مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

Chapter 1

Vectors

physical quantities

- fundamental physical quantities

Quantity	Name	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	I
Temperature	Kelvin	k
Amount of substance	Mole	mol
Luminous intensity	Candela	cd

- derived physical quantities

Vector and Scalar quantities

Is completely specified by a number and appropriate units plus a direction.

Vectors have magnitude and direction

Example:

velocity, displacement, force and acceleration

Is completely specified by a single value with an appropriate unit and has no direction

scalars only have magnitude

Example:

temperature, speed, volume, mass, work, length, distance and time intervals

Displacement and distance

•The direction of the arrowhead (tip the arrow)

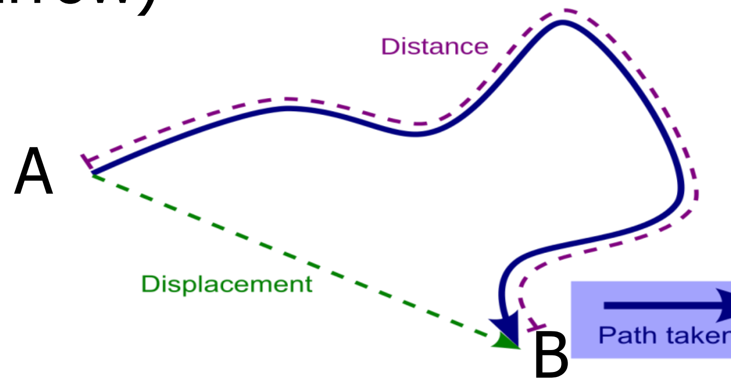
↓

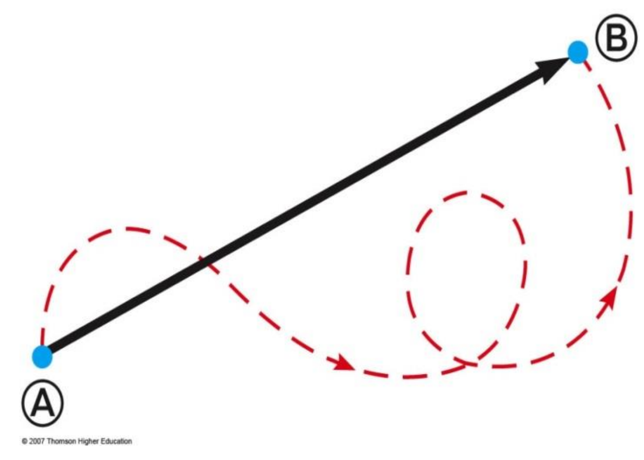
The direction of the displacement

•The length of the arrow

↓

The magnitude of the displacement

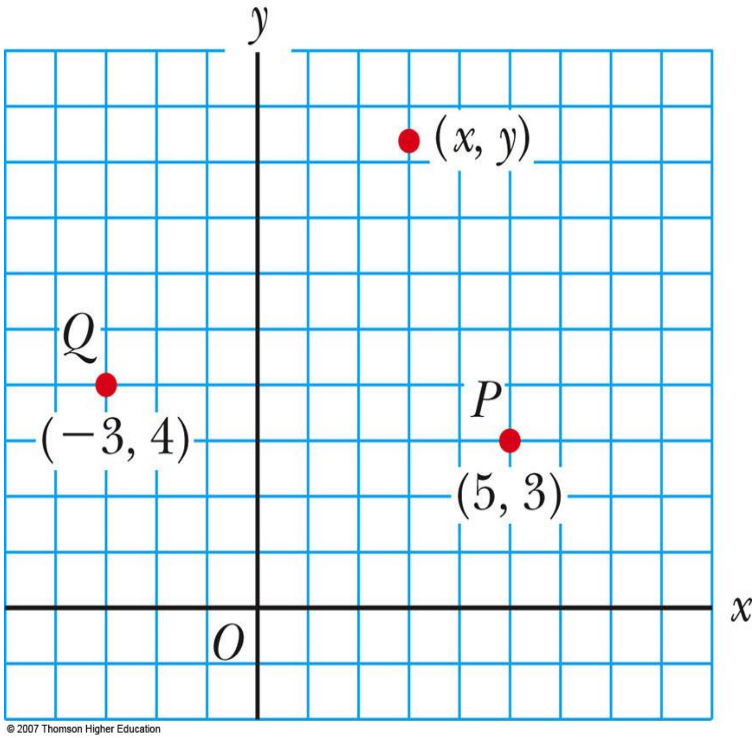




Coordinate Systems

- Used to describe the position of a point in space

horizontal (*x*- axes) and vertical (*y*- axes) axes intersect at a point defined as the origin



Cartesian Coordinate System

or rectangular coordinate system

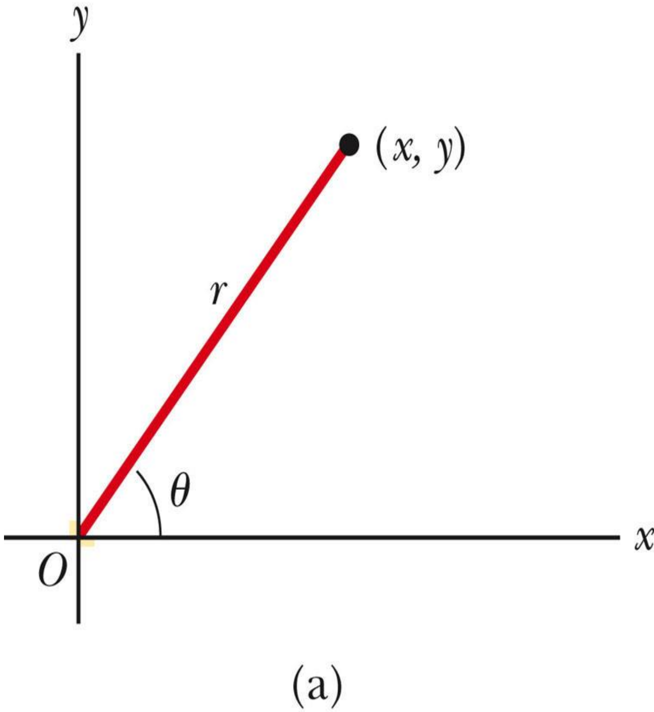
To find the value of x and y

Polar Coordinate System

To find the value of r and θ

r is the distance from the origin to the point having Cartesian coordinates (x, y),

θ is the angle between r and a fixed axis. This fixed axis is usually the positive x axis, and it is usually measured counterclockwise from it.



Polar to Cartesian Coordinates

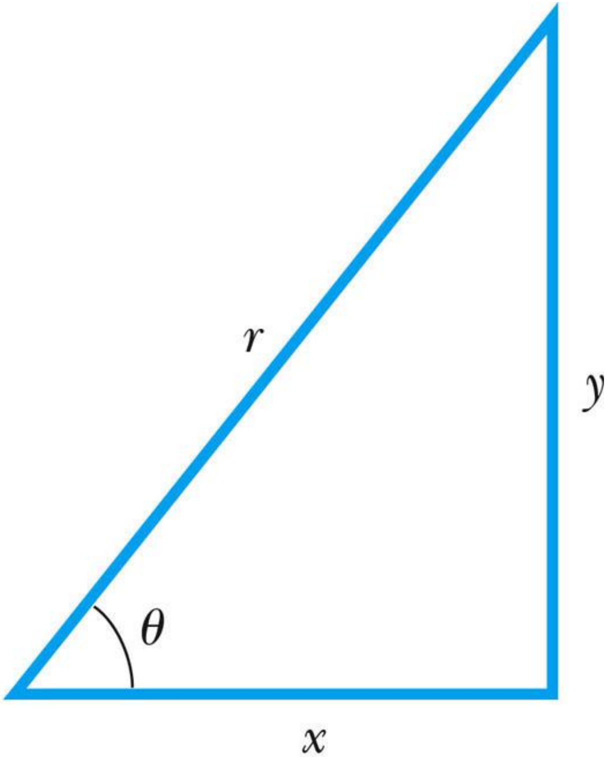
- From the right triangle

- $x = r \cos \theta$
- $y = r \sin \theta$

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



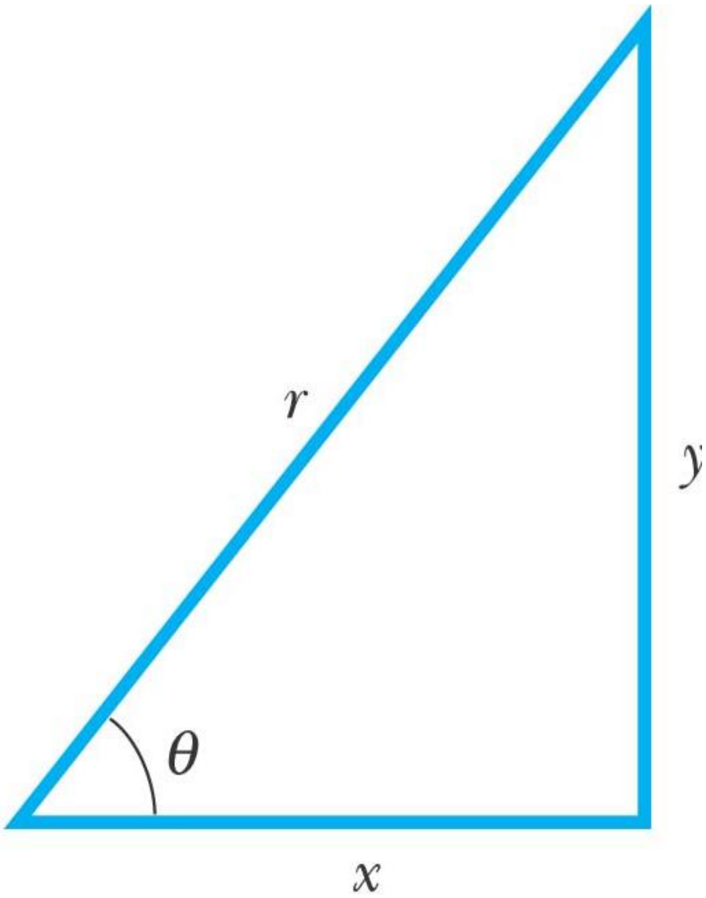
(b)

Cartesian to Polar Coordinates

- r is the hypotenuse and θ an angle

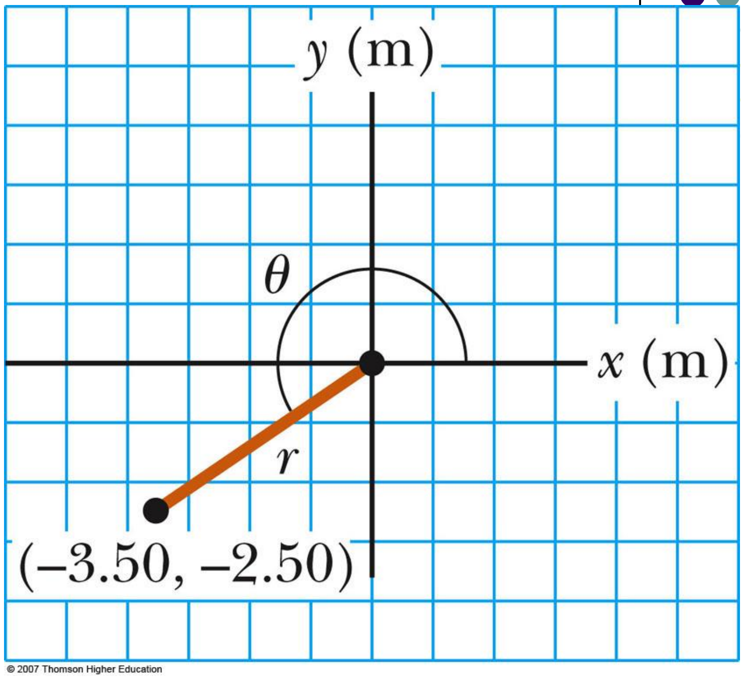
$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$



Example 3.1

- The Cartesian coordinates of a point in the *xy* plane are (*x*,*y*) = (-3.50, -2.50) m, as shown in the figure. Find the polar coordinates of this point.



