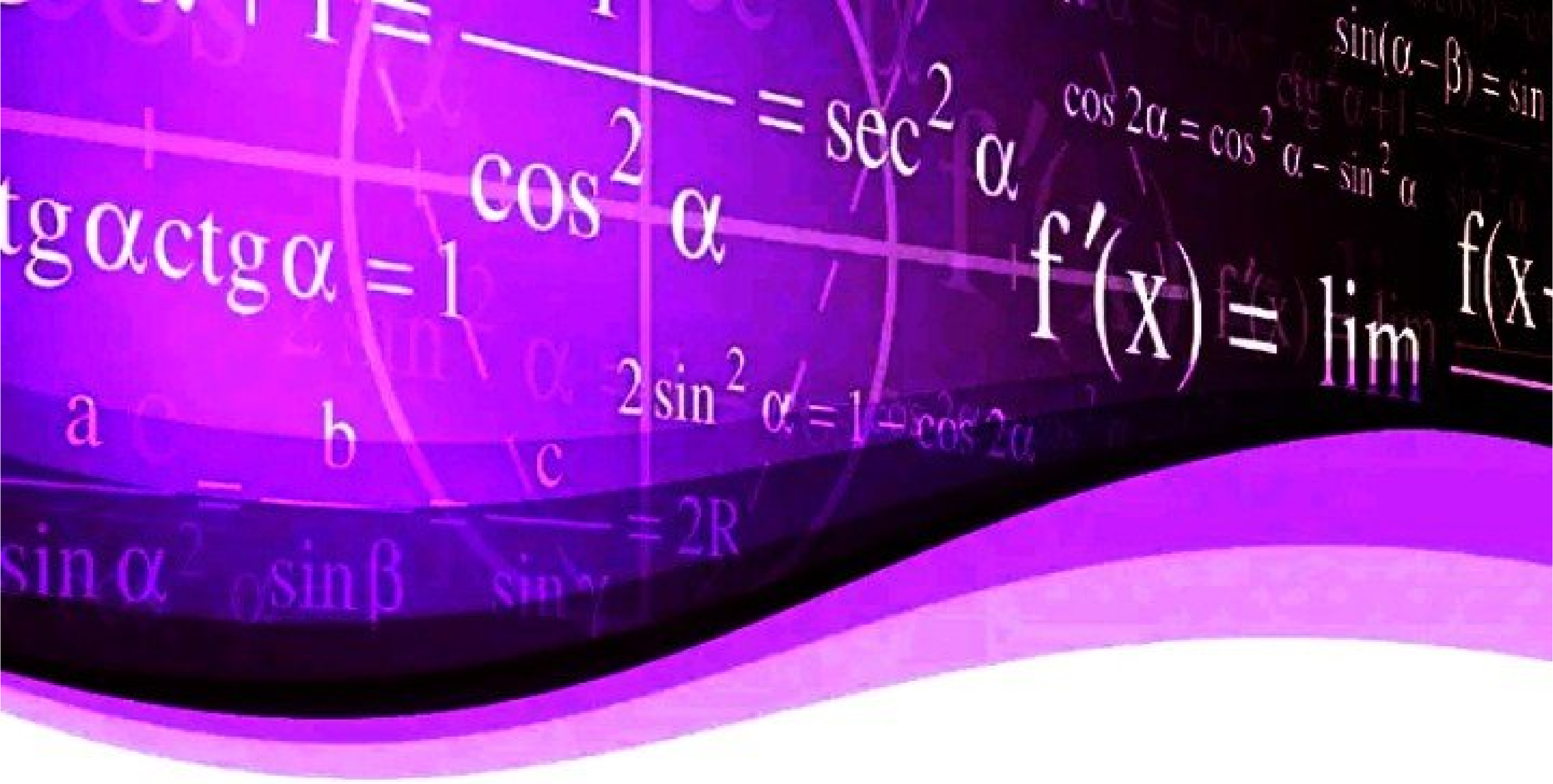

الرياضيات
الثالث الثانوي العلمي

خمسون تمرين في النهايات تشمل الحالات :

(نهايات مثلثية - جذرية - أسية - لوغاريتمية - نهاية تابع الجزء الصحيح - توظيف تعريف العدد المشتق في إيجاد النهاية)

مرفقه بالحل المفصل علها تحمل الفائدة للطلاب الأعزاء

أ.ماهر بربر

1 $f(x)=(x-2)^2.\cot(x-2);x\to 2$

$$\lim_{x\to 2} f(x)=0\times\infty=????$$

$$\boxed{\cot x=\frac{1}{\tan x}\Rightarrow \cot(x-2)=\frac{1}{\tan(x-2)}}\Rightarrow$$

$$f(x)=(x-2)\times\frac{(x-2)}{\tan(x-2)}\quad \boxed{\begin{array}{l}\tau=x-2\\ x\to 2\Rightarrow \tau\to 0\end{array}}\Rightarrow$$

$$\lim_{x\to 2} f(x)=\lim_{\tau\to 0}\tau\times\frac{\tau}{\tan\tau}\quad \boxed{\lim_{\tau\to 0}\frac{\tau}{\tan\tau}=1}\Rightarrow$$

$$\lim_{x\to 2} f(x)=0\times 1=0$$

2 $f(x)=\frac{\sin x}{\sqrt{x^2+x^3}};x\to 0^-$

$$\lim_{x\to 0^-} f(x)=\frac{0}{0}=????$$

$$f(x)=\frac{\sin x}{\sqrt{x^2(1+x)}}\quad \boxed{\begin{array}{l}\sqrt{x^2}=|x|\\ x\to 0^-\Rightarrow\sqrt{x^2}=-x\end{array}}$$

$$f(x)=\frac{\sin x}{-x\sqrt{(1+x)}}=-\frac{\sin x}{x}\times\frac{1}{\sqrt{1+x}}$$

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1}\Rightarrow\lim_{x\to 0^-} f(x)=-(1)(1)=-1$$

