

-
- I
- II
- III
- V

:

-1

□

f

β, α

$f(x) = 80 + \alpha e^{\beta x}$

f (C)

$(0; \vec{i}, \vec{j})$

$B(3; 60) \quad A(0; 53)$

$\beta \quad \alpha$

(C)

10^{-1}

-2

$(n) n$

$u_n = 80 - 27.e^{-0,1n}$:

-

-

-3

72

$n \quad V_n = e^{-0,1n}$:

(V_n)

-

(V_n)

$\lim_{n \rightarrow +\infty} V_n$

-

-

$S = V_1 + V_2 + \dots + V_{10}$:

10

:

-1

$\beta \quad \alpha$

$f(0) = 53$: $A \in (C)$

$f(3) = 60$: $B \in (C)$

:

$$\begin{cases} \alpha = -27 \\ 80 - 27.e^{3\beta} = 60 \end{cases} \quad : \quad \begin{cases} 80 + \alpha = 53 \\ 80 + \alpha e^{3\beta} = 60 \end{cases} :$$

:

$-27.e^{3\beta} = -20$:

<http://www.onefd.edu.dz> $e^{3\beta} = \frac{20}{27}$:

$$3\beta = n \left(\frac{20}{27} \right) : \quad \ln e^{3\beta} = \ln \left(\frac{20}{27} \right) :$$

$$\beta = \frac{1}{3} \ln \left(\frac{20}{27} \right) \quad \alpha = -27 : \quad \beta = \frac{1}{3} \ln \left(\frac{20}{27} \right) :$$

$$:$$

$$\beta \square -0,1 \quad \alpha = -27$$

$$: \quad (u_n) \quad (-2)$$

$$u_{n+1} - u_n = 80 - 27.e^{0,1(n+1)} - (80 - 27.e^{-0,1n})$$

$$u_{n+1} - u_n = -27.e^{-0,1(n+1)} + 27.e^{-0,1n}$$

$$u_{n+1} - u_n = -27e^{-0,1n} \times e^{-0,1} + 27.e^{-0,1n}$$

$$u_{n+1} - u_n = -27.e^{-0,1n} (e^{-0,1} - 1)$$

$$-27.e^{-0,1n} < 0 \quad e^{-0,1} - 1 < 0 :$$

$$. n \quad u_{n+1} - u_n > 0 :$$

$$. \square \quad (u_n)$$

: 72

$$u_n > 72 :$$

$$80 - 27.e^{-0,1n} > 72 :$$

$$e^{-0,1} < \frac{8}{27} : \quad -27.e^{-0,1} > -8 :$$

$$-0,1n < \ln \left(\frac{8}{27} \right) : \quad \ln e^{-0,1n} < \ln \left(\frac{8}{27} \right) :$$

$$. n \geq 13 : \quad n > 12,16 : \quad n > \frac{-\ln \frac{8}{27}}{0,1} :$$

. 72

$$: \quad (V_n) \quad (-3)$$

$$V_{n+1} = V_n \cdot e^{-0,1} :$$

$$q = e^{-0,1} \quad (V_n) :$$

$$\lim_{x \rightarrow \infty} V_n = \lim_{x \rightarrow \infty} e^{-0,1n} = 0 :$$

: S (

$$S = V_1 + V_2 + \dots + V_{10}$$

$$V_1 = e^{-0,1} : \quad S = V_1 \times \frac{1 - q^{10}}{1 - q} :$$

$$S = e^{-0,1} \times \frac{1 - (e^{-0,1})^{10}}{1 - e^{-0,1}} = e^{-0,1} \times \frac{1 - e^{-1}}{1 - e^{-0,1}} :$$

: 10 (

$$S' = u_1 + u_2 + \dots + u_{10} :$$

$$u_n = 80 - 27 \cdot e^{-0,1n} :$$

$$u_n = 80 - 27 \cdot V_n :$$

:

$$S' = (80 - 27V_1) + (80 - 27V_2) + \dots + (80 - 27V_{10})$$

$$S' = 80 + 80 + \dots + 80 - 27(V_1 + V_2 + \dots + V_{10})$$

$$S' = 10 \times 80 - 27 \times S :$$

$$S' = 800 - 27 \times e^{-0,1} \times \frac{1 - e^{-1}}{1 - e^{-0,1}} :$$

$$. S' \square 638 :$$

. 638

$$\begin{aligned}
 & \cdot p(k+1) : \\
 & \cdot n \quad p(n) : \\
 & \quad \quad \quad : 2 \\
 & \quad \quad \quad : n \\
 1^3 + 2^3 + 3^3 + \dots + n^3 &= \frac{1}{4}n^2(n+1)^2
 \end{aligned}$$

$$\begin{aligned}
 p(0) : 1^3 + 2^3 + 3^3 + \dots + n^3 &= \frac{1}{4}n^2(n+1)^2 \quad : \\
 0^3 &= \frac{1}{n}(0)^2(0+1)^2 \quad : \quad : p(0) \quad (1)
 \end{aligned}$$

$$\cdot p(0) : \quad 0 = 0$$

$$p(k+1) \quad p(k) \quad (2)$$

$$p(k) : 1^3 + 2^3 + 3^3 + \dots + k^3 = \frac{1}{4}k^2(k+1)^2$$

$$p(k+1) : 1^3 + 2^3 + 3^3 + \dots + k^3 = \frac{1}{4}(k+1)^2(k+2)^2$$

$$\begin{aligned}
 1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3 &= \frac{1}{4}k^2(k+1)^2 + (k+1)^3 \\
 &= \frac{1}{4}(k+1)^2[k^2 + 4(k+1)] \\
 &= \frac{1}{4}(k+1)^2[k^2 + 4k + 4] \\
 &= \frac{1}{4}(k+1)^2(k+2)^2
 \end{aligned}$$

· n

p(n)

p(k+1)

$$: \quad u_n \quad u_{n+1} \quad (u_n) \\ \cdot \quad f \quad u_{n+1} = f(u_n)$$

: 1

$$: (u_n) \\ \begin{cases} u_0 = 1 \\ u_{n+1} = 2u_n + 1, \quad n \geq 0 \end{cases}$$

· u_1, u_2, u_3 -

$$u_1 = 2u_0 + 1 = 2(1) + 1 = 3 \quad : n = 0 \\ u_2 = 2u_1 + 1 = 2(3) + 1 = 7 \quad : n = 1 \\ u_3 = 2u_2 + 1 = 2(7) + 1 = 15 \quad : n = 2$$

: 2

$$: (V_n) \\ \begin{cases} V_0 = 2 \\ V_1 = 3 \\ V_{n+2} = 2V_{n+1} - V_n, \quad n \geq 0 \end{cases}$$

· V_2, V_3, V_4 -

$$V_2 = 2V_1 - V_0 = 2(3) - 2 = 4 \quad : n = 0 \\ V_3 = 2V_2 - V_1 = 2(4) - 3 = 5 \quad : n = 1 \\ V_4 = 2V_3 - V_2 = 2(5) - 4 = 6 \quad : n = 2$$

: - 1

() $(u_n)_{n \geq n_0}$:

$$u_{n+1} > u_n \quad u_{n+1} \geq u_n \quad n_0$$

. n_0 n (

() $(u_n)_{n \geq n_0}$:

$$u_{n+1} < u_n \quad u_{n+1} \leq u_n \quad n_0$$

. n_0 n (

n_0 $(u_n)_{n \geq n_0}$:

$$n \quad u_{n+1} = u_n \quad n_0$$

) \square I :

