Physics 110
1435-1436 H

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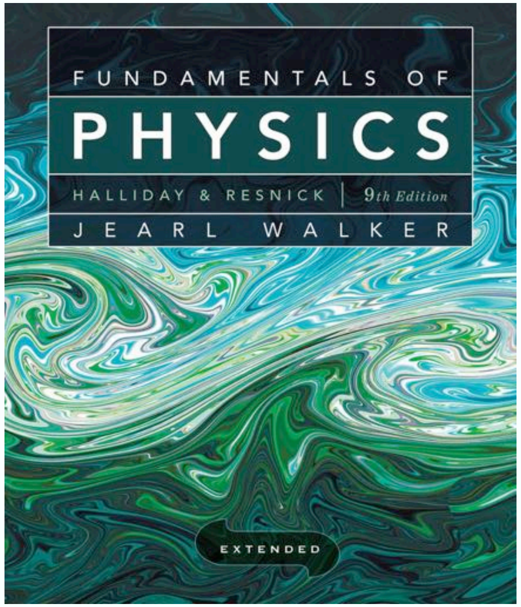


Fundamentals of Physics
9^{th} edition

By: HALLIDAY, RESNICK & WALKER

Exam Dates

مواعيد الدراسة والاختبارات خلال الفصل الثاني

الفصل الدراسي الثاني للعام ١٤٣٦/١٤٣٥	١٤٣٦/٤/٠٥	١٤٣٦/٠٨/١٧
الاختبار الدوري الأول	١٤٣٦/٥/١٠	١٤٣٦/٠٥/١٤
بداية أجازة منتصف الفصل الدراسي الثاني	١٤٣٦/٥/٢٨	١٤٣٦/٠٦/٠٨
الاختبار الدوري الثاني	١٤٣٦/٠٦/٣٠	١٤٣٦/٠٧/٠٤
الاختبارات النهائية	١٤٣٦/٠٧/٢٨	١٤٣٦/٠٨/٠٣

ملحوظة:الرجاء متابعة موقع الجامعة خلال الاختبارات

Course curriculum

Chapter 1: Measurement

Chapter 2: Motion along a straight line

Chapter 3: Vectors

Chapter 4: Motion in 2 & 3 dimensions

Chapter 5: Force & motion I

Chapter 6: Force & motion II

Chapter 7: Kinetic energy & work

Chanper 9: Center of mass & linear momentum

Chapter 1
MEASUREMENT

- The important skills from this chapter:
  1. Identify physical quantities
  2. Differentiate between based & derived quantities
  3. Identify based quantities in Mechanics & their units
  4. Define the international system of unit SI
  5. Convert between different systems of units
  6. Define the standards of length, time & mass

What is physics?

- Experimental observations & quantitative measurements
- How quantities are measured & compared?
- The unit for measurements & comparisons
- Physics purpose:
  Find laws to conduct those experiments & develop theories to predict future results

Measuring things

- Physical quantities:
  1. Independent (base quantities) → time, length, temperature, mass
  2. Dependent (derived quantities) → speed, density, power, work

  Physical quantities are defined in terms of the base quantities & their standards (called base standards)

  Base standards must be both accessible & invariable

- Quantities are measured by "Unit"
  Unit: name assigned to measure a quantity

Measuring things

- Base quantities → based units
- Derived quantities → derived units

- Derived units are defined in terms of the based unit

e.g. the unit of power (watt) is a derived unit; defined in terms of the base units for mass, length & time:

1 watt = 1 w = 1 $kg.m^2/s^3$

$kg.m^2/s^3$ is read as: kilogram-meter squared per second cubed

The international system of unit SI

- Known as *metric system*

Quantity	Unit name	Unit symbol
Length	Meter	m
Time	Second	s
Mass	Kilogram	kg

Scientific notation

Very large & small quantities are expressed by:

- Power of ten
  3,560,000,000 m = 3.56 $\times 10^9$ m
  0.000 000 492 s = 4.92 $\times 10^{-7}$ s

- Exponent of ten (on computer)
  3.56×10^9 m = 3.56 E9
  4.92×10^{-7} s = 4.92 E–7

- Prefixes e.g. centi-, kilo-, micro-, nano-..

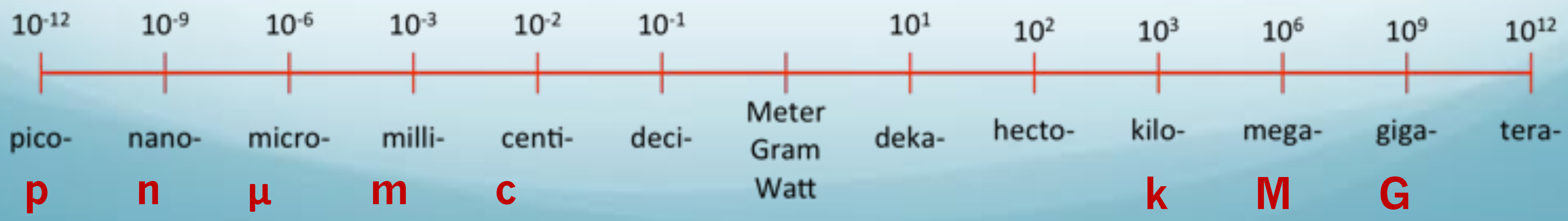


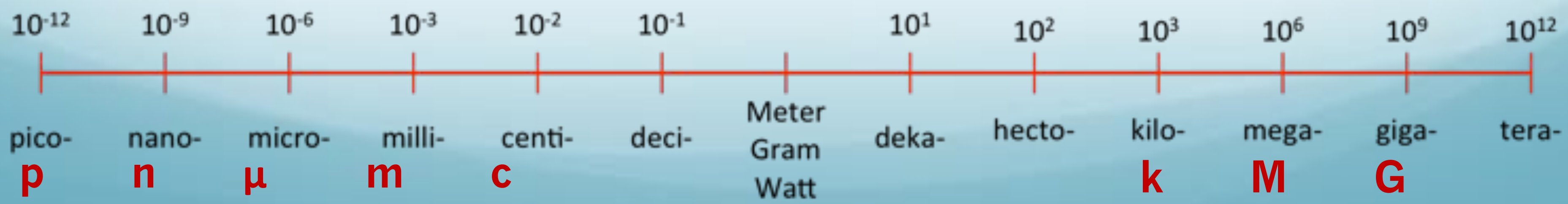
Table 1-2

Prefixes for SI Units

Factor	Prefix[a]	Symbol	Factor	Prefix[a]	Symbol
10^{24}	yotta-	Y	10^{-1}	deci-	d
10^{21}	zetta-	Z	$\mathbf{10^{-2}}$	**centi-**	**c**
10^{18}	exa-	E	$\mathbf{10^{-3}}$	**milli-**	**m**
10^{15}	peta-	P	$\mathbf{10^{-6}}$	**micro-**	**μ**
10^{12}	tera-	T	$\mathbf{10^{-9}}$	**nano-**	**n**
$\mathbf{10^{9}}$	**giga-**	**G**	$\mathbf{10^{-12}}$	**pico-**	**p**
$\mathbf{10^{6}}$	**mega-**	**M**	10^{-15}	femto-	f
$\mathbf{10^{3}}$	**kilo-**	**k**	10^{-18}	atto-	a
10^{2}	hecto-	h	10^{-21}	zepto-	z
10^{1}	deka-	da	10^{-24}	yocto-	y

Examples

- 30 000 g = 30 × 10^3 g = 30 kg
- 3.5 × 10^9 m = 3.5 Gm
- 2.6 × 10^6 m = 2.6 Mm
- 1.48 × 10^{-9} s = 1.48 ns
- 50 km = 50 × 10^3 m
- 0.00000712 watt = 7.12 × 10^{-6} watt
  = 7.12 micro watt = 7.12 μ watt



Changing units

- To change unit, we use a method called "chain-link conversion method"
  → we multiply the original measurement by a "conversion factor"

- Conversion factor: a ratio of units that is equal to unity

- Example: convert 2 min to s:
  Answer: 1min = 60s
  the conversion factor is:

$$\frac{1\,\mathrm{min}}{60\,\mathrm{s}} = 1 \quad \text{or} \quad \frac{60\,\mathrm{s}}{1\,\mathrm{min}} = 1$$

$$2\,\mathrm{min} = 2\,\cancel{\mathrm{min}} \times \frac{60\,\mathrm{s}}{1\,\cancel{\mathrm{min}}} = 120\,\mathrm{s}$$

Standard of length

- Meter (unit of length):
  The length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second

Standard of time

- Second (unit of time):
  The time taken by 9 192 631 770 oscillations of light emitted from a cesium–133 atom

Standard of mass

- Kilogram (unit of mass):
  A platinum–iridium standard mass kept near Paris
- Atomic mass unit (u):
  The carbon–12 atom
  1 u = 1.660 538 86 × 10^{-27} kg
- Density ρ:
  The mass (m) per unit volume (V)

$$\rho = \frac{m}{V}$$

Examples

1. **Convert 60 kg to g**
   1 kg = 1000 g → $60kg = 60kg \times \frac{1000g}{1kg} = 60000g$

2. **Convert 5µm to mm**
   1µm = 10^{-6}m, 1mm = 10^{-3}m

$$5\mu m = 5\mu m \times \frac{10^{-6} m}{1\mu m} \times \frac{1mm}{10^{-3} m} = 5 \times 10^{-3} mm$$

3. **Convert 5 km to m** → 5 km = 5 × 10^3 m

4. **Convert 7000 m to km** → 7000 m = 7 × 10^3 × 10^{-3} km = 7 km

5. **Convert 55.00 km/h to m/s**

$$\frac{55km}{h} = \frac{55 \times 10^3 m}{60 \times 60s} = 15.28m/s$$

6. The basic SI. units of mass is :
(a) m/s^2 (b) m/s (c) kg/m^3 (d) g (e) kg

7. A microsecond is :
(a) 10^6s (b) 10^{-6}s (c) 10^9 s (d) 10^{-9}

8. The SI units of the based quantities (length, mass, time) are:
(a) m, kg, s (b) cm, g, s (c) km, g, s (d) km, kg, s

9. (0.000 000 00636) is equal to:
(a) 6.36×10^{-7} (b) 6.36×10^{-8} (c) 6.36×10^{-9} (d) 6.36×10^{-10}

10. 50 km=
(a) 5×10^5 cm (b) 5×10^6 cm (c) 5×10^7 cm (d) 5×10^8 cm

11. 100 g/cm^3 =
(a)10^3 kg/m^3 (b)10^4 kg/m^3 (c)10^5 kg/m^3 (d)10^6 kg/m^3

12. The density of water is $1 g/cm^3$. This value in SI unit is:
(a) 10^3 kg/cm^3 (b)10 kg/m^3 (c)$10^3 kg/m^3$ (d)10^2 kg/m^3

Assignments:

1- 1 mi is equivalent to 1609 m so 55 mi/h is:

a) 15 m/s b) 25 m/s c) 66 m/s d)88 m/s

2- A cubic box with an edge of exactly 1 cm has a volume of:

a) 10^{-9} m^3 b) 10^{-6} m^3 c) 10^{-3} m^3 d) 10^{6} m^3

3 -The SI base unit for mass is:

a) gram b) pound c) kilogram d) kilopound

4 - A nanosecond is:

a) 10^{9} s b) 10^{-9} s c) 10^{-10} s d) c) 10^{10} s

5 - A gram is:

a). 10^{-6} kg b) 10^{-3} kg c) 1 kg d) 10^{3} kg

6- We can write the speed of light (c = 299,000,000 m/s) using the scientific notation as:

a) 2.99 x 10^{8} b) 29.9 x 10^{8} c) 0.299 x 10^{8} d) 299 x 10^{8}

Problems: 3 (a , b) and 27 (a) .

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•3 The micrometer (1 μm) is often called the *micron.* (a) How many microns make up 1.0 km? (b) What fraction of a centimeter equals 1.0 μm?

••27 Iron has a density of 7.87 g/cm^3, and the mass of an iron atom is 9.27×10^{-26} kg. If the atoms are spherical and tightly packed, (a) what is the volume of an iron atom

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•3 The micrometer (1 μm) is often called the *micron.* (a) How many microns make up 1.0 km? (b) What fraction of a centimeter equals 1.0 μm?

(a) Since 1km $=1\times10^{3}$ m and 1m $= 1\times10^{6}$ μm,

$$1\text{km} = 10^{3}\text{ m} = (10^{3}\text{m})(10^{6}\ \mu\text{m/m}) = 10^{9}\ \mu\text{m}.$$

The given measurement is 1.0 km (two significant figures), which implies our result should be written as 1.0×10^{9} μm.

(b) We calculate the number of microns in 1 centimeter.

Since 1 cm $= 10^{-2}$ m,

$$1\text{cm} = 10^{-2}\text{ m} = (10^{-2}\text{ m})(10^{6}\ \mu\text{ m/m}) = 10^{4}\ \mu\text{m}$$

We conclude that the fraction of one centimeter equal to 1.0 μm is 1.0×10^{-4}

••27 Iron has a density of 7.87 g/cm^3, and the mass of an iron atom is 9.27×10^{-26} kg. If the atoms are spherical and tightly packed, (a) what is the volume of an iron atom

To convert the density ρ unit into kg/m^3

$$\rho = (7.87 g / cm^3)\left(\frac{1kg}{1000g}\right)\left(\frac{100cm}{1m}\right)^3 = 7870 kg / m^3$$

If we ignore the empty spaces between the close-packed spheres, then the density of an individual iron atom will be the same as the density of any iron sample. That is, if *m* is the mass and *V* is the volume of an atom, then

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} = \frac{9.27\times10^{-26} kg}{7870 kg / m^3} = 1.18\times10^{-29} m^3$$

•1 SSM Earth is approximately a sphere of radius 6.37×10^6 m. What are (a) its circumference in kilometers, (b) its surface area in square kilometers

(a) The radius of the Earth

$$R = (6.37\times10^6\text{m})(10^{-3}\text{km}) = 6.37\times10^3 \text{ km}$$

Earth circumference

$$s = 2\pi R = 2\pi\ (6.37 \times 10^3\text{km}) = 4.00 \times10^4 \text{ km}$$

(b) The surface area of Earth is

$$A = 4\pi R^2 = 4\pi\ (6.37\times103 \text{ km})^2 = 5.10\times108 \text{ km}^2$$

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Chapter 2

Motion Along Straight Line

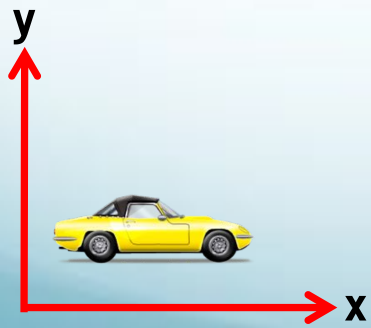
Sections: 2-2, 2-3, 2-4, 2-5

- The important skills from this lecture
  1. Locate the position of a particle with respect to the origin
  2. Identify the positive & negative directions of x & y axes
  3. Calculate the displacement in magnitude & determine its direction
  4. Differentiate between displacement & distance
  5. Define velocity & differentiate between velocity & speed
  6. Calculate the average velocity & speed
  7. Define & calculate the instantaneous velocity & speed
  8. Differentiate between average & instantaneous velocity

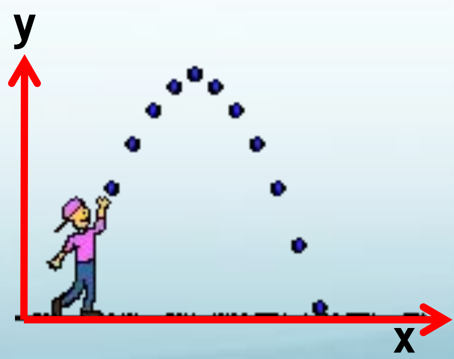
Motion

- Everything in the world moves, even seemingly stationary objects
- The classification & comparison of motions called kinematics, and is often challenging
- Motion could be:

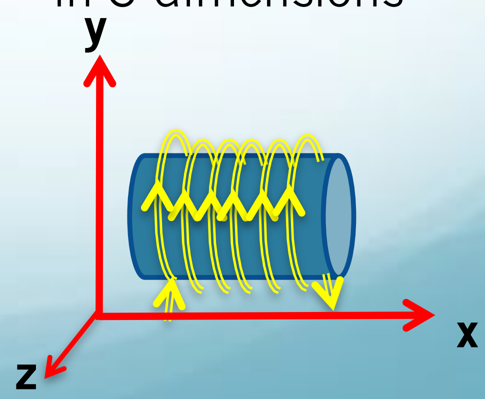
in 1 dimension



in 2 dimensions

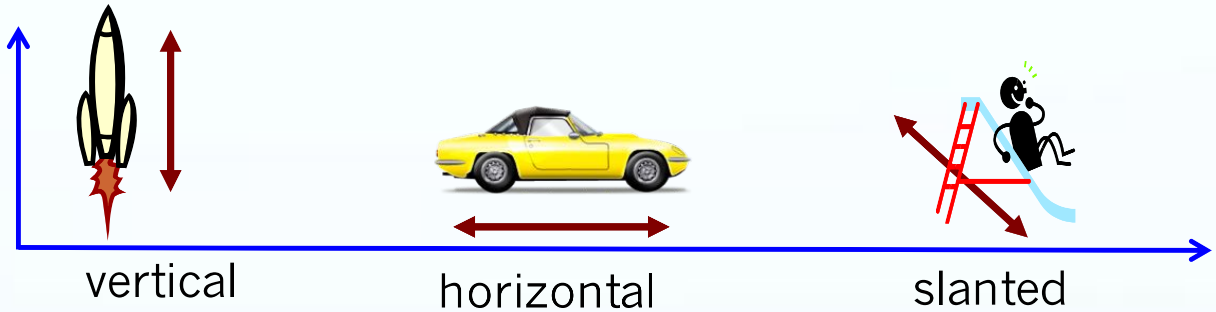


in 3 dimensions



Motion in One Dimension

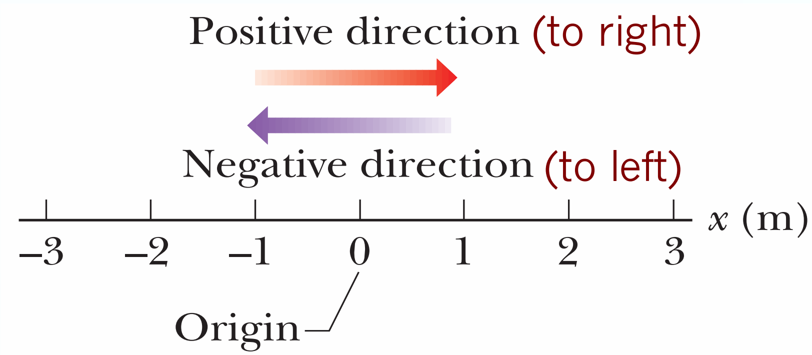
- It is a long a straight line only
  - This straight line might be vertical, horizontal, or slanted



- The moving object speeds up, slow down, stop, or reverse direction
- The motion involves time

Position

- Locate an object means: find its position relative to some reference point, often the origin (zero)



- If the particle is located at:
  x = 3 m → it is 3 m in the +ve direction from the origin
  x = – 3 m → it is 3 m in the –ve direction from the origin
  A coordinate of – 3 m < – 1 m, and both are < +3 m

- \+ sign don't need to be shown
  – sign must always to be shown

Displacement

- Displacement **Δx** (called delta x): the change from a position x_1 to a position x_2

$$\boxed{\Delta x = x_2 - x_1}$$

- Δ: represents a change in a quantity
  Δ = final value – initial value

- Displacement in the right direction always comes out +ve
  Displacement in the left direction always comes out –ve

- If we ignore the sign (the direction) of a displacement
  → magnitude or absolute value of the displacement
  e.g. $\Delta x = -4$ m has a magnitude = $|4 \text{ m}| = 4$ m

- The actual number of meters covered for a trip is irrelevant; displacement involves only the original & final positions

- Displacement has two features:
  1. Its magnitude is the distance between the original & final positions
  2. Its direction, from an original position to a final position, has + or – sign (vector quantity)

CHECKPOINT 1

Here are three pairs of initial and final positions, respectively, along an x axis. Which pairs give a negative displacement: (a) −3 m, +5 m; (b) −3 m, −7 m; (c) 7 m, −3 m?

Solution:

$$\Delta x = x_2 - x_1$$

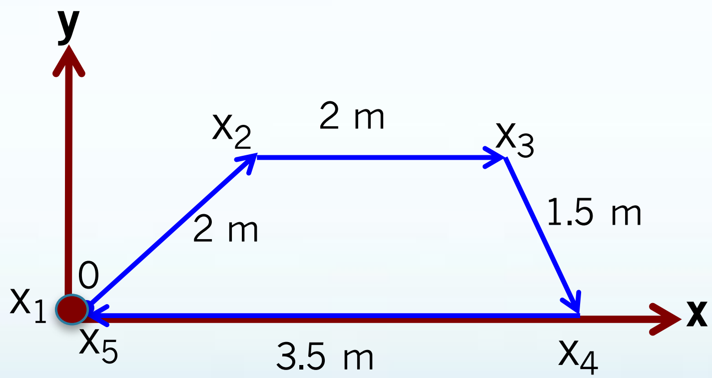
✗ $(a)\ \ x_1 = -3,\ \ x_2 = +5 \Rightarrow\ \ \Delta x = 5 - (-3) = 8m$

✔ $(b)\ \ x_1 = -3,\ \ x_2 = -7 \Rightarrow\ \ \Delta x = -7 - (-3) = -4m$

✔ $(c)\ \ x_1 = 7,\ \ x_2 = -3 \Rightarrow\ \ \Delta x = -3 - 7 = -10m$

Distance

- The actual meters that the object travels
- It is a scalar quantities
- It is not always equal to the magnitude of the displacement e.g. if the particle moves from x_1 to x_5 as follows:



The displacement $= x_5 - x_1 = 0 - 0 = 0$

The distance $= 2 + 2 + 1.5 + 3.5 = 9$ m

Examples

Find the displacement, direction and distance for:

1. **A particle moves from x_1= 5 m to x_2 = 12 m**,
   Δx = 12 m – 5 m = +7 m.
   7 is positive → the motion in + direction
   Distance = 12 + 5 =17 m

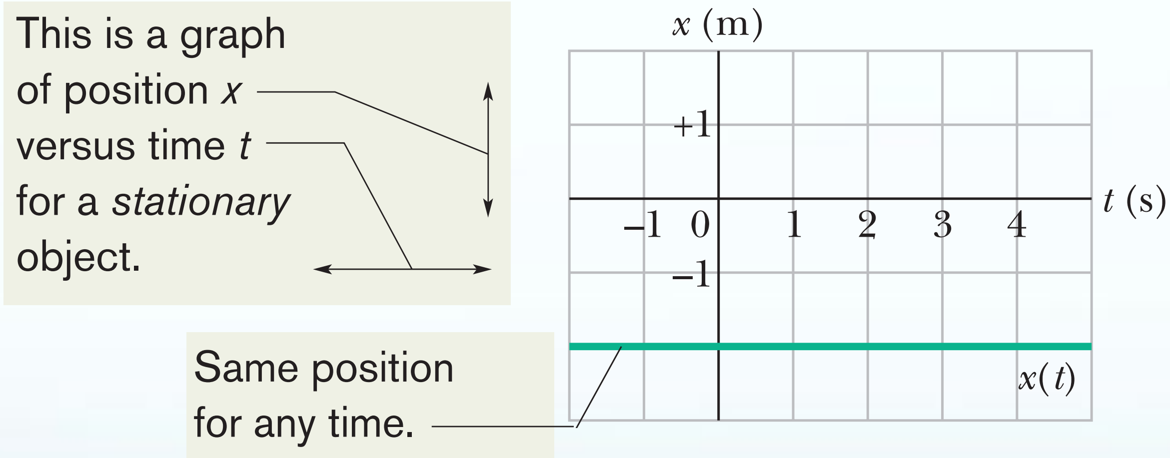
2. **A particle moves from x_1= 5 m to x_2 = 1 m**,
   Δx = 1 m – 5 m = – 4 m.
   4 is negative → the motion in – direction
   Distance = 1 + 5 = 6 m

3. **A particle moves from x_1= 5 m to x_2 = 200 m then back to x_3 = 5**
   Δx = 5 m – 5 m = 0
   Distance = 5 + 200 + 5 =210 m

Graphical Description for Motion

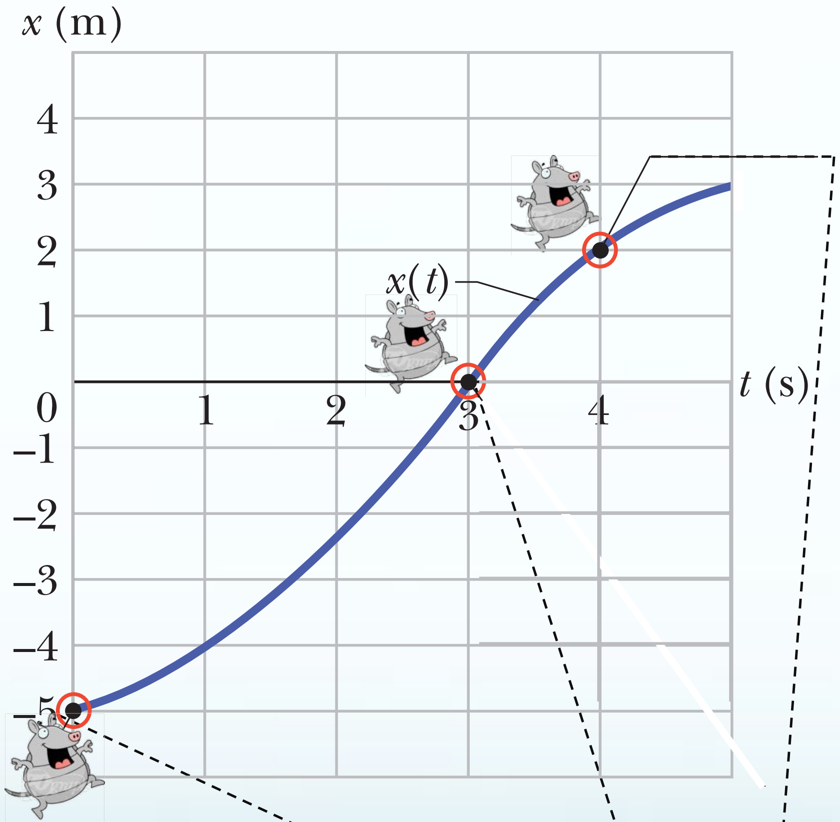
- A position is described also with a graph of position (x) plotted as a function of time (t) – this graph is called a graph of x(t)

1. If the particle is not moving:



The graph shows position x(t) for a stationary object over 7s time interval
The object's position stays at x = – 2 m (not moving)

2. If the object is moving



Time (s)	0	3	4
Position (m)	– 5	0	2

Average Velocity (v_{avg})

- v_{avg}: the ratio of the displacement Δx that occurs during a particular time interval Δt to that interval

$$\boxed{v_{avg} = \frac{\text{displacement}}{\Delta t} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}}$$ length/time e.g. m/s, Km/h

- How we calculate v_{avg} from $x(t)$ graph?
  1. Draw a straight line between the initial & final position of the object
  2. Find the slope of that straight line

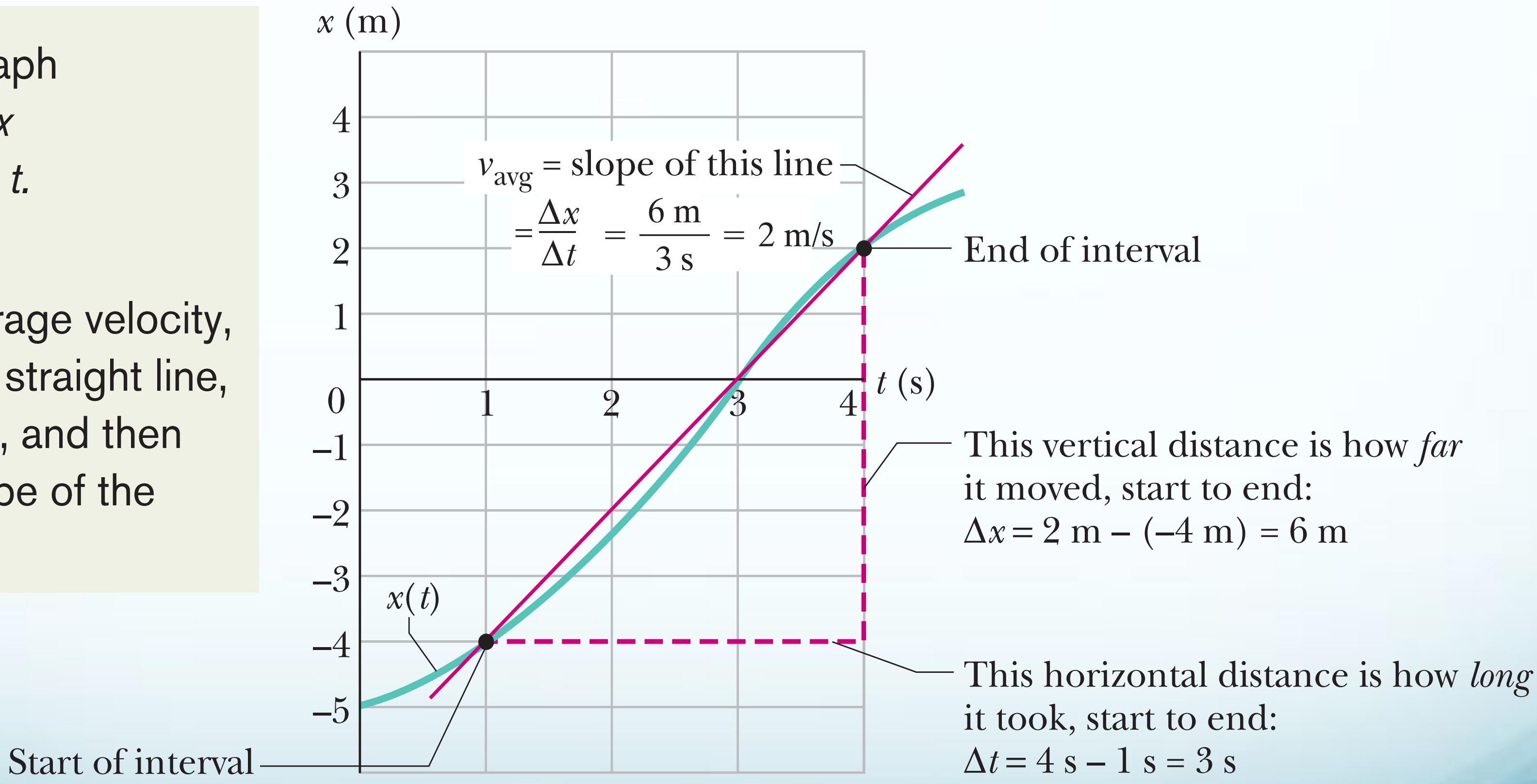
- v_{avg} is a vector quantities (magnitude & direction)
  Magnitude: the slope value
  Direction: + v_{avg} (+ slope) → line slants upward to the right ↗
  – v_{avg} (– slope) → line slants downward to the right ↘

- v_{avg} has the same sign of displacement? Because Δt is always +

Example:
Calculate v_{avg} from x(t) graph

This is a graph of position *x* versus time *t.*

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.



