

السعودي ALSAADI
نسخة جديدة منقحة

1433/32

14

Ch. 5 - Part 2

- The Binomial Distribution.

STAT.110

جمال السعودي
رياضيات - إحصاء

Ch. 5 Part. 2

The Binomial Distribution

A binomial experiment is a probability experiment that satisfies the following four requirements:

1. There must be a fixed number of trials.
2. Each trial has only two outcomes: success or fail.
3. The outcomes of each trial must be independent of each other.
4. The probability of a success must remain the same for each trial.

Mean, Variance, and standard deviation for the binomial distribution

The mean, variance, and standard deviation of a variable that has the binomial distribution can be found by using the following formulas.

- Mean: $\mu = n \cdot p$
- Variance: $\sigma^2 = n \cdot p \cdot q$
- Standard deviation: $\sigma = \sqrt{n \cdot p \cdot q}$

Example:

A dice is rolled 480 times. Find the mean, variance, and standard deviation of the number of 2s that will be rolled.

Solution

Getting a 2 is a success and not getting a 2 is a failure:

- $n = 480$, $P = \frac{1}{6}$, and $q = \frac{5}{6}$
- $\mu = n.p = 480 \cdot \frac{1}{6} = 80$
- $\sigma^2 = n.p.q = 480 \cdot \left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = 66.7$
- $\sigma = \sqrt{n.p.q} = \sqrt{66.7} = 8.2$

Example:

A coin is tossed 4 times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.

Solution

The binomial distribution and

$$n = 4, \quad p = \frac{1}{2} \quad \text{and} \quad q = \frac{1}{2}$$

$$\mu = n \cdot p = 4 \cdot \frac{1}{2} = 2$$

$$\sigma^2 = n \cdot p \cdot q = 4 \cdot \frac{1}{2} \cdot \frac{1}{2} = 1$$

$$\sigma = \sqrt{1} = 1$$

Example:

If 3% of calculators are defective, find the mean, variance, and standard deviation of a lot of 300 calculators.

Solution

$$n = 300$$

$$p = 0.03$$

$$q = 0.97$$

- $\mu = n.p = (300)(0.03) = 9$
- $\sigma^2 = n.p.q = (300)(0.03)(0.97) = 8.7$
- $\sigma = \sqrt{\sigma^2} = \sqrt{8.73} = 2.9 \cong 3$

Example:

In a restaurant, a study found that 42% of all patrons smoked. If the seating capacity of the restaurant is 80 people, find the mean, variance, and standard deviation of the number of smokers. About how many seats should be available. For smoking customers?

Solution

$$n = 80$$

$$p = 0.42$$

$$q = 0.58$$

- $\mu = n.p = (80)(0.42) = 33.6$
- $\sigma^2 = n.p.q = (80)(0.42)(0.58) = 19.5$
- $\sigma = \sqrt{\sigma^2} = \sqrt{19.5} \cong 4.4$

Note

- two outcomes: yes or no → (binomial)
- more than two outcomes → (not binomial)

Which of the following are binomial experiments or can be reduced to binomial experiments?

صابون كثير الرغوة

- a. Surveying 100 People to determine if they like sudsy soap.
(Binomial)
-
- b. Tossing a coin 100 times to see how many heads occur
(Binomial)
-
- c. Asking 1000 people which brand of cigarettes they smoke.
(Not binomial)
-
- d. Testing one brand of aspirin by using 10 people to determine whether it is effective
(Binomial)
-
- f. Asking 100 people if they smoke
(Binomial)
-
- g. Checking 1000 applicants to see whether they were admitted to white Oak college.
(Binomial)
-
- h. Surveying 300 prisoners to see how many different crimes they were convicted of.
(Not binomial)
-
- i. Surveying 300 prisoners to see whether this is their first offense.
(Binomial)

A
L
S
A
A
D
I

Binomial Probability Formula

In a binomial experiment, the probability of exactly X successes in n trials is

$$P(x) = \frac{n!}{(n-x)!x!} \times p^x \times q^{n-x} = nC_x \times p^x \times q^{n-x}$$

Example:

يخمن

A student takes a 20 – question, true/ false exam and guesses on each question. Find the probability of passing if the lowest passing grade is 15 correct out of 20. Would you consider this event likely to occur? Explain your answer.