5 $f(x)=\dfrac{1-\cos x}{x^2}\ ;\ x\to 0$

$$\lim_{x\to 0} f(x)=\frac{0}{0}=????$$

$$\sin^2 x=\frac{1-\cos 2x}{2}\Rightarrow 2\sin^2 x=1-\cos 2x \Rightarrow$$

$$\Rightarrow 2\sin^2\frac{x}{2}=1-\cos x$$

$$f(x)=\frac{2\sin^2\frac{x}{2}}{x^2}=\frac{2\sin^2\frac{x}{2}}{\left(2\times\frac{x}{2}\right)^2}=\frac{1}{2}\times\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\Rightarrow$$

$$\lim_{x\to 0} f(x)=\frac{1}{2}\times(1)^2=\frac{1}{2}$$

6 $f(x)=\dfrac{1-\cos x}{x.\sin x}\ ;\ x\to 0$

$$\lim_{x\to 0} f(x)=\frac{0}{0}=????$$

$$2\sin^2\frac{x}{2}=1-\cos x\ ,\ \sin 2x=2\sin x.\cos x\Rightarrow$$

$$\Rightarrow \sin x=2\sin\frac{x}{2}.\cos\frac{x}{2}$$

$$f(x)=\frac{2\sin^2\frac{x}{2}}{x\times 2\sin\frac{x}{2}\cos\frac{x}{2}}=\frac{\sin\frac{x}{2}}{x.\cos\frac{x}{2}}$$

$$=\frac{1}{x}\times\tan\frac{x}{2}=\frac{\tan\frac{x}{2}}{x}=\frac{\tan\frac{x}{2}}{2\times\frac{x}{2}}=\frac{1}{2}\frac{\tan\frac{x}{2}}{\frac{x}{2}}\ ,\ \lim_{\heartsuit\to 0}\frac{\tan\heartsuit}{\heartsuit}=1$$

$$\Rightarrow \lim_{x\to 0} f(x)=\frac{1}{2}\times(1)=\frac{1}{2}$$

3 $f(x)=\dfrac{\sin x}{\sqrt{1+x}-1};x\to 0$

$\lim\limits_{x\to 0}=\dfrac{0}{0}=???$

نضرب البسط والمقام
بمرافق المقام

$$f(x)=\frac{\sin x\,(\sqrt{1+x}+1)}{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}$$

$$=\frac{\sin x(\sqrt{1+x}+1)}{1+x-1}=\frac{\sin x}{x}\times(\sqrt{1+x}+1)$$

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1}\Rightarrow\lim_{x\to 0}f(x)=1\times 2=2$$

4 $f(x)=\dfrac{\sin(x-1)}{x^2-1};x\to 1$

$\lim\limits_{x\to 1}f(x)=\dfrac{0}{0}=????$

$$f(x)=\frac{\sin(x-1)}{(x^2-1)}=\frac{\sin(x-1)}{(x-1)(x+1)}$$

$$=\frac{1}{x+1}\times\frac{\sin(x-1)}{x-1}$$

$$\lim_{x\to 1}f(x)=\frac{1}{2}\times\lim_{x\to 1}\frac{\sin(x-1)}{x-1}\quad \boxed{\begin{array}{l}\tau=x-1\\ x\to 1\Rightarrow\tau\to 0\end{array}}\Rightarrow$$

$$\lim_{x\to 1}f(x)=\frac{1}{2}\times\lim_{\tau\to 0}\frac{\sin\tau}{\tau}=\frac{1}{2}\times 1=\frac{1}{2}$$

$$\boxed{7}\ f(x)=\frac{(\sqrt{1+x^2}-1)}{\sin x}.\cos x\ ;x\to 0$$

$$\lim_{x\to 0} f(x)=\frac{0}{0}=????$$

$$f(x)=\frac{(\sqrt{1+x^2}-1)(\sqrt{1+x^2}+1)}{(\sqrt{1+x^2}+1)\sin x}.\cos x$$

$$=\frac{1+x^2-1}{(\sqrt{1+x^2}+1)\sin x}.\cos x$$

$$=\frac{x^2}{(\sqrt{1+x^2}+1)\sin x}.\cos x=\frac{x\ .x}{(\sqrt{1+x^2}+1)\sin x}.\ \cos x$$

$$f(x)=\underbrace{\frac{x}{(\sqrt{1+x^2}+1)}}\times\underbrace{\frac{x}{\sin x}}\times\underbrace{\cos x}\Rightarrow$$

$$\lim_{x\to 0}=f(x)=\quad 0\quad\times\ 1\ \times\ 1=0$$

8 $f(x)=\dfrac{1-\sqrt{\cos x}}{x^2}\ ;\ x\to 0$

$$\lim_{x\to 0} f(x)=\frac{0}{0}=????$$

$$f(x)=\frac{(1-\sqrt{\cos x})(1+\sqrt{\cos x})}{(1+\sqrt{\cos x})x^2}$$

$$=\frac{1-\cos x}{(1+\sqrt{\cos x})x^2} \qquad \boxed{1-\cos x=2\sin^2\frac{x}{2}}$$

$$=\frac{2\sin^2\frac{x}{2}}{(1+\sqrt{\cos x})x^2}$$

$$=\frac{1}{(1+\sqrt{\cos x})}\times\frac{2\sin^2\frac{x}{2}}{x^2}$$

$$=2\times\frac{1}{(1+\sqrt{\cos x})}\times\frac{\sin^2\frac{x}{2}}{\left(2\times\frac{x}{2}\right)^2}$$

$$=2\times\frac{1}{(1+\sqrt{\cos x})}\times\frac{1}{4}\times\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}$$

$$=\frac{1}{2}\times\frac{1}{(1+\sqrt{\cos x})}\times\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 \Rightarrow$$

$$\lim_{x\to 0} f(x)=\frac{1}{2}\times\frac{1}{2}\times 1=\frac{1}{4}$$

9 $f(x)=\dfrac{\sin 4x-2\sin 2x}{x^{3}}\;;x\to 0$

$\lim\limits_{x\to 0} f(x)=\dfrac{0}{0}=??$

$$\boxed{\begin{array}{l}\sin 2x=2\sin x\;.\;\cos x\Rightarrow\\ \sin 4x=2\sin 2x\;.\;\cos 2x\end{array}}\Rightarrow$$

$$f(x)=\frac{2in2x.\cos 2x-2\sin 2x}{x^{3}}$$

$$=\frac{2in2x\left(\cos 2x-1\right)}{x^{3}}$$

$$=\frac{2in2x\left(-2\sin^{2}x\right)}{x^{3}}$$

$$=-4\times\frac{\sin 2x}{x}\times\frac{\sin^{2}x}{x^{2}}$$

$$=-4\times\frac{\sin 2x}{\frac{1}{2}\times 2x}\times\left(\frac{\sin x}{x}\right)^{2}$$

$$=-\frac{4}{\frac{1}{2}}\times\frac{\sin 2x}{2x}\times\left(\frac{\sin x}{x}\right)^{2}\Rightarrow$$

$$\lim_{x\to 0} f(x)=-8\times 1\times 1=-8$$

$\boxed{10}\; f(x)=\dfrac{\sin 7x+\sin 3x}{10x\cos 2x}\;;x\to 0$

$$\lim_{x\to 0} f(x)=\frac{0}{0}=??$$

$$\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\Rightarrow$$
$$\sin 7x+\sin 3x=2\sin 5x\cos 2x$$
$\Rightarrow$

$$f(x)=\frac{2in5x.cos2x}{10x\ \cos 2x}=\frac{\sin 5x}{5x}\Rightarrow\lim_{x\to 0} f(x)=1$$

