السعدي ALSAADI

نسخة جديدة منقحة

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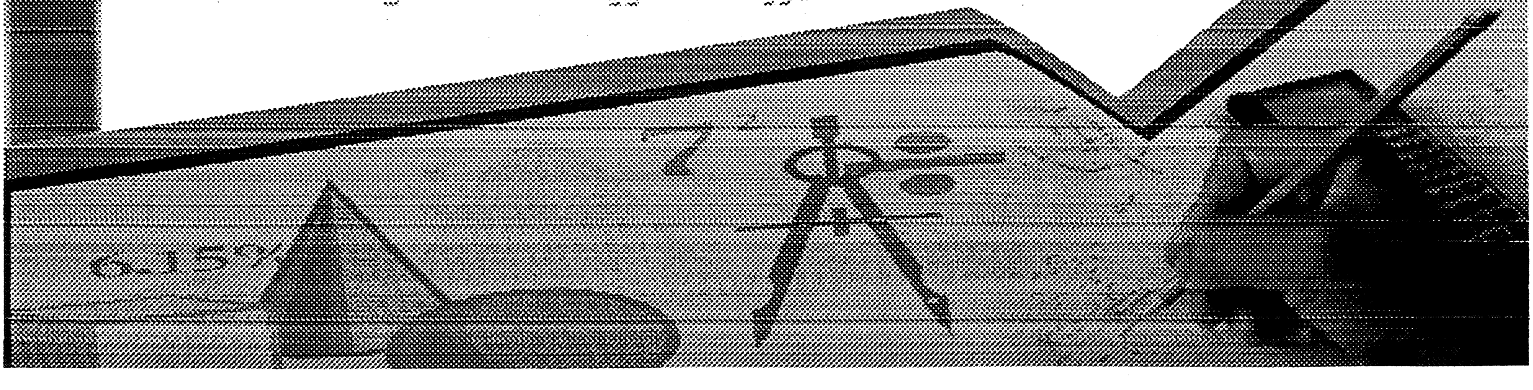
Ch. 4 - Part 3

- The Multiplication Rules and Conditional Probability.

STAT.110

جمال السعدي

رياضيات - إحصاء



Ch. 4 Part. 3

The Multiplication Rules and Conditional Probability

Two events A and B are independent events if :

A occurs does not affect the probability of B occurring.

Multiplication Rule 1

When two events are independent, the probability of both occurring is P(A and B) = P(A) × P(B)

Example:

An urn contains 3 red balls, 2 blue balls, and 5 white balls.
A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.

a. Selecting 2 blue balls.

b. Selecting 1 blue ball and then 1 white ball.

c. Selecting 1 red ball and then 1 blue ball.

Solution

a. P (blue and blue) = P(blue) × P(blue) $= \frac{2}{10} \times \frac{2}{10} = \frac{4}{100} = \frac{1}{25}$

b. P (blue and white) = P(blue) × P(white) $= \frac{2}{10} \times \frac{5}{10} = \frac{10}{100} = \frac{1}{10}$

c. P (red and blue) = P(red) × P(blue) $= \frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = \frac{3}{50}$

Example:

Approximately 9% of men have a type of color blindness that prevents them from distinguishing between red and green. If 3 men are selected at random, find the probability that all of them will have this type of red-green color blindness.

Solution

Let C denote red-green color blindness. Then

$$P(\text{ C and C and C}) = P(C) \times P(C) \times P(C)$$
$$= (0.09)\ (0.09)\ (0.09)$$
$$= 0.000729$$

Example:

A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

Solution

$$P(\text{ head and 4}) = P(\text{head}) \times P(4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Note

- **IF P(A) < 0.5**

A is unlikely to occur

- **If P(A) ≥ 0.5**

A is likely to occur

- **If P(A) = L**

➔ P(none A) = 1 - L

Example:

If 28% of U.S. medical degrees are conferred to women, find the probability that 3 randomly selected medical school graduates are men. Would you consider this event likely or unlikely to occur? Explain your answer.