Average Speed s_{avg}

- s_{avg}: the total distance travelled (e.g. the number of meters moved), independent of the direction

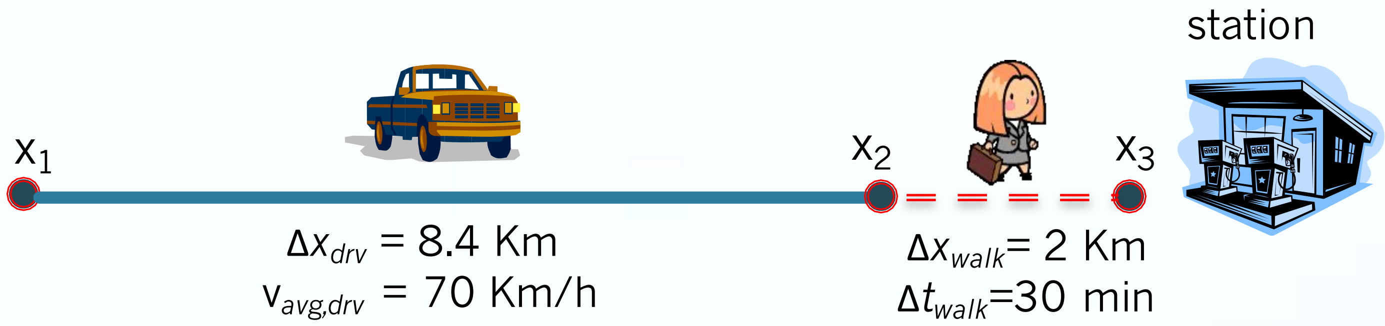
$$\boxed{s_{avg} = \frac{\text{total distance}}{\Delta t}}$$

- It is a scalar quantity

Sample Problem

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?



$$x_1 = 0$$

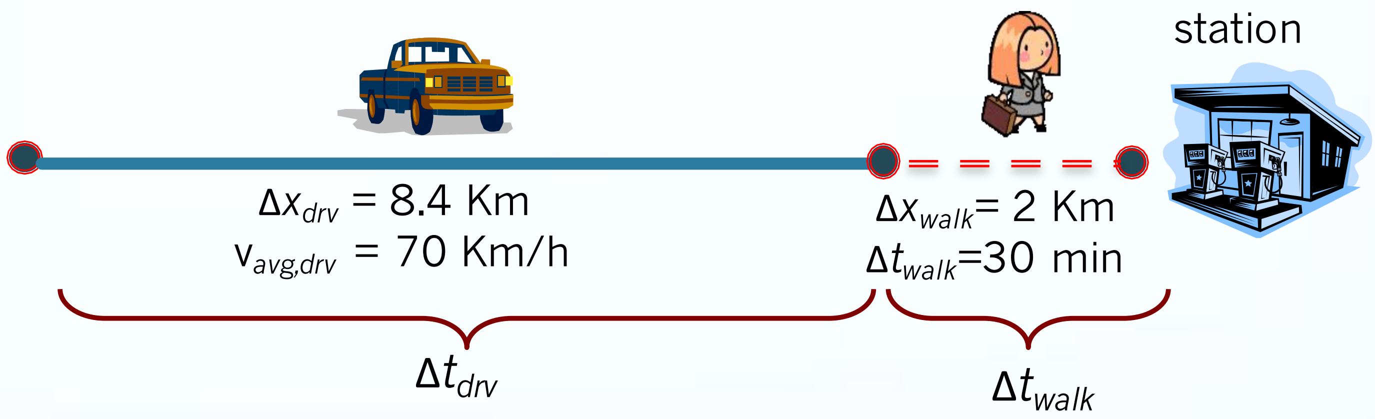
$$x_3 = 8.4km + 2.0km = 10.4km$$

$$\Delta x_{total} = x_3 - x_1 \Rightarrow 10.4 - 0 = 10.4km$$

Another solution:

$$\Delta x_{total} = \Delta x_{drv} + \Delta x_{walk}$$
$$= 8.4km + 2km = 10.4km$$

(b) What is the time interval Δt from the beginning of your drive to your arrival at the station?



$$\Delta t_{total} = \Delta t_{drv} + \Delta t_{walk}$$

$$\Delta t_{walk} = 30\,\text{min} = 0.5h$$

$$v_{avg,drv} = \frac{\Delta x_{drv}}{\Delta t_{drv}}$$

$$\Delta t_{drv} = \frac{\Delta x_{drv}}{v_{svg,drv}} = \frac{8.4km}{70km/h} = 0.12h$$

$$\Delta t_{total} = 0.12h + 0.5h = 0.62h$$

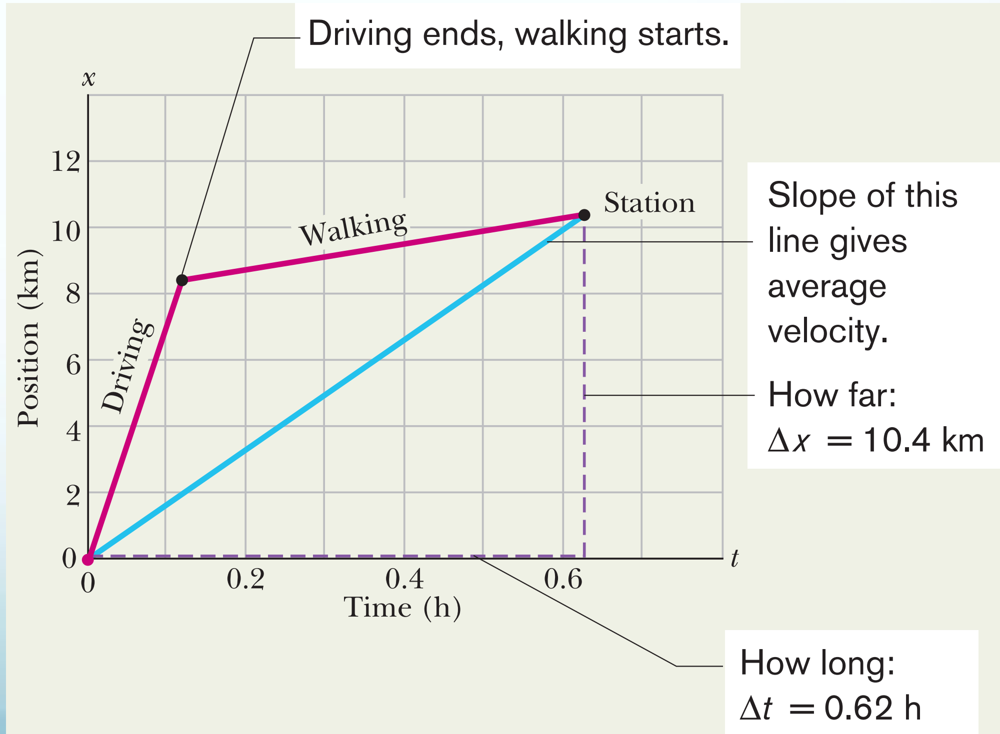
(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

Numerically

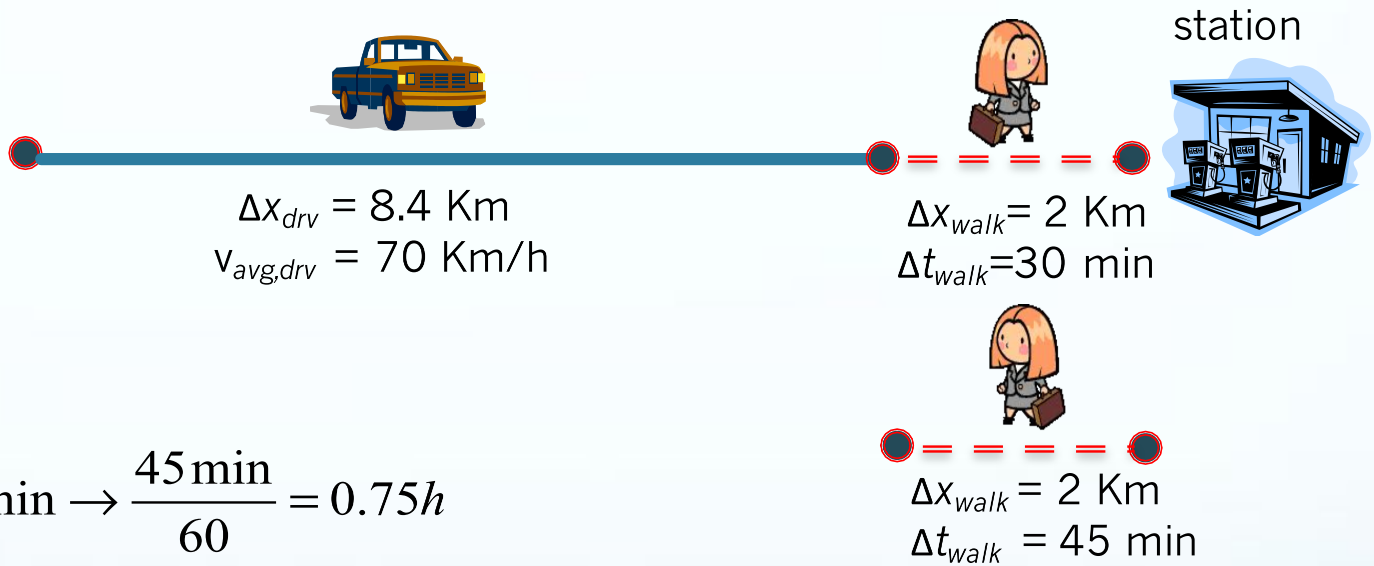
$$v_{avg,total} = \frac{\Delta x_{total}}{\Delta t_{total}} = \frac{10.4km}{0.62h} = 16.6km/h$$

Graphically

x (km)	0	8.4	10.4
t (h)	0	0.12	0.62



(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?



$$45\,\text{min} \rightarrow \frac{45\,\text{min}}{60} = 0.75h$$

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

$$s_{avg} = \frac{8.4 + 2.0 + 2.0}{0.12 + 0.5 + 0.75} = \frac{12.4km}{1.37h} = 9.1km/h$$

Instantaneous Velocity (v)

- The instantaneous velocity (or simply velocity) v refers to velocity of a particle at a given instant

- It is obtained from the v_{avg} by shrinking the time interval Δt closer and closer to 0
  As Δt reduced → v_{avg} approaches a limiting value, which is the velocity at that instant:

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

- v: the rate at which position x is changing with time at a given instant (derivative of x with respect to t)

- Graphically, *v* is given from the slope of the position–time curve ($x(t)$) at the point representing that instant

- *v* is a vector quantity (magnitude & direction)

Instantaneous Speed

- The magnitude of velocity
  (velocity without any indication of direction)

- Notice: speed & average speed are different
  A velocity of + 5 m/s & – 5 m/s both have an associated speed of 5 m/s

- The speedometer in a car measures speed, not velocity (it cannot determine the direction)

CHECKPOINT 2

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?

$$v = \frac{dx}{dt}$$

$$(1)\quad x = 3t - 2,\quad v = \frac{d(3t-2)}{dt} = 3m/s$$

$$(2)\quad x = -4t^2 - 2,\quad v = \frac{d(-4t^2-2)}{dt} = -8tm/s$$

$$(3)\quad x = 2/t^2,\quad v = \frac{d(2t^{-2})}{dt} = -4t^{-3}m/s$$

$$(4)\quad x = -2,\quad v = \frac{d(-2)}{dt} = 0$$

(a) v is constant → (1) & (4)

(b) v is negative → (2) & (3)

Sample Problem

The position of a particle moving on an x axis is given by

$$x = 7.8 + 9.2t - 2.1t^3$$

With x in meter & t in second. (1) What is its velocity at t = 3.5 s? (2) Is the velocity constant, or is it continuously changing?

Answer:

(1)
$$v = \frac{dx}{dt} = \frac{d}{dt}\left(7.8 + 9.2t - 2.1t^3\right)$$

$$= 9.2 - 3(2.1)t^2 = 9.2 - 6.3t^2$$

at t =3.5 s

$$v = 9.2 - 6.3(3.5)^2 = 9.2 - 77.175$$

$$= -67.97 \approx -68 m/s$$

(2) Because v is a function of t, it is not constant but continuously changes with time

Examples:

Q.1 The position of a particle along the *x*-axis is given by

$$x = 3t^3 - 2t^2 - 2$$

where *x* in meters and *t* in second, the velocity of this particle in the time interval from t_1= 1 s to t_2 = 3 s is:

(a) 13 m/s (b) 10 m/s (c) 31 m/s (d) –10 m/s

Answer:

$$t_1 = 1s \quad \Rightarrow \quad x_1 = 3(1)^3 - 2(1)^2 - 2 = -1m$$

$$t_2 = 3s \quad \Rightarrow \quad x_2 = 3(3)^3 - 2(3)^2 - 2 = 61m$$

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{61-(-1)}{3-1} = 31m/s$$

Q.2 A car moves along a straight line with velocity in m/s given by

$$v = t^2 - 16$$

The velocity at *t* = 0 is:
(a) zero (b) 4 m/s (c) –16 m/s (d) -9 m/s

Answer:

$$v(t) = t^2 - 16 \quad \Rightarrow \quad v(0) = (0)^2 - 16 \ = \text{-}16 m/s$$

The change in the velocity between the time interval *t* = 1 and *t* = 5 is:
(a) zero (b) 4 (c) 24 (d) -9

Answer:

$$v(1) = 1 - 16 = -15, \qquad v(5) = (5)^2 - 16 \ = 9 m/s$$

$$\Delta v = 9 - (-15) = 24$$

Q.3 Referring to the previous question, the car stops when *t* equals:
(a) zero (b) 4s (c) 3s (d) 6s (e) 2s

Answer:

When car stops, $v = 0$ →

$$0 = t^2 - 16 \quad \Rightarrow \quad t^2 = 16 \quad \Rightarrow \quad t = 4s$$

Q.4 A bicycle is moving along x-axis according to the equation

$$x(t) = 2t + 3t^2$$

where *x* is in meters and *t* is in second. Its velocity at *t* = 2 s is:
(a) 14 m/s (b) 26 m/s (c) 32 m/s (d) m/s (e) 38 m/s

Answer:

$$v = \frac{dx(t)}{dt} = \frac{d\left(2t + 3t^2\right)}{dt} = 2 + 6t$$

$$\text{at } \; t = 2s, \quad v = 2 + 12 = 14m/s$$

Q.5 The initial and final positions of a particle along the *x*-axis are –3 m, 10 m respectively. Its displacement is:
(a) 7m (b) 13m (c) –13 m (d) –7 m (e) 4.5 m

Answer:

$$\Delta x = 10 - (-3) = 13m$$

Chapter 2
Motion Along Straight Line

Sections 2-6, 2-7

Acceleration
Constant Acceleration

- The important skills from this lecture
  1. Define & calculate the average acceleration
  2. Define & calculate the instantaneous acceleration
  3. Differentiate between average & instantaneous acceleration
  4. Explain motion with constant acceleration
  5. Apply the constant acceleration equations to solve problems

Acceleration (a)

- Changing velocity → acceleration
  If the particle has velocity v_1 at time t_1 & v_2 at time t_2, the average acceleration a_{avg} over a time interval Δt is

$$\boxed{a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}}$$

- The instantaneous acceleration (or simply acceleration) a is the acceleration of a particle at any instant

$$\boxed{a = \frac{dv}{dt}}$$

a is the rate at which the particle's velocity is changing at that instant

- From equations: $v = \dfrac{dx}{dt}, \quad a = \dfrac{dv}{dt}$

$$\rightarrow \; a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

→ The acceleration of a particle at any instant is the second derivative of its position $x(t)$ with respect to time

- Acceleration unit: m/s^2 (length/(time^2))
- Acceleration has both magnitude & direction (vector quantity)
- The sign of an acceleration has a nonscientific meaning:
  +ve acceleration → the speed is increasing ↑
  –ve negative acceleration → the speed is decreasing ↓ (called deceleration)

- If the signs of the v & a are the same → speed increased ↑
  If the signs of the v & a are opposite → speed decreased ↓

CHECKPOINT 3

A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Answer:

(a) movement in +ve direction, ↑speed
→ a & v has same signs → +ve a

(b) movement in +ve direction, ↓speed
→ a & v has different signs → –ve a

(c) movement in –ve direction, ↑speed
→ a & v has same signs → –ve a

(d) movement in –ve direction, ↓speed
→ a & v has same signs → +ve a

Sample Problem

A particle's position on the x axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

Answer:

$$v = \frac{dx}{dt} = -27 + 3t^2 m/s$$

$$a = \frac{dv}{dt} = 6t m/s^2$$

(b) Is there ever a time when $v = 0$?

Answer: $v = 0 = -27 + 3t^2 \quad \Rightarrow \quad 3t^2 = 27 \quad \Rightarrow \quad t = \sqrt{9} = \pm 3s$

The velocity is zero both 3 s before and 3 s after the clock reads 0

Examples:

Q.1 If t_1= 2 s and t_2= 4 s, find the average acceleration when the velocity changes from 8 m/s to 12 m/s.

(a) 1 m/s^2 (b) 3.33 m/s^2 (c) 5 m/s^2 (d) 2 m/s^2 (e) 4.5 m/s^2

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{12-8}{4-2} = \frac{4}{2} = 2m/s^2$$

Q.2 The velocity of a particle starts from the origin as:

$$v(t) = 3t^2 + 5m/s$$

The acceleration of the particle after 2 second is:

(a) 6 m/s^2 (b) 12 m/s^2 (c) 18 m/s^2 (b) 24 m/s^2 (b) 30 m/s^2

$$v(t) = 3t^2 + 5m/s$$

$$a(t) = \frac{dv}{dt} = 6t, \quad \Rightarrow a(2) = 6 \times 2 = 12m/s^2$$

Q.3 The instantaneous acceleration is given by:

$$(a)\frac{dx}{dt} \quad (b)\frac{d}{dt}\left(\frac{dx}{dt}\right) \quad (c)\frac{d^2}{dt^2}\left(\frac{dx}{dt}\right) \quad (d)\frac{d}{dt}\left(\frac{dx}{dt^2}\right) \quad (e)\frac{d^2}{dt^2}\left(\frac{dv}{dt}\right)$$

Q.4 A car is traveling at constant speed of 30 m/s for 3 s:

(1) The acceleration of the car is

(a) 0 **(b) 3 m/s²** **(c) 10 m/s²** **(d) 9 m/s²**

Because the speed is constant → the acceleration is zero

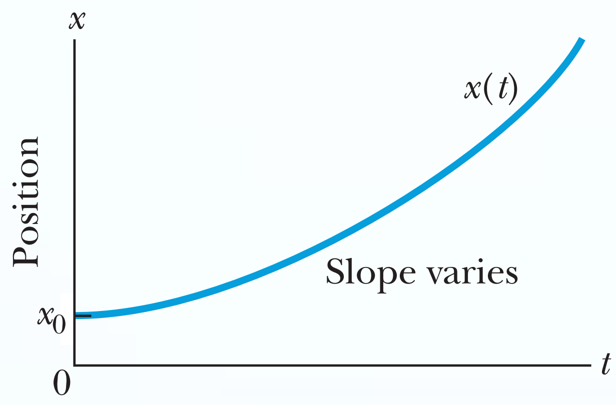
(2) The distance after that time is:

(a) 90 m **(b) 50 m** **(c) 33 m** **(d) 27 m**

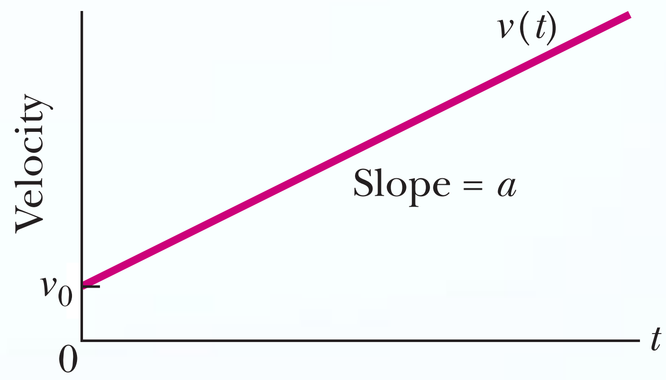
The distance $x = vt = 30 \times 3 = 90$ m

Constant Acceleration: a Special Case

- In many types of motion, the acceleration is either constant or approximately so

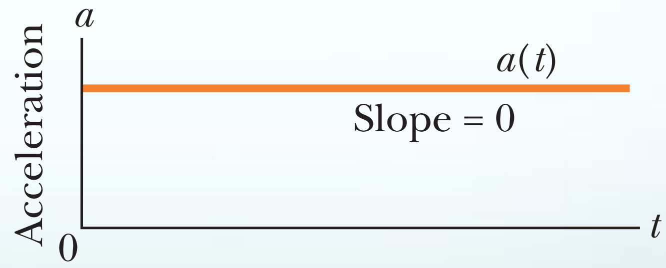


(a)



(b)

t (s)	v (m/s)	x = vt (m)	a = v/t (m/s²)
1	20	20	20
2	40	80	20
3	60	180	20
4	80	320	20
5	100	500	20



(c)

$a(t)$ is constant, which requires that $v(t)$ has a constant slope

Equations of Motion with a Constant Acceleration

When the acceleration is constant → $a_{avg} = a$

$$a = a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v - v_o}{t - 0}$$

since $t_1 = 0, t_2 = t, v_1 = v_0, v_2 = v$

→ $\boxed{v = v_0 + at}$

Similarly, $v_{avg} = \dfrac{x - x_0}{t - 0}$

since $t_1 = 0, t_2 = t, x_1 = x_0, x_2 = x$

→ $x = x_0 + v_{avg} t$

- For the linear velocity function, v_{avg} from t = 0 to t = t) is given by:

$$v_{avg} = \frac{1}{2}(v_0 + v)$$

- Substituting v from $v = v_0 + at$ into the above equation:

$$v_{avg} = v_0 + \frac{1}{2}at$$

- Substituting v_{avg} from the above equation into $x = x_0 + v_{avg}t$ gives:

$$\boxed{x - x_0 = v_0 t + \frac{1}{2}at^2}$$

Table 2-1

Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

The basic 5 quantities that can be involved in any constant acceleration problem are: $x - x_0$, v, t, a, & v_0

CHECKPOINT 4

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

$$(1)\quad x = 3t - 4, \qquad v = \frac{dx}{dt} = 3, \qquad a = \frac{dv}{dt} = 0 \ ✔$$

$$(2)\quad x = -5t^3 + 4t^2 + 6, \qquad v = \frac{dx}{dt} = -15t^2 + 8t, \qquad a = \frac{dv}{dt} = -30t + 8 \ ✗$$

$$(3)\quad x = 2t^{-2} - 4t^{-1}, \qquad v = \frac{dx}{dt} = -4t^{-3} + 4t^{-2}, \qquad a = \frac{dv}{dt} = 12t^{-4} - 8t^{-3} \ ✗$$

$$(4)\quad x = 5t^2 - 3, \qquad v = \frac{dx}{dt} = 10t, \qquad a = \frac{dv}{dt} = 10 \ ✔$$

Sample Problem

The head of a woodpecker is moving forward at speed of 7.49 m/s when the beak makes first contact with a tree limb. The beak stops after penetrating the limb by 1.87 mm. Assuming that acceleration to be constant, find the acceleration magnitude in terms of g

$v_0 = 7.49\, m/s \qquad v = 0$

$x_0 = 0 \qquad x = 1.87\, mm = 1.87\times10^{-3}\, m$

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow 0 = (7.49)^2 + 2a(1.87\times10^{-3} - 0)$$

$$2a(1.87\times10^{-3}) = -(7.49)^2$$

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\rightarrow \quad a = \frac{-(7.49)^2}{2(1.87\times10^{-3})} = -1.5\times10^4\, m/s^2$$

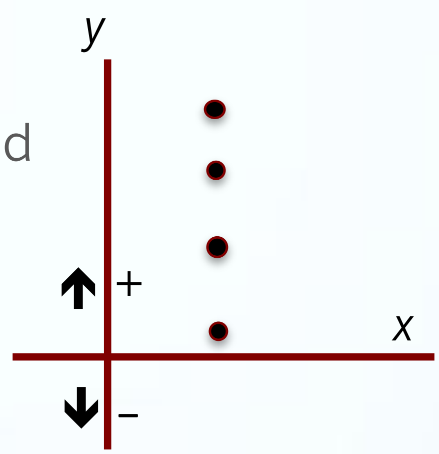
In terms of g, $a = \dfrac{-1.5\times10^4\, m/s^2 \times g}{-9.8\, m/s^2} = 1.53\times10^3\, g$

Free-Fall Object

- If an object is tossed up or down & the effects of air is eliminated (in vacuum)
  → accelerates downward ↓ at a certain constant rate called free-fall acceleration g
- The free-fall acceleration g:
  - Independent of the object's characteristics (mass, density, or shape..)
  - Same for all objects
- In vacuum, feather & apple free-fall at the same g
  The acceleration increases the distance between successive images

- The equations of motion for constant acceleration also applied to free-fall

- For free-fall:
  1. The directions of motion are along y axis instead of the x axis
  2. The free-fall acceleration is always $a = -g = -9.8\ m/s^2$ in the equations, so it is downward ↓

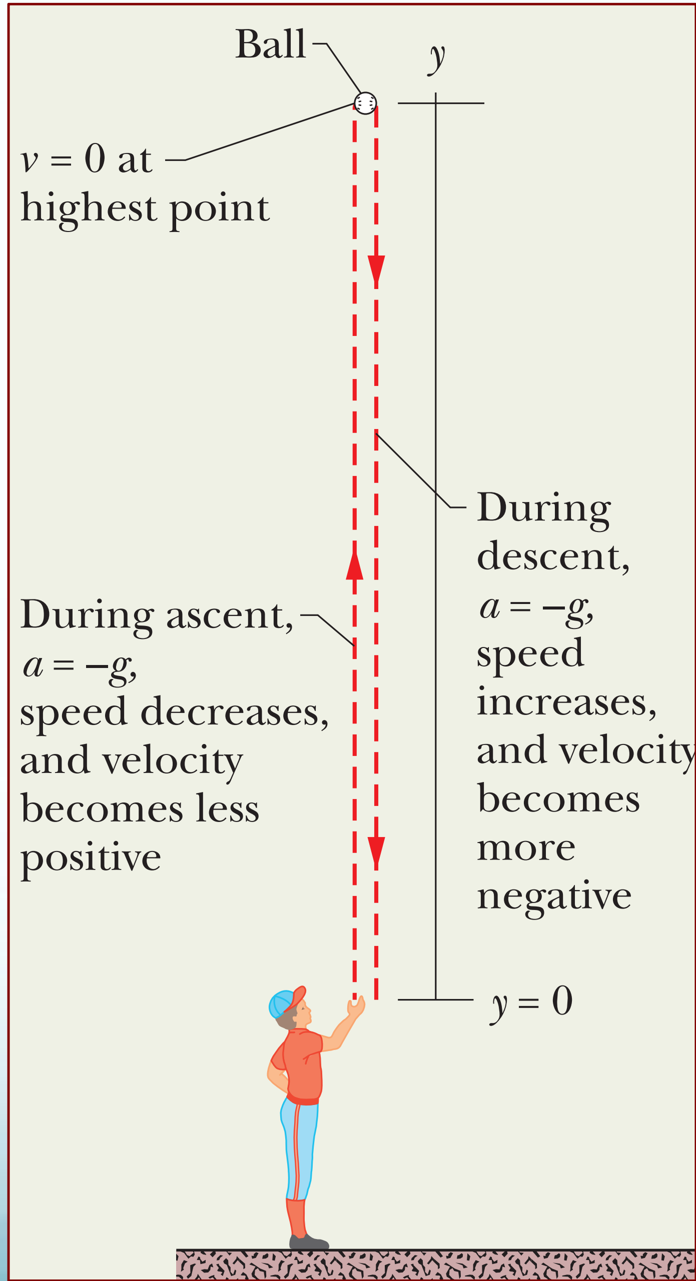


- **Free-Fall Equations of Motion**

$$a = -g, \quad x_0 = y_0, \quad x = y$$

$$\begin{aligned} v &= v_0 + at \\ x - x_0 &= v_0 t + \tfrac{1}{2}at^2 \\ v^2 &= v_0^2 + 2a(x - x_0) \\ x - x_0 &= \tfrac{1}{2}(v_0 + v)t \\ x - x_0 &= vt - \tfrac{1}{2}at^2 \end{aligned} \quad \rightarrow \quad \begin{aligned} v &= v_0 - gt \\ y - y_0 &= v_0 t - \frac{1}{2}gt^2 \\ v^2 &= v_0^2 - 2g(y - y_0) \\ y - y_0 &= \frac{1}{2}(v_0 + v)t \\ y - y_0 &= vt + \frac{1}{2}gt^2 \end{aligned}$$

- The velocity changes in magnitude & direction:
  - During ascent (upward 🡅): +ve direction, v decreases, until it becomes zero
  - The object reaches its maximum height at $v = 0$
  - During descent (downward 🡇): –ve direction, v increases



Sample Problem

On September 26, 1993, Dave Munday went over the Canadian edge of Niagara Falls in a steel ball equipped with an air hole and then fell 48 m to the water (and rocks). Assume his initial velocity was zero, and neglect the effect of the air on the ball during the fall.

(a) How long did Munday fall to reach the water surface?

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v_0 = 0 \qquad y = -48m$$

$$y_0 = 0 \qquad a = g = 9.8\,m/s^2$$

$$y - y_0 = v_0 t - \frac{1}{2}gt^2$$

$$-48 = (0)t - \frac{1}{2}(9.8)t^2$$

$$t^2 = \frac{48}{4.8} \qquad \Rightarrow t = 3.1s$$

(b) Munday could count off the three seconds of free fall but could not see how far he had fallen with each count. Determine his position at each full second.

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

at $t = 1s,\quad y - y_0 = v_0 t - \frac{1}{2}gt^2$

$$\Rightarrow y - 0 = (0)t - \frac{1}{2}(9.8)(1)^2 \Rightarrow y = -4.9m$$

at $t = 2s,\quad y = -19.6m$

at $t = 3s,\quad y = -44.1m$

(c) What was Munday's velocity as he reached the water surface?

$$v^2 = v_0 - 2g\left(y - y_0\right)$$
$$v^2 = (0) - 2(9.8)(-48)$$
$$v^2 = -19.6(-48)$$
$$v = \pm 30.67\, m/s$$

Because the motion in – y → v = -30.67m/s

(d) What was Munday's velocity at each count of one full second? Was he aware of his increasing speed?

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\text{at } t = 1s$$
$$v = v_0 - gt = (-9.8)(1) = -9.8\, m/s$$
$$\text{at } t = 2s$$
$$v = (-9.8)(2) = -19.6\, m/s$$
$$\text{at } t = 3s$$
$$v = (-9.8)(3) = -29.4\, m/s$$

He was unaware of his increasing speed because his acceleration was constant

Sample Problem

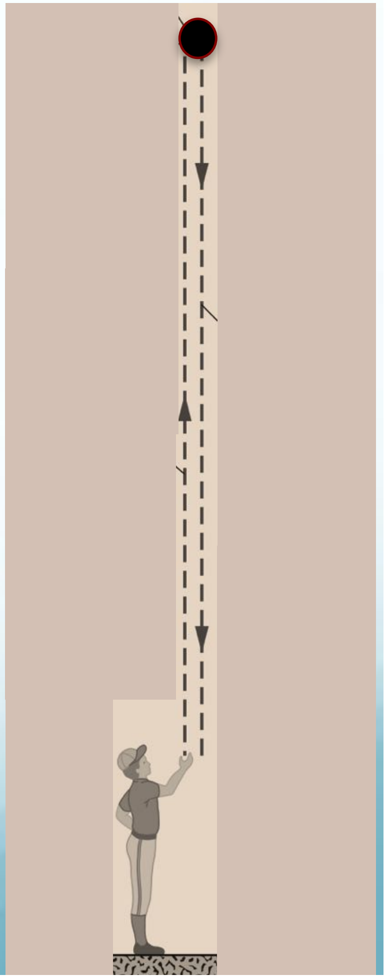
In Fig. 2-11, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v_0 = 12\, m/s$$
$$v = 0$$

$$v = v_0 - gt$$
$$0 = 12 - 9.8t$$
$$t = 1.2s$$



(b) What is the ball's maximum height above its release point?

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$y_0 = 0$$
$$v = 0$$
$$t = 1.2s$$

$$y - y_0 = v_0 t - \frac{1}{2}gt^2$$

$$y - 0 = 12 \times (1.2) - \frac{1}{2}(9.8)(1.2)^2$$

$$y = 7.3m$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

$$5 = 12t - \frac{1}{2}(9.8)(t)^2$$

$$4.9t^2 - 12t + 5 = 0$$

$$t = \frac{12 \pm \sqrt{(12)^2 - 4(4.9)(5)}}{2(4.9)} = \frac{12 \pm 6.7}{9.8}$$

$$\Rightarrow t_1 = 0.53s, \quad t_2 = -1.9s$$

Two times because the ball passes twice through y = 5.0 m, once on the way up and once on the way down

Assignments:

Chapter 2: MOTION ALONG A STRAIGHT LINE

1- Complete the following statement: Displacement is
a) a scalar that indicates the distance between two points.
b) a vector indicating the distance and direction from one point to another.
c) a measure of volume.
d) the same as the distance traveled between two points.

2- A particle moves along the x axis from x_i to x_f . which results in the displacement with the largest magnitude?
a). $x_i = 4m$, $x_f = 6m$
b). $x_i = -4m$, $x_f = -8m$
c). $x_i = -4m$, $x_f = 2m$
d). $x_i = -4m$, $x_f = 4m$

3. Suppose the motion of a particle is described by the equation: $X = 20 + 4t^2$. Find the average velocity of the particle in the time interval t_1=2 s to t_2=5 s ?

a) 29 m/s b) 28 m/s c) 84 m/s d) 10 m/s

4. The following are equations of the position of a particle, in which situation the velocity of the particle is constant ?
a) $x = 4t^2 - 2$ b) $x = -2t^3$ c) $x = -3t - 2$ d) $x = 4t^{-2}$

5. The coordinate of a particle in meters is given by $x(t) = 16t - 3t^3$, where the time t is in seconds. The particle is momentarily at rest at t =

a)0.75 s b) 1.3 s c)5.3 s d)7.3 s

Problems: 23, 45

•23 SSM An electron with an initial velocity $v_0 = 1.50 \times 10^5$ m/s enters a region of length $L = 1.00$ cm where it is electrically accelerated (Fig. 2-23). It emerges with $v = 5.70 \times 10^6$ m/s. What is its acceleration, assumed constant?

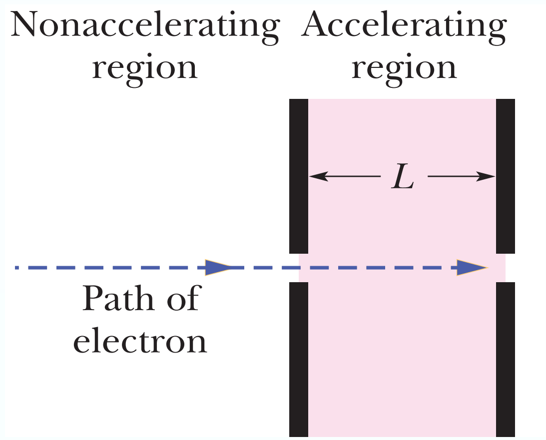


Fig. 2-23 Problem 23.

•45 SSM WWW (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50 m? (b) How long will it be in the air? (c) Sketch graphs of y, v, and a versus t for the ball. On the first two graphs, indicate the time at which 50 m is reached.