Solution: From Equation 3.4,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = 4.30\text{ m}$$

and from Equation 3.3,

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = 216^\circ \quad \text{(signs give quadrant)}$$

magnitude the vector

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

the direction

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

Properties of Vectors

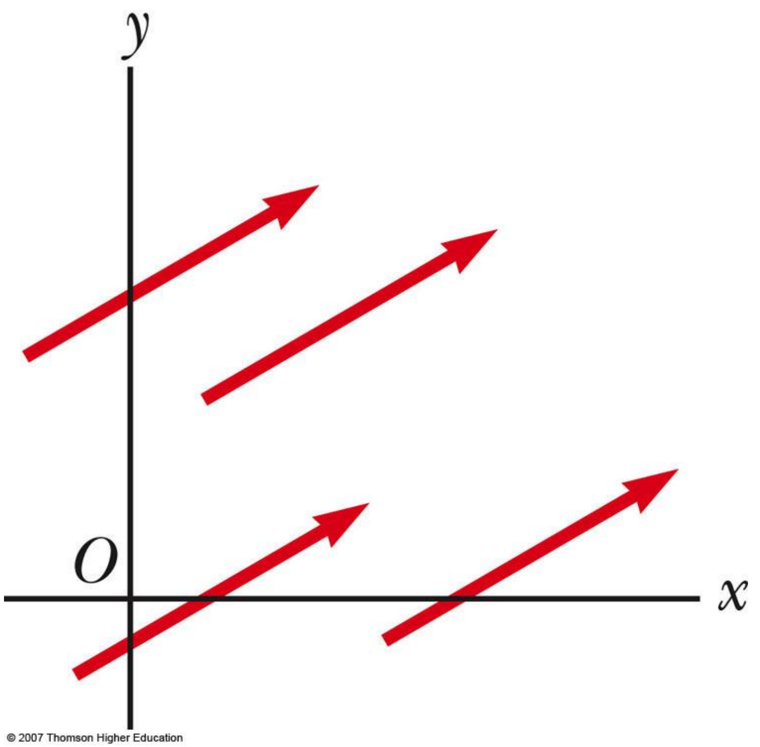
- **Equality of Two Vectors**

  A=B

- Two vectors are **equal** if they have

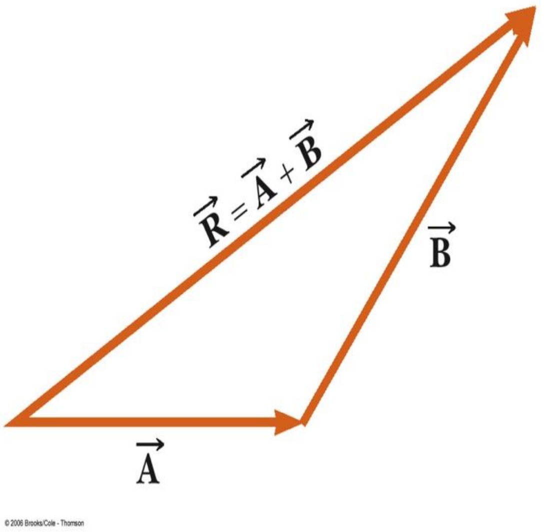
❖ the same magnitude

❖ the same direction along parallel lines



Adding Vectors (Triangle or Polygon Method)

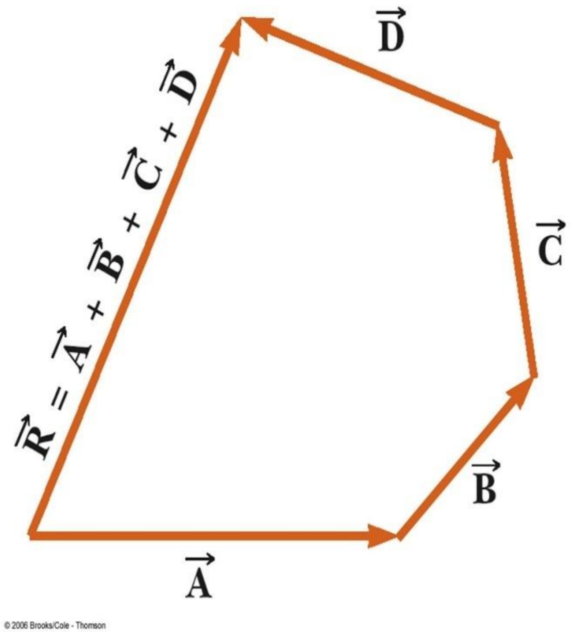
- To add vector **B** to vector **A**, first draw vector **A** on graph paper, with its magnitude represented by a convenient length scale, and then draw vector **B** to the same scale with its **tail** starting from the **tip** of **A**,
- **The resultant vector**
- **R = A+B is the vector drawn from the tail of A to the tip of B.**



Adding Vectors Geometrically (Triangle or Polygon Method)

- A geometric construction can also be used to add more than two vectors.
- In the case of four vectors, The resultant vector **R** = **A** + **B** + **C** + **D**
- is the vector that completes the **polygon**.

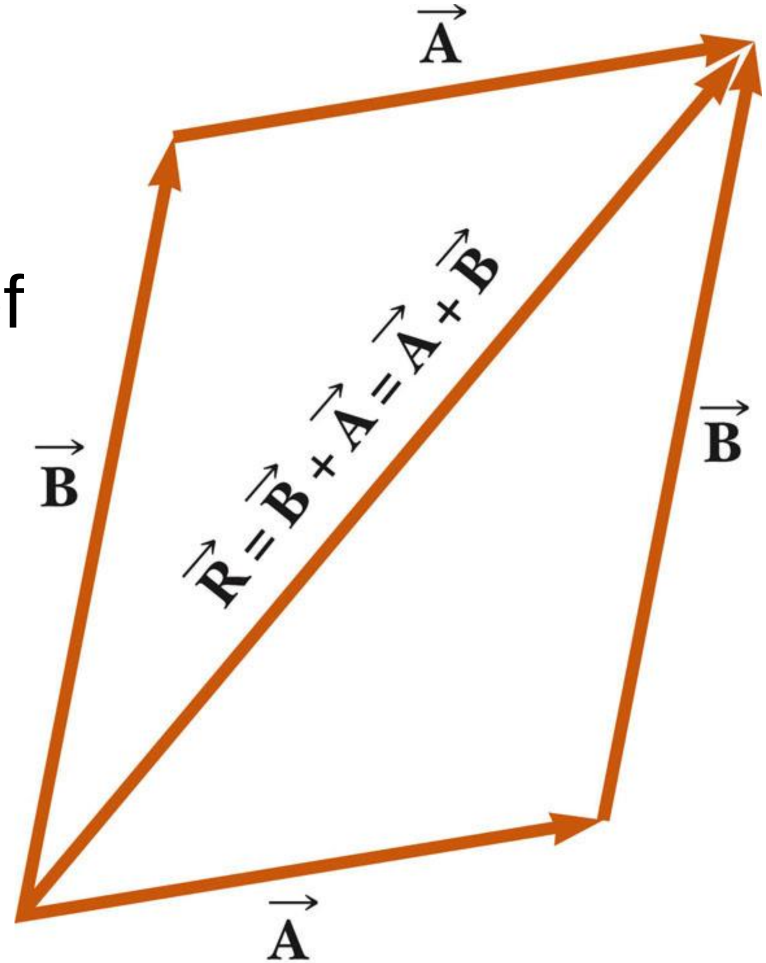
- **R is the vector drawn from the tail of the first vector to the tip of the last vector.**



Commutative Law of Addition

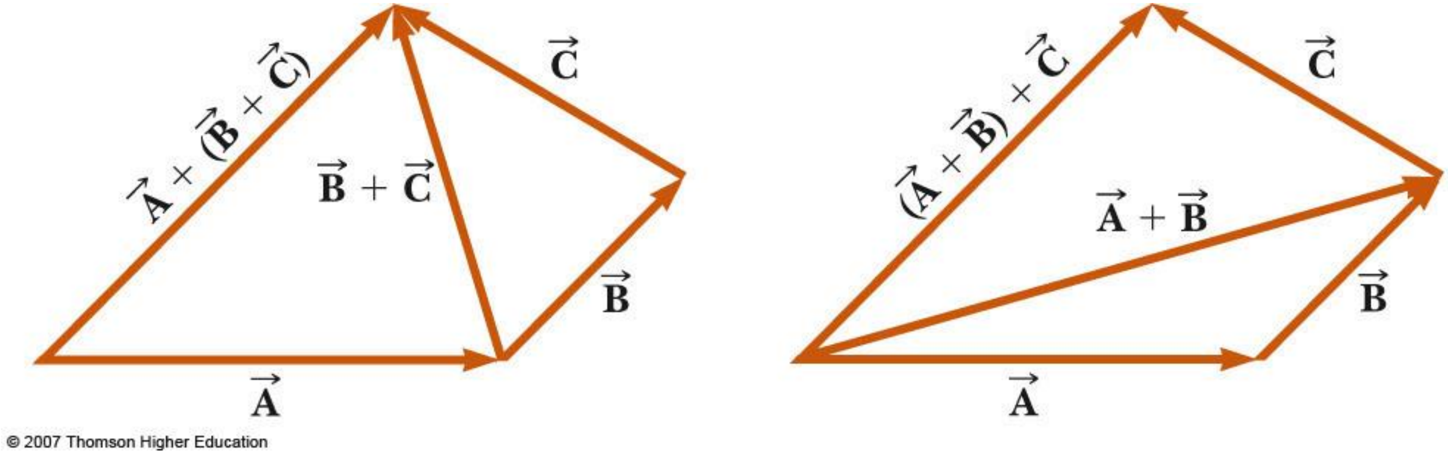
When two vectors are added, the sum is independent of the order of the addition.

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$



The associative law of addition

A + (B + C) = (A + B) + C



Negative Vectors

- Negative of the vector **A is** the vector that when added to **A** gives zero for the vector sum

$$\mathbf{A}+(-\mathbf{A})=0$$

The vectors **A** and **–A** have

❖the same magnitude

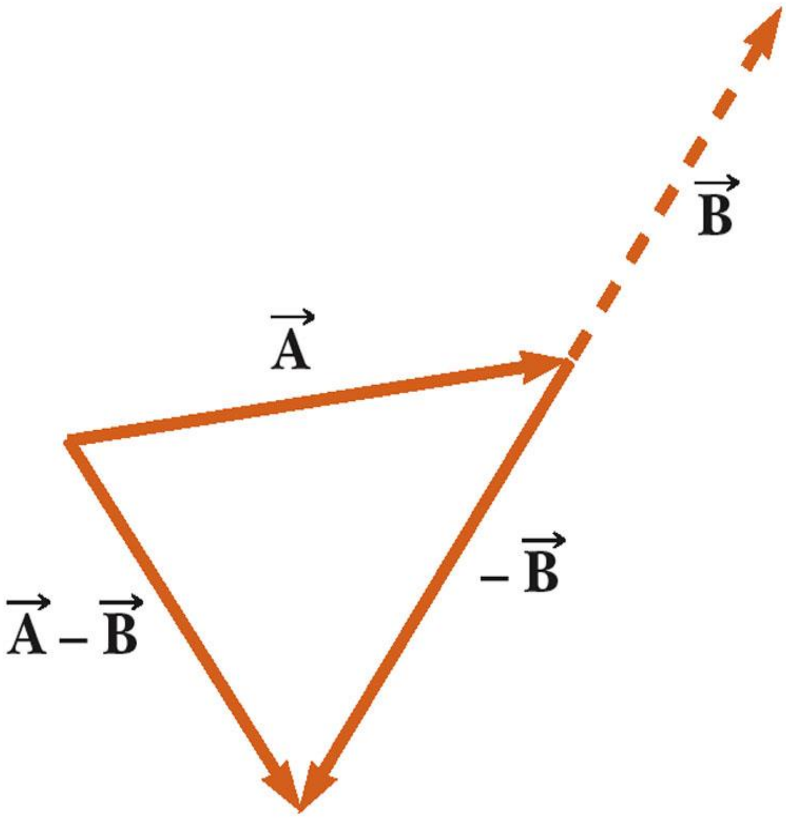
❖but point in opposite directions.

Vector Subtraction

- Add the negative of the subtracted vector

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$$

- Draw A along some convenient axis, place the tail of -B at the tip of A, and C is the difference A - B.