() (

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: - 2

(u_n)

$$. n \quad u_n \leq M : M$$

: - 3

(u_n)

$$. n \quad u_n \geq m : m$$

: - 4

(u_n)

$$p(k+1) \quad p(k)$$

$$u_k = u_0 + kr : p(k)$$

$$u_{k+1} = u_0 + (k+1)r : p(k+1)$$

$$u_{k+1} = u_k + r :$$

$$u_k = u_0 + kr :$$

$$u_{k+1} = u_0 + kr + r :$$

$$u_{k+1} = u_0 + (k+1)r :$$

$$n \quad p(n)$$

$$r \quad u_0 \quad (u_n)$$

$$s = u_0 + u_1 + \dots + u_n :$$

$$s = \frac{n+1}{2}(u_0 + u_n)$$

$$r = 5 \quad u_0 = 3 \quad (u_n)$$

$$s_1 = u_0 + u_1 + \dots + u_n \quad (1)$$

$$s_2 = u_0 + u_1 + \dots + u_{10} \quad (2)$$

$$s_3 = u_3 + u_4 + \dots + u_n \quad (3)$$

$$s_1 = \frac{n+1}{2}(u_0 + u_n) \quad (1)$$

$$u_n = 3 + 5n : \quad u_n = u_0 + nr :$$

$$s_1 = \frac{n+1}{2}(3 + 3 + 5n) = \frac{n+1}{2}(6 + 5n) :$$

$$s_2 = \frac{11}{2}(56) = 308 \quad : \quad (2)$$

$$s_3 = \frac{n-2}{2}(u_3 + u_n) = \frac{n-2}{2}(21 + 5n) \quad : \quad (3)$$

: - 5

$$(u_n) \quad \cdot \quad (u_n)$$

$$n \quad u_{n+1} = \frac{u_n + u_{n+2}}{2} \quad :$$

$$u_{n+2} \quad u_n \quad u_{n+1}$$

: - 2

: -

$$u_0 \quad (u_n)$$

$$: n \quad q$$

$$(u_n) \quad : q \quad u_{n+1} = u_n \times q$$

:

$$u_0 \quad q = 1 \quad -$$

:

$$u_n = 10^n \quad : \quad (u_n)$$

$$u_0 \quad (u_n)$$

:

$$u_{n+1} = 10^{n+1} = 10^n \cdot 10$$

$$(u_n) \quad : \quad u_{n+1} = u_n \cdot 10 \quad :$$

$$u_0 = 10^0 = 1 \quad q = 10$$

:

-

$$q \quad u_0 \quad (u_n)$$

$$u_n = u_0 \cdot q^n : (u_n)$$

. n

():

$$p(n)u_n : u_0 \cdot q^n :$$

: -

p(0)

$$u_0 = u_0 : n = 0$$

: -

p(k+1)

p(k)

$$p(k) : u_k = u_0 \cdot q^k$$

$$p(k+1) : u_{k+1} = u_0 \cdot q^{k+1}$$

$$() u_{k+1} = u_k \cdot q :$$

$$() u_k = u_0 \cdot q^k :$$

$$u_{k+1} = u_0 \cdot q^{k+1} :$$

$$u_{k+1} = u_0 \cdot q^k \cdot q :$$

. n

p(n)

:

$$u_n = u_1 \times q^{n-1}$$

u_1 -

(n

) u_p

-

$$u_n = u_p \times q^{n-p} :$$

. n u_n

-

:

-

. q

u_0

(u_n)

$$s = u_0 + u_1 + \dots + u_n :$$

$$s = (n+1)u_0 : q = 1 -$$

. n

$$s = u_0 \times \frac{1 - q^{n+1}}{1 - q} = u_0 \frac{q^{n+1} - 1}{q - 1} : q \neq 1 :$$

$$\frac{1 - q^{n+1}}{1 - q}$$

$n + 1$

$$q = 2 \quad u_0 = 20 \quad (u_n)$$

$$s_2 = u_0 + u_1 + \dots + u_{10} \quad s_1 = u_0 + u_1 + \dots + u_n$$

$$s_3 = u_{10} + u_{11} + \dots + u_{20}$$

$$u_n = 20 \times 2^n : \quad u_n = u_0 \times q^n : \quad (1)$$

: (2)

$$s_1 = u_0 + u_1 + \dots + u_n : *$$

$$s_1 = u_0 \cdot \frac{1 - q^{n+1}}{1 - q} = 20 \times \frac{1 - 2^{n+1}}{1 - 2} :$$

$$s_1 = 20(2^{n+1} - 1) :$$

$$s_2 = u_0 + u_1 + \dots + u_{10} : *$$

11:

$$s_2 = u_0 \cdot \frac{1 - q^{11}}{1 - q} = 20 \times \frac{1 - 2^{11}}{1 - 2} = 20(2^{11} - 1) :$$

$$s_3 = u_{10} + u_{11} + \dots + u_{20} : *$$

$$20 - 10 + 1 = 11 :$$

$$u_{10} = 20 \times 2^{10} : \quad s_3 = u_{10} \cdot \frac{1 - q^{11}}{1 - q} :$$

$$s_3 = 20 \times 2^{10} \times \frac{1 - 2^{11}}{1 - 2} = 20 \times 2^{10} (2^{11} - 1) :$$

$$(u_{n+1})^2 = u_n \cdot u_{n+2} :$$

$$(u_n)$$

. n

$$\cdot u_{n+2} \quad u_n$$

$$u_{n+1}$$

: - 5

$$\ell (u_n)$$

$$p$$

$$(u_n)$$

$$(u_n)$$

$$\ell \quad I$$

$$\lim_{n \rightarrow +\infty} u_n = \ell :$$

$$(u_n)$$

$$(u_n) :$$

$$]a; +\infty[$$

$$+\infty$$

$$(u_n)$$

:

$$(u_n)$$

$$(a \in \square)$$

$$\lim_{n \rightarrow +\infty} u_n = +\infty$$

$$(u_n)$$

$$]-\infty ; a[$$

$$-\infty$$

$$(u_n)$$

:

$$(u_n)$$

$$(a \in \square)$$

$$\lim_{n \rightarrow +\infty} u_n = -\infty$$

$$(u_n)$$

$$f \quad u_n = f(n) : \quad (u_n)$$

$$\lim_{n \rightarrow +\infty} u_n = \ell : \quad \lim_{x \rightarrow +\infty} f(x) = \ell \quad (1)$$

$$\lim_{n \rightarrow +\infty} u_n = +\infty : \quad \lim_{x \rightarrow +\infty} f(x) = +\infty \quad (2)$$

$$\lim_{n \rightarrow +\infty} u_n = -\infty : \quad \lim_{x \rightarrow +\infty} f(x) = -\infty \quad (3)$$

n

$$n_0 \quad \square$$

$$\lim_{n \rightarrow +\infty} u_n = \ell : \quad \lim_{n \rightarrow +\infty} w_n = \lim_{n \rightarrow +\infty} v_n = \ell$$

$$\lim_{n \rightarrow +\infty} u_n = +\infty : \quad \lim_{n \rightarrow +\infty} v_n = +\infty$$

$$\lim_{n \rightarrow +\infty} u_n = -\infty : \quad \lim_{n \rightarrow +\infty} v_n = -\infty$$

$$q \quad u_0 \quad (u_n)$$

$$\lim_{n \rightarrow +\infty} u_n = +\infty : \quad \mathbf{u_n > 0} \quad q > 1 \quad -$$

.

$$\mathbf{(u_n)}$$

$$\lim_{n \rightarrow +\infty} u_n = -\infty : \quad \mathbf{u_n < 0} \quad q > 1 \quad -$$

.

$$\mathbf{(u_n)}$$

$$\lim_{n \rightarrow +\infty} u_n = 0 : \quad \mathbf{-1 < q < 1} \quad -$$

.

$$\mathbf{(u_n)}$$

$$\mathbf{(u_n)} \quad \mathbf{q \leq -1} \quad -$$

$$\mathbf{u_n = u_0 \cdot e^{n \ln q} : q > 0}$$

$$\lim_{n \rightarrow +\infty} 3^n = +\infty$$

$$\lim_{n \rightarrow +\infty} \left(\frac{1}{2}\right)^n = 0$$

$$\lim_{n \rightarrow +\infty} \left(-\frac{1}{2}\right)^n = 0$$

$$\lim_{n \rightarrow +\infty} (-2)^n$$

- 5

$$\lim_{n \rightarrow +\infty} (u_n - v_n) = 0 :$$

$(u_n) \quad (v_n)$

: $(u_n) \quad (v_n)$

$$v_n = -\frac{2}{n} \quad u_n = \frac{2}{n}$$

$(u_n) \quad (v_n)$

(v_n)

