# تكامل الدوال المثلثية

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sin kx \, dx = -\frac{\cos kx}{k} + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \cos kx \, dx = \frac{\sin kx}{k} + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \csc^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \csc x \cot x \, dx = -\csc x + c$$

① p. 25

ⓐ $\int (c\cos x + \csc^2 x)\, dx =$

$= \sin x - \cot x + c$

ⓑ $\int \sec x (\tan x + \sec x)\, dx =$

$= \int \sec x \tan x \, dx + \int \sec^2 x \, dx$

$= \sec x + \tan x + c$

ⓒ $\int \frac{dx}{\sin^2 x} = \int \csc^2 x \, dx$

$= -\cot x + c$

② P.25

(a) $\int \sin(5x)\,dx = \frac{-1}{5}\cos(5x) + C$

(b) $\int (x^2 + \cos 2x)\,dx = \frac{1}{3}x^3 + \frac{1}{2}\sin 2x + C$

(c) $\int x \sec^2(x^2+2)\,dx =$ $\quad u = x^2+2$

$= \frac{1}{2}\int 2x \sec^2(x^2+2)\,dx \quad du = 2x\,dx$

$= \frac{1}{2}\int \sec^2 u\,du$

$= \frac{1}{2}\tan u + C = \frac{1}{2}\tan(x^2+2) + C$

③ P.26

(a) $\int \sin^3 x \cos x\,dx \quad u = \sin x$

$= \int u^3\,du = \frac{1}{4}u^4 + C \quad du = \cos x\,dx$

$= \frac{1}{4}\sin^4 x + C$

(b) $\int \csc^2 x \cot x\,dx = -\int \cot x(-\csc^2 x)\,dx$

$u = \cot x$

$= \int u\,du = -\frac{1}{2}u^2 + C \quad du = -\csc^2 x\,dx$

$= -\frac{1}{2}(\cot x)^2 + C$

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④ P.27

(a) $\int \cos^3(2x-3)\cdot\sin(2x-3)\,dx$

$= -\frac{1}{2}\int (\cos^3(2x-3))[-2\sin(2x-3)]\,dx$ $\quad u = \cos(2x-3)$

$du = -\sin(2x-3)\,(2)\,dx$

$= -\frac{1}{2}\int u^3\,du = -\frac{1}{2}\cdot\frac{1}{4}u^4 + C$

$= -\frac{1}{8}(\cos(2x-3))^4 + C$

(b) $\int x^2 \sin(x^3-1)\,dx$ $\quad u = x^3 - 1$

$du = 3x^2\,dx$

$= \frac{1}{3}\int 3x^2 \sin(x^3-1)\,dx$

$= \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + C$

$= -\frac{1}{3}\cos(x^3-1) + C$

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(c) $\int (3+\sin 2x)^3 \cos(2x)\,dx$ $\quad u = 3+\sin 2x$

$du = 2\cos 2x\,dx$

$= \frac{1}{2}\int (3+\sin 2x)^3 (2\cos 2x)\,dx$

$= \frac{1}{2}\int u^3\,du = \frac{1}{2}\cdot\frac{1}{4}u^4 + C$

$= \frac{1}{8}(3+\sin 2x)^4 + C$

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$\int \csc^5 x \cot x\,dx$ P. 28

$= -\int \csc^4 x\,(-\csc x \cot x)\,dx$

$u = \csc x$

$= -\int u^4\,du$ $\quad du = -\csc x \cdot \cot x\,dx$

$= -\frac{1}{5}u^5 + C$

$= -\frac{1}{5}\csc^5 x + C$

# الدوال الأسية واللوغاريتمية

## الإشتقاق

$$\frac{d}{dx}a^x = a^x \ln a \quad ، \quad \frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}e^x = e^x \quad ، \quad \frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad ، \quad \frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$$

$$\int e^x dx = e^x + c \quad ; \quad \frac{d}{dx}e^x = e^x$$

$$\int u' e^u dx = e^u + c \quad ; \quad \frac{d}{dx}e^u = e^u \frac{du}{dx} = u' e^u$$

$$\int \frac{1}{x} dx = \ln|x| + c \quad ; \quad \frac{d}{dx}\ln|x| = \frac{1}{x}$$

$$\int \frac{u'}{u} dx = \ln|u| + c \quad ; \quad \frac{d}{dx}\ln|u| = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}$$

$$\int \frac{g'(x)}{g(x)} dx = \ln|g(x)| + c$$

P. 30 ① اوجد مشتقة كل من الدوال التالية:

(a) $f(x) = 10^x$

$f'(x) = (10)^x \ln 10$

(b) $f(x) = 3^{\frac{1}{x}} \rightarrow f'(x) = 3^{\frac{1}{x}} \ln 3 \frac{d}{dx}(\frac{1}{x})$

