ln(3^2) \\ = 2 \ln(3)$$

$$6) \log_a(xy) = \log_a x + \log_a y$$

$$\ln(xy) = \ln(x) + \ln(y)$$

$$7) \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

Note

$$\checkmark \log_a(x \pm y) \neq \log_a x \pm \log_a y$$

Example:

$$\log_2(x^3 \pm y^4) = 3 \log_2 x \pm 4 \log_2 y \quad \checkmark \quad (\text{X})$$

$$\checkmark \ln(x \pm y) \neq \ln(x) \pm \ln(y)$$

$$\text{Example: } \log_3(3x^4) = \log_3 3 + \log_3 x^4 = 1 + 4 \log_3 x$$

$$\ln(3x^4) = \ln(3) + \ln(x^4) = \ln(3) + 4 \ln(x)$$

$$\log_3(15) = \log_3(3 \times 5) = \log_3 3 + \log_3 5 = 1 + \log_3 5$$

$$\ln(15) = \ln(3 \times 5) = \ln(3) + \ln(5)$$

$$\log_5\left(\frac{1}{125}\right) = \log_5(1) - \log_5 125 = 0 - \log_5 5^3 = -3 \log_5 5 = -3(1) = -3$$

$$\log_2 80 - \log_2 5 = \log_2\left(\frac{80}{5}\right) = \log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4(1) = 4$$

$$\ln 80 - \ln 5 = \ln\left(\frac{80}{5}\right)$$

$$= \ln(16)$$

$$= \ln(2^4)$$

$$= 4 \ln(2)$$

$$\textcircled{8} \log_a a^x = x \text{ for all } x \in \mathbb{R}$$

$$a^{\log_a x} = x \text{ for all } x > 0$$

$$\textcircled{9} \ln(e^x) = x \text{ for all } x \in \mathbb{R}$$

$$e^{\ln x} = x \text{ for all } x > 0$$

$$\text{Example: } \ln e^{x+1} = x+1$$

$$e^{\ln(x+1)} = x+1$$

$$e^{\ln x + 1} = e^{\ln x} \cdot e^1 = xe$$

$$\ln e^{x^2} = x^2$$

$$e^{\ln(\frac{1}{2})} = \frac{1}{2}$$

$$\log_{11} 11^5 = 5$$

$$10^{\log \sqrt{2}} = \sqrt{2}$$

$\textcircled{10}$ if $f(x) = \ln(g(x))$ or $f(x) = \log_a(g(x))$ then

$$D_{f(x)} : g(x) > 0$$

Example: Find the Domain of $f(x) = \ln(25 - x^2)$

$$25 - x^2 > 0 \Rightarrow -x^2 > -25 \Rightarrow x^2 < 25$$

$$\Rightarrow \sqrt{x^2} < \sqrt{25}$$

$$\Rightarrow |x| < 5$$

$$\Rightarrow -5 < x < +5$$

$$\therefore D_{f(x)} = (-5, 5)$$

Example: Find the Domain of the following Functions

① $f(x) = \ln(25 - x^2)$

$$25 - x^2 > 0$$

$$-x^2 > -25$$

$$x^2 < 25$$

$$\sqrt{x^2} < \sqrt{25}$$

$$|x| < 5$$

$$-5 < x < 5$$

$$\therefore D_{f(x)} = (-5, 5)$$

$$= \{x \mid -5 < x < 5\}$$

③ $h(x) = \log_5(x^2 + 16)$

$$x^2 + 16 > 0$$

$$x^2 > -16$$

$$\sqrt{x^2} > \sqrt{-16}$$

$$|x| > \sqrt{-16} \notin \mathbb{R}$$

$$\therefore D_{h(x)} = \mathbb{R}$$

5) $g(x) = \ln(x^2 - 6)$
H.W

⑦ $f(x) = \log_2(x^2 + 5)$
H.W

② $\ln(x^2 - 100) = g(x)$

$$x^2 - 100 > 0$$

$$x^2 > 100$$

$$\sqrt{x^2} > \sqrt{100}$$

$$|x| > 10$$

$$x > 10 \text{ or } x < -10$$

$$\therefore D_{g(x)} = (10, \infty) \cup (-\infty, -10)$$



$$\text{or } D_{g(x)} = \{x \mid x > 10 \text{ or } x < -10\}$$

$$\text{or } D_{g(x)} = \mathbb{R} - [-10, 10]$$

④ $f(x) = \log_3(x^3 + 1)$

$$x^3 + 1 > 0$$

$$x^3 > -1$$

$$\sqrt[3]{x^3} > \sqrt[3]{-1}$$

$$x > -1$$

$$\therefore D_{f(x)} = (-1, \infty)$$

⑥ $\log_7(6 - x^2) = h(x)$
H.W

⑧ $f(x) = \ln(x^3 - 8)$
H.W

Example

① Find the Domain of $f(x) = \ln(e^x - 3)$

$$e^x - 3 > 0 \Rightarrow e^x > 3$$

$$\ln e^x > \ln 3$$

$$x > \ln 3$$

$$\therefore D_{f(x)} = (\ln 3, \infty)$$

② Find the range of $f^{-1}(x)$

$$R_{f^{-1}(x)} = D_{f(x)} = (\ln 3, \infty)$$

③ Find $f^{-1}(x)$ and find the Domain of $f^{-1}(x)$

$$y = \ln(e^x - 3)$$

$$e^y = e^{\ln(e^x - 3)}$$

$$e^y = e^x - 3$$

$$e^y + 3 = e^x$$

$$\ln(e^y + 3) = \ln e^x$$

$$\ln(e^y + 3) = x$$

$$\ln(e^x + 3) = f^{-1}(x)$$

$$D_{f^{-1}(x)}: e^x + 3 > 0$$

$$e^x > -3$$

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$$\therefore D_{f^{-1}(x)} = \mathbb{R} = (-\infty, \infty)$$

④ Find $R_{f(x)}$

$$R_{f(x)} = D_{f^{-1}(x)} = \mathbb{R}$$

Example:

$$\log_2 32 = \log_2 2^5 = 5 \log_2 2 = 5(1) = 5$$

$$\ln(32) = \ln(2^5) = 5 \ln(2)$$

$$\log 10000 = 4$$

$$\log 0.01 = -2$$

$$\log_4 2 = \frac{\ln(2)}{\ln(4)} = \frac{\ln(2)}{\ln(2^2)} = \frac{1 \cancel{\ln(2)}}{2 \cancel{\ln(2)}} = \frac{1}{2}$$

$$\log_{27} 3 = \frac{\ln(3)}{\ln(27)} = \frac{\ln(3)}{\ln(3^3)} = \frac{1 \cancel{\ln(3)}}{3 \cancel{\ln(3)}} = \frac{1}{3}$$

$$\log_{32} 2 = \frac{\ln(2)}{\ln(32)} = \frac{\ln(2)}{\ln(2^5)} = \frac{1 \cancel{\ln(2)}}{5 \cancel{\ln(2)}} = \frac{1}{5}$$

$$\begin{aligned} \log 40 + \log 2.5 &= \log(40 \times 2.5) \\ &= \log(\cancel{40} \times \frac{25}{10}) \\ &= \log(4 \times 25) \\ &= \log(100) \\ &= \log_{10}(100) = 2 \end{aligned}$$

$$\begin{aligned} \log_8 60 - \log_8 3 - \log_8 5 &= \log_8 \left(\frac{60}{3}\right) - \log_8 5 \\ &= \log_8 20 - \log_8 5 \\ &= \log_8 \left(\frac{20}{5}\right) = \log_8 4 \\ &= \frac{\ln(4)}{\ln(8)} = \frac{\ln(2^2)}{\ln(2^3)} = \frac{2 \cancel{\ln(2)}}{3 \cancel{\ln(2)}} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned}
 \log_6 36 - \log_5 125 + 3 \log_7 49 &= \log_6 6^2 - \log_5 5^3 + 3 \log_7 7^2 \\
 &= 2 \log_6 6 - 3 \log_5 5 + 3(2) \log_7 7 \\
 &= 2(1) - 3(1) + 3(2)(1) \\
 &= 2 - 3 + 6 \\
 &= -1 + 6 \\
 &= 5
 \end{aligned}$$

$$\ln \sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x$$

$$\ln \left(\frac{1}{e^2} \right) = \ln(1) - \ln e^2 = 0 - 2 = -2$$

$$\frac{1}{2} \ln(3) - \ln(2) = \ln(3^{1/2}) - \ln(2) = \ln \sqrt{3} - \ln 2 = \ln \left(\frac{\sqrt{3}}{2} \right)$$

$$\ln(20) - 2 \ln(2) = \ln 20 - \ln 2^2 = \ln 20 - \ln 4 = \ln \left(\frac{20}{4} \right) = \ln(5)$$

$$\begin{aligned}
 \ln b + 2 \ln c - 3 \ln d &= \ln b + \ln c^2 - \ln d^3 \\
 &= \ln(bc^2) - \ln(d^3) \\
 &= \ln \left(\frac{bc^2}{d^3} \right)
 \end{aligned}$$

$$\begin{aligned}
 \ln \left(\frac{x^3}{\sqrt[5]{y^2}} \right) &= \ln(x^3) - \ln(\sqrt[5]{y^2}) = 3 \ln(x) - \ln y^{2/5} \\
 &= 3 \ln(x) - \frac{2}{5} \ln(y)
 \end{aligned}$$

$$e^{-\ln 7} = e^{\ln 7^{-1}} = 7^{-1} = \frac{1}{7}$$

$$e^{-3 \ln 2} = e^{\ln 2^{-3}} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$e^{\ln(\ln e^3)} = \ln e^3 = 3 \quad \text{or} \quad e^{\ln(\ln e^3)} = e^{\ln(3)} = 3$$

$$\frac{1}{3} \ln(x+2)^3 + \frac{1}{2} [\ln x - \ln(x^2+3x+2)^2]$$

$$\ln[(x+2)^3]^{1/3} + \frac{1}{2} \ln x - \frac{1}{2} \ln(x^2+3x+2)^2$$

$$\ln(x+2) + \ln x^{1/2} - \ln[(x^2+3x+2)^2]^{1/2}$$

$$\ln(x+2) + \ln\sqrt{x} - \ln(x^2+3x+2)$$

$$\ln(x+2) + \ln\sqrt{x} - \ln[(x+1)(x+2)]$$

$$\ln(x+2) + \ln\sqrt{x} - [\ln(x+1) + \ln(x+2)]$$

$$\cancel{\ln(x+2)} + \ln\sqrt{x} - \ln(x+1) - \cancel{\ln(x+2)}$$

$$\ln\sqrt{x} - \ln(x+1)$$

$$\ln\left[\frac{\sqrt{x}}{x+1}\right]$$

Example: Solve each equation for x

$$e^{x-1} = e^2$$

$$x-1=2$$

$$x=2+1$$

$$\boxed{x=3}$$

$$5^{x^2-6} = 25$$

$$5^{x^2-6} = 5^2$$

$$x^2-6=2$$

$$x^2=2+6$$

$$x^2=8$$

$$\sqrt{x^2} = \sqrt{8}$$

$$|x| = 2\sqrt{2}$$

$$\boxed{x = \pm 2\sqrt{2}}$$

$$e^{-x} = 5$$

$$\ln e^{-x} = \ln(5)$$

$$-x = \ln(5)$$

$$x = -\ln(5)$$

$$x = \ln(5^{-1})$$

$$x = \ln\left(\frac{1}{5}\right)$$

$$e^{5-3x} = 10$$

$$\ln(e^{5-3x}) = \ln(10)$$

$$5-3x = \ln(10)$$

$$-3x = \ln(10) - 5$$

$$3x = -\ln(10) + 5$$

$$x = \frac{-\ln(10) + 5}{3}$$

$$= \frac{1}{3}(5 - \ln(10))$$

$$2^{x-5} = 3$$

$$\ln 2^{x-5} = \ln(3)$$

$$(x-5)\ln(2) = \ln(3)$$

$$x-5 = \frac{\ln(3)}{\ln(2)}$$

$$x = \frac{\ln(3)}{\ln(2)} + 5$$

$$\boxed{x = \log_2 3 + 5}$$

$$\begin{aligned} e^{3x} &= 4e^{2x} \Rightarrow \ln(e^{3x}) = \ln(4e^{2x}) \\ &\Rightarrow 3x = \ln(4) + \ln(e^{2x}) \\ &\Rightarrow 3x = \ln(4) + 2x \\ &\Rightarrow 3x - 2x = \ln(4) \\ &\Rightarrow x = \ln 2^2 = 2\ln 2 \end{aligned}$$

$$\log_2 3x - 3 = 0$$

$$\log_2 3x = 3$$

$$2^3 = 3x$$

$$8 = 3x$$

$$\frac{8}{3} = \frac{3x}{3}$$

$$\boxed{\frac{8}{3} = x}$$

$$\log_2(3x-3) = 0$$

$$2^0 = 3x-3$$

$$1 = 3x-3$$

$$1+3 = 3x$$

$$4 = 3x$$

$$\frac{4}{3} = \frac{3x}{3}$$

$$\boxed{\frac{4}{3} = x}$$

$$\log(3x+10) = 2$$

$$10^2 = 3x+10$$

$$100 = 3x+10$$

$$100-10 = 3x$$

$$90 = 3x$$

$$\frac{90}{3} = \frac{3x}{3}$$

$$\boxed{30 = x}$$

$$\ln x - 5 = 0$$

$$\ln x = 5$$

$$e^{\ln x} = e^5$$

$$\boxed{x = e^5}$$

$$\ln(x-5) = 0$$

$$e^{\ln(x-5)} = e^0$$

$$x-5 = 1$$

$$x = 1+5$$

$$\boxed{x = 6}$$

$$\ln x + 2 = 3$$

$$\ln x = 3-2$$

$$\ln x = 1$$

$$e^{\ln x} = e^1$$

$$\boxed{x = e}$$

$$\ln(x+2) = 3$$

$$e^{\ln(x+2)} = e^3$$

$$x+2 = e^3 \Rightarrow \boxed{x = e^3 - 2}$$

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~~7.11.11.2~~

$$2 \ln x - 1 = 0$$

$$2 \ln x = 1$$

$$\ln x = \frac{1}{2}$$

$$e^{\ln x} = e^{1/2}$$

$$x = e^{1/2}$$

$$\boxed{x = \sqrt{e}}$$

$$\ln x^2 - 1 = 3$$

$$\ln x^2 = 3+1$$

$$2 \ln x = 4$$

$$\ln x = \frac{4}{2}$$

$$\ln x = 2$$

$$e^{\ln x} = e^2$$

$$\boxed{x = e^2}$$

H.W

$$\ln(5-2x) = 3$$

$$\ln 5 - 2x = 3$$

Solve: a) $\ln(x^2-1) = 3$

$$e^{\ln(x^2-1)} = e^3$$

$$x^2 - 1 = e^3$$

$$x^2 = e^3 + 1$$

$$\sqrt{x^2} = \sqrt{e^3 + 1}$$

$$|x| = \sqrt{e^3 + 1}$$

$$x = \pm \sqrt{e^3 + 1}$$

b) $e^{2x} - 3e^x + 2 = 0$

$$(e^x)^2 - 3e^x + 2 = 0$$

$$(e^x - 1)(e^x - 2) = 0$$

$$e^x - 1 = 0$$

$$e^x = 1$$

$$\ln e^x = \ln 1$$

$$x = 0$$

$$e^x - 2 = 0$$

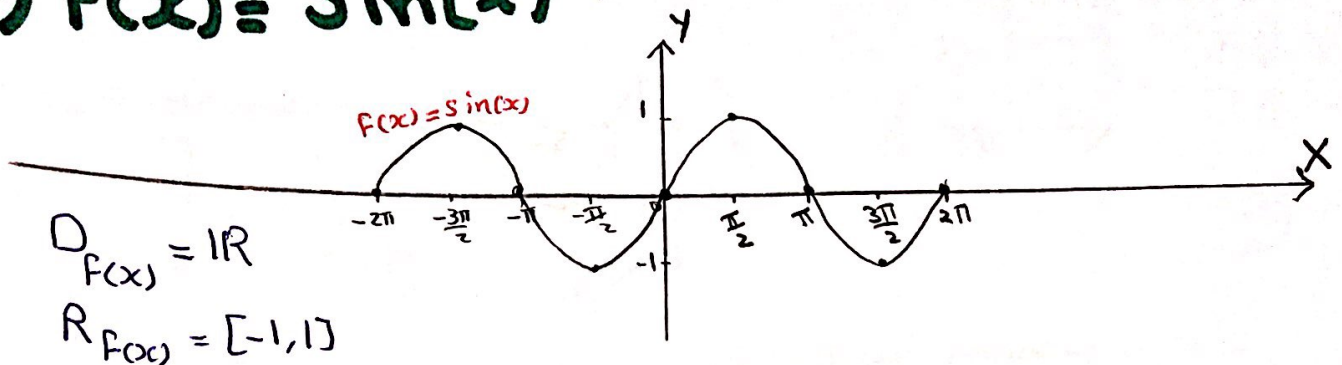
$$e^x = 2$$

$$\ln e^x = \ln 2$$

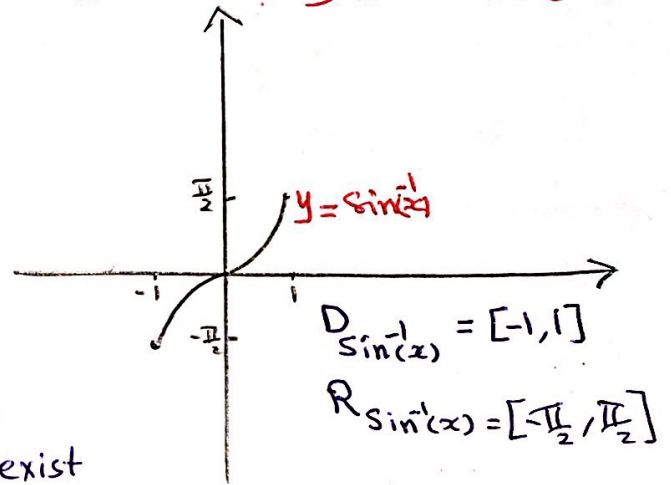
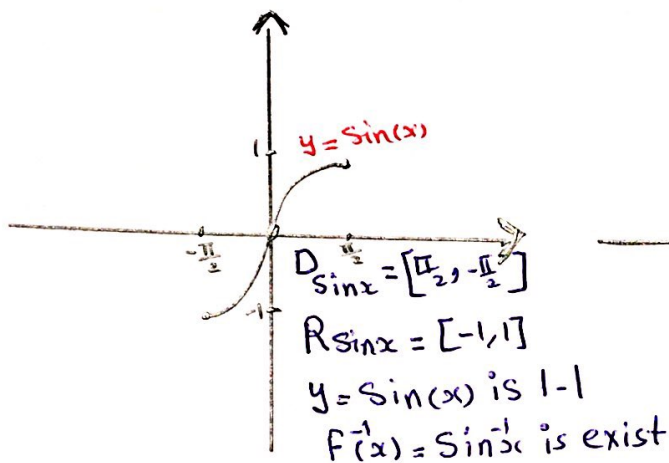
$$x = \ln 2$$

Inverse Trigonometric Functions

1) $f(x) = \sin(x)$ is not 1-1



نقوم بتدعيم مجال لتكوين الدالة 1-1



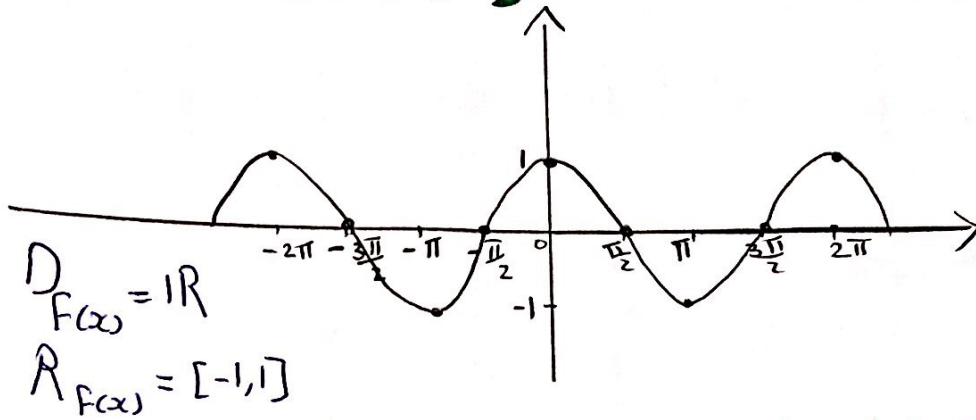
Note

① $y = \sin(x) \Leftrightarrow \sin^{-1}(y) = x$
 $x = \sin(y) \Leftrightarrow \sin^{-1}(x) = y$

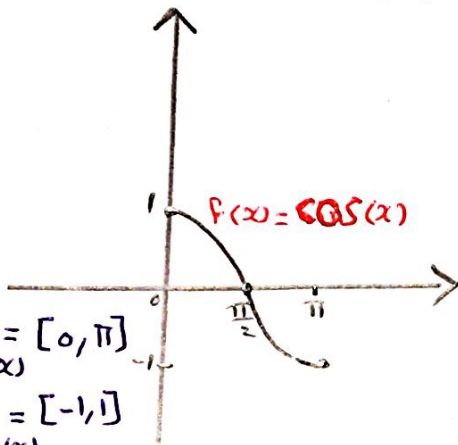
② $f(x) = \sin^{-1}(x)$ is odd function and symmetric about (0,0)
 i.e. $\sin^{-1}(-x) = -\sin^{-1}(x)$

③ $\sin(\sin^{-1}x) = x$ for all $x \in [-1, 1]$ or $-1 \leq x \leq 1$
 $\sin^{-1}(\sin x) = x$ for all $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ or $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

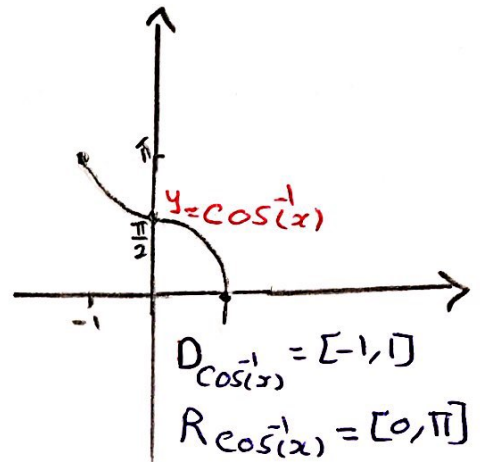
2) $f(x) = \cos(x)$ is not 1-1



نقوم بتصغير المجال لتكون الدالة 1-1



$f(x) = \cos(x)$ is 1-1
 $f^{-1}(x) = \cos^{-1}(x)$ is exist



Note

① $y = \cos(x) \iff \cos^{-1}(y) = x \quad \forall 0 \leq x \leq \pi$

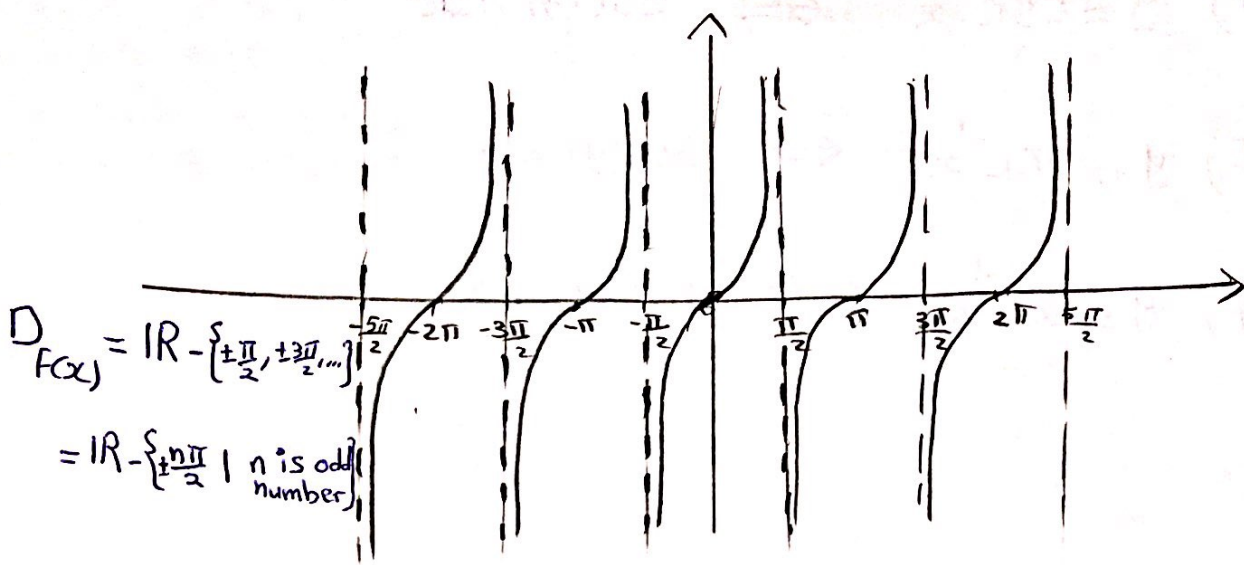
$x = \cos(y) \iff \cos^{-1}(x) = y \quad \forall 0 \leq y \leq \pi$

② $f(x) = \cos^{-1}(x)$ is neither odd nor even

③ $\cos(\cos^{-1}(x)) = x$ For all $x \in [-1, 1]$

$\cos^{-1}(\cos(x)) = x$ For all $x \in [0, \pi]$

3) $f(x) = \tan(x)$ is not 1-1



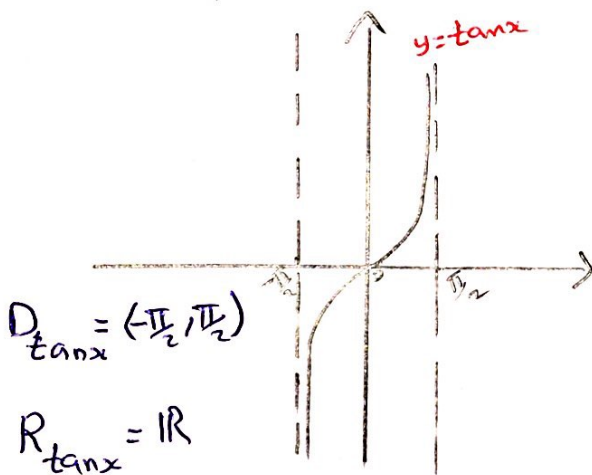
$$D_{f(x)} = \mathbb{R} - \left\{ \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots \right\}$$

$$= \mathbb{R} - \left\{ \pm \frac{n\pi}{2} \mid n \text{ is odd number} \right\}$$

$$= \mathbb{R} - \left\{ (2n+1)\frac{\pi}{2} \mid n \in \mathbb{Z} \right\}$$

$$R_{f(x)} = \mathbb{R}$$

نعوم بتفسير المجال لتكون الدالة 1-1

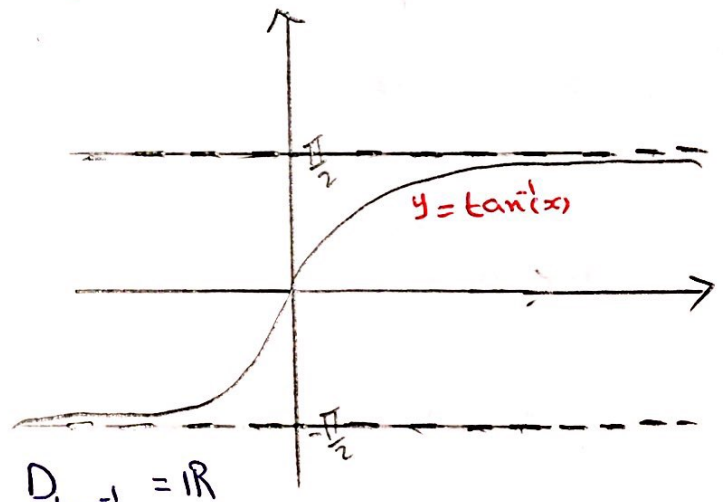


$$D_{\tan x} = \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$R_{\tan x} = \mathbb{R}$$

$y = \tan x$ is 1-1

$f^{-1}(x) = \tan^{-1} x$ exist



$$D_{\tan^{-1}(x)} = \mathbb{R}$$

$$R_{\tan^{-1}(x)} = \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

Note ① $y = \tan x \Leftrightarrow \tan^{-1}(y) = x$
 $x = \tan y \Leftrightarrow \tan^{-1}(x) = y$

② $f(x) = \tan^{-1}(x)$ is odd function and symmetric about origin
 i.e. $\tan^{-1}(-x) = -\tan^{-1}(x)$

$$\textcircled{3} \tan(\tan^{-1}(x)) = x \quad \forall x \in \mathbb{R}$$

$$\tan^{-1}(\tan(x)) = x \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$4) y = \csc^{-1} x \iff \csc(y) = x$$

$$5) y = \sec^{-1} x \iff \sec(y) = x$$

$$6) y = \cot^{-1} x \iff \cot(y) = x$$

Example:

Find the exact value of each expression

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

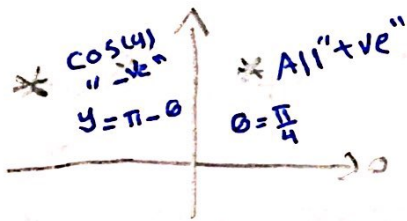
$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

$$\tan^{-1}(-1) = -\tan^{-1}(1) = -\frac{\pi}{4}$$

$$\cos^{-1}(-1) = \pi$$

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$$

$$\cos(y) = -\frac{1}{\sqrt{2}}$$



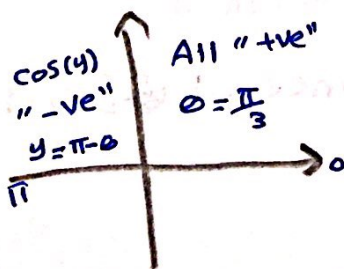
$$y = \pi - \theta$$

$$y = \pi - \frac{\pi}{4}$$

$$y = \frac{4\pi - \pi}{4}$$

$$y = \frac{3\pi}{4}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = y \Leftrightarrow \cos(y) = -\frac{1}{2}$$



$$y = \pi - \theta$$

$$y = \pi - \frac{\pi}{3}$$

$$y = \frac{3\pi - \pi}{3}$$

$$y = \frac{2\pi}{3}$$

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin \theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan \theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$\cot \theta$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$
$\sec \theta$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2
$\csc \theta$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$
	0	$\frac{\pi}{2}$	π
$\sin \theta$	0	1	0
$\cos \theta$	1	0	-1
$\tan \theta$	0	$\frac{1}{0}$	0
$\cot \theta$	$\frac{1}{0}$	0	$-\frac{1}{0}$
$\sec \theta$	1	$\frac{1}{0}$	-1
$\csc \theta$	$\frac{1}{0}$	1	$\frac{1}{0}$

$$\cot^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$$\frac{\sqrt{3}}{3} = \frac{\sqrt{3} \times \sqrt{3}}{3 \times \sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \cot^{-1}\left(\frac{\sqrt{3}}{3}\right) = \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}$$

$$\sec^{-1}(2) = \frac{\pi}{3}$$

$$\csc^{-1}(\sqrt{2}) = \frac{\pi}{4}$$

Example

$$1) \sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$$

$$\sin^{-1}\left(\sin\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$\sin^{-1}(\sin 0) = 0$$

$$\sin^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) = -\frac{\pi}{2}$$

$$\sin^{-1}\left(\sin\frac{7\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

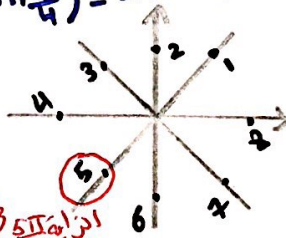
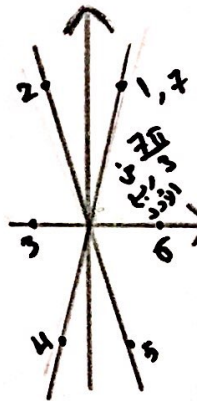
$$\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \sin^{-1}\left(-\sin\frac{\pi}{4}\right) = -\frac{\pi}{4}$$

"Since $\frac{\pi}{6} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ "

"Since $\frac{\pi}{2} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ "

"Since $0 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ "

"Since $-\frac{\pi}{2} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ "



انزاو $\frac{5\pi}{4}$ في
رعة المثال

$$2) \sin\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \frac{2}{3} \quad \text{"Since } \frac{2}{3} \in [-1, 1] \text{"}$$

$$\sin\left(\sin^{-1}\left(\frac{3}{2}\right)\right) \neq \frac{3}{2} \quad \text{"Since } \frac{3}{2} \notin [-1, 1] \text{"}$$

$$\therefore \sin\left(\sin^{-1}\left(\frac{3}{2}\right)\right) = \text{undefined.}$$

$$\sin\left(\sin^{-1}(-1)\right) = -1 \quad \text{"Since } -1 \in [-1, 1] \text{"}$$

3) $\tan^{-1}(\tan \frac{\pi}{4}) = \frac{\pi}{4}$ since $\frac{\pi}{4} \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\tan^{-1}(\tan(-\frac{\pi}{5})) = -\frac{\pi}{5}$ since $-\frac{\pi}{5} \in (-\frac{\pi}{2}, \frac{\pi}{2})$

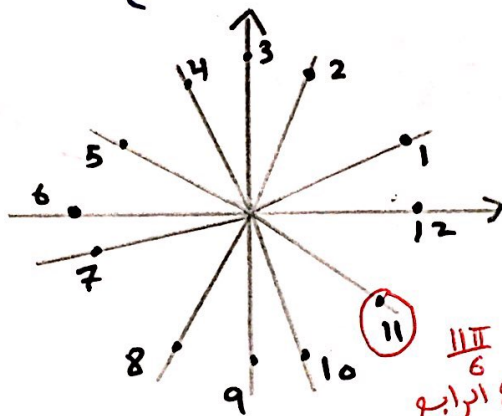
$\tan^{-1}(\tan(0)) = 0$ since $0 \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\tan^{-1}(\tan \frac{\pi}{2}) \neq \frac{\pi}{2}$ since $\frac{\pi}{2} \notin (-\frac{\pi}{2}, \frac{\pi}{2})$

$\therefore \tan^{-1}(\tan \frac{\pi}{2})$ is undefined.