(u_n)

$$\lim_{n \rightarrow +\infty} (u_n - v_n) = \lim_{n \rightarrow +\infty} \left(\frac{2}{n} + \frac{2}{n} \right) = 0 :$$

$$(\mathbf{u}_n) \quad (\mathbf{V}_n)$$

: 8

$$(\mathbf{u}_n) \quad : \quad (\mathbf{u}_n) \quad (\mathbf{V}_n)$$

$$: \quad (\mathbf{V}_n)$$

$$u_n \leq v_n : n \quad (1)$$

$$\lambda \quad (\mathbf{u}_n) \quad (\mathbf{V}_n) \quad (2)$$

$$u_n \leq \lambda \leq v_n : n_0 \quad n \quad (3)$$

:

$$: \quad (\mathbf{u}_n) \quad (\mathbf{V}_n) -$$

$$v_n \leq u_n : n \quad (1)$$

$$. 0 \quad (\mathbf{u}_n) \quad (\mathbf{V}_n) \quad (2)$$

$$v_n \leq 0 \leq u_n : n \quad (3)$$

: 1

: (U_n)

$$U_0 = 0 \quad U_n = \frac{1}{2}U_{n-1} - 2$$

```

Normal Sci Eng
Float 0123456789
Radian Degree
  Par Pol Seq
Connected Dot
Sequential Simul
Real a+by re^θi

```

MODE

:

(1)

Seq :

(2)

```

Plot1 Plot2 Plot3
xMin=0
:u(n)0.5u(n-1)-
2
u(xMin)0{-2}
:u(n)=
v(xMin)=
:w(n)=

```

Y=

:

```

WINDOW
xMin=0
xMax=10
PlotStart=1
PlotStep=1
Xmin=-4
Xmax=10
↓Xscl=1

```

WINDOW

(3)

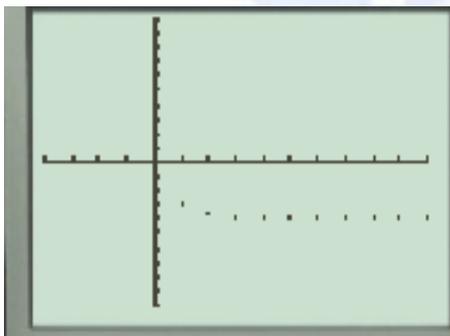
```

Ymin=-10
Ymax=10
Yscl=1

```

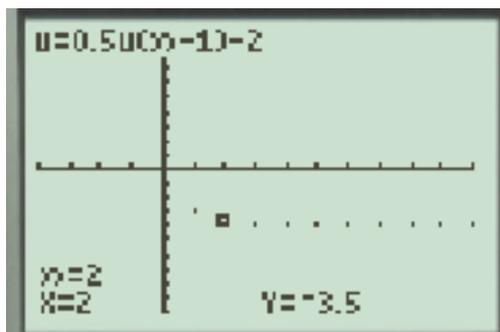
GRAPH

(4)



TRACE

(5)



: 2

:

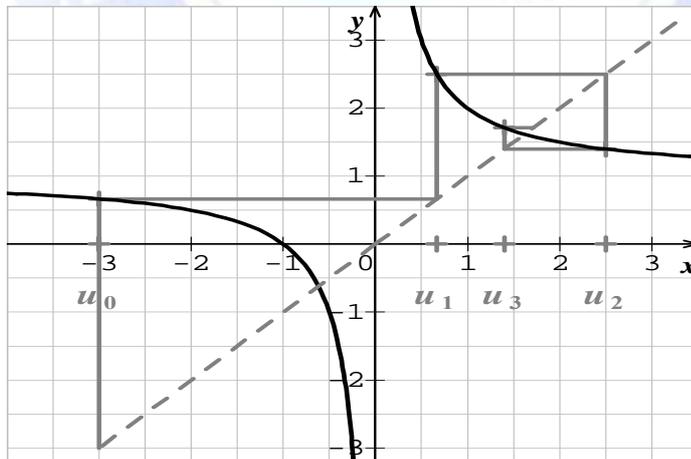
sinequanon

$$\begin{cases} U_0 = -3 \\ U_{n+1} = 1 + \frac{1}{U_n} \end{cases} \quad (1)$$

n	u
0	-3
1	0,666667
2	2,5
3	1,4
4	1,71429
5	1,58333
6	1,63158
7	1,6129
8	1,62
9	1,61728
10	1,61832
11	1,61792
12	1,61808
13	1,61802
14	1,61804

QK

(2)



1

$$p(k) \times p(n) \sqrt{\quad} \quad (1)$$

$$n=1 \quad n=0 \quad p(n) \quad (2)$$

$$n \quad n=2 \quad (3)$$

$$u_n = 4n - 3 : (u_n) \quad (4)$$

$$\begin{cases} u_{n+1} = 4u_n - 1 & n \geq 0 \\ u_0 = 1 \end{cases} : \quad (5)$$

$$u_n = an + b : (v_n) \quad (6)$$

$$\lim_{n \rightarrow +\infty} \left(\frac{1}{1000} \right)^n = +\infty \quad (7)$$

$$\lim_{n \rightarrow +\infty} \left(\frac{3}{840} \right)^n = 0 \quad (8)$$

$$\lim_{n \rightarrow +\infty} \left(-\frac{1}{10} \right)^n : \quad (9)$$

$$\lim_{n \rightarrow +\infty} \left(-\frac{1}{40000} \right)^n = 0 \quad (10)$$

$$(u_n) \quad (v_n) \quad \lim_{n \rightarrow +\infty} (u_n - v_n) = 0 : \quad (11)$$

$$\lim_{n \rightarrow +\infty} \frac{3^n}{4^n} = 0 \quad (12)$$

$$1 + 4 + 4^2 + 4^3 + \dots + 4^{99} = \frac{1 - 4^{100}}{1 - 4} \quad (13)$$

$$3 + 5 + 7 + \dots + (2n + 1) = \frac{(2n + 4)(n)}{2} \quad (14)$$

(15)

(16)

$$\lim_{n \rightarrow +\infty} \frac{1}{\sqrt{n}} = 0 \quad (17)$$

$$u_n = a^n - b : \quad (u_n) \quad (18)$$

$b \neq 0 \quad a \neq 0$

$$u_n = \left(\frac{1}{3}\right)^n : \quad (u_n) \quad (19)$$

$$(u_n) \quad \ell \neq 0, \lim_{n \rightarrow +\infty} = \ell \quad (20)$$

.2

$$1 - 3 + 5 - 7 + \dots + (-1)^{n-1} \cdot (2n - 1) = n (-1)^{n-1}$$

.3

$$f^{(n)}(x) = \sin\left(x + \frac{n\pi}{2}\right) : \quad x \mapsto \sin x : f$$

.4

$$f(x) = \frac{1}{x^2 + 1} : \quad f$$

$f \quad n$

$$f^{(n)}(x) = \frac{p_n(x)}{(x^2 + 1)^{n+1}}$$

$n \geq 1 \quad n$

$p_n(x)$

.5

$$u_0 = 10 \quad u_{n+1} = \sqrt{u_n} \quad (u_n)$$

$$\cdot (u_n) \quad -1$$

$$\cdot u_n \geq 1 : n \quad -2$$

$$\cdot (u_n) \quad -3$$

$$\cdot \quad -4$$

$$\lim_{n \rightarrow +\infty} u_n \quad -5$$

.6

:

$$(u_n)$$

$$\begin{cases} u_0 + u_1 + u_2 = 15 \\ \frac{1}{u_0} + \frac{1}{u_1} + \frac{1}{u_2} = \frac{33}{40} \end{cases}$$

$$\cdot r \quad u_0, u_1, u_2 \quad -$$

$$\cdot S_n = \sum_{i=0}^n u_i : \quad -$$

.7

$$q \quad u_s = 1000, u_1 = 250 : \quad (u_n)$$

$$\cdot q > 0 :$$

$$\cdot q \quad (1)$$

$$\cdot S_n = 2^{\frac{1}{2}} + 2^{\frac{2}{2}} + 2^{\frac{3}{2}} + \dots + 2^{\frac{n}{2}} \quad (2)$$

$$\cdot \lim_{n \rightarrow +\infty} s_n \quad (3)$$

$$: n \quad (4)$$

$$p_n = (\sqrt{2})^1 \cdot (\sqrt{2})^2 \cdot (\sqrt{2})^3 \times \dots \times (\sqrt{2})^n$$

