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KING SAUD UNIVERSITY
College of Science/Department of Mathematics

(Math 151) Discrete Mathematics
Summer Semester 1430/1431

Score

Final Exam

Date: 16/09/1431 Time: 09:00 A.M – 12:00 A.M Time allowed: 3 Hours

STUDENT NAME(IN ENGLISH)	
Registration Number	
Lecture Time	

- There are 14 multiple choice questions in part A and 7 questions in part B. The maximum score is 60 marks.
- Please do not forget to put your name and registration number on your paper.

Put your answers in the following table.

QUESTION	1	2	3	4	5	6	7
ANSWER	C	A	C	C	D	D	D
Question	8	9	10	11	12	13	14
Answer	C	A	B	C	D	B	D

PART - A

$2 \times 14 = 28$

Q1.The proposition $((p \to q) \to q) \to (\neg(p \to q) \vee q)$ is

A. logically equivalent to $(p \wedge \neg q) \leftrightarrow (\neg p \to q)$
B. Contradiction
C. Tautology
D. None of above

Q2. The proposition $(p \to q) \to (\neg p \to r)$ is logically equivalent to

A. $p \vee r$ B. $p \to (r \vee q)$
C. $p \wedge r$ D. $(p \wedge q) \to r$

Q3. The consistent proposition is:

A. $\{p \to (q \wedge \neg r), p \wedge \neg q\}$ B. $\{p \to q, r \to \neg q, \neg r \to s, p \wedge \neg s\}$
C. $\{p \wedge q \to r, p \wedge \neg r\}$ D. None of above

Q4. The relation T defined on $\mathbb{R}$ by $aTb \Leftrightarrow |a-b| \le 5$ is

A. Reflexive and transitive B. Symmetric and antisymmetric

C. Symmetric but not transitive D. not reflexive and not transitive

Q5. If R is an equivalence relation on $A = \mathbb{Z} \times \mathbb{Z}$ defined as
$(a,b) R (c,d) \Leftrightarrow a-c$ is even and $b+d$ is even, then $\left| A/R \right|$ is:

A. 2 B. ∞ C. 6 D. 4

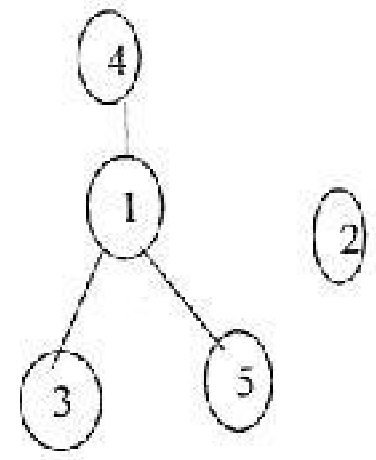
Q6. Let R, S be two relations defined on $\mathbb{Z}^+ = \{1,2,3,...\}$ by
$aRb \Leftrightarrow ab$ is a perfect square and $aSb \Leftrightarrow a$ divides b. Then

A. S is an equivalence relation and R is a partial ordering relation.
B. S and R are both equivalence relation.
C. S and R are both partial ordering relation.
D. S is a partial ordering relation and R is an equivalence relation.

Q7. The transitive closure of the relation $R = \{(a,c),(b,b),(c,b)\}$ defined on $A = \{a,b,c\}$ is

A. $\{(a,c),(b,b),(c,a)\}$ B. $A \times A$
C. $\{(a,b),(b,c),(c,c)\}$ D. $\{(a,b),(a,c),(b,b),(c,b)\}$

Q8. If S is a partial ordering on the set A={1,2,3,4,5} and has for Hasse diagram, the following diagram:



Then,

A. S={(1,1),(2,2),(3,3),(4,4),(5,5)}
B. S={(3,1),(5,1),(1,4)}
C. S={(1,1),(2,2),(3,3),(4,4),(5,5),(3,1),(1,4),(5,1),(3,4),(5,4)}
D. S={(1,1),(2,2),(3,3),(4,4),(5,5),(3,1),(1,4),(5,1)}