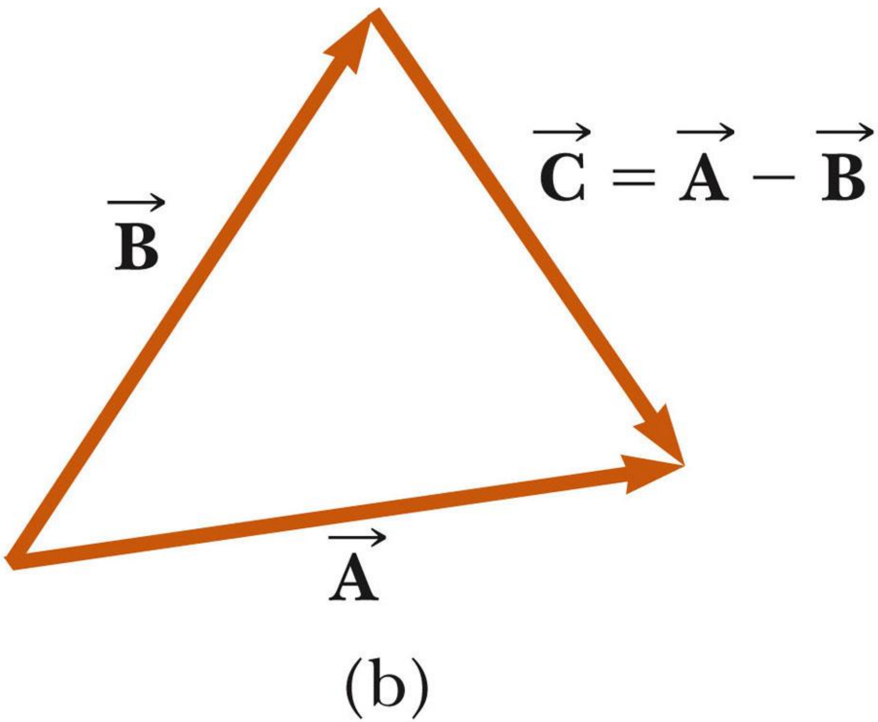


Subtracting Vectors, Method 2

- Find the vector **C**=**A**-**B**

is the vector we must add to the second vector **B** t**o** give the first vector **A**

- As shown, the resultant vector points from the tip of the second to the tip of the first



Multiplying or Dividing a Vector by a Scalar

If vector **A** is multiplied by

- a positive scalar quantity m,

the product m**A** is a vector that has the same direction as **A** and magnitude mA.

- a negative scalar quantity -m,
- the product -m**A** is directed opposite **A**.

For example,

- the vector 5**A** is five times as long as **A** and points in the same direction as **A**;
- the vector -1/3**A** is one-third the length of **A** and points in the direction opposite A.

Component of Vectors

- Graphical addition is not recommended when
  - High accuracy is required
  - If you have a three-dimensional problem
- Component method
  - It uses projections of vectors along coordinate axes

(any vector can be completely described by its components).

Components of a Vector

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$

These three vectors form a right triangle

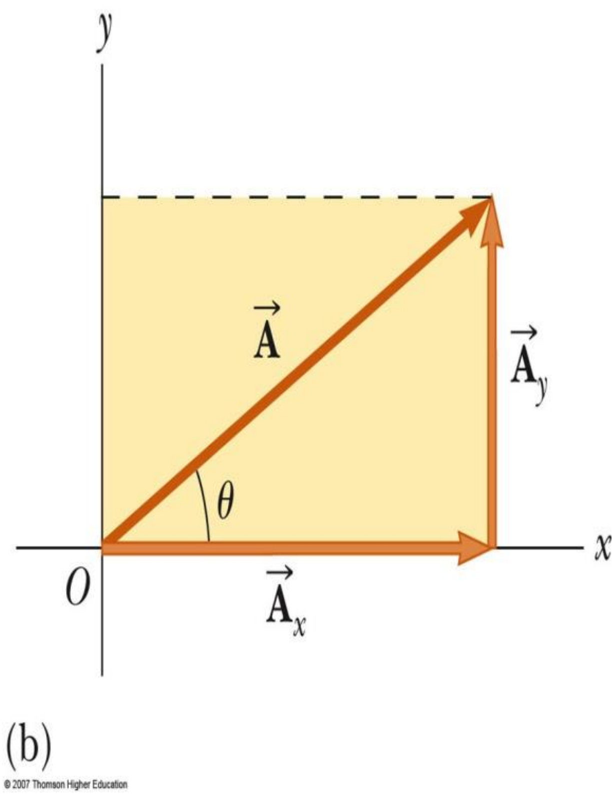
A **component** is a projection of a vector along an axis

- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

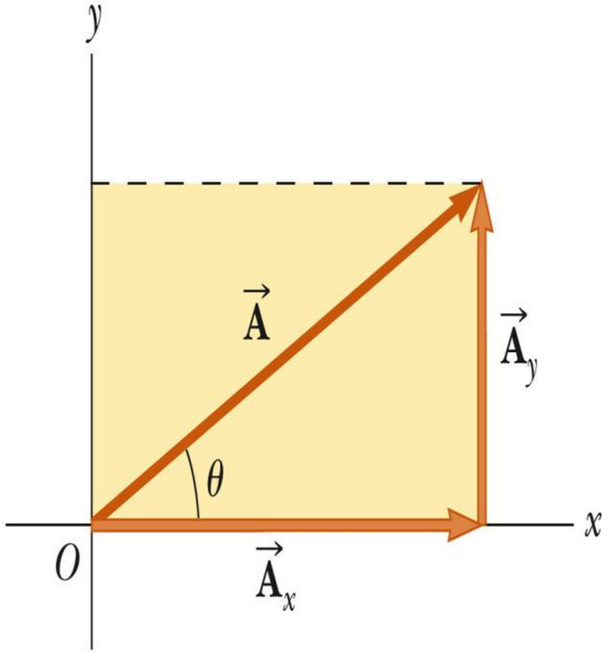
$$A_y = A\sin\theta$$



Components of a Vector

- These components form two sides of a right triangle with a hypotenuse of length *A.*
- The magnitude and direction of A

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$



(b)

The signs of the components (A_x and A_y)

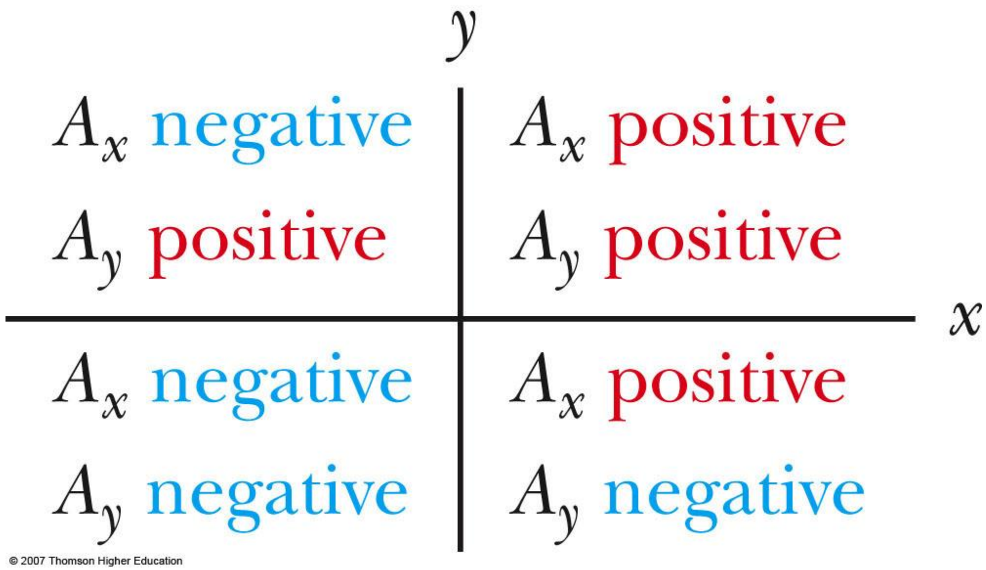
depend on the angle θ.(is an angle between vector and positive x axis)

These components can be positive or negative.

The component A_x is

- positive if A_x points in the positive x direction
- negative if Ax points in the negative x direction.

The same is true for the component A_y .

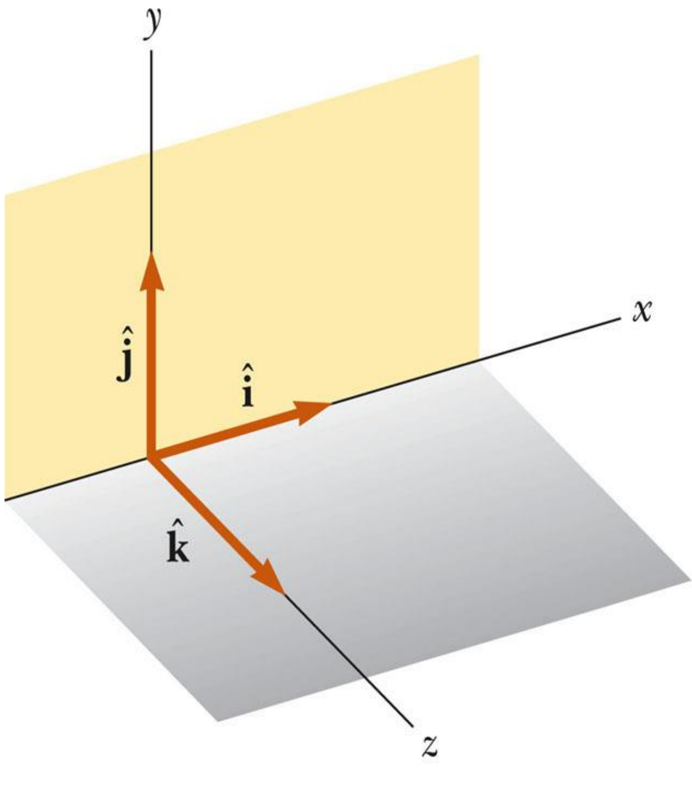


Unit Vectors

- A ***unit vector*** is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction and have no other physical significance
- The symbols $\hat{\mathrm{i}}, \hat{\mathrm{j}},$ and $\hat{\mathrm{k}}$ represent

unit vectors. Remember,

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$



(a)

- They form a set of mutually perpendicular vectors in a right-handed coordinate system

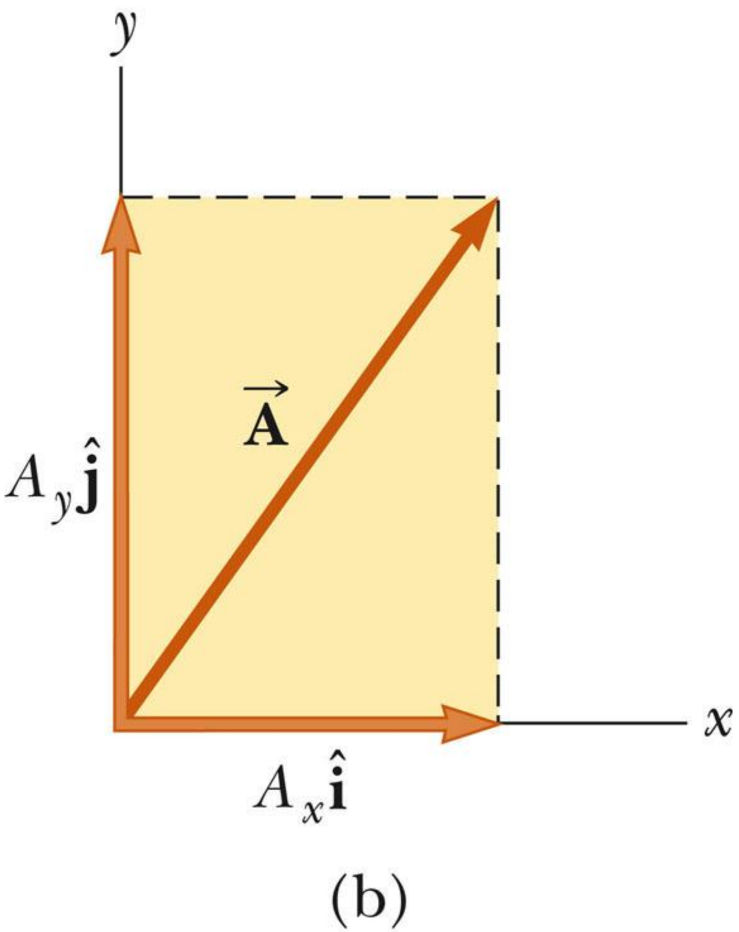
Unit Vectors

Consider a vector **A** lying in the xy plane

- $\mathbf{A_x}$ is the same as $A_x\,\hat{\mathbf{i}}$ and $\mathbf{A_y}$ is the same as $A_y\,\hat{\mathbf{j}}$ etc.