Solution

- P (W) = 0.28
- P (M) = 1 – P (W)

= 1 – 0.28 = 0.72

P (3M) = P (M) . P (M) , P (M)

= (0.72) (0.72) (0.72)

= 0.373

The event is unlikely to occur because P (3M) < 0.5

Example:

Eighty-eight percent of U.S. children are covered by some type of health insurance. If 4 children are selected at random, what is the probability that none are covered?

Solution

P(covered) = 0.88

P(non covered) = 1 – 0.88 = 0.12

P(4 children are non covered) = (0.12) (0.12) (0.12) (0.12) = 0.0002

Multiplication Rule 2

When two events are dependent, the probability of both occurring is

$P(A \text{ and } B) = P(A) \times P(B/A)$

Example:

A person owns a collection of 30 CDs, of which 5 are country music. If 2 CDs are selected at random, find the probability that both are country music.

Solution

Since the events are dependent,

$$P(C_1 \text{ and } C_2) = P(C_1) \times P(C_2 \mid C_1) = \frac{5}{30} \times \frac{4}{29} = \frac{20}{870} = \frac{2}{87}$$

Example:

In a civic organization, there are 38 members; 15 are men and 23 are women. If 3 members are selected to plan the July 4th parade, find the probability that all 3 are women. Would you consider this event likely or unlikely to occur? Explain your answer.

Solution

The total members

= 15 men + 23 women = 38

$$P(3 \text{ women}) = \frac{23}{38} \times \frac{22}{37} \times \frac{21}{36} = 0.21 \qquad < 0.5$$

There for: This event unlikely to occur.

Conditional probability

- P(A/B)

Probability that A occur
After B already occurred

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)} \rightarrow P(A \text{ and } B) = P(B) \times P(A/B)$$

$$P(B/A) = \frac{P(A \text{ and } B)}{P(A)} \rightarrow P(A \text{ and } B) = P(A) \times P(B/A)$$

Example:

A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Find these probabilities.

a. The respondent answered yes, given that the respondent was a female.

b. The respondent was a male, given that the respondent answered no.

Solution

Let: M = Male F = Female
Y = yes N = No

$$a.\ P(Y/F) = \frac{P(Y \text{ and } F)}{P(F)} = \frac{\frac{8}{100}}{\frac{50}{100}} = \underline{\underline{0.16}}$$

$$b.\ P(M/N) = \frac{P(M \text{ and } N)}{P(N)} = \frac{\frac{18}{100}}{\frac{60}{100}} = \underline{\underline{0.3}}$$

Example:

An insurance company classifies drivers as low-risk, medium-risk, and high-risk. Of those insured, 60% are low-risk, 30% are medium-risk, and 10% are high-risk. After a study, the company finds that during a 1-year period, 1% of the low-risk drivers had an accident, 5% of the medium-risk drivers had an accident, and 9% of the high-risk drivers had an accident. If a driver is selected at random, find the probability that the driver will have an accident during the year.

Low
P(A) × P(B/A) ⟹
(0.60) × (0.01)

Medium
P(A) × P(B/A) ⟹
(0.30) × (0.05)

High
P(A) × P(B/A) ⟹
(0.10) × (0.09)

Solution

P(have an accident)

= P(low – risk and have an accident) ➔ (0.6) (0.01)

+ P(medium – risk and have an accident) ➔ (0.3) (0.05)

+ P(high – risk and have an accident) ➔ (0.1) (0.09)

= 0.03

حالة الـ Coin

At least one

Find the probability of getting at least one .....

- (1) A coin is tossed 3 times:
  $N(s) = 2^3 = 8$

$$\therefore P(\text{at least one tail}) = \frac{N(s) - 1}{N(s)} = \frac{8-1}{8} = \frac{7}{8}$$

- (2) A coin is tossed 5 times :
  $N(s) = 2^5 = 32$

$$\therefore P(\text{at least one head}) = \frac{N(s) - 1}{N(s)} = \frac{32-1}{32} = \frac{31}{32}$$

حالة الـنسب المئوية

Rule

- **A: at least one ....**
- **A': no .... ≡ ....**
- **$P(A') = (\ \ )\ (\ \ )\ (\ \ )$........**
- **$P(A) = 1 - P(A')$**

Example:

It has been found that 6% of all automobiles on the road have defective brakes. If 5 automobiles are stopped and checked by the state police, find the probability that at least one will have defective brakes.