أ. ماهر برير

$\boxed{11}\; f(x)=\dfrac{2-2\cos\sqrt{x}}{x}\;;x\to 0$

$$\lim_{x\to 0} f(x)=\frac{0}{0}=??$$

$$f(x)=\frac{2-2cos\sqrt{x}}{x}=\frac{2(1-cos\sqrt{x})}{x}=\frac{2(1-cos\sqrt{x})(1+cos\sqrt{x})}{x(1+cos\sqrt{x})}$$

$$=\frac{2\left(1-cos^2(\sqrt{x})\right)}{x(1+cos\sqrt{x})}=\frac{2sin^2(\sqrt{x})}{x(1+cos\sqrt{x})}=\left(\frac{sin^2(\sqrt{x})}{x}\right)\times\frac{2}{(1+cos\sqrt{x})}$$
$$=\left(\frac{sin\sqrt{x}}{\sqrt{x}}\right)^2\times\frac{2}{(1+cos\sqrt{x})}$$

$$\Rightarrow\lim_{x\to 0} f(x)=(1)^2\times\left(\frac{2}{1+1}\right)=1$$

$$\boxed{12}\ f(x)=\frac{\cos(3x)-\cos x}{x.\sin x}\ ;\ x\longrightarrow 0$$

$$\lim_{x\to 0} f(x)=\frac{0}{0}=???$$

طريقة أولى

$$\cos a-\cos b=-2\sin\left(\frac{a+b}{2}\right).\sin\left(\frac{a-b}{2}\right)\Rightarrow$$

$$f(x)=\frac{-2\sin 2x\times\sin x}{x.\sin x}=\frac{-2\sin 2x}{x}=\frac{-2\sin 2x}{\frac{1}{2}\times 2x}$$

$$=-4\times\frac{\sin 2x}{2x}\Rightarrow \lim_{x\to 0} f(x)=-4\times 1=-4$$

طريقة ثانية

$$\cos 3x=4\cos^3 x-3\cos x\Rightarrow$$

$$f(x)=\frac{4\cos^3 x-3\cos x-\cos x}{x.\sin x}=\frac{4\cos^3 x-4\cos x}{x.\sin x}$$

$$=\frac{4\cos x(\cos^2 x-1)}{x.\sin x}=\frac{4\cos x(-\sin^2 x)}{x.\sin x}$$

$$=-\frac{4\cos x\times\sin^2 x}{x.\sin x}=-4\cos x\times\frac{\sin x}{x}\Rightarrow$$

$$\lim_{x\to 0} f(x)=-4(1)\times 1=-4$$

$$\boxed{13}\ f(x)=\frac{2\cos 2x-1}{\cos 3x}\ ;\ x\longrightarrow\frac{\pi}{6}$$

$$\lim_{x\to\frac{\pi}{6}} f(x)=\frac{0}{0}=???$$

$$\boxed{\begin{array}{l}\cos 3x=4\cos^3 x-3\cos x\\ \cos 2x=1-2\sin^2 x\end{array}}\Rightarrow$$

$$f(x)=\frac{2(1-2\sin^2 x)-1}{4\cos^3 x-3\cos x}=\frac{2-4\sin^2 x-1}{\cos x(4\cos^2 x-3)}$$

$$\boxed{\cos^2 x=1-\sin^2 x}\Rightarrow$$

$$f(x)=\frac{1-4\sin^2 x}{\cos x(4(1-\sin^2 x)-3)}$$

$$=\frac{1-4\sin^2 x}{\cos x(1-4\sin^2 x)}=\frac{1}{\cos x}\Rightarrow$$

$$\lim_{x\to\frac{\pi}{6}} f(x)=\frac{1}{\cos\left(\frac{\pi}{6}\right)}=\frac{1}{\frac{\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}$$

$$=\frac{2\sqrt{3}}{3}$$

استخدام تعريف العدد المشتق

بإزالة عدم التعيين من النمط $\frac{0}{0}$ بحساب النهايات

لإزالة حالة عدم التعيين من النمط $\frac{0}{0}$ لتابع f عند النقطة x_0 نكتب التابع بالشكل

$$f(x) = \frac{g(x) - g(x_0)}{x - x_0}$$

حيث التابع g اشتقاقي عند x_0 وباستخدام تعريف العدد المشتق لـ g عند x_0 فإن :

$$\lim_{x \longrightarrow x_0} f(x) = g'(x_0)$$

$$\boxed{14} \quad \frac{\sqrt{2x^2 - 1} - 1}{x - 1} ; x \longrightarrow 1$$

$$\lim_{x \longrightarrow 0} \frac{\sqrt{2x^2 - 1} - 1}{x - 1} = \frac{0}{0}$$

$$\frac{\sqrt{2x^2 - 1} - 1}{x - 1} , x_0 = 1$$

We write $g(x) = \sqrt{2x^2 - 1} \Rightarrow g(1) = 1$

$$g'(x) = \frac{4x}{2\sqrt{2x^2 - 1}} = \frac{2x}{\sqrt{2x^2 - 1}} , g'(1) = 2$$

$$\Rightarrow \lim_{x \longrightarrow 0} \frac{\sqrt{2x^2 - 1} - 1}{x - 1} = g'(1) = 2$$

15 $\dfrac{2\cos x-1}{x-\frac{\pi}{3}}\ ;x\longrightarrow\dfrac{\pi}{3}$

$$\lim_{x\to\frac{\pi}{3}}\frac{2\cos x-1}{x-\frac{\pi}{3}}=2\lim_{x\to\frac{\pi}{3}}\frac{\cos x-\frac{1}{2}}{x-\frac{\pi}{3}}=\frac{0}{0}$$

$$\frac{\cos x-\frac{1}{2}}{x-\frac{\pi}{3}},\ x_0=\frac{\pi}{3}$$

We write $g(x)=\cos x\Rightarrow g\left(\frac{\pi}{3}\right)=\frac{1}{2}$

$$g'(x)=-\sin x\ ,g'\left(\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

$$\Rightarrow\lim_{x\to\frac{\pi}{3}}=2\underbrace{\lim_{x\to\frac{\pi}{3}}\frac{\cos x-\frac{1}{2}}{x-\frac{\pi}{3}}}$$

$$=2\times\qquad g'\left(\frac{\pi}{3}\right)$$

$$=2\times\qquad -\frac{\sqrt{3}}{2}=-\sqrt{3}$$

16 $\frac{\sin x}{x-\pi}\ ; x \longrightarrow \pi$

$$\lim_{x\to\pi}\frac{\sin x}{x-\pi}=\frac{0}{0}$$

$$\frac{\sin x-0}{x-\pi}\ , x_0=\pi$$

We write $g(x)=\sin x \Rightarrow g(\pi)=0$

$$g'(x)=\cos x\ , g'(\pi)=-1$$

$$\Rightarrow \lim_{x\to\pi}\frac{\sin x}{x-\pi}=\underbrace{\lim_{x\to\pi}\frac{\sin x-0}{x-\pi}}_{g'(\pi)}=-1$$