- The vector can be expressed as

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$



Adding Vectors Using Unit Vectors

- Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$
- Then $\vec{\mathbf{R}} = \left(A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}\right) + \left(B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}}\right)$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

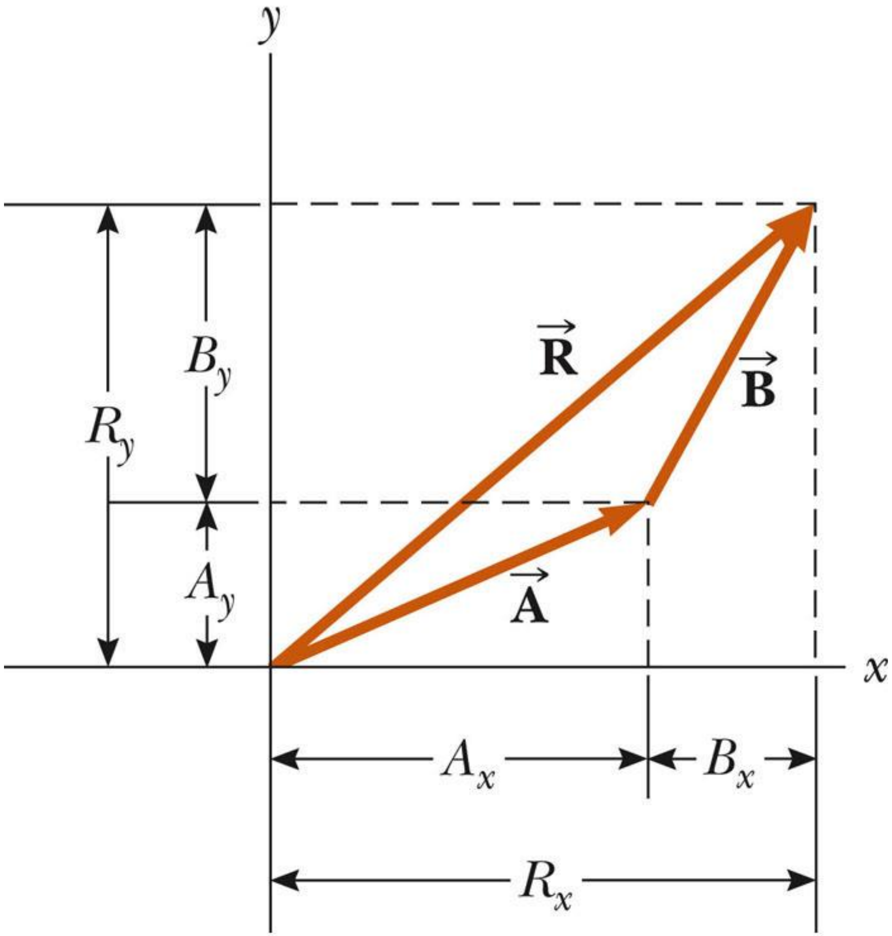
$$\vec{\mathbf{R}} = R_x \hat{\mathbf{i}} + R_y \hat{\mathbf{j}}$$

- and so $R_x = A_x + B_x$ and $R_y = A_y + B_y$
- The magnitude and direction of R

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Check this addition by components with a geometric construction

- $R_x = A_x + B_x$
- $R_y = A_y + B_y$



Three-Dimensional vectors

- Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$
- Then $\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} + \left(A_z + B_z\right)\hat{\mathbf{k}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

- and so $R_x = A_x + B_x$, $R_y = A_y + B_y$, and $R_z = A_z + B_z$
- The magnitude and direction of R

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \theta = \cos^{-1}\frac{R_x}{R}, etc.$$

Example

$\vec{R} = (37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}})$ km

The resultant vector has a magnitude

$$R = \sqrt{37.7^2 + 16.9^2} = 41.3$$

and directed

$$\theta = \tan^{-1} = \frac{16.9}{37.7} = 24.1$$

north of east.

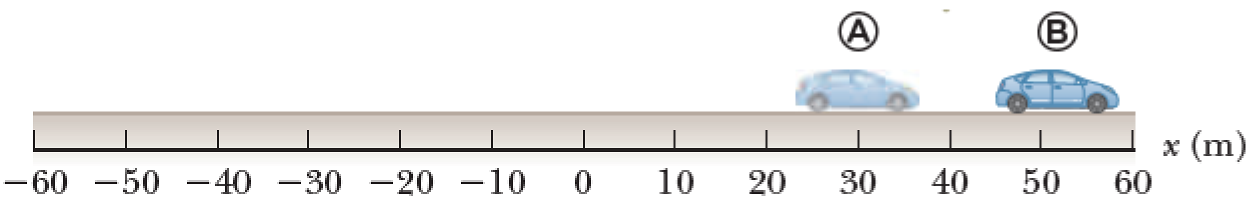
Chapter 2

Motion in One Dimension

Position

The particle's position

is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.



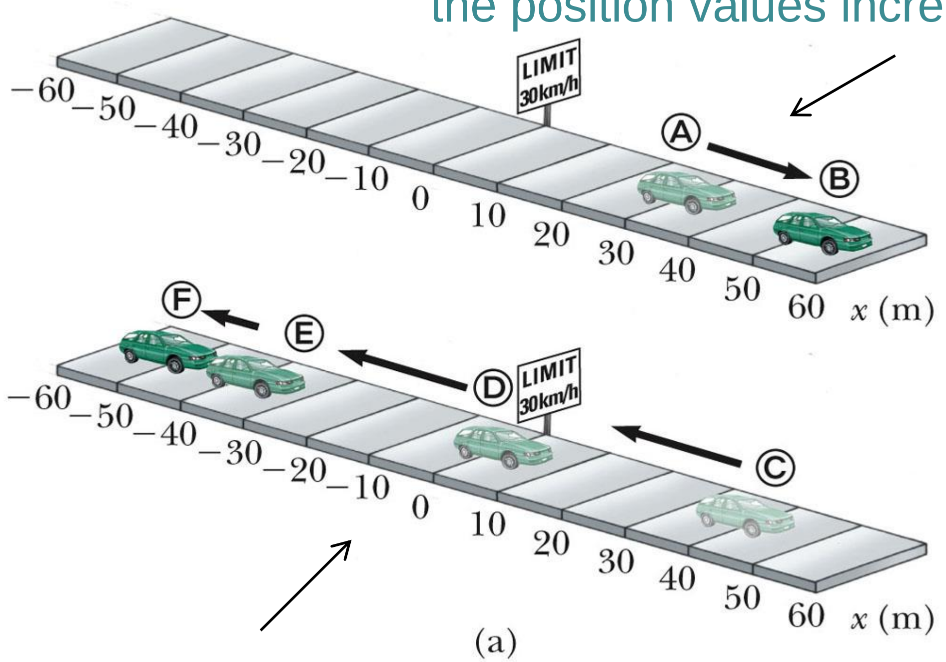
At position A: the car located at 30m from the origin point (0).

At position B: the car located at 52m from the origin point (0).

Position

Consider a car moving back and forth along the x axis as in Figure (a). We start our clock and once every 10 s note the car's position from the table

the position values increase



(a)

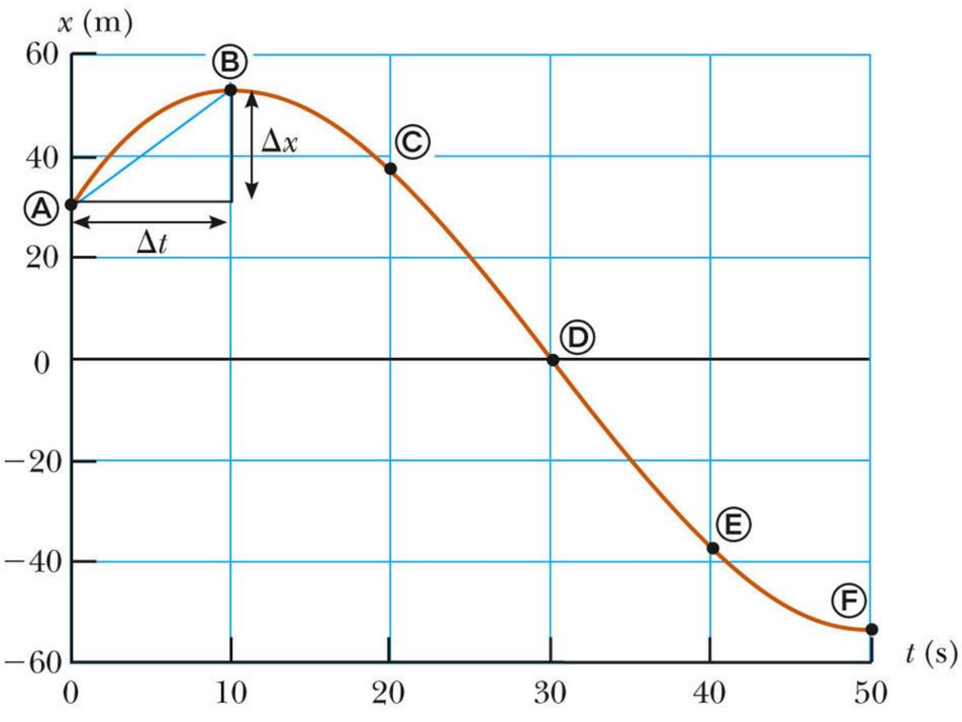
the position values decrease

Position of the Car at Various Times

Position	*t* (s)	*x* (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Position-Time Graph

the change in position of the car for various time intervals.



(b)

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Displacement

- Defined as the change in position in some time interval
- Represented as Δx

$$\Delta x \equiv x_f - x_i$$

SI units are meters (m)

- x_i initial position
- x_f final position

Δx can be

- Positive $\longrightarrow$ $\boldsymbol{x_f > x_i}$
- negative $\longrightarrow$ $\boldsymbol{x_f < x_i}$

Example,

Find the displacement of the car in Figure between

- positions A and B.

At positions A x_i =30m

At positions B x_f =52 m

$\Delta x \equiv x_f - x_i$

= 52-30=22 m

- positions E and D.

At positions D x_i = 0 m

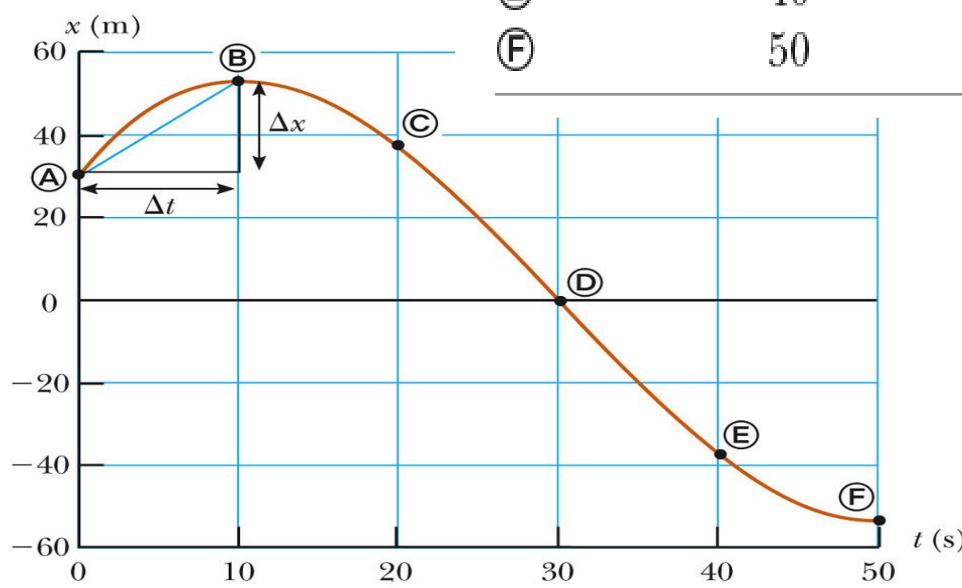
At positions E x_f =-37m

$\Delta x \equiv x_f - x_i$

= -37-0=-37 m

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



(b)

Distance (x) and Displacement ($\Delta x, \bar{x}$)

- A scalar quantity
- Distance is always a positive number,
- Refer to the length of a path followed by a particle.

- A vector quantity
- Displacement can be
  - positive $\Delta x > 0$
  - negative $\Delta x < 0$.
- Refer to the distance between initial position (x_i) to final position (x_f)

- The SI unit of displacement and distance is **meters (m).**
- Will use + and – signs to indicate vector directions

Average Velocity

The average velocity $\bar{v}_x$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$\bar{v}_x = v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The x indicates motion along the x-axis

❖The average velocity **depend on the sign of the displacement Δx. It** can be

➢**Positive** → If the coordinate of the particle increases in time $x_f > x_i$

➢ **negative** → If the coordinate decreases in time $x_f < x_i$

❖The time interval Δt is always positive.

the average velocity has dimensions of length divided by time (L/T) meters per second (m/s) in SI units.

