King Saud University
Department of Mathematics

244
Final Exam, May 2016

NAME:

Group Number/Instructor's Name:

ID:

Question	Grade
I	
II	
III	
IV	
Total	

Question	1	2	3	4	5	6	7	8	9	10
Answer										

I) Choose the correct answer (write it in the table above):

1) If $A^{-1} = \begin{bmatrix} -3 & -2 & 2 \\ 2 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, then the adjoint adj A equals

(a) $\frac{1}{2}\begin{bmatrix} -3 & -2 & 2 \\ 2 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$	(b) $2\begin{bmatrix} -3 & -2 & 2 \\ 2 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$	(c) $\begin{bmatrix} 1 & 2 & 0 \\ -\frac{5}{2} & -4 & \frac{1}{2} \\ -\frac{1}{2} & 0 & 2 \end{bmatrix}$	(d) None of the previous

2) If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 6$, then $\begin{vmatrix} d & 2c \\ b & 2a \end{vmatrix}$ equals

(a) 3	(b) 12	(c) 6	(d) None of the previous

3) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 1 & 3 & 0 \end{bmatrix}$, then its inverse A^{-1} equals

(a) $\begin{bmatrix} -3 & 1 & 1 \\ 6 & -2 & -3 \\ 2 & -3 & -1 \end{bmatrix}$	(b) $\begin{bmatrix} -3 & 6 & 2 \\ 1 & -2 & -1 \\ 1 & -3 & -1 \end{bmatrix}$	(c) $\begin{bmatrix} 3 & -6 & -2 \\ -1 & 2 & 1 \\ -1 & 3 & 1 \end{bmatrix}$	(d) None of the previous

4) If $(2X - I_2)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$, then

(a) $X^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$	(b) $X^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$	(c) $X^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$	(d) None of the previous

5) If the set $\{v_1, v_2.v_3, v_4, v_5, v_6, v_7, v_8\}$ is a basis of the vector space $\mathbb{R}^n$, then

(a) $n > 8$	(b) $n < 8$	(c) $n = 8$	(d) None of the previous

6) If $v_1 = (1,1,0)$, $v_2 = (2,2,0)$, $v_3 = (1,-1,1)$, then the dimension of Span $\{v_1, v_2, v_3\}$ is

(a) 0	(b) 1	(c) 2	(d) 3

7) If $S = \{1+x, 2+x, x^2\}$ is a basis for $\mathcal{P}_2$ and the coordinate vector of $p(x) \in \mathcal{P}_2$ is $(p)_S = (1,2,3)$, then $p(x)$ is

(a) $1+2x+3x^2$	(b) $3+2x+3x^2$	(c) $5+3x+3x^2$	(d) None of the previous

8) If B is a 5×7 matrix and null $(B) = 3$, then null (B^T) equals

(a) 2	(b) 5	(c) 3	(d) 1

9) If $v_1 = (a,1,2,6)$ and $v_2 = (2,2a,1,-1)$ are two orthogonal vectors, then

(a) $a = 1$	(b) $a = -1$	(c) $a = 0$	(d) None of the previous

10) If B is a 3×3 matrix with $\det B = 2$, then

(a) nullity $(B) = 2$, rank $(B) = 1$	(b) nullity $(B) = 0$, rank $(B) = 3$	(c) nullity $(B) = 3$, rank $(B) = 3$	(d) None of the previous

II) A) Let $S = \{v_1 = (1, 2, 2, 1), v_2 = (3, 6, 6, 3), v_3 = (4, 9, 9, 4), v_4 = (5, 8, 9, 5)\}$.

i) Find a subset of S that forms a basis for $\text{span}(S)$.

ii) What is the dimension of $\text{span}(S)$?

B) Let $B = \{v_1 = (1,0,0), v_2 = (1,1,0), v_3 = (1,1,1,)\}$

i) Prove that B is a basis of $\mathbb{R}^3$.

ii) If $v = (0,-1,-1) \in \mathbb{R}^3$, find the coordinate vector $(v)_B$.

iii) Find the vector $w \in \mathbb{R}^3$, if its coordinate vector is $(w)_B = (2,1,-2)$.

C) Prove that the set $\{p_1 = 2 + 5x + x^2, p_2 = -x + 2x^2, p_3 = 3 + x^2\} \subset \mathcal{P}_2$ is linearly independent.

III) A) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by $T(x, y) = (3x, 2x + 4y)$.

i) Find the standard matrix of T.

ii) Is T one-to-one? Justify your answer.

iii) Compute T^{-1}.

iv) Find $(T \circ T)(x, y)$.

B) Find the standard matrix for the composed transformation in $\mathbb{R}^2$ given by a reflection about the line $y = x$, followed by a counterclockwise rotation about O, through $\theta = \dfrac{\pi}{6}$, followed by a reflection about the x-axis.

IV) A) Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{bmatrix}$. Is the matrix A invertible?

Justify your answer.

B) Let $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Find the eigenspace of B that corresponds to the eigenvalue $\lambda = 2$.

Scrap paper. It will be not be graded.