King Saud University
Department of Mathematics

244
Final Exam, May 2014

NAME:

Group Number:

ID:

Question	Grade
I	
II	
III	
IV	
V	
Total	

Question	1	2	3	4	5	6
Answer						

I) Choose the correct answer:

1) If A is a 3×3 matrix, such that $\det(2A)^2 = 64$, then $\det A$ equals

(A) ± 8	(B) ± 4	(C) ± 1	(D) None of the previous

2) If u and v are orthogonal vectors of $\mathbb{R}^n$, such that $\|u\| = 3$ and $\|v\| = 4$, then the distance $d(u, v)$ between u and v is

(A) 1	(B) 5	(C) 7	(D) None of the previous

3) If A is a 3×6 matrix, such that $rank(A) = 3$, then $nullity(A)$ is

(A) 2	(B) 6	(C) 3	(D) None of the previous

4) If $T_A : \mathbb{R}^4 \to \mathbb{R}^3$ and $T_B : \mathbb{R}^4 \to \mathbb{R}^3$ are matrix transformations, such that $T_A(x) = T_B(x)$ for all $x \in \mathbb{R}^4$, then

(A) $\det(A) \neq \det(B)$	(B) $A = B$	(C) T_A and T_B are one-to-one	(D) None

5) The values of k that make the linear system $\begin{cases} 9x + ky = a \\ kx + y = b \end{cases}$ consistent, for all $a, b \in \mathbb{R}$ are

(A) $\{-3, 3\}$	(B) $\mathbb{R} \setminus \{-3, 3\}$	(C) $\{-3, 0, 3\}$	(D) None of the previous

6) If $S = \{v_1, v_2, v_3, v_4\}$ is a linear independent set of vectors in $\mathcal{P}_4[x]$, then

(A) S is a basis of $\mathcal{P}_4[x]$	(B) $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors	(C) $\{v_1, v_2, v_3\}$ is a linearly dependent set of vectors	(D) None of the previous

II) A) For the linear system of equations

$$\begin{cases} x - y + z &= 1 \\ -x + 2y + z &= 0 \\ -x + y + 2z &= 0 \end{cases}$$

find the unknown x, **using Cramer's rule**.

B) If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, write the matrix $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ as a linear combination of A and A^2.

C) If u, v, w are pairwise orthogonal vectors in $\mathbb{R}^n$, prove that

$$(u + 3v + w) \cdot (2u - 6v) = 2\|u\|^2 - 18\|v\|^2.$$

D) If $\{u, v\}$ is a linearly independent set of vectors, prove that $\{u - v, u + v\}$ is a linearly independent set of vectors.

III) A) i) Give the definition of a *symmetric matrix*;

ii) If B is a 3×3 symmetric matrix with $\det(B) = -2$, find $\det[2B^2B^T + (B^T)^2B]$.

B) Prove that

$$W = \{A \in \mathcal{M}_{22}, \quad A = A^T\}$$

is a subspace of $\mathcal{M}_{22}$.

IV) A) Let

$$A = \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 1 & -1 & 3 & 0 & 5 \\ 2 & -2 & 6 & 1 & 9 \end{bmatrix}.$$

i) Find a basis for the null space of A;

ii) Find a basis for the row space of A;

iii) Find a basis for the column space of A;

iv) Find the rank and the nullity of A.

B) If $B = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$, find the eigenvalues of B.

C) If $M = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$, find the eigenspace of M that corresponds to the eigenvalue $\lambda = 1$.

V) A) Suppose that $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ is a matrix transformation, with standard matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

a) Show that T_A is one-to-one;

b) Find the inverse matrix $[T_A^{-1}]$;

c) Evaluate $T_A^{-1}(1, 2, 3)$.

B) Find the standard 2×2 matrix that first rotates a vector counterclockwise about the origin, through an angle $\theta = \dfrac{\pi}{4}$, then reflects the resulting vector about the y-axis and then projects that vector orthogonally on the x-axis.