Solution

$$n = 20$$

$$p = \frac{1}{2}$$

$$q = \frac{1}{2}$$

$$p(\text{passing}) = p(x \geq 15)$$

$$= p(x = 15) + p(x = 16) + p(x = 17) + p(x = 18) + p(x = 19) + p(x = 20)$$

$$= 20C_{15} \left(\frac{1}{2}\right)^{15} \left(\frac{1}{2}\right)^5 + 20C_{16} \left(\frac{1}{2}\right)^{16} \left(\frac{1}{2}\right)^4$$

$$+ 20C_{17} \left(\frac{1}{2}\right)^{17} \left(\frac{1}{2}\right)^3 + 20C_{18} \left(\frac{1}{2}\right)^{18} \left(\frac{1}{2}\right)^2$$

$$+ 20C_{19} \left(\frac{1}{2}\right)^{19} \left(\frac{1}{2}\right)^1 + 20C_{20} \left(\frac{1}{2}\right)^{20} \left(\frac{1}{2}\right)^0$$

$$= 0.015 + 0.005 + 0.001 + \dots = 0.021 < 0.5$$

There for $P(\text{passing})$ unlikely to occur.

Example:

ضحية
 A survey found that 86% of Americans have never been a victim
 جريمة العنف
 of violent crime. If a sample of 12 Americans is selected at
 معقول يبدو
 random, find the probability that 10 or more have never been
 victims of violent crime. Does it seem reasonable that 10 or
 more have never been victims of violent crime?

Solution

$$n = 12 \quad p = 0.86 \quad q = 0.14$$

$$p(x \geq 10) = p(x = 10) + p(x = 11) + p(x = 12)$$

$$= 12C_{10} (0.86)^{10} (0.14)^2 + 12C_{11} (0.86)^{11} (0.14)^1 + 12C_{12} (0.86)^{12} (0.14)^0$$

$$= 0.77 > 0.5$$

Yes: it seem reasonable.....

Example:

مجتمع

خدمة

If 80% of the people in a community have internet access from their homes, find these probabilities for a sample of 10 people.

- على الأكثر
- At most 6 have internet access.
 - Exactly 6 have internet access.
 - At least 6 have internet access.
 - Which event a, b, or c is most likely to occur? Explain why?

Solution

$$n = 10$$

$$p = 0.8$$

$$q = 0.2$$

$$(a) P(\text{at most } 6) = P(X \leq 6)$$

$$= P(X = 6) + P(X = 5) + P(X = 4) + P(X = 3) + P(X = 2) + P(X = 1) + P(X = 0)$$

$$= 10 C_6 (0.8)^6 (0.2)^4 + 10 C_5 (0.8)^5 (0.2)^5 + 10 C_4 (0.8)^4 (0.2)^6$$

$$+ 10 C_3 (0.8)^3 (0.2)^7 + 10 C_2 (0.8)^2 (0.2)^8 + 10 C_1 (0.8)^1 (0.2)^9$$

$$+ 10 C_0 (0.8)^0 (0.2)^{10} = 0.121$$

$$(b) P(X = 6) = 10 C_6 (0.8)^6 (0.2)^4 = 0.088$$

$$(c) P(\text{at least } 6) = P(X \geq 6) = \dots = 0.967.$$

(d) Event c is most likely to occur because it's > 0.5

A die is rolled 5 times. the probability of getting a number 4 one time only is
 A) 0.402 B) 0.167 C) 0.015 D) 0.386

$$n = 5 \quad p = \frac{1}{6} \quad q = \frac{5}{6}$$

$$P(X = ?) = n C_x \cdot p^x \cdot q^{n-x} \rightarrow \text{قانونه}$$

$$\begin{aligned} P(X = 1) &= 5 C_1 \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^{5-1} \\ &= 0.4018 \approx \underline{\underline{0.402}} \end{aligned}$$