Physics 110
1435-1436 H

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Chapter 3
Vectors

Sections 3-2, 3-3

Vectors & Scalars
Adding Vectors Geometrically

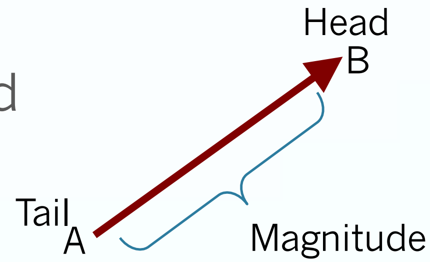
- The important skills from this lecture:
  1. Define and differentiate between vector & scalar quantities
  2. Add vectors geometrically
  3. Identify the addition properties of vectors: commutative law, associative law, and subtraction

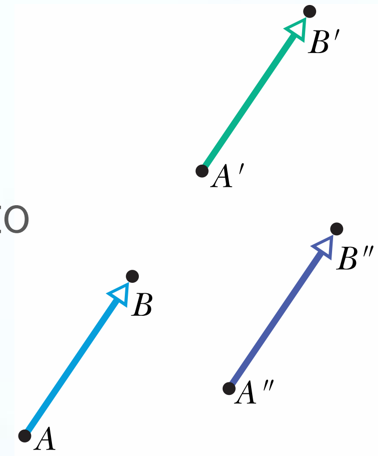
Vectors & Scalars

- A particle moving along a straight line can move in only two directions, +ve & –ve
- For a particle moving in 3D, + & – signs is not enough to indicate a direction → we use a vector
- Physical quantities could be vector or scalar:
  - **Vector quantity**: has magnitude & direction
    → represented by vector
    e.g. displacement, velocity, & acceleration
  - A vector that represents a displacement is called displacement vector
    (Similarly, velocity vectors & acceleration vectors)
  - **Scalar quantity:** no direction
    e.g. temperature, pressure, energy, mass..
  - Note: A single value, with a sign specifies a scalar (e.g., a temperature of –40°F)

Example: Displacement Vector

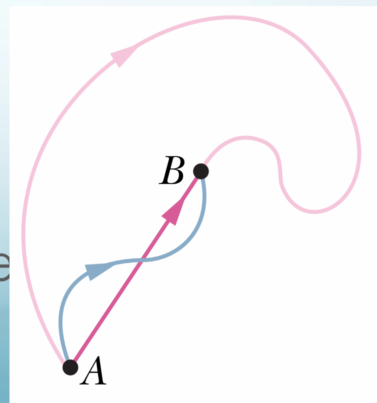
- If a particle moves from A to B → its displacement represented by an arrow from A to B





- The arrows from A to B, from A’ to B’, and from A’’ to B’’ have the same magnitude & direction
→ identical displacement vectors
→ represent the same change in position

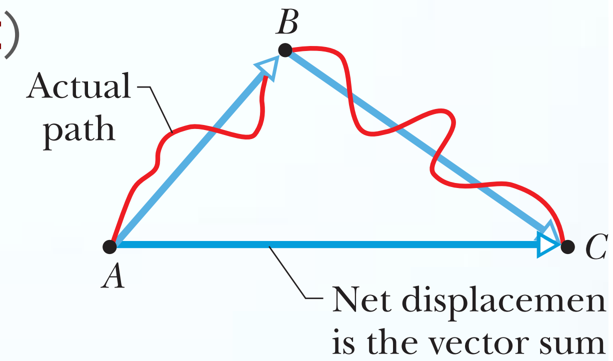
- A vector can be shifted without changing its value if its length & direction are not changed



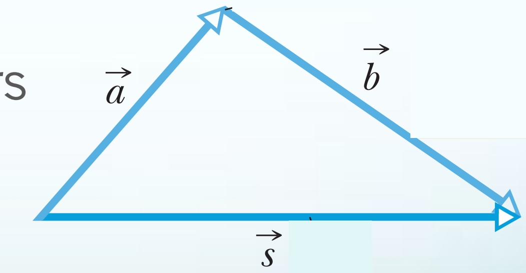
- Displacement vectors represent only the overall effe of the motion, not the path of motion itself

Adding Vectors Geometrically

- If a particle moves from A to B, then to C
  → the overall displacement (**net displacement**) is the sum of AB & BC vectors
  AB + BC → displacement from A to C
  AC called vector sum (or **resultant**)



- $\vec{a}, \vec{b}$, and $\vec{s}$ represent the displacement vectors AB, BC, & AC
  a, b, and s indicate only the magnitude

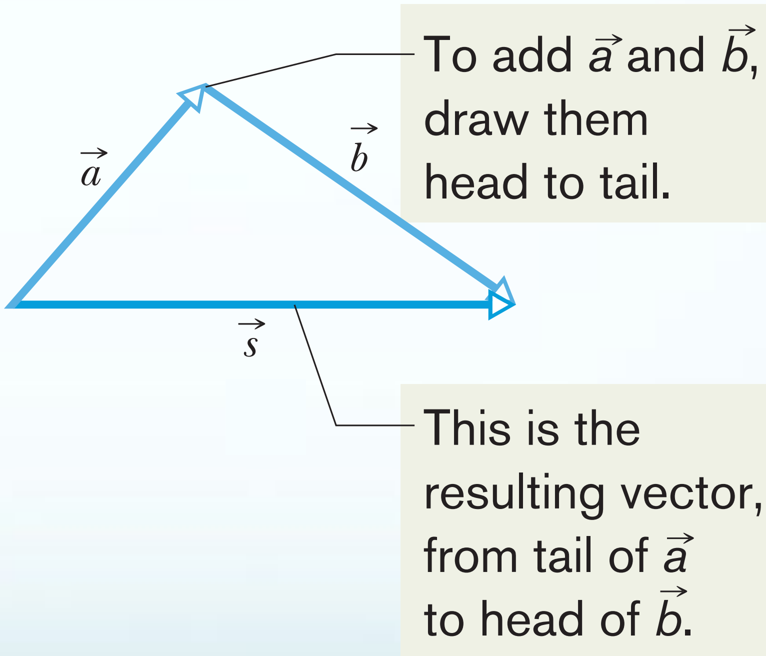


- We can represent the relation among the three vectors with the vector equation

$$\boxed{\vec{s} = \vec{a} + \vec{b}}$$

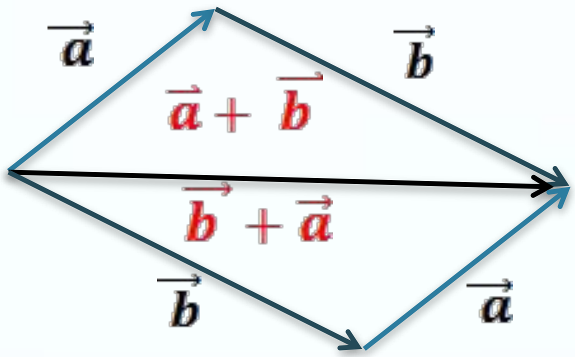
How to Add Vectors

1. Draw the 1^{st} vector with proper length & orientation
2. Draw the 2^{nd} vector with proper length & orientation originating from the head of the 1^{st} vector
3. The vector sum is the vector that extends from the tail of 1^{st} vector to the head of the 2^{nd} one
4. Measure the length & orientation angle of the resultant



Vector Addition Properties

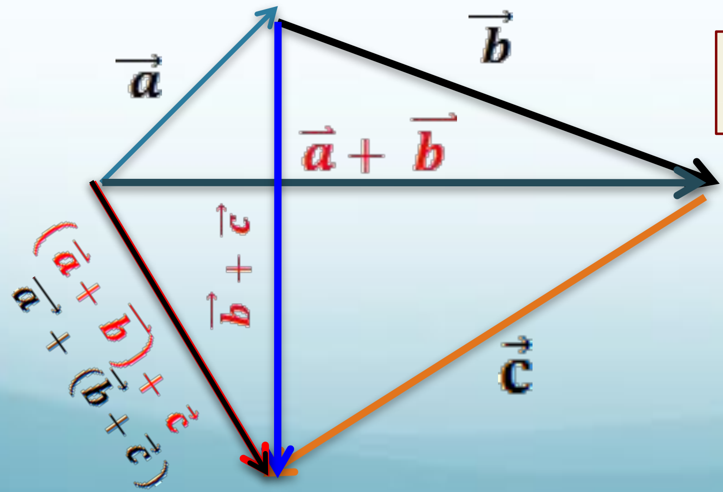
- **Vector addition is commutative:**
  The addition order does not matter



$$\boxed{\vec{a} + \vec{b} = \vec{b} + \vec{a}}$$

Commutative Law

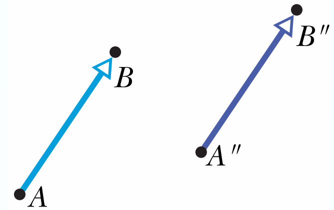
- **Vector addition is Associative:**
  For adding more than two vectors, we can group them in any order



$$\boxed{(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})}$$

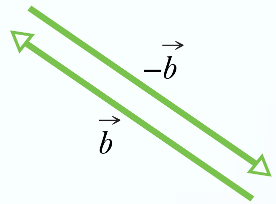
Associative Law

- If the vector *a* & *b* have the same direction & magnitude
  → *a* = b, *a* & b are **parallel**



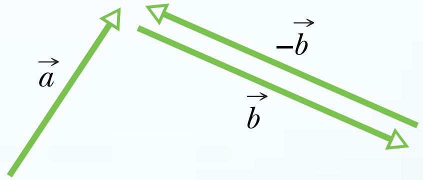
- If the vector b & b have the same magnitude but opposite direction
  → b = – b, b & – b are **antiparallel**

$$\vec{b} + (-\vec{b}) = 0$$



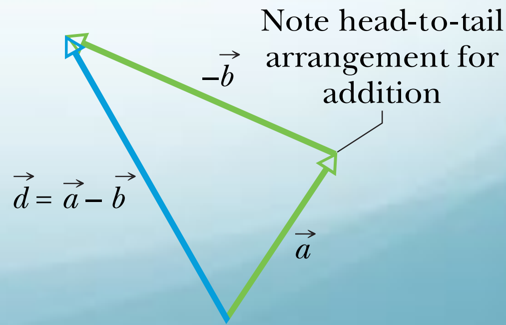
- Vector **subtraction**

$$\boxed{\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})}$$



As in algebra, we can move a term that includes a vector symbol from one side of a vector equation to the other, but we must change its signs

$$\vec{d} + \vec{b} = \vec{a} \quad \text{or} \quad \vec{a} = \vec{d} + \vec{b}$$



CHECKPOINT 1

The magnitudes of displacements $\vec{a}$ and $\vec{b}$ are 3 m and 4 m, respectively, and $\vec{c} = \vec{a} + \vec{b}$. Considering various orientations of $\vec{a}$ and $\vec{b}$, what is (a) the maximum possible magnitude for $\vec{c}$ and (b) the minimum possible magnitude?

(a) The maximum possible magnitude for c

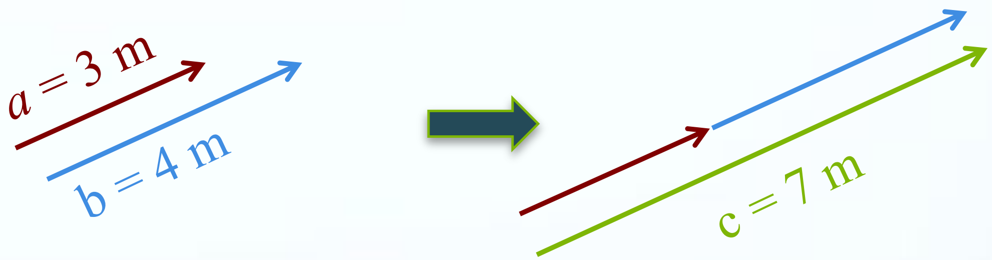
$$\vec{a} = 3m$$

$$\vec{b} = 4m$$

$$\vec{c} = \vec{a} + \vec{b}$$

$$\vec{c} = 3 + 4 = 7m$$

$$|\vec{c}| = 7m$$

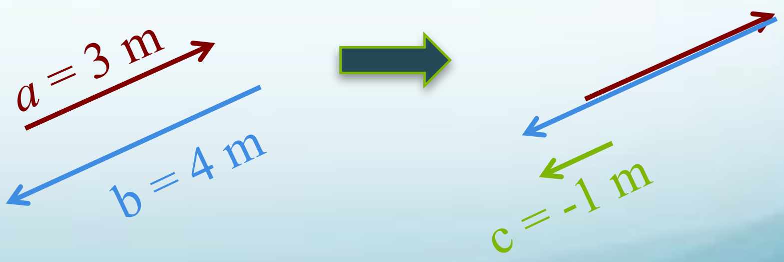


(b) The minimum possible magnitude for c

$$\vec{c} = \vec{a} - \vec{b}$$

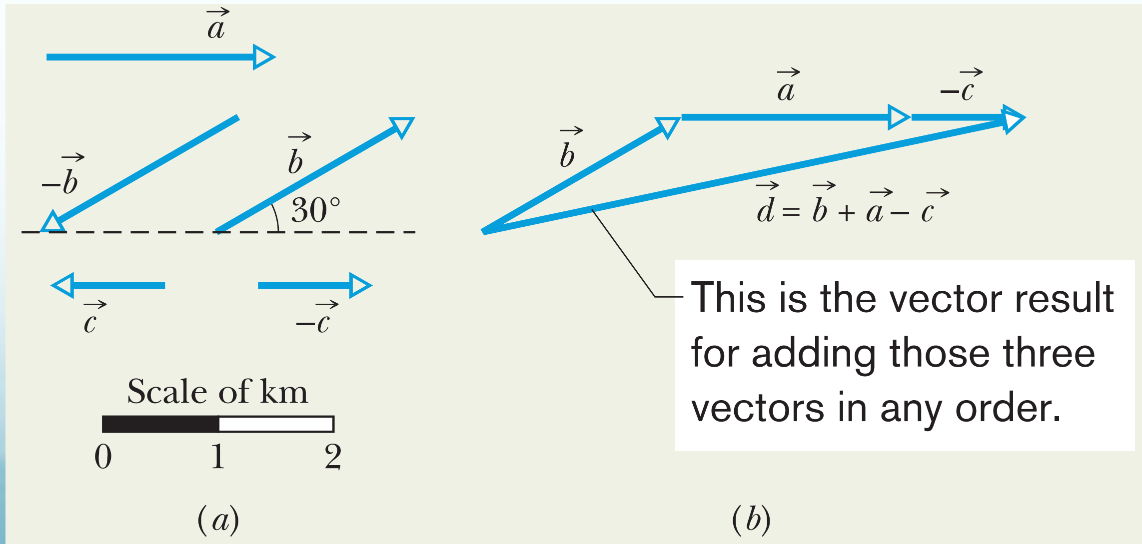
$$\vec{c} = 3 - 4 = -1m$$

$$|\vec{c}| = 1m$$



Sample Problem

In an orienteering class, you have the goal of moving as far (straight-line distance) from base camp as possible by making three straight-line moves. You may use the following displacements in any order: (a) $\vec{a}$, 2.0 km due east (directly toward the east); (b) $\vec{b}$, 2.0 km 30° north of east (at an angle of 30° toward the north from due east); (c) $\vec{c}$, 1.0 km due west. Alternatively, you may substitute either $-\vec{b}$ for $\vec{b}$ or $-\vec{c}$ for $\vec{c}$. What is the greatest distance you can be from base camp at the end of the third displacement?



Homework ☺

Chapter 3
Vectors

Sections 3-4, 3-5, 3-6

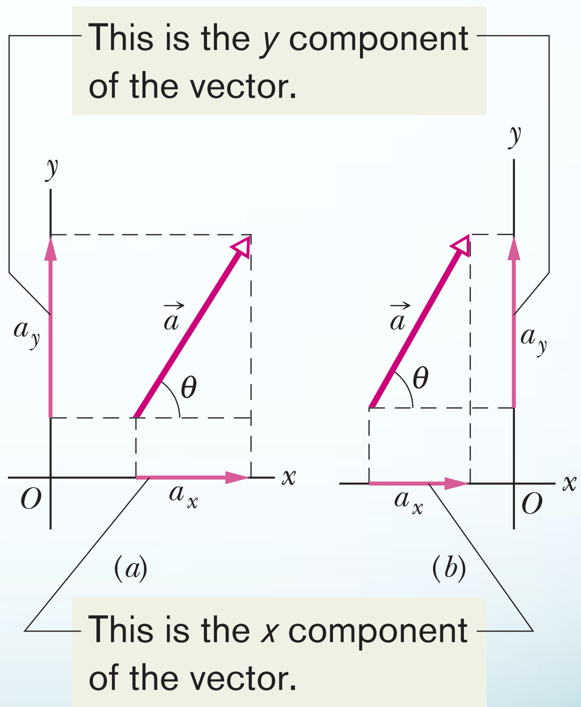
Component of Vectors
Unit Vector
Adding Vectors by Components

- The important skills from this lecture
  1. Find the inverse of any vector
  2. Resolve any vector and find its x and y component
  3. Calculate both magnitude and direction of vector
  4. Identify the unit vector
  5. Write a vector in unit vector notation
  6. Adding vectors by components

Component of Vectors

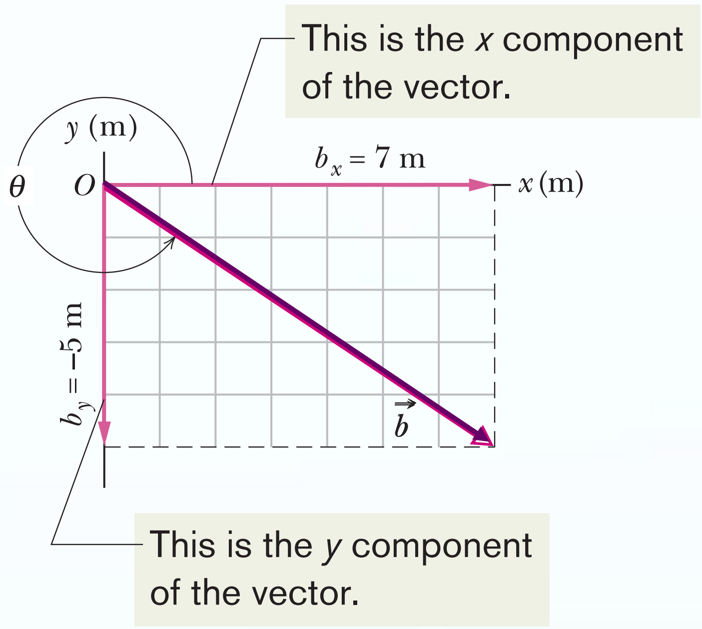
- **A vector component:**
  The projection of the vector on an axis

- **How we find a vector projection along an axis (Resolving vector)?**
  - We draw perpendicular lines from the two ends of the vector to the axis
  - Projection of the vector on x axis → x component of the vector
    Projection of the vector on y axis → y component of the vector
    - a_x → vector component along x axis
      a_y → vector component along y axis



- The vector component of has the same direction of the vector
  - a_x & a_y are both in +ve direction of x & y because $\vec{a}$ extends in +ve direction of both axes
  - If the vector a is reversed, its components would point toward – x & – y

- **Example: the components of vector b?**



- In general, a vector has 3 components in x, y, z
  In the previous examples, the component along the z axis is zero

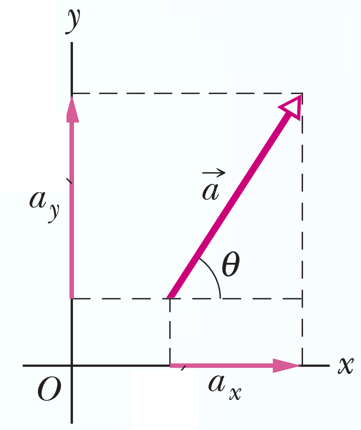
- If the vector is shifted without changing its direction
  → its components do not change

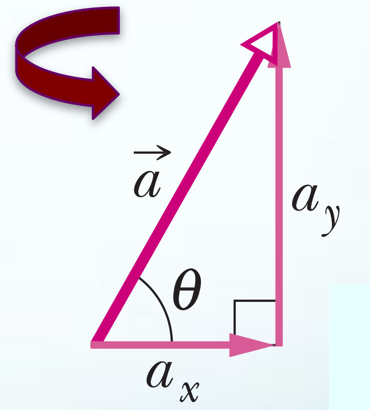
Finding Vector Components Geometrically

- How to find the components of $\vec{a}$ geometrically from the right triangle?
  1. Arrange vector components <u>head to tail</u>
  2. Complete a right triangle by the vector itself
  3. The vector forms the *hypotenuse*, from the tail of one component to the head of the other component

$$\boxed{a_x = a\cos\theta} \quad \boxed{a_y = a\sin\theta}$$

ϑ: the angle between vector a and +ve direction of x axis
a: the magnitude of vector a



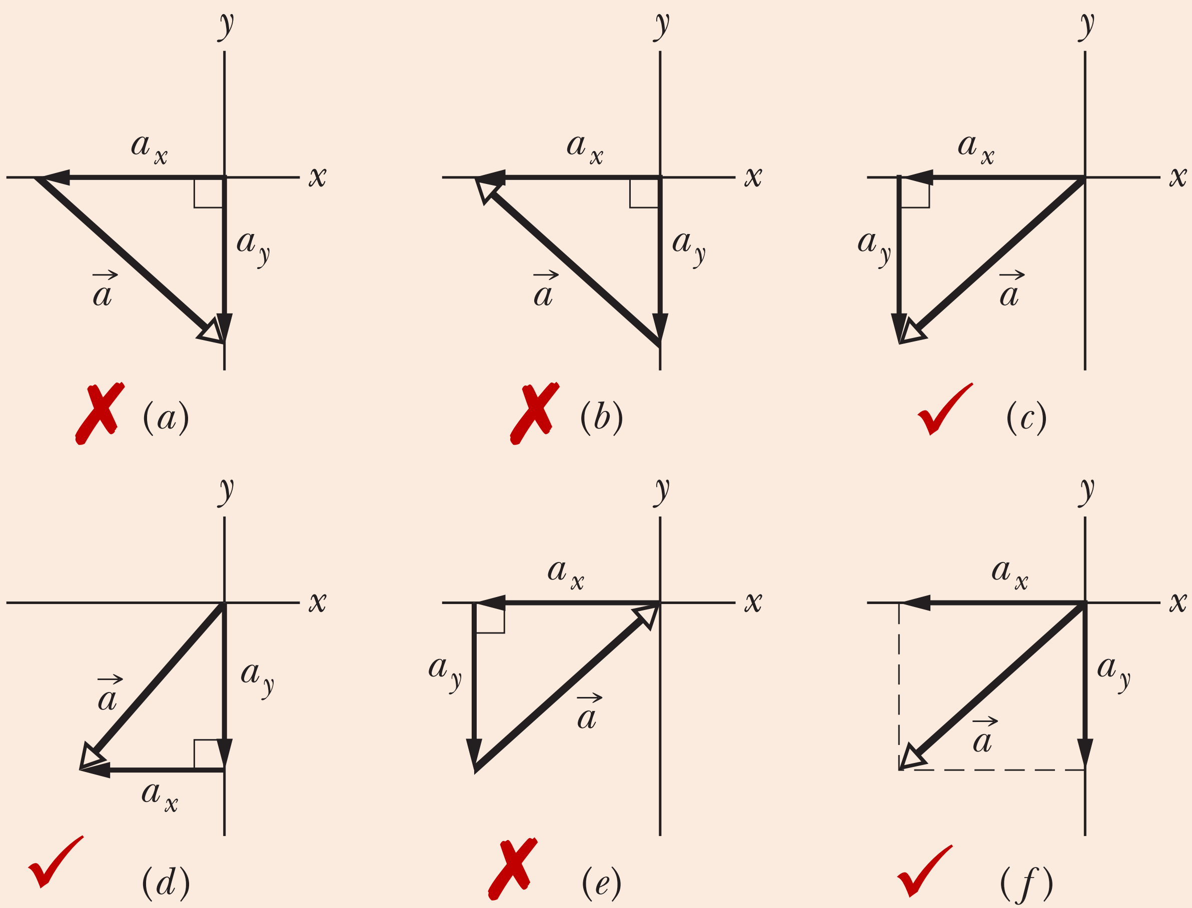


- Vector a could be completely determined by 2 ways:
  1. Component notation (a_x & a_y)
  2. Magnitude-angle notation (a & θ)
     If we know the components a_x & a_y , both a & θ are given by

$$\boxed{a = \sqrt{a_x^2 + a_y^2}} \quad \boxed{\tan\theta = \frac{a_y}{a_x}}$$

CHECKPOINT 2

In the figure, which of the indicated methods for combining the *x* and *y* components of vector $\vec{a}$ are proper to determine that vector?



Sample Problem

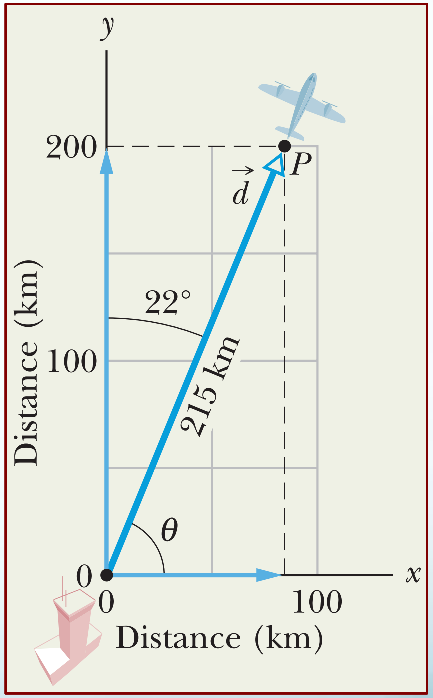
A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?

To find the components of $\vec{d}$, we use Eq. 3-5 with $\theta = 68°\ (= 90° - 22°)$:

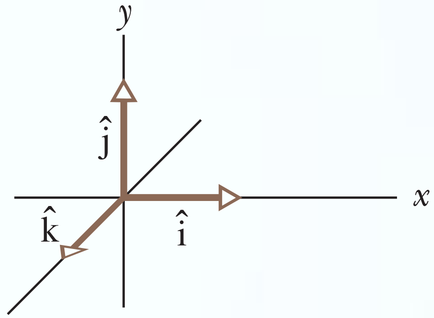
$$d_x = d\cos\theta = (215\text{ km})(\cos 68°) = 81\text{ km} \qquad \text{(Answer)}$$

$$d_y = d\sin\theta = (215\text{ km})(\sin 68°) = 199\text{ km} \approx 2.0 \times 10^2\text{ km.} \qquad \text{(Answer)}$$

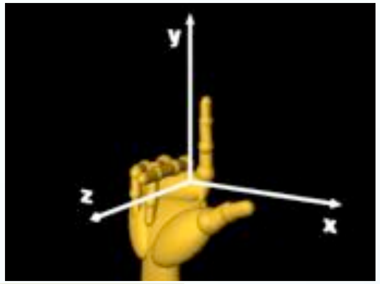
Thus, the airplane is 81 km east and 2.0×10^2 km north of the airport.



Unit Vectors



- Unit vector: vector that has a magnitude of exactly 1 and points in a particular direction

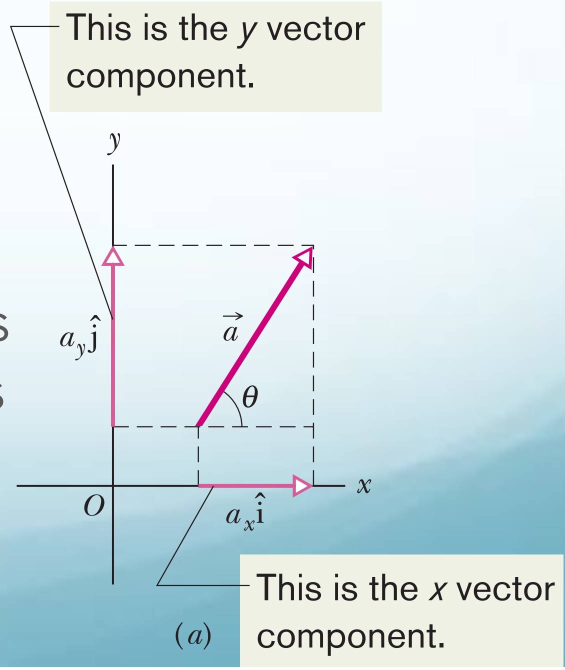


- The unit vectors in the +ve directions of the x, y, and z axes are $\hat{\mathrm{i}}, \hat{\mathrm{j}}$, and $\hat{\mathrm{k}}$

- The arrangement of axes is named: **a right-handed coordinate system**

- Unit vectors are used to express vectors; e.g. $\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}$

- $a_x\hat{\mathrm{i}}$ and $a_y\hat{\mathrm{j}}$ are vectors → vector components
  a_x and a_y are scalars → scalar components of vector a (or simply its components)



Examples:

Q1. A vector a in the xy plane, if its direction is 230° counterclockwise from the positive direction of the x axis, and its magnitude is 7.3m

(1) the x-component is:

(a) – 4.7î (b) –4.7 (c) 2.3î (d) –2.3

$a_x = a \cos\theta = 7.3 \cos 230^{o} = -4.7$

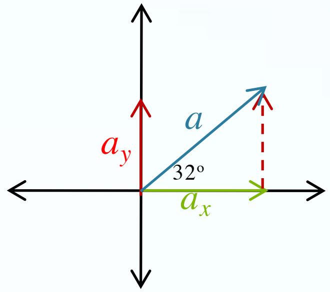
(2) the y-component is:

(a) –5.6ĵ (b) –5.6 (c) –4.2ĵ (d) –4.2

$a_y = a \sin\theta = 7.3 \sin 230^{o} = -5.6$

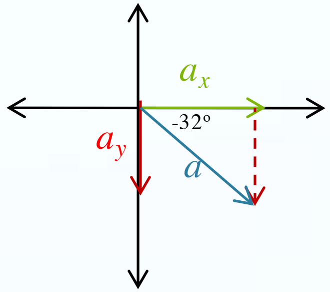
Q.2 If the components of vector a is given by:
a_x = 8 cm and a_y = 5 cm, find the direction of this vector

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{5}{8} = 32^o$$



Q.3 In the previous question, if a_y = –5 cm, find its direction

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{-5}{8} = -32^o$$



Because the vector a located in the 4th quarter, –32° means that θ= 360° – 32° = 328°

Q.4 The angle between vector D = 2î + 2ĵ and the +ve y-axis is:
(a) 63° (b) 19° (c) 30° (d) 45° (e) 11°

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{2}{2} = 45^o$$

The vector D is located in the 1st quarter and its components are equal

Q.5 The x component of vector a is –20 m, and the y component is +15 m.

(1) Vector a in unit vector is:

(a) –20î + 15ĵ (a) 15î – 20ĵ (c) 5î – 10ĵ (d) 20î – 15ĵ

(2) The magnitude of vector a is:

(a) –5 (b) 35 (c) 25 (d) 1.25

$$a = \sqrt{(-20)^2 + (15)^2} = 25$$

(3) The angle between the direction of vector a and the +ve x axis is:

(a) 37^o (b) 143^o (c) 120^o (d) 215^o

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{15}{-20} = -37^o$$

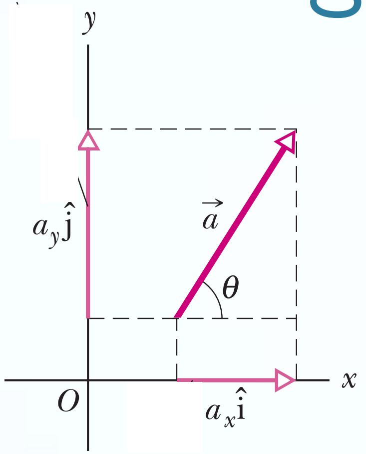
Because the vector a located in the 2nd quarter, -37^o means that $\theta = 180^o - 37^o = 143^o$

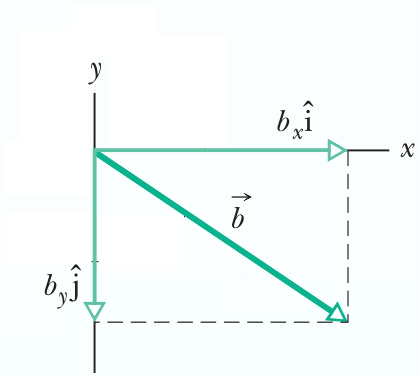
Q.6 Vector C starts at point (4, 1, 2) and ends at point (4, 3, 2). Its magnitude is:

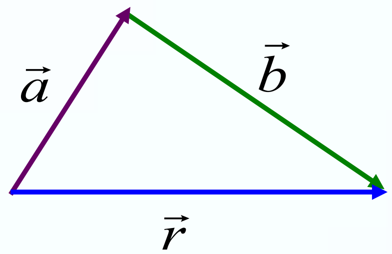
(a) 5 (b) 6 (c) 2 (d) 8 (e) 4

$$C = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = \sqrt{(4-4)^2 + (3-1)^2 + (2-2)^2} = 2$$

Adding Vectors by Components







$$\vec{r} = \vec{a} + \vec{b}$$

- To add vectors by component:
  1. Resolve the vectors into their scalar components
  2. Combine these scalar components, axis by axis, to get the components of the sum $\vec{r}$
  3. Combine the components of $\vec{r}$ to get $\vec{r}$ itself

$$\vec{a} = a_x\hat{i} + a_y\hat{j}, \qquad \vec{b} = b_x\hat{i} + b_y\hat{j}$$

$$\vec{r} = \vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$$

$$= r_x\hat{i} + r_y\hat{j}$$

Vectors are equal if their corresponding components are equal

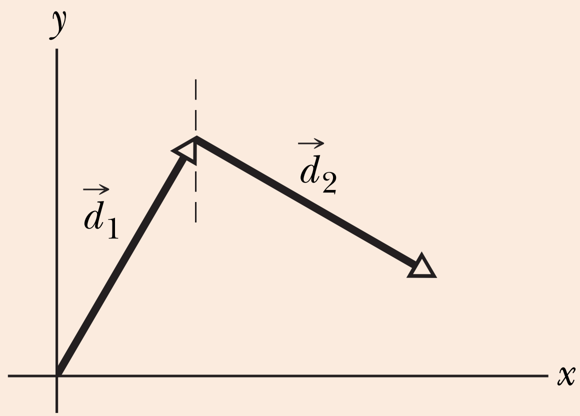
$$r_x = a_x + b_x$$
$$r_y = a_y + b_y$$
$$r_z = a_z + b_z$$

- Adding vectors by components also applies to vector subtractions

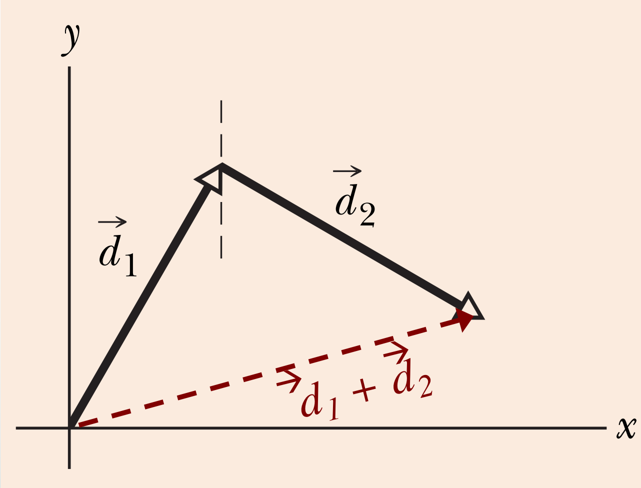
- Remember! There are 2 ways to express any vector:
  1. In unit-vector notation (î, ĵ)
  2. In magnitude-angle notation (r,θ)

CHECKPOINT 3

(a) In the figure here, what are the signs of the x components of $\vec{d}_1$ and $\vec{d}_2$? (b) What are the signs of the y components of $\vec{d}_1$ and $\vec{d}_2$? (c) What are the signs of the x and y components of $\vec{d}_1 + \vec{d}_2$?