$\tan^{-1}(\tan \frac{11\pi}{6}) \neq \frac{11\pi}{6}$

$\therefore \tan^{-1}(\tan \frac{11\pi}{6}) = \tan^{-1}(\boxed{-} \tan \frac{\pi}{6}) = -\tan^{-1}(\tan \frac{\pi}{6}) = -\frac{\pi}{6}$



الزاوية III في الربع الرابع

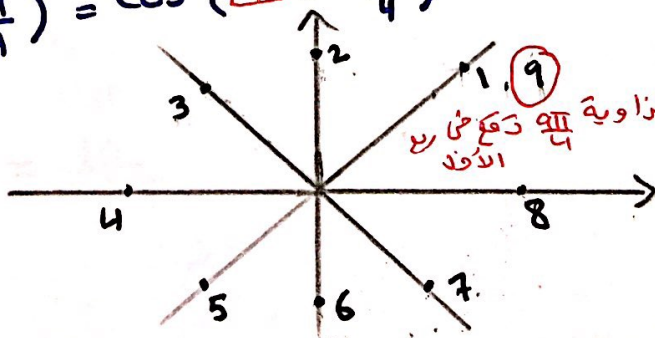
4) $\tan(\tan^{-1}(5)) = 5$ since $5 \in \mathbb{R}$

5) $\cos^{-1}(\cos \frac{2\pi}{3}) = \frac{2\pi}{3}$ since $\frac{2\pi}{3} \in [0, \pi]$

$\cos^{-1}(\cos \frac{5\pi}{6}) = \frac{5\pi}{6}$ since $\frac{5\pi}{6} \in [0, \pi]$

$\cos^{-1}(\cos \frac{9\pi}{4}) \neq \frac{9\pi}{4}$

$\therefore \cos^{-1}(\cos \frac{9\pi}{4}) = \cos^{-1}(\boxed{+} \cos \frac{\pi}{4}) = \cos^{-1}(\cos \frac{\pi}{4}) = \frac{\pi}{4}$



الزاوية III في الربع الأول

$$\textcircled{6} \cos(\cos^{-1}(\frac{1}{2})) = \frac{1}{2} \text{ Since } \frac{1}{2} \in [-1, 1]$$

$$\cos(\cos^{-1}(2)) \neq 2 \text{ Since } 2 \notin [-1, 1]$$

$$\therefore \cos(\cos^{-1}(2)) \text{ is undefined.}$$

$$\cos(\cos^{-1}(1)) = 1 \text{ Since } 1 \in [-1, 1]$$

$$\cos(\cos^{-1}(-1)) = -1 \text{ Since } -1 \in [-1, 1]$$

$$\cos(\cos^{-1}(0)) = 0 \text{ Since } 0 \in [-1, 1]$$

Example

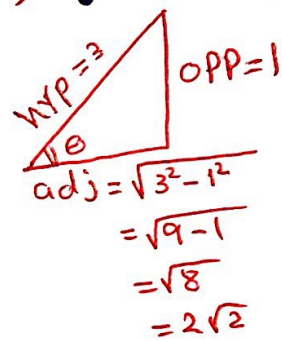
$$\textcircled{1} \tan(\sin^{-1}(\frac{1}{3}))$$

$$\text{let } \theta = \sin^{-1}(\frac{1}{3}) \Leftrightarrow \sin \theta = \frac{1}{3} \begin{matrix} \rightarrow \text{opp} \\ \rightarrow \text{hyp} \end{matrix}$$

$$\therefore \tan(\sin^{-1}(\frac{1}{3})) = \tan(\theta)$$

$$= \frac{\text{opp}}{\text{adj}} = \frac{1}{2\sqrt{2}}$$

$$= \frac{1 \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2(2)} = \frac{\sqrt{2}}{4}$$



$$\textcircled{2} \cos(\tan^{-1}(\frac{\pi}{10}))$$

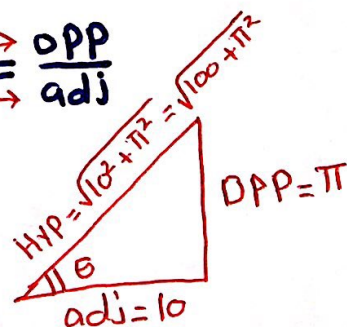
$$\text{let } \theta = \tan^{-1}(\frac{\pi}{10}) \Rightarrow \tan(\theta) = \frac{\pi}{10} \begin{matrix} \rightarrow \text{opp} \\ \rightarrow \text{adj} \end{matrix}$$

$$\therefore \cos(\tan^{-1}(\frac{\pi}{10})) = \cos(\theta)$$

$$= \frac{\text{adj}}{\text{Hyp}}$$

$$= \frac{10}{\sqrt{100 + \pi^2}}$$

$$= \frac{10 \sqrt{100 + \pi^2}}{\sqrt{100 + \pi^2} \sqrt{100 + \pi^2}} = \frac{10 \sqrt{100 + \pi^2}}{(100 + \pi^2)}$$



$$\textcircled{3} \tan(\sin^{-1}x)$$

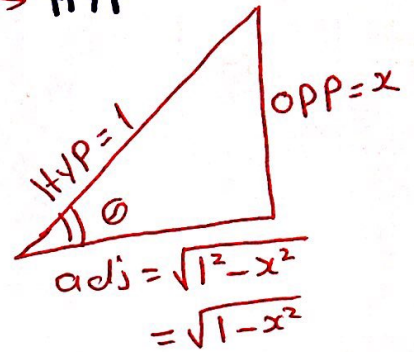
$$\text{let } \theta = \sin^{-1}x \iff \sin\theta = \frac{x}{1} \implies \frac{\text{opp}}{\text{hyp}}$$

$$\therefore \tan(\sin^{-1}x) = \tan(\theta)$$

$$= \frac{\text{opp}}{\text{adj}}$$

$$= \frac{x}{\sqrt{1-x^2}}$$

$$= \frac{x \times \sqrt{1-x^2}}{\sqrt{1-x^2} \times \sqrt{1-x^2}} = \frac{x\sqrt{1-x^2}}{(1-x^2)}$$



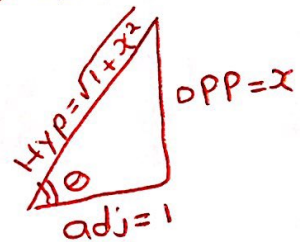
$$\textcircled{4} \sin(\tan^{-1}x)$$

$$\text{let } \theta = \tan^{-1}x \iff \tan\theta = \frac{x}{1} \implies \frac{\text{opp}}{\text{adj}}$$

$$\therefore \sin(\tan^{-1}x) = \sin(\theta)$$

$$= \frac{\text{opp}}{\text{hyp}}$$

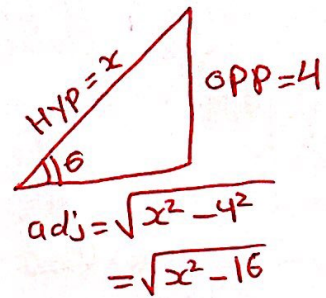
$$= \frac{x}{\sqrt{1+x^2}}$$



$$\textcircled{5} \sec(\csc^{-1} \frac{x}{4})$$

$$\text{let } \theta = \csc^{-1} \frac{x}{4} \iff \csc \theta = \frac{x}{4} \begin{matrix} \rightarrow \text{HYP} \\ \rightarrow \text{OPP} \end{matrix}$$

$$\begin{aligned} \therefore \sec(\csc^{-1} \frac{x}{4}) &= \sec(\theta) \\ &= \frac{\text{HYP}}{\text{adj}} \\ &= \frac{x}{\sqrt{x^2 - 16}} \end{aligned}$$



$$\textcircled{6} \sin(2\cos^{-1} x)$$

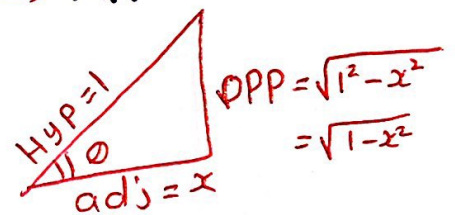
$$\text{let } \theta = \cos^{-1} x \iff \cos \theta = \frac{x}{1} \begin{matrix} \rightarrow \text{adj} \\ \rightarrow \text{HYP} \end{matrix}$$

$$\begin{aligned} \therefore \sin(2\cos^{-1} x) &= \sin(2\theta) \\ &= 2 \sin \theta \cdot \cos \theta \end{aligned}$$

$$= 2 \left(\frac{\text{OPP}}{\text{HYP}} \right) \left(\frac{\text{adj}}{\text{HYP}} \right)$$

$$= 2 \left(\frac{\sqrt{1-x^2}}{1} \right) \left(\frac{x}{1} \right)$$

$$= 2x\sqrt{1-x^2}$$



$$\textcircled{7} \cos\left(2 \arcsin\left(\frac{5}{13}\right)\right) = \cos\left(2 \sin^{-1}\left(\frac{5}{13}\right)\right)$$

$$\text{let } \theta = \sin^{-1}\left(\frac{5}{13}\right) \Leftrightarrow \sin \theta = \frac{5}{13} \begin{matrix} \xrightarrow{\text{OPP}} \\ \xrightarrow{\text{HYP}} \end{matrix}$$

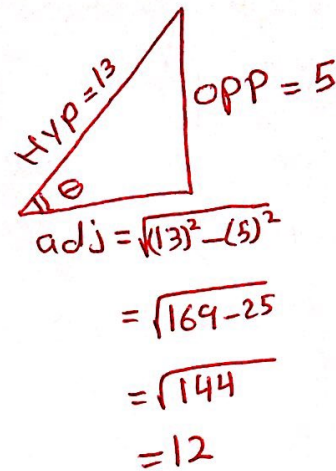
$$\begin{aligned} \therefore \cos\left(2 \sin^{-1}\left(\frac{5}{13}\right)\right) &= \cos(2\theta) \\ &= \cos^2 \theta - \sin^2 \theta \end{aligned}$$

$$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$$

$$= \frac{144}{169} - \frac{25}{169}$$

$$= \frac{144-25}{169}$$

$$= \frac{119}{169}$$



Example:

Find the Domain of the following

$$\textcircled{1} f(x) = \tan^{-1}(\sqrt{5-x^2})$$

$$D_{f(x)} = D_{\tan^{-1}x} \cap D_{\sqrt{5-x^2}}$$

$$= \mathbb{R} \cap [-\sqrt{5}, \sqrt{5}]$$

$$= [-\sqrt{5}, \sqrt{5}]$$

$$\textcircled{2} \quad g(x) = \sin^{-1}(3x+1)$$

$$-1 \leq 3x+1 \leq 1$$

$$-1-1 \leq 3x \leq 1-1$$

$$-2 \leq 3x \leq 0$$

$$-\frac{2}{3} \leq \frac{3x}{3} \leq \frac{0}{3}$$

$$-\frac{2}{3} \leq x \leq 0$$

$$\therefore D_{g(x)} = \left[-\frac{2}{3}, 0\right]$$

Example

if $f(x) = \cos(2x-6) + 1$ then find $f^{-1}(x)$ =

$$1) \quad y = \cos(2x-6) + 1$$

$$2) \quad y-1 = \cos(2x-6)$$

$$\cos^{-1}(y-1) = 2x-6$$

$$\cos^{-1}(y-1) + 6 = 2x$$

$$\frac{1}{2} [\cos^{-1}(y-1) + 6] = \frac{2x}{2}$$

$$\frac{1}{2} [\cos^{-1}(y-1) + 6] = x$$

$$3) \quad \frac{1}{2} [\cos^{-1}(x-1) + 6] = y$$

$$4) \quad f^{-1}(x) = \frac{1}{2} [\cos^{-1}(x-1) + 6] = \frac{1}{2} \cos^{-1}(x-1) + \frac{6}{2} = \frac{1}{2} \cos^{-1}(x-1) + 3 \quad \#$$

Transcendental Functions: "Not Algebraic"

1] Exponential Functions

The Natural Exponential Function

$$f(x) = e^x$$

The General Exponential Function

$$f(x) = a^x \text{ for all } a \in \mathbb{R}^+$$

Example:

$$f(x) = 2^x$$

$$f(x) = \left(\frac{1}{3}\right)^x$$

$$f(x) = \left(\frac{3}{2}\right)^x$$

$$f(x) = \pi^x$$

$$f(x) = (\sqrt{2})^x$$

Exponential Function

$$f(x) = x^2$$

$$f(x) = x^{1/3}$$

$$f(x) = x^{3/2}$$

$$f(x) = x^\pi$$

$$f(x) = x^{\sqrt{2}}$$

Power function

Example:

✓ $f(x) = (-2)^x$ is a General Exponential Function

Since: $a = -2 \notin \mathbb{R}^+$

⊗

✓ $f(x) = \pi^{-x}$

is a General Exponential Function

Since: $f(x) = \pi^{-x} = \frac{1}{\pi^x}$
 $f(x) = \left(\frac{1}{\pi}\right)^x$

⊗

X

✓ $f(x) = 5^{\frac{x}{2}}$ is not Exponential function

✓ (X)

Since: $f(x) = 5^{\frac{x}{2}}$

$$= (5^{\frac{1}{2}})^x$$

$= (\sqrt{5})^x$ is a Exponential Function

✓ $f(x) = 8^{\frac{x}{3}}$ is a Exponential function

✓

X

Since: $f(x) = 8^{\frac{x}{3}}$

$$= (2^3)^{\frac{x}{3}}$$

$$= 2^{\frac{3x}{3}}$$

$= 2^x$ is a Exponential Function.

✓ $f(x) = \sqrt{e^{2x}}$ is not Natural Exponential function

✓ (X)

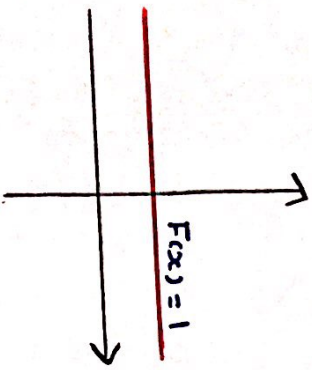
Since: $f(x) = \sqrt{e^{2x}} = (e^{2x})^{\frac{1}{2}} = e^{2x \cdot \frac{1}{2}}$

$= e^x$ is natural Exponential Function.

$a = 1$

$f(x) = 1$

- 1) $f(x)$ is constant function
- 2) $f(x)$ is polynomial of degree zero



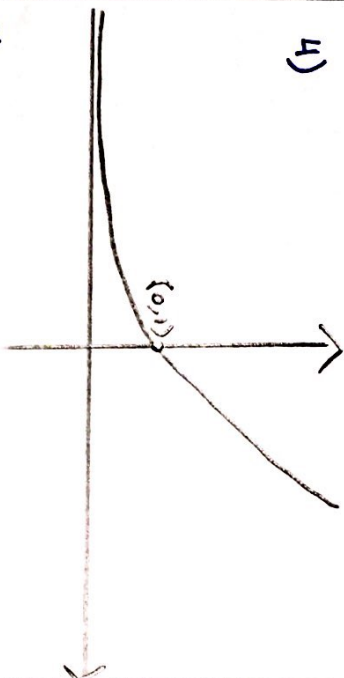
- 3)
- 4) $f(x)$ is horizontal line with slope: 0 and y-intercept: 1
- 5) $D_{f(x)} = \mathbb{R}$; $R_{f(x)} = \{1\}$
- 6) $f(x)$ is even function
- 7) $f(x)$ is symmetric about y-axis "x=0"

$f(x) = a^x$ For all $a \in \mathbb{R}^+$

$a > 1$

$f(x) = a^x$

- 1) $f(x)$ is Exponential Function
- 2) $f(x)$ is not algebraic
- 3) $f(x)$ is Transcendental Function
- 4)

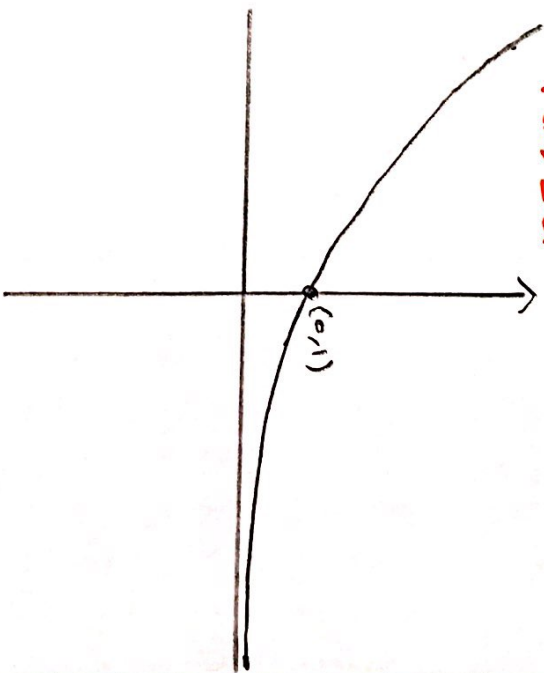


- 5) $f(x)$ is increasing on \mathbb{R}
- 6) $f(x)$ is neither even nor odd
- 7) $D_{f(x)} = \mathbb{R}$; $R_{f(x)} = (0, \infty)$

$0 < a < 1$

$f(x) = a^x$

- 1) $f(x)$ is decreasing on \mathbb{R}
- 2) $f(x)$ is neither even nor odd
- 3) $D_{f(x)} = \mathbb{R}$; $R_{f(x)} = (0, \infty)$



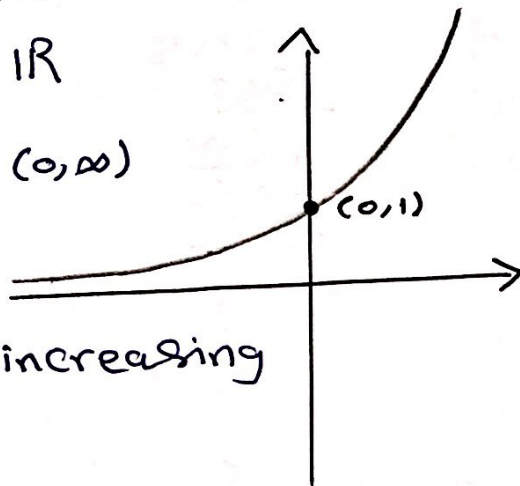
Example:

$$f(x) = 2^x \quad "2 > 1"$$

$f(x)$ is a Exponential function

$$D_{f(x)} = \mathbb{R}$$

$$R_{f(x)} = (0, \infty)$$



$f(x)$ is increasing on \mathbb{R} .

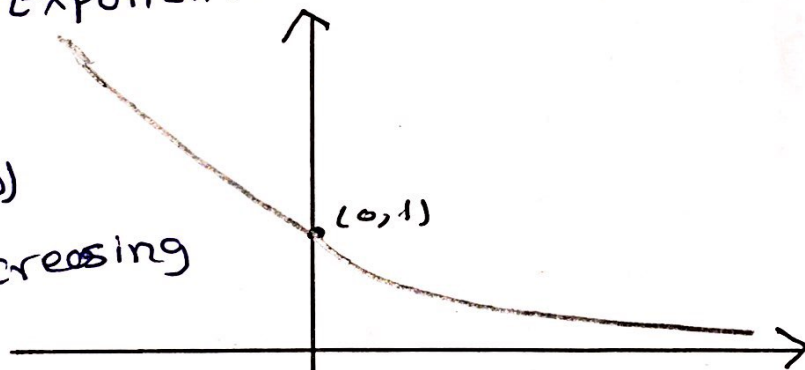
$$f(x) = \left(\frac{4}{3}\right)^{-x} = \left(\frac{3}{4}\right)^x \quad "0 < \frac{3}{4} < 1"$$

$f(x)$ is a Exponential function

$$D_{f(x)} = \mathbb{R}$$

$$R_{f(x)} = (0, \infty)$$

$f(x)$ is decreasing on \mathbb{R}

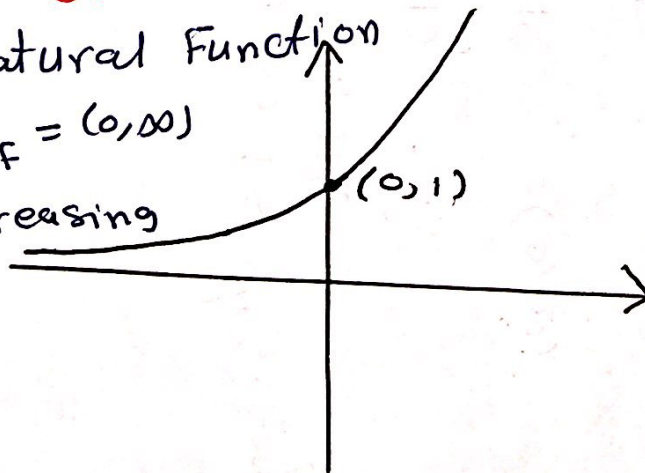


$$f(x) = e^x \quad "e \approx 2.7 > 1"$$

$f(x)$ is a Natural Function

$$D_f = \mathbb{R} \quad ; \quad R_f = (0, \infty)$$

$f(x)$ is increasing on \mathbb{R}



H.W ::

1) The Domain of $f(x) = 3^x$ is

$\{3\}$

$(0, \infty)$

$(3, \infty)$

\mathbb{R}

2) The Range of $f(x) = \left(\frac{5}{6}\right)^x$ is ...

$\left\{\frac{5}{6}\right\}$

$(0, \infty)$

\mathbb{R}

$(-\infty, \frac{5}{6})$

3) $f(x) = \left(\frac{\pi}{3}\right)^x$ is decreasing on \mathbb{R}

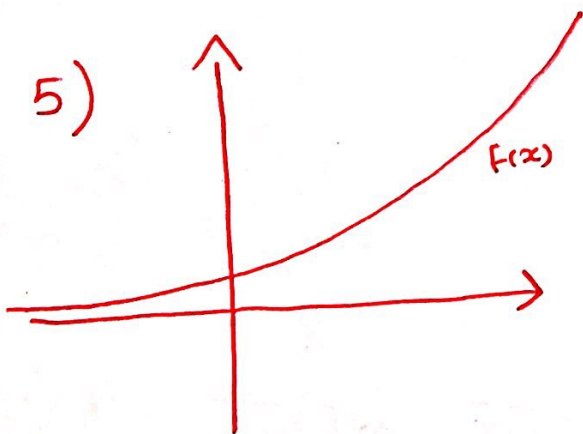


\times

4) $f(x) = e^{-x}$ is increasing on \mathbb{R}



\times



a) $f(x) = \left(\frac{2}{3}\right)^x$

b) $f(x) = \frac{2}{3}$

c) $f(x) = x^{2/3}$

\textcircled{d} $f(x) = \left(\frac{3}{2}\right)^x$

Laws of Exponents :

$$1) a^0 = 1 \quad ; \quad e^0 = 1$$

$$2) a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x \quad ; \quad e^{-x} = \frac{1}{e^x} = \left(\frac{1}{e}\right)^x$$

$$3) (a^x)^y = a^{xy}$$

$$; (e^x)^y = e^{xy}$$

$$4) a^x \cdot a^y = a^{x+y}$$

$$; e^x \cdot e^y = e^{x+y}$$

$$5) \frac{a^x}{a^y} = a^{x-y}$$

$$; \frac{e^x}{e^y} = e^{x-y}$$

$$6) (ab)^x = a^x b^x$$

$$7) \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$8) \left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$$

$$9) a^{p/q} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

$$10) \frac{a^{-x}}{b^{-y}} = \frac{b^y}{a^x}$$

$$11) \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$12) \sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}$$

$$13) \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$14) a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ times}}$$

$$15) a^x > 0$$

Example:.

Simplify of the following:.

$$1) a) \frac{4^{-3}}{2^{-8}} = \frac{2^8}{4^3} = \frac{2^8}{(2^2)^3} = \frac{2^8}{2^6} = 2^{8-6} = 2^2 = 4$$

$$b) \frac{1}{\sqrt[3]{x^4}} = \frac{1}{x^{4/3}} = x^{-4/3}$$

$$2) a) 8^{4/3} = (2^3)^{4/3} = 2^{3(4/3)} = 2^4 = 16$$

$$b) x(3x^2)^3 = x(3^3 \cdot (x^2)^3) \\ = x(27 \cdot x^6) \\ = 27x^1 \cdot x^6 \\ = 27x^7$$

$$3) a) b^8(2b)^4 = b^8(2^4 b^4) = b^8(16b^4) = 16b^4 b^8 \\ = 16b^{4+8} = 16b^{12}$$

$$b) \frac{(6y^3)^4}{2y^5} = \frac{6^4(y^3)^4}{2y^5} = \frac{(2^4)(3^4)y^{12}}{(2)^1 y^5} = (2^3)(3^4)y^{12-5} \\ = (8)(81)y^7 = 648y^7$$

$$\begin{aligned}
 4) \text{ a) } \frac{x^{2n} \cdot x^{2n-1}}{x^{n+2}} &= \frac{x^{2n+2n-1}}{x^{n+2}} \\
 &= \frac{x^{4n-1}}{x^{n+2}} \\
 &= x^{(4n-1)-(n+2)} \\
 &= x^{(4n-1-n-2)} \\
 &= x^{4n-n-1-2} \\
 &= x^{3n-3} = x^{3(n-1)}
 \end{aligned}$$

$$\begin{aligned}
 b) \frac{\sqrt{a \cdot \sqrt{b}}}{\sqrt[3]{ab}} &= \frac{\sqrt{a} \cdot \sqrt[2]{\sqrt{b}}}{\sqrt[3]{a} \cdot \sqrt[3]{b}} = \frac{\sqrt{a} \cdot \sqrt[4]{b}}{\sqrt[3]{a} \cdot \sqrt[3]{b}} \\
 &= \frac{a^{1/2} \cdot b^{1/4}}{a^{1/3} \cdot b^{1/3}} = a^{1/2-1/3} \cdot b^{1/4-1/3} \\
 &= a^{1/6} \cdot b^{-1/12} \\
 &= \frac{a^{1/6}}{b^{1/12}} = \frac{\sqrt[6]{a}}{\sqrt[12]{b}}
 \end{aligned}$$

$$\begin{aligned}
 c) 2^x \cdot 8^{2x} &= 2^x \cdot (2^3)^{2x} = 2^x \cdot 2^{6x} = 2^{x+6x} \\
 &= 2^{7x} = (2^7)^x
 \end{aligned}$$

3] Logarithmic Functions

The Natural logarithmic Functions

$$f(x) = \log_e x = \ln(x)$$

$f(x) = \ln(x)$ is the inverse function of the $f^{-1}(x) = e^x$

Domain of $f(x) = \ln(x)$ is $(0, \infty)$

Range of $f(x) = \ln(x)$ is \mathbb{R}

The General logarithmic Functions

$$f(x) = \log_a x \text{ for all } a \in \mathbb{R}^+$$

$f(x) = \log_a x$ is the inverse function of the $f^{-1}(x) = a^x$

Domain of $f(x) = \log_a x$ is $(0, \infty)$

Range of $f(x) = \log_a x$ is \mathbb{R}

Example:

✓ $f(x) = \log_2 x$ is logarithmic function

✓ $D_{f(x)} = (0, \infty)$; $R_{f(x)} = \mathbb{R}$

✓ $f(x) = \log_2 x$ is inverse of the function $f^{-1}(x) = 2^x$

Example

The inverse function of $(\frac{3}{2})^x$ is

a) $x^{3/2}$

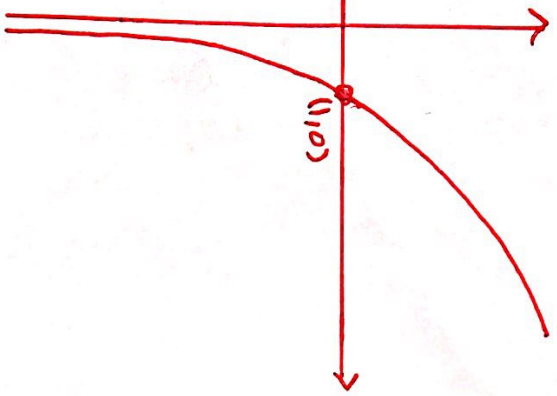
b) $\ln(\frac{3x}{2})$

c) $(\frac{2}{3})^x$

d) $\log_{\frac{3}{2}} x$

$$F(x) = \log_a x \quad \forall a \in \mathbb{R}^+$$

$$f(x) = \log_a x \quad \forall a > 1$$

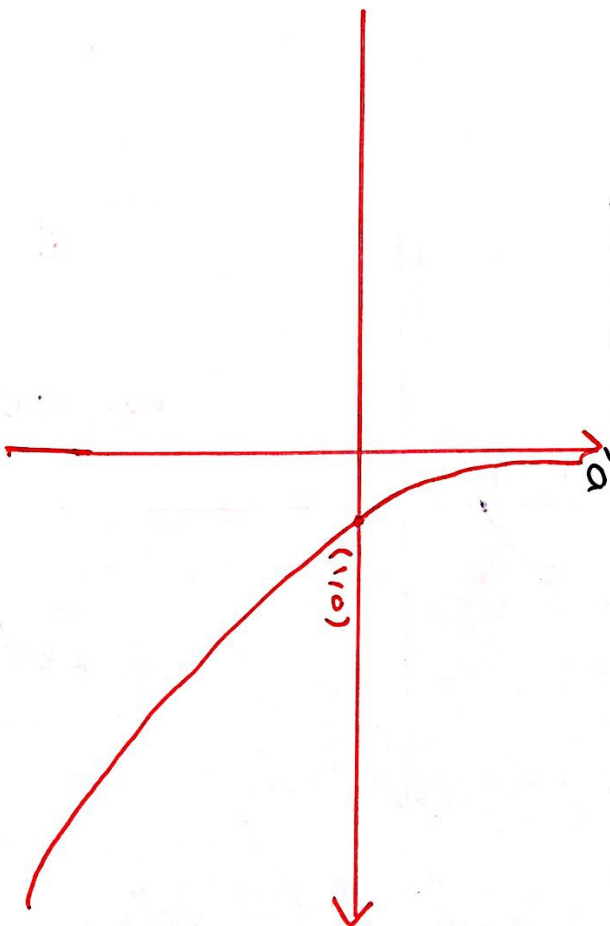


✓ $f(x)$ is increasing function $\forall x \in (0, \infty)$

✓ $f(x)$ is neither even nor odd

✓ $x=0$ is asymptote of $-f(x)$

$$f(x) = \log_a x \quad \forall 0 < a < 1$$



✓ $f(x)$ is decreasing function $\forall x \in (0, \infty)$

✓ $f(x)$ is neither even nor odd

✓ $x=0$ is asymptote of $f(x)$.