.8

$$\begin{aligned}
 & : \quad (v_n) \quad (u_n) \\
 n > 1, \quad v_n &= \frac{1}{\ln n}, \quad u_n = \frac{-2}{n} \\
 & \cdot (v_n) \quad (u_n) \quad -1 \\
 & \cdot \lim_{n \rightarrow +\infty} (u_n - v_n) \quad -2 \\
 & (v_n) \quad (u_n) \quad -3
 \end{aligned}$$

.9

$$\begin{aligned}
 & : \quad (v_n) \quad (u_n) \\
 n > 0 : \quad v_n &= \ln(n+1), \quad u_n = \ln(n) \\
 & \cdot (v_n) \quad (u_n) \quad -1 \\
 & \cdot \lim_{n \rightarrow +\infty} (u_n - v_n) \quad -2 \\
 & \cdot (v_n) \quad (u_n) \quad -3
 \end{aligned}$$

.10

$$\begin{aligned}
 & 0 \leq \theta \leq \frac{\pi}{2} : \quad \theta \\
 & \left\{ \begin{aligned} u_0 &= 2 \cos \theta \\ u_{n+1} &= \sqrt{2 + u_n} \end{aligned} \right., n \geq 0 : \quad (u_n) \\
 (\cos 2\theta &= 2 \cos^2 \theta - 1) \cdot \theta \quad u_0, u_1, u_2, u_3 \quad -1 \\
 & n \quad -2 \\
 & \cdot u_n = 2 \cos \left(\frac{\theta}{2^n} \right) : \\
 v_n &= \frac{\theta}{2^n} : \quad \square \quad (v_n) \quad -3 \\
 & \cdot (v_n) \\
 & \cdot (u_n) \quad -4
 \end{aligned}$$

.11

: (u_n)

$$\begin{cases} u_0 = -1 \\ u_{n+1} = \frac{3 + 2u_n}{2 + u_n} \end{cases}, u \in \mathbb{Q}$$

. u_1, u_2, u_3, u_4 : -1

$u_n > 0$: n -2

\mathbb{Q} \mathbb{Q} (u_n)

$u_n \leq \sqrt{3}$: n -3

(u_n) -4

$$V_n = \frac{u_n - \sqrt{3}}{u_n + \sqrt{3}} : \mathbb{Q} (v_n) -5$$

(v_n) -

. q

. $\lim_{n \rightarrow +\infty} u_n$ $\lim_{n \rightarrow +\infty} V_n$ -

.12

α

$$u_{n+1} = \alpha u_n + 3 : u_0 (u_n)$$

$n \in \mathbb{Q}$

(u_n) α α_0 -1

. (u_n) u_0 . $\alpha \neq \alpha_0$ -2

(u_n) -3

$$V_n = \beta u_n + \delta, n \geq 0 : (v_n)$$

q (v_n) -

$$\beta \quad \alpha \quad \delta \quad \cdot \quad \alpha$$

$$(\nu_n) \quad \alpha \quad -$$

$$\lim_{n \rightarrow +\infty} u_n$$

.13

$$u_0 = 1 \quad (u_n)$$

$$u_{n+1} = u_n + \frac{1}{2^{n+1}} :$$

$$S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^n} : S_n \quad -1$$

$$\cdot n \quad u_n \quad -2$$

$$\cdot \lim_{n \rightarrow +\infty} u_n \quad -3$$

$$\cdot (u_n) \quad -4$$

.14

$$\begin{cases} u_0 = -2 \\ u_{n+1} = \sqrt{2 + u_n} \end{cases}, n \geq 0 : (u_n)$$

$$2 - u_n \geq 0 \quad n \quad -1$$

$$n \quad -$$

$$\cdot u_n \geq 0 : \quad -$$

$$\cdot (u_n) \quad -2$$

$$\cdot (u_n) \quad -3$$

.15

$$: u_0 = 60 \quad (u_n) (1)$$

$$u_{n+1} - u_n = u_n \times 0,06, n \geq 0$$

$$(u_n) \quad -$$

$$S_n = \sum_{i=0}^n u_i : -$$

$$. 2000 \quad 1 \quad 600 \quad (2)$$

$$60\%$$

$$. 2007 \quad 1$$

.16

$$: (u_n) \quad \lambda$$

$$\begin{cases} u_0 = 1, & u_1 = 2 \\ u_{n+2} = (\lambda + 1)u_{n+1} - \lambda u_n, & n \geq 0 \end{cases}$$

$$V_{n+1} = u_{n+1} - u_n, \quad n \geq 0 : (V_n)$$

$$\lambda \quad n \quad V_n \quad (V_n) \quad -1$$

$$S_n = V_0 + V_1 + \dots + V_{n-1} : \lambda \quad n \quad S_n \quad -2$$

$$S_n = u_n - 1 \quad .$$

$$\lambda \quad n \quad u_n \quad -$$

$$: n \quad \lambda = 3 \quad -3$$

$$S'_n = \frac{1}{4}n + 225 : n \quad S'_n = u_0^2 + u_1^2 + \dots + u_{n-1}^2$$

1

×	(4	×	(3	×	(2	×	(1
√	(8	×	(7	√	(6	√	(5
√	(12	×	(11	√	(10	×	(9
×	(16	×	(15	√	(14	√	(13
√	(20	√	(19	×	(18	√	(17

2

$p(n)$

$$1 - 3 + 5 - 7 + \dots + (-1)^n \cdot (2n - 1) = n(-1)^{n-1}$$

$$1 = 1(-1)^0 = 1 : p(1) \quad -$$

$p(1)$

$p(k+1)$

$p(k)$

$$p(k) : 1 - 3 + 5 - 7 + \dots + (-1)^{k-1} (2k - 1) = k(-1)^{k-1}$$

$$p(k+1) : 1 - 3 + 5 - 7 + \dots + (-1)^{k-1} (2k - 1) + (-1)^k \cdot (2k + 1) = (k+1)(-1)^k$$

$$1 - 3 + 5 - 7 + \dots + (-1)^{k-1} \cdot (2k - 1) + (-1)^k \cdot (2k + 1) = k(-1)^{k-1} + (-1)^k (2k + 1)$$

$$= k(-1)^{k-1} + (-1)^{k-1} \times (-1)(2k + 1)$$

$$= (-1)^{k-1} [k - 2k - 1] = (-1)^{k-1} (-k - 1)$$

$$= -(-1)^{k-1} (k + 1) = (-1)^k (k + 1)$$

$p(k+1)$

$p(n)$

3

$$f^{(n)}(x) = \sin\left(x + \frac{n\pi}{2}\right) : p(n)$$

$$f^{(1)}(x) = \sin\left(x + \frac{\pi}{2}\right) = \cos x : p(1) \quad -$$

$$p(1) : f'(x) = \cos x :$$

$$p(k+1) \quad p(k) \quad -$$

$$f^{(k)}(x) = \sin\left(x + \frac{k\pi}{2}\right) :$$

$$f^{(k+1)}(x) = \sin\left(x + \frac{(k+1)\pi}{2}\right) :$$

$$f^{(k)}(x) = \sin\left(x + \frac{k\pi}{2}\right) :$$

$$f^{(k+1)}(x) = (f^{(k)})'(x) = \cos\left(x + \frac{k\pi}{2}\right) :$$

$$= \sin\left(x + \frac{k\pi}{2} + \frac{\pi}{2}\right) = \sin\left(x + \frac{(k+1)\pi}{2}\right)$$

$$p(k+1) :$$

n

$p(n)$

4

$$f^{(n)}(x) = \frac{p_n(x)}{(x^2 + 1)^{p+1}} : p(n)$$

$$: p(1)$$

- 1

$$f^{(1)}(x) = \frac{-2x}{(x^2 + 1)^2} = \frac{-2x}{(x^2 + 1)^{1+1}}$$

$$p_1(n) = -2x$$

$$p(k+1)$$

$$p(k)$$

- 2

$$p(k) : f^{(k)}(x) = \frac{p_k(x)}{(x^2 + 1)^{k+1}} :$$

$k+1$

$p_k(x) :$

$$p(k) : f^{(k+1)}(x) = \frac{p_{k+1}(x)}{(x^2 + 1)^{k+2}} :$$

$$f^{(k)}(x) = \frac{p_k(x)}{(x^2 + 1)^{k+1}} :$$

$$: f^{(k+1)}(x) = (f^k)'(x) :$$

$$(f^k)'(x) = \frac{p'_k(x) \cdot (x^2 + 1)^{k+1} - (k+1) \cdot 2x(x^2 + 1)^k \cdot p_k(x)}{\left[(x^2 + 1)^{k+1} \right]^2}$$