Q9. If B is a Boolean algebra and $x, y \in B$ such $xy = y$, then

A. $x + y = x$ B. $x + y = y$
C. $x = 1$ D. None of above

Q10. If $f(x,y,z,w) = (y' + z)w' + xy'z' + x'y'z$ then $CSP(f)$ is equal to

A. $xy'z'w + xy'z'w' + x'y'zw + x'y'zw' + xy'zw' + x'y'z'w'$
B. $xy'z'w + xy'z'w' + x'y'zw + x'y'zw' + xy'zw' + x'y'z'w' + xyzw' + x'yzw'$
C. $xy'z'w + xy'z'w' + x'y'zw + x'y'zw' + xy'zw' + x'y'z'w' + xyzw'$
D. $xy'z'w + xy'z'w' + x'y'zw + x'y'zw' + xy'zw' + x'y'z'w' + x'yz'w' + x'yz'w$

Q11. The MSP form of the Boolean function f in **Q10** is

A. $zw' + y'w' + x'y'z$ B. $zw' + y'w' + xy'z'$
C. $zw' + y'w' + x'y'z + xy'z'$ D. $zw' + y'w'$

Q12. The MPS form of the Boolean function f in **Q10** is

A. $(x'+z'+w')(x+z+w')$ B. $(x'+z'+w')(x+z+w')(y'+z)$
C. $(x'+z'+w')(x+z+w')(y'+w')$ D. $(x'+z'+w')(x+z+w')(y'+z)(y'+w')$

Q13. If G is a simple graph with n vertices and 56 edges and $\overline{G}$ has 80 edges then n is equal to

A. 16 B. 17 C. 18 D. None of above

Q14. If a connected planar simple graph G has 11 vertices with degrees 2,2,2,2,3,3,3,3,4,4,6 then the number of regions in a planar representation is :

A. 25 B. 22
C. 6 D. 8

PART-B

3 Marks

Q1. Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge (\neg(\neg p) \vee \neg q) \quad ①$$
$$\equiv \neg p \wedge (p \vee \neg q)$$
$$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \quad ①$$
$$\equiv F \vee (\neg p \wedge \neg q)$$
$$① \quad \equiv \neg p \wedge \neg q$$

4 Marks

Q2. Show that $(3n+2)$ is odd number if and only if $(9n+5)$ is even number for an integer number n.

$(3n+2)$ is odd $\Leftrightarrow$ $(9n+5)$ is even

"$\Rightarrow$" We suppose that $(3n+2)$ is odd.
Then there exists an integer $k \in \mathbb{N}$ such $3n+2 = 2k+1$,

$9n+6 = 6k+3$

② $9n+5 = 6k+2 = 2(3k+1) = 2m$

$(m = 3k+1 \in \mathbb{N})$

So $(9n+5)$ is even.

"$\Leftarrow$" We suppose that $(9n+5)$ is even. Then there exists $\ell \in \mathbb{N}$ such $9n+5 = 2\ell$

$3n+2 = 2\ell - 6n - 3 = 2(\ell - 3n - 2) + 1$

② Now we put $j = \ell - 3n - 2 \in \mathbb{Z}$, We get
$3n+2 = 2j+1$; That means $(3n+2)$ is odd.

4 Marks

Q3. If sequence $\{a_n\}_{n=1}^{\infty}$ is defined as follows:

$a_1 = 1, a_2 = 5$

$a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$,

By using mathematical induction, show that $a_n < 2\times 3^{n-1}$ for all integers $n \geq 1$.

we put P_n : " $a_n < 2\times 3^{n-1}$ $\forall n \geq 1$ "

Basis step: $n=1$ $\qquad$ $n=2$ $\qquad$ (1)

$a_1 = 1 < 2\times 3^0$ $\qquad$ $a_2 = 5 < 2\times 3$

P_1 is true $\qquad$ P_2 is true

Inductive step: Let $k \geq 1$. We suppose that $P_1, P_2, \ldots, P_k$ are true and we will show that P_{k+1} remains true.

(1) As $a_{k+1} = 2a_k + 3a_{k-1}$ and

(1) $a_k < 2\times 3^{k-1}$ (P_k is true)

$a_{k-1} < 2\times 3^{k-2}$ (P_{k-1} is true)

(1) then $a_{k+1} < 2^2 \times 3^{k-1} + 3\times 2\times 3^{k-2}$, So $a_{k+1} < 2\times 3^k$

$< 2\times 3^{k-1}[2+1]$ $\qquad$ That means P_{k+1} is true.