17 $\frac{\ln(x-1)}{x-2}\ ; x \longrightarrow 2$ $\lim_{x\to\pi}\frac{\ln(x-1)}{x-2}=\frac{0}{0}$

$$\frac{\ln(x-1)-0}{x-2}\quad x_0=2$$

We write $g(x)=\ln(x-1) \Rightarrow g(2)=0$

$$g'(x)=\frac{1}{x-1}\ , g'(2)=1$$

$$\Rightarrow \lim_{x\to 2}\frac{\ln(x-1)}{x-2}=\underbrace{\lim_{x\to 2}\frac{\ln(x-1)-0}{x-2}}_{g'(2)}=0$$

$$\boxed{18}\quad \frac{\tan x - 1}{x - \frac{\pi}{4}} ; x \longrightarrow \frac{\pi}{4}$$

$$\lim_{x \longrightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}} = \frac{0}{0}$$

$$\frac{\tan x - 1}{x - \frac{\pi}{4}} ; x_0 = \frac{\pi}{4}$$

We write $g(x) = \tan x \Rightarrow g\left(\frac{\pi}{4}\right) = 1$

$$g'(x) = 1 + \tan^2 x \ , g'\left(\frac{\pi}{4}\right) = 2$$

$$\Rightarrow \lim_{x \longrightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}} = g'\left(\frac{\pi}{4}\right) = 2$$

$$\boxed{18}\quad \frac{\cos x}{x-\frac{\pi}{2}}\ ;x\longrightarrow\frac{\pi}{2}$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{x-\frac{\pi}{2}}=\frac{0}{0}$$

$$\frac{\cos x-0}{x-\frac{\pi}{2}}\ ;\ x_0=\frac{\pi}{2}$$

We write $g(x)=\cos x\Rightarrow g\left(\frac{\pi}{2}\right)=0$

$$g'(x)=-\sin x\ ,g'\left(\frac{\pi}{2}\right)=-1$$

$$\Rightarrow\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{x-\frac{\pi}{2}}=\underbrace{\lim_{x\to\frac{\pi}{2}}\frac{\cos x-0}{x-\frac{\pi}{2}}}_{g'\left(\frac{\pi}{2}\right)}=-1$$

$\boxed{19}$ $\dfrac{\sqrt{x+4}-2}{x}\ ; x \longrightarrow 0$

$$\lim_{x\longrightarrow 0}\frac{\sqrt{x+4}-2}{x}=\frac{0}{0}$$

$$\frac{\sqrt{x+4}-1}{x-0}\ , x_0=0$$

We write $g(x)=\sqrt{x+4} \Rightarrow g(0)=2$

$$g'(x)=\frac{1}{2\sqrt{x+4}}\ , g'(0)=\frac{1}{4}$$

$$\Rightarrow \lim_{x\longrightarrow 0}\frac{\sqrt{x+4}-2}{x}=\underbrace{\lim_{x\longrightarrow 0}\frac{\sqrt{x+4}-2}{x-0}}_{g'(0)}=\frac{1}{4}$$

$\boxed{20}$ $f(x)=\dfrac{\sin(x-1)}{\sqrt{x-1}}\ ; x \longrightarrow 1$

$$\lim_{x\longrightarrow 1}=\frac{0}{0}=???$$

$$f(x)=\frac{\sin(x-1)}{\sqrt{x-1}}$$

$\tau = x-1$

$x \longrightarrow 1 \Rightarrow \tau \longrightarrow 0$

$$\lim_{x\longrightarrow 1}f(x)=\lim_{\tau\longrightarrow 1}\frac{\sin\tau}{\sqrt{\tau}}=\lim_{\tau\longrightarrow 1}\frac{\sin\tau}{\sqrt{\tau}}\times\frac{\sqrt{\tau}}{\sqrt{\tau}}$$

$$=\lim_{\tau\longrightarrow 1}\frac{\sin\tau}{\tau}\times\sqrt{\tau}=1\times 0=0$$

$$\boxed{21}\ f(x)=\frac{\sqrt{x}}{\ln x}\ ; x \longrightarrow +\infty$$

$$\lim_{x\longrightarrow+\infty} f(x)=\frac{+\infty}{+\infty}=??$$

$$f(x)=\frac{\sqrt{x}}{\ln x}=\frac{\sqrt{x}}{\underbrace{\ln(\sqrt{x})^2}}=\frac{\sqrt{x}}{2\ln\sqrt{x}}\Rightarrow$$

$$\ln(a)^b=b\ln a$$

$$\lim_{x\longrightarrow+\infty} f(x)=\lim_{x\longrightarrow+\infty}\frac{1}{2}\times\frac{\sqrt{x}}{\ln\sqrt{x}}$$

$$\boxed{\lim_{\heartsuit\longrightarrow\infty}\frac{\heartsuit}{\ln\heartsuit}=+\infty}$$

$$=\frac{1}{2}\times+\infty$$

$$\boxed{22}\ f(x)=\left(x^2\times\frac{1}{\ln(1+x^2)}\right); x\longrightarrow \begin{matrix}0\\+\infty\end{matrix}$$

$$\lim_{x\longrightarrow0} f(x)=0\times\infty=??$$

$$f(x)=\frac{x^2}{\ln(1+x^2)}\qquad \boxed{\begin{array}{l}\lim\limits_{\heartsuit\longrightarrow0}\dfrac{\heartsuit}{\ln(1+\heartsuit)}=1\\[2ex]\lim\limits_{\heartsuit\longrightarrow0}\dfrac{\ln(1+\heartsuit)}{\heartsuit}=1\end{array}}\qquad\Rightarrow$$

$$\lim_{x\longrightarrow0} f(x)=\lim_{x\longrightarrow0}\frac{x^2}{\ln(1+x^2)}=1$$

$$f(x)= x^2\times\frac{1}{\ln(1+x^2)}$$

$$\lim_{x\longrightarrow+\infty} f(x)=\infty\times 0=??$$

$$f(x)=x^2\times\frac{1}{\ln(1+x^2)}=\frac{x^2}{1+x^2}\times\frac{1+x^2}{\ln(1+x^2)}$$

$$\boxed{\lim_{\heartsuit\longrightarrow+\infty}\frac{\heartsuit}{\ln\heartsuit}=+\infty\ ,\ \lim_{\heartsuit\longrightarrow+\infty}\frac{\ln\heartsuit}{\heartsuit}=0}\Rightarrow$$

$$\lim_{x\longrightarrow+\infty} f(x)=\lim_{x\longrightarrow+\infty}\underbrace{\frac{x^2}{1+x^2}}_{1}\times\underbrace{\frac{1+x^2}{\ln(1+x^2)}}_{+\infty}=+\infty$$