Average velocity geometrically

We can interpret average velocity geometrically by drawing a straight line between any two points on the position–time graph This line forms the hypotenuse of a right triangle of height Δx *and* base Δt. *The slope of this line is*

$$\text{slop} = \bar{v}_x = \frac{\Delta x}{\Delta t}$$

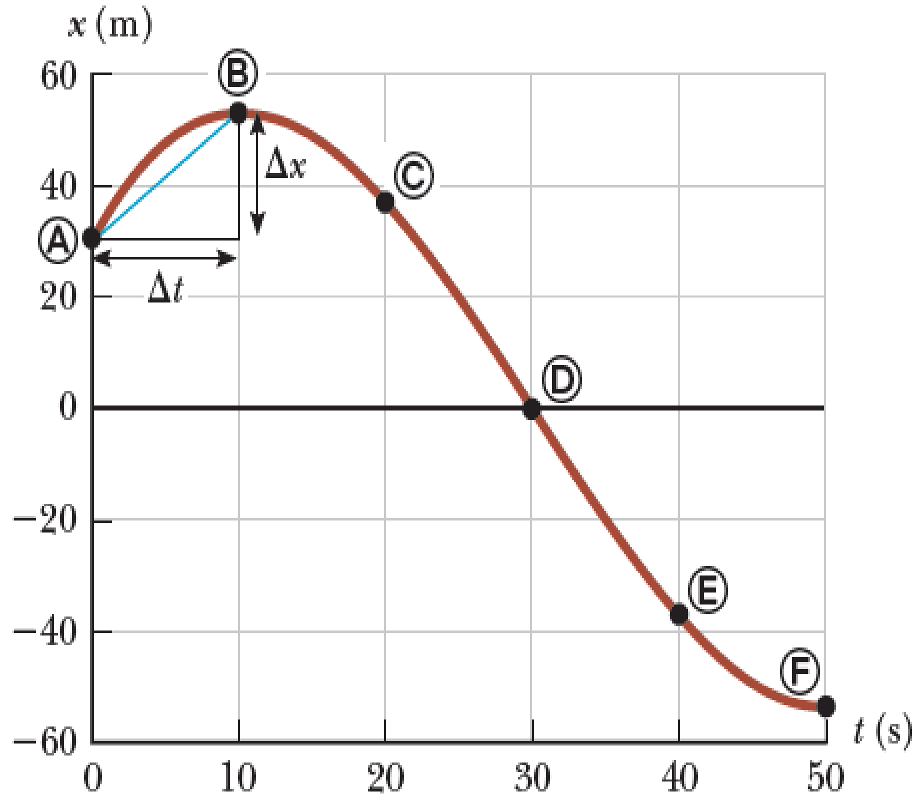
Example,

- The line between positions A and B has a slope equal to the average velocity of the car between those two times

$$\text{slop} = \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{52 - 30}{10 - 0} = 2.2\ m/s$$

- The average velocity between positions D and E equal

$$\text{slop} = \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{0 - 37}{30 - 40} = -3.7\ m/s$$



The average speed

The average speed of a particle, a scalar quantity, is defined as the total distance traveled divided by the total time interval required to travel that distance:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$v = \frac{d}{t}$$

The SI unit of average speed and the average velocity is **meters per second (m/s)**.

Unlike average velocity, however, **average speed has no direction and is always expressed as a positive number.**

Average velocity and Average speed

• Average velocity is the displacement divided by the time interval,

• **has direction**

• **It** can be **Positive or negative**

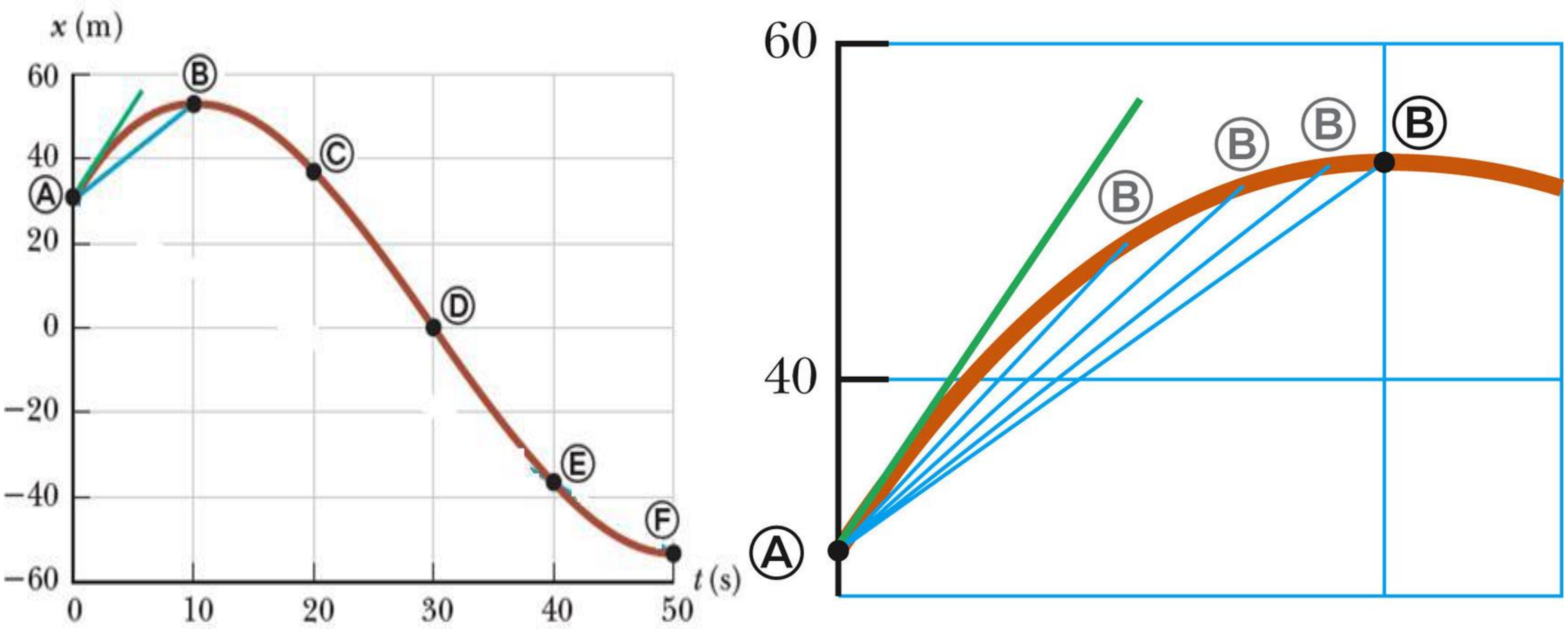
• average speed is the distance divided by the time interval.

• **has no direction**

• **it always expressed as a positive number.**

Instantaneous Velocity and speed

we need to know the velocity of a particle at a particular instant in time,



(b)

Instantaneous Velocity

the instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The instantaneous velocity can be positive, negative, or zero.

Velocity $\longrightarrow$ instantaneous velocity

Instantaneous Speed

Instantaneous Speed is the magnitude of its instantaneous velocity.

has no direction and no algebraic sign

Example,

if one particle has an instantaneous velocity of +25 m/s along a given line and another particle has an instantaneous velocity of -25 m/s along the same line, both have a speed of 25 m/s.

Speed ⟶ Instantaneous Speed

Acceleration

The average acceleration $\bar{a}_x$ of the particle is defined as the change in velocity Δv_x divided by the time interval Δt during which that change occurs:

$$a_{x,avg} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

•we can use positive and negative signs to indicate the direction of the acceleration

•The SI unit of acceleration is meters per second squared (m/s^2).

Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero

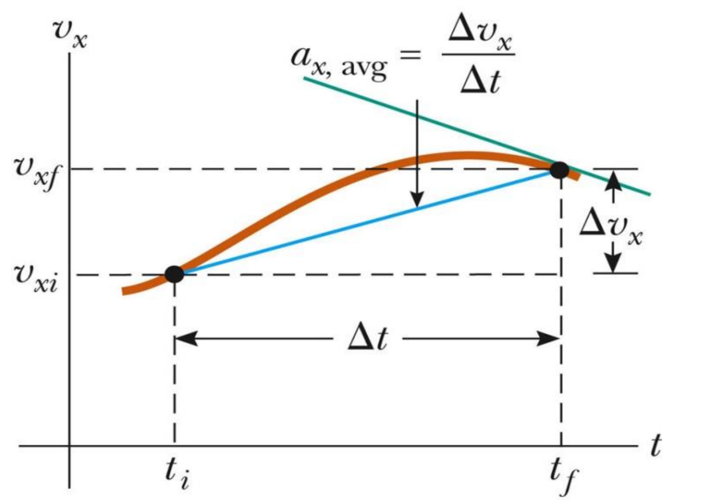
$$\bar{a}_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

The instantaneous acceleration equals the derivative of the velocity with respect to time,

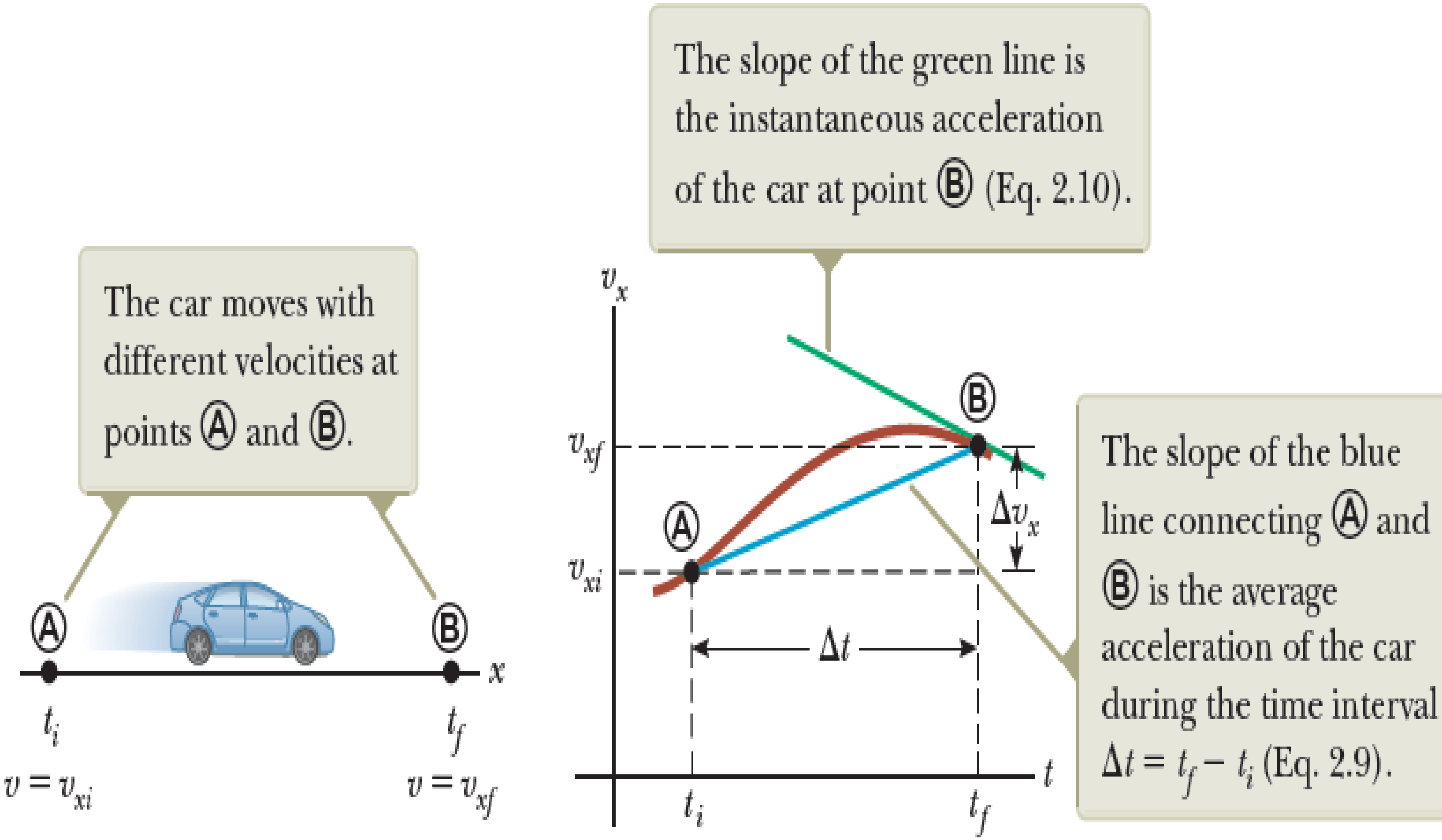
$$\bar{a}_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

The acceleration equals the second derivative of x with respect to time.

acceleration $\rightarrow$ instantaneous acceleration



(b)


The car moves with different velocities at points Ⓐ and Ⓑ.
Ⓐ
Ⓑ
x
t_i
$v = v_{xi}$
t_f
$v = v_{xf}$
The slope of the green line is the instantaneous acceleration of the car at point Ⓑ (Eq. 2.10).
v_x
v_{xf}
v_{xi}
Ⓐ
Ⓑ
Δv_x
Δt
t
t_i
t_f
The slope of the blue line connecting Ⓐ and Ⓑ is the average acceleration of the car during the time interval $\Delta t = t_f - t_i$ (Eq. 2.9).