	Sing of x component	Sing of y component
$\vec{d}_1$	+	+
$\vec{d}_2$	+	−
$\vec{d}_1 + \vec{d}_2$	+	+



Sample Problem

Figure 3-15*a* shows the following three vectors:

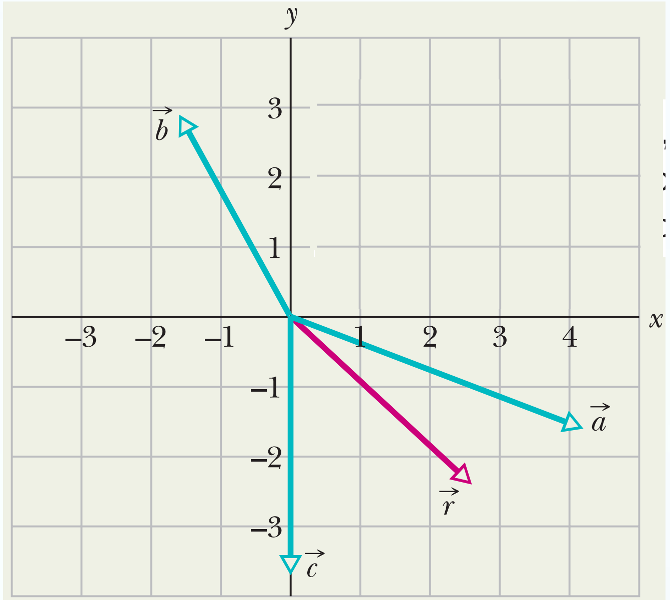
$$\vec{a} = (4.2\text{ m})\hat{i} - (1.5\text{ m})\hat{j},$$

$$\vec{b} = (-1.6\text{ m})\hat{i} + (2.9\text{ m})\hat{j},$$

and

$$\vec{c} = (-3.7\text{ m})\hat{j}.$$

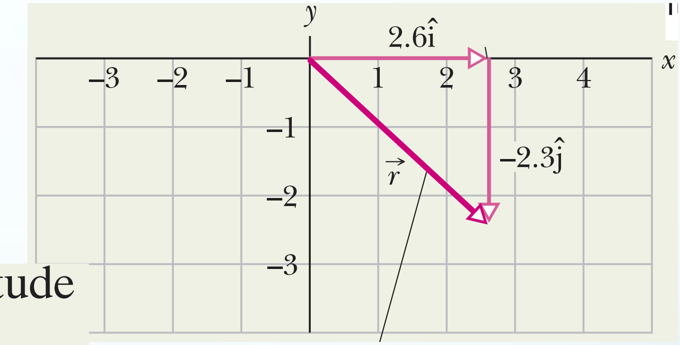
What is their vector sum $\vec{r}$ which is also shown?



$$r_x = a_x + b_x + c_x$$
$$= 4.2\text{ m} - 1.6\text{ m} + 0 = 2.6\text{ m}$$

$$r_y = a_y + b_y + c_y$$
$$= -1.5\text{ m} + 2.9\text{ m} - 3.7\text{ m} = -2.3\text{ m}$$

$$\vec{r} = (2.6\text{ m})\hat{i} - (2.3\text{ m})\hat{j}.$$



We can also answer the question by giving the magnitude and an angle for $\vec{r}$. From Eq. 3-6, the magnitude is

$$r = \sqrt{(2.6\text{ m})^2 + (-2.3\text{ m})^2} \approx 3.5\text{ m} \qquad \text{(Answer)}$$

and the angle (measured from the $+x$ direction) is

$$\theta = \tan^{-1}\left(\frac{-2.3\text{ m}}{2.6\text{ m}}\right) = -41^\circ, \qquad \text{(Answer)}$$

where the minus sign means clockwise. $\theta = 360^o - 41^o = 319^o$

Examples:

Q.1 Two vectors are given as $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$, and $\vec{b} = 2\hat{i} + 4\hat{j} + 2\hat{k}$
find vector c that satisfies the relation $\vec{a} - \vec{b} + \vec{c} = 3\hat{i}$
(a) Î + 3ĵ (b) –Î + 5ĵ (c) –Î + ĵ (d) 4Î + 2ĵ (b) –Î + 2ĵ

$$\vec{a} - \vec{b} + \vec{c} = 3\hat{i} \Rightarrow \vec{c} = 3\hat{i} + \vec{b} - \vec{a}$$

$$\vec{b} - \vec{a} = (2-1)\hat{i} + (4-2)\hat{j} + (2-2)\hat{k} = \hat{i} + 2\hat{j}$$

$$\vec{c} = 3\hat{i} + (\hat{i} + 2\hat{j}) = 4\hat{i} + 2\hat{j}$$

Q.2 Vector A has a magnitude of 5.0 m and is directed 30º north of east. Vector B has a magnitude of 6.0 m and is directed north. The magnitude of $\vec{A} + \vec{B}$ is:
(a) 7.4 m (b) 6.8 m (c) 5.4 m (d) 9.5 m (e) 3.2 m

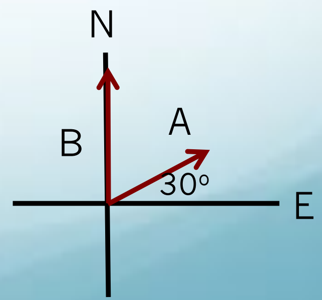
$$A_x = 5\cos 30 = 4.3, \qquad A_y = 5\sin 30 = 2.5$$

$$\vec{A} = 4.3\hat{i} + 2.5\hat{j}$$

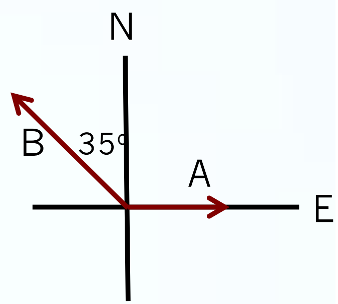
$$B_x = 6\cos 90 = 0, \qquad B_y = 6\sin 90 = 6$$

$$\vec{B} = 0\hat{i} + 6\hat{j}$$

$$\vec{A} + \vec{B} = 4.3\hat{i} + 8.5\hat{j}, \qquad \left|\vec{A} + \vec{B}\right| = \sqrt{4.3^2 + 8.5^2} = 9.5$$



Q.3 vector A has a magnitude of 3 m and is directed east, vector B has a magnitude of 5 m and directed 35° west of north



(1) Vector A in unit-vector notation is:

(a) 3î (b) 3î – 2ĵ + 0k (c) 0î + 3ĵ + 0k (d) 5î – 2ĵ + k

(2) Vector B in unit-vector notation is:

(a) 0.1î + 4.1ĵ (b) –0.1î + 5ĵ (c) –2.9î + 4.1ĵ (d) –2.9î + 4.1ĵ + k

$$\theta = 90^o + 35^o = 125^o$$

$$B_x = 5\cos 125 = -2.9, \qquad B_y = 5\sin 125 = 4.1, \quad \vec{B} = -2.9\hat{i} + 4.1\hat{j}$$

(3) Vector $\vec{A} + \vec{B}$ is:

(a) 0.1î + 4.1ĵ (b) –0.1î + 4.1ĵ (c) 2.5î + 0ĵ (d) 0.1î – 2.5ĵ + k

(4) The magnitude and direction of $\vec{A} + \vec{B}$ is:

(a) 4.1, 88.6° (b) 7.2, 325° (c) 5.5, 325° (d) 13.5, 34°

$$\left|\vec{A} + \vec{B}\right| = \sqrt{0.1^2 + 4.1^2} = 4.1, \qquad \theta = \tan^{-1}\frac{4.1}{0.1} = 88.6^o$$

(5) Vector $\vec{A} - \vec{B}$ is:

(a) 5.9î – 4.1ĵ (b) –5.9î + 4.1ĵ (c) 2.1î – 2.5ĵ (d) 5î – 2.5ĵ

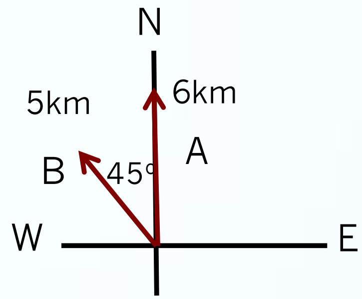
(6) The magnitude and direction of $\vec{A} - \vec{B}$ is:

(a) 13, 34° (b) 7.2, –35° (c) 5.5, 325° (d) 13.5, 34°

$$\left|\vec{A} - \vec{B}\right| = \sqrt{(5.9^2 + 4.1)^2} = 7.2, \qquad \theta = \tan^{-1}\frac{-4.1}{5.9} = -35^o$$

Q.4 You drive 6 km north and then 5 km northwest. The magnitude of the resultant displacement is:
(a) 9.24 km (b) 12.07 km (c) 6.57 km (d) 8.32 km (e) 10.17 km

$$A_x = 6\cos 90 = 0, \qquad A_y = 6\sin 90 = 6$$
$$\vec{A} = 0\hat{i} + 6\hat{j}$$
$$B_x = 5\cos 135 = -3.5, \qquad B_y = 5\sin 135 = 3.5$$
$$\vec{B} = -3.5\hat{i} + 3.5\hat{j}$$
$$\vec{A} + \vec{B} = -3.5\hat{i} + 9.5\hat{j}, \qquad \left|\vec{A} + \vec{B}\right| = \sqrt{(-3.5)^2 + 9.5^2} = 10.12$$



Q.5 Two vectors A = xÎ + 6Ĵ and B = 2Î + yĴ. The values of x and y satisfying the relation A + B = 4Î + Ĵ are:
(a) (-1, -2) (b) (-3, 2) (c) (2, -5) (d) (1, -4) (e) (0, -3)

$$\vec{A} = x\hat{i} + 6\hat{j}$$
$$\vec{B} = 2\hat{i} + y\hat{j}$$
$$\vec{A} + \vec{B} = (x+2)\hat{i} + (6+y)\hat{j} = 4\hat{i} + \hat{j}$$
$$\Rightarrow x + 2 = 4, \qquad \Rightarrow x = 2$$
$$\Rightarrow 6 + y = 1, \qquad \Rightarrow y = -5$$

Q.6 The sum of two vectors $\overrightarrow{A} + \overrightarrow{B}$ is $4\hat{i} + \hat{j}$, and their difference $\overrightarrow{A} - \overrightarrow{B}$ is $-2\hat{i} + \hat{j}$, the magnitude of vector A is:

(a) 1.8 (b) 2.8 (c) 4.1 (d) 2 (e) 1.4

$$\begin{array}{rl} & \vec{A} + \vec{B} = 4\hat{i} + \hat{j} \\ + & \vec{A} - \vec{B} = -2\hat{i} + \hat{j} \\ \hline & 2\vec{A} = 2\hat{i} + 2\hat{j} \end{array}$$

$$\Rightarrow \vec{A} = \hat{i} + \hat{j}$$

$$\Rightarrow \left|\vec{A}\right| = \sqrt{1+1} = \sqrt{2} = 1.4$$

Q.7 The magnitude of vector A is 5 units, and its x-component is 2.5 unit, find the angle θ between the vector A and the x-axis.

$$A_x = A\cos\theta$$

$$2.5 = 5\cos\theta$$

$$\theta = \cos^{-1}\frac{2.5}{5}$$

Chapter 3
Vectors

Section 3-8

Multiplying Vectors
By Scalar
By vectors: Scalar Product
Vector Product

- The important skills from this lecture
  1. Multiply vector by scalar
  2. Identify the two kinds of multiplications of vector by another vector
  3. Calculate the scalar product of unit vectors
  4. Identify the properties of the scalar product
  5. Calculate the scalar product of two vectors when they are written in unit-vector notation, and in angle-magnitude notation
  6. Calculate the vector product of unit vectors
  7. Identify the properties of the vector product
  8. Calculate the vector product of two vectors when they are written in unit-vector notation, and in angle-magnitude notation

Multiplying a Vector by a Scalar

- If we have vector a & scalar s → $s\vec{a} = \vec{r}$
  $\vec{r}$ is a new vector, $|\vec{r}| = s|\vec{a}|$

- $\vec{r}$ direction: if s is +ve → $\vec{r}$ in the same direction of vector a,
  if s is –ve → $\vec{r}$ in the opposite direction of vector a

- To divide vector a by scalar s → $\vec{r} = \vec{a}(1/s)$
  $\vec{r}$ is a new vector,
  $\vec{r}$ direction: if s is +ve → $\vec{r}$ in the same direction of vector a,
  if s is –ve → $\vec{r}$ in the opposite direction of vector a

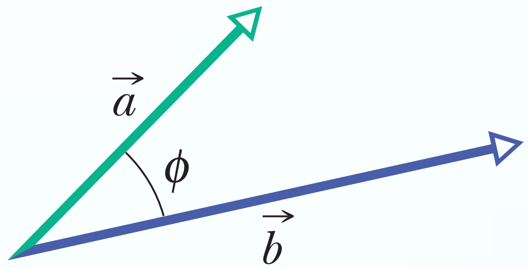
Multiplying a Vector by a Vector

scalar product $\rightarrow$ scalar quantity

vector product $\rightarrow$ vector quantity

Scalar Product
(dot product)

$$\boxed{\vec{a} \cdot \vec{b} = ab \cos \phi}$$



a: magnitude of $\vec{a}$
b: magnitude of $\vec{b}$
Φ: angle between $\vec{a}$ & $\vec{b}$

Notice: there are two such angles between $\vec{a}$ & $\vec{b}$, the small one, Φ, and the big one $360° - \Phi$. Both of them can be used because their cosines are the same ($\cos \Phi = \cos 360^{o} - \Phi$)

A dot product is a product of 2 quantities:
(1) the magnitude of one of the vectors and
(2) the scalar component of the 2nd vector along the direction of the 1st one

Scalar product properties:

$$\phi = 0 \rightarrow \bar{a}.\bar{b} = ab\cos\phi = ab$$

$$\phi = 90 \rightarrow \bar{a}.\bar{b} = ab\cos\phi = 0$$

$$\phi = 180 \rightarrow \bar{a}.\bar{b} = ab\cos\phi = -ab$$

If the angle ϕ between two vectors is $0°$, the component of one vector along the other is maximum, and so also is the dot product of the vectors. If, instead, ϕ is $90°$, the component of one vector along the other is zero, and so is the dot product.

- Commutative law: $\boxed{\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}}$
- When two vectors are in unit-vector notation:

$$\vec{a} \cdot \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \cdot (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1, \qquad (\phi = 0)$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0, \qquad (\phi = 90^o)$$

$$\rightarrow \qquad \boxed{\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z}$$

CHECKPOINT 4

Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if $\vec{C} \cdot \vec{D}$ equals (a) zero, (b) 12 units, and (c) -12 units?

$C = 3 \qquad D = 4$

$$\vec{C}.\vec{D} = CD\cos\theta, \Rightarrow \cos\theta = \frac{\vec{C}.\vec{D}}{CD}$$

$$(a) \quad \vec{C}.\vec{D} = 0, \Rightarrow \theta = \cos^{-1}\frac{0}{12} = 90^{o}$$

$$(b) \quad \vec{C}.\vec{D} = 12, \Rightarrow \theta = \cos^{-1}\frac{12}{12} = 0$$

$$(d) \quad \vec{C}.\vec{D} = -12, \Rightarrow \theta = \cos^{-1}\frac{-12}{12} = 180$$

Sample Problem

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$? (*Caution:* Although many of the following steps can be bypassed with a vector-capable calculator, you will learn more about scalar products if, at least here, you use these steps.)

$$\vec{a} \cdot \vec{b} = ab \cos \phi.$$

$$a = \sqrt{3.0^2 + (-4.0)^2} = 5.00, \quad b = \sqrt{(-2.0)^2 + 3.0^2} = 3.61$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (3.0\hat{i} - 4.0\hat{j}) \cdot (-2.0\hat{i} + 3.0\hat{k}) \\ &= (3.0\hat{i}) \cdot (-2.0\hat{i}) + (3.0\hat{i}) \cdot (3.0\hat{k}) \\ &\quad + (-4.0\hat{j}) \cdot (-2.0\hat{i}) + (-4.0\hat{j}) \cdot (3.0\hat{k}).\end{aligned}$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= -(6.0)(1) + (9.0)(0) + (8.0)(0) - (12)(0) \\ &= -6.0.\end{aligned}$$

$$-6.0 = (5.00)(3.61) \cos \phi,$$

$$\phi = \cos^{-1} \frac{-6.0}{(5.00)(3.61)} = 109° \approx 110°.$$

Examples:

Q.1 If $\vec{A}\cdot\vec{B}=0$, then the angle between vector A and vector B is:
(a) zero (b) 90º (c) 180º (d) 45º (e) 360º

Q.2 The vectors A and B are in x-y plane. Their magnitude are 4.5 and 7.3 units, respectively whereas their direction are 320º and 85º measured counterclockwise from the psitive x-axis. The A dot B is:
(a) 3.45î–2.9ĵ (b) –18.8 (c) 0.6î+7.3ĵ (d)2.2î–21ĵ

$$\theta = 320^o - 85^o = 235^o$$

$$\vec{A}\cdot\vec{B} = AB\cos\theta = 4.5\times7.3\times\cos 235 = -18.8$$

Q.3 Given $\vec{A}=2\hat{i}-4\hat{j}$ the vector that is perpendicular to vector A is:
(a) 2î–4ĵ (b) 4î+2ĵ (c) 2î+4ĵ (d)2î–6ĵ

For vector A that is normal to vector B, $\vec{A}\cdot\vec{B}=0$. This condition is applied when vector B is 4Î+2ĵ

Q.4 For vectors $\vec{A} = 3\hat{i} - 4\hat{j}$ and $\vec{B} = -5\hat{i} + 4\hat{j}$, $\vec{A} \cdot \vec{B}$ is:

(a) -31 **(b) 31** **(c) -31Î** **(d) –Î**

$$\vec{A} \cdot \vec{B} = -15 - 16 = -31$$

Q.5 Three vectors $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = 5\hat{i} + 2\hat{j} - 6\hat{k}$ and $\vec{C} = 2\hat{i} + 3\hat{j}$

The value of $(\vec{A} + \vec{B}) \cdot \vec{C}$ is:

(a) 18 **(b) 12** **(c) 14** **(d) 7**

$$\vec{A} + \vec{B} = 6\hat{i} + 0 - 5\hat{k}$$

$$\vec{C} = 2\hat{i} + 3\hat{j} + 0$$

$$(\vec{A} + \vec{B}) \cdot \vec{C} = 12 + 0 + 0 = 12$$

Vector product (cross product)

- The vector product of $\vec{a}$ & $\vec{b}$, $\vec{a} \times \vec{b} \rightarrow$ third vector $\vec{c}$ whose <u>magnitude</u> is:

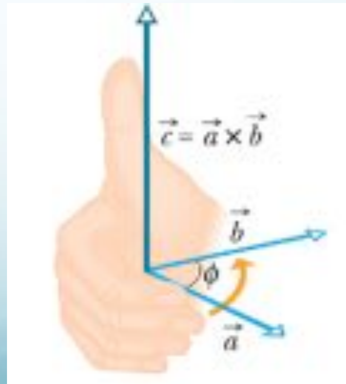
$$\boxed{c = ab \sin \phi,}$$

  where Φ is the <u>smaller</u> of the 2 angles between $\vec{a}$ & $\vec{b}$

Notice: Φ is the smaller angle between the vectors because

$\sin\Phi \neq \sin(360° - \Phi)$

- The direction of $\vec{c}$ is perpendicular to the plane that contains $\vec{a}$ & $\vec{b}$
- The direction of $\vec{c} = \vec{a} \times \vec{b}$ is determined by the **right-hand rule**:
  - Place the vectors tail to tail without altering their orientations
  - Imagine a line that is perpendicular to their plane where they meet
  - Pretend to place your right hand around that line in such a way that your fingers would sweep $\vec{a}$ into $\vec{b}$ through the smaller angle between them
  - The thumb points in the direction of $\vec{c}$



Vector product properties:

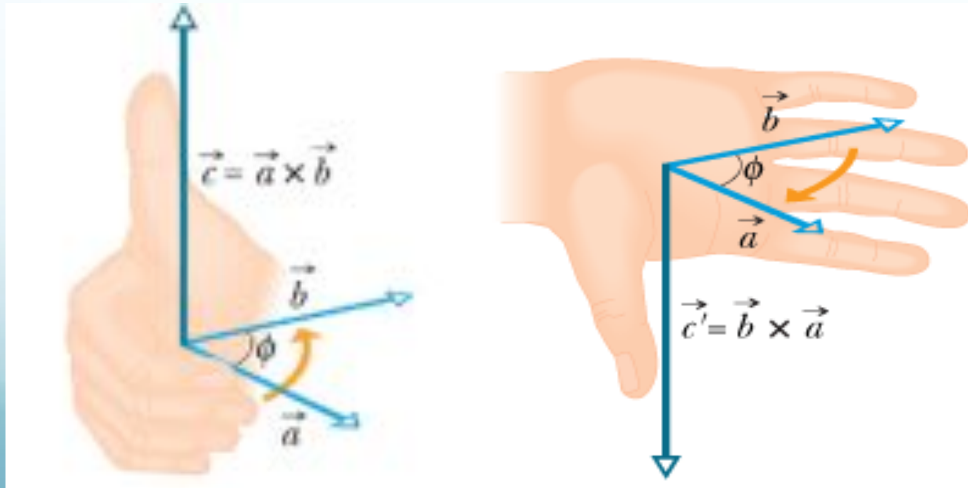
$$\phi = 0 \rightarrow \bar{a} \times \bar{b} = ab\sin\phi = 0$$

$$\phi = 90 \rightarrow \bar{a} \times \bar{b} = ab\sin\phi = ab$$

$$\phi = 180 \rightarrow \bar{a} \times \bar{b} = ab\sin\phi = 0$$

If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

- The **order** of the vector multiplication is **important**



$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$$

commutative law does not apply

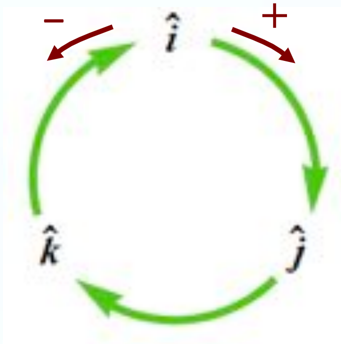
- When two vectors $\vec{a}$ & $\vec{b}$ are in unit-vector notation:

$$\vec{a} \times \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \times (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0, \qquad (\phi = 0)$$

$$\left|\hat{i} \times \hat{j}\right| = \left|\hat{j} \times \hat{k}\right| = \left|\hat{k} \times \hat{i}\right| = 1 \qquad (\phi = 90^o)$$

$$\left|\hat{j} \times \hat{i}\right| = \left|\hat{i} \times \hat{k}\right| = \left|\hat{k} \times \hat{j}\right| = -1 \qquad (\phi = 90^o)$$



→ $\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{\mathrm{i}} + (a_z b_x - b_z a_x)\hat{\mathrm{j}} + (a_x b_y - b_x a_y)\hat{\mathrm{k}}$

- $\vec{c} = \vec{a} \times \vec{b}$ can also be calculated by taking the determinant of the following matrix:

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\vec{c} = \vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{i} - (a_x b_z - b_x a_z)\hat{j} + (a_x b_y - b_x a_y)\hat{k}$$

$$\begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \end{array} \qquad \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \end{array} \qquad \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \end{array}$$

CHECKPOINT 5

Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if the magnitude of the vector product $\vec{C} \times \vec{D}$ is (a) zero and (b) 12 units?

$$C = 3 \qquad D = 4$$

$$\vec{C} \times \vec{D} = CD\sin\theta, \Rightarrow \sin\theta = \frac{\vec{C} \times \vec{D}}{CD}$$

$$(a) \quad \vec{C} \times \vec{D} = 0, \Rightarrow \theta = \sin^{-1}\frac{0}{12} = 0$$

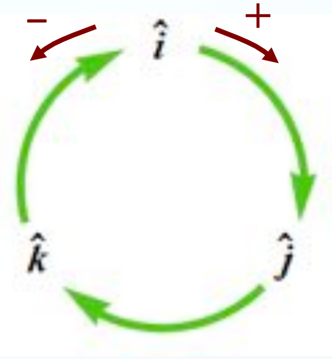
$$(b) \quad \vec{C} \times \vec{D} = 12, \Rightarrow \theta = \sin^{-1}\frac{12}{12} = 90^{o}$$

Sample Problem

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

$$\begin{aligned}\vec{c} &= (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k}) \\ &= 3\hat{i} \times (-2\hat{i}) + 3\hat{i} \times 3\hat{k} + (-4\hat{j}) \times (-2\hat{i}) \\ &\quad + (-4\hat{j}) \times 3\hat{k}.\end{aligned}$$

$$\begin{aligned}\vec{c} &= -6(0) + 9(-\hat{j}) + 8(-\hat{k}) - 12\hat{i} \\ &= -12\hat{i} - 9\hat{j} - 8\hat{k}.\end{aligned}$$



Examples:

Q.1 For any two vectors A and B, if $\vec{A}\times\vec{B}=0$, then the angle between them is:
(a) 60º (b) 90º (c) zero (d) 30º (e) 270º

Q.2 Two vectors $\vec{A}=3\hat{i}-7\hat{j}$ and $\vec{B}=2\hat{i}+3\hat{j}-2\hat{k}$. The vector that is perpendicular to the plane of A and B vectors is:
(a) 12Î-20ĵ+k (b) 14Î+6ĵ+23k (c) -14Î-6ĵ+23k (d) 5Î-2ĵ+13k

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -7 & 0\\ 2 & 3 & -2\end{vmatrix}=[14-0]\hat{i}+[0-(-6)]\hat{j}+[9-(-14)]\hat{k}=14\hat{i}+6\hat{j}+23\hat{k}$$

Q.3 The vector perpendicular to vector $\vec{A} = 2\hat{i} + 2\hat{k}$ and vector $\vec{B} = 5\hat{i} + 6\hat{k}$ is:

(a) 11Î (b) -9k (c) -2ĵ (d) 6Î (e) 4k

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 2 \\ 5 & 0 & 6 \end{vmatrix} = 0\hat{i} + [10 - 12]\hat{j} + 0\hat{k} = -2\hat{j}$$

Q.4 A vector A of magnitude 10 units and another vector b of magnitude 5 units differ in directions by 60°

(1) The scalar product of the two vectors is:

(a) 13Î (b) 15 (c) 25 (d) 25ĵ

$$\vec{A} \cdot \vec{B} = AB\cos\theta = 5(10)\cos 60^{o} = 25$$

(2) The magnitude of the vector product is:

(a) 43.3 (b) 43.3k (c) 15.5Î (d) 16.6

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta = 5(10)\sin 60^{o} = 43.3$$

Assignments:

1- A vector has two components (Ax = 3 cm and Ay = - 4 cm). What is the magnitude of A?

a) 4 cm b) 5 cm c) 1 cm d) 7 cm

2-Let A = (2m) i+ (6m) j + (3m) k and B = (4m) i+(2m) j - (1m) k. the vector sum S = A +B is:

a). (6 m) i + (8m) j + (2m) k

b). (-2m) i + (4m) j + (4m) k

c). (2 m) i+ (4m) j + (4m) k

d). (8m) i + (12m) j + (3m) k

3- The value of k . (k × i) is

a) zero b) +1 c) -1 d) 3

4- What is the cross product of a = (1, 2, 3) and b = (4, 5, 6)?

Problems:

•1 SSM What are (a) the x component and (b) the y component of a vector $\vec{a}$ in the xy plane if its direction is 250° counterclockwise from the positive direction of the x axis and its magnitude is 7.3 m?

•3 SSM The x component of vector $\vec{A}$ is -25.0 m and the y component is $+40.0$ m. (a) What is the magnitude of $\vec{A}$? (b) What is the angle between the direction of $\vec{A}$ and the positive direction of x?

•9 Two vectors are given by

$$\vec{a} = (4.0\text{ m})\hat{\text{i}} - (3.0\text{ m})\hat{\text{j}} + (1.0\text{ m})\hat{\text{k}}$$

and

$$\vec{b} = (-1.0\text{ m})\hat{\text{i}} + (1.0\text{ m})\hat{\text{j}} + (4.0\text{ m})\hat{\text{k}}.$$

In unit-vector notation, find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$, and (c) a third vector $\vec{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$.

Physics 110
1435-1436 H

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Chapter 4

MOTION IN TWO AND THREE DIMENSIONS

Sections 4-2, 4-3, 4-4

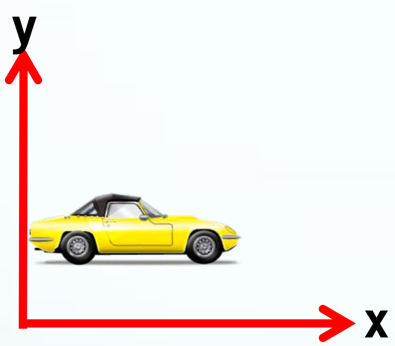
Position & Displacement
Average Velocity & Instantaneous Velocity
Average Acceleration & Instantaneous Acceleration

Important skills from this lecture:

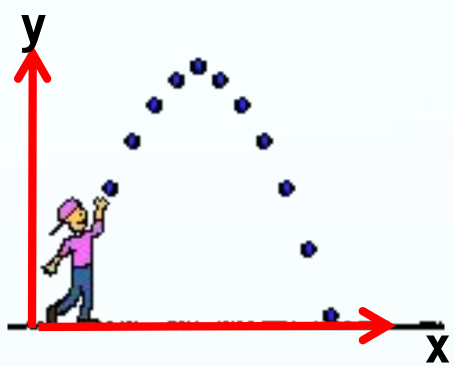
1. Define the motion in 2 and 3 dimensions
2. Locate the particle position in 2 and 3 dimensions
3. Calculate the displacement vector
4. Calculate the average velocity
5. Calculate the instantaneous velocity
6. Write all the preceding vectors in magnitude-direction and unit-vector notation
7. Calculate the average acceleration
8. Calculate the instantaneous acceleration
9. Write all the preceding vectors in magnitude-direction and unit-vector notation

- Motion could be:

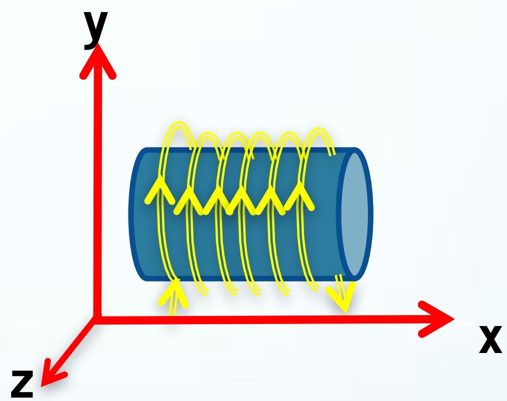
in 1 dimension



in 2 dimensions



in 3 dimensions



Position in 3 Dimensions

- To locate a particle → position vector $\vec{r}$.

$$\boxed{\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}}$$

$x\hat{\mathrm{i}}, y\hat{\mathrm{j}}$, and $z\hat{\mathrm{k}}$ → vector components,
x, y, and z (called coefficient) → scalar components

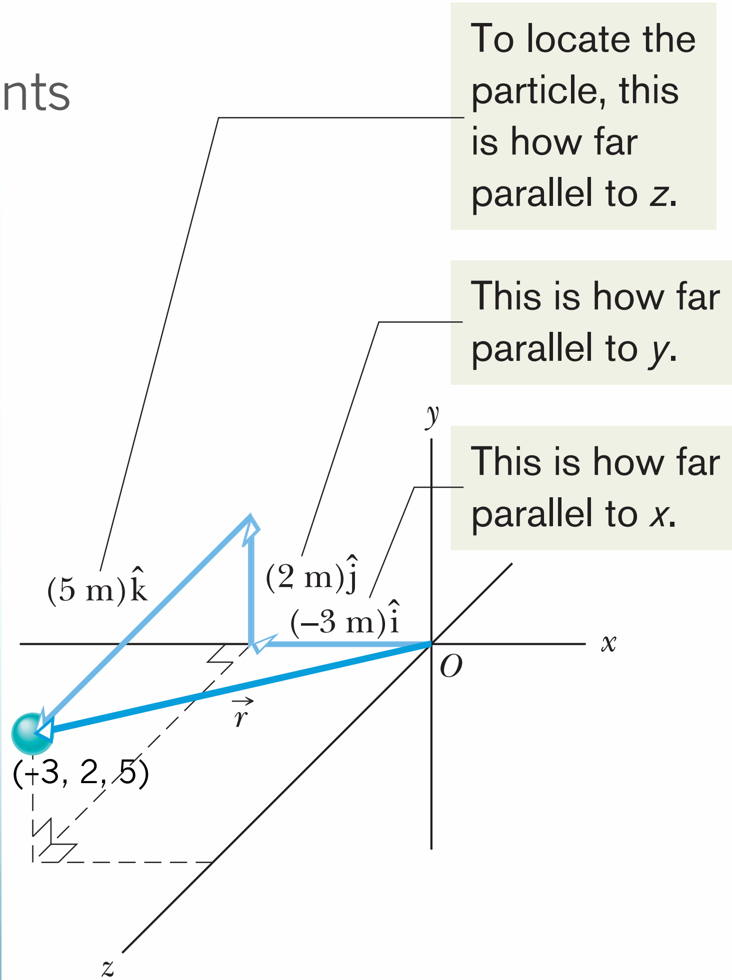
- e.g. a particle with position vector

$$\vec{r} = (-3\text{ m})\hat{\mathrm{i}} + (2\text{ m})\hat{\mathrm{j}} + (5\text{ m})\hat{\mathrm{k}}$$

has rectangular coordinates

(x, y, z) = (–3 m, 2 m, 5 m)

- The location of this particle is:
Along the x axis, 3 m from the origin, in the $-\hat{\mathrm{i}}$ direction
Along the y axis, 2 m from the origin, in the $+\hat{\mathrm{j}}$ direction
Along the z axis, 5 m from the origin, in the + k^ direction



Displacement in 3 Dimensions

- As a particle moves → position vector changes
- If the position vector changes from $\vec{r}_1$ to $\vec{r}_2$ during a certain time interval → the particle's **displacement** $\Delta\vec{r}$ is given by:

$$\boxed{\Delta\vec{r} = \vec{r}_2 - \vec{r}_1}$$

- Using the unit-vector notation:

$$\Delta\vec{r} = (x_2\hat{\mathrm{i}} + y_2\hat{\mathrm{j}} + z_2\hat{\mathrm{k}}) - (x_1\hat{\mathrm{i}} + y_1\hat{\mathrm{j}} + z_1\hat{\mathrm{k}})$$

$$\rightarrow \quad \boxed{\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\hat{\mathrm{k}}}$$

$$\rightarrow \quad \Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}$$

Sample Problem

The position vector for a particle initially is:

$$\vec{r}_1 = (-3.0m)\hat{i} + (2.0m)\hat{j} + (5.0m)\hat{k}$$

and then later is:

$$\vec{r}_2 = (9.0m)\hat{i} + (2.0m)\hat{j} + (8.0m)\hat{k}$$

what is the particle's displacement?

$$\begin{aligned}\Delta\vec{r} &= \vec{r}_2 - \vec{r}_1 \\ &= [9.0-(-3.0)]\hat{i} + [2.0-2.0]\hat{j} + [8.0-5.0]\hat{k} \\ &= (12m)\hat{i} + (3m)\hat{k}\end{aligned}$$

Sample Problem

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \qquad (4\text{-}5)$$

and

$$y = 0.22t^2 - 9.1t + 30. \qquad (4\text{-}6)$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

$\vec{r}(t) = x(t)\hat{\text{i}} + y(t)\hat{\text{j}}$

At $t = 15$ s, the scalar components are

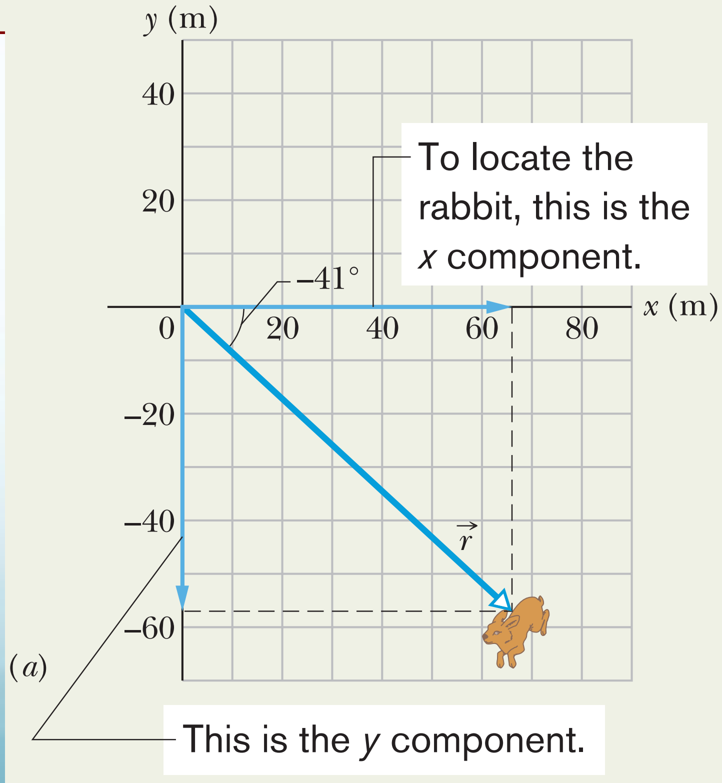
$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66 \text{ m}$$

$$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57 \text{ m},$$

$$\boxed{\vec{r} = (66 \text{ m})\hat{\text{i}} - (57 \text{ m})\hat{\text{j}}}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(66 \text{ m})^2 + (-57 \text{ m})^2}$$
$$= 87 \text{ m},$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57 \text{ m}}{66 \text{ m}}\right) = -41^\circ = 320^\circ$$

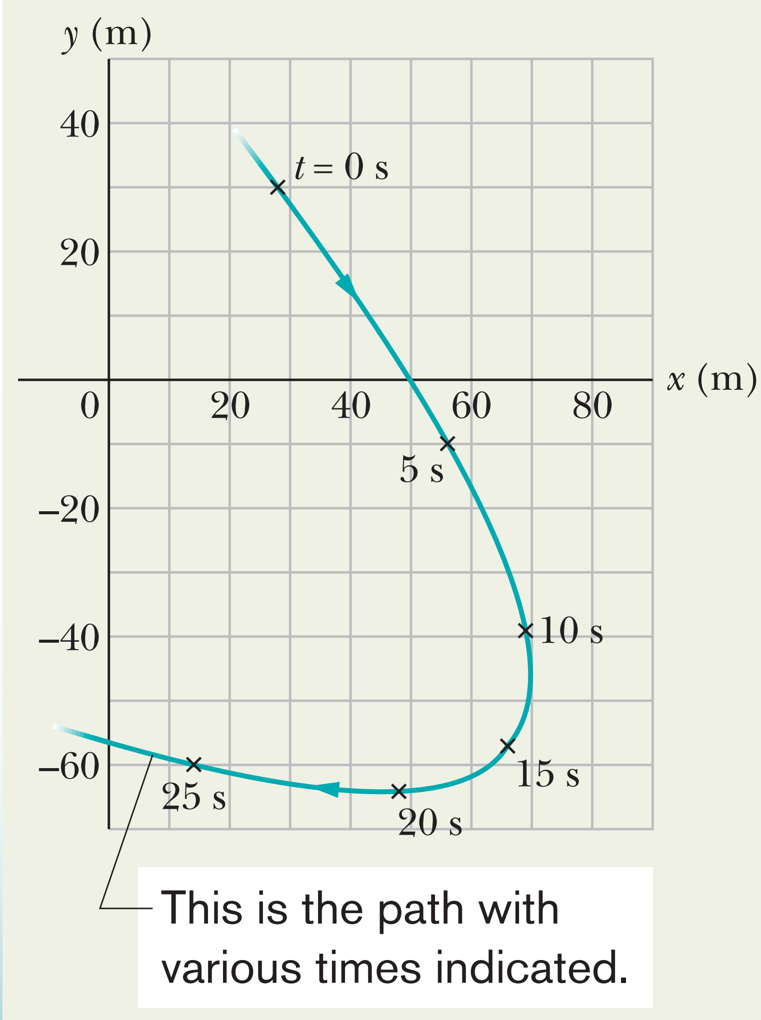


(b) Graph the rabbit's path for $t = 0$ to $t = 25$ s

we repeat part (a) for several values of t and then plot the results.