Example

$$f(x) = \ln(x)$$

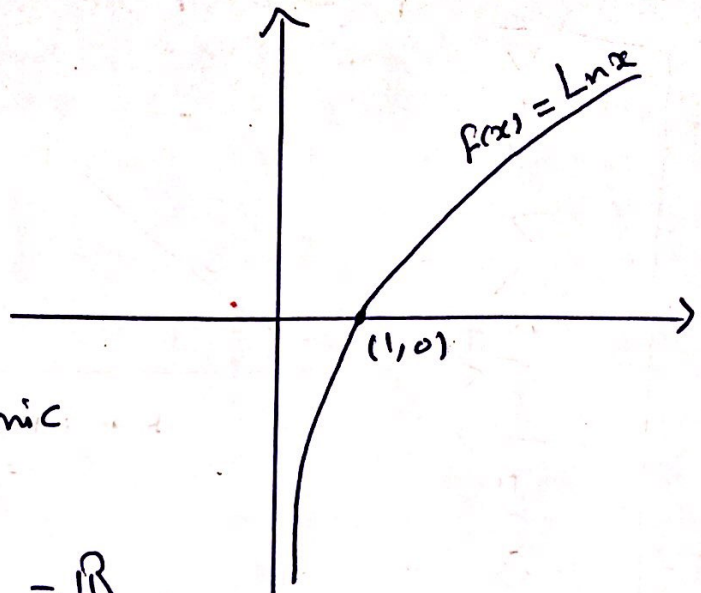
$$f(x) = \ln(x) = \log_e x$$

$$e \approx 2.7 > 1$$

✓ $f(x)$ is the logarithmic function

$$D_{f(x)} = (0, \infty) ; R_{f(x)} = \mathbb{R}$$

✓ $f(x)$ is increasing on $(0, \infty)$



Trigonometric Functions

$$y = \sin x$$

Domain: \mathbb{R}

Range: $[-1, 1]$

Period: 2π

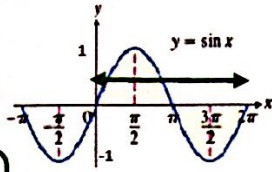
$$\sin(\theta + 2\pi) = \sin \theta$$

Odd function:

$$\sin(-\theta) = -\sin \theta$$

$$-1 \leq \sin x \leq 1 \Leftrightarrow |\sin x| \leq 1$$

$$\sin x = 0 \text{ when } x = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots$$



$$y = \cos x$$

Domain: \mathbb{R}

Range: $[-1, 1]$

Period: 2π

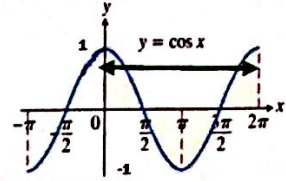
$$\cos(\theta + 2\pi) = \cos \theta$$

Even function:

$$\cos(-\theta) = \cos \theta$$

$$-1 \leq \cos x \leq 1 \Leftrightarrow |\cos x| \leq 1$$

$$\cos x = 0 \text{ when } x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$$



$$y = \tan x$$

Domain:

$$\mathbb{R} - \left\{ \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots \right\}$$

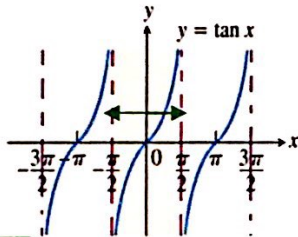
Range: \mathbb{R}

Period: π

$$\tan(\theta + \pi) = \tan \theta$$

Odd function:

$$\tan(-\theta) = -\tan \theta$$



$$y = \cot x$$

Domain:

$$\mathbb{R} - \{0, \pm\pi, \pm2\pi, \dots\}$$

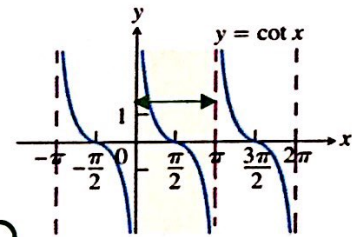
Range: \mathbb{R}

Period: π

$$\cot(\theta + \pi) = \cot \theta$$

Odd function:

$$\cot(-\theta) = -\cot \theta$$



$$y = \csc x$$

Domain:

$$\mathbb{R} - \{0, \pm\pi, \pm2\pi, \dots\}$$

Range:

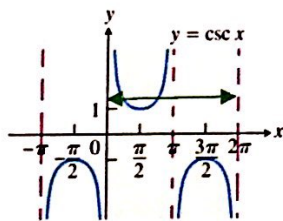
$$\mathbb{R} - (-1, 1)$$

Period: 2π

$$\csc(\theta + 2\pi) = \csc \theta$$

Odd function:

$$\csc(-\theta) = -\csc \theta$$



$$y = \sec x$$

Domain:

$$\mathbb{R} - \left\{ \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots \right\}$$

Range:

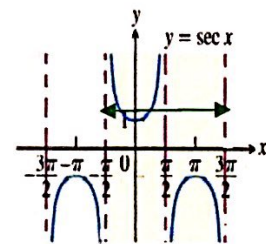
$$\mathbb{R} - (-1, 1)$$

Period: 2π

$$\sec(\theta + 2\pi) = \sec \theta$$

Even function:

$$\sec(-\theta) = \sec \theta$$



😊 أ/ هناء الخطامي

Example

Classify the following as one types of functions

a) $5^x = f(x)$ exponential function

b) $g(x) = \frac{x^5}{1}$ Power function or Polynomial of degree 5 or rational function

c) $h(x) = \frac{1+x}{1-\sqrt{x}}$ algebraic function

d) $u(t) = \frac{1-t+5t^4}{1}$ Polynomial of degree 4 or rational function

e) $y = \pi^x$
exponential
function

f) $y = x^\pi$
power function

g) $y = x^2(2-x^3)$
 $y = 2x^2 - x^5$
Polynomial of degree 5

h) $y = \tan t - \cos t$
trigonometric

i) $y = \frac{5}{1+5}$
rational function

j) $y = \frac{\sqrt{x^3-1}}{1+\sqrt[3]{x}}$ algebraic
function

k) $\log_2 x = f(x)$
logarithmic function

l) $g(x) = \sqrt[4]{x} = x^{1/4}$
Root function or Power

m) $h(x) = \frac{2x^3}{1-x^2}$
rational function

n) $u(t) = 1 - 16t + 254t^2$
Polynomial of
degree 2

o) $w(\theta) = \sin \theta \cos^2 \theta$
trigonometric

Appendix D : Trigonometry

الهدف الأول

كيفية التحويل الزاوية من درجات الى راديان ومن راديان الى درجات

Exercise (1)

convert from degrees to radians

$$1) 210^\circ = \frac{210 \times \pi}{180} = \frac{21\pi}{18} = \frac{(3)(7)\pi}{(2)(3)(3)} = \frac{7\pi}{(2)(3)} = \frac{7\pi}{6}$$

$$2) 300^\circ = \frac{300 \times \pi}{180} = \frac{30\pi}{18} = \frac{(2)(3)(5)\pi}{(2)(3)(3)} = \frac{5\pi}{3}$$

$$3) 9^\circ = \frac{9 \times \pi}{180} = \frac{(3)(3)\pi}{(2)(2)(3)(3)(5)} = \frac{\pi}{(2)(2)(5)} = \frac{\pi}{20}$$

$$4) -315^\circ = -\frac{315 \times \pi}{180} = -\frac{(3)(3)(5)(7)\pi}{(2)(2)(3)(3)(5)} = -\frac{7\pi}{(2)(2)} = -\frac{7\pi}{4}$$

$$5) 900^\circ = \frac{900 \times \pi}{180} = \frac{90\pi}{18} = \frac{(2)(3)(3)(5)\pi}{(2)(3)(3)} = 5\pi$$

$$6) 36^\circ = \frac{36 \times \pi}{180} = \frac{(2)(2)(3)(3)\pi}{(2)(2)(3)(3)(5)} = \frac{\pi}{5}$$

Exercise (2)

convert from radians to degrees

$$1) 4\pi = 4 \times 180 = 720^\circ$$

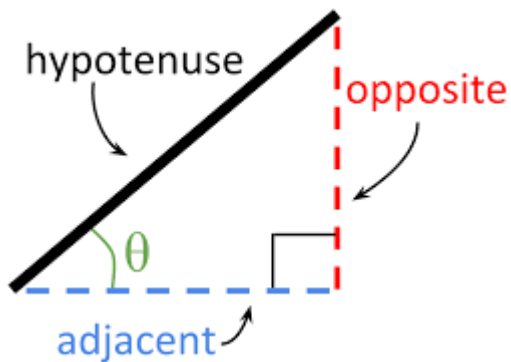
$$2) -\frac{7\pi}{2} = -\frac{7 \times 180}{2} = -7 \times 90 = -630^\circ$$

$$3) \frac{8\pi}{3} = \frac{8 \times 180}{3} = 8 \times 60 = 480^\circ$$

$$4) \frac{5\pi}{12} = -\frac{5 \times 180}{12} = 5 \times 15 = 75^\circ$$

الهدف الثاني

معرفة قوانين الدوال المثلثية وكيفية استخدامها

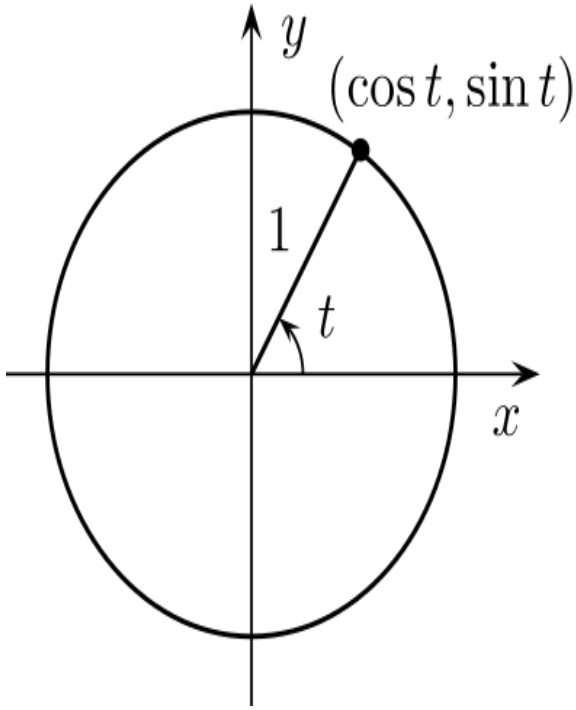


$$\sin(\theta) = \frac{opp}{hyp} \quad \cos(\theta) = \frac{adj}{hyp}$$

$$\csc(\theta) = \frac{hyp}{opp} \quad \sec(\theta) = \frac{hyp}{adj}$$

$$\tan(\theta) = \frac{opp}{adj} \quad \cot(\theta) = \frac{adj}{opp}$$

Trig Functions using Unit Circle



$$\sin \theta = \frac{y}{1} = y \quad \csc \theta = \frac{1}{y}$$

$$\cos \theta = \frac{x}{1} = x \quad \sec \theta = \frac{1}{x}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$

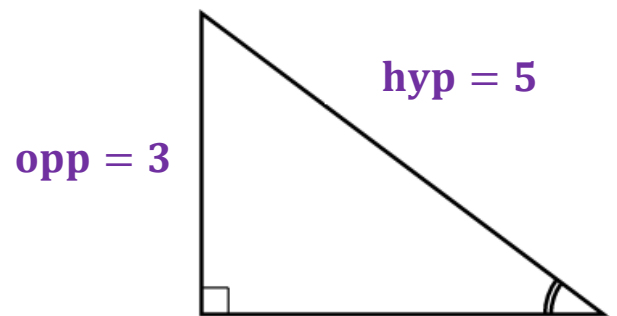
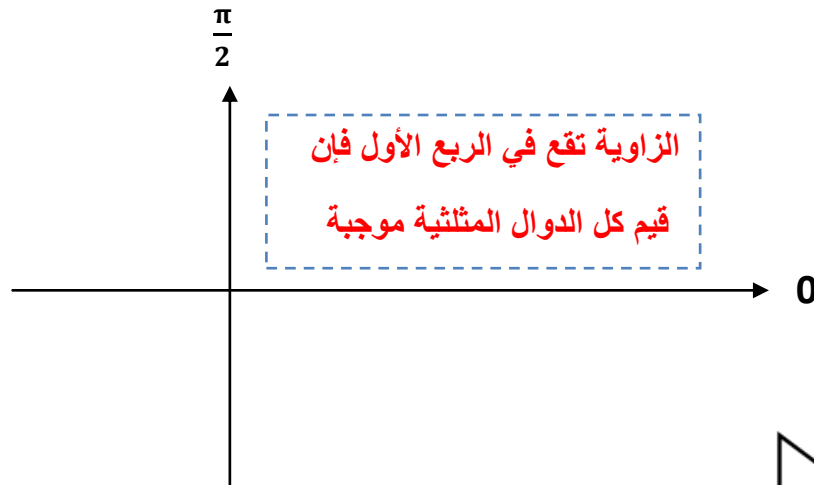
قاعدة الاشارات

<p>فقط في ربع الثاني قيم الـ $\sin(x)$ و $\csc(x)$ موجبة وباقى الدوال المثلثية قيمها سالبة</p>	<p>الربع الأول قيم الدوال المثلثية موجبة</p>
<p>فقط في ربع الثالث قيم الـ $\tan(x)$ و $\cot(x)$ موجبة وباقى الدوال المثلثية قيمها سالبة</p>	<p>فقط في ربع الرابع قيم الـ $\cos(x)$ و $\sec(x)$ موجبة وباقى الدوال المثلثية قيمها سالبة</p>

Exercise (3)

Find the remaining trigonometric ratio

$$1) \sin(\theta) = \frac{3}{5}, 0 < \theta < \frac{\pi}{2}$$



$$\sin(\theta) = \frac{3}{5} = \frac{\text{opp}}{\text{hyp}}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

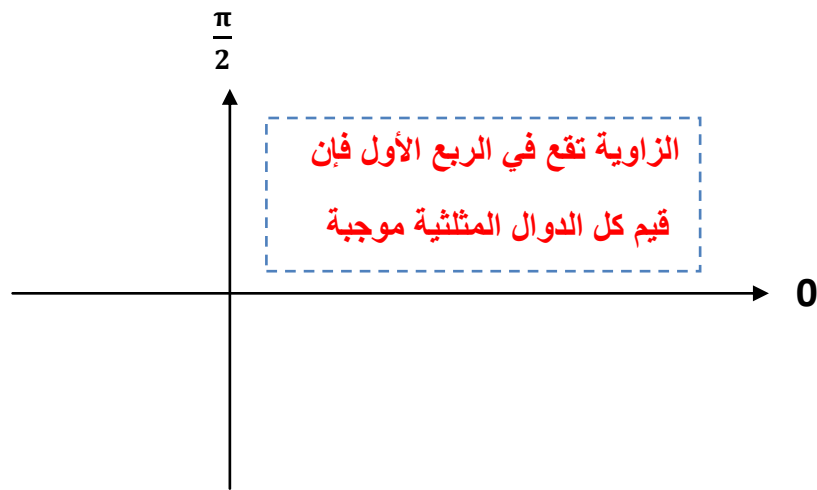
$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$$\text{adj} = \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9}$$

$$= \sqrt{16} = 4$$

$$2) \tan(\theta) = 2, 0 < \theta < \frac{\pi}{2}$$



$$\tan(\theta) = \frac{2}{1} = \frac{\text{opp}}{\text{adj}}$$

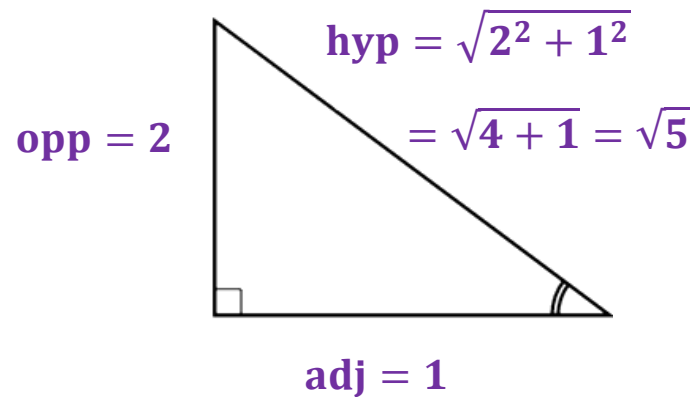
$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{5}}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{5}}$$

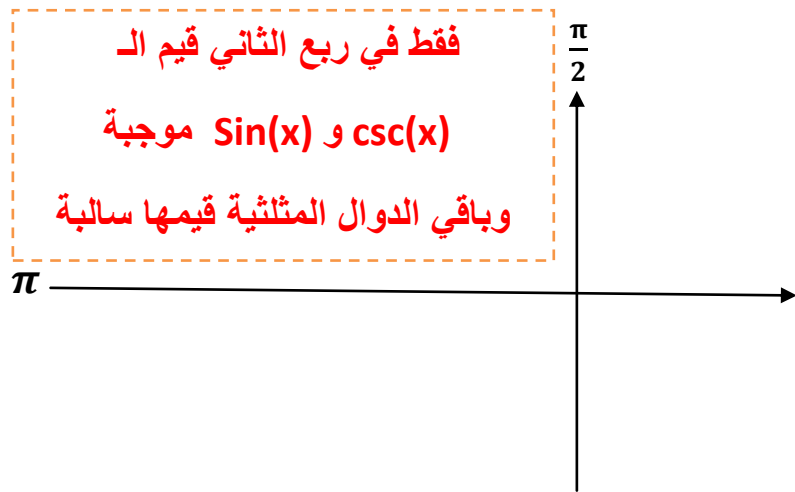
$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = \frac{1}{2}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{2}$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \sqrt{5}$$

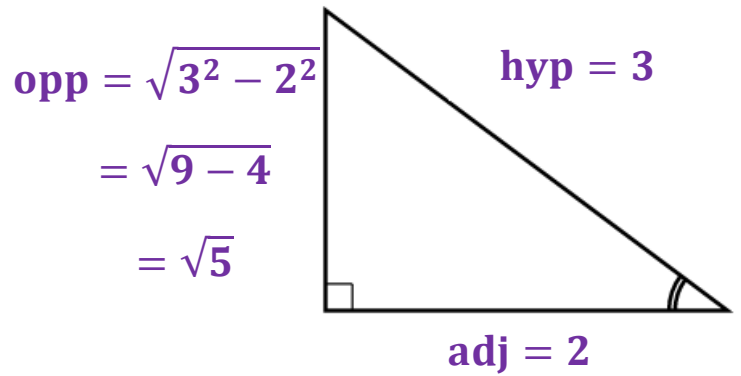


$$3) \sec(\theta) = -1.5, \frac{\pi}{2} < \theta < \pi$$



$$\sec(\theta) = -1.5 = -\frac{15}{10} = -\frac{3 \times 5}{2 \times 5} = -\frac{3}{2}$$

$$\sec(\theta) = -\frac{3}{2} = \frac{\text{hyp}}{\text{adj}}$$



$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = -\frac{2}{3}$$

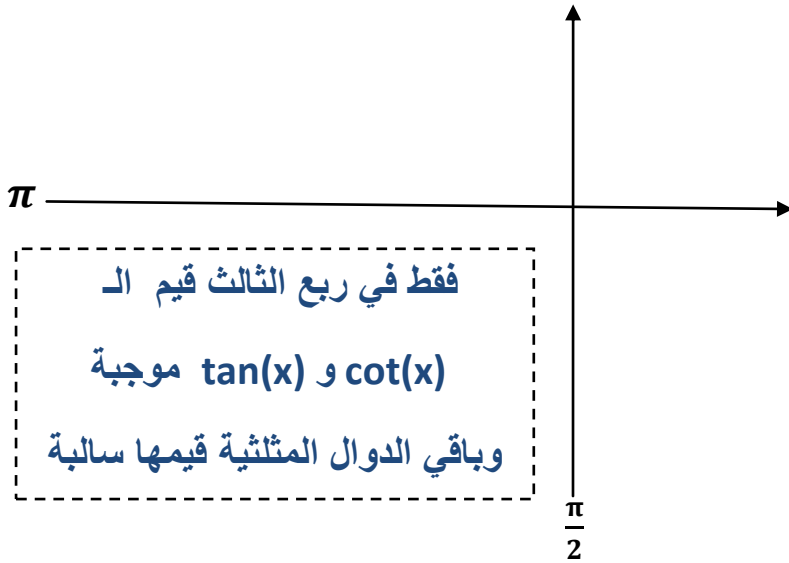
$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = -\frac{\sqrt{5}}{2}$$

$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = -\frac{2}{\sqrt{5}}$$

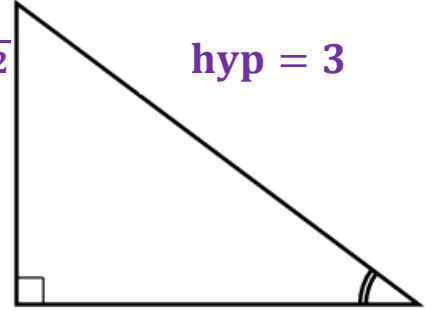
$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = \frac{3}{\sqrt{5}}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{5}}{3}$$

$$4) \cos(\theta) = -\frac{1}{3}, \pi < \theta < \frac{3\pi}{2}$$



$$\begin{aligned} \text{opp} &= \sqrt{3^2 - 1^2} \\ &= \sqrt{9 - 1} \\ &= \sqrt{8} = 2\sqrt{2} \end{aligned}$$



$$\text{adj} = 1$$

$$\cos(\theta) = -\frac{1}{3} = \frac{\text{adj}}{\text{hyp}}$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = -3$$

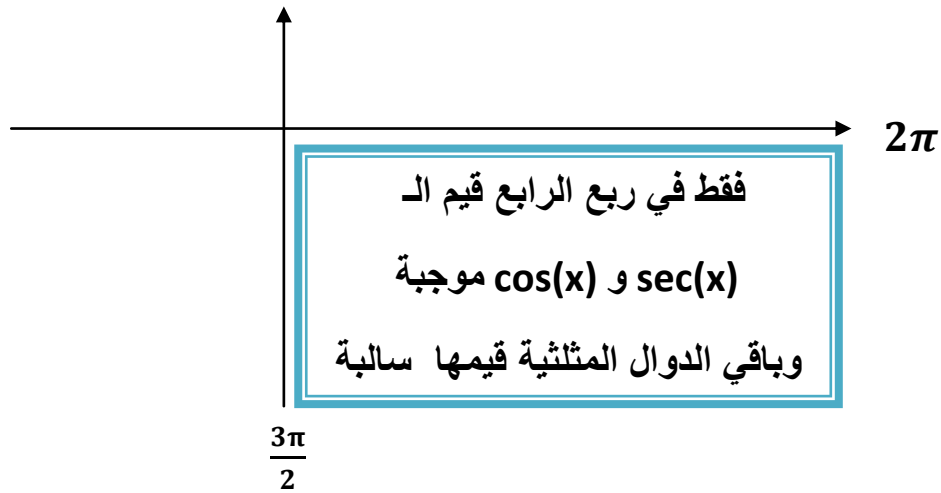
$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = 2\sqrt{2}$$

$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = \frac{1}{2\sqrt{2}} = \frac{1 \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = -\frac{3}{2\sqrt{2}} = -\frac{3 \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = -\frac{2\sqrt{2}}{3}$$

$$5) \csc(\theta) = -\frac{4}{3}, \quad \frac{3\pi}{2} < \theta < 2\pi$$



$$\csc(\theta) = -\frac{4}{3} = \frac{\text{hyp}}{\text{opp}}$$

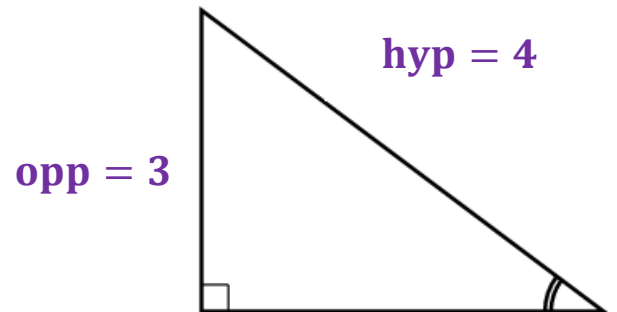
$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{7}}{4}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = -\frac{3}{\sqrt{7}}$$

$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = -\frac{\sqrt{7}}{3}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = -\frac{3}{4}$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \frac{4}{\sqrt{7}}$$



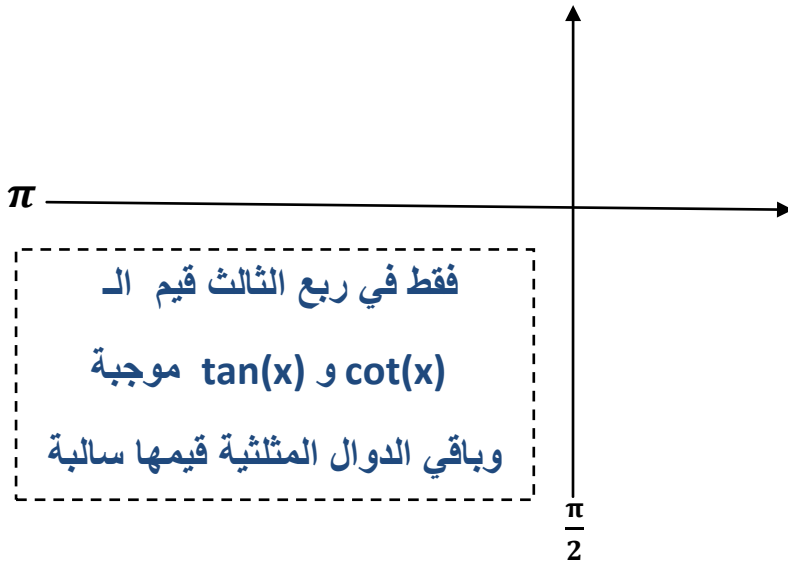
$$\text{adj} = \sqrt{4^2 - 3^2}$$

$$= \sqrt{16 - 9}$$

$$= \sqrt{7}$$

$$6) \cot(\theta) = 3, \quad \pi < \theta < 2\pi$$

الزاوية تقع في الربع الثالث والرابع ولكن بحكم أن قيمة الـ $\cot(\theta)$ موجبة إذن الزاوية تقع في ربع الثالث



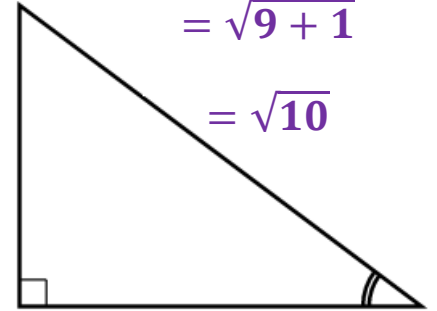
$$\cot(\theta) = \frac{3}{1} = \frac{\text{adj}}{\text{opp}}$$

$$\text{hyp} = \sqrt{3^2 + 1^2}$$

$$= \sqrt{9 + 1}$$

$$= \sqrt{10}$$

$$\text{opp} = 1$$



$$\text{adj} = 3$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = -\frac{\sqrt{10}}{3}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{1}{3}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = -\frac{3}{\sqrt{10}} = -\frac{3 \times \sqrt{10}}{\sqrt{10} \times \sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = -\sqrt{10}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = -\frac{1}{\sqrt{10}} = -\frac{1 \times \sqrt{10}}{\sqrt{10} \times \sqrt{10}} = -\frac{\sqrt{10}}{10}$$

الهدف الثالث

ايجاد قيم بعض الدوال المثلثية للزوايا الأساسية

الزوايا الأساسية هي :

Degrees	radians	Degrees	radians
0°	0	90°	$\frac{\pi}{2}$
30°	$\frac{\pi}{6}$	180°	π
45°	$\frac{\pi}{4}$	270°	$\frac{3\pi}{2}$
60°	$\frac{\pi}{3}$	360°	2π

	$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$
$\sin(\theta)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan(\theta)$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

	$0^\circ = 0$	$90^\circ = \frac{\pi}{2}$	$180^\circ = \pi$	$270^\circ = \frac{3\pi}{2}$	$360^\circ = 2\pi$
$\sin(\theta)$	0	1	0	-1	0
$\cos(\theta)$	1	0	-1	0	1
$\tan(\theta)$	0	undefined	0	undefined	0

Exercise (4)

$$1) \sin(0) = 0$$

$$\sin(\pi) = 0$$

$$\sin(2\pi) = 0$$

Note: $\sin(n\pi) = 0$ for all $n \in \mathbb{Z}$

For example: $\sin(4\pi) = 0$, $\sin(5\pi) = 0$

$$2) \cos(0) = 1$$

$$\cos(\pi) = -1$$

$$\cos(2\pi) = 1$$

Note: $\cos(n\pi) = \begin{cases} 1 & \text{if } n \text{ is even number} \\ -1 & \text{if } n \text{ is odd number} \end{cases}$

For example: $\cos(4\pi) = 1$, $\cos(5\pi) = -1$

$$3) \cos\left(\frac{\pi}{2}\right) = 0$$

$$\cos\left(\frac{3\pi}{2}\right) = 0$$

$$\cos\left(\frac{5\pi}{2}\right) = 0$$

Note: $\cos\left(\frac{n\pi}{2}\right) = 0$ for all n is odd number

Or : $\cos\left(\frac{(2n+1)\pi}{2}\right) = 0$ for all $n \in \mathbb{Z}$

For example: $\cos\left(\frac{9\pi}{2}\right) = 0$, $\cos\left(\frac{15\pi}{2}\right) = 0$

$$4) \sin\left(\frac{\pi}{2}\right) = 1$$

$$\sin\left(\frac{3\pi}{2}\right) = -1$$

$$\sin\left(\frac{5\pi}{2}\right) = 1$$

Note:

$$\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 1 & \text{if } n \text{ is odd number and } n=1,5,9,13,\dots \\ -1 & \text{if } n \text{ is odd number and } n=3,7,11,15,\dots \end{cases}$$

For example: $\sin\left(\frac{9\pi}{2}\right) = 1$, $\sin\left(\frac{15\pi}{2}\right) = -1$