$$= \frac{(x^2 + 1)^k \left[p'_k(x) \cdot (x^2 + 1) - 2(k+1)x p_k(x) \right]}{(x^2 + 1)^{2k+2}}$$

$$= \frac{p'_k(x) \cdot (x^2 + 1) - 2(k+1)x \cdot p_k(x)}{(x^2 + 1)^{2k+2-k}}$$

$$= \frac{-2(k+1)x p_k(x) - p'_k(x)}{(x^2 + 1)^{k+2}}$$

$$k-1 : p'_k(x) \quad k : p_k(x)$$

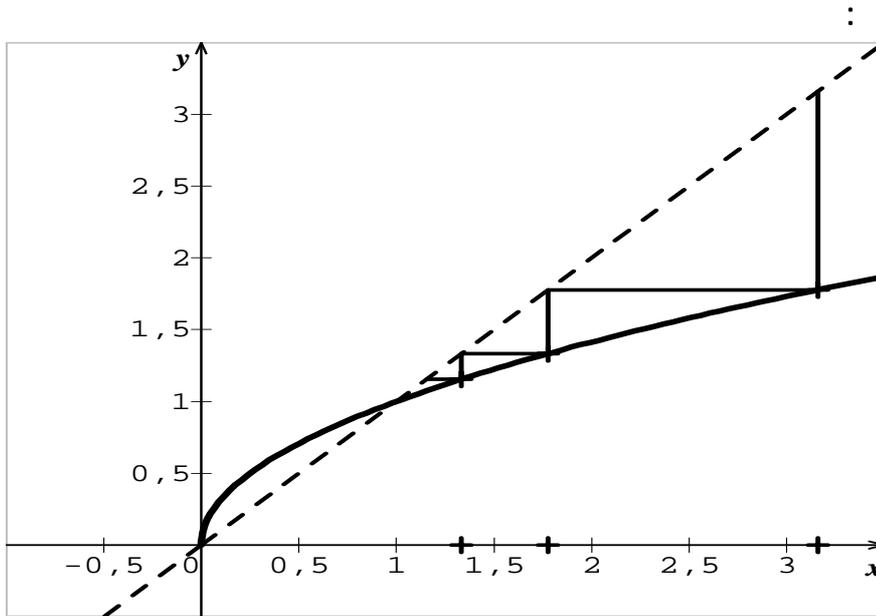
$$k+1 : xp_k(x)$$

$$k+1 : -2(k+1)x p_k(x) - p'_k(x) \cdot (x^2 + 1)$$

$$p_{k+1}(x) = 2(k+1)x p_k(x) - p'_k(x) \cdot (x^2 + 1) :$$

$$p(k+1) : f^{(k+1)}(x) = \frac{p_{k+1}(x)}{(x^2 + 1)^{k+2}} :$$

$$. n \quad p(n)$$



. 1 :

$$U_n \geq 1 : p(n) \tag{2}$$

$$U_0 = 10 : U_0 \geq 1 : p(0) \quad -$$

$$p(k+1) \quad p(k) \quad -$$

$$p(k) : U_k \geq 1 :$$

$$p(k+1) : U_{k+1} \geq 1 :$$

$$\sqrt{U_k} \geq 1 : U_k \geq 1 :$$

$$p(k+1) : U_{k+1} \geq 1 :$$

$$: (U_n) \quad -3$$

$$U_{n+1} - U_n = \sqrt{U_n} - U_n = \frac{(\sqrt{U_n} - U_n)(\sqrt{U_n} + U_n)}{\sqrt{U_n} + U_n}$$

$$= \frac{U_n - U_n^2}{\sqrt{U_n} + U_n} = \frac{U_n(1 - U_n)}{\sqrt{U_n} + U_n}$$

$$: \sqrt{U_n} > 0 \quad U_n > 0 \quad 1 - U_n \leq 0 : U_n \geq 1 :$$

$$: (U_n) : U_{n+1} - U_n \leq 0$$

$$(U_n) \quad -4$$

1

$$(U_n)$$

$$(U_n) \quad (U_n > 1)$$

$$\lim_{n \rightarrow +\infty} U_n \quad -5$$

$$\lim_{n \rightarrow +\infty} U_n = \ell \quad : \quad n+1 \rightarrow +\infty : n \rightarrow +\infty$$

$$\lim_{n \rightarrow +\infty} U_{n+1} = \lim_{n \rightarrow +\infty} \sqrt{U_n} \quad : \quad \lim_{n \rightarrow +\infty} U_{n+1} = \ell$$

$$\ell^2 - \ell = 0 \quad \ell^2 = \ell \quad : \quad \ell = \sqrt{\ell} \quad :$$

$$(U_n \geq 1) \quad \ell = 0 \quad : \quad \ell(\ell - 1) = 0 \quad :$$

$$\lim_{n \rightarrow +\infty} U_n = 1 \quad : \quad \ell = 1 \quad :$$

.6

$$U_0 + U_2 = 2U_1 \quad : \quad (U_n)$$

$$U_1 = 5 \quad : \quad 3U_1 = 15 \quad : \quad U_0 + U_1 + U_2 = 15 \quad :$$

$$\begin{cases} U_0 + U_2 = 10 \\ \frac{1}{U_0} + \frac{1}{U_2} = \frac{5}{8} \end{cases} \quad : \quad \begin{cases} U_0 + 5 + U_2 = 15 \\ \frac{1}{U_0} + \frac{1}{5} + \frac{1}{U_2} = \frac{33}{40} \end{cases} \quad :$$

$$\begin{cases} U_0 + U_2 = 10 \\ U_0 \times U_2 = 16 \end{cases} \quad : \quad \begin{cases} U_0 + U_2 = 10 \\ \frac{U_0 + U_2}{U_0 \cdot U_2} = \frac{5}{8} \end{cases} \quad :$$

$$x^2 - 10x + 16 = 0 \quad : \quad U_0, U_2 \quad :$$

$$\Delta = 36 \quad : \quad \Delta = (10)^2 - 4(16) \quad :$$

$$U_2 = 8 \quad U_0 = 2 \quad : \quad x_2 = 8 \quad x_1 = 2 \quad :$$

$$r = U_1 - U_0 = 5 - 2 = 3 \quad :$$

$$S_n = U_0 + U_1 + \dots + U_n \quad : \quad S_n$$

$$S_n = \frac{(n+1)}{2}(U_0 + U_n) : n+1$$

$$S_n = \frac{(3n+4)(n+1)}{2} :$$

.7

$$U_5 = U_1 \times q^4 : q \quad -1$$

$$q^4 = \frac{1000}{250} : q^4 = \frac{U_5}{U_1}$$

$$q = \sqrt{2} : q^2 = 2 : q^4 = 4 :$$

$$S_n = 2^{\frac{1}{2}} + 2^{\frac{2}{2}} + 2^{\frac{3}{2}} + \dots + 2^{\frac{n}{2}} : \quad -2$$

$$S_n = \sqrt{2} + (\sqrt{2})^2 + (\sqrt{2})^3 + \dots + (\sqrt{2})^n :$$

$$q = \sqrt{2} : S_n :$$

$$S_n = \sqrt{2} \times \frac{1 - q^n}{1 - q} = \sqrt{2} \times \frac{1 - (\sqrt{2})^n}{1 - \sqrt{2}}$$

$$S_n = \sqrt{2} \times \frac{[1 - (\sqrt{2})^n][1 + \sqrt{2}]}{[1 - \sqrt{2}][1 + \sqrt{2}]}$$

$$S_n = \sqrt{2} \times \frac{[1 - (\sqrt{2})^n]^n [1 + \sqrt{2}]}{1 - 2} = -\sqrt{2} [1 - (\sqrt{2})^n] (1 + \sqrt{2})$$