A
L
S
A
A
D
I

A die is rolled 5 times, the probability of getting a number 5 exactly two times only is
 A) 0.598 B) 0.161 C) 0.839 D) 0.402

$$n = 5 \quad p = \frac{1}{6} \Rightarrow q = \frac{5}{6} \quad x=2$$

$$P(X) = n C_x p^x q^{n-x} \quad \text{قانونه}$$

$$P(X=2) = 5 C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{5-2} \quad \text{بالا} = 0.161$$

$$n = 7$$

$$p = \frac{1}{4} \quad \therefore q = \frac{3}{4}$$

A student takes a 7 question multiple choice quiz with 4 choices for each question. If the student guesses at random on each question, what is the probability that the student gets exactly 3 questions correct?

$$?$$

$$x = 3$$

- A) 0.130 B) 0.346 C) 0.173 D) 0.043

$$P(x) = {}^n C_x p^x q^{n-x}$$

$$P(x=3) = {}^7 C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{7-3} = 0.173$$

The outcomes of each trial in a binomial experiment

- A) are unlimited B) are independent C) are dependent D) must be fixed

من ضمن الخواص الاربعة (مفتوحاً)
binomial

The number of trials in a binomial experiment ... من ضمن خواص binomial

- A) are independent. B) are dependent. C) must be fixed. D) are unlimited.

A study shows that 70% of drivers consider themselves above average in driving ability. If 10ⁿ drivers at random are chosen, what is the mean and variance of the number of drivers who consider themselves above average?

- A) mean = 7 and variance = 7. C) mean = 7 and variance = 2.1.
 B) mean = 10 and variance = 1.45 D) mean = 10 and variance = 10.

$$p = 0.70 = 0.7 \Rightarrow q = 0.3, n = 10$$

$$\text{mean: } \mu = n \cdot p = (10)(0.7) = \boxed{7}$$

$$\text{Variance: } \sigma^2 = n \cdot p \cdot q = (10)(0.7)(0.3) = \boxed{2.1}$$

A
L
S
A
A
D
I

"A distribution using the means computed from all possible random samples of a specific size taken from a population." The previous statement is the definition of

- A) central limit theorem B) sampling distribution C) sampling error D) empirical distribution

Chapter Quiz

Determine whether each statement is true or false. If the statement is false explain why.

①. The expected value of a random variable can be thought of as a long – run average.
 فترة طويلة

②. The number of courses a students is taking this semester is an example of a continuous random variable.

③. when the multinomial distribution is used, the outcomes must be dependent.

④. A binomial experiment has a fixed number of trials.

Complete these statements with the best answer:

⑤. Random variable values are determined by **chance**.

⑥. The mean for a binomial variable can be found by using the formula **$\mu = n \cdot p$** .

⑦. One requirement for a probability distribution is that the sum of all the events in the sample space must equal **1**.

** A probability distribution can be graphed using **bar char.**

A
L
S
A
A
D
I

Select the best answer:

8. What is the sum of the probabilities of all outcomes in a probability distribution?

a. 0

c. 1

b. 1/2

d. It cannot be determined.

9. How many outcomes are there in a binomial experiment?

a. 0

c. 2

b. 1

d. It varies

For questions 11 through 14, determine if the distribution represents a probability distribution. If not, state why.

11

X	1	2	3	4	5
P(X)	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{2}{7}$

→ No where $\sum P(x) > 1$

12

X	3	6	9	12	15
P(X)	0.3	0.5	0.1	0.08	0.02

→ yes

13

X	50	75	100
P(X)	0.5	0.2	0.3

→ yes

14

X	4	8	12	16
P(X)	$\frac{1}{6}$	$\frac{3}{12}$	$\frac{1}{2}$	$\frac{1}{12}$

→ yes

إندري 5 Ch.

كل الأمنيات بالنجاح والتوفيق

السعدى