Exercise (5)

$$\sec(7\pi) = \frac{1}{\cos(7\pi)} = \frac{1}{-1} = -1$$

$$\cot\left(\frac{5\pi}{2}\right) = \frac{\cos\left(\frac{5\pi}{2}\right)}{\sin\left(\frac{5\pi}{2}\right)} = \frac{0}{1} = 0$$

$$\csc(6\pi) = \frac{\cos(6\pi)}{\sin(6\pi)} = \frac{1}{0} = \text{undefined}$$

Exercise (6)

$$1) \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2} \text{ or } \frac{2}{\sqrt{2}}$$

$$2) \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\cot\left(\frac{\pi}{6}\right) = \frac{\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

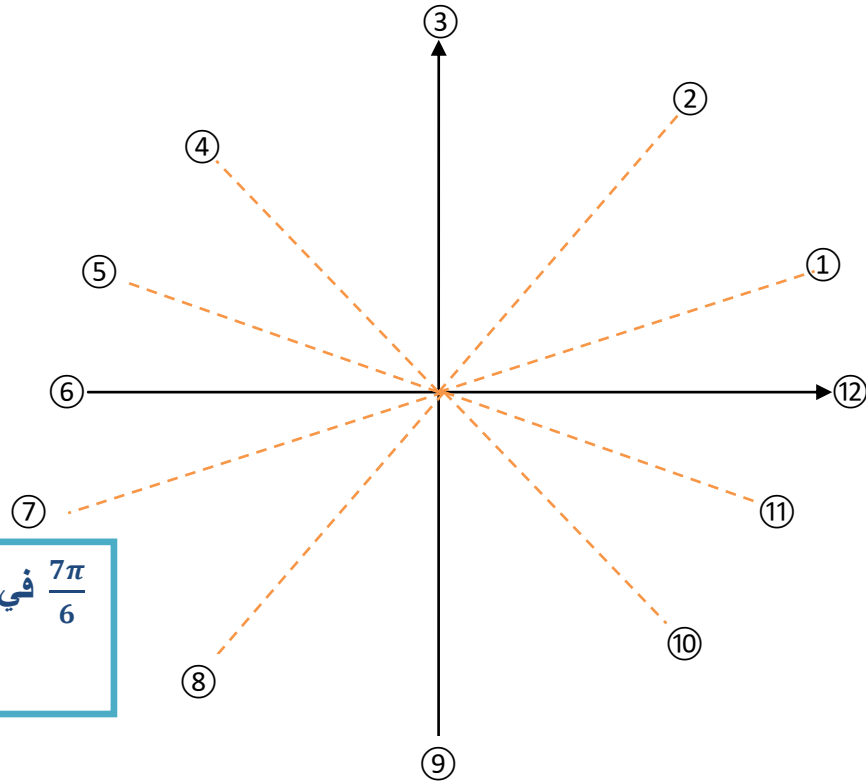
$$3) \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

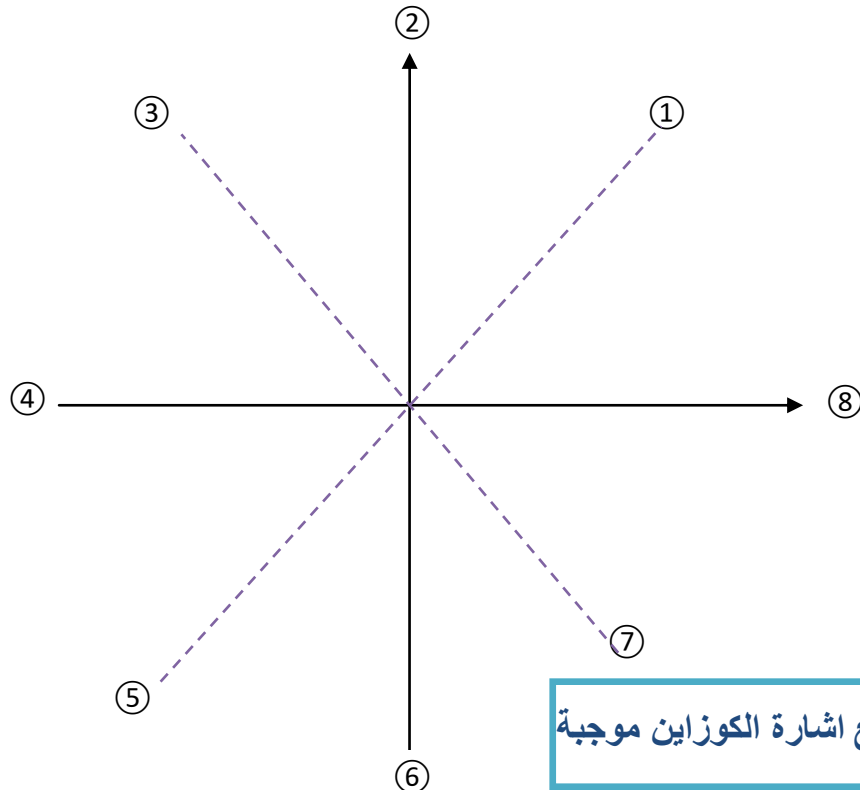
$$\csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Exercise (7)

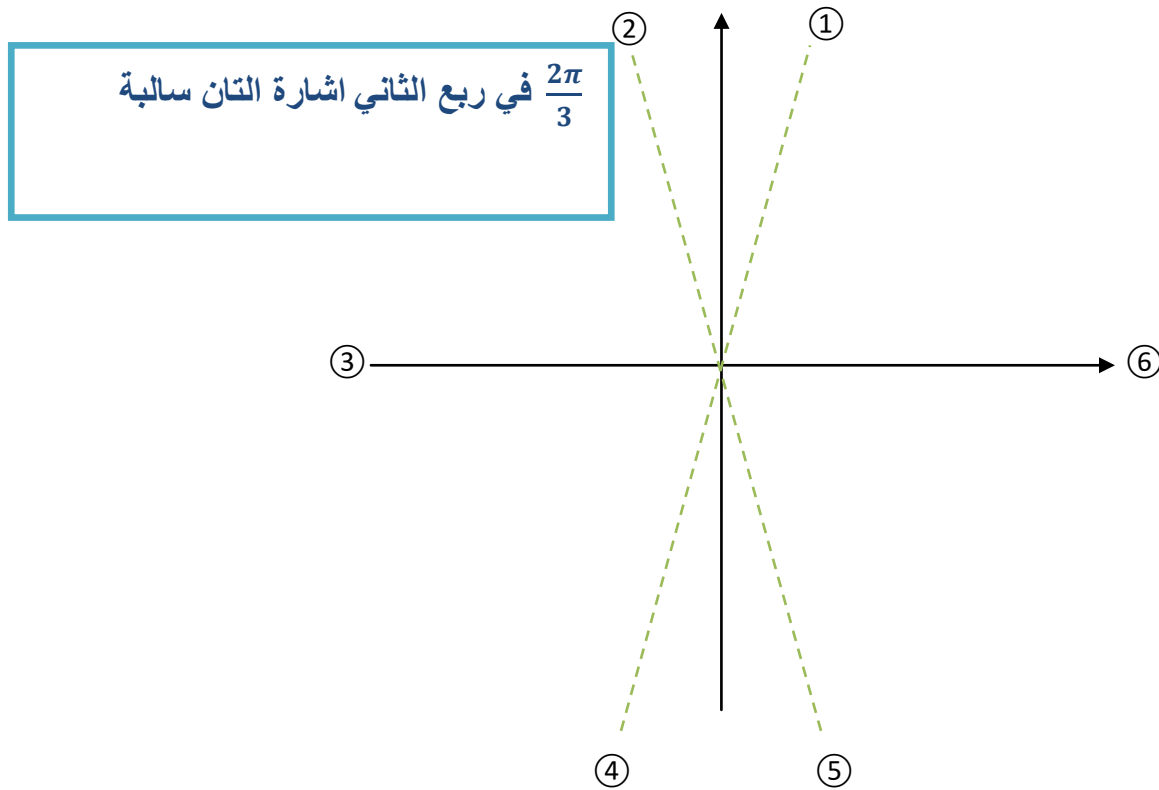
$$1) \sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$



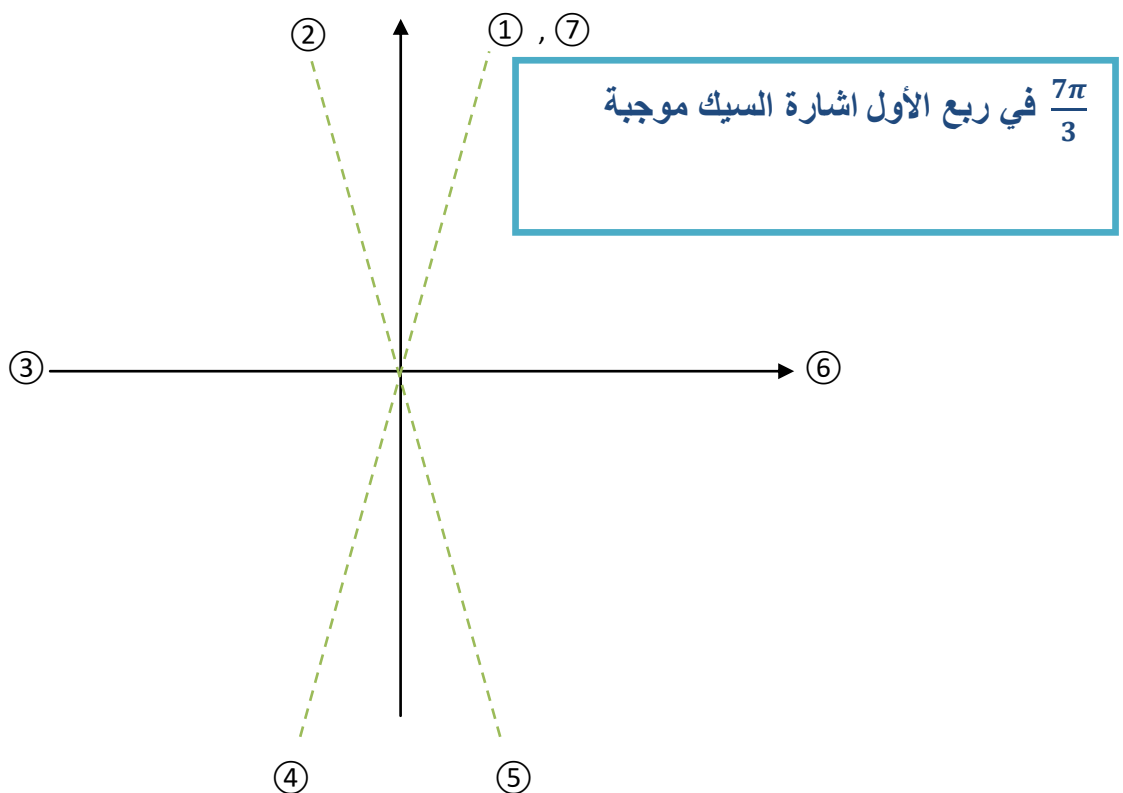
$$2) \cos\left(\frac{7\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$



$$3) \tan\left(\frac{2\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)} = -\frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = -\frac{\sqrt{3}}{2} \times \frac{2}{1} = -\sqrt{3}$$



$$\text{Sec}\left(\frac{7\pi}{3}\right) = \text{Sec}\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = \frac{2}{1} = 2$$



Trigonometric identities

Trigonometric Identities

Reciprocal Identities $\cot \theta = \frac{1}{\tan \theta}$ $\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$	Quotient Identities $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$
Pythagorean Identities $\sin^2 \theta + \cos^2 \theta = 1$ $\tan^2 \theta + 1 = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$	

Exercise (8)

$$\sec(\theta) \cos(\theta) = \mathbf{1}$$

$$\tan(\theta) \cos(\theta) = \mathbf{\sin(\theta)}$$

$$\mathbf{1} - \csc^2(\theta) = \mathbf{-\cot^2(\theta)}$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\csc(-\theta) = -\csc(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\cot(-\theta) = -\cot(\theta)$$

The odd functions

$$\cos(-\theta) = \cos(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

The even functions

Exercise (9)

$$\cos(-\pi) = \cos(\pi) = -1$$

$$\sin\left(-\frac{3\pi}{2}\right) = -\sin\left(\frac{3\pi}{2}\right) = -(-1) = 1$$

$$\cos(\theta+2\pi) = \cos(\theta)$$

$$\sec(\theta+2\pi) = \sec(\theta)$$

$$\sin(\theta+2\pi) = \sin(\theta)$$

$$\csc(\theta+2\pi) = \csc(\theta)$$

2π : period لجميع الدوال المثلثية

معدا الـ $\tan(\theta)$ و الـ $\cot(\theta)$ الـ π : period

$$\tan(\theta+\pi) = \tan(\theta)$$

$$\cot(\theta+\pi) = \cot(\theta)$$

Exercise (10)

$$\tan\left(\frac{\pi}{4}+\pi\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\csc\left(\frac{\pi}{6}+2\pi\right) = \csc\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{6}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

Exercise (11)

If $\sin(\theta) = \frac{3}{4}$ and $\cos(\theta) = \frac{\sqrt{7}}{4}$ then find $\sin(2\theta)$ and $\cos(2\theta)$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\sin(2\theta) = 2 \times \frac{3}{4} \times \frac{\sqrt{7}}{4} = \frac{3}{2} \times \frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{8}$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\cos(2\theta) = \left(\frac{\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = \frac{7}{16} - \frac{9}{16} = -\frac{2}{16} = -\frac{1}{8}$$

Trigonometric Functions

$$y = \sin x$$

Domain: \mathbb{R}

Range: $[-1, 1]$

Period: 2π

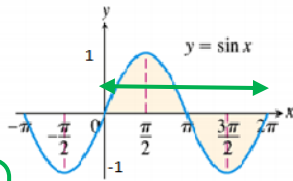
$$\sin(\theta + 2\pi) = \sin \theta$$

Odd function:

$$\sin(-\theta) = -\sin \theta$$

$$-1 \leq \sin x \leq 1 \Leftrightarrow |\sin x| \leq 1$$

$$\sin x = 0 \text{ when } x = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots$$



$$y = \cos x$$

Domain: \mathbb{R}

Range: $[-1, 1]$

Period: 2π

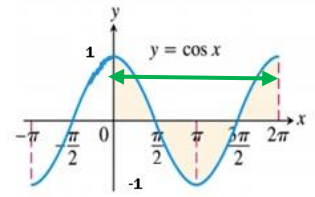
$$\cos(\theta + 2\pi) = \cos \theta$$

Even function:

$$\cos(-\theta) = \cos \theta$$

$$-1 \leq \cos x \leq 1 \Leftrightarrow |\cos x| \leq 1$$

$$\cos x = 0 \text{ when } x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$$



$$y = \tan x$$

Domain:

$$\mathbb{R} - \left\{ \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots \right\}$$

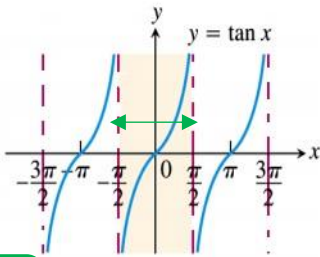
Range: \mathbb{R}

Period: π

$$\tan(\theta + \pi) = \tan \theta$$

Odd function:

$$\tan(-\theta) = -\tan \theta$$



$$y = \cot x$$

Domain:

$$\mathbb{R} - \{0, \pm\pi, \pm2\pi, \dots\}$$

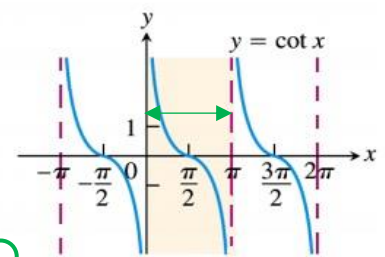
Range: \mathbb{R}

Period: π

$$\cot(\theta + \pi) = \cot \theta$$

Odd function:

$$\cot(-\theta) = -\cot \theta$$



$$y = \csc x$$

Domain:

$$\mathbb{R} - \{0, \pm\pi, \pm2\pi, \dots\}$$

Range:

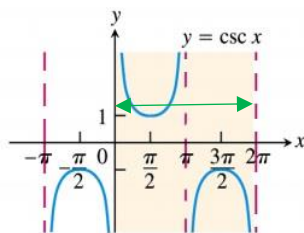
$$\mathbb{R} - (-1, 1)$$

Period: 2π

$$\csc(\theta + 2\pi) = \csc \theta$$

Odd function:

$$\csc(-\theta) = -\csc \theta$$



$$y = \sec x$$

Domain:

$$\mathbb{R} - \left\{ \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots \right\}$$

Range:

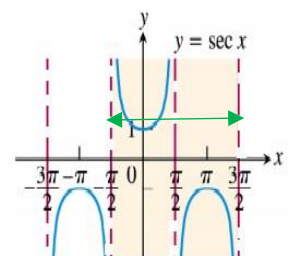
$$\mathbb{R} - (-1, 1)$$

Period: 2π

$$\sec(\theta + 2\pi) = \sec \theta$$

Even function:

$$\sec(-\theta) = \sec \theta$$



Instructions : (33 points). Solve each of the following problems and choose the correct answer.

- (1) The domain of the function $f(x) = |3x - 6|$ is
- (a) $\mathbb{R} - \{2\}$
 - (b) $[2, \infty)$
 - (c) $\mathbb{R} - \{-2\}$
 - (d) \mathbb{R} *
- (2) The domain of the function $f(x) = \frac{x+2}{x^2+x-6}$ is
- (a) $\mathbb{R} - \{-2, 3\}$
 - (b) $\mathbb{R} - \{-2, -3\}$
 - (c) $\mathbb{R} - \{2, -3\}$ *
 - (d) $\mathbb{R} - \{2, 3\}$
- (3) The domain of the function $f(x) = \sqrt{4 - x^2}$ is
- (a) $(-2, 2)$
 - (b) $[-2, 2]$ *
 - (c) $(-\infty, -2] \cup [2, \infty)$
 - (d) $(2, \infty)$
- (4) The range of the function $f(x) = \sqrt{25 + x^2}$ is
- (a) $(-\infty, 5]$
 - (b) $(-\infty, 5)$
 - (c) $(5, \infty)$
 - (d) $[5, \infty)$ *
- (5) The range of the function $f(x) = 9 - x^2$ is
- (a) $(-\infty, 9]$ *
 - (b) $[9, \infty)$
 - (c) $(-\infty, -9]$
 - (d) $[-9, \infty)$
- (6) The function $f(x) = 10 - x^3$ is even.
- (a) True
 - (b) False *
- (7) The function $f(x) = x^{\frac{2}{3}} + x^2$ is
- (a) Algebraic function *
 - (b) Power function
 - (c) Polynomial function
 - (d) Exponential function
- (8) If $h(x) = |\cos x|$, $f(x) = \cos x$, $g(x) = |x|$, then
- (a) $h = f \circ g$
 - (b) $h = g \circ f$ *
 - (c) $h = f.g$
 - (d) $h = f \circ f$
- (9) The function $f(x) = \frac{7 - x^2}{x^3 + 3x}$ is symmetric about the origin.
- (a) True *
 - (b) False
- (10) The function $f(x) = (x - 1)^2$ is

- (a) increasing on $(1, \infty)$ *
- (b) increasing on $(-\infty, 1)$
- (c) decreasing on $(1, \infty)$
- (d) decreasing on $(-1, \infty)$

(11) The degree measure of $\theta = \frac{7\pi}{6}$ is

- (a) 100°
- (b) 120°
- (c) 210° *
- (d) 75°

(12) The radian measure of $\theta = 150^\circ$ is

- (a) $\frac{5\pi}{6}$ *
- (b) $\frac{10\pi}{3}$
- (c) $\frac{10\pi}{9}$
- (d) $\frac{4\pi}{3}$

(13) If $f(x) = x^2$ and $g(x) = \sqrt{2+x}$, then $(f \circ g)(x) =$

- (a) $2 + x^2$
- (b) $\sqrt{2 + x^2}$
- (c) $(2 + x)^2$
- (d) $2 + x$ *

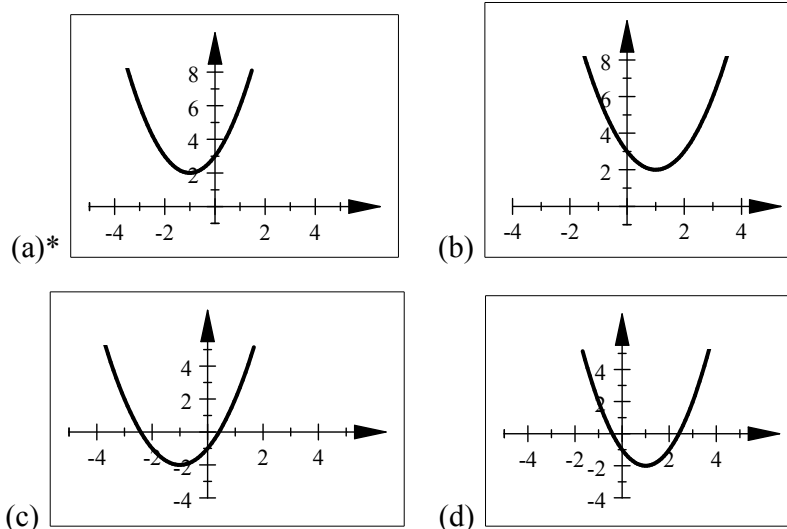
(14) If $f(x) = x$ and $g(x) = 3x^2 + x$, then $\left(\frac{f}{g}\right)(x) =$

- (a) $\frac{x}{3x^2 - 1}$
- (b) $\frac{1}{3x + 1}$ *
- (c) $\frac{1}{3x - 1}$
- (d) $\frac{x}{3x^2 + 1}$

(15) If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, then the domain of $(f+g)(x)$ is

- (a) $(-\infty, 2]$
- (b) $[0, 2]$ *
- (c) $[0, \infty)$
- (d) $(0, 2)$

(16) The graph of the function $f(x) = (x+1)^2 + 2$ is



- (17) The graph of $g(x) = |x - 4|$ is a shifting of the graph of $f(x) = |x|$
- (a) 4 units to the left
 - (b) 4 units to the right *
 - (c) 4 units downward
 - (d) 4 units upward
- (18) If the graph of $f(x) = 3^x$ is reflected about the y -axis, then the equation of the new function is
- (a) $(\frac{1}{3})^{-x}$
 - (b) $(-3)^x$
 - (c) $(\frac{1}{3})^x$ *
 - (d) $-(3^x)$
- (19) If $\cos x = \frac{3}{2}$, $\sin x = \frac{1}{2}$, then $\sin(2x) =$
- (a) $\frac{3}{2}$ *
 - (b) 2
 - (c) 4
 - (d) $\frac{3}{4}$
- (20) The function $f(x) = (\frac{1}{2})^x$ is increasing on \mathbb{R} .
- (a) True
 - (b) False *
- (21) If $\sin \theta = \frac{3}{4}$ and $0 < \theta < \frac{\pi}{2}$, then $\cos \theta =$
- (a) $\frac{-3}{\sqrt{7}}$
 - (b) $-\frac{\sqrt{7}}{4}$
 - (c) $\frac{3}{\sqrt{7}}$
 - (d) $\frac{\sqrt{7}}{4}$ *
- (22) If $\theta = \frac{-\pi}{3}$, then $\sin \theta =$
- (a) $\frac{1}{2}$
 - (b) $\frac{\sqrt{3}}{2}$

- (c) $\frac{-\sqrt{3}}{2}$ *
- (d) $\frac{-1}{2}$

(23) The range of the function $f(x) = \sin x$ is

- (a) \mathbb{R}
- (b) $(-1, 1)$
- (c) $\mathbb{R} - (-1, 1)$
- (d) $[-1, 1]$ *

(24) The function $f(x) = \cot x$ is

- (a) even
- (b) odd *
- (c) even and odd
- (d) neither even nor odd

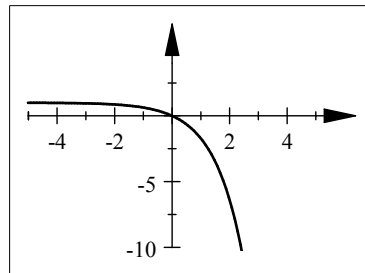
(25) If a is a positive number and x, y are real numbers, then $(a^x)^y =$

- (a) a^{x+y}
- (b) $a^{x \cdot y}$ *
- (c) $a^x \cdot a^y$
- (d) $a^{x/y}$

(26) The range of the function $y = 2^x + 1$ is

- (a) $(1, \infty)$ *
- (b) $[1, \infty)$
- (c) $(-\infty, 1)$
- (d) $(-\infty, 1]$

(27) The following graph represents the function $f(x) =$



- (a) $-e^x - 1$
- (b) $e^{-x} + 1$
- (c) $e^{-x} - 1$
- (d) $1 - e^x$ *

(28) The domain of the function $f(x) = \frac{1}{1 - e^{2x}}$ is

- (a) $\mathbb{R} - \{0\}$ *
- (b) $\mathbb{R} - \{1\}$
- (c) $\mathbb{R} - \{0, 1\}$
- (d) \mathbb{R}

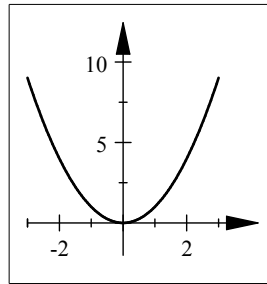
(29) If $f(x) = 3x + 2$, then $f^{-1}(x) =$

- (a) $\frac{x-3}{2}$
- (b) $\frac{x+3}{2}$

(c) $\frac{x-2}{3}$ *

(d) $\frac{x+2}{3}$

(30) The following graph represents one - to - one function



1. (a) true
(b) false *

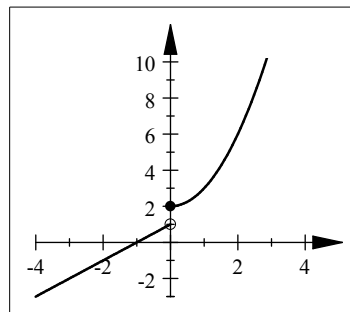
(31) The range of the function $f(x) = \sqrt{x}$ is

- (a) \mathbb{R}
(b) $\mathbb{R} - \{0\}$
(c) $[0, \infty)$ *
(d) $(0, \infty)$

(32) One of the following identities is true

- (a) $\cos(2x) = \cos^2 x - \sin^2 x$ *
(b) $\cos(2x) = \cos^2 x + \sin^2 x$
(c) $\cos(2x) = \cos^2(2x) - \sin^2(2x)$
(d) $\cos(2x) = 2 \sin x \cdot \cos x$

(33) The following graph



represents the function :

(a) $f(x) = \begin{cases} x^2 + 2 & \text{if } x > 0 \\ x + 1 & \text{if } x \leq 0 \end{cases}$

(b) $f(x) = \begin{cases} x^2 + 2 & \text{if } x \geq 0 \\ x + 1 & \text{if } x < 0 \end{cases}$ *

(c) $f(x) = \begin{cases} x^2 + 2 & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$

(d) $f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

المحاضرة الأولى في مساند

(1.1) Four Ways to Represent a Function

الهدف الأول من الفصل الأول

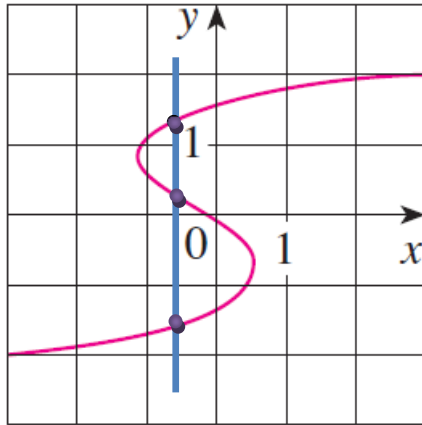
كيفية معرفة المنحنى أنه دالة (function) ام لا (not function)

الهدف الثاني من الفصل الأول

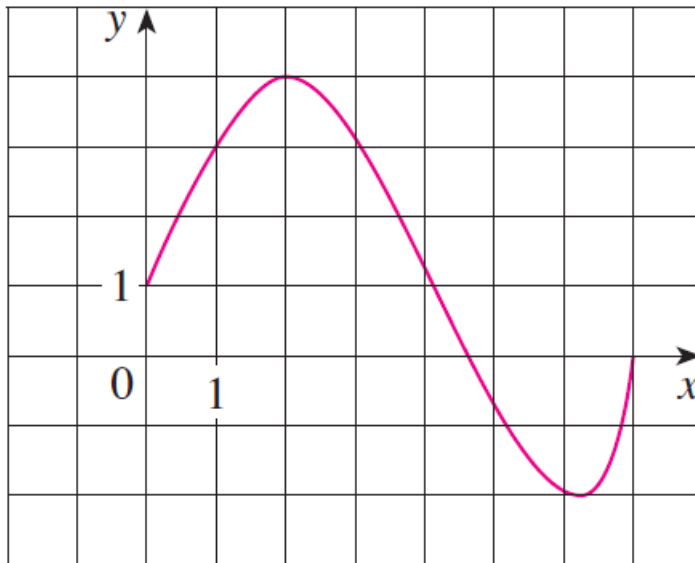
كيفية ايجاد المجال والمدى من الرسم البياني

Exercise (1)

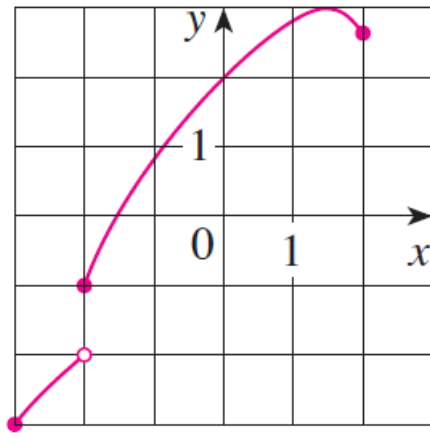
Determine whether the curve is the graph of a function of x . if it is, state the domain and range of the function.