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

t	X(t)	Y(t)
0	28	30
5	56.25	-10
10	69	-39
15	66	-57
20	48	-64
25	14	-60



Average Velocity

- If a particle moves through a displacement $\Delta\vec{r}$ in a time interval Δt →

$$\text{average velocity} = \frac{\text{displacement}}{\text{time interval}} \rightarrow \boxed{\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}}$$

$$\vec{v}_{\text{avg}} = \frac{\Delta x\hat{\text{i}} + \Delta y\hat{\text{j}} + \Delta z\hat{\text{k}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\text{i}} + \frac{\Delta y}{\Delta t}\hat{\text{j}} + \frac{\Delta z}{\Delta t}\hat{\text{k}}$$

- The direction of $\vec{v}_{\text{avg}}$ is the same as that of $\Delta\vec{r}$

Sample Problem

If a particle's displacement is given by

$$\Delta\vec{r} = (12m)\hat{i} + (3m)\hat{k}$$

Find its velocity during the time interval of 2 s.

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(12m)\hat{i} + (3m)\hat{k}}{2\sec} = (6m/s)\hat{i} + (1.5m/s)\hat{k}$$

The average velocity has a component of 6.0 m/s along the x axis and a component of 1.5 m /s along the z axis

Instantaneous Velocity

- The instantaneous velocity $\vec{v}$ (or simply velocity) is the value that $\vec{v}_{avg}$ approaches when the time interval Δt reached to 0

$$\Delta t \longrightarrow 0 \qquad \vec{v}_{avg} \longrightarrow \vec{v} \quad \rightarrow \quad \boxed{\vec{v} = \frac{d\vec{r}}{dt}}$$

- In three dimensions

$$\vec{v} = \frac{d}{dt}(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}$$

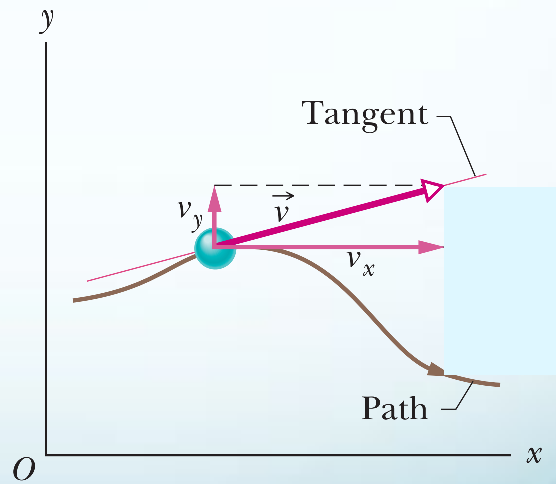
$$\boxed{\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}}$$

$$\boxed{v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}}$$

scalar component of $\vec{v}$ along the x axis

scalar component of $\vec{v}$ along the y axis

scalar component of $\vec{v}$ along the z axis



The direction of the velocity of a particle is always tangent to the particle's path at the particle's position.

Sample Problem

For the rabbit in the preceding Sample Problem, find the velocity $\vec{v}$ at time $t = 15$ s.

$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30.$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$

$$= -0.62t + 7.2.$$

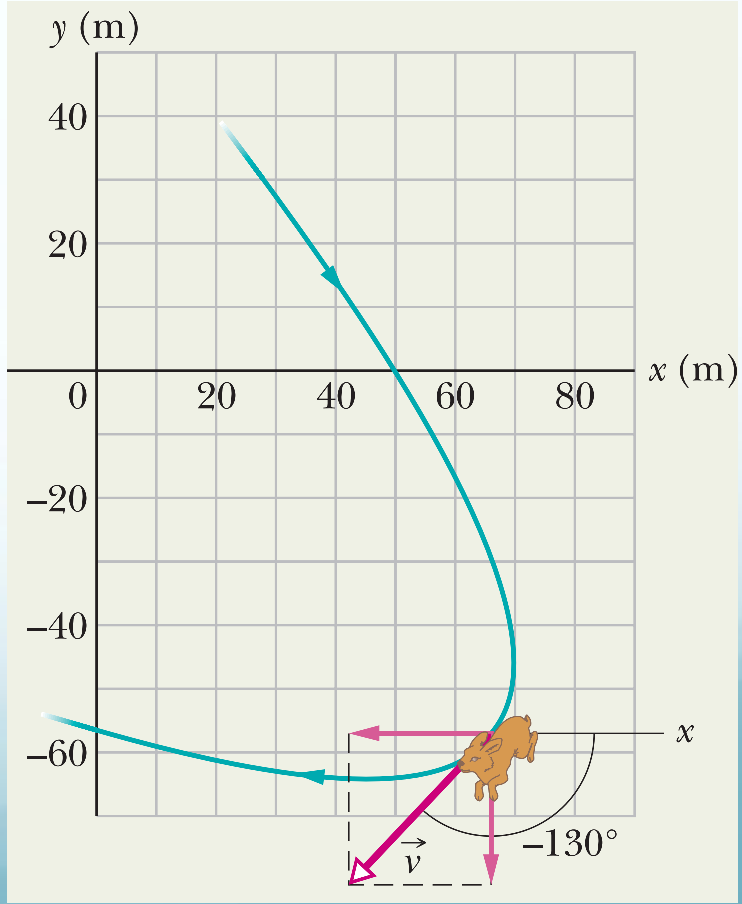
At $t = 15$ s, this gives $v_x = -2.1$ m/s.

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$

$$= 0.44t - 9.1.$$

$t = 15$ s, this gives $v_y = -2.5$ m/s.

$$\vec{v} = (-2.1 \text{ m/s})\hat{\mathrm{i}} + (-2.5 \text{ m/s})\hat{\mathrm{j}}$$



$$\boxed{\vec{v} = (-2.1\ \text{m/s})\hat{\text{i}} + (-2.5\ \text{m/s})\hat{\text{j}}}$$

To get the magnitude and angle of $\vec{v}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1\ \text{m/s})^2 + (-2.5\ \text{m/s})^2}$$
$$= 3.3\ \text{m/s} \qquad \text{(Answer)}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-2.5\ \text{m/s}}{-2.1\ \text{m/s}}\right)$$
$$= \tan^{-1} 1.19 = -130°. \qquad \text{(Answer)}$$

Examples:

Q.1 If the x-component of a vector r is 3.2 m and the y-component is 6.2 m, then vector r in unit vector notation is:
(a) 2.6Î–2.3Ĵ (b) -2.3Î+2.6Ĵ (c) 6.2Î+3.2Ĵ (d) 3.2Î+6.2Ĵ

Q.2 The displacement of a particle moving from
$\vec{r}_1 = -5\hat{i} + 2\hat{j} + 2\hat{k}$ **to** $\vec{r}_2 = -8\hat{i} + 2\hat{j} - 2\hat{k}$ **is:**

(a) -7Î+12Ĵ (b) 3Î+4k^ (c) 7Î–12Ĵ (d) -3Î–4k^

$$\begin{aligned} \Delta\vec{r} &= \vec{r}_2 - \vec{r}_1 \\ &= [-8-(-5)]\hat{i} + [2-2]\hat{j} + [-2-2]\hat{k} \\ &= -3\hat{i} - 4\hat{k} \end{aligned}$$

Q.3 The components of a car's velocity as a function of time are given by: $v_x = 2t + 3, \qquad v_y = 3t^2 + 3$

Its velocity vector at t = 2s is:
(a) 9Î+11Ĵ (b) 5Î+3Ĵ (c) 7Î+7Ĵ (d) 7Î+15Ĵ

Average Acceleration & Instantaneous Acceleration

- If the particle's velocity changed from $\vec{v}_1$ to $\vec{v}_2$ at time interval Δt, then its average acceleration is:

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time interval}} \rightarrow \boxed{\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}}$$

- If $\Delta t \rightarrow$ zero, then average acceleration = instantaneous acceleration (or acceleration) $\vec{a}$:

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}}$$

- Remember: If the velocity changes in either magnitude or direction (or both), the particle must have an acceleration

- In unit-vector notation:

$$\vec{a} = \frac{d}{dt}(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}})$$

$$= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}$$

$$\boxed{\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}}$$

- The scalar components of $\vec{a}$:

$$\boxed{a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}}$$

CHECKPOINT 2

Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$ (3) $\vec{r} = 2t^2\hat{\mathrm{i}} - (4t + 3)\hat{\mathrm{j}}$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$ (4) $\vec{r} = (4t^3 - 2t)\hat{\mathrm{i}} + 3\hat{\mathrm{j}}$

Are the x and y acceleration components constant? Is acceleration $\vec{a}$ constant?

(1)

$$x = -3t^2 + 4t - 2$$

$$v_x = \frac{dx}{dt} \rightarrow v_x = -6t + 4$$

$$a_x = \frac{dv_x}{dt} \rightarrow a_x = -6$$

$$y = 6t^2 - 4t$$

$$v_y = \frac{dy}{dt} \rightarrow v_y = 12t - 4$$

$$a_y = \frac{dv_y}{dt} \rightarrow a_y = 12$$

$$\boxed{\vec{a} = (-6)\hat{i} + (12)\hat{j}}$$

constant

(2)

$$x = -3t^3 - 4t \qquad y = -5t^2 + 6$$

$$v_x = \frac{dx}{dt} \rightarrow v_x = -9t^2 - 4 \qquad v_y = \frac{dy}{dt} \rightarrow v_y = -10t$$

$$a_x = \frac{dv_x}{dt} \rightarrow a_x = -18t \qquad a_y = \frac{dv_y}{dt} \rightarrow a_y = -10$$

$$\boxed{\vec{a} = (-18t)\hat{i} + (-10)\hat{j}}$$ not constant

(3)

$$\vec{r} = 2t^2\hat{i} - (4t+3)\hat{j}$$

$$\vec{v}_r = \frac{dr}{dt} \rightarrow \vec{v}_r = (4t)\hat{i} - 4\hat{j}$$

$$\vec{a}_r = \frac{dv_r}{dt} \rightarrow \vec{a}_r = 4\hat{i}$$ constant

(4)

$$\vec{r} = (4t^3 - 2t)\hat{i} + 3\hat{j}$$

$$\vec{v}_r = \frac{dr}{dt} \rightarrow \vec{v}_r = (12t^2 - 2)\hat{i}$$

$$\vec{a}_r = \frac{dv_r}{dt} \rightarrow \vec{a}_r = 24t\hat{i}$$ not constant

Sample Problem

For the rabbit in the preceding two Sample Problems, find the acceleration $\vec{a}$ at time $t = 15$ s.

$$v_x = -0.62t + 7.2 \qquad v_y = 0.44t - 9.1$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 \text{ m/s}^2$$

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2$$

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{\text{i}} + (0.44 \text{ m/s}^2)\hat{\text{j}}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.62 \text{ m/s}^2)^2 + (0.44 \text{ m/s}^2)^2}$$

$$= 0.76 \text{ m/s}^2.$$

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\left(\frac{0.44 \text{ m/s}^2}{-0.62 \text{ m/s}^2}\right) = -35°.$$

From a_x & a_y, $\vec{a}$ is located in the 2nd quarter → θ= 180° – 35° = 145°

Sample Problem

A particle with velocity $\vec{v}_o = -2\hat{i} + 4\hat{j}$ (in meters per second) at t = 0 undergoes a constant acceleration of a magnitude a = 3 m/s^2 at an angle θ= 130^o from the positive direction of the x axis. What is the particle's velocity at t = 5 s?

Because the acceleration is constant, we can apply the equations of motion

$$v_x = v_{ox} + a_x t \qquad \text{and} \qquad v_y = v_{oy} + a_y t$$

$$v_{ox} = -2m/s \qquad \text{and} \qquad v_{oy} = 4m/s$$

$$a_x = a\cos\theta = 3\cos 130^o = -1.93m/s^2$$

$$a_y = a\sin\theta = 3\sin 130^o = 2.3m/s^2$$

$$v_x = -2 + (-1.9)(5) = -11.65m/s$$

$$v_y = 4 + (2.3)(5) = 15.5m/s$$

$$\rightarrow$$

$$v = \sqrt{v_x^2 + v_y^2} = 19.4m/s$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = 127^o$$

$$\vec{v} = \left(-12m/s\right)\hat{i} + \left(16m/s\right)\hat{j}$$

Examples:

Q.1 if the position of a particle is given by $\vec{r} = (3t^2 + 2t)\hat{i} + (t^3 + 1)\hat{j}$

(a) Find its velocity vector at t = 1s, and the magnitude and direction

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = (6t + 2)\hat{i} + (3t^2)\hat{j} \qquad \vec{v}(1) = 8\hat{i} + 3\hat{j}$$

$$v = \sqrt{8^2 + 3^2} = \sqrt{73} \qquad \theta = \tan^{-1}\frac{3}{8} = 20.6^o$$

(b) Find the average acceleration from t = 1s to t = 2s

$$\vec{v}_1 = 8\hat{i} + 3\hat{j} \qquad \vec{v}_2 = 14\hat{i} + 12\hat{j}$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{6\hat{i} + 9\hat{j}}{1}$$

(c) Find the acceleration at t = 2s

$$\vec{v}(t) = (6t + 2)\hat{i} + (3t^2)\hat{j}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = 6\hat{i} + (6t)\hat{j} \qquad \vec{a}(2) = 6\hat{i} + 12\hat{j}$$

Q.2 The components of a car's velocity as a function of time are given by $v_x = 5t^2 - 5$, $v_y = -4t^3$. The acceleration components are:

(a) $a_x = 10t$, $a_y = -12t^2$ (b) $a_x = 4t$, $a_y = -6t^2$

(c) $a_x = 6t$, $a_y = -15t$ (d) $a_x = 12t$, $a_y = -9t^2$

Q.3 Acceleration is equal to:

(a) dr/dt (b) dv/dt (c) Δr/Δt

Q.4 A particle is moving with initial velocity $\vec{v}_o = 2\hat{i} + 4\hat{j}\, m/s$ and acceleration $\vec{a} = 5\hat{i} + 8\hat{j}\, m/s^2$, the x-component v_x of the final velocity at t = 7s is?

(a) 7m/s (b) 17m/s (c) 27m/s (d) 37m/s

$$v_{ox} = 2, \qquad a_x = 5$$

$$v_x = v_{ox} + a_x t \quad = 2 + 5(7) = 37$$

Chapter 4
MOTION IN TWO AND THREE DIMENSIONS

Section 4-5, 4-6

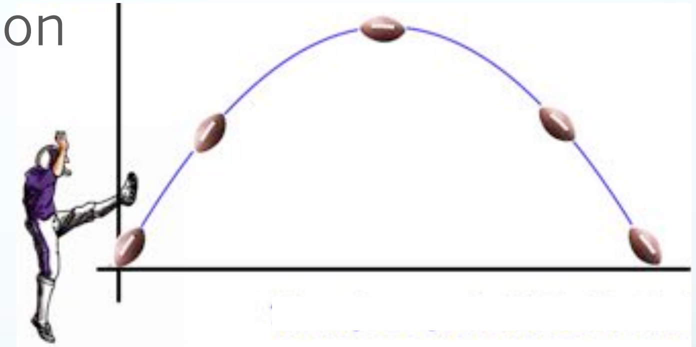
Projectile Motion
Projectile Motion Analyzed

- Important skills from this lecture:
  1. Identify the projectile motion and its velocity and acceleration components
  2. Analyze the projectile motion into horizontal and vertical motion
  3. Describe the projectile path
  4. Calculate its horizontal & vertical components of the final velocity after time t
  5. Calculate its horizontal & vertical displacement
  6. Calculate its horizontal range & maximum height
  7. Calculate the time projectile spend to reach any position

Projectile Motion ?

- A projectile: particle launched or projected near the earth's surface with initial velocity (launched velocity) $\vec{v}_0$, and it moves along a curved path under the action of **gravity** only

- Such motion is called a **projectile motion**
  It is a special case of two-dimensional motion



- Examples: tennis ball or baseball
  It is not the motion of a flying airplane
  or a flying duck

- The projectile acceleration is always the free-fall acceleration ,
  (downward $\vec{g}$)

- The projectile's position vector $\vec{r}$ & velocity vector $\vec{v}$
  change during its two-dimensional motion

- Because a projectile motion is a two-dimensional motion → we break up its problem into 2 separated one-dimensional problems:
  1. The horizontal motion ⟶
  2. The vertical motion ↑

- The projectile's launched velocity vector $\vec{v}_0$ has two components:

$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}$$

- The components of its velocity:

$$v_{0x} = v_0 \cos\theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin\theta_0$$

- The projectile's acceleration:
  $a = a_y + a_x$
  $a_x = 0 \rightarrow v_x = \text{constant}$
  $a_y = -g$ (↓)

- In projectile motion, the horizontal motion & the vertical motion are independent of each other; that is, neither motion affects the other

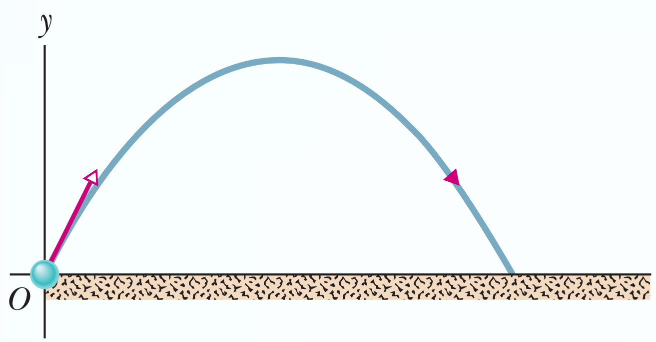
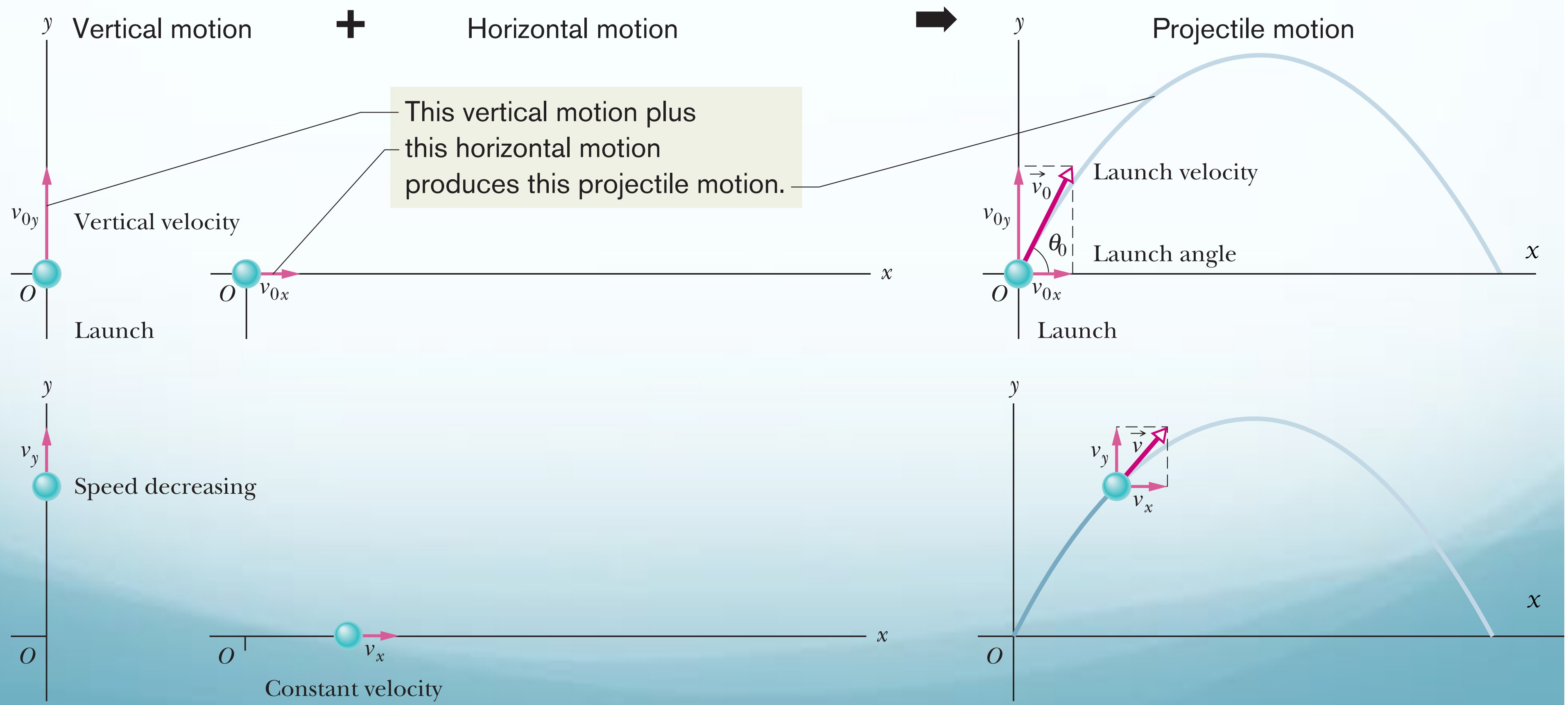
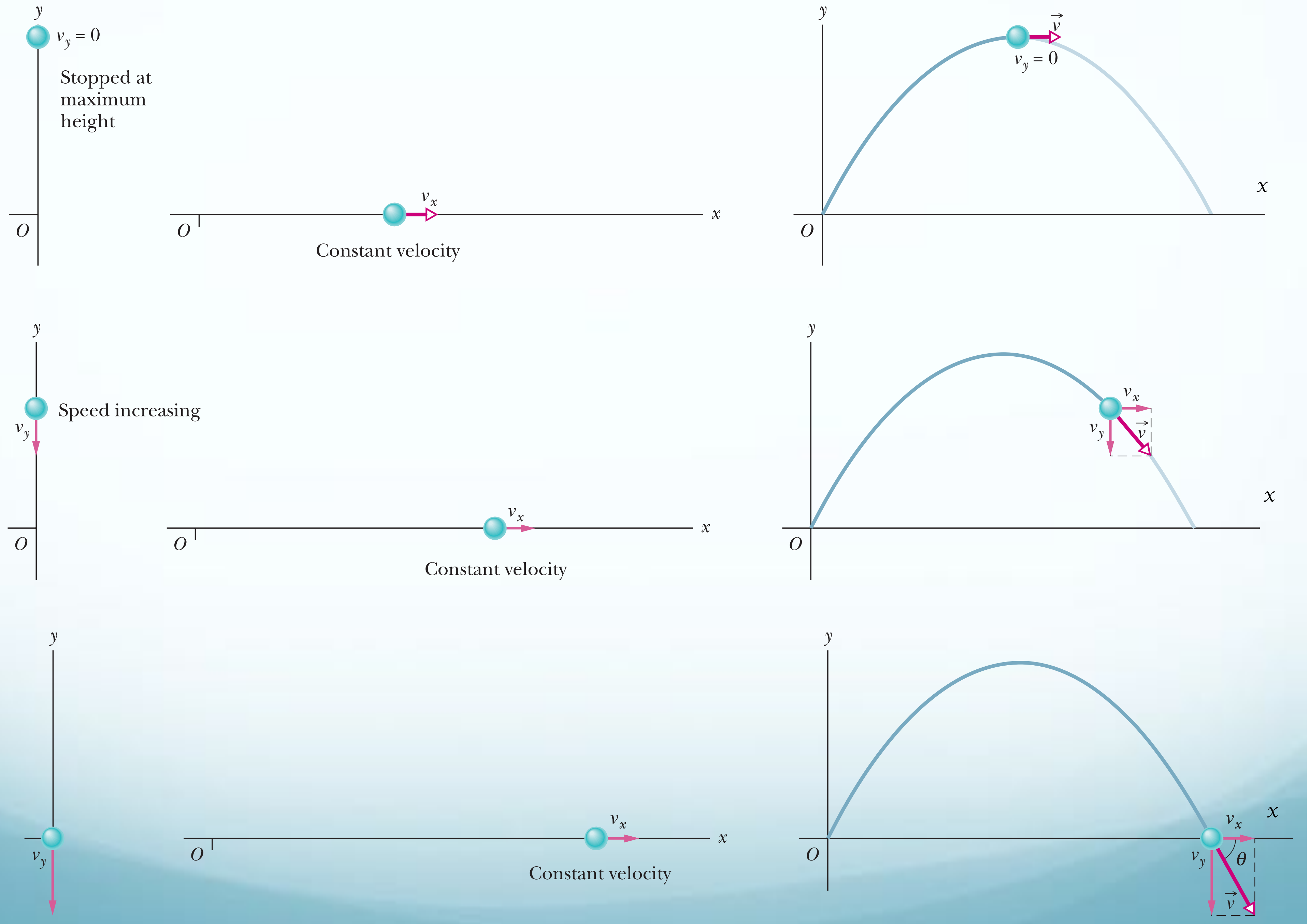


Fig. 4-9 The *projectile motion* of an object launched into the air at the origin of a coordinate system and with launch velocity $\vec{v}_0$ at angle θ_0. The motion is a combination of vertical motion (constant acceleration) and horizontal motion (constant velocity), as shown by the velocity components.





y
$v_y = 0$
Stopped at maximum height
O
v_x
O
x
Constant velocity
y
$\vec{v}$
$v_y = 0$
x
O
y
Speed increasing
v_y
O
v_x
O
x
Constant velocity
y
v_x
v_y
$\vec{v}$
x
O
y
v_y
v_x
O
x
Constant velocity
y
x
v_x
O
v_y
θ
$\vec{v}$

Projectile Motion Analyzed

- We analyze projectile motion horizontally & vertically

Horizontal analysis:

- No acceleration ($a_x = 0$) → $v_x = v_{ox}$ (constant), we apply the equations of motion with a constant acceleration
- $x - x_o$ (the horizontal displacement from an initial position x_o) is given by:

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2$$

$a_x = 0$ → $$x - x_0 = v_{0x} t.$$

Because $$v_{0x} = v_0 \cos \theta_0,$$

→ $$\boxed{x - x_0 = (v_0 \cos \theta_0) t.}$$

Vertical analysis:

- It is a free fall motion → constant acceleration (a_y = −g)
  we apply the equations of motion with a constant acceleration:

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

- The vertical displacement is:

$$\boxed{\begin{aligned} y - y_0 &= v_{0y}t - \tfrac{1}{2}gt^2 \\ &= (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2 \end{aligned}}$$

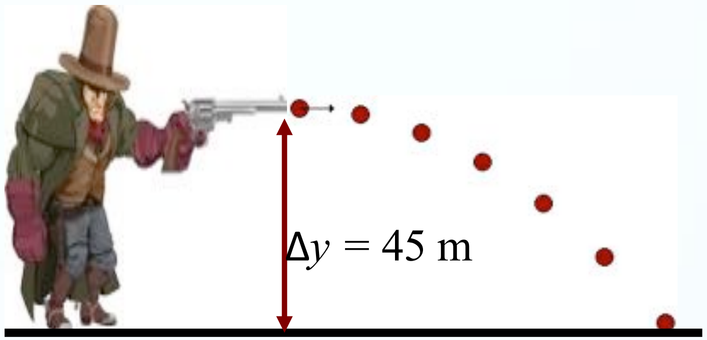
  and the vertical velocity $\boxed{v_y = v_0 \sin\theta_0 - gt}$

$$\boxed{v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)}$$

- Initially, the vertical velocity is directed upward ↑
  & its magnitude steadily decreases to zero (at the maximum height of the path), then its component reverses direction ↓
  & its magnitude increases with time

Problems:

•23 A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air? (b) At what horizontal distance from the firing point does it strike the ground? (c) What is the magnitude of the vertical component of its velocity as it strikes the ground?



(a) $h = \; y - y_0 = v_{0y}t - \frac{1}{2}gt^2$

$$h = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2$$

$\theta_0 = 0$ $\quad t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(45.0 \text{ m})}{9.80 \text{ m/s}^2}} = 3.03 \text{ s}$

(b) The horizontal distance traveled is given by Eq. 4-21:

$x - x_0 = (v_0 \cos \theta_0)t.$ $\quad \Delta x = v_0 t = (250 \text{ m/s})(3.03 \text{ s}) = 758 \text{ m}.$

(c) And from Eq. 4-23, we find

$v_y = v_0 \sin \theta_0 - gt$ $\quad |v_y| = gt = (9.80 \text{ m/s}^2)(3.03 \text{ s}) = 29.7 \text{ m/s}.$

••32 GO You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-35). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

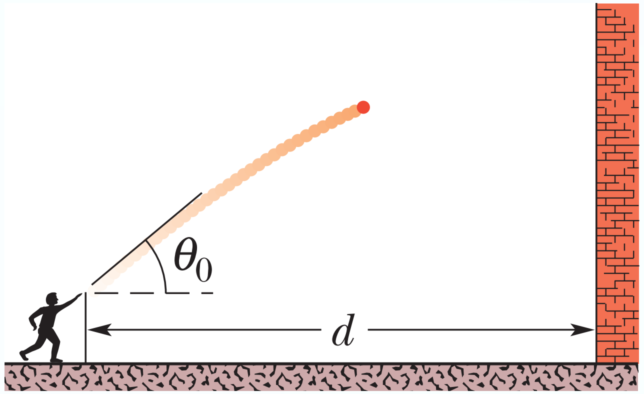


Fig. 4-35 Problem 32.

$$x - x_0 = (v_0 \cos \theta_0)t.$$

$$t = \frac{\Delta x}{v_x} = \frac{22.0 \text{ m}}{(25.0 \text{ m/s})\cos 40.0°} = 1.15 \text{ s}.$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 = (25.0 \text{ m/s})\sin 40.0°(1.15 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.15 \text{ s})^2 = 12.0 \text{ m}.$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0° = 19.2$ m/s.

(c) The vertical component becomes (using Eq. 4-23)

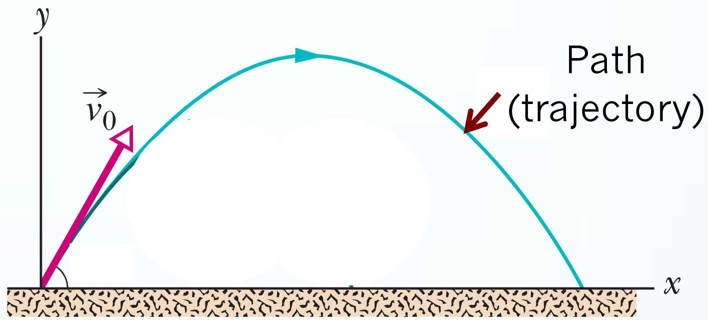
$$v_y = v_0 \sin\theta_0 - gt = (25.0 \text{ m/s}) \sin 40.0° - (9.80 \text{ m/s}^2)(1.15 \text{ s}) = 4.80 \text{ m/s}.$$

(d) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

The Equation of the Path (trajectory):

- We can find the equation of the projectile's path (its **trajectory**) by eliminating time t between Eqs:

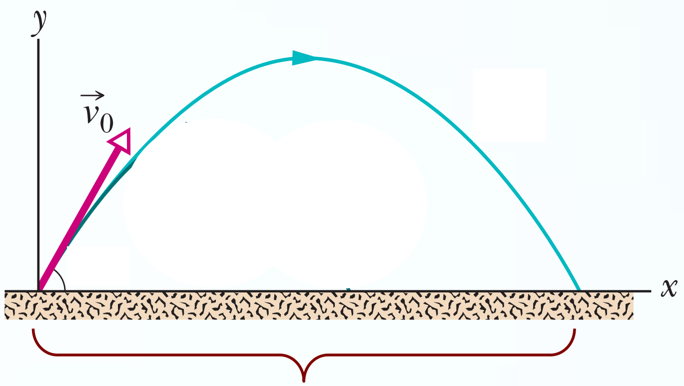
$$x - x_0 = (v_0 \cos \theta_0)t$$
$$y - y_0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2$$



→ $\boxed{y = (\tan \theta_0)x - \dfrac{gx^2}{2(v_0 \cos \theta_0)^2}}$ **trajectory**

- Since θ_0, v_0 and g constants, this equation in the form of $y = ax + bx^2$, (equation of a parabola)
  → the path is **parabolic**

The Horizontal Range (R):



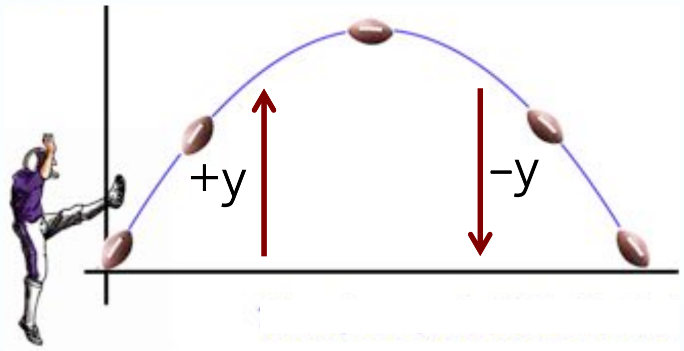
- It is the horizontal distance traveled by the projectile when it returns to its initial height
- To find R, we put $x - x_0 = R$ and $y - y_0 = 0$ in Eqs $x - x_0 = (v_0 \cos\theta_0)t$ and $y - y_0 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$, respectively, then eliminate t between the new equations

$$\rightarrow \boxed{R = \frac{v_0^2}{g}\sin 2\theta_0.} \quad \text{(horizontal range)}$$

- Note: This equation gives the horizontal distance traveled by a projectile **only** when the final height is as the same as the launched height
- R is maximum when the launched angle is 45°
  ($\sin 2\theta_0 = 1 \rightarrow \theta_0 = 45^o$)

CHECKPOINT 3

At a certain instant, a fly ball has velocity $\vec{v} = 25\hat{i} - 4.9\hat{j}$ (the x axis is horizontal, the y axis is upward, and $\vec{v}$ is in meters per second). Has the ball passed its highest point?



Yes, because v_y = –4.9 is in the negative direction of the y-axes, so the ball started to move downward after it passed the maximum height (at v_y = 0)

The total time (t):

The y component of the projectile velocity is:

$$v_y = v_0 \sin \theta_0 - gt$$

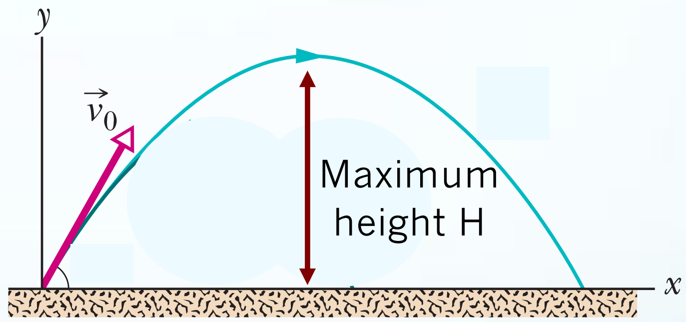
at maximum height, $v_y = 0$ → $\boxed{t = \dfrac{v_o \sin \theta_o}{g}}$

Note: this *t* is the projectile time to reach the **maximum height**
the **total time** for the projectile (time for fight) is ***2t***

The maximum height (H):

$$H = y - y_0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2$$

→ $\boxed{H = \dfrac{v_o^2 \sin^2 \theta_o}{2g}}$



The relation between R & H:

$$\boxed{H = \frac{R \tan \theta}{4}}$$

CHECKPOINT 4

A fly ball is hit to the outfield. During its flight (ignore the effects of the air), what happens to its (a) horizontal and (b) vertical components of velocity? What are the (c) horizontal and (d) vertical components of its acceleration during ascent, during descent, and at the topmost point of its flight?

a. Horizontal velocity v_x is constant all the time

b. Vertical velocity v_y :
   - Initially positive and maximum
   - Then decreases to zero
   - Then becomes negative & increases to maximum

c. Horizontal acceleration a_x= 0

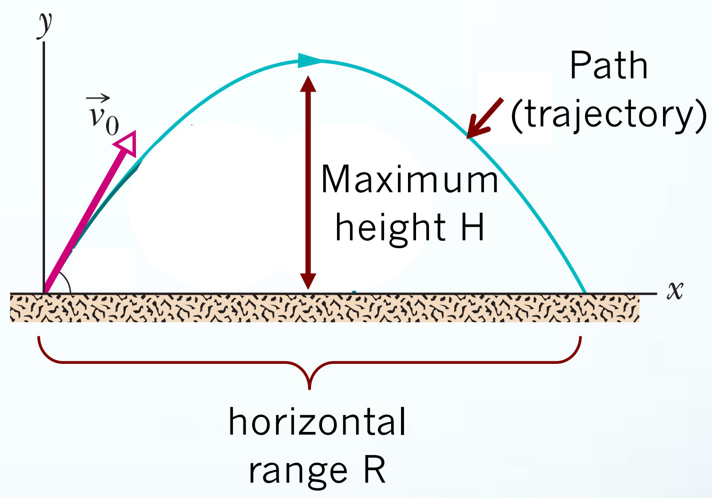
d. Vertical acceleration a_y =– g during both ascent and descent

Projectile motion review

- Horizontal displacement $x - x_0 = (v_0 \cos \theta_0)t.$
- Horizontal velocity $v_x = v_{ox}$
- Vertical displacement $y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2$
- Vertical velocity $v_y = v_0 \sin \theta_0 - gt$

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0)$$

- Trajectory $y = (\tan \theta_0)x - \dfrac{gx^2}{2(v_0 \cos \theta_0)^2}$
- Horizontal Range $R = \dfrac{v_0^2}{g} \sin 2\theta_0.$
- Total time $2t$, $t = \dfrac{v_o \sin \theta_o}{g}$
- Maximum height $H = \dfrac{v_o^2 \sin^2 \theta_o}{2g}$
- Relation between R & H $H = \dfrac{R \tan \theta}{4}$



Sample Problem

Figure 4-15 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?