$\boxed{23}$ $f(x)=x.\ln\left(1+\frac{1}{x}\right)\ ;x\longrightarrow\infty$

$$\lim_{x\longrightarrow+\infty} f(x)=+\infty\times 0=???$$

$$f(x)=x.\ln\left(1+\frac{1}{x}\right),\ \boxed{\begin{array}{l}\frac{1}{x}=\tau\ \Rightarrow x=\frac{1}{\tau}\\ x\longrightarrow+\infty\Rightarrow\tau\ \longrightarrow 0\end{array}}\Rightarrow$$

$$\lim_{x\longrightarrow+\infty} f(x)=\lim_{\tau\longrightarrow 0}\frac{1}{\tau}\times\ln(1+\tau)$$

$$=\lim_{\tau\longrightarrow 0}\frac{\ln(1+\tau)}{\tau}=1$$

$\boxed{24}\ f(x)=x.\ln^2 x\ ;\ x\longrightarrow 0^+$

$$\lim_{x\longrightarrow 0^+} f(x)=0\times(+\infty)=???$$

$$f(x)=x.\ln^2 x\ ,\quad \boxed{\ln x^2=2\ln x\neq \ln^2 x}$$

$$=x(\ln x)^2=(\sqrt{x})^2\times(\ln(\sqrt{x})^2)^2$$

$$=(\sqrt{x})^2(2\ln\sqrt{x})^2\quad \boxed{(a\times b)^n=a^n\times b^n}$$

$$=(\sqrt{x})^2(4)\ln^2\sqrt{x}$$

$$=4(\sqrt{x})^2\ln^2\sqrt{x}$$

$$=4(\sqrt{x}\times\ln\sqrt{x})^2\quad \boxed{\begin{array}{l}\sqrt{x}=\tau\\ x\longrightarrow 0^+\Rightarrow \tau\longrightarrow 0^+\end{array}}$$

$$=4(\tau\times\ln\tau)^2\quad \boxed{\lim_{x\longrightarrow 0^+} x\ln x=0^-}\Rightarrow$$

$$\lim_{x\longrightarrow 0^+} f(x)=\lim_{\tau\longrightarrow 0^+} 4\times(\tau\times\ln\tau)^2$$

$$=\quad 4\times 0^2=0$$

$$\boxed{25}\ f(x)=\left(\frac{\ln(x+1)}{\ln(x-1)}-1\right);\ x\longrightarrow+\infty$$

$$\lim_{x\longrightarrow+\infty}=\frac{+\infty}{+\infty}=??$$

$$f(x)=\frac{\ln(x+1)-\ln(x-1)}{\ln(x-1)}$$ $\ln\left(\frac{a}{b}\right)=\ln a-\ln b$

$$=\frac{\ln\left(\frac{x+1}{x-1}\right)}{\ln(x-1)}\Rightarrow$$

$$\lim_{x\longrightarrow+\infty}f(x)=\frac{\ln(1)}{\ln(+\infty)}=0$$

$$\boxed{26}\ f(x)=\sqrt{x}-\ln x\ ;\ x\longrightarrow+\infty$$

$$\lim_{x\longrightarrow+\infty}=+\infty-\infty=??$$

$$f(x)=\sqrt{x}\times\left(1-\frac{\ln x}{\sqrt{x}}\right)$$

$$=\sqrt{x}\times\left(1-\frac{\ln(\sqrt{x})^2}{\sqrt{x}}\right)=\sqrt{x}\times\left(1-\frac{2\ln\sqrt{x}}{\sqrt{x}}\right)$$

$$\Rightarrow\lim_{x\longrightarrow+\infty}f(x)=+\infty\times(1-2(0))=+\infty$$

$$\boxed{27}\ f(x)=\frac{\ln(2x+1)}{\sin 3x}\ ;\ x\longrightarrow 0$$

$$\lim_{x\longrightarrow 0}=\frac{0}{0}=??$$

$$f(x)=\frac{1}{\sin 3x}\times\ln(2x+1)$$

$$=\frac{1}{3x}\times\frac{3x}{\sin 3x}\times 2x\times\frac{\ln(2x+1)}{2x}$$

$$=\frac{2x}{3x}\times\frac{3x}{\sin 3x}\times\frac{\ln(2x+1)}{2x}\Rightarrow$$

$$\lim_{x\longrightarrow 0}f(x)=\frac{2}{3}\times 1\times 1=\frac{2}{3}$$

$$\boxed{28}\ f(x)=\frac{\ln x^2}{x}\ ;\ x\longrightarrow -\infty$$

$$\lim_{x\longrightarrow -\infty}f(x)=\frac{+\infty}{-\infty}=??$$

$$x^2=t\Rightarrow x=-\sqrt{t}\ ;x\longrightarrow -\infty$$

$$x\longrightarrow -\infty\Rightarrow t\longrightarrow +\infty$$

$$\lim_{x\longrightarrow -\infty}f(x)=\lim_{t\longrightarrow +\infty}\frac{\ln t}{-\sqrt{t}}$$

$$=\lim_{t\longrightarrow +\infty}-\frac{\ln(\sqrt{t})^2}{\sqrt{t}}=\lim_{t\longrightarrow +\infty}-2\frac{\ln\sqrt{t}}{\sqrt{t}}=0$$

$$\boxed{29}\ f(x)=\frac{\ln x}{x^2-2x+1}\ ;x\longrightarrow 1^+$$

$$\lim_{x\longrightarrow 1^+} f(x)=\frac{0}{0}=??$$

$$f(x)=\frac{\ln x}{(x-1)^2}$$

$$=\frac{1}{x-1}\times\frac{\ln x}{x-1}\quad \boxed{\lim_{\heartsuit\longrightarrow 1}\frac{\ln\heartsuit}{\heartsuit-1}=1}\Rightarrow$$

$$\lim_{x\longrightarrow 1^+} f(x)=+\infty\times 1=+\infty$$

$$\boxed{30}\ f(x)=\frac{x}{x+1}-\ln(x+1);\ x\longrightarrow -1^+$$

$$\lim_{x\longrightarrow 1^+} f(x)=-\infty+\infty=??$$

$$f(x)=\frac{x-(x+1)\ln(x+1)}{x+1}$$

$$\boxed{\begin{array}{l} t=x+1\Rightarrow x=t-1\\ x\longrightarrow -1^+\Rightarrow t\longrightarrow 0^+\end{array}}\ ,\ \boxed{\lim_{x\longrightarrow 0^+} x\ln x=0^-}$$