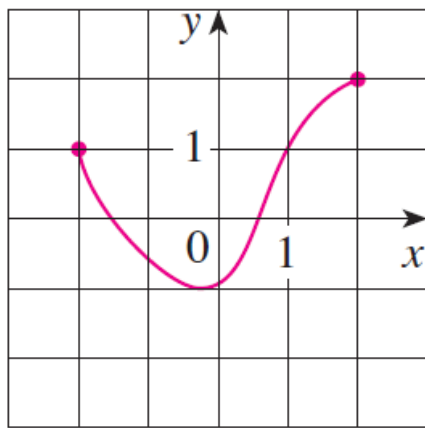
This curve does not represent a function because, as you can see, there are vertical lines that intersect the curve more than once



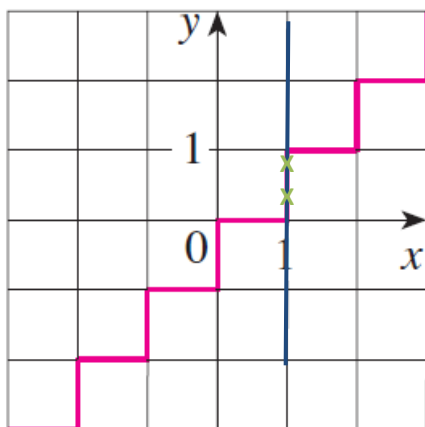
This curve represents a function
The Domain: $[0 , 7]$
The Range: $[-2 , 4]$



This curve represents a function
 The Domain: $[-3, 2]$
 The Range: $[-3, -2) \cup [-1, 3]$



This curve represents a function
 The Domain: $[-2, 2]$
 The Range: $[-1, 2]$



This curve does not represent a function because, as you can see, there are vertical lines that intersect the curve more than once

الهدف الثالث من الفصل الأول

كيفية ايجاد مجال الدالة الخطية ومداهما وتمثيلها بيانياً

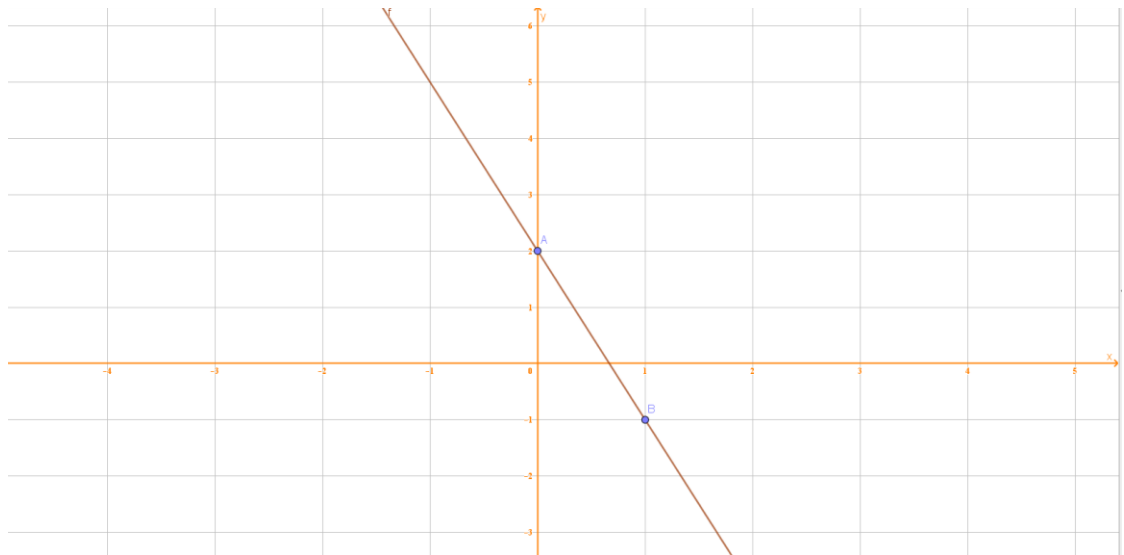
Exercise (2)

Find the domain , range and sketch the graph of the function.

$$f(x) = 2 - 3x$$

$f(x) = 2 - 3x$ is a linear function

x	0	1
$y = 2 - 3x$	$y(0) = 2 - 3(0)$ $= 2$	$y(1) = 2 - 3(1)$ $= 2 - 3 = -1$
(x, y)	(0, 2)	(1, -1)



Domain : $R = (-\infty, \infty)$

Range : $R = (-\infty, \infty)$

Exercise (3)

Find the domain of the function.

1) $f(x) = \frac{x+3}{x+1}$

- نوجد اصفار المقام

$$x + 1 = 0$$

$$x = -1$$

- $D_{f(x)} = R - \{-1\}$

$$\text{Or } = \{ x \mid x \neq -1 \}$$

$$\text{Or } = (-\infty, -1) \cup (-1, \infty)$$

2) $f(x) = \frac{x}{2x-6}$

- نوجد اصفار المقام

$$2x - 6 = 0$$

$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

- $D_{f(x)} = R - \{3\}$

$$\text{Or } = \{ x \mid x \neq 3 \}$$

$$\text{Or } = (-\infty, 3) \cup (3, \infty)$$

$$3) f(x) = \frac{x+4}{x^2-9}$$

- نوجد اصفار المقام

$$x^2 - 9 = 0 \Rightarrow x^2 = 9$$

$$\sqrt{x^2} = \sqrt{9} \Rightarrow |x| = 3 \Rightarrow x = \pm 3$$

- $D_{f(x)} = R - \{-3, 3\}$

Or $= \{ x \mid x \neq -3, x \neq 3 \}$

Or $= (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

$$4) f(x) = \frac{1}{x^2+5}$$

- نوجد اصفار المقام

$$x^2 + 5 = 0 \Rightarrow x^2 = -5$$

$$\sqrt{x^2} = \sqrt{-5} \Rightarrow |x| = \sqrt{-5} \notin R$$

إذن لا يوجد أصفار مقام

- $D_{f(x)} = R$

$$5) f(x) = \frac{2x^3-5}{x^2+x-6}$$

- نوجد اصفار المقام

$$x^2 + x - 6 = 0$$

$$(x - 2)(x + 3) = 0$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$x + 3 = 0 \Rightarrow x = -3$$

- $D_{f(x)} = R - \{-3, 2\}$

Or $= \{ x \mid x \neq -3, x \neq 2 \}$

Or $= (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

$$6) f(x) = \frac{4x}{x^2 - 5x + 6}$$

- نوجد اصفار المقام

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$x - 3 = 0 \Rightarrow x = 3$$

- $D_{f(x)} = R - \{2, 3\}$

$$\text{Or } = \{ x \mid x \neq 2, x \neq 3 \}$$

$$\text{Or } = (-\infty, 2) \cup (2, 3) \cup (3, \infty)$$

المحاضرة الثالثة في مساند

(1.1) Four Ways to Represent a Function

الهدف التاسع من الفصل الأول

كيفية ايجاد مجال و مدى ورسم وصور العناصر لدوال المتعددة التعريف

(Piecewise Defined Functions)

Exercise (1)

A) If $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 1 - x & \text{if } x \geq 0 \end{cases}$

1) Find $f(-3)$, $f(2)$ and $f(0)$

$f(-3) = -3 + 2 = -1$ (a)
عوضنا عن -3 في الدالة الأولى لأن $-3 \in (-\infty, 0)$

$f(2) = 1 - 2 = -1$ (b)
عوضنا عن 2 في الدالة الثانية لأن $2 \in [0, \infty)$

$f(0) = 1 - 0 = 1$ (c)
عوضنا عن 0 في الدالة الثانية لأن $0 \in [0, \infty)$

2) Find the domain of the function $f(x)$

$$\begin{aligned} D_{f(x)} &= \{x \mid x < 0\} \cup \{x \mid x \geq 0\} \\ &= (-\infty, 0) \cup [0, \infty) \\ &= (-\infty, \infty) = \mathbb{R} \end{aligned}$$

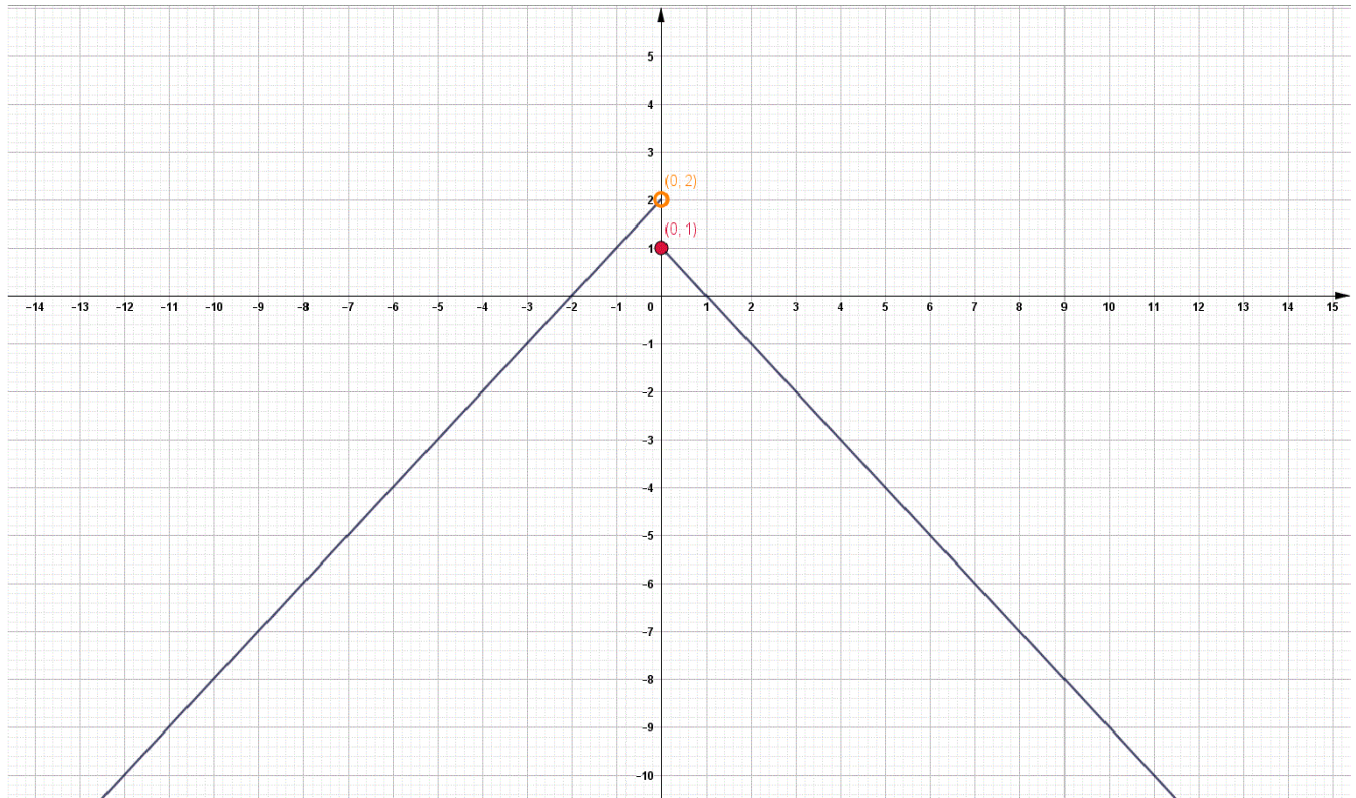
3) Sketch the graph of the function $f(x)$ and find the range

let $f_1(x) = x + 2$ for all $x \in (-\infty, 0)$

x	0	-1
$y = f_1(x)$	2	1
(x, y)	(0,2)	(-1,1)

let $f_2(x) = 1 - x$ for all $x \in [0, \infty)$

x	0	1
$y = f_2(x)$	1	0
(x, y)	(0,1)	(1,0)



The range : $(-\infty, 2)$

B) If $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x < 2 \\ 2x - 5 & \text{if } x \geq 2 \end{cases}$

1) Find $f(-3)$, $f(2)$ and $f(0)$

$f(-3) = 3 - \frac{1}{2}(-3) = \frac{3}{1} + \frac{3}{2} = \frac{3 \times 2}{1 \times 2} + \frac{3 \times 1}{2 \times 1} = \frac{6}{2} + \frac{3}{2} = \frac{6+3}{2} = \frac{9}{2}$ (a)

عوضنا عن -3 في الدالة الأولى لأن $-3 \in (-\infty, 2)$

$f(2) = 2(2) - 5 = 4 - 5 = -1$ (b)

عوضنا عن 2 في الدالة الثانية لأن $2 \in [2, \infty)$

$f(0) = 3 - \frac{1}{2}(0) = 3 - 0 = 3$ (c)

عوضنا عن 0 في الدالة الأولى لأن $0 \in (-\infty, 2)$

2) Find the domain of the function $f(x)$

$$\begin{aligned} D_{f(x)} &= \{x \mid x < 2\} \cup \{x \mid x \geq 2\} \\ &= (-\infty, 2) \cup [2, \infty) \\ &= (-\infty, \infty) = \mathbb{R} \end{aligned}$$

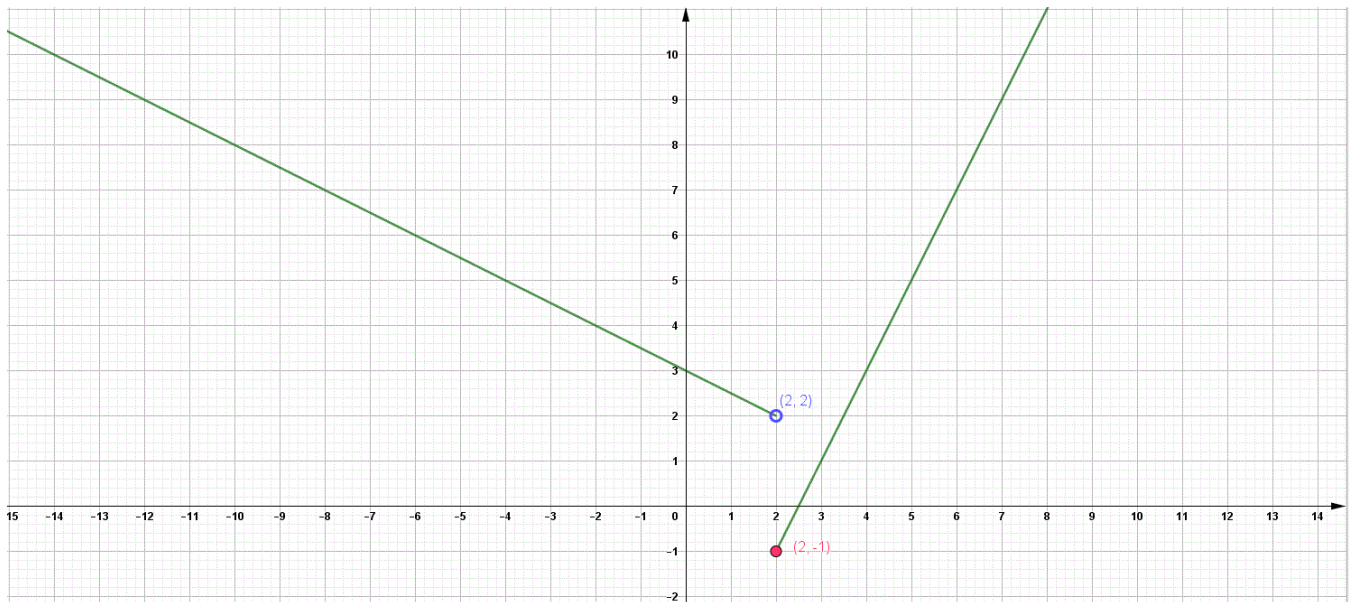
3) Sketch the graph of the function $f(x)$ and find the range

let $f_1(x) = 3 - \frac{1}{2}x$ for all $x \in (-\infty, 2)$

x	2	0
$y = f_1(x)$	2	3
(x, y)	(2,2)	(0,3)

let $f_2(x) = 2x - 5$ for all $x \in [2, \infty)$

x	2	3
$y = f_2(x)$	-1	1
(x, y)	(2,-1)	(3,1)



The range of $f(x) = [-1, \infty)$

c) If $f(x) = \begin{cases} x + 1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

4) Find $f(-3)$, $f(2)$ and $f(0)$

$$f(-3) = -3 + 1 = -2 \quad (\text{a})$$

عوضنا عن -3 في الدالة الأولى لأن $-3 \in (-\infty, -1]$

$$f(2) = (2)^2 = 4 \quad (\text{b})$$

عوضنا عن 2 في الدالة الثانية لأن $2 \in (-1, \infty)$

$$f(0) = (0)^2 = 0 \quad (\text{c})$$

عوضنا عن 0 في الدالة الثانية لأن $0 \in (-1, \infty)$

5) Find the domain of the function $f(x)$

$$\begin{aligned} D_{f(x)} &= \{x \mid x \leq -1\} \cup \{x \mid x > -1\} \\ &= (-\infty, -1] \cup (-1, \infty) \\ &= (-\infty, \infty) = \mathbb{R} \end{aligned}$$

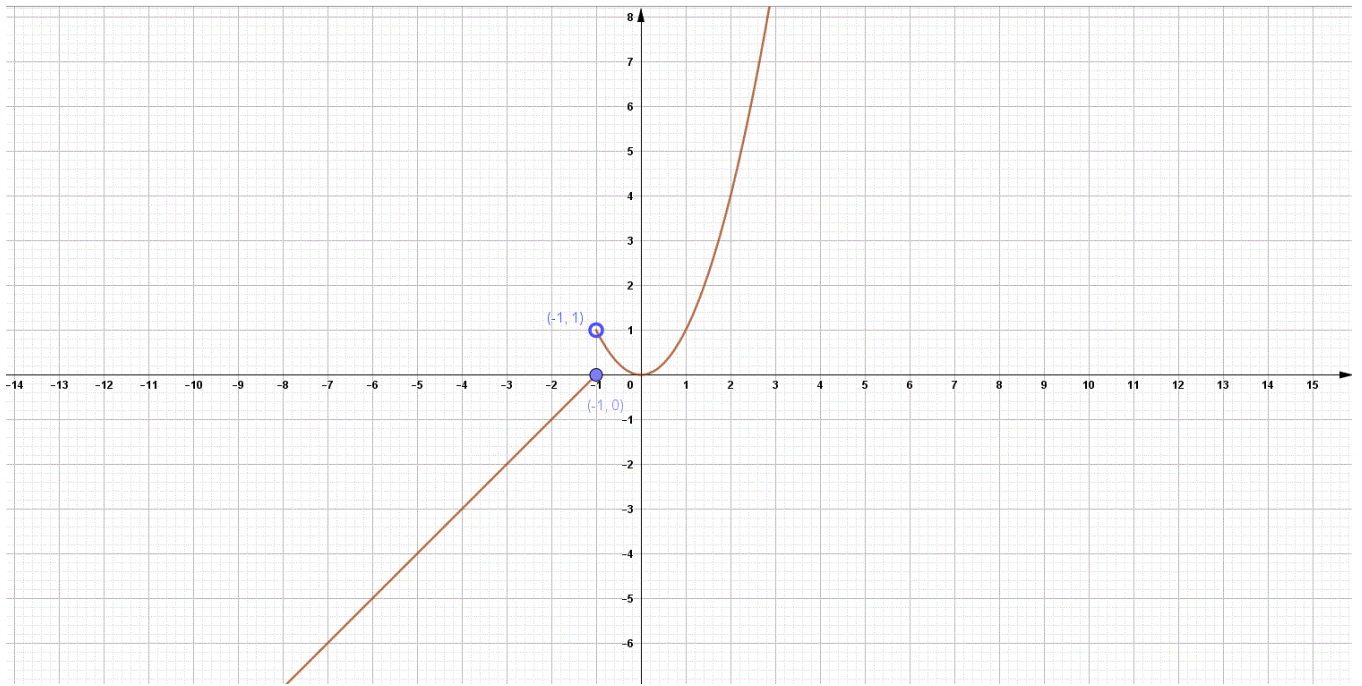
6) Sketch the graph of the function $f(x)$ and find the range

let $f_1(x) = x + 1$ for all $x \in (-\infty, -1]$

x	-1	-2
$y = f_1(x)$	0	-1
(x, y)	(-1, 0)	(-2, -1)

let $f_2(x) = x^2$ for all $x \in (-1, \infty)$

x	-1	0	1	2
$y = f_2(x)$	1	0	1	4
(x, y)	(-1, 1)	(0, 0)	(1, 1)	(2, 4)



The range of $f(x) = \mathbb{R}$

D) If $f(x) = \begin{cases} -1 & \text{if } x \leq 1 \\ 7 - 2x & \text{if } x > 1 \end{cases}$

1) Find $f(-3)$, $f(2)$ and $f(0)$

$$f(-3) = -1 \quad (\text{d})$$

عوضنا عن -3 في الدالة الأولى لأن $-3 \in (-\infty, 1]$

$$f(2) = 7 - 2(2) = 7 - 4 = 3 \quad (\text{e})$$

عوضنا عن 2 في الدالة الثانية لأن $2 \in (1, \infty)$

$$f(0) = -1 \quad (\text{f})$$

عوضنا عن 0 في الدالة الأولى لأن $0 \in (-\infty, 1]$

2) Find the domain of the function $f(x)$

$$\begin{aligned} D_{f(x)} &= \{x \mid x \leq 1\} \cup \{x \mid x > 1\} \\ &= (-\infty, 1] \cup (1, \infty) \\ &= (-\infty, \infty) = \mathbb{R} \end{aligned}$$

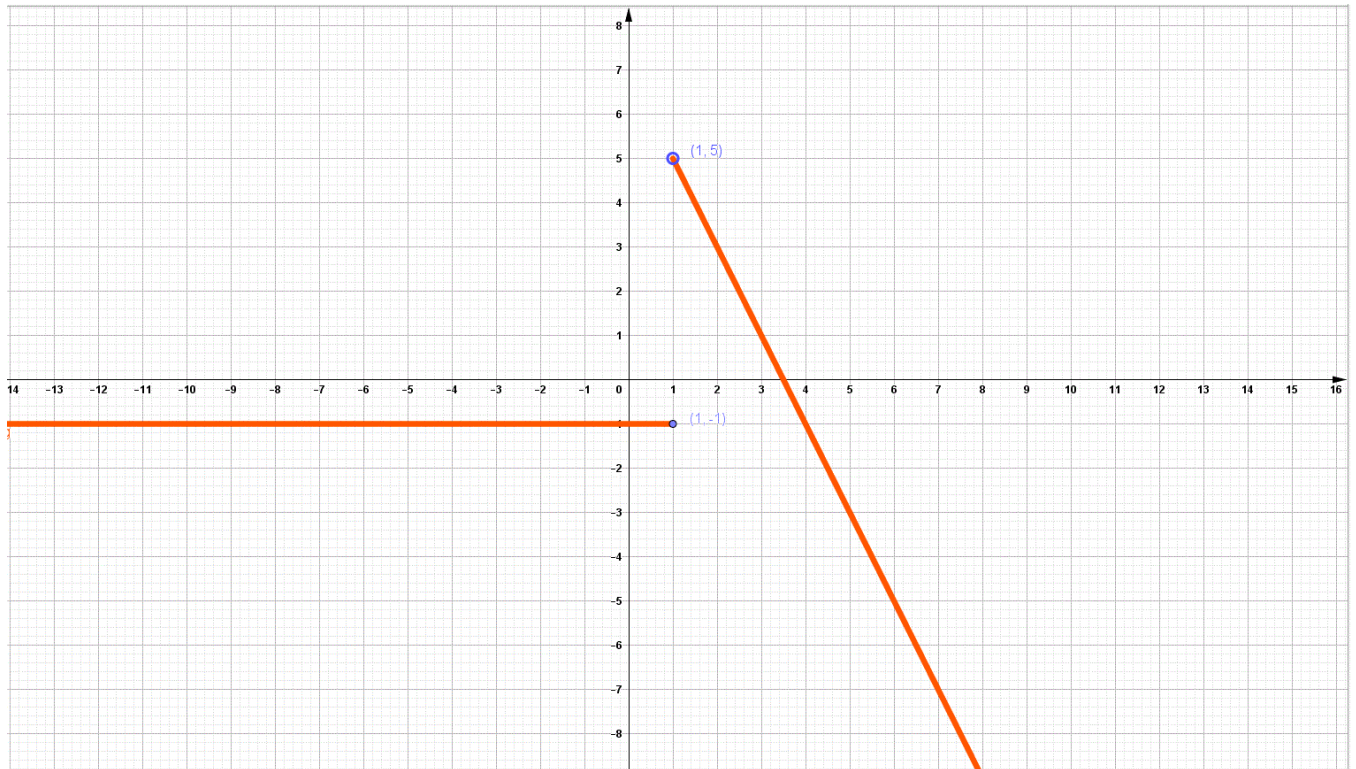
3) Sketch the graph of the function $f(x)$ and find the range

let $f_1(x) = -1$ for all $x \in (-\infty, 1]$

x	1	0
$y = f_1(x)$	-1	-1
(x, y)	(1, -1)	(0, -1)

let $f_2(x) = 7 - 2x$ for all $x \in \cup (1, \infty)$

x	1	2
$y = f_2(x)$	5	3
(x, y)	(1, 5)	(2, 3)



The range of $f(x) = (-\infty, 5)$

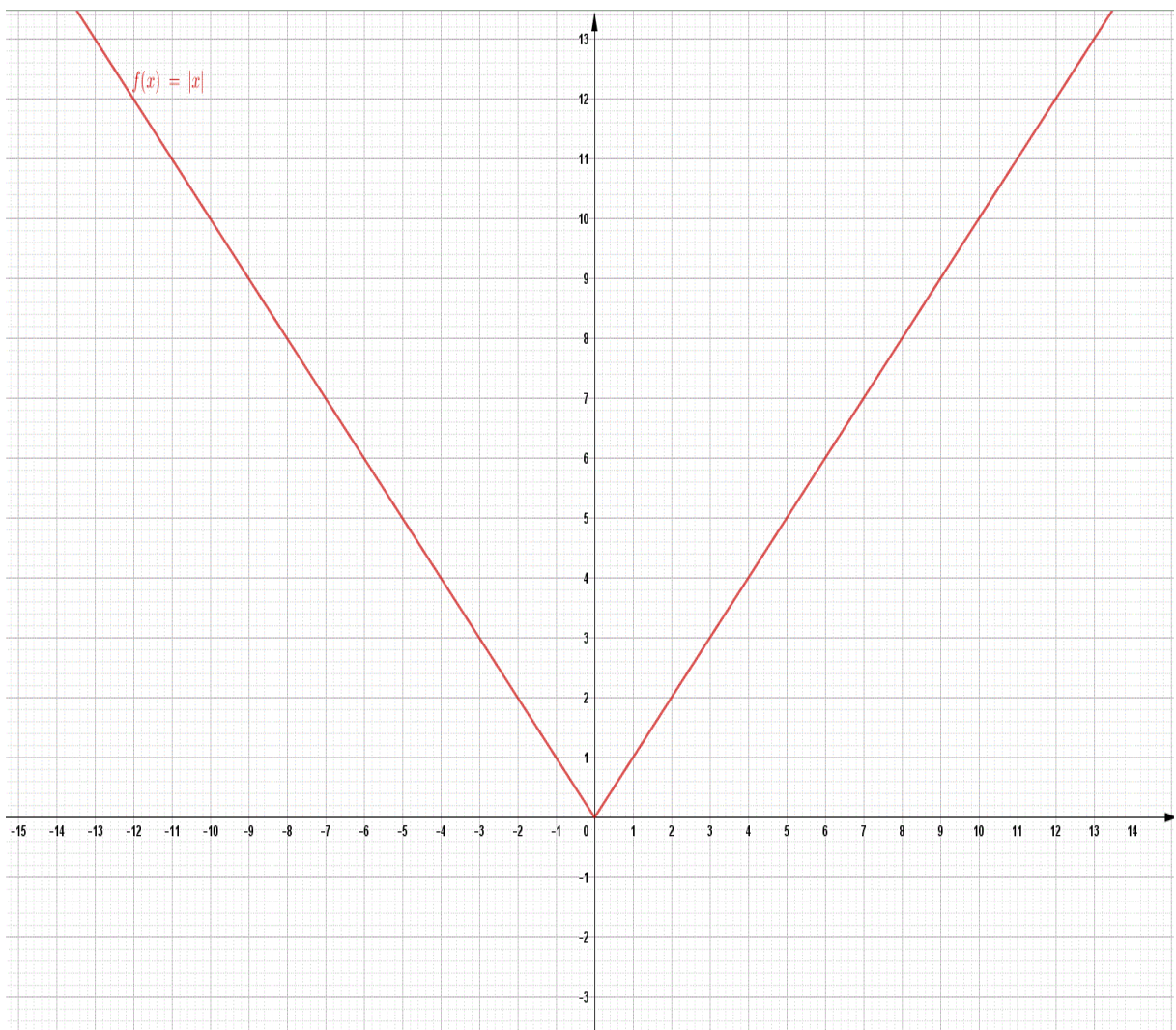
كيفية ايجاد مجال و مدى ورسم وفك دالة القيمة المطلقة

(absolute value Function)

Exercise (1)A) If $f(x) = |x|$

then sketch the graph of the function and find the domain and range

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

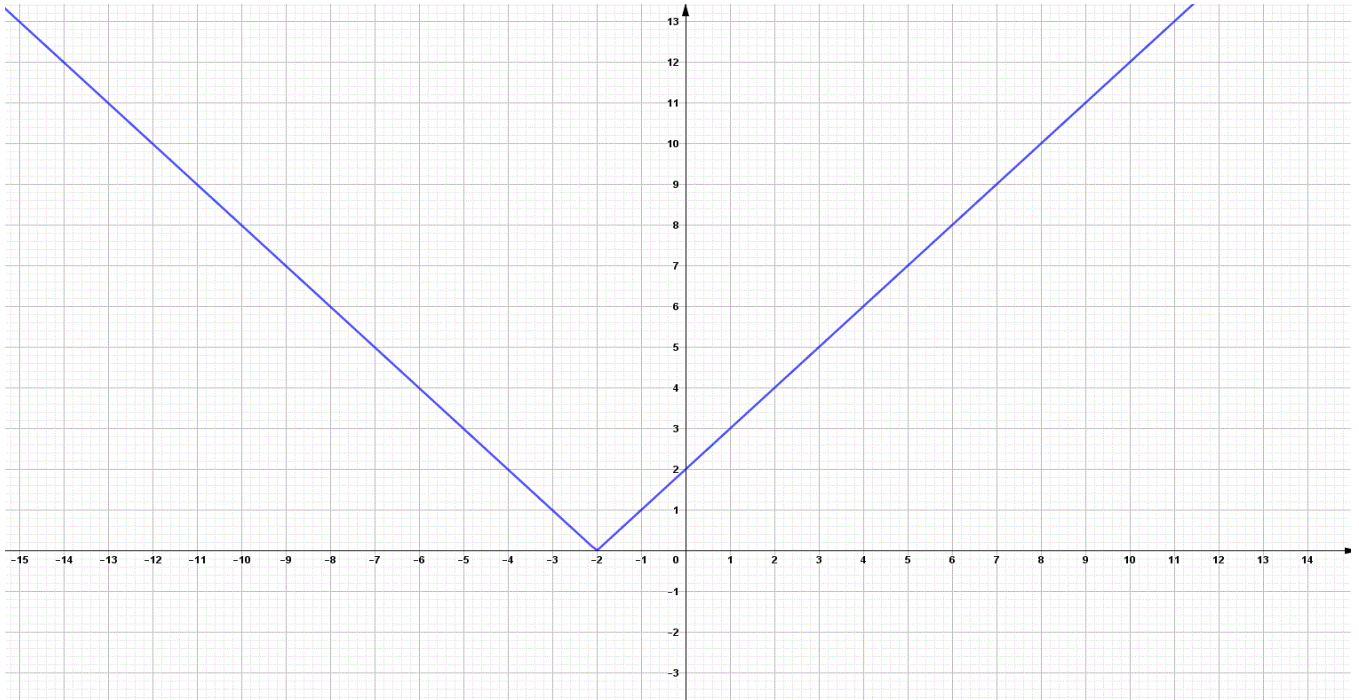
The domain of $f(x) = (-\infty, \infty)$ The range of $f(x) = [0, \infty)$

B) If $f(x) = |x + 2|$

then sketch the graph of the function and find the domain and range

$$f(x) = |x + 2| = \begin{cases} x + 2 & \text{if } x + 2 \geq 0 \\ -(x + 2) & \text{if } x + 2 < 0 \end{cases}$$

$$= \begin{cases} x + 2 & \text{if } x \geq -2 \\ -x - 2 & \text{if } x < -2 \end{cases}$$