Conclusion $\forall n \geq 1, a_n < 2\times 3^{n-1}$

6 Marks

Q4. Let S be the relation defined on the set $\mathbb{Z}-\{0\}$ of non zero integers numbers as follows

$xSy \Leftrightarrow \exists k \in \{0,1,2,3,\ldots\}$ such $x = y^{2k+1}$.

i) Show that S is a partial ordering relation on $\mathbb{Z}-\{0\}$.

(1) S is a partial ordering relation if S is reflexive, antisymmetric and transitive.

(1) . S is reflexive : If we take $k=0$, $x = x^{2\times 0+1}$ then xSx.

. S is antisymmetric : we suppose that xSy and ySx, we will show that $x=y$.

As xSy then $\exists k \in \mathbb{N}$ / $x = y^{2k+1}$ (1)

As ySx then $\exists m \in \mathbb{N}$ / $y = x^{2m+1}$ (2)

By substitution (1) into (2), we get $y = (y^{2k+1})^{(2m+1)} = y^{4km+2k+2m+1}$

(1) $\Leftrightarrow y\left(1 - y^{2(2km+k+m)}\right) = 0$

As $y \neq 0$ ($y \in \mathbb{Z}-\{0\}$) then $2km+k+m=0$. As $k, m \in \mathbb{N}$, we get $k=m=0$. We deduce that $x=y$.

• S is transitive: We suppose $x S y$ and $y S z$, we will show that $x S z$.
As $x S y$ then $\exists k \in \mathbb{N}$ such $x = y^{2k+1}$. Also $y S z$ then $\exists m \in \mathbb{N}$
1 such $y = z^{2m+1}$. By substitution, we get $x = z^{2(2km+m+k)+1}$
We take now $l = 2km+m+k \in \mathbb{N}$, we have $x = z^{2l+1} \Leftrightarrow x S z$.

ii) Is it S a totally ordering relation on $\mathbb{Z}-\{0\}$ or not?

We take for example $x=2$ and $y=3$.
2 As $2 \not S 3$ and $3 \not S 2$ then 2 and 3 are not comparable
Therefore S is not a totally ordering relation on $\mathbb{Z}-\{0\}$.

8 Marks

Q5. Construct the least and & or logic network for
$f(x,y,z,w) = xyz'w' + xyz'w + xy'zw' + xy'z'w' + x'y'z'w' + x'y'z'w + x'yz'w' + x'yz'w$.

