مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

تمارين على تفاضل الدوال البارامترية

$y = t^2 + 1 \qquad x = 3t^2 + t$

$\frac{dy}{dt} = 2t \qquad \frac{dx}{dt} = 6t + 1$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{2t}{6t+1}$

$y = 3t^3 + t \qquad x = t^2 + 1$

$\frac{dy}{dt} = 9t^2 + 1 \qquad \frac{dx}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{9t^2+1}{2t}$

$y = t^2 + 1 \qquad x = e^{2t}$

$\frac{dy}{dt} = 2t \qquad \frac{dx}{dt} = 2e^{2t}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{2t}{2e^{2t}} = \frac{t}{e^{2t}}$

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(2)

$y = e^{2t}$ $\quad X = t^{2+1}$

$\frac{dy}{dt} = 2e^{2t}$ $\quad \frac{dx}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{2e^{2t}}{2t} = \frac{e^{2t}}{t}$

$y = \sqrt{t^2+1}$ $\quad X = \frac{1}{t}$

$\frac{dy}{dt} = \frac{2t}{\sqrt{t^2+1}}$ $\quad \frac{dx}{dt} = -t^{-2} = \frac{-1}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{2t}{\sqrt{t^2+1}} \cdot -t^2$

$= \frac{-2t^3}{\sqrt{t^2+1}}$

$y = \frac{1}{t}$ $\quad X = \sqrt{t^2+1}$

$\frac{dy}{dt} = \frac{-1}{t^2}$ $\quad \frac{dx}{dt} = \frac{2t}{\sqrt{t^2+1}}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{-1}{t^2} \cdot \frac{\sqrt{t^2+1}}{2t}$

$= \frac{-\sqrt{t^2+1}}{2t^3}$

$y = \ln(t+1) \qquad x = t^3$

$\frac{dy}{dt} = \frac{1}{t+1} \qquad \frac{dx}{dt} = 3t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{1}{t+1} \cdot \frac{1}{3t^2}$

$= \frac{1}{3t^2(t+1)}$

$y = t^3 \qquad x = \ln(t+1)$

$\frac{dy}{dt} = 3t^2 \qquad \frac{dx}{dt} = \frac{1}{t+1}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 3t^2(t+1)$

$y = \sin t \qquad x = \cos t$

$\frac{dy}{dt} = \cos t \qquad \frac{dx}{dt} = -\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \cos t \cdot \frac{1}{-\sin t} = -\cot t$

$y = \cos t \qquad x = \sin t$

$\frac{dy}{dt} = -\sin t \qquad \frac{dx}{dt} = \cos t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = -\sin t \cdot \frac{1}{\cos t}$

$= -\tan t$

(4)

$y = \tan t \qquad x = \sec t$

$\frac{dy}{dt} = \sec^2 t \qquad \frac{dx}{dt} = \sec t \tan t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \sec^2 t \cdot \frac{1}{\sec t \tan t}$

$= \frac{\sec t}{\tan t}$

$= \frac{1}{\cos t} \cdot \frac{\cos t}{\sin t} = \csc t$

~~y = csc~~ $y = \sec t$ ~~dy~~ $x = \tan t$

$\frac{dy}{dt} = \sec t \tan t \qquad \frac{dx}{dt} = \sec^2 t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \sec t \tan t \cdot \frac{1}{\sec^2 t} = \frac{\tan t}{\sec t}$

$\frac{\sin t}{\cos t} \cdot \sin t = \frac{\sin^2 t}{\cos t}$

$y = \csc t$ $\quad x = \cot t$

$\frac{dy}{dt} = -\csc t \cot t$ $\quad \frac{dx}{dt} = -\csc^2 t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= +\csc t \cot t \cdot \frac{1}{+\csc^2 t} = \frac{\cot t}{\csc t}$

$y = \cot t$ $\quad x = \csc t$

$\frac{dy}{dt} = -\csc^2 x$ $\quad \frac{dx}{dt} = -\csc t \cot t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = -\csc^2 x \cdot \frac{1}{-\csc t \cot t}$

$= \frac{\csc x}{\cot x}$

$y = \cosh t$ $\quad x = \sinh t$

$\frac{dy}{dt} = \sinh t$ $\quad \frac{dx}{dt} = \cosh t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \frac{\sinh t}{\cosh t} = \tanh t$

$y = \sinh t \qquad x = \cosh t$

$\frac{dy}{dt} = \cosh t \qquad \frac{dx}{dt} = \sinh t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\cosh t}{\sinh t} = \coth t$

$y = \operatorname{sech} t \qquad x = \tanh t$

$\frac{dy}{dt} = -\operatorname{sech} t \tanh t \qquad \frac{dx}{dt} = \operatorname{sech}^2 t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \frac{-\operatorname{sech} t \tanh t}{\operatorname{sech}^2 t} = -\frac{\tanh t}{\operatorname{sech} t}$

$y = \tanh t \qquad x = \operatorname{sech} t$

$\frac{dy}{dt} = \operatorname{sech}^2 t \qquad \frac{dx}{dt} = -\operatorname{sech} t \tanh t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \operatorname{sech}^2 t \cdot \frac{1}{-\operatorname{sech} t \tanh t}$

$= \frac{-\operatorname{sech} t}{\tanh t}$

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$y = \operatorname{csch} t$ $\quad$ $x = \coth t$