: -3

$$\lim_{n \rightarrow +\infty} S_n = \lim_{n \rightarrow +\infty} \left[-\sqrt{2} (1 - (\sqrt{2})^n) \right] (1 + \sqrt{2}) = +\infty$$

$$\lim_{n \rightarrow +\infty} (\sqrt{2})^n = +\infty :$$

: P_n -4

$$P_n = (\sqrt{2})^1 \times (\sqrt{2})^2 \times (\sqrt{2})^3 \times \dots \times (\sqrt{2})^n$$

$$p_n = (\sqrt{2})^{1+2+3+\dots+n} = (\sqrt{2})^{\frac{(1+n)n}{2}}$$

.8

:(V_n) (U_n)

- 1

$$U_{n+1} - U_n = \frac{-2}{n+1} + \frac{2}{n} = \frac{-2n + 2(n+1)}{n(n+1)} = \frac{2}{n(n+1)}$$

(U_n) $U_{n+1} - U_n > 0$:

$$V_{n+1} - V_n = \frac{1}{\ln(n+1)} - \frac{1}{\ln(n)} = \frac{\ln(n) - \ln(n+1)}{\ln(n+1) \cdot \ln(n)}$$

$$= \frac{\ln\left(\frac{n}{n+1}\right)}{\ln(n+1) \cdot \ln(n)}$$

$\ln(n+1) > 0$ $\ln(n) > 0$: $n > 1$:

$$\ln\left(\frac{n}{n+1}\right) < 0 : \frac{n}{n+1} < 1 :$$

(V_n) : $V_{n+1} - V_n < 0$:

$$\lim_{n \rightarrow +\infty} (U_n - V_n) = \lim_{n \rightarrow +\infty} \left(\frac{-2}{n} - \frac{1}{\ln(n)} \right) = 0 : -2$$

(V_n) (U_n) - 3

$$\lim_{n \rightarrow +\infty} (U_n - V_n) = 0 :$$

-1

$$U_{n+1} - U_n = \ln(n+1) - \ln(n) = \ln\left(\frac{n+1}{n}\right) :$$

$$\ln\left(\frac{n+1}{n}\right) > 0 \quad : \quad \frac{n+1}{n} > 1 \quad :$$

$$(U_n) \quad U_{n+1} - U_n > 0 \quad :$$

$$V_{n+1} - V_n = \ln(n+2) - \ln(n+1) = \ln\left(\frac{n+2}{n+1}\right)$$

$$\ln\left(\frac{n+2}{n+1}\right) > 0 \quad : \quad \frac{n+2}{n+1} > 1 \quad :$$

$$(V_n) \quad V_{n+1} - V_n > 0 \quad :$$

- 2

$$\lim_{n \rightarrow +\infty} (U_n - V_n) = \lim_{n \rightarrow +\infty} [\ln(n+1) - \ln(n+2)]$$

$$= \lim_{n \rightarrow +\infty} \ln\left(\frac{n+1}{n+2}\right) = \lim_{n \rightarrow +\infty} \ln\left(\frac{n\left(1 + \frac{1}{n}\right)}{n\left(1 + \frac{2}{n}\right)}\right)$$

$$= \lim_{n \rightarrow +\infty} \ln\left(\frac{1 + \frac{1}{n}}{1 + \frac{2}{n}}\right) = 0$$

- 3

$$\begin{aligned}
 U_1 &= \sqrt{2 + U_0} = \sqrt{2 + 2 \left(2 \cos^2 \frac{\theta}{2} - 1 \right)} \\
 &= \sqrt{2 + 4 \cos^2 \frac{\theta}{2} - 2} = \sqrt{4 \cos^2 \frac{\theta}{2}} = 2 \cos \frac{\theta}{2} \\
 0 \leq \frac{\theta}{2} \leq \frac{\pi}{4} & : \quad \frac{\theta}{2} > 0 \quad \cos \frac{\theta}{2} > 0 :
 \end{aligned}$$

$$\begin{aligned}
 U_2 &= \sqrt{2 + U_1} = \sqrt{2 + 2 \cos \frac{\theta}{2}} = \sqrt{2 + 2 \left(2 \cos^2 \frac{\theta}{4} - 1 \right)} \\
 &= \sqrt{4 \times \cos^2 \frac{\theta}{4}} = 2 \cos \frac{\theta}{4} = 2 \cos \frac{\theta}{2^2}
 \end{aligned}$$

$$U_3 = \sqrt{2 + U_2} = \sqrt{2 + 2 \cos \frac{\theta}{4}} = 2 \cos \frac{\theta}{8} = 2 \cos \frac{\theta}{2^3}$$

$$U_n = 2 \cos \left(\frac{\theta}{2^n} \right) : p(n) \quad -2$$

$$p(0) \quad U_0 = 2 \cos \theta : p(0) \quad -$$

$$p(k) : U_k = 2 \cos \left(\frac{\theta}{2^k} \right) :$$

$$p(k+1) : U_{k+1} = 2 \cos \left(\frac{\theta}{2^{k+1}} \right)$$

$$U_{k+1} = \sqrt{2 + U_k} = \sqrt{2 + 2 \cos \left(\frac{\theta}{2^k} \right)}$$

$$U_{k+1} = \sqrt{2 + 2 \left(2 \cos^2 \frac{\theta}{2^{k+1}} - 1 \right)} = \sqrt{4 \cos^2 \frac{\theta}{2^{k+1}}} = 2 \cos \frac{\theta}{2^{k+1}}$$

$$0 \leq \theta \leq \frac{\pi}{2} : \quad \cos \frac{\theta}{2^{k+1}} > 0 :$$

$$. n \quad p(n) : \quad p(k+1)$$

$$\lim_{n \rightarrow +\infty} 2^n = 0 : \quad \lim_{n \rightarrow +\infty} V_n = \lim_{n \rightarrow +\infty} \frac{\theta}{2^n} = 0 \quad -3$$

$$\text{http://www.onefd.edu.dz} \quad U_n = 2 \cos V_n : (U_n) \quad \text{جميع الحقوق محفوظة} \quad -4$$

$$= \lim_{n \rightarrow +\infty} U_n = \lim_{n \rightarrow +\infty} 2 \cos V_n = 2$$

$$. 2 \quad (U_n) \quad \lim_{n \rightarrow +\infty} V_n = 0 \quad :$$

.11

: - 1

$$U_1 = \frac{3 + 2U_0}{2 + U_0} = \frac{3 - 2}{2 - 1} = 1$$

$$U_2 = \frac{3 + 2U_1}{2 + U_1} = \frac{3 + 2}{2 + 1} = \frac{5}{3}$$

$$U_3 = \frac{3 + 2U_2}{2 + U_2} = \frac{3 + \frac{10}{3}}{2 + \frac{5}{3}} = \frac{19}{11}$$

$$U_4 = \frac{3 + 2U_3}{2 + U_3} = \frac{3 + \frac{38}{11}}{2 + \frac{19}{11}} = \frac{71}{41}$$

$$U_n > 0 : p(n) \quad - 2$$

$$p(1) \quad U_1 > 0 : p(1) \quad -$$

$$p(k+1) \quad p(k) \quad -$$

$$p(k) : U_k > 0 \quad :$$

$$p(k+1) : U_{k+1} > 0 \quad :$$

$$U_{k+1} > 0 \quad \frac{3 + 2U_k}{2 + U_k} > 0 \quad : \quad U_k > 0 \quad :$$

$$. n \quad p(n) \quad . \quad p(k+1) :$$

$$U_{n+1} = \frac{3 + 2U_n}{2 + U_n} :$$

$$. \square \quad \square \quad U_{k+1} : \quad 2 + U_n \neq 0 : \quad U_n > 0 :$$