$$\lim_{x\longrightarrow -1^+} f(x)=\lim_{t\longrightarrow 0^+}\frac{t-1-t\ln t}{t}$$

$$=\frac{0-1-0}{0^+}=-\infty$$

31 $\dfrac{\ln\sqrt{x+1}-\ln\sqrt{2}}{x-1}\;;x\longrightarrow 1$

$$\lim_{x\longrightarrow 1}\frac{\ln\sqrt{x+1}-\ln\sqrt{2}}{x-1}=\frac{0}{0}$$

$x_0=1$

We write $g(x)=\ln\sqrt{x+1}$

$$\Rightarrow g(1)=\ln\sqrt{2}$$

$$g'(x)=\frac{\frac{1}{2\sqrt{x+1}}}{\sqrt{x+1}}=\frac{1}{2}\times\frac{1}{x+1}$$

$$g'(1)=\frac{1}{4}$$

$$f(x)=\frac{g(x)-g(x_0)}{x-x_0}$$

$$=\frac{\ln\sqrt{x+1}-\ln\sqrt{2}}{x-1}\Rightarrow$$

$$\lim_{x\longrightarrow 1}f(x)=\underbrace{\lim_{x\longrightarrow 1}\frac{\ln\sqrt{x+1}-\ln\sqrt{2}}{x-1}}_{g'(1)}=\frac{1}{4}$$

32 $\frac{\ln(\cos x)}{x}\ ; x \longrightarrow 0$

$$\lim_{x\to 0}\frac{\ln(\cos x)}{x}=\frac{0}{0}$$

$$x_0=0$$

We write $g(x)=\ln(\cos x)$

$$\Rightarrow g(0)=0$$

$$g'(x)=\frac{-\sin x}{\cos x}$$

$$g'(0)=0$$

$$f(x)=\frac{g(x)-g(x_0)}{x-x_0}=\frac{\ln(\cos x)-0}{x-0}\Rightarrow$$

$$\lim_{x\to 0}f(x)=\underbrace{\lim_{x\to 0}\frac{\ln(\cos x)-0}{x-0}}_{g'(0)}=0$$

33 $f(x)=e^x-\ln x\ ;\ x\longrightarrow+\infty$

$$\lim_{x\longrightarrow+\infty} f(x)=+\infty-\infty=??$$

$$f(x)=e^x-\ln x=e^x\left(1-\frac{\ln x}{e^x}\right)$$

$$=e^x\left(1-\frac{x}{e^x}\times\frac{\ln x}{x}\right)$$

$$\boxed{\lim_{x\longrightarrow+\infty}\frac{x}{e^x}=0,\ \lim_{x\longrightarrow+\infty}\frac{\ln x}{x}=0}\Rightarrow$$

$$\lim_{x\longrightarrow+\infty} f(x)=+\infty(1-0\times0)=+\infty$$

34 $f(x)=\dfrac{\ln(1+4x)}{1-e^{3x}};x\longrightarrow+0$

$$\lim_{x\longrightarrow0} f(x)=\frac{0}{0}=??$$

$$f(x)=\frac{\ln(1+4x)}{1-e^{3x}}=-\frac{\ln(4x+1)}{e^{3x}-1}$$

$$=-\frac{4x\times\dfrac{\ln(4x+1)}{4x}}{3x\times\dfrac{e^{3x}-1}{3x}}=-\frac{4}{3}\frac{\dfrac{\ln(4x+1)}{4x}}{\dfrac{e^{3x}-1}{3x}}$$

$$\boxed{\lim_{\heartsuit\longrightarrow0}\frac{\ln(\heartsuit+1)}{\heartsuit}=1,\lim_{\heartsuit\longrightarrow0}\frac{e^{\heartsuit}-1}{\heartsuit}=1}\Rightarrow$$

$$\lim_{x\longrightarrow+\infty} f(x)=-\frac{4}{3}\times1=-\frac{4}{3}$$

$\boxed{35}\ f(x)=\frac{\ln(x+2)}{x.e^{x}};\ x\longrightarrow+\infty$

$$\lim_{x\longrightarrow+\infty} f(x)=\frac{+\infty}{+\infty}=??$$

$$f(x)=\frac{\ln(x+2)}{x.e^{x}}=\frac{1}{e^{x}}\times\frac{1}{x}\ln(x+2)$$

$$=\frac{1}{e^{x}}\times\frac{1}{x}\ln\left(x\left(1+\frac{2}{x}\right)\right)$$

$$=\frac{1}{e^{x}}\times\frac{1}{x}\left(\ln x+\ln\left(1+\frac{2}{x}\right)\right)=\frac{1}{e^{x}}\left(\frac{\ln x}{x}+\frac{\ln\left(1+\frac{2}{x}\right)}{x}\right)$$

$$\Rightarrow \lim_{x\longrightarrow+\infty} f(x)=0(0+0)=0$$

$\boxed{36}\ f(x)=\ln(e^{2x}-e^{x}+1)\ ;x\longrightarrow+\infty$

$$\lim_{x\longrightarrow+\infty}=+\infty-\infty=??$$

$$f(x)=\ln(e^{2x}-e^{x}+1)\quad \boxed{e^{-nx}=\frac{1}{e^{nx}}}$$

$$=\ln(e^{2x}(1-e^{-x}+e^{-2x}))\quad \boxed{\ln(a.b)=\ln a+\ln b}$$

$$=\ln e^{2x}+\ln(1-e^{-x}+e^{-2x})\quad \boxed{\ln e^{\alpha x}=\alpha x}$$

$$=2x+\ln(1-e^{-x}+e^{-2x})$$

$$\boxed{e^{-\infty}=0}\Rightarrow$$

$$\lim_{x\longrightarrow+\infty} f(x)=+\infty+\ln(1-0+0)=+\infty$$

37 $f(x)=\dfrac{\sqrt{e^x+1}-\sqrt{2}}{x}\ ;x\longrightarrow 0$

$$\lim_{x\longrightarrow 0} f(x)=\frac{0}{0}=??$$

$$f(x)=\frac{\sqrt{e^x+1}-\sqrt{2}}{x}=\frac{e^x+1-2}{x(\sqrt{e^x+1}+\sqrt{2})}$$

$$=\frac{e^x-1}{x}\times\frac{1}{\sqrt{e^x+1}+\sqrt{2}}\ \boxed{\lim_{\heartsuit\longrightarrow 0}\frac{e^{\heartsuit}-1}{\heartsuit}=1}\Rightarrow$$