Because the cannon and the ship are at the same height, the horizontal displacement is the range.

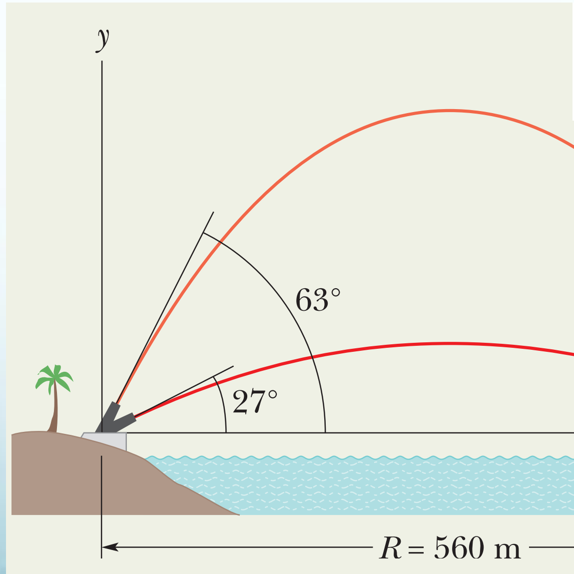
$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{Rg}{v_0^2}$$

$$2\theta = \sin^{-1}\frac{Rg}{v_0^2} \rightarrow \theta = \frac{1}{2}\sin^{-1}\frac{(9.8)(560)}{(82)^2}$$

$$\theta = \frac{1}{2}\sin^{-1}(.816) = \frac{1}{2}(54.7)$$

$$\theta = 27^o$$



Either launch angle gives a hit.

(b) What is the maximum range of the cannonballs?

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$R_{\max} \rightarrow \theta = 45$$

$$R = \frac{(82)^2}{9.8} \sin(2 \times 45)$$

$$R = \frac{6724}{9.8} = 686.7m$$

Examples:

Q.1: The maximum range of a projectile is at a launch angle:

(a) 35° **(b) 45°** **(c) 55°** **(d) 25°**

Q.2: In the projectile motion, the horizontal velocity component v_x remains constant because the acceleration in the horizontal direction is:

(a) x_x = g **(b) a_x >g** **(c) a_x = 0** **(d) a_x > 0**

Q.3: The range of a ball that is thrown at angle of 30° above the horizontal with an initial speed of 65 m/s is:

(a) 318.1m **(b) 266.3m** **(c) 373.4m** **(d) 220m**

$$R = \frac{V_o^2 \sin(2\theta)}{g} = \frac{65^2 \sin 60}{9.8} = 373.4m$$

Q.4: An object is projected from the ground with an initial velocity of 15m/s at an angle of 30° above the horizontal. The maximum height the object reaches above the ground is:

(a) 11.48m (b) 16.3m (c) 2.87m (d) 5.1m

$$H = \frac{V_o^2 \sin^2 \theta_o}{2g} = \frac{15^2 \sin^2(30)}{2(9.8)} = 2.87m$$

Q.5: Cannon is firing a ball from ground level at an angle of 15° above the horizontal. If the ball speed is 200m/s, the horizontal distance of the ball just before it hits the ground is:

(a) 4.59km (b) 3.19km (c) 6.25km (d) 2.04km

$$R = \frac{V_o^2 \sin 2\theta}{g} = \frac{200^2 \sin(30)}{9.8} = 2040.8m$$

Q.6: A projectile is fired from a ground at angle 45° above the horizontal. If it reaches the ground at 60m from the starting point, the initial velocity is:

(a) 24.2m/s (b) 16m/s (c) 9.8m/s 31.3m/s

$$R=\frac{V_o^2\sin 2\theta}{g}\Rightarrow V_o=\sqrt{\frac{Rg}{\sin 2\theta}}=\sqrt{\frac{60\times 9.8}{\sin 90}}=24.3m/s$$

Q.7: A baseball leaves the bat with initial velocity of v_o= 10î+20ĵ m/s, its range is:

(a) 40.8m (b) 102m (c) 20.4m (d) 61.2m

$$V_o=\sqrt{10^2+20^2}=\sqrt{500} \qquad \theta=\tan^{-1}\frac{20}{10}=63.43^0$$

$$R=\frac{V_o^2\sin 2\theta}{g}=\frac{500\sin 126.86}{9.8}=40.8m$$

Q.8: A ball is projected above the horizontal with an initial velocity V_o= 25î+25ĵ m/s. The maximum height the ball rises is:

(a) 1m (b) 20.4m (c) 2.4m (d) 31.89m

$$V_o = \sqrt{25^2 + 25^2} = 35.35 m/s \qquad \theta = \tan^{-1}\frac{25}{25} = 45^0$$

$$H = \frac{V_o^2 \sin^2\theta_o}{2g} = \frac{1250\sin^2(45)}{2(9.8)} = 31.89m$$

Q.9: A ball is kicked with speed of 25m/s at an angle of 35° above the ground. Its time of flight is:

(a) 5.9s (b) 11s (c) 3.25s (d) 2.93s

$$t = 2\left(\frac{V_o \sin\theta}{g}\right) = \frac{2(25)\sin 35}{9.8} = 2.93s$$

Q.10: A ball is kicked from the ground with an initial speed of 4m/s at an upward angle of 30°. The time the ball takes to reach its maximum height is:

(a) 0.2s (b) 0.31s (c) 0.41s (d) 0.51s

$$t = \frac{V_o \sin\theta}{g} = \frac{4\sin 30}{9.8} = 0.2s$$

Q.11: A ball is kicked from the ground with initial speed of 15m/s, the maximum horizontal distance the ball travels is:

(a) 40.8m (b) 22.96m (c) 25.5m (d) 63.8m

When a projectile is thrown and reach the maximum horizontal distance, this means that θ= 45°

$$R = \frac{V_o^2 \sin 2\theta}{g} = \frac{15^2 \sin 90}{9.8} = 22.96m$$

Q.12: A projectile is launched at an angle such that the maximum height reached equals the horizontal range. The launch angle is:

(a) 22.5° **(b) 45°** **(c) 30°** **(d) 76°**

$$H = \frac{R\tan\theta}{4}$$

$$H = R$$

$$\theta = \tan^{-1}\frac{4H}{R} = \tan^{-1}4 = 76^{o}$$

Chapter 4
MOTION IN TWO AND THREE DIMENSIONS

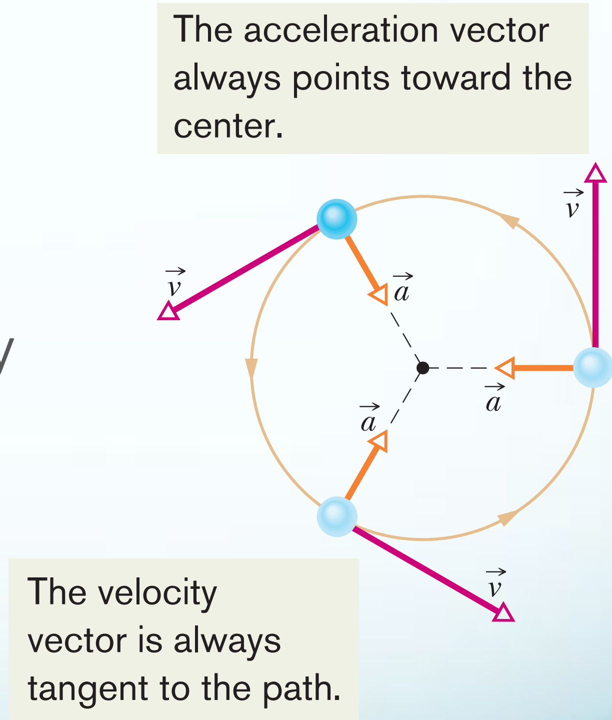
Section 4-7

Uniform Circular Motion

- Important skills from this lecture:
  1. Define the uniform circular motion
  2. Define the direction for both velocity & acceleration in the circular motion
  3. Calculate the particle's acceleration in the uniform circular motion
  4. Calculate the period of the revolution

Uniform Circular Motion

- **Uniform circular motion**: when a particle travels around a circle at constant (uniform) speed
  - Its speed does not vary
  - Its acceleration changes**?**
    because its velocity changes in direction
- The relationship between the particle's velocity & acceleration:
  - Both vectors have constant magnitude, but their directions change continuously
  - The velocity is always directed tangent to the circle in the direction of motion
  - The acceleration is always directed radially inward



- **Centripetal acceleration:** it is the acceleration that is associated with the uniform circular motion
- The magnitude of the centripetal acceleration a is

$$\boxed{a = \frac{v^2}{r} \qquad \text{(centripetal acceleration)}}$$

where r : the radius of the circle
v : the speed of the particle

- During this acceleration, the particle travels the circumference of the circle (a distance of $2\pi r$) in time T

$$\boxed{T = \frac{2\pi r}{v} \qquad \text{(period)}}$$

- **Period of revolution** or **period (T)**: the time for a particle to go around a closed path exactly once

CHECKPOINT 5

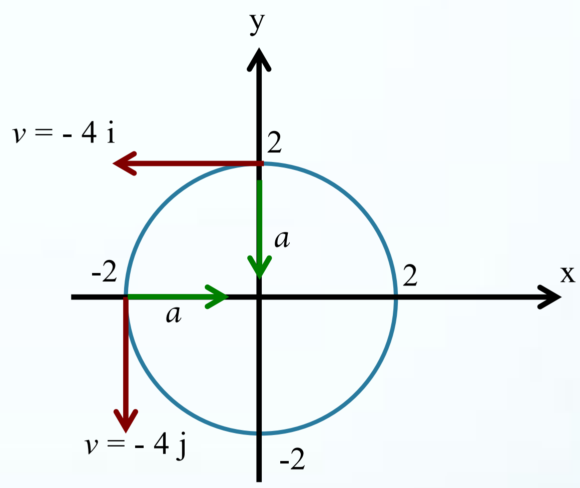
An object moves at constant speed along a circular path in a horizontal xy plane, with the center at the origin. When the object is at $x = -2$ m, its velocity is $-(4 \text{ m/s})\hat{\mathrm{j}}$. Give the object's (a) velocity and (b) acceleration at $y = 2$ m.

(a) $v = -4\hat{\mathrm{i}}$

(b) $a = v^2/r$

$= 16/2 = 8 \text{ m/s}^2$

a direction: $-8\,\hat{\mathrm{j}}$ m/s²



Sample Problem

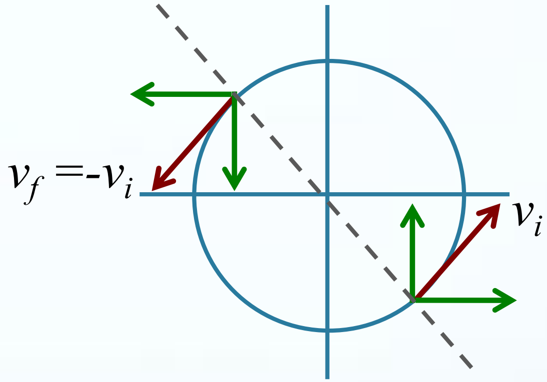
"Top gun" pilots have long worried about taking a turn too tightly. As a pilot's body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is $2g$ or $3g$, the pilot feels heavy. At about $4g$, the pilot's vision switches to black and white and narrows to "tunnel vision." If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as g-LOC for "g-induced loss of consciousness."

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{\mathrm{i}} + 500\hat{\mathrm{j}})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{\mathrm{i}} - 500\hat{\mathrm{j}})$ m/s?

$\vec{v}_i = (400\hat{\mathrm{i}} + 500\hat{\mathrm{j}})$ $\vec{v}_f = (-400\hat{\mathrm{i}} - 500\hat{\mathrm{j}})$

The final velocity is the reverse of the initial velocity → the aircraft leaves on the opposite side of the circle from the initial point and must have completed half a circle in the given 24.0 s. Thus a full circle would have taken *T* = 48.0 s.



$$T = \frac{2\pi r}{v} \Rightarrow r = \frac{Tv}{2\pi}$$

$$a = \frac{v^2}{r} = \frac{2\pi v^2}{Tv} = \frac{2\pi v}{T}$$

$$v = \sqrt{(400)^2 + (500)^2} = 640.31 m/s$$

$$a = \frac{2(3.14)(640.31)}{48} = 83.81 = \frac{83.81 \times g}{9.8} = 8.6g$$

Examples:

Q.1: a player runs in a circular tract has a radius of 50m with a constant speed of 10m/s. The magnitude of his centripetal acceleration is:

(a) 0.2m/s^2 (b) 2m/s^2 (c) 5m/s^2 (d) 20m/s^2

$$a = \frac{v^2}{r} = \frac{100}{50} = 2m / s^2$$

Chapter (4) MOTION IN TWO AND THREE DIMENSIONS

- A projectile is fired from the ground level over level ground with an initial velocity that has a vertical component of 20m/s and a horizontal component of 30m/s.

1- The distance from launching to landing points is:

(a). 40m (b) 60m (c) 20.4m (d) 122m

2-The maximum height the projectile reached is :

(a). 40m (b) 60m (c) 20 .4m (d) 122m

3-The time the projectile takes to reach its maximum height is:

(a). 4.1 s (b) 2.05 s (c) 1.05 s (d)0.5 s

Problem: 3 and 17

•3 A positron undergoes a displacement $\Delta\vec{r} = 2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k}$, ending with the position vector $\vec{r} = 3.0\hat{j} - 4.0\hat{k}$, in meters. What was the positron's initial position vector?

••17 A cart is propelled over an xy plane with acceleration components $a_x = 4.0\ \text{m/s}^2$ and $a_y = -2.0\ \text{m/s}^2$. Its initial velocity has components $v_{0x} = 8.0$ m/s and $v_{0y} = 12$ m/s. In unit-vector notation, what is the velocity of the cart when it reaches its greatest y coordinate?

Chapter (4) MOTION IN TWO AND THREE DIMENSIONS

- A projectile is fired from the ground level over level ground with an initial velocity that has a vertical component of 20m/s and a horizontal component of 30m/s.

1- The distance from launching to landing points is:

(a). 40m (b) 60m (c) 20.4m (d) 122m

$$v_o = \sqrt{v_{ox}^2 + v_{oy}^2} = 36.1 m/s$$

$$\theta = \tan^{-1}\frac{v_{oy}}{v_{ox}} = 33.68^o$$

$$R = \frac{v_o^2}{g}\sin 2\theta = 122m$$

2-The maximum height the projectile reached is :

(a). 40m **(b) 60m** **(c) 20 .4m** **(d) 122m**

$$H = \frac{v_o^2 \sin^2 \theta_o}{2g} = \frac{1300(0.55)^2}{2(9.8)} = 20.4m$$

3-The time the projectile takes to reach its maximum height is:

(a). 4.1 s **(b) 2.05 s** **(c) 1.05 s** **(d)0.5 s**

$$t = \frac{v_o \sin \theta_o}{g} = 2.04\text{s}$$

•3 A positron undergoes a displacement $\Delta\vec{r} = 2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k}$, ending with the position vector $\vec{r} = 3.0\hat{j} - 4.0\hat{k}$, in meters. What was the positron's initial position vector?

3. The initial position vector $\vec{r}_o$ satisfies $\vec{r} - \vec{r}_o = \Delta\vec{r}$, which results in

$$\vec{r}_o = \vec{r} - \Delta\vec{r} = (3.0\hat{j} - 4.0\hat{k})\text{m} - (2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k})\text{m} = (-2.0\text{ m})\hat{i} + (6.0\text{ m})\hat{j} + (-10\text{ m})\hat{k}$$

•17 A cart is propelled over an xy plane with acceleration components $a_x = 4.0\ \text{m/s}^2$ and $a_y = -2.0\ \text{m/s}^2$. Its initial velocity has components $v_{0x} = 8.0$ m/s and $v_{0y} = 12$ m/s. In unit-vector notation, what is the velocity of the cart when it reaches its greatest y coordinate?

2-11	$v = v_0 + at$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$
2-16	$v^2 = v_0^2 + 2a(x - x_0)$
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$
2-18	$x - x_0 = vt - \frac{1}{2}at^2$

17. We find t by applying Eq. 2-11 to motion along the y axis (with $v_y = 0$ characterizing $y = y_{\max}$):

$$v = v_0 + at$$

$$0 = (12\ \text{m/s}) + (-2.0\ \text{m/s}^2)t \quad \Rightarrow \quad t = 6.0\ \text{s}.$$

Then, Eq. 2-11 applies to motion along the x axis to determine the answer:

$$v_x = (8.0\ \text{m/s}) + (4.0\ \text{m/s}^2)(6.0\ \text{s}) = 32\ \text{m/s}.$$

Therefore, the velocity of the cart, when it reaches $y = y_{\max}$, is $(32\ \text{m/s})\hat{\text{i}}$.

Physics 110
1435-1436 H

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Chapter 5
FORCE AND MOTION -I

Sections 5-2, 5-3, 5-4, 5-5, 5-6

Newtonian Mechanics
Newton's First Law
Force
Mass
Newton's Second Law

- Important skills from this lecture:
  1. Explain Newton's first law
  2. Define the force and its unit
  3. Resolve forces and find the resultant along x & y axes
  4. Define the mass and its relation to force
  5. Explain newton second law
  6. Relate the force component along an axis to its acceleration
  7. Define Newton unit
  8. Draw free-body diagram
  9. Apply Newton 2^{nd} law in one & two dimensions

Newtonian Mechanics

- The study of the relation between force & acceleration is called **Newtonian mechanics**

- Newtonian mechanics does not apply to all situations;
  - If the speeds of the interacting bodies are very large (near the speed of light) → Einstein's special theory of relativity applied
  - If the interacting bodies are on the scale of atomic structure, e.g. electrons → quantum mechanics is applied

- Newtonian mechanics is applied to the motion of objects ranging in size from the very small to astronomical objects

Newton's First Law

- When there is no force acting on a body:
  - If the body is at rest, it stays at rest
  - If the body is moving, it continues to move with the same velocity (same magnitude & direction)

Newton's First Law: If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.

Force

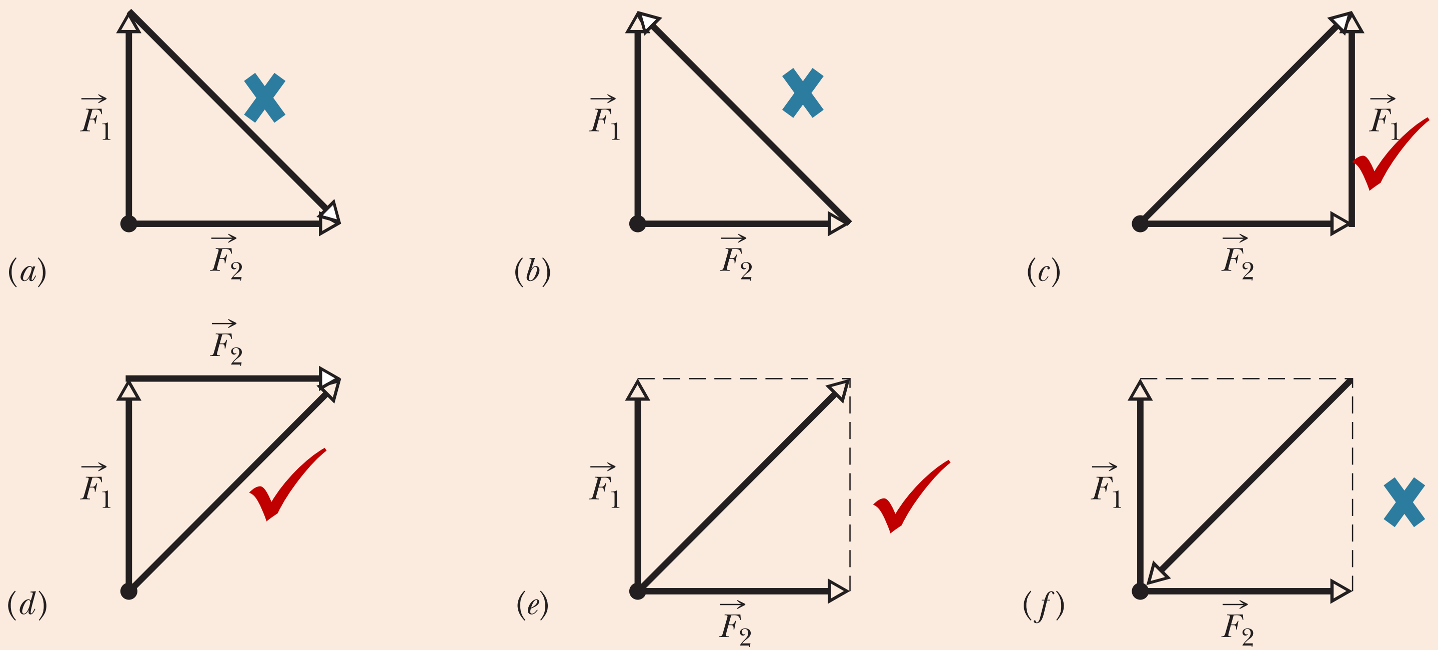
- A force is measured by the acceleration it produces
  - Acceleration is a vector quantity → force is a vector quantity ( ) $\vec{F}$,
  - Force unit is Newton (N)
- **Superposition principle for forces:**
  - When two or more forces act on a body, their net force or resultant force ($\vec{F}_{net}$) are the vector addition of the individual forces
  - A single force that has the magnitude & direction of the net force has the same effect on the body as all the individual forces together
  - $\vec{F}_{net}$ can have many components along coordinate axes
  - When forces act only along a single axis, they are single-component forces → the arrows could be replaced by signs to indicate the forces directions

- If many forces are acting on a body, and their net force is zero→ the body cannot accelerate

Newton's First Law: If no *net* force acts on a body ($\vec{F}_{net} = 0$), the body's velocity cannot change; that is, the body cannot accelerate.

CHECKPOINT 1

Which of the figure's six arrangements correctly show the vector addition of forces $\vec{F}_1$ and $\vec{F}_2$ to yield the third vector, which is meant to represent their net force $\vec{F}_{net}$?



Mass

- Force produces different magnitudes of acceleration for different bodies
- e.g., if a baseball  & a bowling ball  are given the same kick
  → baseball acceleration > bowling ball**?**
  - Because the mass of the baseball differs from the mass of the bowling
  - A *less massive* baseball receives a greater acceleration than a *more massive* bowling ball when the same force is applied to both
- The mass of a body is the characteristic that relates a force on the body to the resulting acceleration
- Mass is:
  - an intrinsic characteristic of a body
  - a scalar quantity

Newton's Second Law

Newton's Second Law: The net force on a body is equal to the product of the body's mass and its acceleration.

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \text{(Newton's second law)}$$

- This equation is equivalent to three component equations, one for each axis of an *xyz* coordinate system

$$F_{\text{net},x} = ma_x, \quad F_{\text{net},y} = ma_y, \quad \text{and} \quad F_{\text{net},z} = ma_z$$

- Each of these equations relates the net force component along an axis to the acceleration along that same axis

The acceleration component along a given axis is caused *only by* the sum of the force components along that *same* axis, and not by force components along any other axis.

- From Newton's 2nd law, if $\vec{F}_{net} = 0 \quad \Rightarrow \vec{a} = 0$
  - If the body is at rest (v = 0 & a = 0), it stays at rest
  - If the body is moving ($v \neq$ 0 & a = 0), it continues to move at constant velocity
  - Any forces on such body balance one another, and both the forces and the body are said to be in equilibrium
  - The forces could cancel one another, but still act on the body

- In SI units

$$\boxed{1\ \mathrm{N} = (1\ \mathrm{kg})(1\ \mathrm{m/s^2}) = 1\ \mathrm{kg \cdot m/s^2}}$$

- Some force units in other systems of units are given in Table 5-1

Table 5-1

Units in Newton's Second Law (Eqs. 5-1 and 5-2)

System	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
CGS[a]	dyne	gram (g)	cm/s^2
British[b]	pound (lb)	slug	ft/s^2

[a] 1 dyne = $1\ g \cdot cm/s^2$.

[b] 1 lb = $1\ slug \cdot ft/s^2$.

Calculate an unknown mass by knowing another mass and their accelerations

If a force F = 1 N is applied on 2 bodies, a standard body, whose mass m_o = 1.0 kg, and acceleration a_o = 1.0 m/s^2, and the 2nd body X whose mass (m_X) is not known, and its acceleration a_X = 0.25 m/s^2. To find m_X :

$$F_o = F_x$$

$$\frac{m_X}{m_0} = \frac{a_0}{a_X}$$

$$m_X = m_0 \frac{a_0}{a_X} = (1.0\ \text{kg}) \frac{1.0\ \text{m/s}^2}{0.25\ \text{m/s}^2} = 4.0\ \text{kg}.$$

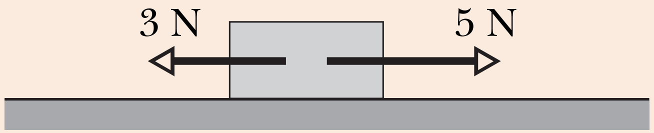
Free-body Diagram

- To solve problems with Newton's second law, a **free-body diagram** is drawn
- A free-body diagram is a diagram that contains the only body under the forces
- In the free-body diagram, the body is represented with a dot, and each force on the body is drawn as a vector arrow with its tail on the body

- A **system** consists of one or more bodies
- Types of forces that affect any system:
  - **External force:** any force on the bodies that comes from outside the system
  - **Internal forces:** forces between two bodies inside the system
  - If the bodies making up a system are rigidly connected to one another,
    → the system is treated as one composite body, and $\vec{F}_{\text{net}}$,
    on the system is the vector sum of all external forces
  - e.g., a system of a connected railroad engine & car
    If a tow line pulls on the front of the engine
    → tow force acts on the whole engine – car system
    → $\vec{F}_{\text{net}}$, on the system can be related to its acceleration using Newton's 2nd law
    (m) will be the total mass of the system

CHECKPOINT 2

The figure here shows two horizontal forces acting on a block on a frictionless floor. If a third horizontal force $\vec{F}_3$ also acts on the block, what are the magnitude and direction of $\vec{F}_3$ when the block is (a) stationary and (b) moving to the left with a constant speed of 5 m/s?



$F_1 = 5N$, $F_2 = -3N$

From Newton's 2nd law, $F_{net} = ma$

(a) The block is stationary → $v = 0$, $a = 0$, → $F_{net} = 0$

$F_3 + F_1 + F_2 = 0$

→ $F_3 + 5 - 3 = 0$

→ $F_3 = -2N$

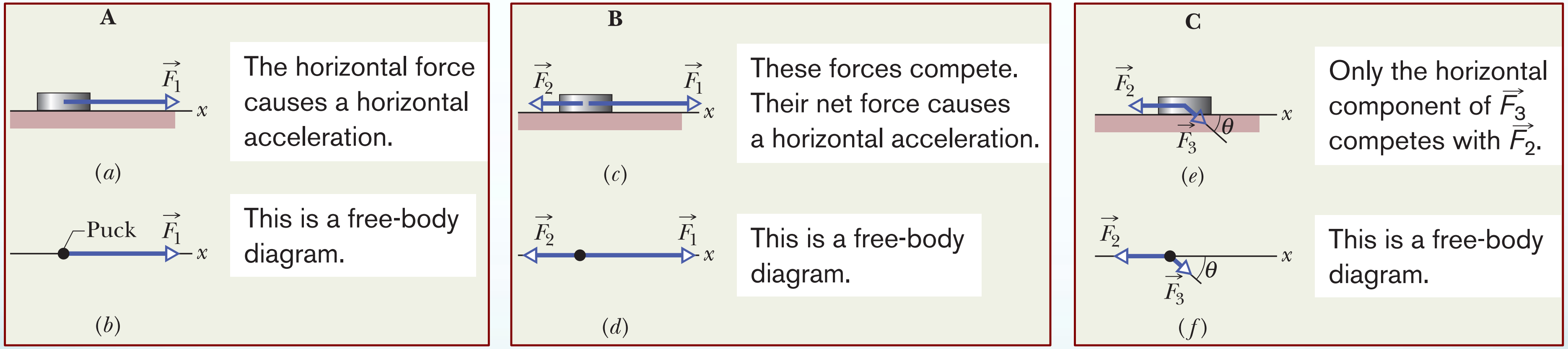
(b) The body is moving with a constant velocity → $a = 0$

From Newton's 2nd law, $F_{net} = 0$

→ $F_3 = -2N$

Sample Problem

Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is $m = 0.20$ kg. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\vec{F}_3$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?



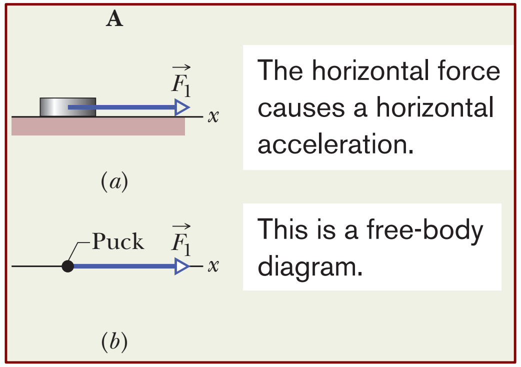
$$m = 0.20kg, \quad F_1 = 4N, \quad F_2 = 2N, \quad F_3 = 1N, \quad \theta = 30^o$$

a)

$$F_{net,x} = ma_x$$

$$F_1 = ma_x$$

$$a_x = \frac{F_1}{m} = \frac{4N}{0.20Kg} = 20m/s^2$$

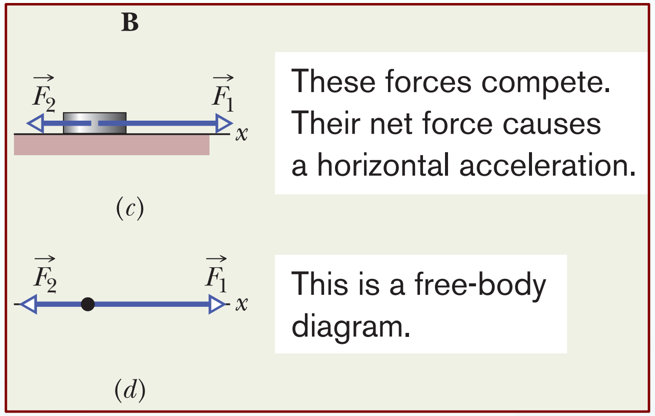


b)

$$F_{net,x} = ma_x$$

$$F_1 - F_2 = ma_x$$

$$a_x = \frac{F_1 - F_2}{m} = \frac{4N - 2N}{0.20Kg} = 10m/s^2$$



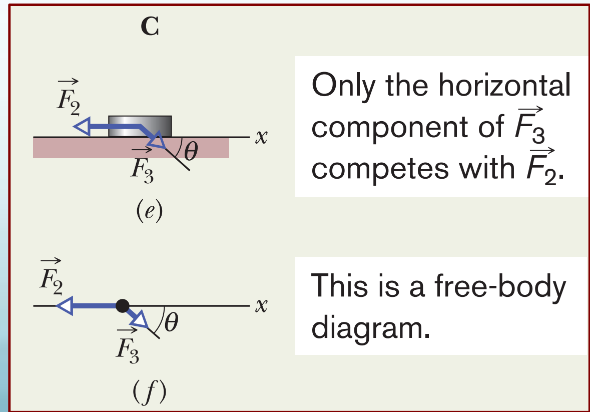
c)

$$F_{net,x} = ma_x$$

$$F_{3,x} - F_2 = ma_x$$

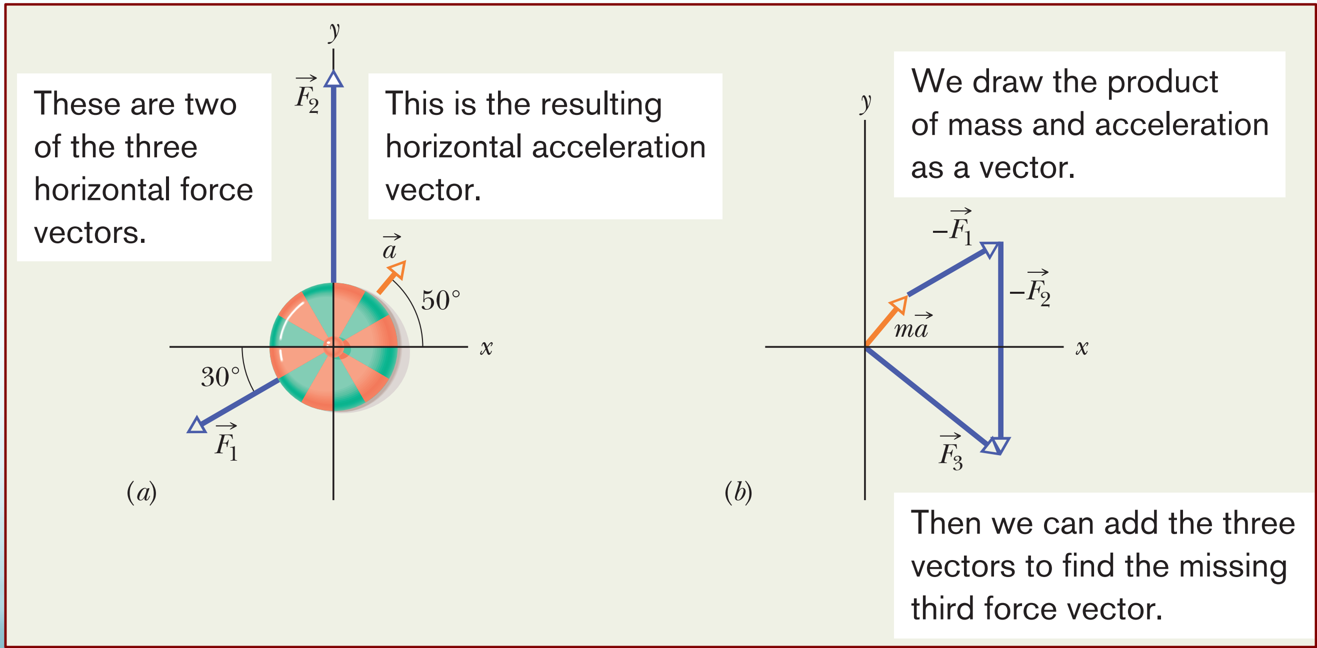
$$a_x = \frac{F_{3,x} - F_2}{m} = \frac{F_3 \cos\theta - F_2}{m} =$$

$$a_x = \frac{(1)\cos(-30^o) - 2}{0.20kg} = -5.7m/s^2$$



Sample Problem

In the overhead view of Fig. 5-4a, a 2.0 kg cookie tin is accelerated at 3.0 m/s^2 in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. What is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation?