<http://www.onefd.edu.dz>

$$U_n \leq \sqrt{3} : p(n)$$

- 3

$$U_0 = -1 \quad U_0 \leq \sqrt{3} : n = 0 \quad -$$

$$p(0) :$$

$$p(k+1) \quad p(k) \quad -$$

$$p(k) : U_k \leq \sqrt{3} \quad :$$

$$p(k+1) : U_{k+1} \leq \sqrt{3} \quad :$$

$$U_{k+1} - \sqrt{3} = \frac{3 + 2U_k}{2 + U_k} - \sqrt{3} = \frac{3 + 2U_k - 2\sqrt{3} - U_k\sqrt{3}}{2 + U_k}$$

$$= \frac{2(U_k - \sqrt{3}) + 3 - U_k\sqrt{3}}{2 + U_k} = \frac{2(U_k - \sqrt{3}) \leq 3(U_k - \sqrt{3})}{2 + U_k}$$

$$= \frac{(U_k - \sqrt{3})(2 - \sqrt{3})}{2 + U_n}$$

$$U_{k+1} - \sqrt{3} \leq 0 : \quad U_k - \sqrt{3} \leq 0 \quad U_k \leq \sqrt{3}$$

$$U_{k+1} \leq \sqrt{3} :$$

$\cdot n$

$$p(n) :$$

$$p(k+1) :$$

:

$$(V_n) \quad - 5$$

$$V_{n+1} = \frac{U_{n+1} - \sqrt{3}}{U_{n+1} + \sqrt{3}} = \frac{\frac{3 + 2U_n}{2 + U_n} - \sqrt{3}}{\frac{3 + 2U_n}{2 + U_n} + \sqrt{3}}$$

$$= \frac{3 + 2U_n - 2\sqrt{3} - U_n\sqrt{3}}{3 + 2U_n + 2\sqrt{3} + U_n\sqrt{3}}$$

$$= \frac{(2 - \sqrt{3})U_n + 3 - 2\sqrt{3}}{(2 + \sqrt{3})U_n + 3 + 2\sqrt{3}} = \frac{(2 - \sqrt{3})U_n - \sqrt{3}(2 - \sqrt{3})}{(2 + \sqrt{3})U_n + \sqrt{3}(2 + \sqrt{3})}$$

$$= \frac{(2 - \sqrt{3})[U_n - \sqrt{3}]}{(2 + \sqrt{3})[U_n + \sqrt{3}]} = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \cdot V_n$$

$$V_{n+1} = \frac{(2 - \sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} \cdot V_n \quad :$$

$$V_{n+1} = \frac{(2 - \sqrt{3})^2}{4 - 3} \cdot V_n \quad :$$

$$V_{n+1} = (2 - \sqrt{3})^2 \cdot V_n \quad :$$

$$V_{n+1} = (7 - 4\sqrt{3}) V_n \quad :$$

$$q = 7 - 4\sqrt{3} \quad (V_n)$$

$$V_0 = \frac{U_0 - \sqrt{3}}{U_0 + \sqrt{3}} = \frac{-1 - \sqrt{3}}{-1 + \sqrt{3}} \quad :$$

$$V_0 = \frac{(-1 - \sqrt{3})(-1 - \sqrt{3})}{(-1 + \sqrt{3})(-1 - \sqrt{3})} = \frac{(1 + \sqrt{3})}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2}$$

$$V_0 = -2 - \sqrt{3}$$

$$: \lim_{n \rightarrow +\infty} V_n \quad -$$

$$V_n = V_0 \cdot q^n = -(2 + \sqrt{3})(7 - 4\sqrt{3})^n$$

$$\begin{aligned} \lim_{n \rightarrow +\infty} V_n &= \lim_{n \rightarrow +\infty} \left[-(2 + \sqrt{3})(7 - 4\sqrt{3})^n \right] \\ &= \lim_{n \rightarrow +\infty} - (2 + \sqrt{3})(2 - \sqrt{3})^{2n} = 0 \end{aligned}$$

$$0 < 2 - \sqrt{3} < 1 \quad :$$

$$: \lim_{n \rightarrow +\infty} U_n$$

$$V_n (U_n + \sqrt{3}) = U_n - \sqrt{3} \quad : \quad V_n = \frac{U_n - \sqrt{3}}{U_n + \sqrt{3}}$$

$$U_n V_n + V_n \sqrt{3} = U_n - \sqrt{3} \quad :$$

$$: \quad U_n V_n - U_n = -V_n \sqrt{3} - \sqrt{3} \quad :$$

$$U_n = \frac{-\sqrt{3}(V_n + 1)}{V_n - 1} : \quad U_n (V_n - 1) = -\sqrt{3}(V_n + 1)$$

$$\lim_{n \rightarrow +\infty} U_n = \lim_{n \rightarrow +\infty} \frac{-\sqrt{3}(V_n + 1)}{V_n - 1} = \sqrt{3} :$$

$$\cdot \lim_{n \rightarrow +\infty} V_n = 0 :$$

.12

$$: (U_n) : \alpha_0 \quad -1$$

$$U_{n+1} - U_n = r$$

$$U_{n+1} - U_n = \alpha U_n + 3 - U_n = U_n(\alpha - 1) + 3 :$$

$$\alpha = 1 : \quad \alpha - 1 = 0 :$$

$$\cdot \alpha = 1 : \quad U_{n+1} - U_n = 3 :$$

$$\cdot \alpha = 1 : \quad 3 \quad (U_n)$$

$$\alpha \neq 1 : \quad -2$$

$$: (U_n)$$

$$U_{n+1} = U_n : n$$

$$(\alpha - 1)U_n = -3 : \quad \alpha U_n + 3 = U_n :$$

$$U_0 = U_n : (U_n) \quad U_n = \frac{-3}{\alpha - 1} :$$

$$\cdot U_0 = \frac{-3}{\alpha - 1} :$$

$$: q = \alpha \quad -3$$

$$V_{n+1} = \beta U_{n+1} + \delta = \beta(\alpha U_n + 3) + \delta$$

$$= \alpha\beta U_n + 3\beta + \delta$$

$$= \alpha\beta U_n + \alpha\delta - \alpha\delta + 3\beta + \delta$$

$$= \alpha(\beta U_n + \delta) - \alpha\delta + 3\beta + \delta$$

$$= \alpha V_n - \alpha \delta + 3\beta + \delta$$

$$\delta(1-\alpha) = -3\beta \quad : \quad -\alpha\delta + 3\beta + \delta = 0 \quad :$$

$$\delta = \frac{-\beta}{1-\alpha} \quad q = \alpha \quad : \quad \delta = \frac{-3\beta}{1-\alpha} \quad :$$

$$-1 < \alpha < 1 \quad : \quad -1 < q < 1 \quad (V_n) \quad -$$

$$: \lim_{n \rightarrow +\infty} U_n \quad : \quad -$$

$$U_n = \frac{1}{\beta}(V_n - \delta) \quad : \quad V_n = \beta U_n + \delta \quad :$$

$$\lim_{n \rightarrow +\infty} U_n = \lim_{n \rightarrow +\infty} \frac{1}{\beta}(V_n - \delta) = \frac{-\delta}{\beta}$$

.13

$$: S_n \quad -1$$

$$n+1 \quad S_n$$

$$S_n = 1 \cdot \frac{1 - \left(\frac{1}{2}\right)^{n+1}}{1 - \frac{1}{2}} \quad : \quad q = \frac{1}{2}$$

$$S_n = 2 \left[1 - \left(\frac{1}{2}\right)^{n+1} \right] \quad :$$

$$: n \quad U_n \quad -2$$

$$U_1 = U_0 + \frac{1}{2} \quad :$$

$$U_2 = U_1 + \frac{1}{2^2}$$

$$U_3 = U_2 + \frac{1}{2^3}$$

⋮

$$U_{n-1} = U_{n-2} + \frac{1}{2^{n-1}}$$

$$U_n = U_{n-1} + \frac{1}{2^n}$$

$$U_n = U_0 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^n} :$$

$$U_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^n} :$$

$$U_n = 2 \left[1 - \left(\frac{1}{2} \right)^{n+1} \right] : \quad U_n = S_n :$$

$$: (U_n) \quad - 3$$

$$\lim_{n \rightarrow +\infty} U_n = \lim_{n \rightarrow +\infty} 2 \left[1 - \left(\frac{1}{2} \right)^{n+1} \right] = 2$$

$$\lim_{n \rightarrow +\infty} \left(\frac{1}{2} \right)^{n+1} = 0 :$$

$$: (U_n) \quad - 4$$

$$U_{n+1} - U_n = 2 \left[1 - \left(\frac{1}{2} \right)^{n+2} \right] - 2 \left[1 - \left(\frac{1}{2} \right)^{n+1} \right]$$

$$= -2 \left(\frac{1}{2} \right)^{n+2} + 2 \left(\frac{1}{2} \right)^{n+1}$$

$$= 2 \left(\frac{1}{2} \right)^{n+1} \left[-\frac{1}{2} + 1 \right] = \left(\frac{1}{2} \right)^{n+1}$$