$$\lim_{x\longrightarrow 0} f(x)=\lim_{x\longrightarrow 0}\Big(\underbrace{\frac{e^x-1}{x}}\times\underbrace{\frac{1}{\sqrt{e^x+1}+\sqrt{2}}}\Big)$$

$$=\quad 1\quad\times\frac{1}{2\sqrt{2}}=\frac{1}{2\sqrt{2}}$$

تستطيع توظيف تعريف العدد المشتق في حساب هذه النهاية

38 $f(x)=x(e^{\frac{1}{x}}-1)\ ;x\longrightarrow +\infty$

$$\lim_{x\longrightarrow +\infty} f(x)=+\infty\times 0=???$$

$$\boxed{t=\frac{1}{x}\Rightarrow x=\frac{1}{t}\ ,x\longrightarrow +\infty\Rightarrow t\longrightarrow 0}\Rightarrow$$

$$\lim_{x\longrightarrow +\infty} f(x)=\lim_{t\longrightarrow 0}\left(\frac{1}{t}(e^t-1)\right)=\lim_{t\longrightarrow 0}\frac{e^t-1}{t}=1$$

$$\boxed{39}\ f(x)=\frac{e^{x}-e^{4x}}{x}\ ;x\longrightarrow 0$$

$$\lim_{x\longrightarrow 0} f(x)=\frac{0}{0}=??$$

$$f(x)=\frac{e^{x}-e^{4x}}{x}=-\frac{e^{4x}-e^{x}}{x}=-e^{x}\left(\frac{e^{3x}-1}{x}\right)$$

$$=-3e^{x}\left(\frac{e^{3x}-1}{3x}\right)$$

$$\Rightarrow \lim_{x\longrightarrow 0} f(x)=-3\lim_{x\longrightarrow 0}\underbrace{e^{x}}_{1}\left(\underbrace{\frac{e^{3x}-1}{3x}}_{1}\right)=-3$$

$$\lim_{t\longrightarrow 0}(1+t)^{\frac{1}{t}}=e\ ,\ \lim_{t\longrightarrow +\infty}\left(1+\frac{1}{t}\right)^{t}=e$$

$$\boxed{40}\ f(x)=(3+x)^{\frac{2}{x+2}},\ x\longrightarrow -2$$

$$\lim_{x\longrightarrow -2} f(x)=1^{\infty}=???$$

$$f(x)=\left(1+\underbrace{x+2}_{t}\right)^{\frac{2}{x+2}}$$

$$t=x+2\Rightarrow \frac{1}{t}=\frac{1}{x+2}\ ,x\longrightarrow -2\Rightarrow t\longrightarrow 0\Rightarrow$$

$$\lim_{x\longrightarrow -2} f(x)=\lim_{t\longrightarrow 0}(1+t)^{\frac{2}{t}}=\lim_{t\longrightarrow 0}\left(\underbrace{(1+t)^{\frac{1}{t}}}_{e}\right)^{2}=e^{2}$$

$$\boxed{41}\ f(x)=\left(\frac{x+3}{x-1}\right)^{\frac{x}{2}} \quad ;x\longrightarrow+\infty$$

$$\lim_{x\longrightarrow+\infty} f(x)=1^{\infty}=??$$

$$f(x)=\left(\frac{x+3}{x-1}\right)^{\frac{x}{2}}$$

$$=\left(\frac{x-1+4}{x-1}\right)^{\frac{x}{2}}$$

$$=\left(1+\underbrace{\frac{4}{x-1}}_{t}\right)^{\frac{x}{2}}$$

$$t=\frac{4}{x-1}\Rightarrow\frac{1}{t}=\frac{x-1}{4}\Rightarrow$$

$$\frac{1}{t}=\frac{x}{4}-\frac{1}{4}\Rightarrow\frac{x}{2}=\frac{2}{t}+\frac{1}{2}\Rightarrow$$

$$x\longrightarrow+\infty\Rightarrow t\longrightarrow 0$$

$$\lim_{x\longrightarrow+\infty} f(x)=\lim_{t\longrightarrow 0}(1+t)^{\frac{2}{t}+\frac{1}{2}}$$

$$=\lim_{t\longrightarrow 0}\left(\underbrace{(1+t)^{\frac{1}{t}}}\right)^{2}\ \underbrace{\times}_{x^{\frac{1}{2}}=\sqrt{x}}\ \sqrt{1+t}$$

$$=\quad e^{2}\quad\times\quad 1=e^{2}$$

42 $f(x)=(2-x)^{\frac{3}{x-1}}$; $x\longrightarrow 1$

$$\lim_{x\to 1} f(x)=1^{\infty}=??$$

$$f(x)=(2-x)^{\frac{3}{x-1}}=\Big(1+\underbrace{1-x}_{t}\Big)^{\frac{3}{x-1}}$$

$$t=1-x=-(x-1)$$

$$\Rightarrow \frac{1}{t}=-\frac{1}{x-1}=-\frac{1}{x-1}\Rightarrow \frac{-3}{t}=\frac{3}{x-1}\Rightarrow$$

$$x\longrightarrow 1\Rightarrow t\longrightarrow 0$$

$$\lim_{x\to 1} f(x)=\lim_{t\to 0}(1+t)^{\frac{-3}{t}}=\lim_{t\to 0}\Big(\underbrace{(1+t)^{\frac{1}{t}}}\Big)^{-3}$$

$$= \qquad e^{-3}=\frac{1}{e^{3}}$$

43 $f(x)=\left(\frac{x-2}{x+1}\right)^{\frac{x+1}{3}}$; $x\longrightarrow +\infty$

$$\lim_{x\to +\infty} f(x)=1^{\infty}=??$$

$$f(x)=\left(\frac{x-2}{x+1}\right)^{\frac{x+1}{3}}=\left(\frac{x+1-1-2}{x+1}\right)^{\frac{x+1}{3}}=\Bigg(1+\underbrace{\frac{-3}{x+1}}_{t}\Bigg)^{\frac{x+1}{3}}$$

$$t=\frac{-3}{x+1}\Rightarrow -t=\frac{3}{x+1}$$

$$\Rightarrow -\frac{1}{t}=\frac{x+1}{3}\Rightarrow$$

$$x\longrightarrow +\infty\Rightarrow t\longrightarrow 0$$

$$\lim_{x\to +\infty} f(x)=\lim_{t\to 0}(1+t)^{-\frac{1}{t}}=\lim_{t\to 0}\Big(\underbrace{(1+t)^{\frac{1}{t}}}\Big)^{-1}$$

$$= \qquad e^{-1}=\frac{1}{e}$$

$$\boxed{44}\ f(x)=\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}\ ;\ x\longrightarrow+\infty$$

$$\lim_{x\longrightarrow+\infty} f(x)=\frac{+\infty}{+\infty}=???$$

$$f(x)=\sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}}=\sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{x}}$$

$$x\longrightarrow+\infty \Rightarrow x>0 \Rightarrow x=|x|=\sqrt{x^2} \Rightarrow$$

$$f(x)=\sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{x}}$$

$$=\sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x^2}}}$$

$$=\sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}$$

$$=\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}} \Rightarrow$$

$$\lim_{x\longrightarrow+\infty} f(x)=\lim_{x\longrightarrow+\infty}\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}=1$$