$= -3^{\frac{1}{x}} \ln 3 \frac{1}{x^2}$

(c) $f(x) = 5^{\cos x}$

$f'(x) = 5^{\cos x} \ln 5 \frac{d}{dx} \cos x$

$= -5^{\cos x} \ln 5 \sin x$

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P. 31 ② اوجد مشتقة كل من الدوال التالية:

(a) $f(x) = e^{\sqrt{x}}$

$f'(x) = e^{\sqrt{x}} \frac{1}{2\sqrt{x}}$

(b) $f(x) = e^{x^2-4}$

$f'(x) = e^{x^2-4} \frac{d}{dx}(x^2-4)$

$= e^{x^2-4}(2x)$

(c) $f(x) = e^{\tan x} \rightarrow f'(x) = e^{\tan x} \frac{d}{dx}(\tan x)$

$= e^{\tan x} \sec^2 x$

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P. 32 ③ اوجد مشتقة كل من الدوال التالية

(a) $f(x) = \ln(2x + x^3)$

$f'(x) = \frac{1}{2x+x^3} \cdot \frac{d}{dx}(2x+x^3)$

$= \frac{2+3x^2}{2x+x^3}$

(b) $g(x) = \ln\left(\frac{1}{2x+1}\right)$

$$g'(x) = \frac{1}{\frac{1}{2x+1}} \cdot \frac{d}{dx}\frac{1}{2x+1}$$

$$= (2x+1) \cdot \frac{-2}{(2x+1)^2} = \frac{-2}{2x+1}$$

(c) $h(x) = \ln(1+\sqrt{3}x)$

$$h'(x) = \frac{1}{1+\sqrt{3}x} \cdot \frac{d}{dx}(1+\sqrt{3}x)$$

$$= \frac{1}{1+\sqrt{3}x} \cdot (\sqrt{3}) = \frac{\sqrt{3}}{1+\sqrt{3}x}$$

(d) $h(x) = \ln(\sin x)$

$$h'(x) = \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) = \frac{\cos x}{\sin x} = \cot x$$

(a) $\int e^{3x}\,dx = \frac{1}{3}\int 3e^{3x}\,dx$ : راجع ④ P. 33

$u = 3x$

$du = 3\,dx$

$$= \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + c$$

$$= \frac{1}{3}e^{3x} + c$$

(b) $\int (2x-1)\, e^{x^2-x+3}\,dx$

$u = x^2 - x + 3$

$du = (2x-1)\,dx$

$$= \int e^u\,du = e^u + c$$

$$= e^{x^2-x+3} + c$$

⑤ P. 34 أوجد

(a) $\int \frac{-5}{3x-2}\,dx$

$= \frac{-5}{3}\int \frac{3}{3x-2}\,dx$ $\quad u = 3x-2$, $du = 3\,dx$

$= \frac{-5}{3}\int \frac{du}{u} = \frac{-5}{3}\ln|u| + c$

$= \frac{-5}{3}\ln|3x-2| + c$

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(b) $\int \frac{3t^2-6t}{t^3-3t^2+8}\,dt$ $\quad u = t^3-3t^2+8$, $du = (3t^2-6t)\,dt$

$= \int \frac{du}{u} = \ln|u| + c$

$= \ln|t^3-3t^2+8| + c$

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(c) $\int \frac{x^3+4}{x}\,dx = \int \frac{x^3}{x}\,dx + \int \frac{4}{x}\,dx$

$= \int x^2\,dx + 4\int \frac{1}{x}\,dx = \frac{1}{3}x^3 + 4\ln|x| + c$

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⑥ P. 35 أوجد

$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$

$= \int \frac{du}{u} = \ln|u| + c$ $\quad u = \sin x$, $du = \cos x\,dx$

$= \ln|\sin x| + c$