The domain of $f(x) = (-\infty, \infty)$

The range of $f(x) = [0, \infty)$

C) If $f(x) = |6 - 2x|$

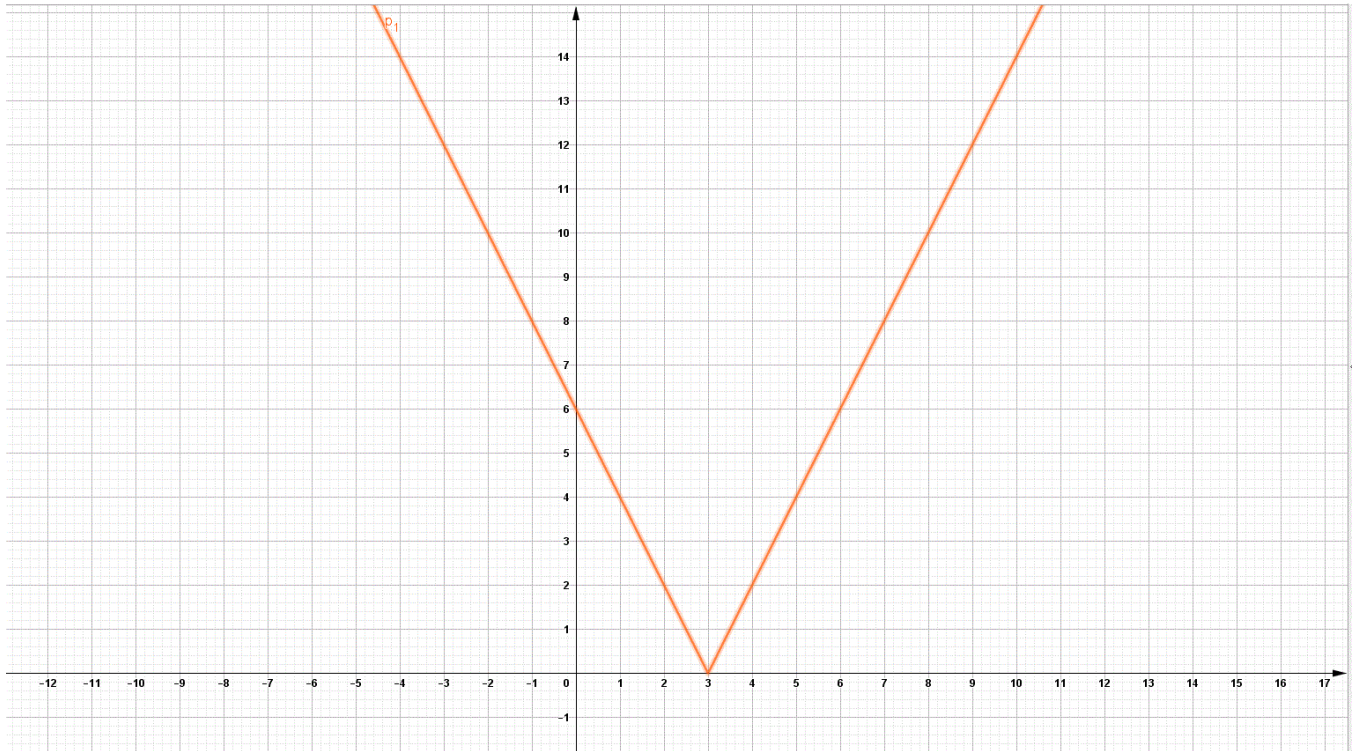
then sketch the graph of the function and find the domain and range

$$f(x) = |6 - 2x| = \begin{cases} 6 - 2x & \text{if } 6 - 2x \geq 0 \\ -(6 - 2x) & \text{if } 6 - 2x < 0 \end{cases}$$

$$= \begin{cases} 6 - 2x & \text{if } -2x \geq -6 \\ -6 + 2x & \text{if } -2x < -6 \end{cases}$$

$$= \begin{cases} 6 - 2x & \text{if } \frac{-2x}{-2} \leq \frac{-6}{-2} \\ 2x - 6 & \text{if } \frac{-2x}{-2} > \frac{-6}{-2} \end{cases}$$

$$= \begin{cases} 6 - 2x & \text{if } x \leq 3 \\ 2x - 6 & \text{if } x > 3 \end{cases}$$



The domain of $f(x) = (-\infty, \infty)$

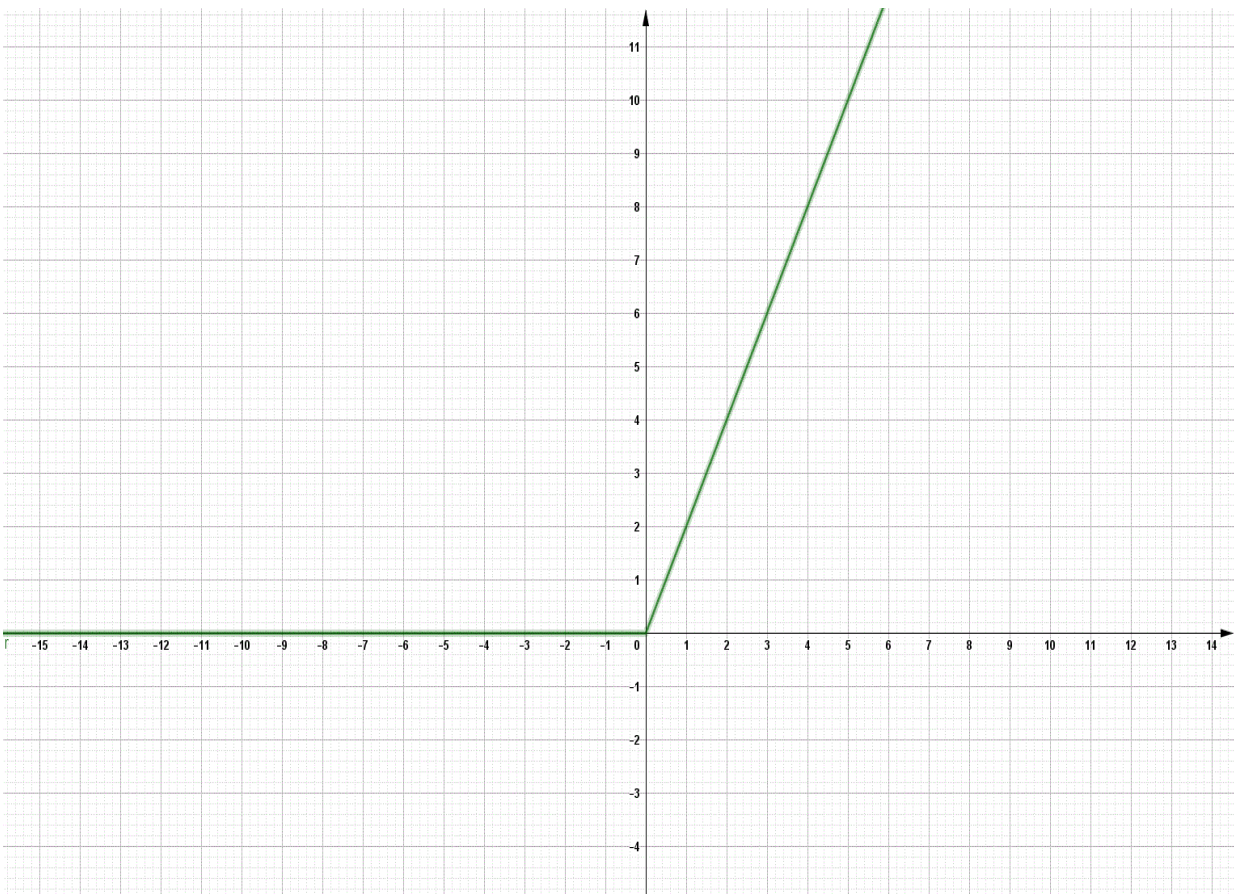
The range of $f(x) = [0, \infty)$

D) If $f(x) = x + |x|$

then sketch the graph of the function and find the domain and range

$$f(x) = x + |x| = \begin{cases} x + x & \text{if } x \geq 0 \\ x - x & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$



The domain of $f(x) = (-\infty, \infty)$

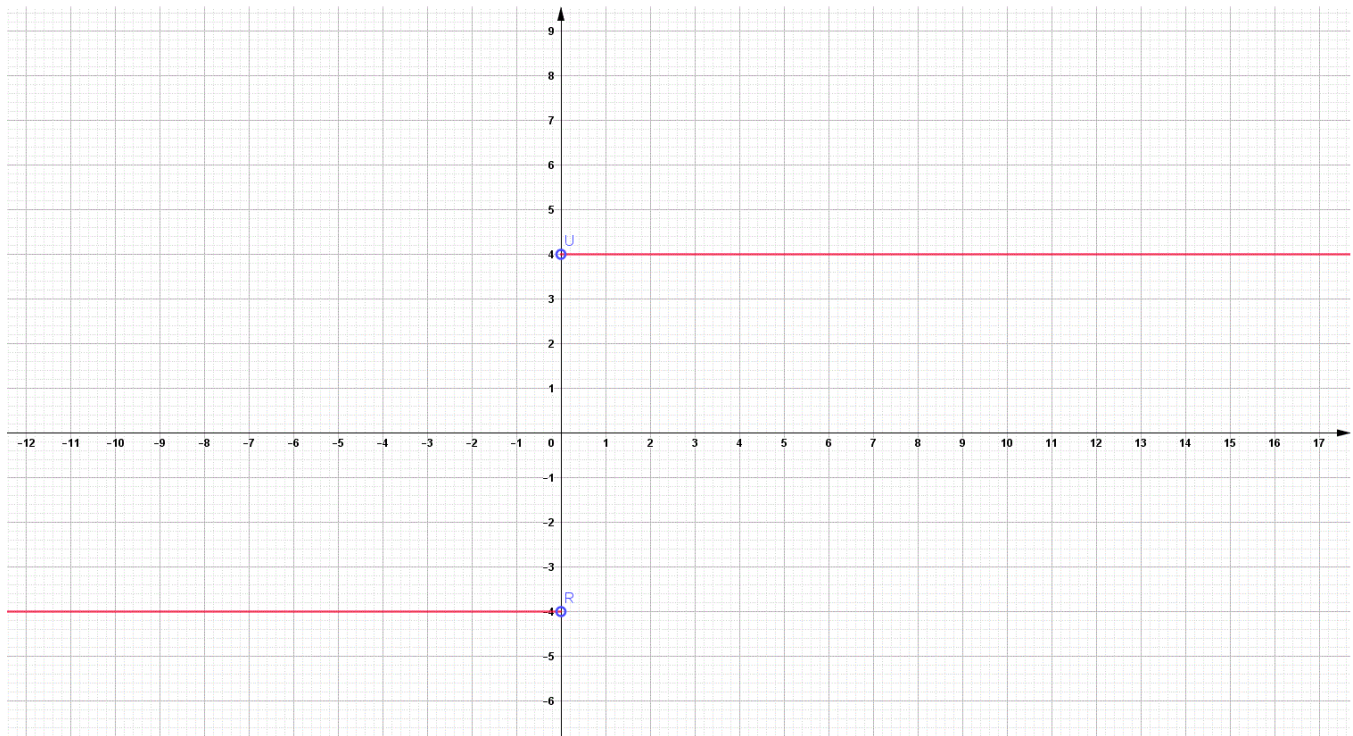
The range of $f(x) = [0, \infty)$

E) If $f(x) = \frac{4x}{|x|}$

then sketch the graph of the function and find the domain and range

$$f(x) = \frac{4x}{|x|} = \begin{cases} \frac{4x}{x} & \text{if } x > 0 \\ \frac{4x}{-x} & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} 4 & \text{if } x > 0 \\ -4 & \text{if } x < 0 \end{cases}$$



The domain of $f(x) = (-\infty, 0) \cup (0, \infty) = \mathbb{R} - \{0\}$

The range of $f(x) = \{-4, 4\}$

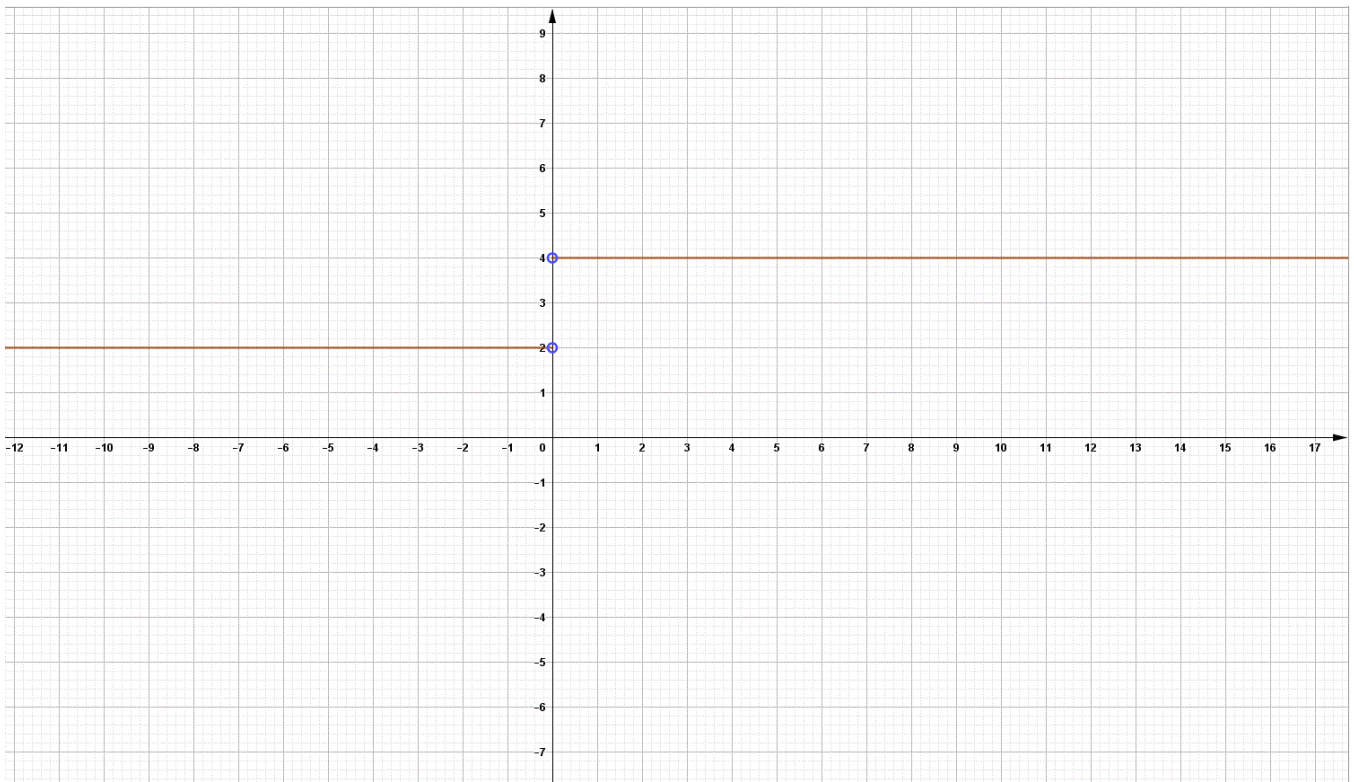
F) If $f(x) = \frac{3x+|x|}{x}$

then sketch the graph of the function and find the domain and range

$$f(x) = \frac{3x + |x|}{x} = \begin{cases} \frac{3x + x}{x} & \text{if } x > 0 \\ \frac{3x - x}{x} & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} \frac{4x}{x} & \text{if } x > 0 \\ \frac{2x}{x} & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} 4 & \text{if } x > 0 \\ 2 & \text{if } x < 0 \end{cases}$$



The domain of $f(x) = (-\infty, 0) \cup (0, \infty) = \mathbb{R} - \{0\}$

The range of $f(x) = \{2, 4\}$

G) If $f(x) = \begin{cases} |x| & \text{if } |x| \leq 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

then sketch the graph of the function and find the domain and range

$$f(x) = \begin{cases} |x| & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \text{ or } x < -1 \end{cases}$$

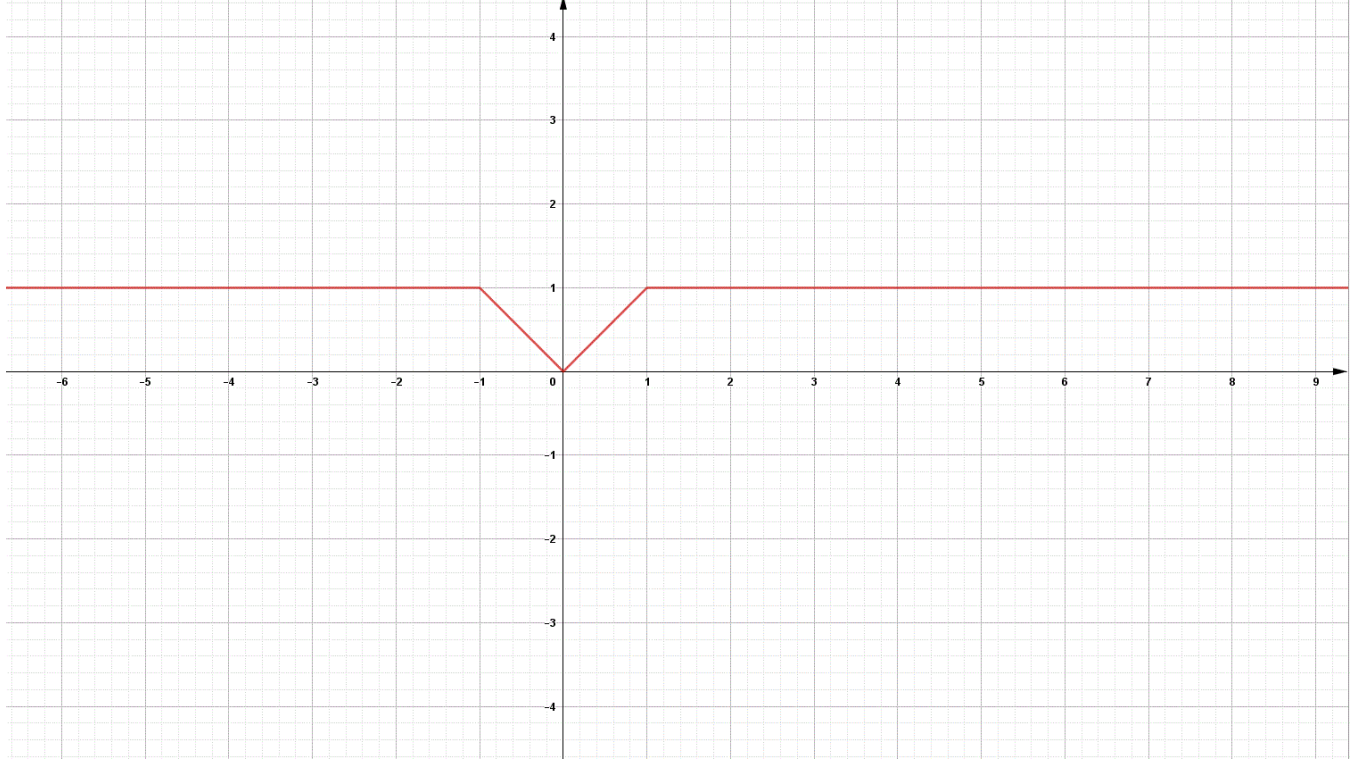
$$\begin{aligned} D_{f(x)} &= \{x \mid -1 \leq x \leq 1\} \cup \{x \mid x > 1 \text{ or } x < -1\} \\ &= [-1, 1] \cup \{(1, \infty) \cup (-\infty, -1)\} \\ &= (-\infty, \infty) = \mathbb{R} \end{aligned}$$

let $f_1(x) = |x|$ for all $x \in [-1, 1]$

x	-1	0	1
$y = f_1(x)$	1	0	1
(x, y)	(-1,1)	(0,0)	(1,1)

let $f_2(x) = 1$ for all $x \in \mathbb{U} (1, \infty) \cup (-\infty, -1)$

x	1	2	-1	-2
$y = f_2(x)$	1	1	1	1
(x, y)	(1,1)	(2,1)	(-1,1)	(-2,1)



The range of $f(x) = [0, 1]$

ملاحظة

$$e \cong 2.7 ; \quad \pi \cong 3.14 \quad (1)$$

(2) لإيجاد قيمة $\sqrt{2}$

نحصر العدد الذي بداخل الجذر بين أقرب عددين مربعين

$$1 < 2 < 4$$

ثم نأخذ الجذر التربيعي للمترابحة السابقة

$$\sqrt{1} < \sqrt{2} < \sqrt{4}$$

$$1 < \sqrt{2} < 2$$

إذن قيمة جذر 2 عدد عشري أكبر من العدد 1 وأصغر من العدد 2

إيجاد قيمة $\sqrt{10}$

نحصر العدد الذي بداخل الجذر بين أقرب عددين مربعين

$$9 < 10 < 16$$

ثم نأخذ الجذر التربيعي للمترابحة السابقة

$$\sqrt{9} < \sqrt{10} < \sqrt{16}$$

$$3 < \sqrt{10} < 4$$

إذن قيمة جذر ١٠ عدد عشري أكبر من العدد ٣ وأصغر من العدد ٤

Exercise (2)

$$|2 - \pi| = \left| \begin{array}{l} 2 - \pi \\ \text{نتاج العملية عدد سالب} \end{array} \right| = -(2 - \pi) = -2 + \pi = \pi - 2$$

$$|3 - \pi| = \left| \begin{array}{l} 3 - \pi \\ \text{نتاج العملية عدد سالب} \end{array} \right| = -(3 - \pi) = -3 + \pi = \pi - 3$$

$$|4 - \pi| = \left| \begin{array}{l} 4 - \pi \\ \text{نتاج العملية عدد موجب} \end{array} \right| = 4 - \pi$$

$$|2 - e| = \left| \begin{array}{l} 2 - e \\ \text{نتاج العملية عدد سالب} \end{array} \right| = -(2 - e) = -2 + e = e - 2$$

$$|-3 - e| = \left| \begin{array}{l} -3 - e \\ \text{نتاج العملية عدد سالب} \end{array} \right| = -(-3 - e) = 3 + e$$

$$|4 - e| = \left| \begin{array}{l} 4 - e \\ \text{نتاج العملية عدد موجب} \end{array} \right| = 4 - e$$

$$|1 - \sqrt{2}| = \left| \begin{array}{c} 1 - \sqrt{2} \\ \text{نتاج العملية عدد سالب} \end{array} \right| = -(1 - \sqrt{2}) = -1 + \sqrt{2} \\ = \sqrt{2} - 1$$

$$|-4 - \sqrt{2}| = \left| \begin{array}{c} -4 - \sqrt{2} \\ \text{نتاج العملية عدد سالب} \end{array} \right| = -(-4 - \sqrt{2}) = 4 + \sqrt{2}$$

$$|3 - \sqrt{2}| = \left| \begin{array}{c} 3 - \sqrt{2} \\ \text{نتاج العملية عدد موجب} \end{array} \right| = 3 - \sqrt{2}$$

المحاضرة الثانية في مساند

(1.1) Four Ways to Represent a Function

الهدف الخامس من الفصل الأول

كيفية ايجاد مدى بعض الدوال الكسرية ورسمها (Rational function)

Exercise (1)

find the domain , range and sketch the graph of the function

1) $f(x) = \frac{x^2-1}{x+1}$

The domain of $f(x)$:

- نوجد اصفار المقام
 $x + 1 = 0$
 $x = -1$
- $D_{f(x)} = R - \{-1\}$
Or $= \{ x \mid x \neq -1 \}$
Or $= (-\infty, -1) \cup (-1, \infty)$

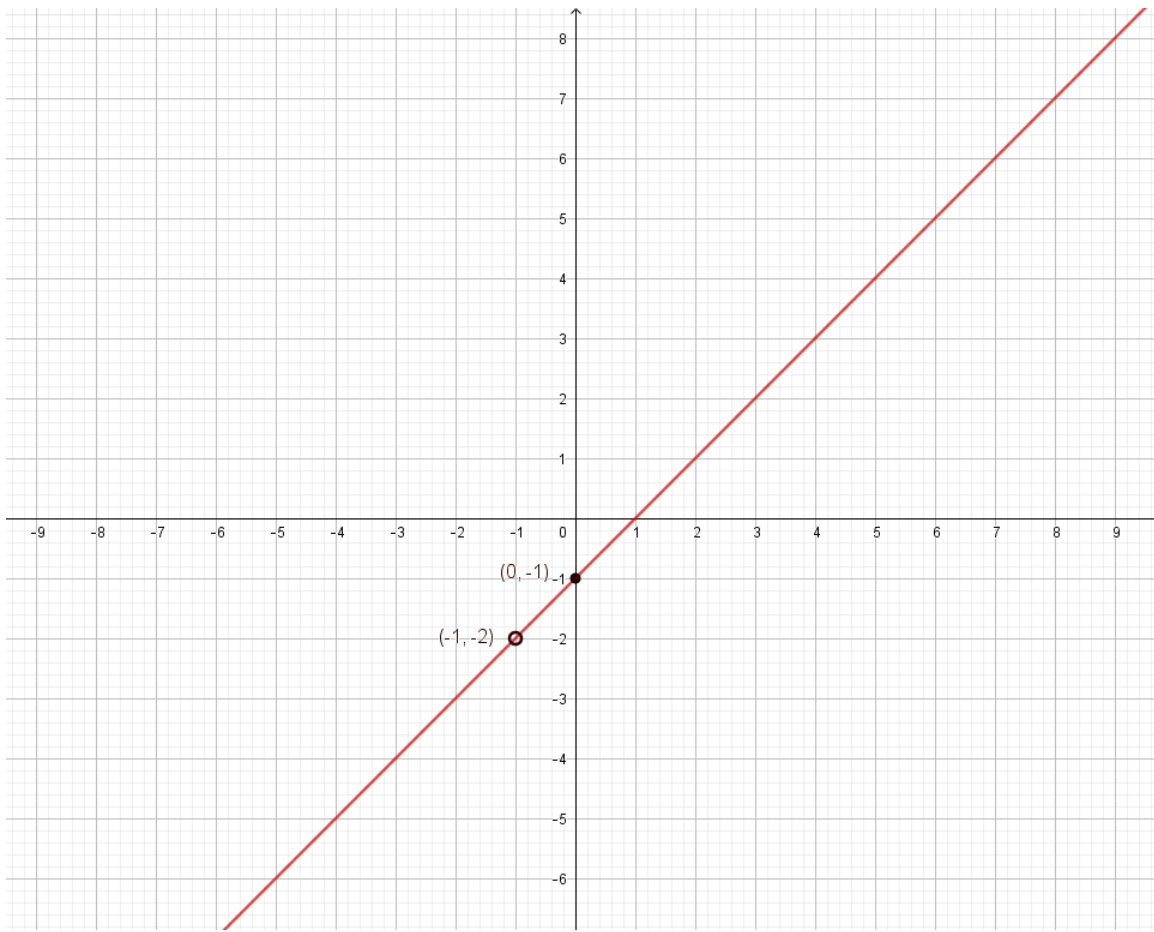
The graph of $f(x)$:

$$f(x) = \frac{x^2-1}{x+1} \text{ for all } x \neq -1$$

$$f(x) = \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}} \text{ for all } x \neq -1$$

$$f(x) = x - 1 \text{ for all } x \neq -1$$

x	0	-1
$y = x - 1$	$y(0) = 0 - 1$ $= -1$	$y(1) = -1 - 1$ $= -2$
(x, y)	$(0, -1)$	$(-1, -2)$



The range of $f(x) = R - \{-2\}$

الهدف السادس من الفصل الأول

كيفية إيجاد مجال الدالة الجذرية ($f(x) = \sqrt[n]{g(x)}$)

(١) لو كان n عدد فردي فإن مجال الدالة الجذرية R

(٢) لو n عدد زوجي فإنه يجب ما بداخل الجذر أكبر من أو يساوي الصفر

Exercise (2)

Find the domain of the function.

$$1) \left. \begin{aligned} f(x) &= \sqrt[3]{2x - 1} \\ f(x) &= \sqrt[5]{x^2 - 4} \\ f(x) &= \sqrt[7]{3 - 7x - x^2} \\ f(x) &= \sqrt[9]{4x^5 + 3x} \end{aligned} \right\} \text{The domain of } f(x) : R$$

$$2) f(x) = \sqrt[4]{15 - 3x}$$

$$15 - 3x \geq 0$$

$$-3x \geq -15$$

$$\frac{-3x}{-3} \leq \frac{-15}{-3}$$

$$x \leq 5$$

$$\text{The domain of } f(x) = \{x \mid x \leq 5\} = (-\infty, 5]$$

$$3) f(x) = \sqrt[6]{2x + 3}$$

$$2x + 3 \geq 0$$

$$2x \geq -3$$

$$\frac{2x}{2} \geq \frac{-3}{2}$$

$$x \geq -\frac{3}{2}$$

$$\text{The domain of } f(x) = \left\{x \mid x \geq -\frac{3}{2}\right\} = \left[-\frac{3}{2}, \infty\right)$$

$$4) f(x) = \sqrt{4 - x^2}$$

$$4 - x^2 \geq 0$$

$$-x^2 \geq -4$$

$$\frac{-1x^2}{-1} \leq \frac{-4}{-1}$$

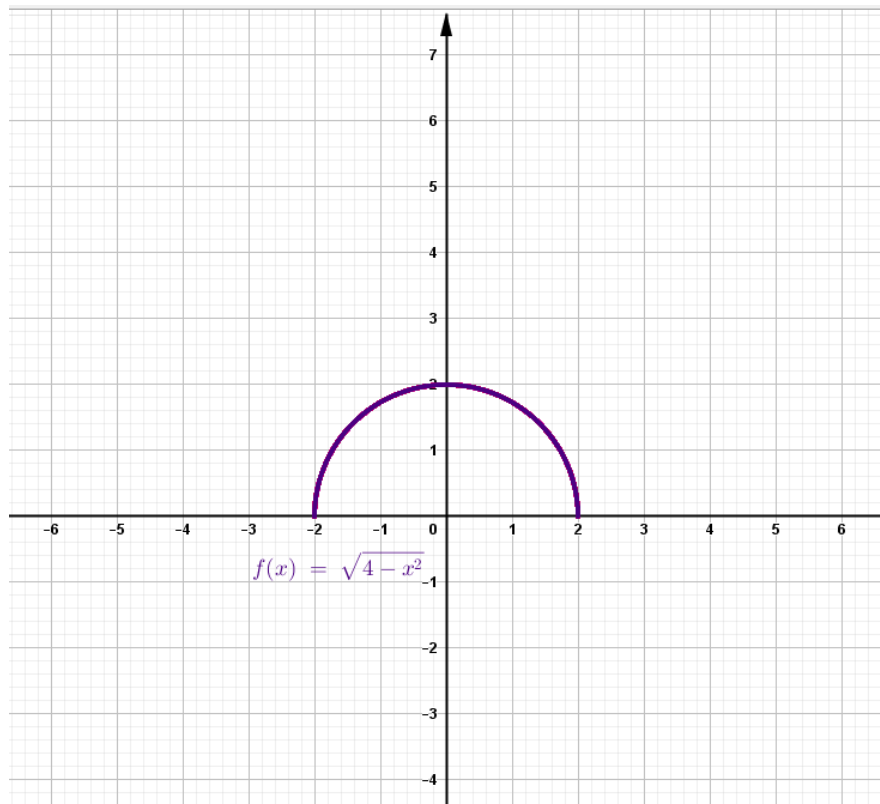
$$x^2 \leq 4$$

$$\sqrt{x^2} \leq \sqrt{4}$$

$$|x| \leq 2 \Leftrightarrow -2 \leq x \leq 2$$

$$\text{The domain of } f(x) = \{x \mid -2 \leq x \leq 2\} = [-2, 2]$$

الطريقة الجبرية



The domain of $f(x) = [-2, 2]$

The range of $f(x) = [0, 2]$

5) $f(x) = \sqrt{x^2 - 4}$

$$x^2 - 4 \geq 0$$

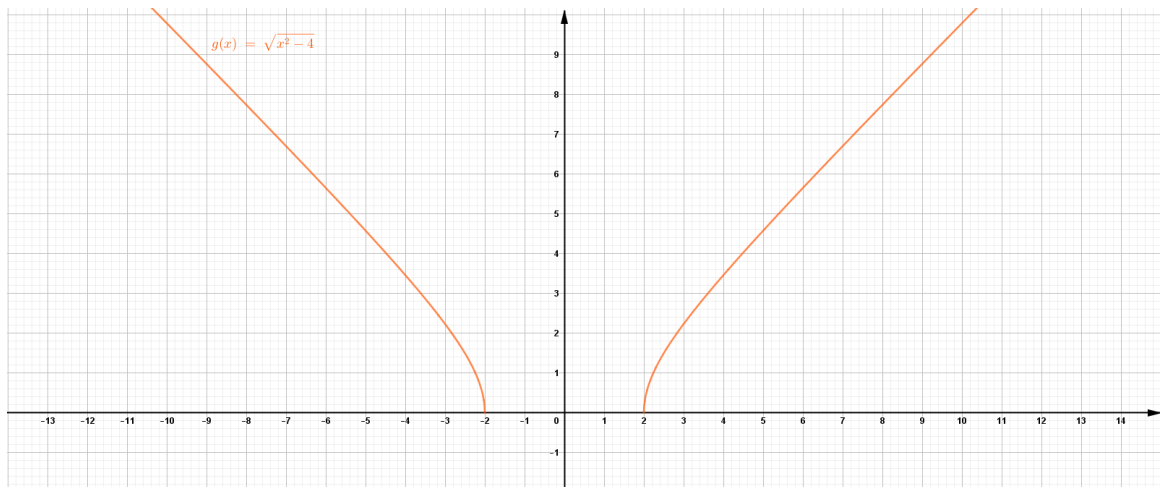
$$x^2 \geq 4$$

$$\sqrt{x^2} \geq \sqrt{4}$$

$$|x| \geq 2 \Leftrightarrow x \leq -2 \text{ or } x \geq 2$$

$$D_{f(x)} = \{x \mid x \leq -2 \text{ or } x \geq 2\} = (-\infty, -2] \cup [2, \infty) = \mathbf{R} - (-2, 2)$$