$$m = 2kg, \quad a = 3m/s^2, \quad F_1 = 10N, \quad F_2 = 20N$$

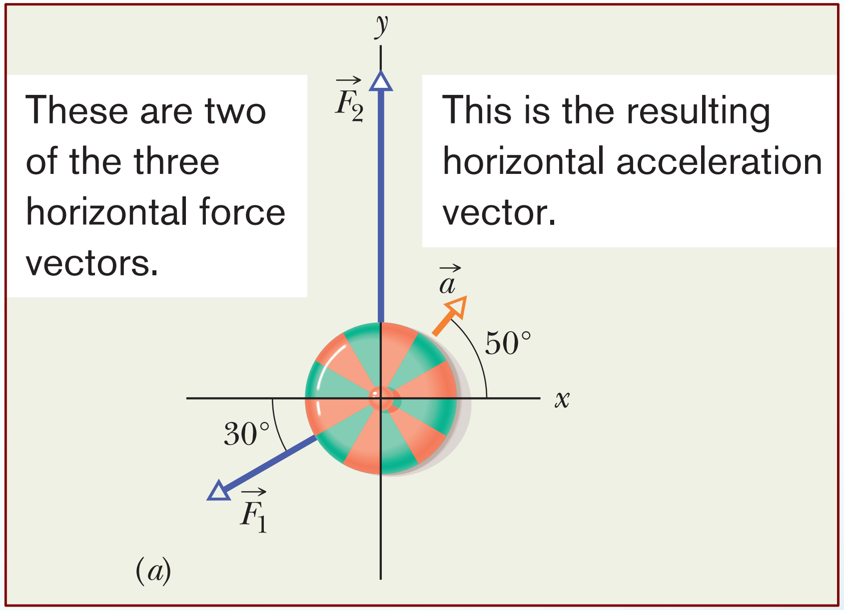
$$F = ma$$
$$F_1 + F_2 + F_3 = ma$$
$$F_3 = ma - F_1 - F_2$$

For x component

$$F_{3,x} = ma_x - F_{1,x} - F_{2,x}$$
$$F_{3,x} = m(a\cos 50) - (F_1 \cos(30-180)) - (F_2 \cos 90)$$
$$F_{3,x} = 2(3\cos 50) - 10\cos(-150) - 20\cos 90$$
$$F_{3,x} = 12.5N$$

For y component

$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$
$$F_{3,y} = m(a\sin 50) - (F_1 \sin(-150)) - (F_2 \sin 90)$$
$$F_{3,y} = 2(3\sin 50) - 10\sin(-150) - 20\sin 90$$
$$F_{3,y} = -10.4N$$



F_3 in unit vector

$$\vec{F}_3 = F_{3,x}\hat{i} + F_{3,y}\hat{j}$$

$$\vec{F}_3 = 12.5\hat{i} - 10.4\hat{j}$$

The magnitude

$$F_3 = \sqrt{(12.50^2 + (-10.4)^2}$$

$$F_3 = 16N$$

The angle

$$\theta = \tan^{-1}\frac{F_{3.y}}{F_{3,x}} = \tan^{-1}\frac{-10.4}{12.5}$$

$$\theta = -40^o$$

Examples:

Q.1: One Newton equals:

(a) Kg.m **(b) kg.m/s^2** **(c) kg/s^2** **(d) m/s^2**

$F = ma \rightarrow N = kg.m/s^2$

Q.2: The basic SI unit of the force is:

(a) Kg.m **(b) kg.m/s^2** **(c) kg/s^2** **(d) m/s^2**

Q.3: A box is moving with a constant speed of 24.7 m/s. The net force on the box is:

(a) Zero **(b) 4N** **(c) 5N** **(d) 45N**

If $\Sigma F = 0 \rightarrow a = 0$

Q.4: Three forces act on a particle of mass m, $F_1 = 80i + 60j$, $F_2 = 40i + 100j$. If the particle moves with a constant speed of 4m/s, then F_3 is:

(a) 80i + 60j (b) 80i – 60j (c) –80i + 60j (d) –120i–160j

v constant → $a = 0$ → $\Sigma F = 0$

$F_1 + F_2 + F_3 = 0 \rightarrow F_3 = -(F_1 + F_2) = -120i - 160j$

Q.5: Two forces act on a particle that moves with constant velocity. If $F_1 = 6i - 2j$, then F_2 is:

(a) 6i – 2k (b) – 2i + 6k (c) – 6i + 2j (d) – 2i + 6j

v constant → $a = 0$ → $\Sigma F = 0$

$F_1 + F_2 = 0 \rightarrow F_2 = -F_1 = -6i + 2j$

Chapter 5
FORCE AND MOTION -I

Sections 5-7

Some Particular Forces

- Important skills from this lecture:
  1. Define the gravitational force & write it in unit vector notation and magnitude and angle notation
  2. Define the weight and differentiate between mass and weight
  3. Define the normal force and calculate it
  4. Define the frictional force
  5. Calculate the tension force

The Gravitational Force

- **A gravitational force $\vec{F}_g$ on a body:** a force that pulls on the body directly toward the center of Earth (downward ↓)

- In free fall motion, when the effects of air is neglected, the only force acting on a body of mass m is the gravitational force F_g

  From Newton's 2nd law → $F_{\text{net},y} = ma_y$, or $-F_g = m(-g)$

$$\boxed{F_g = mg}$$

In words, the magnitude of the gravitational force is equal to the product mg.

- Using unit vector notation: $\boxed{\vec{F}_g = -F_g\hat{\mathrm{j}} = -mg\hat{\mathrm{j}} = m\vec{g}}$

- The gravitational force acts on the body even when the body is not in the free fall situation

- For the gravitational force to disappear, Earth has to disappear

Weight

- **The weight (W) of a body:** is the magnitude of the net force required to prevent the body from falling freely, as measured by someone on the ground

- e.g., if the magnitude of the gravitational force on a ball is 2.0 N. To keep a ball at rest, an upward force with magnitude of 2.0 N has to be applied to balance the gravitational one
→ the weight W of the ball is 2.0 N

- If another ball requires a greater force to keep it at rest
→ the 2nd ball is *heavier* than the 1st one

- If two forces act on a body; $\vec{F}_g$(downward) & a balancing upward force of magnitude W
→ from Newton's 2nd law:

$$\rightarrow F_{\text{net},y} = ma_y \rightarrow W - F_g = m(0)$$

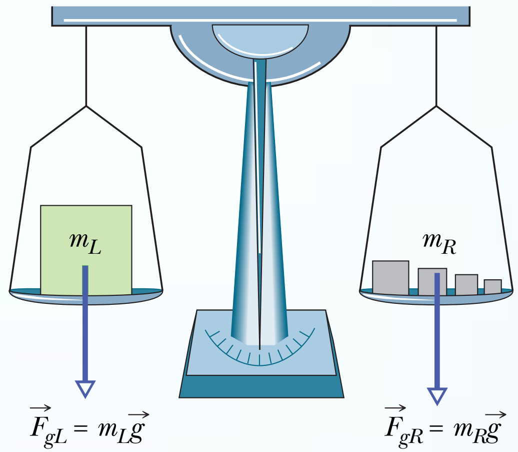
$$\rightarrow W = F_g \rightarrow \boxed{W = mg}$$

The weight W of a body is equal to the magnitude F_g of the gravitational force on the body.

How to weight a body

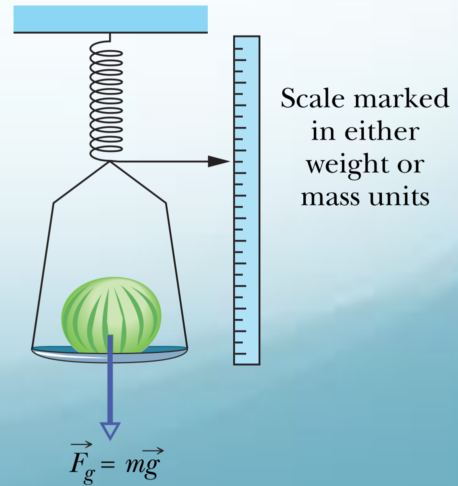
Two ways for weighting a body:

1. **Equal-arm balance:** the body is placed on one of the pans, a reference bodies (with known masses) is placed on the other pan
   when a balance is obtained:
   → the gravitational forces on the two sides match
   → the masses on the pans match
   → we know the mass of the body



1. **Spring scale:** the body stretches a spring, which causes a movement of a pointer along a scale. The scale has been calibrated & marked in either mass or weight units

   It is accurate only when the value of g is the same as when the scale was calibrated



- The weight of a body must be measured when the body is not accelerating vertically relative to the ground
- A body's weight is not its mass
  - Weight is the magnitude of a force
  - Weight is related to mass by Newton's 2nd law
  - If a body is moved to a point where the value of g is different:
    - The body's mass will not differ
    - The body's weight will differ

e.g., for a bowling ball of mass 0.3 kg	
On Earth	On Moon
M = 0.3 Kg	M = 0.3 Kg
g = 9.8 m/s^2	g = 1.6 m/s^2
W= (0.3) (9.8) = 2.9 N	W= (0.3) (1.6) = 0.49 N

The Normal Force

e.g., if you stand on a floor or a mattress
→ they deform (compressed, bent, or buckled slightly)
Earth pulls you down, and they pushes you up
→ you remain stationary

When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force $\vec{F}_N$ that is perpendicular to the surface.

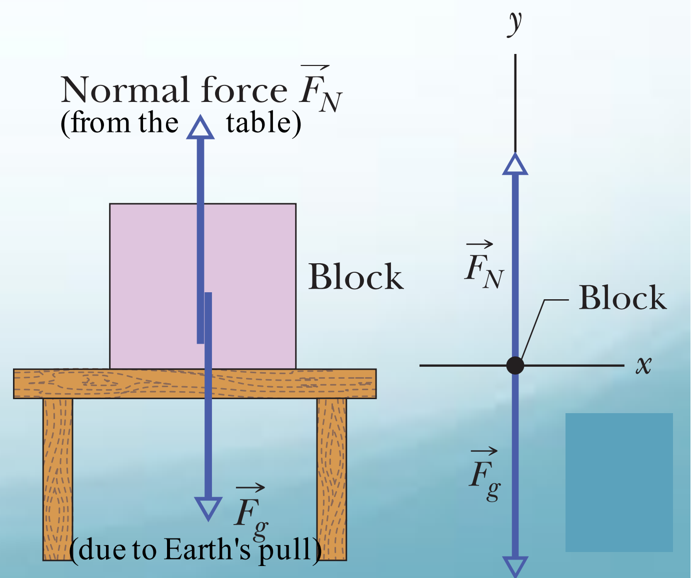
$\vec{F}_g$ and $\vec{F}_N$ are the only forces on the block
they are both vertical
→ Newton's 2nd law in y axis:

$$F_{\text{net},y} = ma_y$$

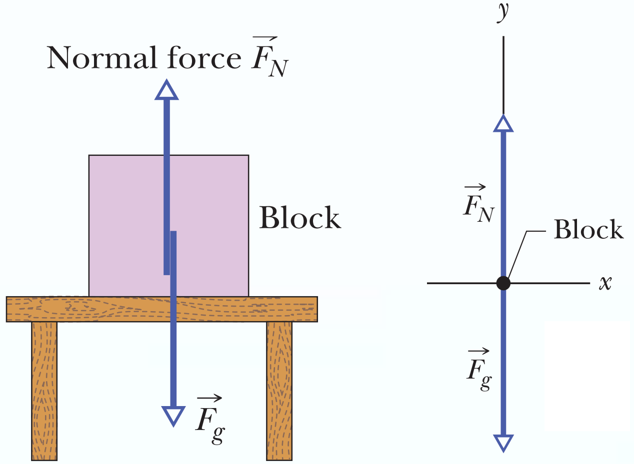
$$F_N - F_g = ma_y$$

$$\rightarrow F_N - mg = ma_y$$

$$\rightarrow \boxed{F_N = mg + ma_y = m(g + a_y)}$$



The Normal Force



$$\boxed{F_N = mg \pm ma_y = m(g \pm a_y)}$$

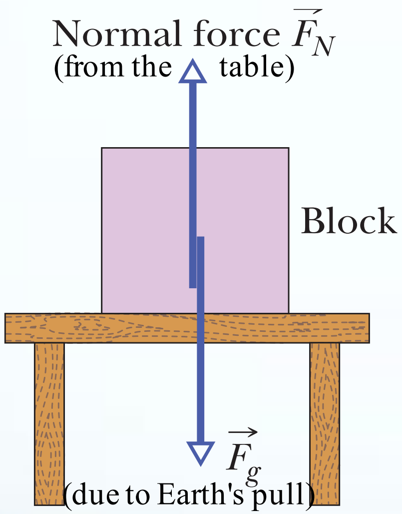
1. If the table and block are not accelerating relative to the ground,
   → $a_y = 0$ → $F_N = mg$.

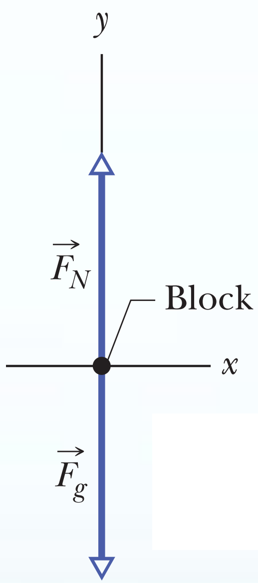
2. If the table and block are accelerating up-ward relative to the ground,
   → a_y is +ve → $F_N = m(g + a_y)$

3. If the table and block are accelerating down-ward relative to the ground,
   → a_y is –ve → $F_N = m(g - a_y)$

CHECKPOINT 3

In Fig. 5-7, is the magnitude of the normal force $\vec{F}_N$ greater than, less than, or equal to mg if the block and table are in an elevator moving upward (a) at constant speed and (b) at increasing speed?





a)

$$F_N = mg + ma_y$$

$$a_y = 0$$

$$F_N = mg$$

b)

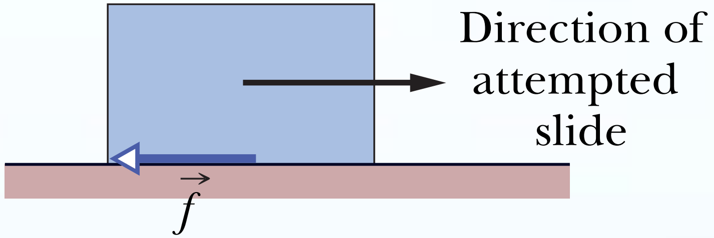
$$F_N = ma_y + mg$$

$$a_y = +ve$$

$$F_N > mg$$

Friction

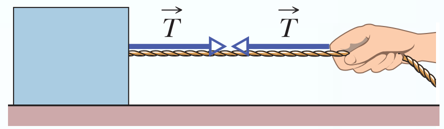
- If a body is slid over a surface, the motion is resisted by a bonding between the body and the surface
- This resistance is a single force $\vec{f}$, called **frictional force** or simply **friction**

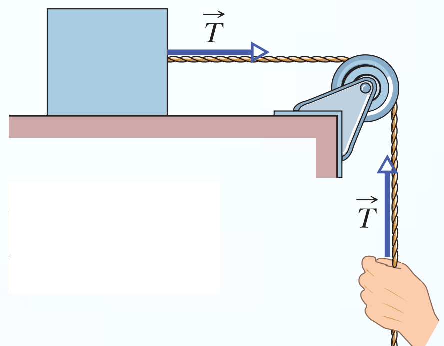


- Frictional force is directed along the surface, opposite the direction of the motion
- To simplify a situation, sometimes a friction is assumed to be negligible (the surface is **frictionless**)

Tension

- **Tension force ($\vec{T}$):** If a cord is attached to a body and pulled, the cord pulls on the body with tension force
  - The direction of $\vec{T}$ is along the cord, away from the body



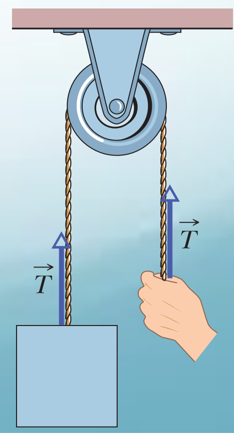


The forces at the two ends of the cord are equal in magnitude

- **Tension (T):** is the magnitude of the force $\vec{T}$
  e.g., if $\vec{T}$ has magnitude T = 50 N
  → the tension in the cord is 50 N

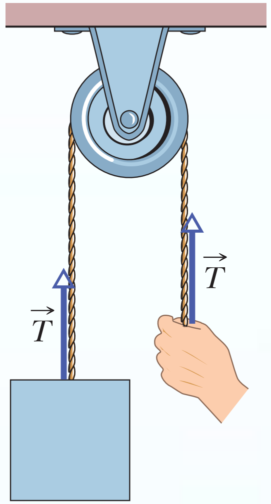
- The cord properties:
  - Unstretchable
  - Massless (its mass is negligible compared to the body's mass)
  - Exists only as a connection between 2 bodies
  - Pulls on both bodies with the same force magnitude T

- If the cord wraps halfway around a pulley, the net force on the pulley from the cord has the magnitude 2T



CHECKPOINT 4

The suspended body in Fig. 5-9*c* weighs 75 N. Is T equal to, greater than, or less than 75 N when the body is moving upward (a) at constant speed, (b) at increasing speed, and (c) at decreasing speed?



a)
$$F_y = ma_y$$
$$T - W = ma$$
$$a = 0$$
$$T - W = 0m$$
$$T = W$$
$$T = 75N$$

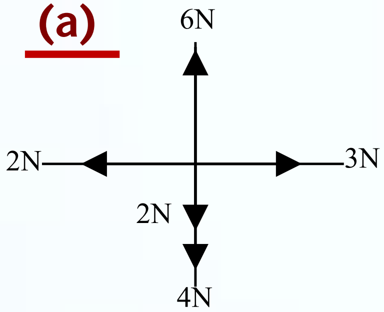
b)
$$F_y = ma_y$$
$$T - W = ma$$
$$T = W + ma$$
$$T = 75 + ma$$
$$T > W$$

c)
$$F_y = ma_y$$
$$T - W = m(-a)$$
$$T = W - ma$$
$$T = 75 - ma$$
$$T < W$$

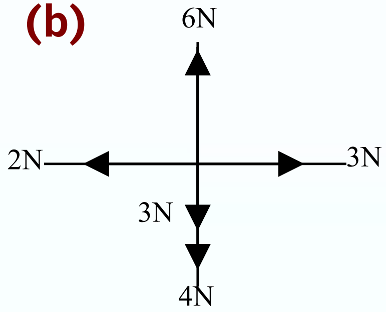
Examples:

Q.1: In which figure of the following the y-component of the net force is zero?

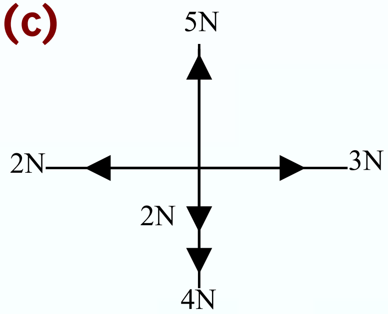
(a)



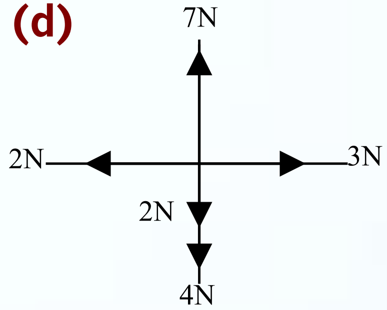
(b)



(c)

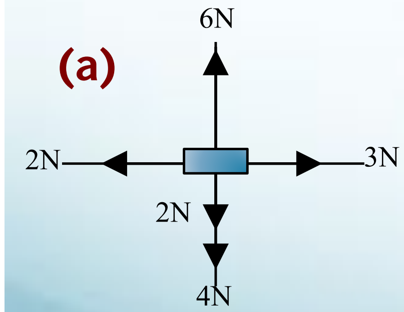


(d)

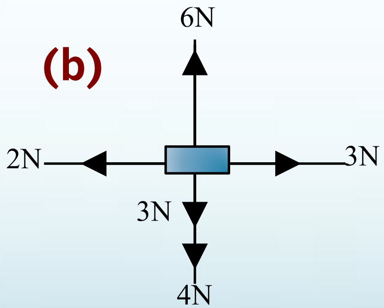


Q.2: In which figure of the following the particle moves with a constant velocity?

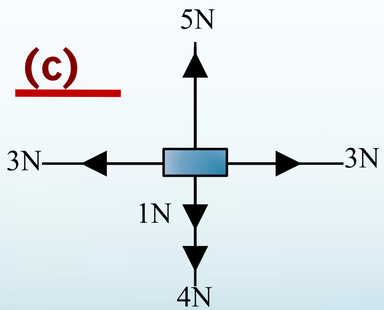
(a)



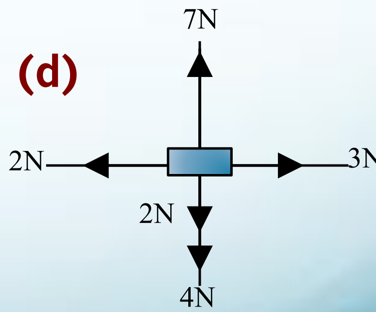
(b)



(c)



(d)



$\Sigma F_x = 0, \Sigma F_y = 0$

Q.3: A particle of mass 2kg at a point where g = 9.8m/s^2, the weight of this particle at point where g = 0 is:

(a) 49N **(b) 98N** **(c) zero** **(d) 9.8N**

$W = mg = 2 \times 0 = 0$

Q.4: The direction of the acceleration of a body is:

(a) Opposite to the net force

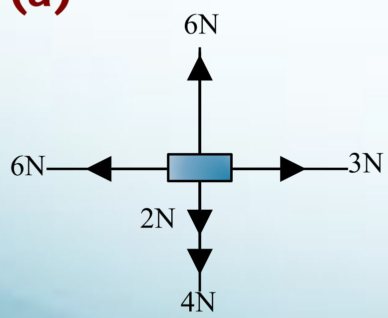
(b) The same direction of the net force

(c) Perpendicular to the direction of the net force

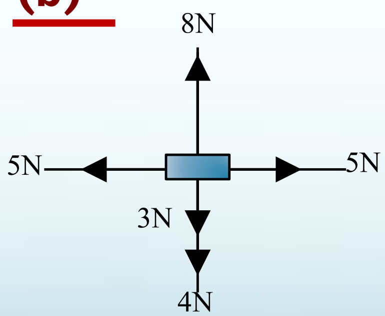
(d) The same of the initial velocity

Q.5: In which figure of the following the particle moves up if it starts from rest?

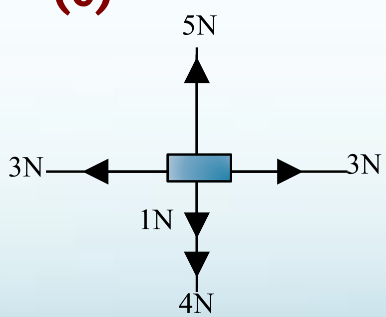
(a)



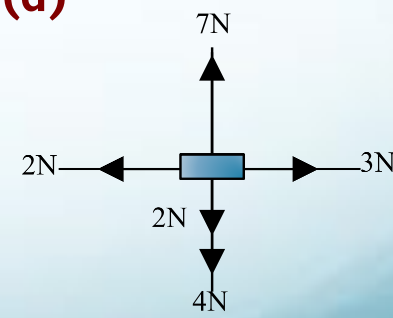
(b)



(c)

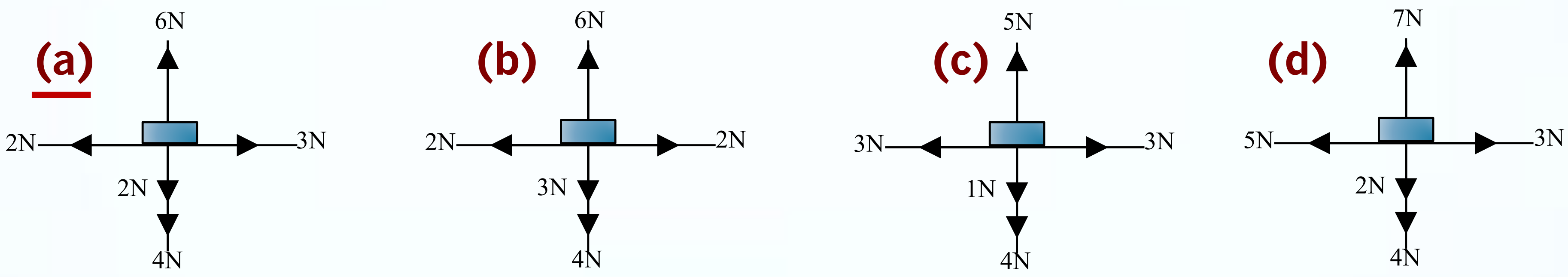


(d)



$\Sigma F_x = 0, \Sigma F_y = +ve$

Q.6: In which figure of the following the acceleration of the particle moves to right?



(a) $\Sigma F_x = 1, \Sigma F_y = 0$ (b) $\Sigma F_x = 0, \Sigma F_y = -1$

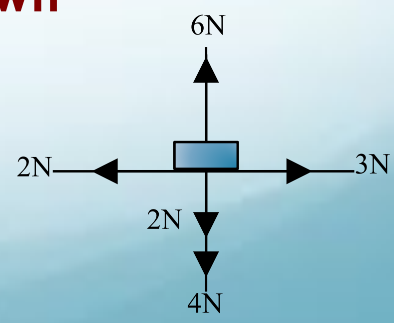
(c) $\Sigma F_x = 0, \Sigma F_y = 0$ (d) $\Sigma F_x = -2, \Sigma F_y = 1$

the correct answer is (a) because the direction of a has to be as the same direction as F_{net}

Q.7: In the figure, the net force on the block is:

(a) 1N right **(b) 6N up** **(c) 2N left** **(d) 4N down**

$\Sigma F_x = 3 - 2 = 1N, \Sigma F_y = 6 - 2 - 4 = 0N$



Q.8: When a force of 10N is applied to a body, the body accelerates with $2 m/s^2$. The mass of the body is:

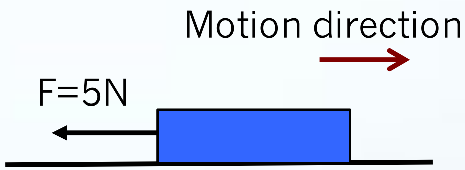
(a) 20kg (b) 10kg (c) 0.5kg (d) 5kg

$m = F/a = 10/2 = 5kg$

Q.9: From the figure, the acceleration of the block of mass = 0.5kg moving along the x-axis on a horizontal frictionless table is:

(a) $10 m/s^2$ (b) $-10 m/s^2$ (c) $-6.3 m/s^2$ (d) $-8.3 m/s^2$

$a = F/m = -5/0.5 = -10 m/s^2$



Q.10: A force of 7N is applied to a mass of 7kg, the resulting acceleration is:

(a) $3 m/s^2$ (b) $1 m/s^2$ (c) $2 m/s^2$ (d) $4 m/s^2$

$a = F/m = 7/7 = 1 m/s^2$

Q.11: A force accelerate a 5kg particle from rest to a speed of 12m/s in 4s. The magnitude of this force is:

(a) 10N (b) zero (c) 20N (d) 15N

$$M = 5kg,\ \ v_o = 0,\ \ v = 12m/s,\ \ t = 4s$$

$$v = v_o + at \Rightarrow a = 3m/s^2$$

$$F = Ma = 3(5) = 15N$$

Q.12: A body of mass 1kg is accelerating by a = 3i + 4jm/s², the magnitude of the acting force F on the body is:

(a) 2.5N (b) 7.5N (c) 12N (d) 5N

$$\vec{F} = m\vec{a} = (1)(3i + 4j) = 3i + 4j$$

$$\left|\vec{F}\right| = \sqrt{9+16} = 5N$$

Q.13: A net force of 15N acts on a body of weight 29.4N. The acceleration of the body is:

(a) $9.8 m/s^2$ (b) $5 m/s^2$ (c) $6.5 m/s^2$ (d) $2.4 m/s^2$

$$W = mg \Rightarrow m = \frac{W}{g} = \frac{29.4}{9.8} = 3kg$$

$$a = \frac{F}{m} = \frac{15}{3} = 5m/s^2$$

Q.14: Only two forces are acing on a particle of mass 2kg that moves with an acceleration of $3 m/s^2$ in the positive direction of y axis. If F_1 = 8i N, the magnitude of F_2 is:

(a) 12N (b) 10N (c) 17N (d) 15N

$$\vec{F}_1 + \vec{F}_2 = ma$$

$$8i + \vec{F}_2 = 2(3j) \Rightarrow \vec{F}_2 = -8i + 6j$$

$$\left|\vec{F}_2\right| = \sqrt{64 + 36} = 10N$$

Chapter 5
FORCE AND MOTION -I

Sections 5-8, 5-9

Newton's Third Law
Applying Newton's Laws

- Important skills from this lecture:
  1. Explain Newton's third law and apply it to different cases
  2. Apply Newton's laws to solve problems for one and two body system

Newton's Third Law

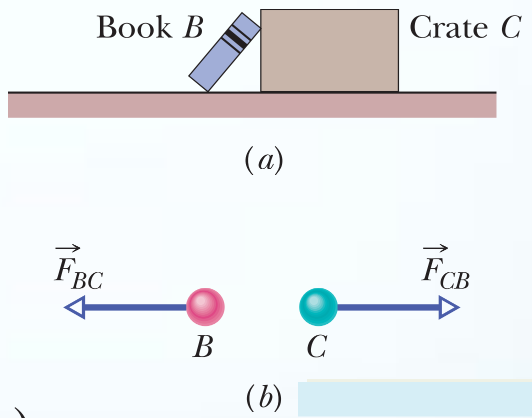
Newton's Third Law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

- Both book & crate in the figure interact: There is a horizontal force $\vec{F}_{BC}$ on the book from the crate and a horizontal force $\vec{F}_{CB}$ on the crate from the book

$$F_{BC} = F_{CB} \qquad \text{(equal magnitudes)}$$

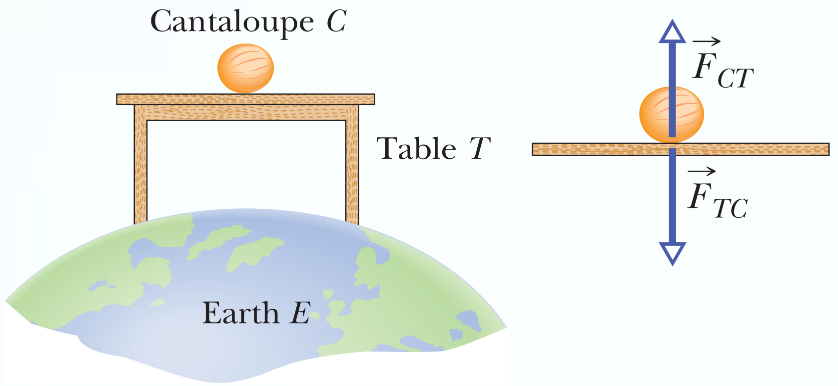
$$\vec{F}_{BC} = -\vec{F}_{CB} \qquad \text{(equal magnitudes and opposite directions)}$$

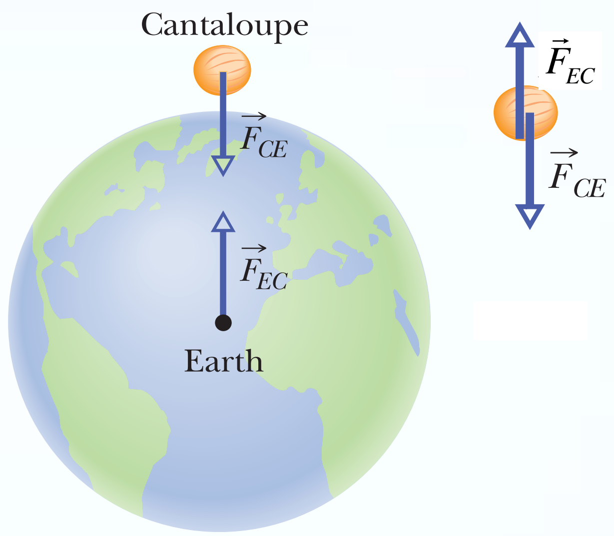
- These two forces are called ***action*** & ***reaction*** forces



The force on B due to C has the same magnitude as the force on C due to B.

- **A third-law force pair:** the forces between two interacting bodies
  - When any two bodies interact <u>in any situation</u> (stationary, moving, accelerating), a third-law force pair is <u>present</u>





- The cantaloupe interacts with the table & with Earth
- The cantaloupe–table interaction:
  - $\vec{F}_{CT}$ (normal force on the cantaloupe from the table)
  - $\vec{F}_{TC}$ (force on the table from the cantaloupe)

$$\vec{F}_{CT} = -\vec{F}_{TC} \qquad \text{(cantaloupe – table interaction)}$$

- The cantaloupe–Earth interaction:
  - $\vec{F}_{CE}$ (Earth pulls on the cantaloupe with a gravitational force)
  - $\vec{F}_{EC}$ (cantaloupe pulls on Earth with a gravitational force)

$$\vec{F}_{CE} = -\vec{F}_{EC} \qquad \text{(cantaloupe – Earth interaction)}$$

Example:

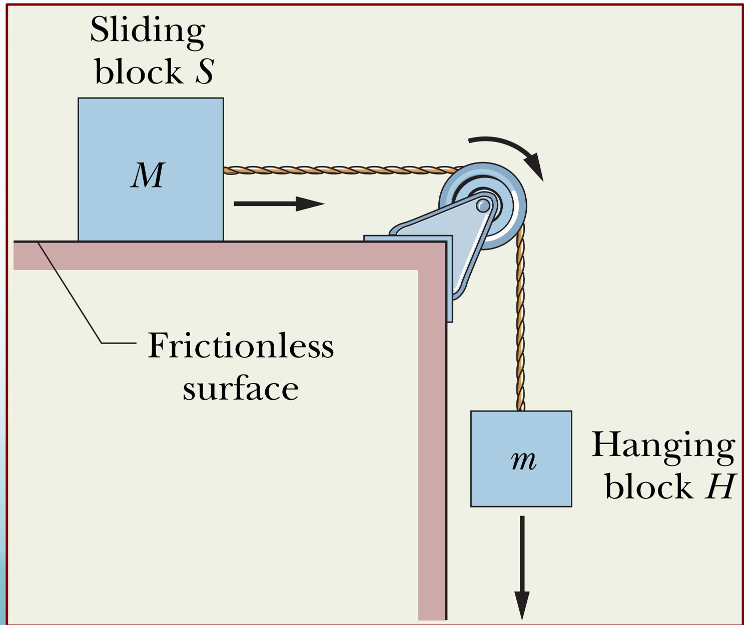
Q.1: A book rests on a table, exerting a downward force on it. The reaction to this force is:

(a) Force from the Earth on the table
(b) Force from the book on Earth
(c) Force from the Earth on the book
(d) Force from the the table on the book

Applying Newton's Laws

Sample Problem

Figure 5-12 shows a block S (the *sliding block*) with mass $M = 3.3$ kg. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block H (the *hanging block*), with mass $m = 2.1$ kg. The cord and pulley have negligible masses compared to the blocks (they are "massless"). The hanging block H falls as the sliding block S accelerates to the right. Find (a) the acceleration of block S, (b) the acceleration of block H, and (c) the tension in the cord.