Acceleration

- If a_x is positive, the acceleration is in the positive x direction;
- if a_x is negative, the acceleration is in the negative x direction.

- When the object's velocity and acceleration are in the same direction, the object is speeding up.
- when the object's velocity and acceleration are in opposite directions, the object is slowing down.

Example 4,5,6

A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s,

find (a) the position of the particle,

(b) its velocity,

(c) its acceleration.

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(2 + 3t - t^2\right) = 3 - 2t \quad \text{and} \quad a_x = \frac{dv}{dt} = \frac{d}{dt}(3 - 2t) = -2$$

Now we can evaluate x, v, and a at $t = 3.00$ s.

(a) $x = 2.00 + 3.00(3.00) - (3.00)^2 = 2.00$ m

(b) $v_x = 3.00 - 2(3.00) = -3.00$ m/s

(c) $a_x = -2.00$ m/s^2

force and acceleration

Force is proportional to acceleration

$$F \propto a$$

acceleration and forceare in

- The same direction. ⟶ The object speeds up
- The opposite directions ⟶ The object slows down

Example

If a car is traveling eastward and slowing down, what is the direction of the force on the car that causes it to slow down?

(a) eastward

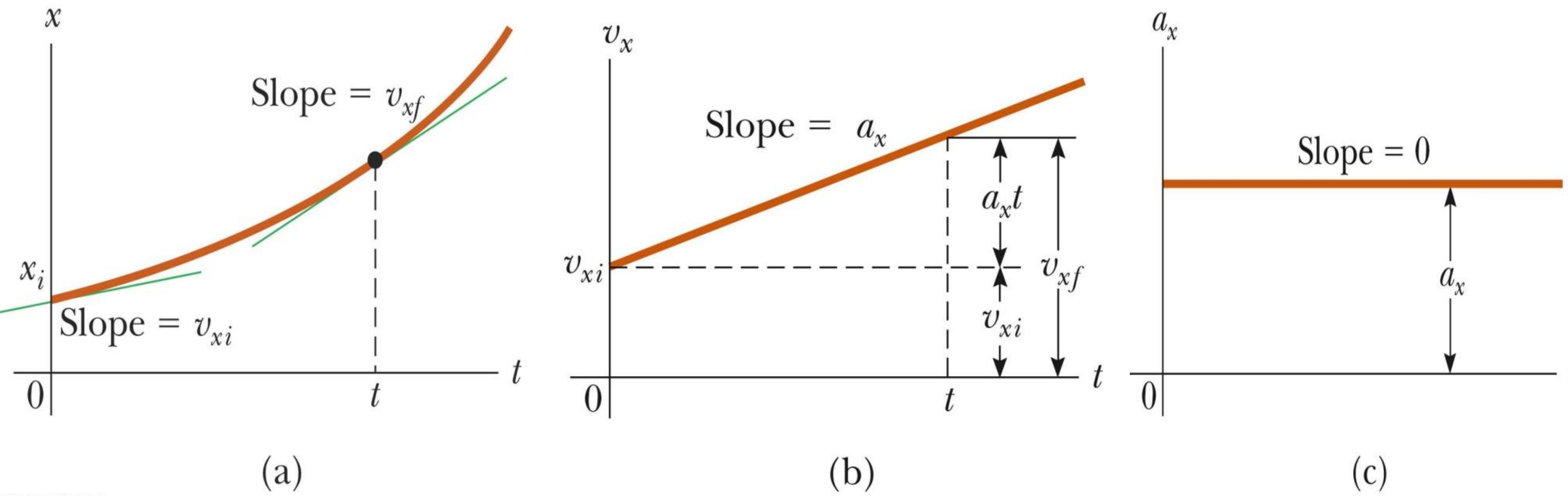
(b) westward

(c) neither of these.

One-Dimensional Motion with Constant Acceleration

❖The acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the acceleration is constant.

❖When the acceleration is constant, the graph of acceleration versus time is a straight line having a slope of zero.



Particle moving along the x axis with constant acceleration a_x;

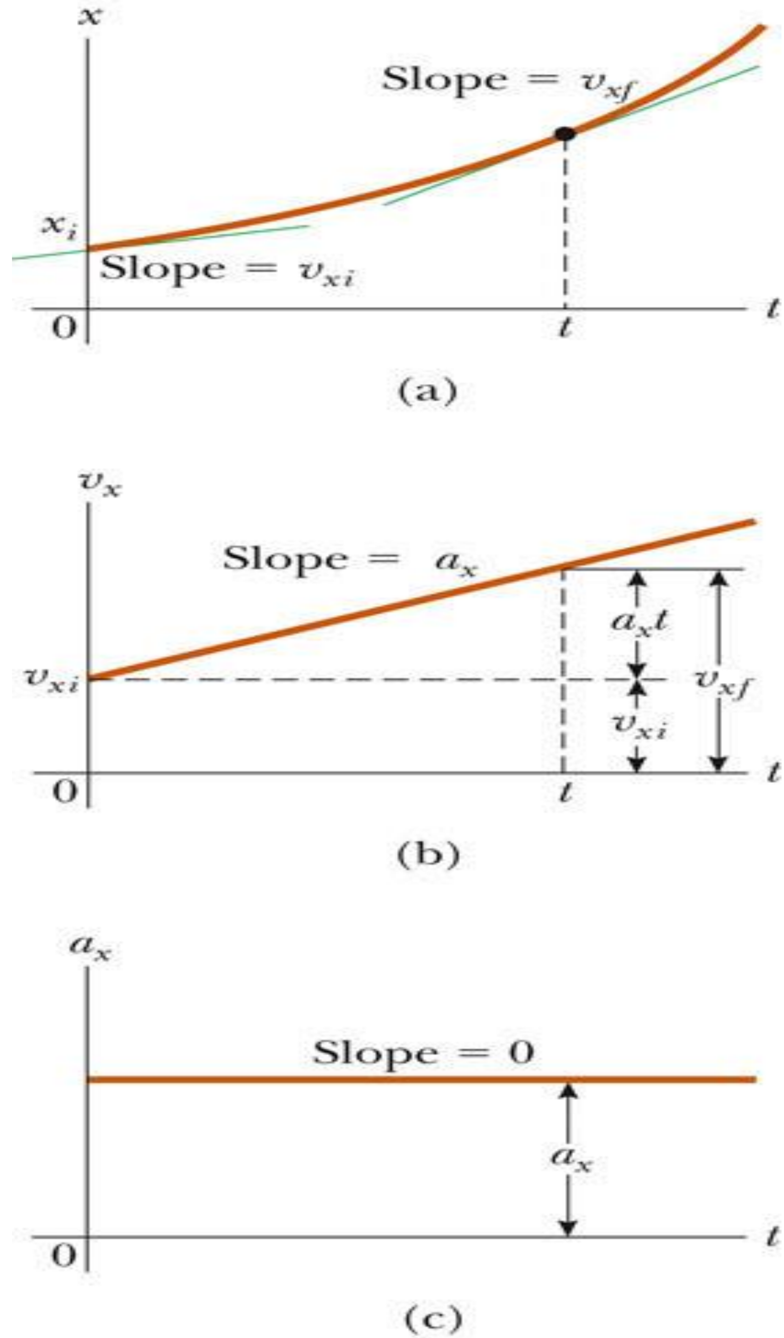
The position– time graph,

The velocity–time graph,

The velocity at constant acceleration varies linearly in time

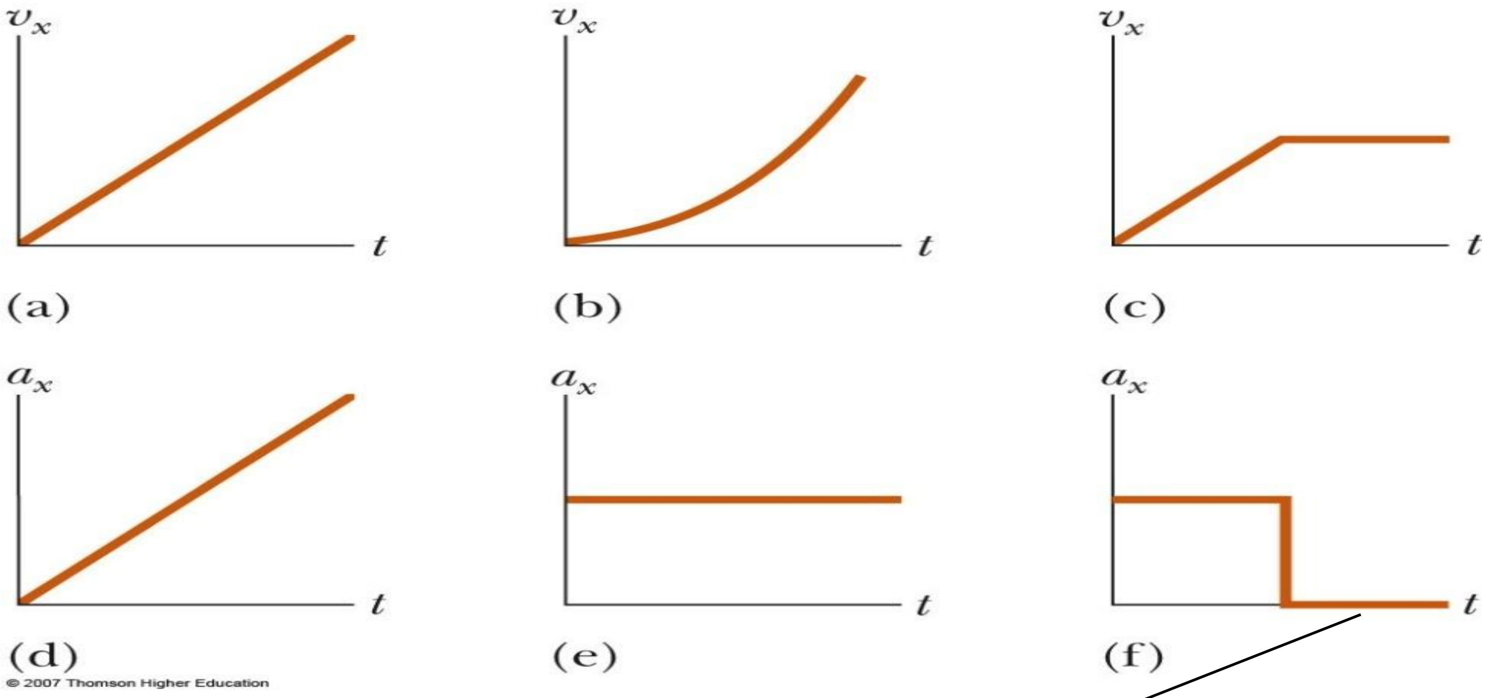
The acceleration–time graph.

When the acceleration is constant, the graph of acceleration versus time is a straight line having a slope of zero.



Example

match each (v_x –t) graph on the left with the (a_x –t) graph on the right that best describes the motion.



The acceleration of a particle is zero, its velocity is constant and its position changes linearly with time

Kinematic Equations

TABLE 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^{\ 2} = v_{xi}^{\ 2} + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

Example11,12:

A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

We have

$x = 40\text{ m}$, $t = 8.50\text{ s}$, $v_f = 2.80\text{ m/s}$,

(a) Find its original speed.

$$x = \frac{1}{2}(v_i + v_f)t$$

$$40\text{ m} = \frac{1}{2}(v_i + 2.80\text{ m/s})(8.50\text{ s})$$

$$80\text{ m} = 8.50\, v_i + 23.80$$

$$v_i = \boxed{6.61\text{ m/s}}.$$

(b) Find its acceleration

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t} = \frac{2.80\text{ m/s} - 6.61\text{ m/s}}{8.50\text{ s}} = \boxed{-0.448\text{ m/s}^2}$$

Freely Falling Objects

•It is well known that, in the absence of air resistance, all objects dropped near the Earth’s surface fall toward the Earth with the same constant acceleration under the influence of the Earth’s gravity.