Solution

A = at least one have defective brakes
A' = no have defective brakes
P(defective) = 0.06 P(undefective) = 1 – 0.06 = 0.94
$P(A') = (0.94)\ (0.94)\ (0.94)\ (0.94)\ (0.94) = \underline{\underline{0.7339}}$
$P(A) = 1 - P(A') = 1 - 0.7339 = \underline{\underline{0.266}}$

Use the following to answer questions

An apartment building has the following apartments:

	1st Floor	2nd Floor	3rd Floor	Total
2 Bedroom	3	1	2	6
3 Bedroom	1	$\bigcirc\!\!\!\!3$	2	6
Total	4	4	4	12

If an apartment is selected at random,

what is the probability that it is on $\overset{A}{\text{the 2nd floor}}$ $\overset{\cup}{\text{or}}$ has $\overset{B}{\text{3 bedrooms}}$?

A) 4/12 B) 6/12 [✓ C) 7/12] D) 3/12

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{12} + \frac{6}{12} - \frac{3}{12} = \boxed{\frac{7}{12}}$$

what is the probability that it is $\overset{A}{\text{a 3 bedroom apartment}}$ given / that it is $\overset{B}{\text{on the 3rd floor}}$?

A) 2/12 [✓ B) 6/12] C) 4/12 D) 1/12

** كلمة given that

تدل على الاحتمال المشروط

ما قبل كلمة given that يمثل A

ما بعد // // // // B

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{\cancel{12}}}{\frac{4}{\cancel{12}}} =$$

$$= \frac{2}{4} = \frac{1}{2} = \boxed{\frac{6}{12}}$$

Use the following to answer questions

In a recent study, the following data was obtained in response to the question, "Do you favor recycling in your neighborhood? "

	Yes	No	*No opinion*	Total
Males	23	17	8	48
Females	7	8	12	27
Total	30	25	20	75

If a person is selected at random, use the above table to answer the following questions.

F / X

The probability that a person is a female given that she answered yes regarding recycling is:
A) 0.68 B) 0.32 C) 0.767 D) 0.233

$$P(F/y) = \frac{P(F\cap y)}{P(y)} = \frac{7/75}{30/75} = \frac{7}{30} \approx \boxed{0.233}$$

What is the probability that a person has no opinion regarding recycling?
A) 0.267 B) 0.333 C) 0.4 D) 0.64

$$P(\text{no opinion}) = \frac{20}{75} \approx \boxed{0.267}$$

What is the probability that a person is a male and he answered no regarding recycling?
A) 0.107 B) 0.227 C) 0.093 D) 0.16

$$P(m \cap No) = \frac{17}{75} \approx \boxed{0.227}$$

The probability that a person is a male or he has no opinion regarding recycling is:
A) 0.467 B) 0.8 C) 0.747 D) 0.587

$$P(m \cup \text{no opinion})$$
$$= P(m) + P(\text{no opinion}) - P(m \cap \text{no opinion})$$
$$= \frac{48}{75} + \frac{20}{75} - \frac{8}{75} = \boxed{0.8}$$

The manager of a bank recorded the amount of time each customer spent waiting in line during الذروة peak business hours one Monday. The frequency table below summarizes the results.

Waiting Time (minutes)	Number of Customers
0-3	14
4-7	9
8-11	11
12-15	6
16-19	7
20-23	3
24-27	2
	$n = \Sigma f = 52$

If we randomly select one of the customers represented in the table, what is the probability that the waiting time is at least 12 minutes or between 8 and 15 minutes?

A) 0.519 B) 0.63 C) 0.558 D) 0.2

$$P(\underbrace{\text{at least 12 minutes}}_{A} \overset{\cup}{\text{or}} \underbrace{\text{between 8 and 15 minutes}}_{B})$$

$$= P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{18}{52} + \frac{17}{52} - \frac{6}{52} \quad \text{بالآلة} \quad = \frac{29}{52} \simeq \boxed{0.558}$$

The probability that a student has a computer is 0.91 and the probability that he has a car is 0.49 while the probability that he has both is 0.46. Find the probability that the student has a computer given that he has a car.