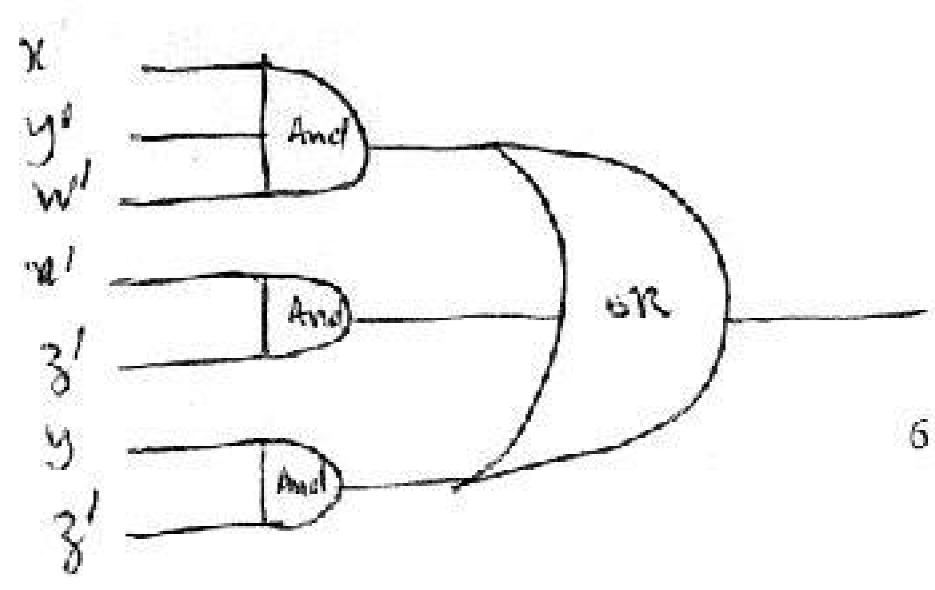
Karnaugh map

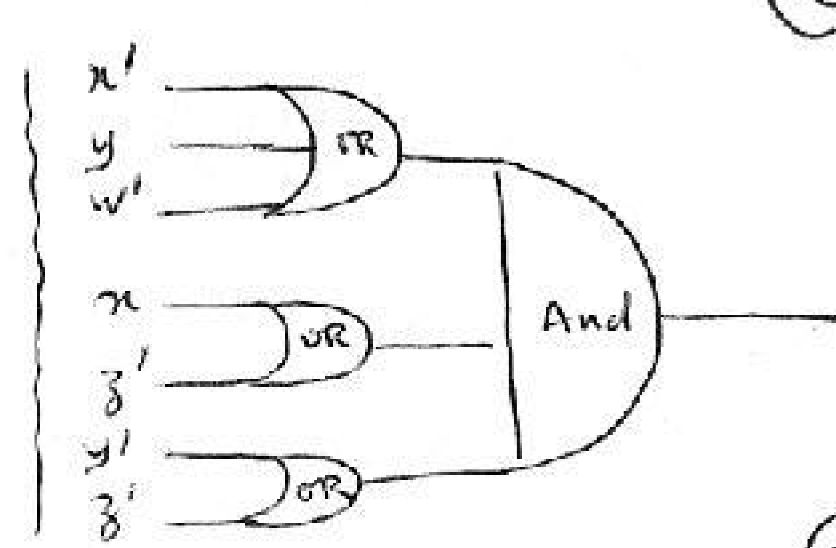
	zw	zw'	$z'w'$	$z'w$
xy	0	0	1	1
xy'	0	1	1	0
$x'y'$	0	0	1	1
$x'y$	0	0	1	1

2

$$MSP(f) = yz' + x'z' + xy'w'$$ 2

As $MSP(f') = xy'w + x'z + yz$
then we deduce $MPS(f) = (x'+y+w')(x+z')(y'+z')$
2



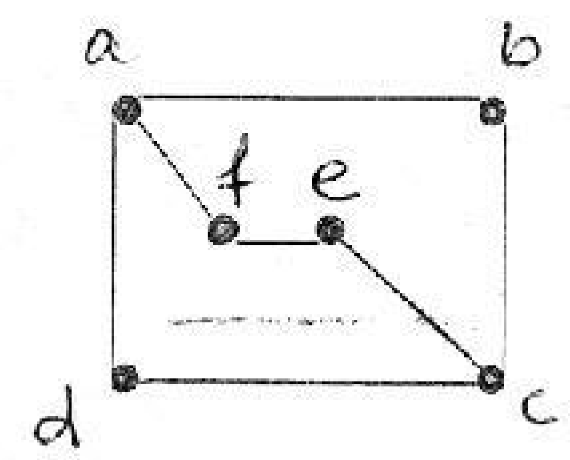


1 1

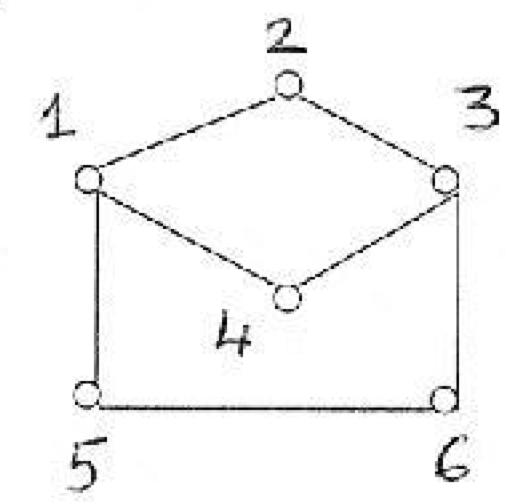
4 gates 4 gates
The two cases are the least and & or logic network for f

4 Marks

Q6. Show whether the graphs G and H are isomorphic or not?



G



H

(2)

x	a	b	c	d	e	f
$f(x)$	1	2	3	4	6	5

As f is bijective then $G \simeq H$ (G and H are isomorphic)

(2)

3 Marks

Q7. If G is a simple graph with n vertices, show that $|E|+|\bar{E}|=\frac{n(n-1)}{2}$ such $\bar{E}$ is the set of edges for the complementary graph $\bar{G}$.

$G = (V, E)$ a simple graph with $|V| = n$

$\bar{G} = (\bar{V}, \bar{E})$ its complementary.

(2) So $G \cup \bar{G} = (V, E \cup \bar{E})$ is $(n-1)$-regular graph.

(1) Then $|E \cup \bar{E}| = |E| + |\bar{E}| = \frac{n(n-1)}{2}$.