$$: (U_n) : \quad U_{n+1} - U_n > 0 :$$

.14

$$2 - U_n \geq 0 : p(n) \quad - 1$$

$$4 \geq 0 : 2 - U_0 \geq 0 : n = 0 \quad -$$

$$: p(0) :$$

$$: p(k+1) \quad p(k) \quad -$$

$$p(k) : 2 - U_k \geq 0 :$$

<http://www.onepd.edu> $p(k+1) : 2 - U_{k+1} \geq 0$ جميع الحقوق محفوظة

$$U_k \leq 2 : 2 - U_k \geq 0 :$$

$$\begin{aligned} \sqrt{2+U_k} \leq 2 & : & 2+U_k \leq 4 & : \\ 2-U_{k+1} \geq 0 & : & U_{k+1} \leq 2 & : \\ & & p(k+1) & : \\ & \cdot n & p(n) & \end{aligned}$$

$$U_n \geq 0 :$$

$$p(n) : U_n \geq 0$$

$$U_1 \geq 0 : n = 1$$

$$p(1) : U_1 = \sqrt{2+U_0} = \sqrt{2-2} = 0 :$$

$$p(k+1) \quad p(k)$$

$$p(k) : U_k \geq 0 :$$

$$p(k+1) : U_{k+1} \geq 0 :$$

$$U_k + 2 \geq 0 : U_k \geq 0 :$$

$$U_{k+1} \geq 0 : \sqrt{U_k + 2} \geq 0 :$$

$$\cdot n \quad p(n)$$

$$: (U_n) \quad -2$$

$$\begin{aligned} U_{n+1} - U_n &= \sqrt{2+U_n} - U_n = \frac{[\sqrt{2+U_n} - U_n] \sqrt{2+U_n} + U_n}{\sqrt{2+U_n} + U_n} \\ &= \frac{2+U_n - U_n^2}{\sqrt{2+U_n} + U_n} = \frac{-U_n^2 + U_n + 2}{\sqrt{2+U_n} + U_n} \\ &\quad -U_n^2 + U_n + 2 : \end{aligned}$$

$$: U_n = -1 \quad U_n = 2 : \Delta = 9 :$$

$$-U_n^2 + U_n + 2 = -(U_n - 2)(U_n + 1) = (2 - U_n)(U_n + 1)$$

$$U_{n+1} - U_n = \frac{(2 - U_n)(U_n + 1)}{\sqrt{2+U_n} + U_n} :$$

$$U_{n+1} - U_n \geq 0 : U_n \geq 0 \quad 2 - U_n \geq 0 : (U_n) :$$

$$: (U_n) \quad -3$$

$$U_n \leq 2 : \quad 2 - U_n \geq 0 : (U_n)$$

.15

$$(U_n) \quad (1-1)$$

$$U_{n+1} - U_n = U_n \times 0,06 :$$

$$U_{n+1} = U_n + U_n \times 0,06 :$$

$$U_{n+1} = U_n (1 + 0,06)$$

$$U_{n+1} = 1,06 \times U_n :$$

$$q = 1,06 \quad (U_n) :$$

$$: n \quad U_n \quad (2)$$

$$U_n = 60 \cdot (1,06)^n : \quad U_n = U_0 \times q^n :$$

$$: (3)$$

$$S_n = U_0 + U_1 + \dots + U_n :$$

$$S_n = U_0 \times \frac{1 - q^{n+1}}{1 - q} = 60 \times \frac{1 - (1,06)^{n+1}}{1 - 1,06}$$

$$S_n = 60 \times \frac{1 - (1,06)^{n+1}}{0,06} = 60 \times \frac{1 - (1,06)^{n+1}}{\frac{6}{100}}$$

$$S_n = \frac{6000}{6} \times [1 - (1,06)^{n+1}]$$

$$S_n = 1000 \times [1 - (1,06)^{n+1}]$$

$$n \quad U_n \quad - \text{II}$$

$$n+1 \quad U_{n+1}$$

$$U_{n+1} = U_n + U_n \times \frac{6}{100} :$$

$$U_{n+1} = U_n + U_n \times 0,06$$

$$U_{n+1} = 1,06 \cdot U_n :$$

$$U_0 = 60 : \quad U_n = 60(1,06)^n :$$

2000

$$U_7 \quad 2007$$

$$U_7 \approx 90,22 : \quad U_7 = 60(1,06)^7 :$$

90,22 \quad 2007

16

(v_n) - 1

$$V_{n+1} = U_{n+2} - U_{n+1} = (\lambda + 1)U_{n+1} - \lambda U_n - U_{n+1}$$

$$= \lambda U_{n+1} - \lambda U_n = \lambda (U_{n+1} - U_n) = \lambda V_n$$

$$q = \lambda \quad (V_n) \quad V_{n+1} - \lambda V_n :$$

: \lambda \quad n \quad V_n \quad -

$$V_n = V_0 \cdot q^n, \quad V_0 = U_1 - U_0 = 1 :$$

$$V_n = \lambda^n :$$

: S_n - 2

$$S_n = V_0 \times \frac{1 - q^n}{1 - q} = \frac{1 - \lambda^n}{1 - \lambda} :$$

$$S_n = U_n - 1 :$$

$$V_0 = U_1 - U_0 \quad :$$

$$V_1 = U_2 - U_1$$

$$V_2 = U_3 - U_2$$

$$\vdots$$

$$V_{n-2} = U_{n-1} - U_{n-2}$$

$$V_{n-1} = U_n - U_{n-1}$$

$$V_0 + V_1 + \dots + V_{n-1} = U_n - U_0 \quad :$$

$$U_n = \frac{1 - \lambda^n}{1 - \lambda} + 1 \quad : \quad S_n = U_n - 1 \quad :$$

$$S'_n \quad :$$

$$S'_n = U_0^2 + U_1^2 + \dots + U_{n-1}^2$$

$$U_n = \frac{1 - 3^n}{1 - 3} + 1 \quad : \quad U_n = \frac{1 - \lambda}{1 - \lambda} + 1 \quad :$$

$$U_n = -\frac{1}{2} + \frac{1}{2} \cdot 3^n \quad : \quad U_n = -\frac{1}{2}(1 - 3^n) + 1 \quad :$$

$$U_n = -\frac{1}{2}(1 + 3^n)$$

$$S'_n = \left(\frac{1}{2}\right)^2 (1 + 3^0) + \left(\frac{1}{2}\right)^2 (1 + 3^1)^2 + \dots + \left(\frac{1}{2}\right)^2 (1 + 3^{n-1})^2$$

$$S'_n = \frac{1}{4} \left[1 + 2 \cdot 3^0 + (3^0)^2 \right] + \frac{1}{4} \left[1 + 2 \cdot 3^1 + (3^1)^2 \right] + \dots$$

$$\dots + \frac{1}{4} \left[1 + 2 \cdot 3^{n-1} + (3^{n-1})^2 \right]$$

$$S'_n = \frac{1}{4} \left[1 + 1 + \dots + 1 + 2(3^0 + 3^1 + \dots + 3^{n-1}) \right]$$

$$+ \left(3^0 + 3^2 + \dots + 3^{2(n-1)} \right) \Big]$$

$$S'_n = \frac{1}{4} \left[n + 2 \times \frac{1 - 3^n}{1 - 3} + \frac{1 - 3^{2n}}{1 - 3^2} \right]$$

$$= \frac{1}{4} \left[n - 1 + 3^n + \frac{1 - 3^{2n}}{-8} \right]$$

$$= \frac{1}{4} \left[n + 3^n - 1 - \frac{1}{8} + \frac{1}{8} 3^{2n} \right]$$

$$= \frac{1}{4} \left[n + \frac{1}{8} 3^{2n} + 3^n - \frac{9}{8} \right]$$

: n

$$\frac{1}{4} \left(\frac{1}{8} 3^{2n} + 3^n - \frac{9}{8} \right) = 225 :$$

$$: 3^n = t : 3^{2n} + 8 \cdot 3^n - 7209 = 0 :$$

$$3^n = 81 : t = 81 : t^2 - 8t - 7209 = 0$$

. $n = 4 :$