A) 0.82 B) 0.51 C) 0.94 D) 0.65

$P(\text{has computer}) = 0.91$ ، $P(\text{has car}) = 0.49$ ، $P(\text{has both}) = 0.46$

(has computer given that has car)

$$= P(\text{has computer} / \text{has car})$$

كلمة given that تعني احتمال مشروط

$$= \frac{P(\text{has comp. and has car})}{P(\text{has car})}$$

$$= \frac{P(\text{both})}{P(\text{car})} = \frac{0.46}{0.49} = 0.938 \simeq \boxed{0.94}$$

Use the following to answer questions

The table below shows the number of earned degrees in the year 2008 in a university by level and gender. A person who earned a degree in the year 2008 from this university is randomly selected. Find the probability of selecting someone who

		Male	Female
Level of Degree	Bachelor's	240	180
	Master's	35	15
	PhD's	25	5

is a female given that the person earned a bachelor's degree.
A) 0.83 B) 0.57 C) 0.17 D) 0.43

* مجموع الدرجات العلمية

$$240 + 180 + 35 + 15 + 25 + 5 = \underline{\underline{500}}$$

* كلمة given that
تعني احتمال مشروط
Conditional Probability

$$P(\text{Female} / \text{bach.})$$

$$= \frac{P(\text{Female} \cap \text{bach.})}{P(\text{bach.})}$$

* لإيجاد التقاطع خط افقي مع خط رأسي

$$= \frac{\frac{180}{\cancel{500}}}{\frac{240+180}{\cancel{500}}} = \frac{180}{240+180} = 0.428 \simeq \boxed{0.43}$$

earned a master's degree or is a female.
A) 0.47 B) 0.63 C) 0.45 D) 0.61

$$P(\text{master} \cup \text{Female})$$

$$= P(\text{mas.}) + P(\text{F.}) - P(\text{mas.} \cap \text{F.})$$

خط افقي مع خط رأسي

or → $\cup$
and → $\cap$
given that → /

$$= \frac{50}{500} + \frac{200}{500} - \frac{15}{500}$$

$$= \frac{235}{500} = \boxed{0.47}$$

A B

The probability that a student has a computer is 0.82 and the probability that he has a car is 0.48 while the probability that he has either a computer or a car is 0.86. Find the probability that the student has both.

A ∪ B

A) 0.38 B) 0.44 C) 0.34 D) 0.52

* تطبيق قانون
ثم حل معادله

$$P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}_{\text{has both}}$$

$$0.68 = 0.82 + 0.48 - P(A \cap B)$$

$$P(A \cap B) = 0.82 + 0.48 - 0.68 = \boxed{0.44}$$

Use the following to answer questions

A supermarket employs cashiers, managers and cleaner. The distribution of employees according to marital status is shown here.

	Cashier	Manager	Cleaner	Total
Married	9	12	3	24
Not married	5	15	2	22
Total	14	27	5	46

Find the probability that ...

$P(A \cup B)$

the employee is a manager or married

A) 24/46 B) 12/46 C) 27/46 D) 39/46

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{27}{46} + \frac{24}{46} - \frac{12}{46} = \frac{39}{46}$$

$P(A / B)$

the employee is a cashier given that he is married.

A) 9/24 B) 9/22 C) 9/46 D) 9/14

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{9}{\cancel{46}}}{\frac{24}{\cancel{46}}} = \frac{9}{24}$$

Use the following to answer questions

Two dice are rolled. Let X represents the summation of the two faces that will appear.

المتغير X يمثل

مجموع الوجهين الظاهرين

		Die 1					
	Sums	1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

The probability of X= 4 is

A) 0.083 B) 0.833 C) 0 D) 0.028

$$X : \text{sum}4 = \{ (1,3) , (2,2) , (3,1) \}$$

$$P(X=4) = \frac{3}{36} = \underline{\underline{0.083}}$$

T

The probability of X=15 is

A) 0.056 B) 0.028 C) 0.083 D) 0

$$P(X=15)$$

$$= P(\phi) = \underline{\underline{\text{Zero}}}$$

T

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