45 $f(x)=x+\sqrt{|4x^2-1|}\ ;\begin{cases}x\longrightarrow+\infty\\x\longrightarrow-\infty\end{cases}$

- $\lim\limits_{x\to+\infty} f(x)=+\infty+\infty=+\infty$
- $\lim\limits_{x\to-\infty} f(x)=-\infty+\infty=???$

$$f(x)=x+\sqrt{\left|x^2\left(4-\frac{1}{x^2}\right)\right|}$$

$$=x+|x|\times\sqrt{\left|4-\frac{1}{x^2}\right|}$$ $x\longrightarrow-\infty\Rightarrow|x|=-x$

$$=x-x\times\sqrt{\left|4-\frac{1}{x^2}\right|}=x\left(1-\sqrt{\left|4-\frac{1}{x^2}\right|}\right)\Rightarrow$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty} x\left(1-\sqrt{\left|4-\frac{1}{x^2}\right|}\right)=-\infty(1-2)=+\infty$$

46 $f(x)=\sqrt{|4x^2-1|}-2x\ \ ;x\longrightarrow+\infty$

$\lim\limits_{x\to-\infty} f(x)=+\infty-\infty=???$

$$f(x)=\sqrt{|4x^2-1|}-2x$$

$$=\frac{(\sqrt{|4x^2-1|}-2x)(\sqrt{|4x^2-1|}+2x)}{\sqrt{|4x^2-1|}+2x}$$

$$=\frac{|4x^2-1|-4x^2}{\sqrt{4x^2-1}+2x}$$ $x\longrightarrow+\infty\Rightarrow|4x^2-1|=4x^2-1$

$$=\frac{4x^2-1-4x^2}{\sqrt{4x^2-1}+2x}=\frac{-1}{\sqrt{4x^2-1}+2x}\Rightarrow$$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{-1}{\sqrt{4x^2-1}+2x}=\frac{-1}{+\infty}=0$$

$$\boxed{47}\; f(x)=\frac{x.\cos(x^2+1)}{x^2+1}\;;x\longrightarrow+\infty$$

$$\lim_{x\longrightarrow+\infty} f(x)=\frac{\infty}{\infty}=??$$

$$-1\le\cos(x^2+1)\le+1 \qquad\Rightarrow$$

$$-x\le\cos(x^2+1)\le x \quad ,x>0\Rightarrow$$

$$\frac{-x}{x^2+1}\le\frac{\cos(x^2+1)}{x^2+1}\le\frac{x}{x^2+1} \qquad\Rightarrow$$

$$\underbrace{\lim_{x\longrightarrow+\infty}\frac{-x}{x^2+1}}_{0}\le\lim_{x\longrightarrow+\infty}\frac{\cos(x^2+1)}{x^2+1}\le\underbrace{\lim_{x\longrightarrow+\infty}\frac{x}{x^2+1}}_{0}$$

$$\Rightarrow\lim_{x\longrightarrow+\infty} f(x)=0$$

$$\boxed{48}\; f(x)=\frac{\sqrt{x+1}+E(x)}{x}\;;x\longrightarrow+\infty$$

$$x-1<E(x)\le x\Rightarrow$$

$$x-1+\sqrt{x+1}<\sqrt{x+1}+E(x)\le x+\sqrt{x+1}\quad ;x>0\Rightarrow$$

$$\frac{x-1+\sqrt{x+1}}{x}<\frac{\sqrt{x+1}+E(x)}{x}\le\frac{x+\sqrt{x+1}}{x}$$

$$1-\frac{1}{x}+\sqrt{\frac{x+1}{x^2}}<\frac{\sqrt{x+1}+E(x)}{x}\le1+\frac{1}{x}+\sqrt{\frac{x+1}{x^2}}\Rightarrow$$

$$\lim_{x\longrightarrow+\infty}\left(1-\frac{1}{x}+\sqrt{\frac{x+1}{x^2}}\right)=1$$

$$\lim_{x\longrightarrow+\infty}\left(1+\frac{1}{x}+\sqrt{\frac{x+1}{x^2}}\right)=1$$

$$\Rightarrow\lim_{x\longrightarrow+\infty} f(x)=1$$

$\boxed{49}\ f(x)=\dfrac{\sqrt{x+1}+E(x)}{x}\ ;x\longrightarrow 0^-$

$$\frac{\sqrt{x+1}+E(x)}{x}=\frac{\sqrt{x+1}-1}{x}\ ;E(x)=-1;x\in[-1,0[$$

$$\lim_{x\longrightarrow 0^-} f(x)=\lim_{x\longrightarrow 0^-}\frac{\sqrt{x+1}-1}{x}=\frac{0}{0}=??$$

$$\frac{\sqrt{x+1}-1}{x}=\frac{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}{x(\sqrt{x+1}-1)}=\frac{x}{x(\sqrt{x+1}+1)}$$

$$\Rightarrow \lim_{x\longrightarrow 0^-} f(x)=\lim_{x\longrightarrow 0^-}\frac{1}{(\sqrt{x+1}+1)}=\frac{1}{2}$$

$\boxed{50}\ f(x)=\dfrac{x.E(x)}{1-x^2}\ ;x\longrightarrow +\infty$

$$x-1<E(x)\le x\Rightarrow x^2-x<x\ .E(x)\le x^2$$

$$x\longrightarrow +\infty\Rightarrow 1-x^2<0\Rightarrow$$

$$\frac{x^2-x}{1-x^2}>\frac{x\ .E(x)}{1-x^2}\ge\frac{x^2}{1-x^2}\Rightarrow$$

$$\underbrace{\lim_{x\longrightarrow +\infty}\frac{x^2-x}{1-x^2}}_{-1}>\lim_{x\longrightarrow +\infty}\frac{x\ .E(x)}{1-x^2}\ge\underbrace{\lim_{x\longrightarrow +\infty}\frac{x^2}{1-x^2}}_{-1}$$

$$\Rightarrow \lim_{x\longrightarrow +\infty} f(x)=-1$$

تمت وبالخير عمت