•A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion.
Objects thrown upward or downward and those released from rest are all falling freely once they

•We shall denote the magnitude of the free-fall acceleration, also called the acceleration due to gravity, by the symbol g.

Kinematic Equations for Freely Falling Objects

Objects thrown upward

1-has initial velocity(v_i)

2-Final velocity(v_f)=0

Objects thrown downward

1-has final velocity(v_f)

2-initial velocity(v_i)=0

The value of gravity = -9.8m/s2

$$v_{yf} = v_{yi} + gt$$

$$y = v_{yi}t + \frac{1}{2}gt^2$$

$$v_{yf}^2 = v_{yi}^2 + 2g\,y$$

$$y = \frac{1}{2}(v_{yi} + v_{yf})t$$

$$v_{y,avg} = \frac{1}{2}(v_{yi} + v_{yf})$$

Example 2.10: Describing the Motion of a Tossed Ball

A ball is tossed straight up at 25 m/s. Estimate its velocity at 1-s intervals. (g=-10m/s²)

$$v_f = v_i + at$$

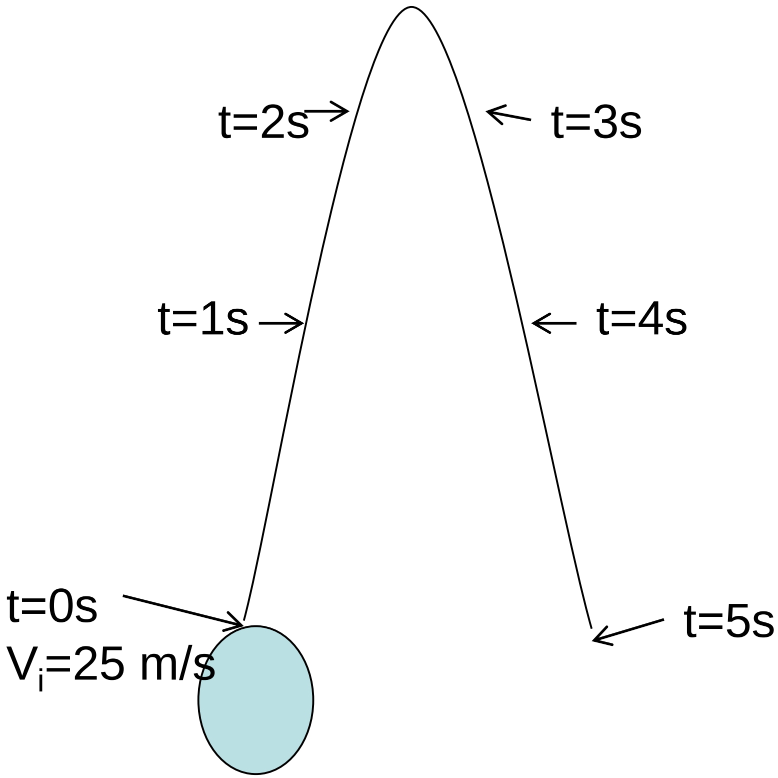
At t=1s $v_f = 25 + (-10) \times 1 = 15 m/s$

At t=2s $v_f = 25 + (-10) \times 2 = 5 m/s$

At t=3s $v_f = 25 + (-10) \times 3 = -5 m/s$

At t=4s $v_f = 25 + (-10) \times 4 = -15 m/s$

At t=5s $v_f = 25 + (-10) \times 5 = -25 m/s$



Example 22:

An arrow is shot straight up in the air at an initial speed of 15.0 m/s. After how much time is the arrow moving downward at a speed of 8.00 m/s?

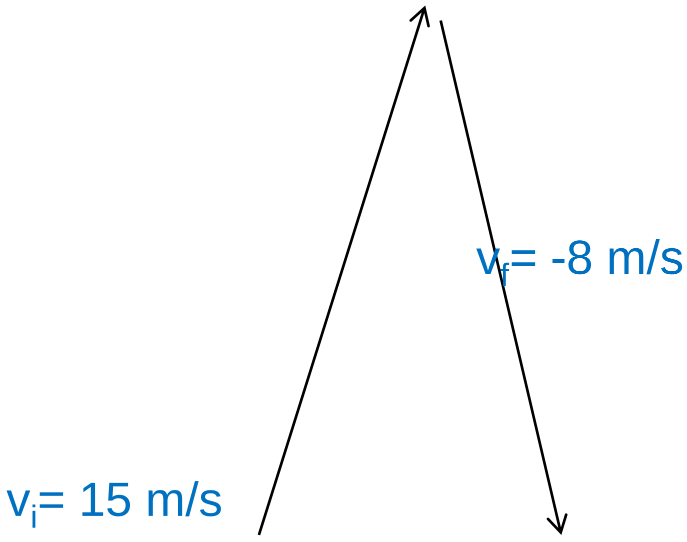
(a) 0.714 s (b) 1.24 s (c) 1.87 s (d) 2.35 s (e) 3.22

We have v_i= 15 m/s, v_f= -8 m/s, g = -9.8m/s^2, t= ??

$$v_f = v_i + at$$

$$-8 = 15 + (-9.8)t$$

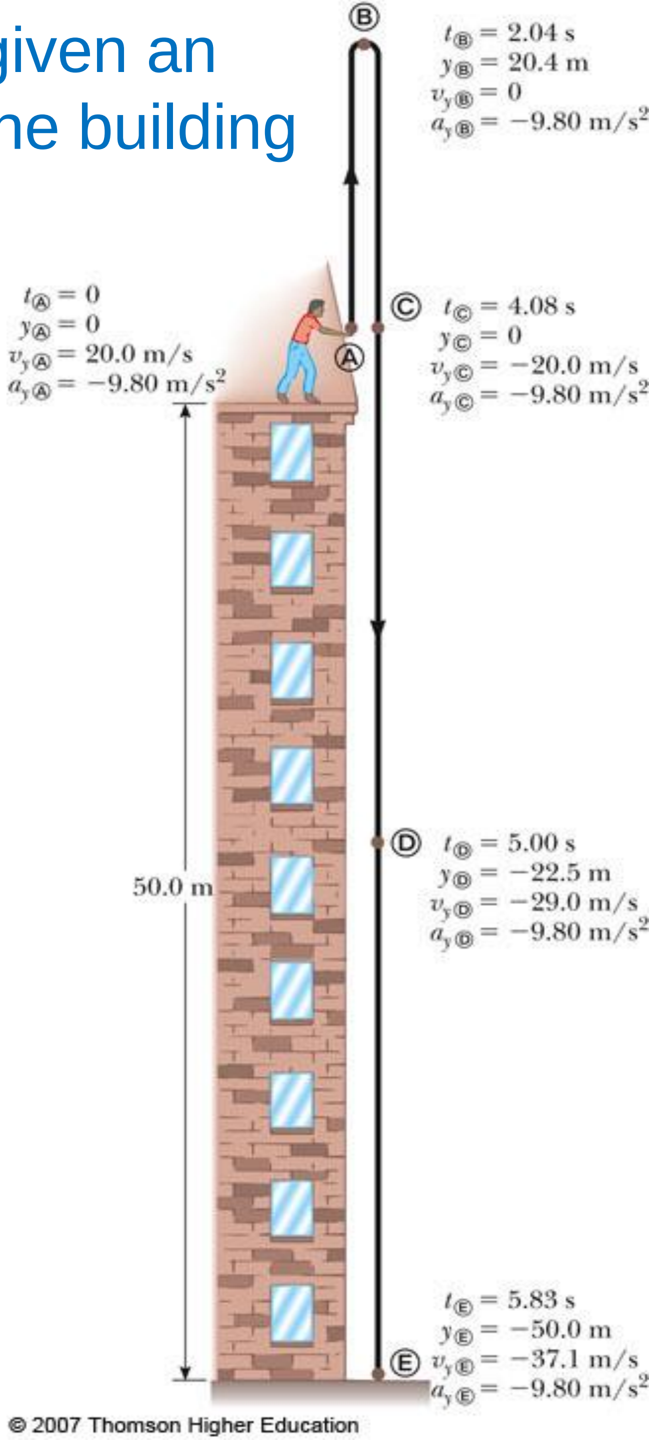
$$\rightarrow t = 2.35s$$



A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high,

We have velocity(vi)=20m/s

The high of the building = 50.0 m



At position A

$t_A=0$, $y_A=0$, $g=-9.8 m/s^2$

$v_A=20m/s$

At position B

$v_B=0$

(A) the time at which the stone reaches its maximum height, → $t_B=?$

We have $v_i=20m/s$, $v_f=0$, $g=-9.8m/s^2$

$$v_f = v_i + at$$

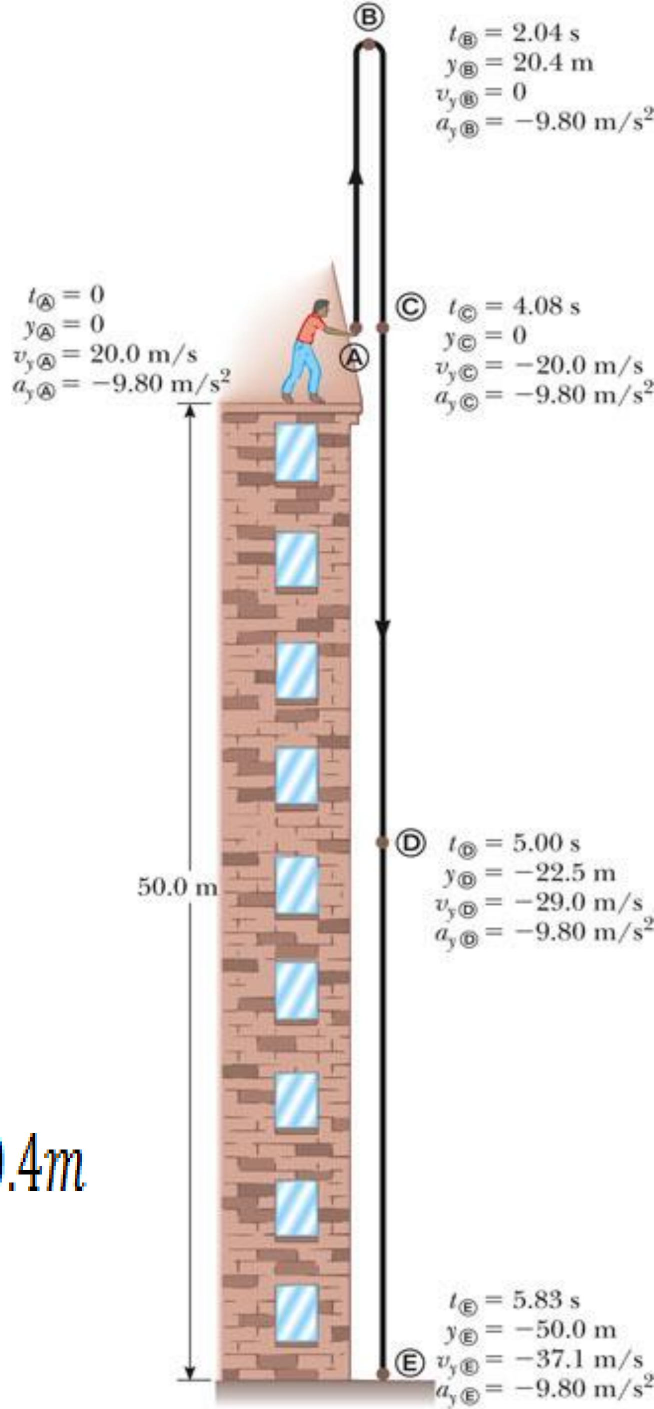
$$0 = 20 + (-9.8)t \rightarrow t = 2.04s$$