الطريقة البيانية

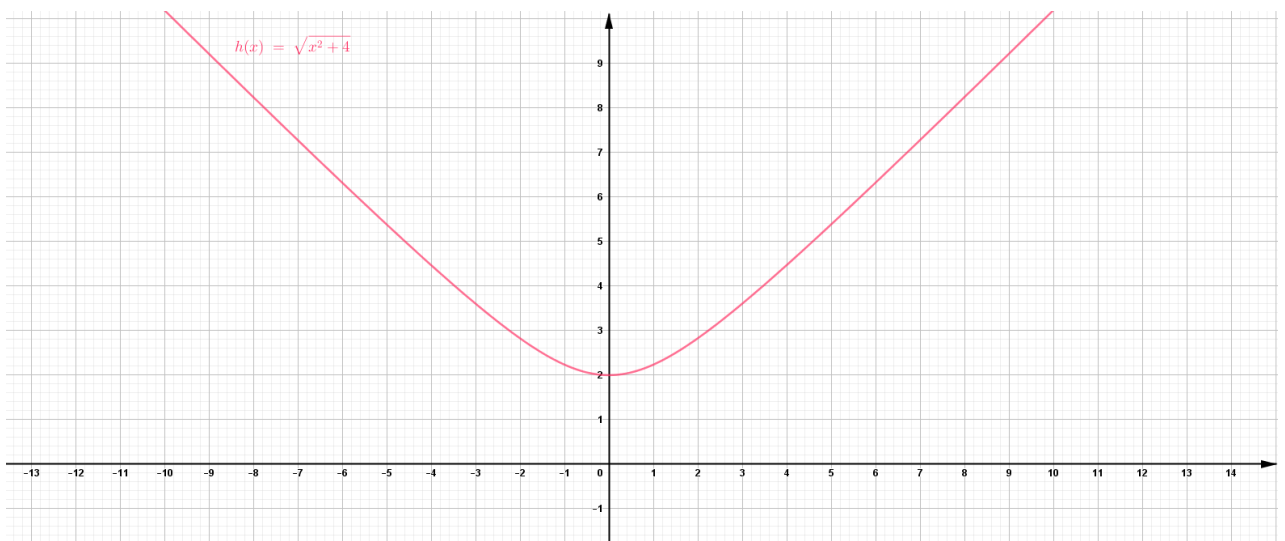


$$D_{f(x)} = (-\infty, -2] \cup [2, \infty) = \mathbb{R} - (-2, 2)$$

$$R_{f(x)} = [0, \infty)$$

6) $f(x) = \sqrt{x^2 + 4}$ or $\sqrt{4 + x^2}$

الطريقة البيانية



$$D_{f(x)} = (-\infty, \infty)$$

$$R_{f(x)} = [2, \infty)$$

$$x^2 + 4 \geq 0$$

$$x^2 \geq -4$$

$$\sqrt{x^2} \geq \sqrt{-4}$$

$$|x| \geq \sqrt{-4}$$

$$x \geq \pm\sqrt{-4} \notin R$$

$$D_{f(x)} = (-\infty, \infty)$$

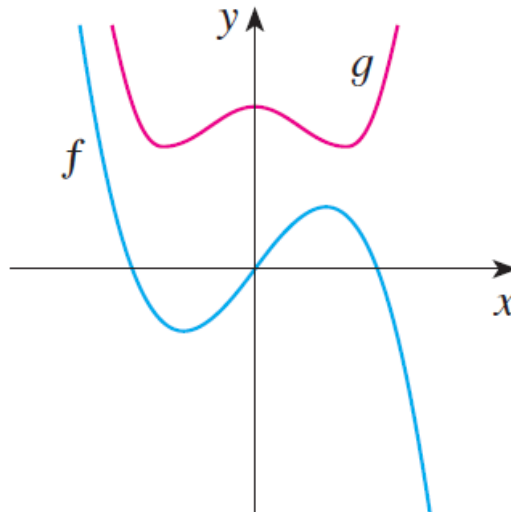
الهدف السابع من الفصل الأول

كيفية تحديد انه الدالة زوجية ام فردية ام ليست فردية او زوجية بيانيا او جبريا

Exercise (3)

Graphs of f and g are shown. Decide whether each function is even , odd , or neither.

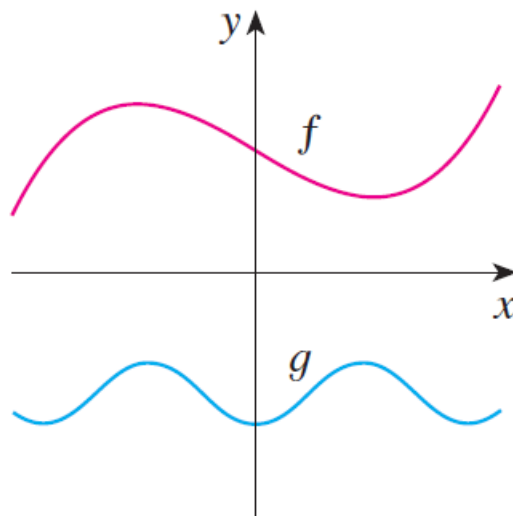
1)



$f(x)$ is an odd function and symmetric about $(0, 0)$ or the origin

$g(x)$ is an even function and symmetric about y -axis

2)

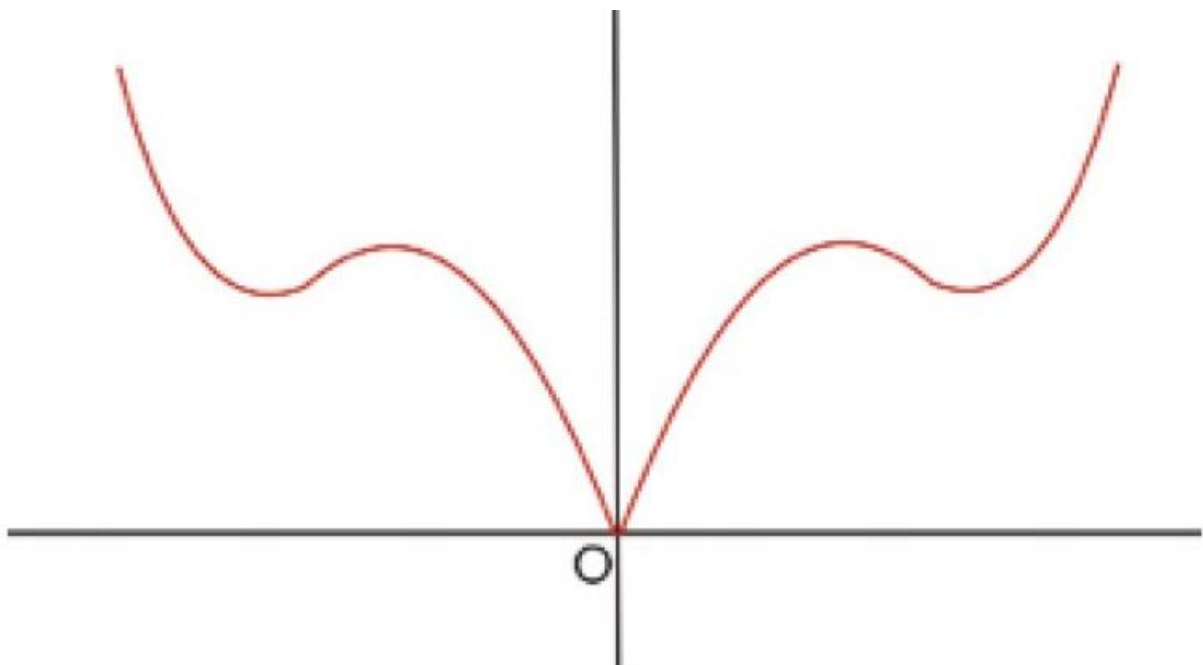


$f(x)$ is a neither

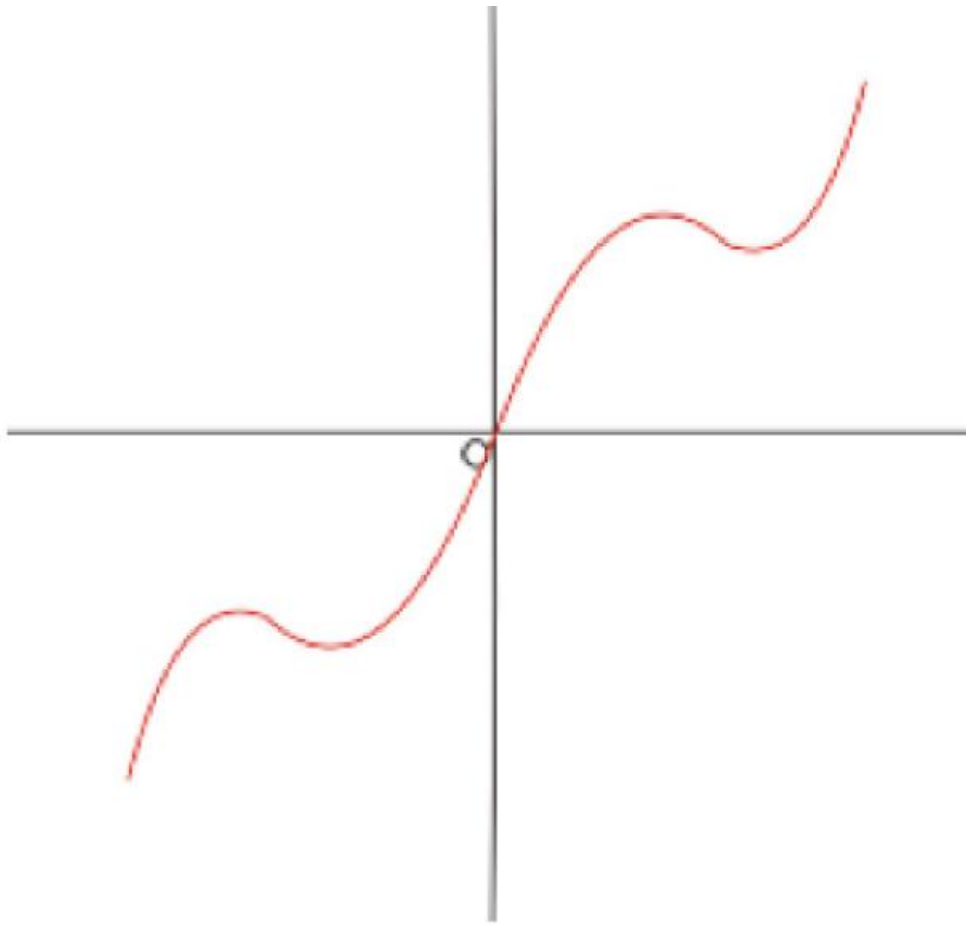
$g(x)$ is an even function and symmetric about y –axis

Exercise (4)

Graphs are shown. Decide whether each function is even , odd , or neither.



The graph is an even function and symmetric about y –axis



The graph is an odd function and symmetric about $(0, 0)$ or the origin

Exercise (5)

Determine whether f is even , odd , or neither.

1) $f(x) = 3$

$$f(x) = 3(1)$$

$$f(x) = 3x^0 \rightarrow \text{even number}$$

$\therefore f(x)$ is an even function and symmetric about y-axis

2) $f(x) = 2x$

$$f(x) = 2x^1 \rightarrow \text{odd number}$$

$\therefore f(x)$ is an odd function and symmetric about the origin

3) $f(x) = 5x^{100}$

$$f(x) = 5x^{100} \rightarrow \text{even number}$$

$\therefore f(x)$ is an even function and symmetric about y-axis

$$4) f(x) = 4x^{21}$$

$$f(x) = 2x^{21 \rightarrow \text{odd number}}$$

$\therefore f(x)$ is an odd function and symmetric about the origin

$$5) f(x) = \frac{7}{x^{10}}$$

$$f(x) = \frac{7}{x^{10 \rightarrow \text{even number}}}$$

$\therefore f(x)$ is an even function and symmetric about y-axis

$$6) f(x) = \frac{6}{x^5}$$

$$f(x) = \frac{6}{x^{5 \rightarrow \text{odd number}}}$$

$\therefore f(x)$ is an odd function and symmetric about the origin

$$\left. \begin{aligned} 7) f(x) &= \sin(\theta) \\ f(x) &= \csc(\theta) \\ f(x) &= \tan(\theta) \\ f(x) &= \cot(\theta) \end{aligned} \right\}$$

Are an odd functions
Symmetric about (0,0)

$$\left. \begin{aligned} 8) f(x) &= \cos(\theta) \\ f(x) &= \sec(\theta) \end{aligned} \right\}$$

Are an even functions
Symmetric about y-axis

$$9) f(x) = |x|$$

$f(x)$ is an even function and symmetric about y-axis

Note :

odd \pm odd = odd , even \pm even = even , odd \pm even = neither

$$10) f(x) = 1 + 3x^2 - x^4$$

$$f(x) = 1x^0 + \underbrace{3x^2 - x^4}$$

Are an even functions

$\therefore f(x)$ is an even function and symmetric about y-axis

$$11) \quad f(x) = |x| + \cos(x) - 9x^4$$

$$f(x) = |x| + \cos(x) - 9x^4$$

Are an even functions

$\therefore f(x)$ is an even function and symmetric about y-axis

$$12) \quad f(x) = 7 - 3x^2$$

$$f(x) = 7x^0 - 3x^2$$

Are an even functions

$\therefore f(x)$ is an even function and symmetric about y-axis

$$13) \quad f(x) = x + \csc(x) - \frac{2}{x^3}$$

$$f(x) = x + \csc(x) - \frac{2}{x^3}$$

Are an odd functions

$\therefore f(x)$ is an odd function and symmetric about (0,0)

$$14) \quad f(x) = 1 + 3x^2 - x^4$$

$$f(x) = 1x^0 + 3x^2 - x^5$$

Are an even functions is an odd function

$\therefore f(x)$ is a neither even nor odd

$$15) \quad f(x) = x \pm 3$$

$$f(x) = x^1 \pm 3x^0$$

odd \pm even

$\therefore f(x)$ is a neither even nor odd

Note :

$$\text{odd} \times \text{odd} = \text{even}$$

$$\text{odd} \times \text{even} = \text{odd}$$

$$\text{even} \times \text{neither} = \text{neither}$$

$$\text{even} \times \text{even} = \text{even}$$

$$\text{odd} \times \text{neither} = \text{neither}$$

$$\text{neither} \times \text{neither} = \text{neither}$$

$$\text{odd} \div \text{odd} = \text{even}$$

$$\text{odd} \div \text{even} = \text{odd}$$

$$\text{even} \div \text{neither} = \text{neither}$$

$$\text{even} \div \text{even} = \text{even}$$

$$\text{odd} \div \text{neither} = \text{neither}$$

$$\text{neither} \div \text{neither} = \text{neither}$$

16) $f(x) = x|x|$

$$f(x) = x \times |x|$$

$$\text{odd} \times \text{even} = \text{odd}$$

$\therefore f(x)$ is an odd function and symmetric about (0,0)

17) $f(x) = x \sin(x)$

$$f(x) = x \times \sin(x)$$

$$\text{odd} \times \text{odd} = \text{even}$$

$\therefore f(x)$ is an even function and symmetric about y-axis

18) $f(x) = |x| \sec(x)$

$$f(x) = |x| \times \sec(x)$$

$$\text{even} \times \text{even} = \text{even}$$

$\therefore f(x)$ is an even function and symmetric about y-axis

19) $f(x) = (x + 5) \cos(x)$

$$f(x) = (x^1 + 5x^0) \times \cos(x)$$

$$\text{neither} \times \text{even} = \text{neither}$$

$\therefore f(x)$ is a neither even nor odd

20) $f(x) = \frac{x}{\cot(x)}$

$$f(x) = \frac{x}{\cot(x)} = \frac{\text{odd}}{\text{odd}} = \text{even}$$

$\therefore f(x)$ is an even function and symmetric about y-axis

$$21) f(x) = \frac{|x|}{\sin(x)}$$

$$f(x) = \frac{|x|}{\sin(x)} = \frac{\text{even}}{\text{odd}} = \text{odd}$$

∴ $f(x)$ is an odd function and symmetric about (0,0)

$$22) f(x) = \frac{x^4 + \cos(x)}{\sec(x)}$$

$$f(x) = \frac{x^4 + \cos(x)}{\sec(x)} = \frac{\text{even} + \text{even}}{\text{even}} = \frac{\text{even}}{\text{even}} = \text{even}$$

∴ $f(x)$ is an even function and symmetric about y-axis

$$23) f(x) = \frac{x^2}{x^4 + 1}$$

$$f(x) = \frac{x^2}{x^4 + 1x^0} = \frac{\text{even}}{\text{even} + \text{even}} = \frac{\text{even}}{\text{even}} = \text{even}$$

∴ $f(x)$ is an even function and symmetric about y-axis

$$24) f(x) = \frac{x}{x^2 + 1}$$

$$f(x) = \frac{x^1}{x^2 + 1x^0} = \frac{\text{odd}}{\text{even} + \text{even}} = \frac{\text{odd}}{\text{even}} = \text{odd}$$

∴ $f(x)$ is an odd function and symmetric about (0,0)

$$25) f(x) = \frac{x}{x+1}$$

$$f(x) = \frac{x^1}{x^1 + 1x^0} = \frac{\text{odd}}{\text{odd} + \text{even}} = \frac{\text{odd}}{\text{neither}} = \text{nither}$$

∴ $f(x)$ is a neither even nor odd

Note :

$$|\text{odd}| = \text{even} , \quad |\text{even}| = \text{even} , \quad |\text{neither}| = \text{neither}$$

$${}^{\text{even}}\sqrt{\text{even}} = \text{even} , \quad {}^{\text{even}}\sqrt{\text{odd}} = \text{neither} , \quad {}^{\text{even}}\sqrt{\text{neither}} = \text{neither}$$

$${}^{\text{odd}}\sqrt{\text{odd}} = \text{odd} , \quad {}^{\text{odd}}\sqrt{\text{even}} = \text{even} , \quad {}^{\text{odd}}\sqrt{\text{neither}} = \text{neither}$$

- 26) $f(x) = |x^3 - \sin(x)|$
 $|\text{odd} - \text{odd}| = |\text{odd}| = \text{even}$
 $\therefore f(x)$ is an even function and symmetric about y-axis
- 27) $f(x) = |x^4 + |x||$
 $|\text{even} + \text{even}| = |\text{even}| = \text{even}$
 $\therefore f(x)$ is an even function and symmetric about y-axis
- 28) $f(x) = |(x - 1) \tan(x)|$
 $|\text{neither} \times \text{odd}| = |\text{neither}| = \text{neither}$
 $\therefore f(x)$ is a neither even nor odd
- 29) $f(x) = \sqrt[3]{x^2 - 4}$
 $f(x) = \sqrt[3]{\text{even} - \text{even}} = \sqrt[3]{\text{even}} = \text{even}$
 $\therefore f(x)$ is an even function and symmetric about y-axis
- 30) $f(x) = \sqrt[3]{x^7 + \csc(x)}$
 $f(x) = \sqrt[3]{\text{odd} + \text{odd}} = \sqrt[3]{\text{odd}} = \text{odd}$
 $\therefore f(x)$ is an odd function and symmetric about (0,0)
- 31) $f(x) = \sqrt[3]{x^3 + |x|}$
 $f(x) = \sqrt[3]{\text{odd} + \text{even}} = \sqrt[3]{\text{neither}} = \text{neither}$
 $\therefore f(x)$ is a neither even nor odd
- 32) $f(x) = \sqrt[4]{x^2 - 4}$
 $f(x) = \sqrt[4]{\text{even} - \text{even}} = \sqrt[4]{\text{even}} = \text{even}$
 $\therefore f(x)$ is an even function and symmetric about y-axis
- 33) $f(x) = \sqrt[4]{x^7 + \csc(x)}$
 $f(x) = \sqrt[4]{\text{odd} + \text{odd}} = \sqrt[4]{\text{odd}} = \text{neither}$
 $\therefore f(x)$ is a neither even nor odd

$$34) f(x) = \sqrt[4]{x^3 + |x|}$$

$$f(x) = \sqrt[4]{\text{odd} + \text{even}} = \sqrt[4]{\text{neither}} = \text{neither}$$

$\therefore f(x)$ is a neither even nor odd

Note :

$(\text{odd})^{\text{even}} = \text{even}$, $(\text{even})^{\text{even}} = \text{even}$, $(\text{neither})^{\text{even}} = \text{neither}$

$(\text{odd})^{\text{odd}} = \text{odd}$, $(\text{even})^{\text{odd}} = \text{even}$, $(\text{neither})^{\text{odd}} = \text{neither}$

$$35) f(x) = \sin^2(x)$$

$$f(x) = (\sin(x))^2 = (\text{odd})^2 = \text{even}$$

$\therefore f(x)$ is an even function and symmetric about y-axis

$$36) f(x) = \tan^3(x)$$

$$f(x) = (\tan(x))^3 = (\text{odd})^3 = \text{odd}$$

$\therefore f(x)$ is an odd function and symmetric about (0,0)

$$37) f(x) = \cos^4(x)$$

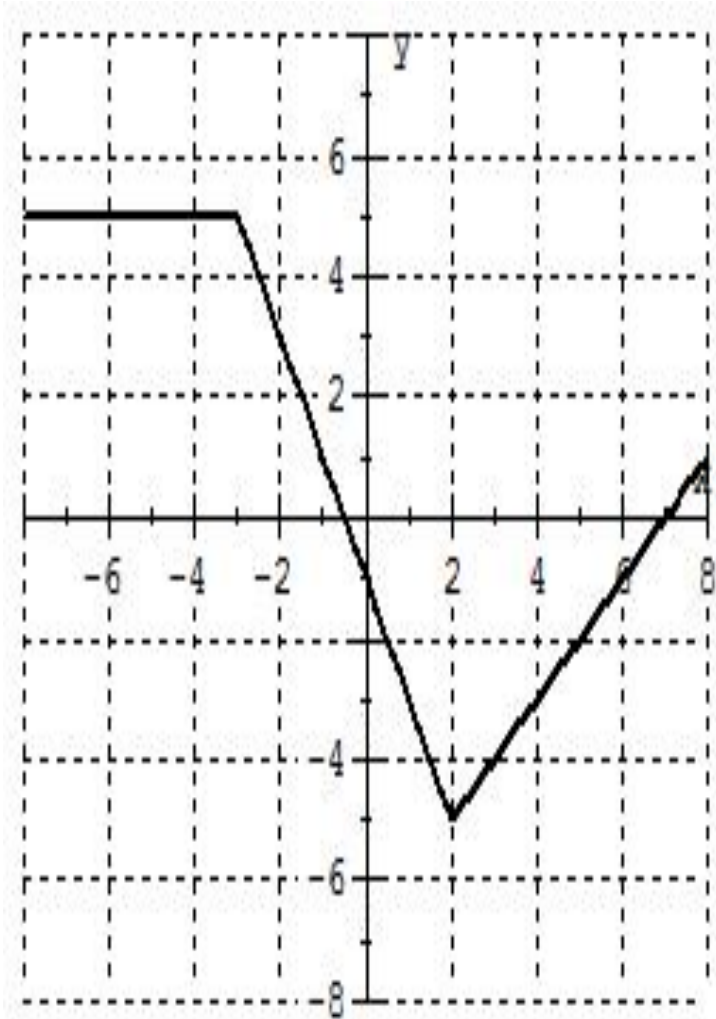
$$f(x) = (\cos(x))^4 = (\text{even})^2 = \text{even}$$

$\therefore f(x)$ is an even function and symmetric about y-axis

$$38) f(x) = \sec^5(x)$$

$$f(x) = (\sec(x))^5 = (\text{even})^5 = \text{even}$$

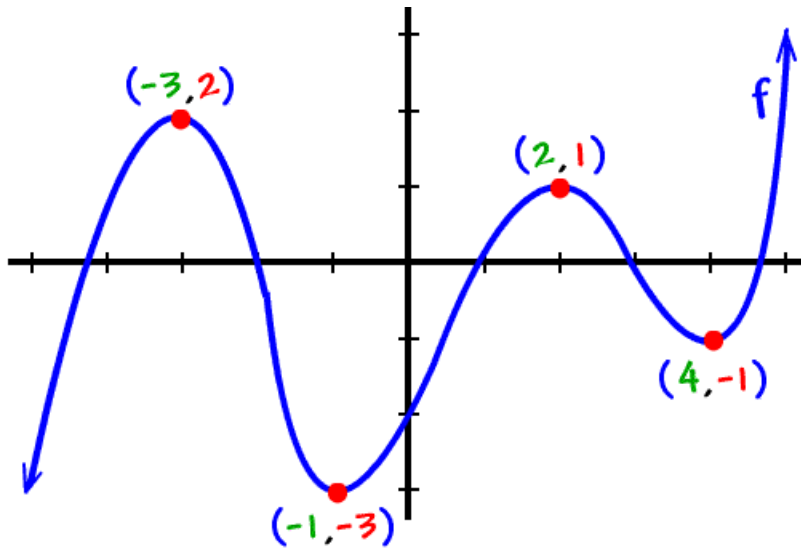
$\therefore f(x)$ is an even function and symmetric about y-axis

Exercise (6)**The graph of a function f is given**

- a) $f(x)$ is decreasing on $[-3, 2]$ or $(-3, 2)$
- b) $f(x)$ is increasing on $[2, 8]$ or $(2, 8)$
- c) $f(x)$ is constant on $[-7, -3]$ or $(-7, -3)$

Exercise (7)

The graph of a function f is given



a) $f(x)$ is increasing on $(-\infty, -3] \cup [-1, 2] \cup [4, \infty)$
or $f(x)$ is increasing on $(-\infty, -3) \cup (-1, 2) \cup (4, \infty)$

b) $f(x)$ is decreasing on $[-3, -1] \cup [2, 4]$
Or $f(x)$ is decreasing on $(-3, -1) \cup (2, 4)$

1.2 - Types of The Functions .

Type 1: Polynomials .

Example: If $P(x) = 2x^6 - x^4 + \frac{2}{5}x^3 + \sqrt{2}$ then

a) Name: Polynomial

b) The Degree: Polynomial of degree 6

c) Leading Coefficient: 2
or coefficient of x^6
or a_6

d) Coefficient of x^3 : $a_3 = \frac{2}{5}$

e) Constant term: $a_0 = \sqrt{2}$
or coefficient of x^0

f) $a_4 = -1$

g) $a_2 = 0$

Example: $P(x) = 5x^\pi - \frac{1}{3}x^e + 2x^{\sqrt{3}} - 2 \cdot 12^{-5} - \sqrt{2}x^{\frac{3}{4}} + 3x^{3.5} \sin(x) - \frac{1}{x} + \ln(4) - x^{\log}$ is not Polynomial

a) T

b) F

Example: $P(x) = \frac{1}{3}x - \sqrt{2}x^2 + \pi x^3 - x^7 + \frac{3}{e}$ is Polynomial
of degree

- a) 1 b) 2 c) 3 **d) 7**

H.W: $P(x) = -\frac{2}{3}x^3 + 5x - 8$

a) Name:

b) degree:

c) Leading coefficient:

d) constant term:

e) coefficient of x :

f) $a_2 = \dots$

Note: The Domain of any Polynomial is
 \mathbb{R} or $(-\infty, \infty)$

Example: The Domain of $f(x) = x^2 - 16$ is....