$\frac{dy}{dt} = -\operatorname{csch} t \coth t$ $\quad$ $\frac{dx}{dt} = -\operatorname{csch}^2 t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\frac{-\operatorname{csch} t \coth t}{-\operatorname{csch}^2 t} = \frac{\coth t}{\operatorname{csch} t}$

$y = \coth t$ $\quad$ $x = \operatorname{csch} t$

$\frac{dy}{dt} = -\operatorname{csch}^2 t$ $\quad$ $\frac{dx}{dt} = -\operatorname{csch} t \coth t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \frac{-\operatorname{csch}^2 t}{-\operatorname{csch} t \coth t} = \frac{\operatorname{csch} t}{\coth t}$

تفاضل الدوال اللوغارتمية

①

$$y = \log_2 (x^2 - 2)$$

$$\frac{dy}{dx} = \frac{2x}{(x^2-2)\ln 2}$$

$$y = \log_4 (3x - 2)$$

$$\frac{dy}{dx} = \frac{3}{(3x-2)\ln 4}$$

$$y = \log_2 (2 - 3x)$$

$$\frac{dy}{dx} = \frac{-3}{(2-3x)\ln 2}$$

$$f(x) = \ln\left(\frac{x^2+1}{x^3+3}\right)$$

$$= \ln(x^2+1) - \ln(x^3+3)$$

$$f'(x) = \frac{2x}{x^2+1} - \frac{3x^2}{x^3+3}$$

$$f(x) = \log_2 (4x - 2)$$

$$f'(x) = \frac{4}{(4x-2)\ln 2}$$

(2)

$y = \ln \sqrt{x^2+1}$

$y = \ln (x^2+1)^{\frac{1}{2}}$

$= \frac{1}{2} \ln (x^2+1)$

$= \frac{1}{2} \frac{2x}{x^2+1} = \frac{x}{x^2+1}$

$y = \log_4 x^2$

$\frac{dy}{dx} = \frac{2x}{x^2 \ln 4}$

$f(x) = \ln \sqrt{\frac{x^2+1}{x^2+1}} = \ln \left(\frac{x^2+1}{x^2+1}\right)^{\frac{1}{2}}$

$\frac{1}{2} [\ln (x+1) - \ln (x^2+1)$

$\frac{1}{2} \frac{1}{x+1} - \frac{1}{2} \frac{2x}{x^2+1}$

$\frac{1}{2(x+1)} - \frac{x}{x^2+1}$

$y = \ln \sqrt[3]{x^3+3x^2-6} = \ln (x^3+3x^2-6)^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3} \frac{3x^2+6x}{x^3+3x^2-6}$

$= \frac{x^2+2x}{x^3+3x^2-6}$

(3)

$$y = \log_4 \{3 - 5x\}$$

$$\frac{dy}{dx} = \frac{-5}{(3-5x)\ln 4}$$

$$y = \ln \sqrt[4]{x^4 + 4x^3}$$

$$\frac{1}{4} \ln (x^4 + 4x^3)$$

$$f'(x) = \frac{1}{4} \frac{4x^3 + 12x^2}{x^4 + 4x^3} = \frac{x^3 + 3x^2}{x^4 + 4x^3}$$

$$y = \ln \left(\frac{7x+6}{x^2+2} \right)$$

$$y = \ln (7x+6) - \ln (x^2+2)$$

$$= \frac{7}{7x+6} - \frac{2x}{x^2+2}$$

$$y = \log_3 \sqrt{x^2+2x}$$

$$= \log_3 (x^2+2x)^{\frac{1}{2}}$$

$$= \frac{1}{2} \frac{2x+2}{(x^2+2x)\ln 3} = \frac{x+1}{(x^2+2x)\ln 3}$$

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(4)

$y = \ln \sqrt[4]{2x^2+1}$

$y = \frac{1}{4} \ln(2x^2+1)$

$\frac{1}{4} \frac{4x}{2x^2+1} = \frac{x}{2x^2+1}$

$y = \log_3 (10x^2+4)$

$y' = \frac{20x}{(10x^2+4)(\ln 3)}$

$y = \log_5 \{x^2+2x\}$

$\frac{dy}{dy} = \frac{2x+2}{(x^2+2x)\ln 5}$

$y = \ln\left(\frac{x+1}{x^2+1}\right)$

$y = \ln(x+1) - \ln(x^2+1)$

$\frac{1}{x+1} - \frac{2x}{x^2+1}$

$y = \log_4 \sqrt[3]{x+5}$

$y = \frac{1}{3} \log_4 (x+5) = \frac{1}{3} \frac{1}{(x+5)\ln 4}$

(5)

$$y = \ln\left(\frac{x}{x+1}\right)$$

$$= \ln x - \ln(x+1)$$

$$\frac{1}{x} - \frac{1}{x+1}$$

$$y = \log\{x^2 + x^4\}$$

$$\frac{dy}{dx} = \frac{2x + 4x^3}{x^2 + x^4}$$

$$y = \log_4(x^2 - x)$$

$$y' = \frac{2x-1}{(x^2-x)\ln 4}$$

$$y = \ln(x^2+1)(x^4+1)$$

$$\ln(x^2+1) + \ln(x^4+1)$$

$$\frac{2x}{x^2+1} + \frac{4x^3}{x^4+1}$$

$$y = \log_3(4x+2)$$

$$\frac{4}{(4x+2)\ln 3}$$