(B) the maximum height → $y_B=?$

$$y_f = y_i + v_i t + \frac{1}{2} g t^2$$

$$y_f = 0 + (20)(2.04) + \frac{1}{2}(-9.8)(2.04^2) \longrightarrow y_f = 20.4m$$

OR $y_f = y_i + \frac{1}{2}(v_i + v_f)t$



At position C

$y_c = 0$, $v_C = -20 m/s$

C) the time at which the stone returns to the height from which it was thrown,

$$y_f = y_i + v_i t + \frac{1}{2} g t^2$$

$$0 = 0 + (20)t + \frac{1}{2}(-9.8)t^2$$

$$t(20 - 4.9t) = 0$$

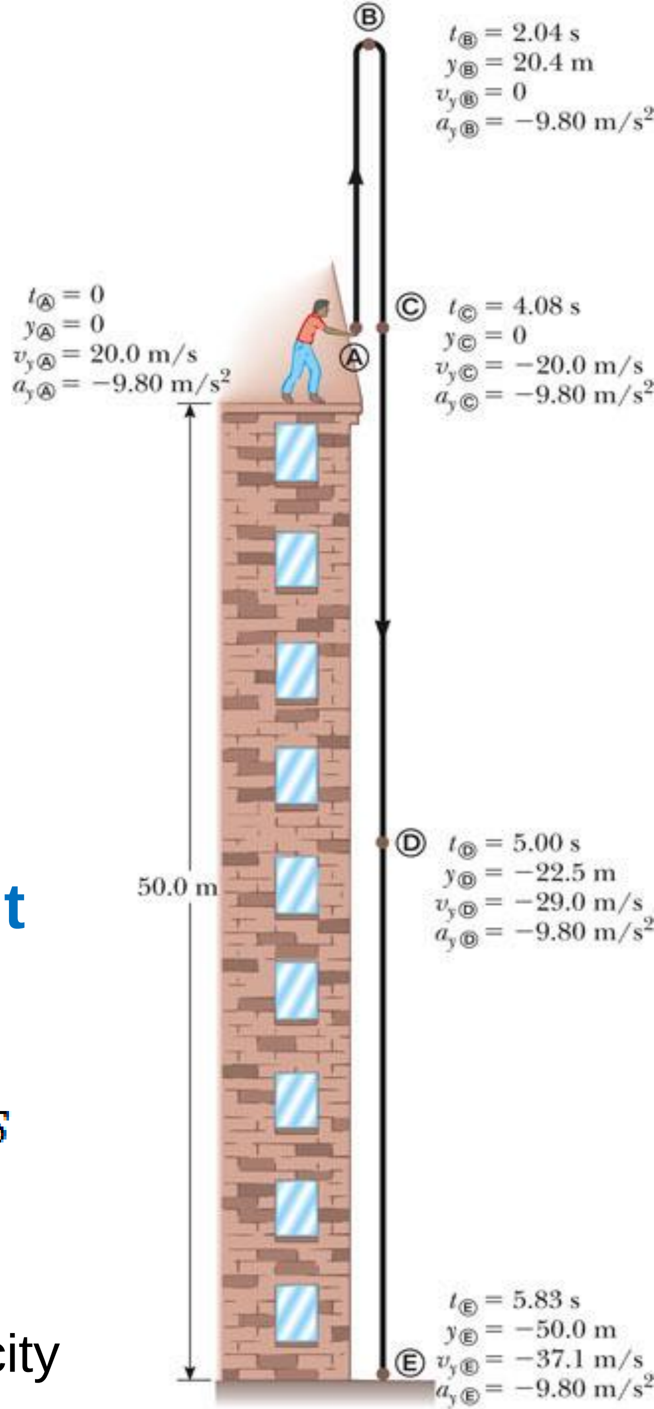
$$t = 0 \ or \quad 20 - 4.9t = 0 \longrightarrow t = 4.08\ s$$

(D) the velocity of the stone at this instant

$$v_f = v_i + at$$

$$v_f = 20 + (-9.8)(4.08) \longrightarrow v_f = -20 m/s$$

The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction.



At position D

E) the velocity and position of the stone at t = 5s

The initial **position A**

$$v_f = v_i + at$$

$$v_f = 20 + (-9.8)(5)$$

$$v_f = -29 m/s$$

The initial **position B**

$$v_f = v_i + at$$

$$v_f = 0 + (-9.8)(5 - 2.04)$$

$$v_f = -29 m/s$$

From the initial **position c**

$$y_f = y_i + v_i t + \frac{1}{2} g t^2$$

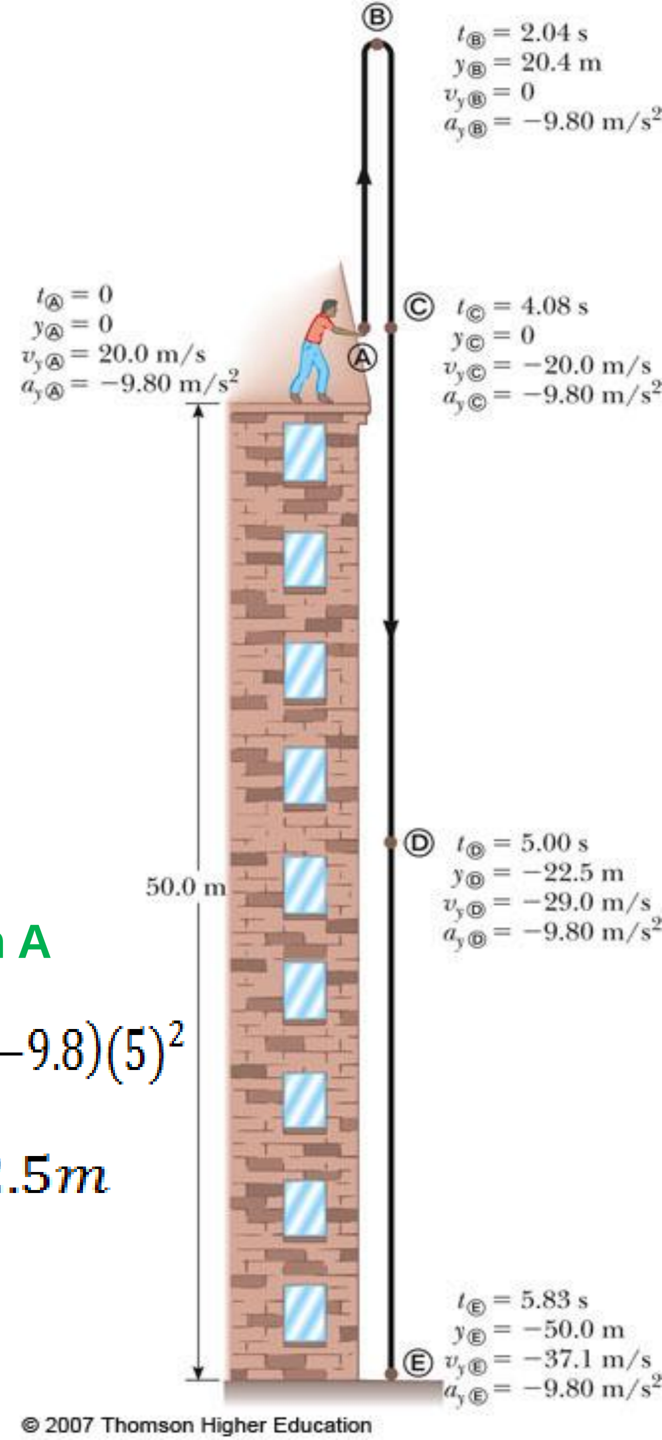
$$y_f = 0 + (-20)(5 - 4.08) + \frac{1}{2}(-9.8)(5 - 4.08)^2$$

$$y_f = -22.5 m$$

The initial **position A**

$$y_f = 0 + (20)(5) + \frac{1}{2}(-9.8)(5)^2$$

$$y_f = -22.5 m$$



Example 2-1

The position of an object is 35 meters when the time was 2 seconds and then the position changes to 87 meters at 15 seconds.

Calculate the average velocity of the object?

Solution

$$\upsilon_{Ave} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$= \frac{87 - 35}{15 - 2} = \frac{52}{13} = 4\,m/s$$

Example 2-2

A Car travels 10 km at velocity of 90 km/h, and then the car stopped because the car run out of the fuel. This is causing the driver to walk a distance of 5 km to reach the nearest petrol station for half an hour.

What is the average velocity for the driver?

Solution

The distance x_1 equal to zero because the driver was still at home, x_2 is the distance covered by car until the fuel run out, and x_3 is the distance that the driver walked to the petrol station.

Convert the distance from kilometers to meters, and convert the units of other velocity from km/h to m/s as follows

$x_f = x_2 + x_3 = 10\times10^3 + 5\times10^3 = 10,000 + 5,000 = 15,000\ m$

$$\upsilon = 90\frac{km}{h} = \frac{90\times1000}{60\times60} = 25\ m/s$$

the time taken before the fuel run out

$$t_1 = \frac{x}{\upsilon} = \frac{10000}{25} = 400\,s$$

the time that the driver takes to reach the petrol station

$t_2 = 30 \times 60 = 1800\ s$

Total time for the car and the driver is

$t_{Total} = t_f = 400 + 1800 = 2200\ s$

The average speed is

$$\bar{\upsilon} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$= \frac{x_f - 0}{t_f - 0} = \frac{15000}{2200} = 6.8\, m/s$$

Example 2-3

Moving object in the positive direction of the x-axis with a relationship, as the following $x(t) = 8 + 2t + 3t^2$ *where the distance is measured in meters and the time in second.*

A) *Find the instantaneous velocity of the object after two seconds?*

B) *Find the instantaneous acceleration of the object after two seconds?*

C) *Find the distance of the object after two seconds?*

Solution

A)*The instantaneous velocity of the object after two seconds is:*

$$\upsilon = \frac{dx}{dt} = \frac{d}{dt}(8 + 2t + 3t^2)$$

$$= 2 + 3 \times 2t = 2 + 6 \times 2 = 14\, m/s$$

***B)**The instantaneous acceleration of the object after two seconds is:*

$$a = \frac{d\upsilon}{dt} = \frac{d}{dt}(2 + 6t)$$

$$= 6\, m/s^2$$

***C)** The distance of the object after two seconds is:*

$$x = 8 + 2t + 3t^2$$

$$= 8 + 2 \times 2 + 3 \times (2)^2$$

$$= 8 + 4 + 12 = 24\, m$$

Example 2-4

A car travels with a velocity of 20 m/s, the driver increased the velocity until it reaches 100 km/h in three seconds. Then the driver decided to stop, the car stopped after four seconds.

Find the average acceleration in both cases?

Solution

First: convert the units from km /h to m /s.

$$\upsilon_f = \frac{100 \times 1000}{60 \times 60} = 27.8\, m/s$$

$$a_{Ave.1} = \frac{\Delta\upsilon}{\Delta t}$$

$$= \frac{27.8 - 20}{3} = 2.6\, m/s$$

$$a_{Ave.2} = \frac{\Delta\upsilon}{\Delta t}$$

$$= \frac{0 - 27.8}{4} = -6.95\, m/s^2$$

Example 2-5

An object moves in x-axis according to the following formula: $x = t^3 + 5$

Note that the distance and the time are measured in meters and seconds, respectively.

Find:

A) *The velocity and the acceleration of the object?*

B) *The velocity and the acceleration when* $t = 3s$, $t = 2s$?

C) *Average of the velocity and the acceleration at* $= 3s$, $t = 2s$?

Solution

A) *Defines the distance equation*

$$x = t^3 + 5 \; \ldots \ldots \ldots \; (1)$$

The velocity is:

$$\upsilon = \frac{dx}{dt} = \frac{d}{dt}(t^3 + 5) = 3t^2 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

The acceleration is

$$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2) = 6t \quad \ldots\ldots\ldots\ldots\ldots (3)$$

B) When the time (t = 2 s) the distance, velocity and the acceleration will be

The distance after 2 sec.

$$x_1 = t^3 + 5 = (2)^3 + 5 = 13\,m$$

The velocity after 2 sec.

$$\upsilon_1 = \frac{dx}{dt} = 3t^2 = 3(2)^2 = 12\,m/s$$

The acceleration after 2 sec.

$$a_1 = \frac{d\upsilon}{dt} = 6t = 6(2) = 12\ m/s^2$$

When the time (t=3 s), the distance, velocity and the acceleration, as what we have *done in the previous (السابق) answers we get:*

$x = 32\ m,\quad v = 27\ m/s \quad a = 18\ m/s^2$

C) *The average velocity and acceleration are*

$$\upsilon_{Ave.} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{32 - 13}{3 - 2} = 19\ m/s$$

$$a_{Ave.} = \frac{\Delta \upsilon}{\Delta t} = \frac{\upsilon_2 - \upsilon 1}{t_2 - t_1}$$

$$= \frac{18 - 12}{3 - 2} = 6\ m/s^2$$