- a) $\{4, -4\}$ b) $\mathbb{R} - \{\pm 4\}$ c) $\mathbb{R} - [4, 4]$
d) $[-4, 4]$ e) $(-4, 4)$ **f) \mathbb{R}**

Type of Polynomials :

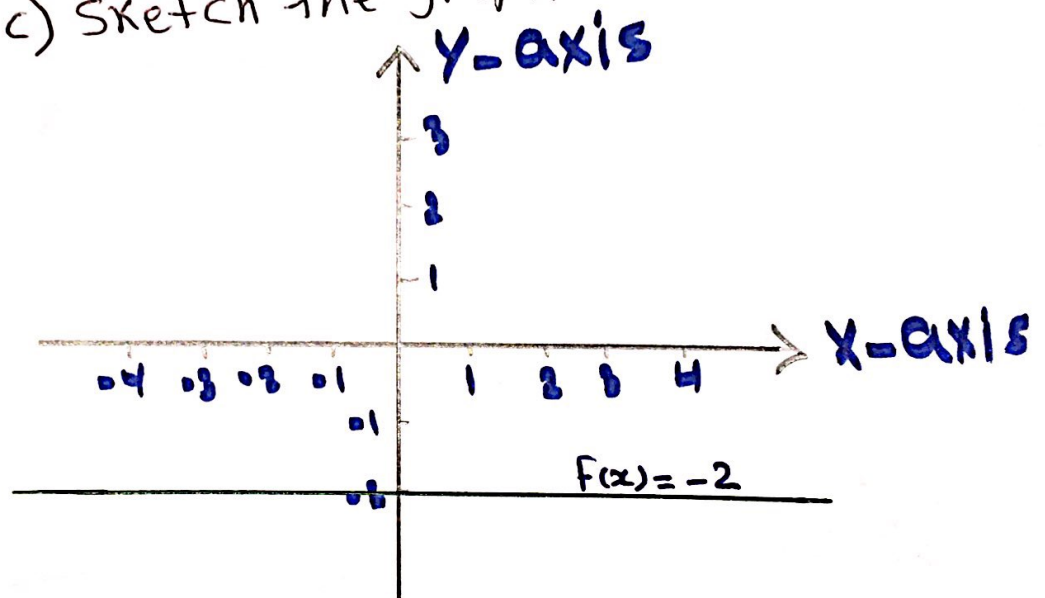
A) Constant function: $P(x) = a$ is polynomial of degree zero for all $a \in \mathbb{R}$

Example: If $f(x) = -2$ then...

a) $f(x)$ is polynomial of degree zero

b) $f(x)$ is called constant function

c) Sketch the graph of $f(x) = -2$



d) The graph of $f(x) = -2$ is Horizontal line
with slope: 0

y-intercept: -2

Parallel to x-axis

e) $D_{f(x)} = \mathbb{R}$

$R_f = \{-2\}$

B) Linear function: $P(x) = ax + b$ is Polynomial of degree 1 with slope: a and y-intercept: b

Example: $f(x) = 2 - 5x$

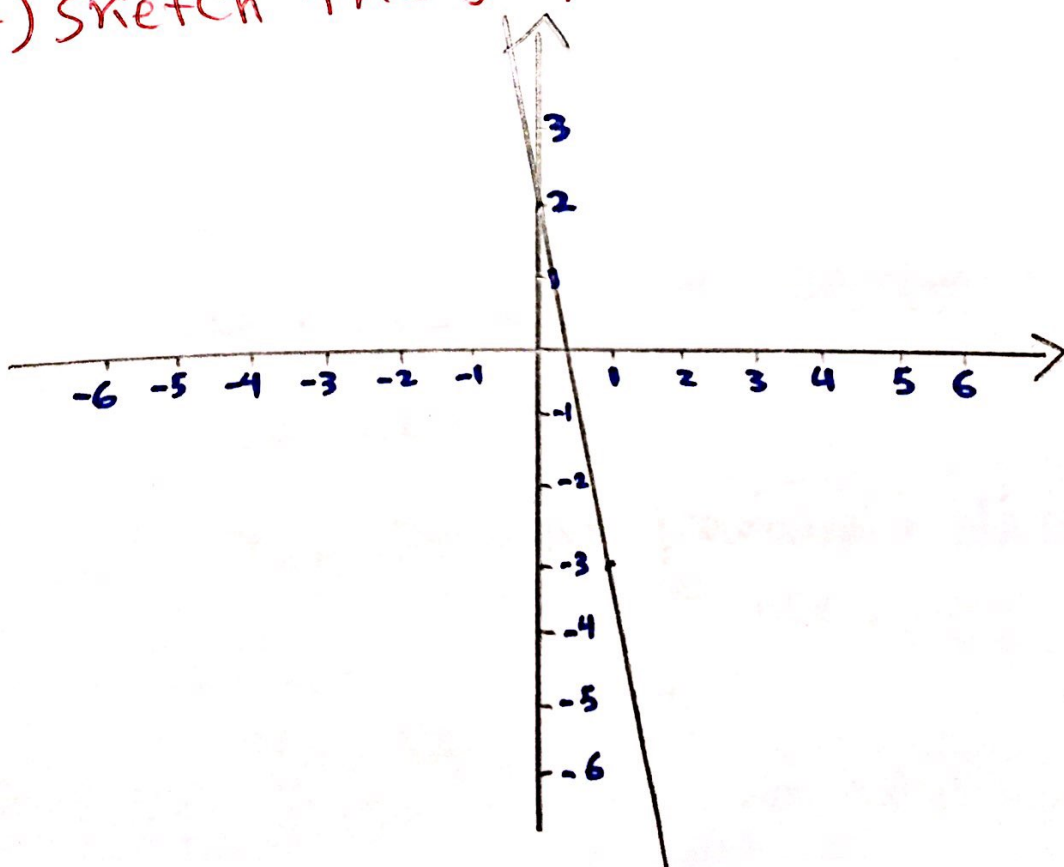
a) $f(x)$ is Polynomial of degree 1

b) $f(x)$ is called a linear function

c) The slope: -5 y-intercept: 2

d) $D_{f(x)} = \mathbb{R}$ $R_{f(x)} = \mathbb{R}$

e) sketch the graph of $f(x) = 2 - 5x$



c) Quadratic function: $P(x) = ax^2 + bx + c$ is
Polynomial of degree 2
It's graph is always
a parabola

Note: if $a > 0$ then the parabola opens
upward
if $a < 0$ then the parabola opens
downward.

Example: $y = x^2 + x + 1$ is polynomial of
degree 2 or Quadratic function

The Graph: Parabola opens up
since: $a = 1 > 0$

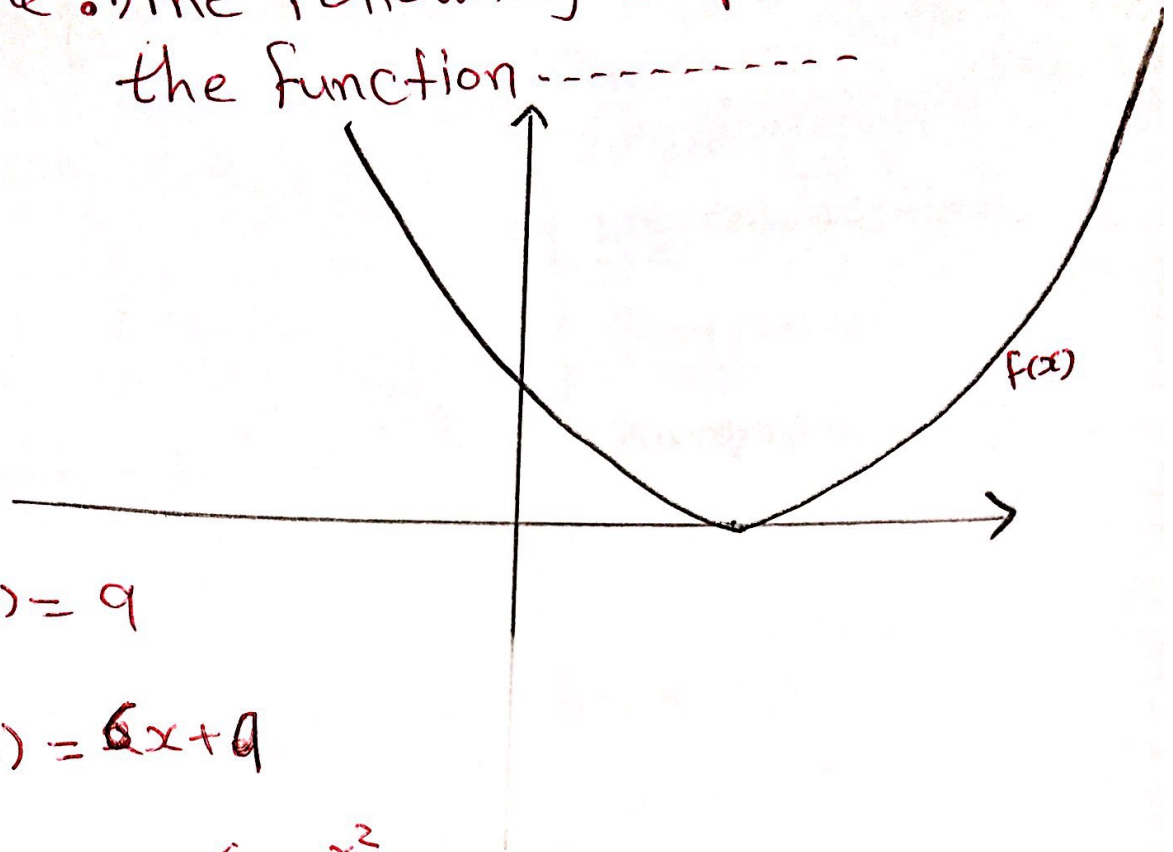
The Domain of $y = x^2 + x + 1$: $(-\infty, \infty)$

Example: $f(x) = 1 - 2x^2$ is polynomial of
degree 2 or Quadratic function

The Graph: Parabola opens down
since: $a = -2 < 0$

* The $D_{f(x)} = \mathbb{R}$

Example: 1) The following graph represents the function -----



a) $f(x) = 9$

b) $f(x) = 6x + 9$

c) $f(x) = 9 + 6x - x^2$

d) $f(x) = x^2 - 6x + 9$

2) $D_{f(x)} = \mathbb{R}$ $R_{f(x)} = [0, \infty)$

D) The Cubic function: $P(x) = ax^3 + bx^2 + cx + d$
is Polynomial of degree 3
Domain: \mathbb{R} Rang: \mathbb{R}

Example:

$$f(x) = 3x^3 + 2x + 1$$

$$y = x^3 + 1$$

$$H(x) = \frac{1}{2}x^3 - \sqrt{2}x^2$$

$$G(x) = 4x^3$$

Polynomial of degree 3 or Cubic Function

Domain: $\mathbb{R} = (-\infty, \infty)$

Range: $\mathbb{R} = (-\infty, \infty)$

Type 2: The Power Function

$$f(x) = x^a \text{ for all } a \in \mathbb{R}$$

Example:

$$f(x) = x^7$$

$$f(x) = x^{-8}$$

$$f(x) = x^{3/2}$$

$$f(x) = x^\pi$$

$$f(x) = \sqrt[5]{x^{11}} = x^{11/5}$$

$$f(x) = \frac{1}{x^{-4}} = x^4$$

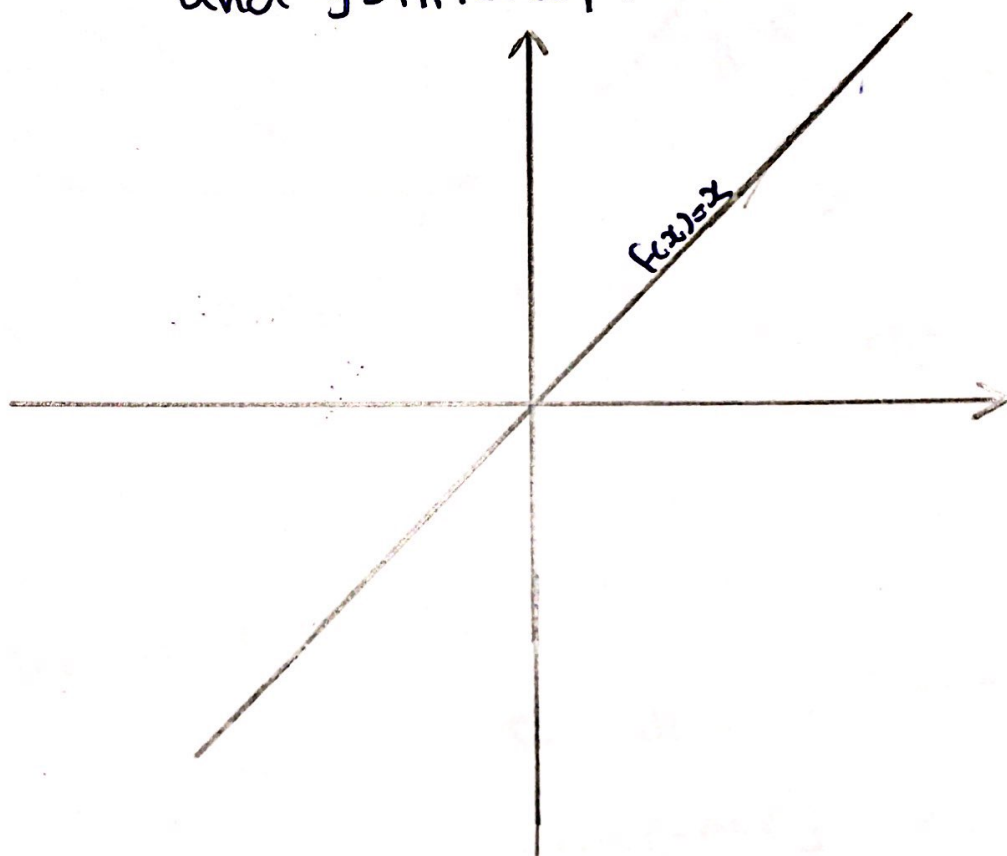
$$f(x) = \frac{1}{x^{10}} = x^{-10}$$

The Power Functions

Example: 1) $f(x) = x$

a) Name: Power function or linear function or Polynomial of degree 1 or Unit function

b) Graph: The Straight line with slope: 1 and y-intercept: 0



c) $f(x) = x$ is odd function

d) $f(x) = x$ is Symmetric about The origin or $(0, 0)$

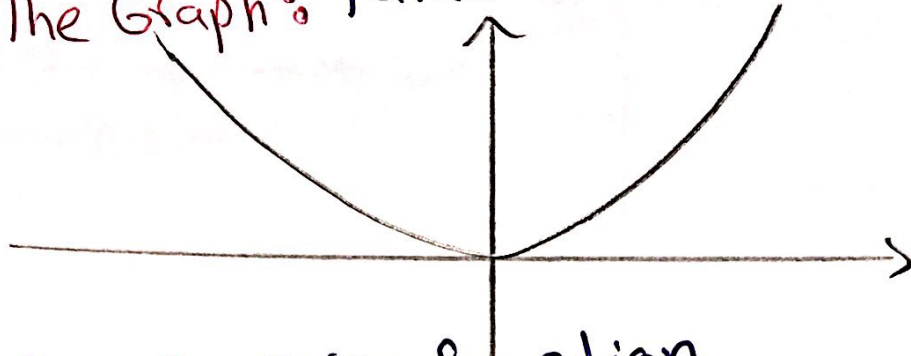
e) $f(x) = x$ is increasing for all $x \in \mathbb{R}$

f) $D_{f(x)} = \mathbb{R}$ $R_{f(x)} = \mathbb{R}$

2) $f(x) = x^2$

a) Name: The power function or Polynomial of Degree 2 or Quadratic function

b) The Graph: Parabola



c) $f(x)$ is even function

d) $f(x)$ is Symmetric about y -axis

e) $f(x)$ is increasing for all $x \geq 0$

or $f(x)$ is increasing on $[0, \infty)$

f) $f(x)$ is decreasing for all $x \leq 0$

or $f(x)$ is decreasing on $(-\infty, 0]$

3) $f(x) = x^3$

a) Name: The power function or Cubic function or Polynomial of degree 3

b) $f(x)$ is odd function

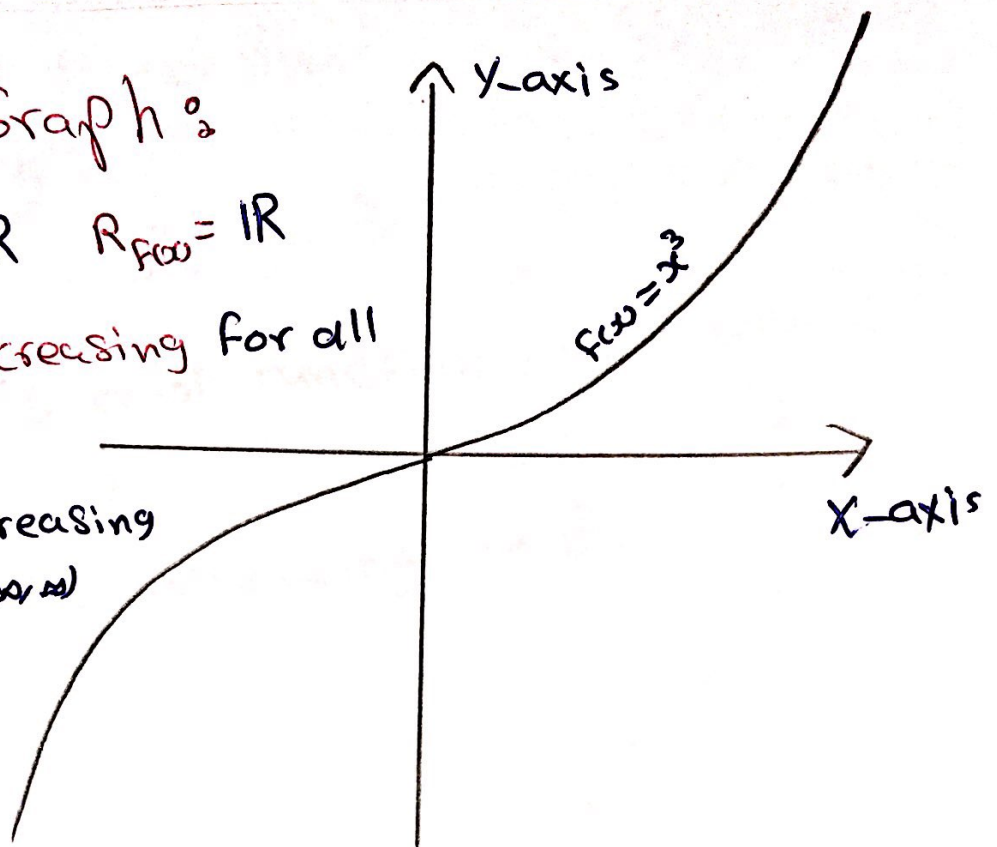
c) $f(x)$ is Symmetric about $(0,0)$ or The origin

d) The Graph:

e) $D_{f(x)} = \mathbb{R}$ $R_{f(x)} = \mathbb{R}$

f) $f(x)$ is increasing for all $x \in \mathbb{R}$

or $f(x)$ increasing
on $\mathbb{R} = (-\infty, \infty)$



4) $f(x) = x^4$ is Power function or Polynomial
of degree 4

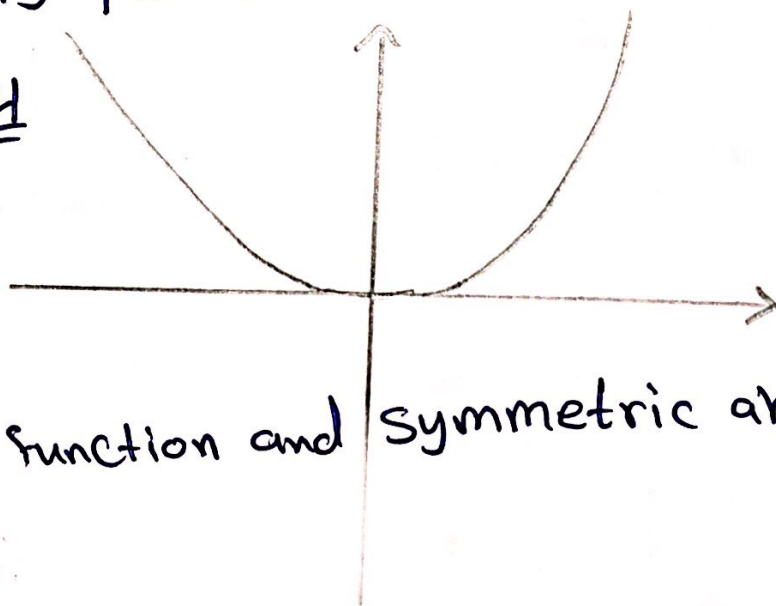
$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [0, \infty)$

$f(x)$ is even function and symmetric about
y-axis

$f(x)$ is increasing on $[0, \infty)$

$f(x)$ is decreasing on $(-\infty, 0]$

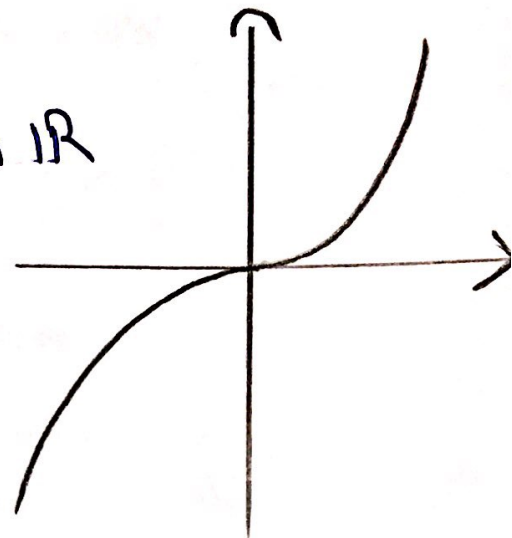


5) $f(x) = x^5$ is Power function or Polynomial of Degree 5

$$D_{f(x)} = \mathbb{R} ; R_{f(x)} = \mathbb{R}$$

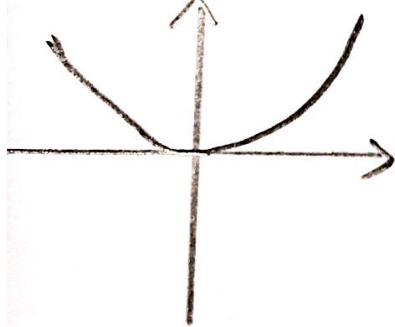
$f(x) = x^5$ is odd function and Symmetric about $(0,0)$

$f(x) = x^5$ is increasing on \mathbb{R}



$f(x) = x^n$ for all $n = 2, 3, 4, 5, 6, 7, \dots$

n : even number



Domain: \mathbb{R}

Range: $[0, \infty)$

Power function

Polynomial of degree n

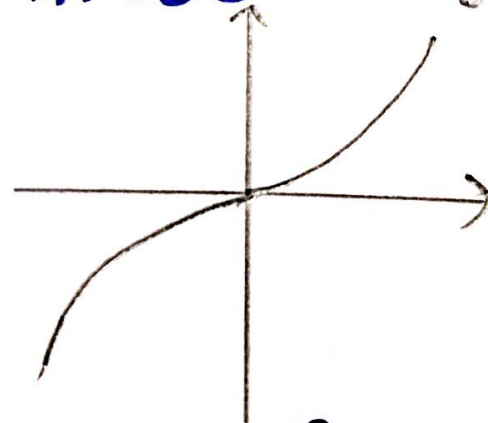
$f(x)$ is even

$f(x)$ is increasing on $[0, \infty)$

$f(x)$ is decreasing on $(-\infty, 0]$

EX: $f(x) = x^{12}$

n : odd num



Domain: \mathbb{R}

Range: \mathbb{R}

Power function

Polynomial of Degree n

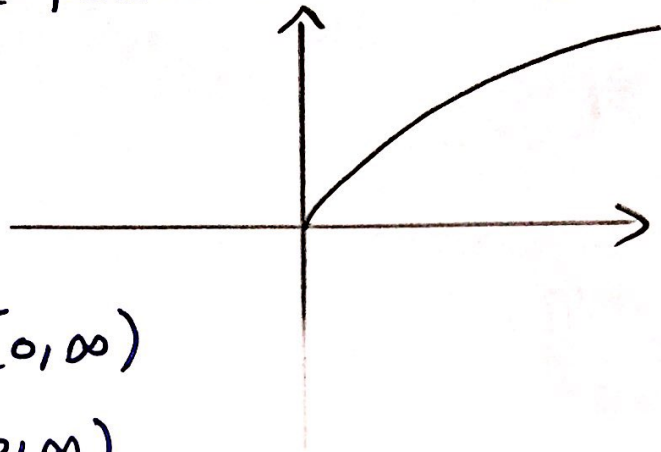
$f(x)$ is odd

$f(x)$ is increasing on \mathbb{R}

EX: $f(x) = x^9$

6) $f(x) = x^{\frac{1}{2}}$ or $f(x) = \sqrt{x}$

Name: The power function or Root function



Domain: $[0, \infty)$

Range: $[0, \infty)$

$f(x) = \sqrt{x}$ is increasing on $[0, \infty)$

$f(x) = \sqrt{x}$ is neither odd nor even

7) $f(x) = \sqrt[3]{x}$ or $x^{\frac{1}{3}}$

Name: The Power function or Root function

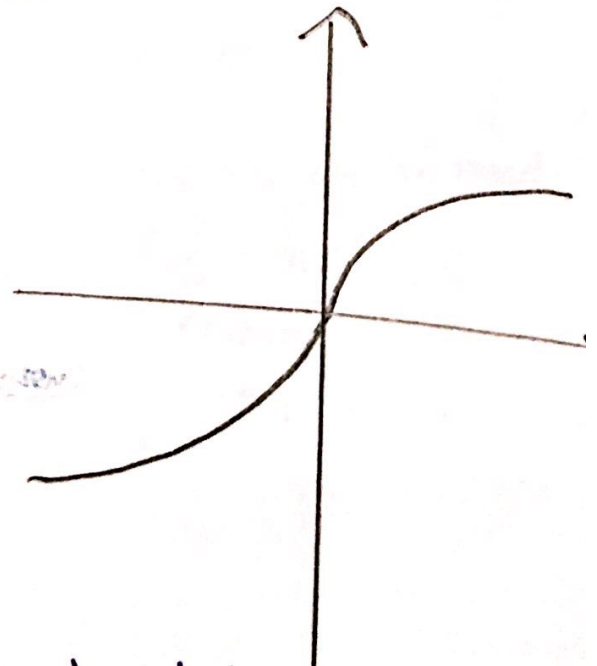
Domain: \mathbb{R}

Range: \mathbb{R}

$f(x) = x^{\frac{1}{3}}$ is increasing on \mathbb{R}

$f(x) = x^{\frac{1}{3}}$ is an odd function

$f(x) = x^{\frac{1}{3}}$ is symmetric about $(0,0)$

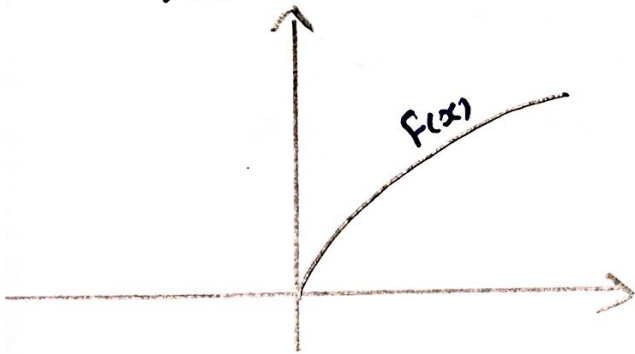


$$f(x) = \sqrt[n]{x} \text{ or } x^{\frac{1}{n}} \text{ For all } n=2,3,4,5,\dots$$

n is even number

$$D_{f(x)} = [0, \infty)$$

$$R_{f(x)} = [0, \infty)$$



$f(x)$ is increasing on $[0, \infty)$

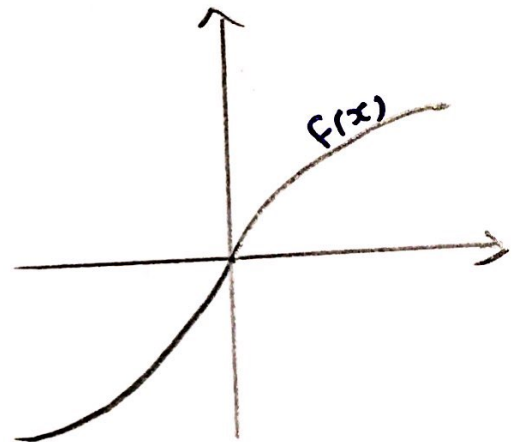
$f(x)$ is neither odd nor even

Ex : $f(x) = \sqrt[6]{x}$

n is odd number

$$D_{f(x)} = \mathbb{R}$$

$$R_{f(x)} = \mathbb{R}$$



$f(x)$ is increasing on \mathbb{R}

$f(x)$ is odd and symmetric about the origin

Ex : $f(x) = \sqrt[7]{x}$

⑧ $f(x) = x^{-1}$ or $\frac{1}{x}$

a) Name : The Power function or

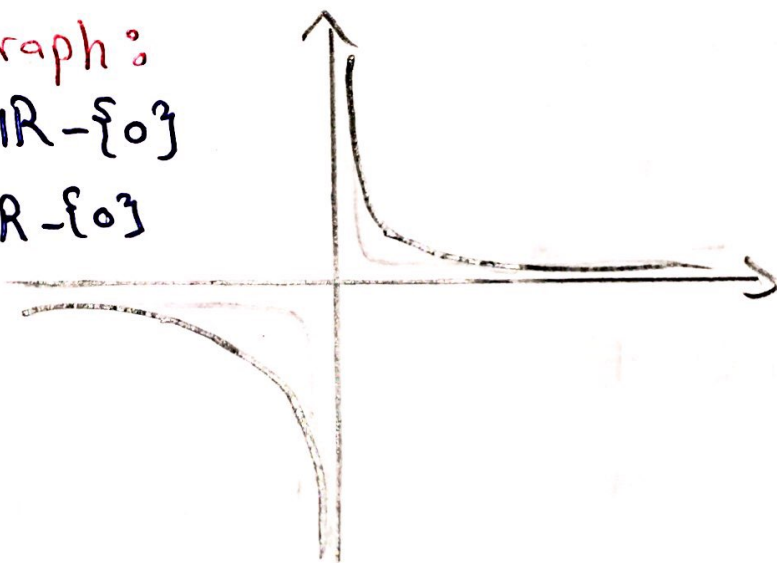
Rational function or reciprocal function

b) $f(x) = \frac{1}{x}$ is odd function and Symmetric about $(0,0)$

c) The Graph :

$$D_{f(x)} = \mathbb{R} - \{0\}$$

$$R_{f(x)} = \mathbb{R} - \{0\}$$



d) $f(x) = \frac{1}{x}$ is decreasing on $\mathbb{R} - \{0\}$

e) $x = 0$ and $y = 0$ are asymptotes

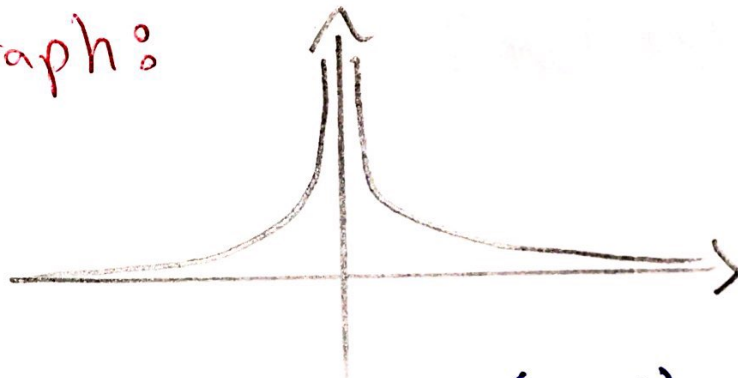
H.W : $f(x) = \frac{1}{x^5}$; $f(x) = \frac{1}{x^3}$

$$\textcircled{9} f(x) = x^{-2} \text{ or } \frac{1}{x^2}$$

Name: The Power function or Rational function

$f(x) = \frac{1}{x^2}$ is even function and symmetric about y-axis

The Graph:



$$D_{f(x)} = \mathbb{R} - \{0\}$$

$$R_{f(x)} = (0, \infty)$$

$x=0$ and $y=0$ are asymptotes.

$f(x)$ is decreasing on $(0, \infty)$

$f(x)$ is increasing on $(-\infty, 0)$

H.W $f(x) = x^{-4}$; $f(x) = x^{-10}$

Type 3 : Radical function

$$f(x) = \sqrt[n]{p(x)} \quad \text{for all } p(x) \text{ is Polynomial.}$$

Example :

$$\left. \begin{aligned} f(x) &= \sqrt[7]{x^3+1} \\ f(x) &= \sqrt{x^2+12} \end{aligned} \right\} \text{The Radical functions}$$

Type 4 : Rational function

$$f(x) = \frac{P(x)}{Q(x)} \quad \text{for all } P(x) \text{ and } Q(x)$$

are polynomials.

$$D_{f(x)} = \mathbb{R} - \left\{ \text{المقامات} \right\}$$

Example :

$$\left. \begin{aligned} f(x) &= \frac{x}{x^3+27} \\ f(x) &= \frac{x-1}{x^2+100} \end{aligned} \right\} \text{The Rational functions.}$$

Type 5 : Absolute Value Function

$$f(x) = |p(x)| \text{ for all } p(x) \text{ is Polynomial}$$

$$D_{f(x)} = \mathbb{R} \quad ; \quad R_{f(x)} = [0, \infty)$$

Example : $f(x) = |x|$ } Absolute value functions
 $f(x) = |x-4|$ }
 $f(x) = |x^2-9|$ } $D_f = \mathbb{R}$
 $R_f = [0, \infty)$

Type 6 : Algebraic function

- ① Polynomial
- ② Power function
- ③ Radical function
- ④ Rational function
- ⑤ absolute value function
- ⑥ Add, Subtract, multiple, divided and taking for above function.

Example:

$$g(x) = 3x^7 - 3x^2 - 1$$

$$g(x) = x^{-20}$$

$$g(x) = \sqrt{x}$$

$$g(x) = x^3$$

$$g(x) = \frac{1}{x^{12}}$$

$$g(x) = \sqrt{x^2 + 7x + 1}$$

$$g(x) = \frac{x}{x^2 + x + 7}$$

$$g(x) = |x + 1|$$

$$g(x) = x^2 + \sqrt{x} + x^{-3} - |x^2 + 1| + \frac{1}{2}$$

$$g(x) = \frac{x^4 - 16x^2}{x + \sqrt{x}}$$

$$g(x) = (x - 2) \sqrt[3]{x - 1}$$

$$g(x) = |x - 1| + \sqrt{x}$$

Algebraic Function