- We need to find F_{net} in x & y directions for both blocks of masses M & m

- For block S of mass M:

$$F_{net,x} = Ma_x \Rightarrow T = Ma_x = Ma \qquad (1)$$

$$F_{net,y} = Ma_y \Rightarrow F_N - F_{gS} = M(0) = 0$$

$$\Rightarrow F_N = F_{gS} = Mg \qquad (2)$$

- For block H of mass m:

$$F_{net,x} = 0$$

$$F_{net,y} = ma_y \Rightarrow T - F_{gS} = ma_y$$

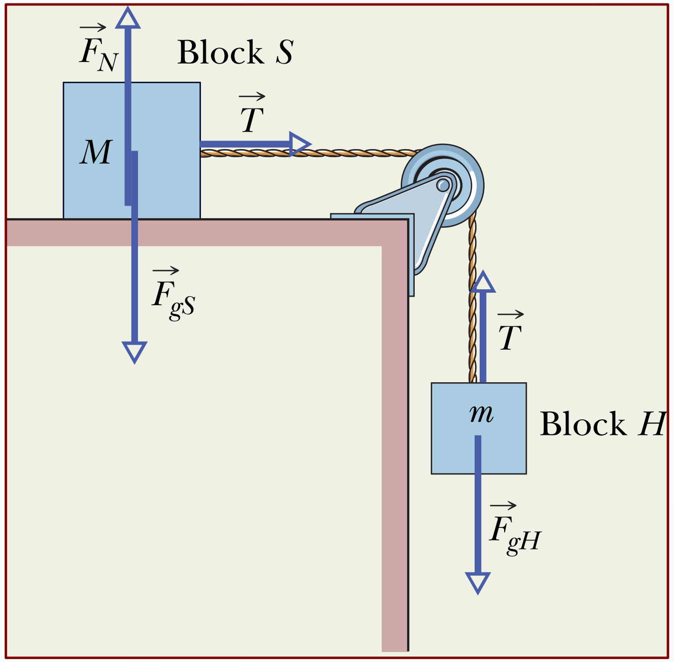
$$\Rightarrow T - mg = -ma \qquad (3)$$

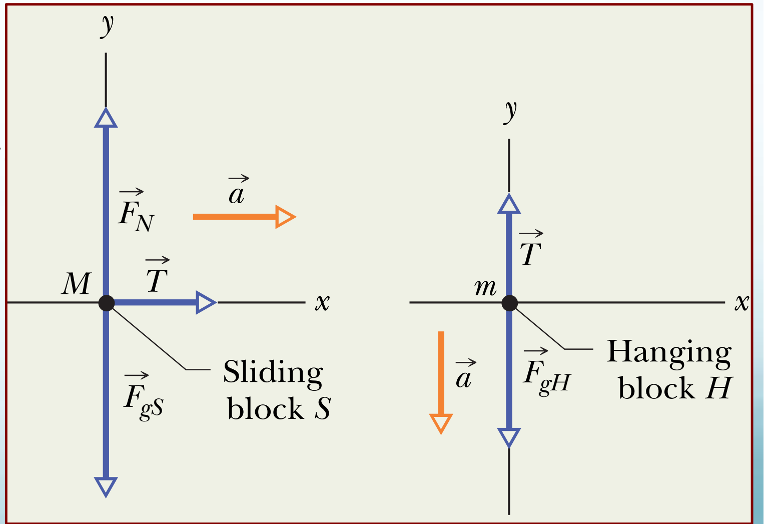
The $-a$ because block H accelerates in the negativ direction of the y axis

Both blocks M & m accelerate with the same magnitude a

By substituting 1 into 3:

$$Ma - mg = -ma \Rightarrow a = \frac{m}{M+m}g \qquad (4)$$





By substituting 4 into 1:

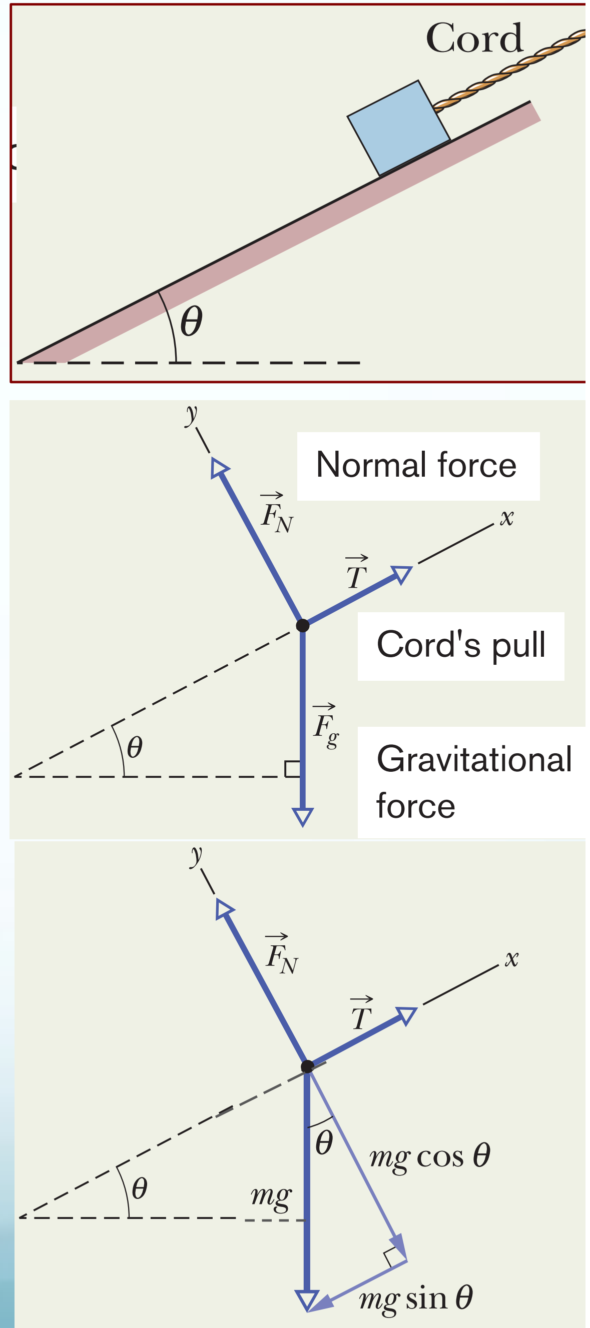
$$T = \frac{Mm}{M+m} g$$

Putting in the values of M, m, and g from the problem, we obtain:

$$a = \frac{m}{M+m} g = \frac{2.1 \text{ kg}}{3.3 \text{ kg} + 2.1 \text{ kg}} (9.8 \text{ m/s}^2)$$
$$= 3.8 \text{ m/s}^2$$
$$T = \frac{Mm}{M+m} g = \frac{(3.3 \text{ kg})(2.1 \text{ kg})}{3.3 \text{ kg} + 2.1 \text{ kg}} (9.8 \text{ m/s}^2)$$
$$= 13 \text{ N.}$$

Sample Problem

In Fig. 5-15*a*, a cord pulls on a box of sea biscuits up along a frictionless plane inclined at $\theta = 30°$. The box has mass $m = 5.00$ kg, and the force from the cord has magnitude $T = 25.0$ N. What is the box's acceleration component a along the inclined plane?



The box is moving in the +ve direction of x axes

The acceleration component along the inclined plain is required

→ We need to find F_{net} in the x direction

$$F_{net,x} = ma_x$$

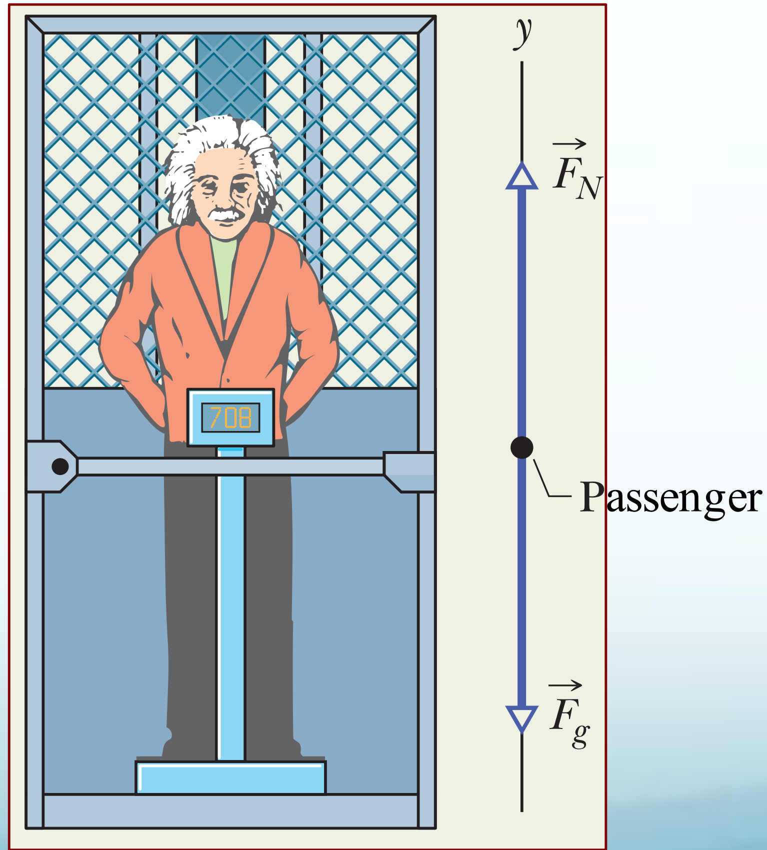
$$\Rightarrow T - mg\sin\theta = ma$$

$$a = \frac{T}{m} - g\sin\theta$$

$$= \frac{25}{5}(9.8)\sin 30 = 0.1 m/s^2$$

Sample Problem

In Fig. 5-17*a*, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.



(a) Find a general solution for the scale reading, whatever the vertical motion of the cab.

$$F_{net,y} = ma_y \Rightarrow F_N - F_g = ma \Rightarrow F_N = mg + ma$$
$$\Rightarrow F_N = m(a+g)$$

(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

When the cab is stationary or moving with a constant speed, $a = 0$

$$F_N = m(a+g) \rightarrow F_N = mg = (72.2)(9.8) = 708N$$

(c) What does the scale read if the cab accelerates upward at 3.20 m/s^2 and downward at 3.20 m/s^2?

Accelerates upward $F_N = m(a+g) \rightarrow F_N = 72.2(3.2+9.8) = 939N$

Accelerates downward $F_N = m(-a+g) \rightarrow F_N = 72.2(-3.2+9.8) = 476.52N$

(d) During the upward acceleration in part (c), what is the magnitude F_{net} of the net force on the passenger, and what is the magnitude $a_{p,cab}$ of his acceleration as measured in the frame of the cab? Does $\vec{F}_{net} = m\vec{a}_{p,cab}$?

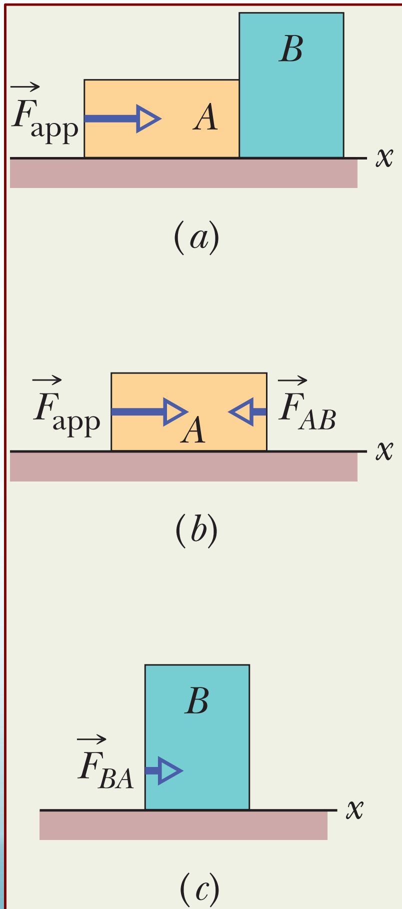
$$F_{net} = F_N - F_g$$

$$F_{net} = 939 - (72.2)(9.8)$$

$$F_{net} = 231N$$

Sample Problem

In Fig. 5-18*a*, a constant horizontal force $\vec{F}_{app}$ of magnitude 20 N is applied to block A of mass $m_A = 4.0$ kg, which pushes against block B of mass $m_B = 6.0$ kg. The blocks slide over a frictionless surface, along an x axis.



This force causes the acceleration of the full two-block system.

These are the two forces acting on just block *A*. Their net force causes its acceleration.

This is the only force causing the acceleration of block *B*.

(a) What is the acceleration of the blocks?

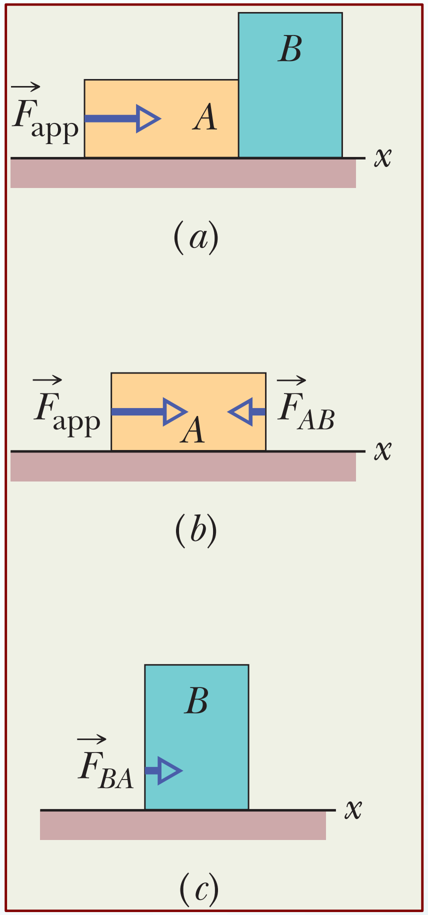
$$F_{app} = 20N, \quad m_A = 4Kg, \quad m_B = 6Kg$$

$$F_{net,x} = (m_A + m_B)a$$

$$F_{app} + F_{BA} - F_{AB} = (m_A + m_B)a$$

$$20 + 0 = (4 + 6)a$$

$$a = 2m/s^2$$

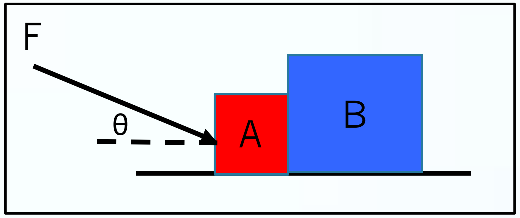


(b) What is the (horizontal) force $\vec{F}_{BA}$ on block B from block A (Fig. 5-18c)?

$$F_{BA} = m_B a = 6 \times 2 = 12N$$

Examples:

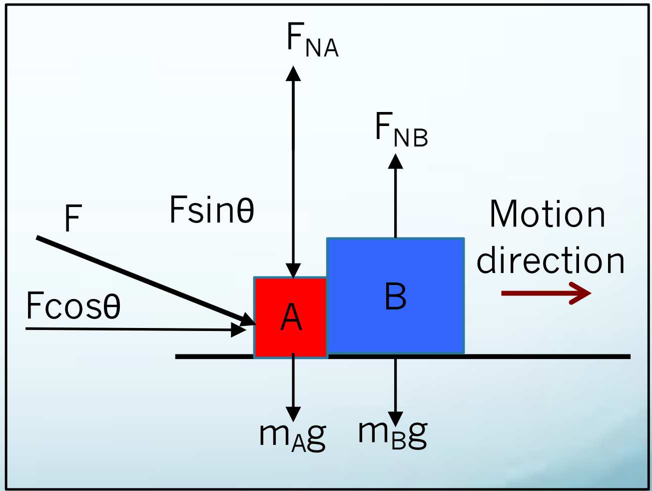
Q.1: A constant force of 46N is applied at an angle of 60^o to a block A of a mass 10kg as shown in the figure. Block A pushes another block B of mass 36kg. Assuming a frictionless surface, the total acceleration of the blocks along the x-axes is:



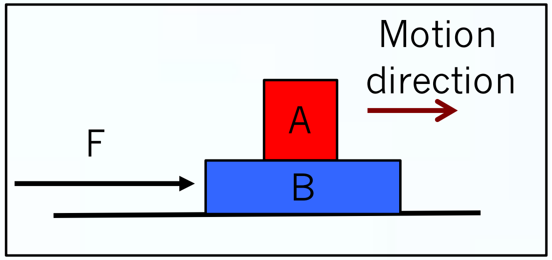
(a) $1.5 m/s^2$ (b) $0.25 m/s^2$ (c) $0.5 m/s^2$ (d) $2 m/s^2$

$$F_{net,x} = (m_A + m_B)a$$

$$a = \frac{F\cos\theta}{m_A + m_B} = \frac{46\cos 60^o}{10+36} = 0.5 m/s^2$$



Q.2: A 3kg box is placed in the top of a 10kg box. The bottom box is pushed with a force F. The two boxes moves together with an acceleration of 2m/s². The horizontal force F is:



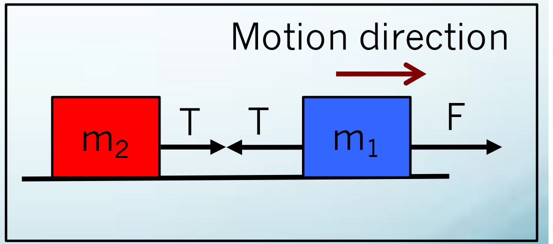
(a) 3N (b) 26N (c) 1N (d) 5N

$$F_{net,x} = (m_A + m_B)a = (10+3)2 = 26N$$

Q.3 In the figure, two blocks are connected and pulled on a horizontal table by a force with a magnitude of 20N. If the m_1 =3kg, m_2 = 2kg, then *T* and *a* are:

(a) 5N, 4m/s² (b) 8N, 4m/s² (c) 5N, 4m/s² (d) 10N, 3m/s²

$$F_{net,x} = (m_1 + m_2)a \quad \Rightarrow a = \frac{F}{m_1 + m_2} = \frac{20}{5} = 4m/s^2$$



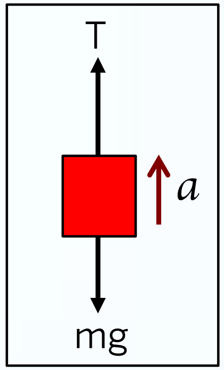
To find *T* we apply Newton's 2nd law on one of the mass:

$$F - T = m_1 a \Rightarrow T = F - m_1 a = 20 - 12 = 8N$$

$$\text{or} \quad T = m_2 a = 2(4) = 8N$$

Q.4: An elevator of total mass 2000kg moves upward. The tension in the cable pulling it is 24000N. The acceleration of the elevator is:

(a) 2.2m/s² **(b) 9.8m/s²** **(c) 12m/s²** **(d) 3.6m/s²**

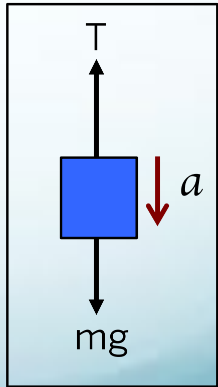


$$F_{net,y} = ma$$

$$T - mg = ma \quad \Rightarrow a = \frac{T - mg}{m} = \frac{24000 - 2000(9.8)}{2000} = 2.2 m/s^2$$

Q.5: A 70kg man stands on a spring scale in an elevator that has a downward acceleration of 2.8m/s². The scale will read:

(a) 980N **(b) 680N** **(c) 490N** **(d) 343N**

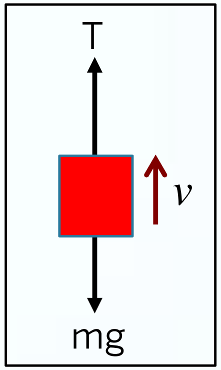


$$F_{net,y} = ma$$

$$T - mg = -ma \quad \Rightarrow T = m(g - a) = 70(9.8 - 2.8) = 490N$$

Q.6: An elevator has a body of 10kg. The tension in the cable when the elevator is moving upward at a constant speed of 10m/s is:

(a) zero (b) 98N (c) 1.5N (d) 7.3N

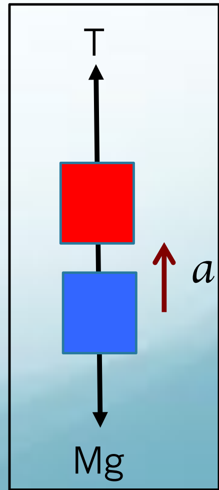


$$F_{net,y} = ma$$

$$T - mg = 0 \quad \Rightarrow T = ma = 10(9.8) = 98N$$

Q.7: Two masses m_1 = 4kg and m_2 = 6kg are connected to a rope of a negligible mass. An upward force of 198N is applied as shown. The magnitude of the acceleration of the system is:

(a) 10m/s² (b) 40.2m/s² (c) 50.2m/s² (d) 70.2/s²



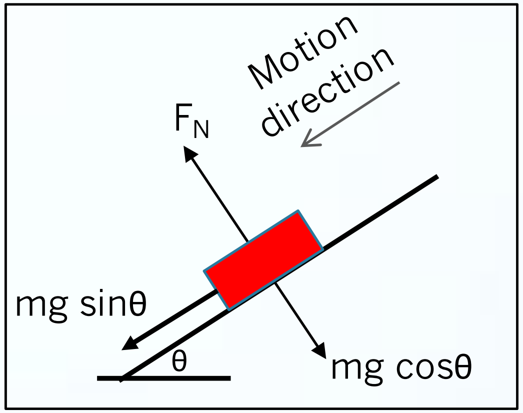
$$F_{net,y} = Ma$$

$$T - (m_1 + m_2)g = (m_1 + m_2)a \quad \Rightarrow a = \frac{T - (m_1 + m_2)g}{m_1 + m_2}$$

$$= \frac{198 - (10)9.8}{10} = 10m/s^2$$

Q.8: A block slides down a frictionless inclined plane with an acceleration of 4.9m/s². The angle between the plane and the horizontal is:

(a) 30º (b) 26º (c) 21.55º (d)14.32º



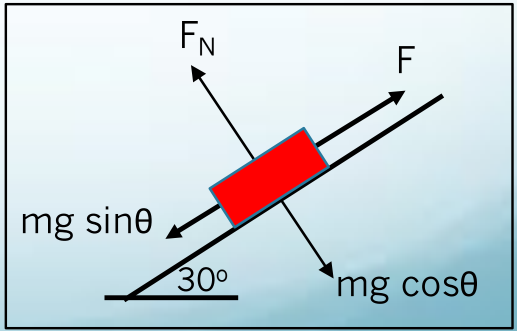
$$F_{net,x} = ma_x$$

$$\Rightarrow -mg\sin\theta = -ma \quad \Rightarrow a = g\sin\theta$$

$$\Rightarrow \theta = \sin^{-1}\frac{a}{g} = 30^o$$

Q.9: A 40kg crate is held at rest on a frictionless incline by a force parallel to the incline. If the incline is 30º above the horizontal, the magnitude of the applied force is:

(a) 20N (b) 40N (c) 23.5N (d) 10N

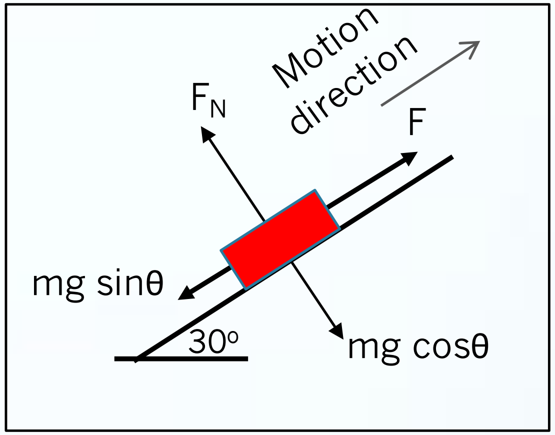


$F_{net,x} = ma_x$ at rest → a = 0

$$\Rightarrow F - mg\sin\theta = 0 \quad \Rightarrow F = mg\sin\theta = 40(9.8)\sin 30^o = 20N$$

Q.10: A block of mass 4kg is pushed up a smooth 30° inclined plane by a constant force of magnitude 40N and parallel to the incline. The magnitude of the acceleration of the block is:

(a) zero (b) 9.8m/s² (c) 1.2m/s² (d) <u>5.1m/s²</u>

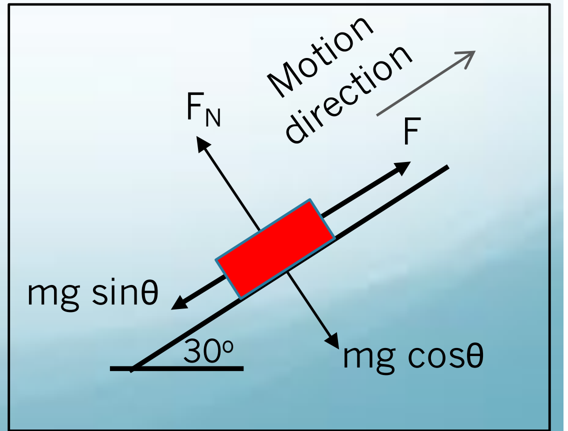


$$F_{net,x} = ma_x \Rightarrow F - mg\sin\theta = ma$$

$$\Rightarrow a = \frac{F - mg\sin\theta}{m} = \frac{40 - 4(9.8)\sin 30^o}{4} = 5.1 m/s^2$$

Q.11: If the mass of the block is 5kg. Find T if the block moves with a constant velocity upward the smooth inclined plane (or at rest).

(a) 45N (b) <u>24.5N</u> (c) 42N (d) 25N

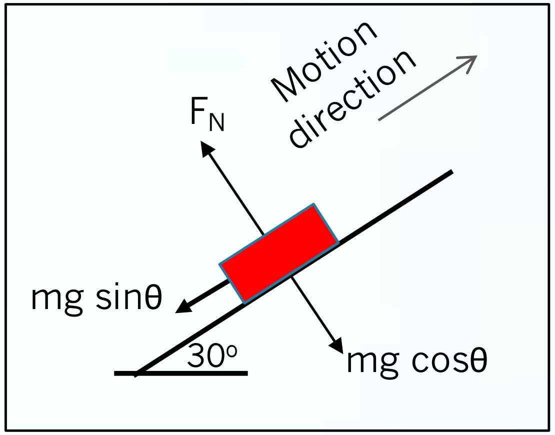


$$F_{net,x} = ma_x \Rightarrow T - mg\sin\theta = 0$$

$$\Rightarrow T = mg\sin\theta = 5(9.8)\sin 30^o = 24.5N$$

Q.12: A 5kg block is pushed upward 30º inclined plane with initial velocity of 14m/s. The distance that the block goes is:

(a) 20m (b) 10m (c) 18m(d) 24m



$$v^2 = v_0^2 + 2a(x - x_0)$$

$$F_{net,x} = ma_x \Rightarrow ma = -mg\sin\theta$$

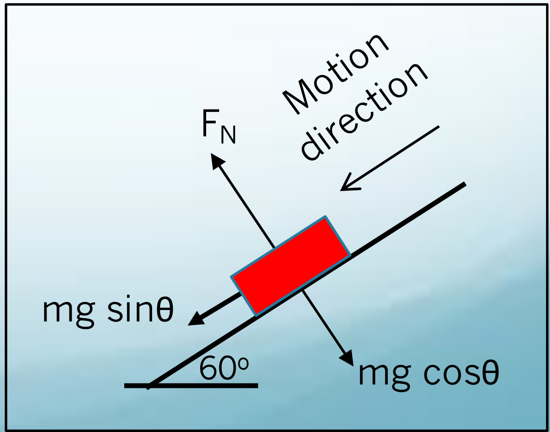
$$\Rightarrow a = -g\sin\theta = -4.4 m/s^2$$

$$v_o = 14 m/s, \quad v = 0$$

$$\Delta x = -\frac{v_o^2}{a} = \frac{14^2}{4.4} = 20m$$

Q.13: From the figure, the normal force F_N on a block of weight 60N sliding down a frictionless plane is:

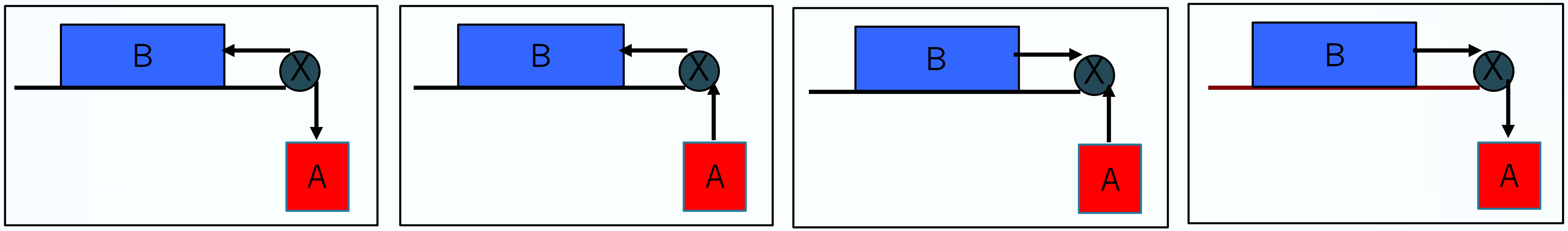
(a) 50N (b) 30N (c) 25N (d) 40N



$$F_{net,y} = ma_y \Rightarrow F_N - mg\cos\theta = 0$$

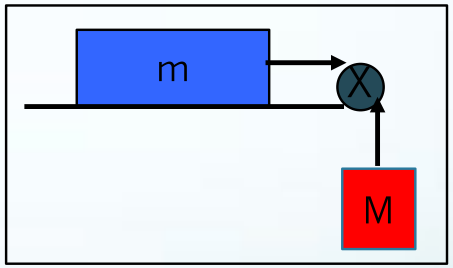
$$\Rightarrow F_N = mg\cos\theta = 60\cos 60^o = 30N$$

Q.14: Show the correct direction of the tension force T:

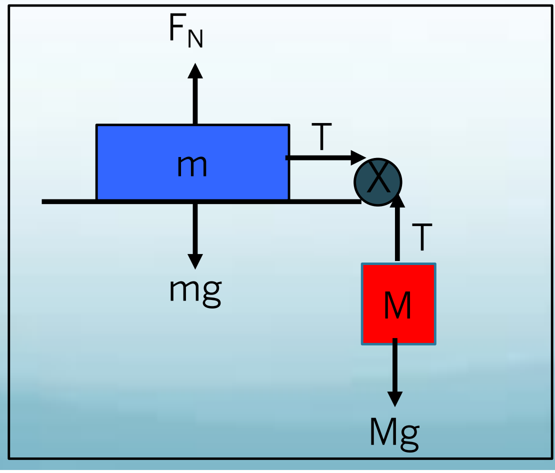


(a) (b) (c) (d)

Q.15: A block of mass m is connected to a block of mass M as shown. The normal force on the block m is:



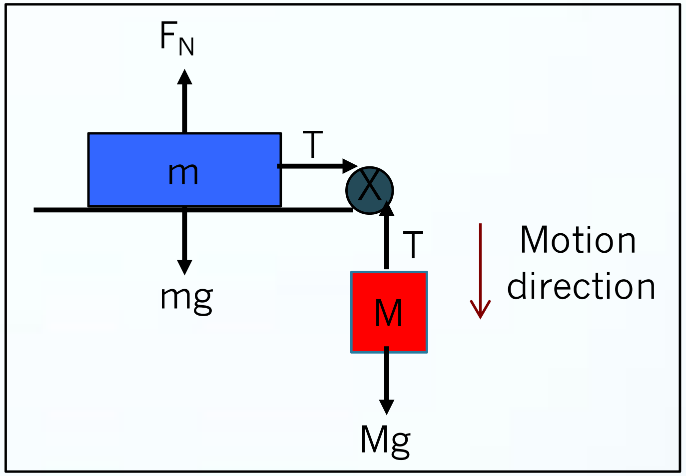
(a) mg – T (b) mg (c) Mg – T (d) Mg



Q.16: Referring to the last example, if the block M is moving downward, the net force acting on it is:

(a) Ma – T = Mg (b) T = Ma (c)T = Mg (d) T – Mg = – Ma

$$F_{net,y} = Ma \quad \Rightarrow T - Mg = -Ma$$



Problems:

•2 Two horizontal forces act on a 2.0 kg chopping block that can slide over a frictionless kitchen counter, which lies in an xy plane. One force is $\vec{F}_1 = (3.0\text{ N})\hat{\text{i}} + (4.0\text{ N})\hat{\text{j}}$. Find the acceleration of the chopping block in unit-vector notation when the other force is (a) $\vec{F}_2 = (-3.0\text{ N})\hat{\text{i}} + (-4.0\text{ N})\hat{\text{j}}$, (b) $\vec{F}_2 = (-3.0\text{ N})\hat{\text{i}} + (4.0\text{ N})\hat{\text{j}}$, and (c) $\vec{F}_2 = (3.0\text{ N})\hat{\text{i}} + (-4.0\text{ N})\hat{\text{j}}$.

•3 If the 1 kg standard body has an acceleration of 2.00 m/s^2 at 20.0° to the positive direction of an x axis, what are (a) the x component and (b) the y component of the net force acting on the body, and (c) what is the net force in unit-vector notation?

••4 While two forces act on it, a particle is to move at the constant velocity $\vec{v} = (3\text{ m/s})\hat{\text{i}} - (4\text{ m/s})\hat{\text{j}}$. One of the forces is $\vec{F}_1 = (2\text{ N})\hat{\text{i}} + (-6\text{ N})\hat{\text{j}}$. What is the other force?

••53 In Fig. 5-48, three connected blocks are pulled to the right on a horizontal frictionless table by a force of magnitude $T_3 = 65.0$ N. If $m_1 = 12.0$ kg, $m_2 = 24.0$ kg, and $m_3 = 31.0$ kg, calculate (a) the magnitude of the system's acceleration, (b) the tension T_1, and (c) the tension T_2.

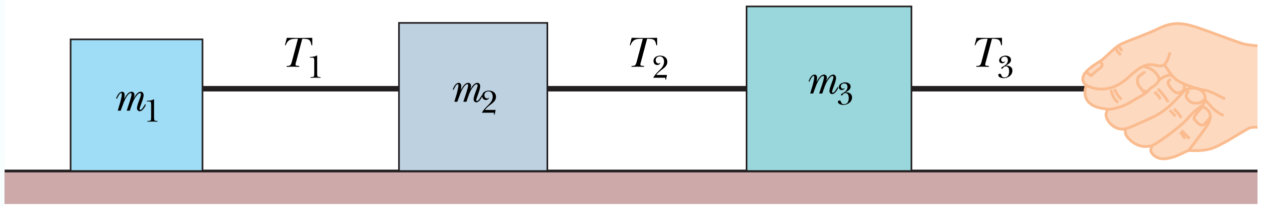


Fig. 5-48 Problem 53.

•2 Two horizontal forces act on a 2.0 kg chopping block that can slide over a frictionless kitchen counter, which lies in an xy plane. One force is $\vec{F}_1 = (3.0\text{ N})\hat{\text{i}} + (4.0\text{ N})\hat{\text{j}}$. Find the acceleration of the chopping block in unit-vector notation when the other force is (a) $\vec{F}_2 = (-3.0\text{ N})\hat{\text{i}} + (-4.0\text{ N})\hat{\text{j}}$, (b) $\vec{F}_2 = (-3.0\text{ N})\hat{\text{i}} + (4.0\text{ N})\hat{\text{j}}$, and (c) $\vec{F}_2 = (3.0\text{ N})\hat{\text{i}} + (-4.0\text{ N})\hat{\text{j}}$.

2. We apply Newton's second law (Eq. 5-1 or, equivalently, Eq. 5-2). The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = \left(\vec{F}_1 + \vec{F}_2\right)/m$.

(a) In the first case

$$\vec{F}_1 + \vec{F}_2 = \left[(3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right] + \left[(-3.0\text{N})\hat{\text{i}} + (-4.0\text{N})\hat{\text{j}}\right] = 0$$

so $\vec{a} = 0$.

(b) In the second case, the acceleration $\vec{a}$ equals

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right) + \left((-3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right)}{2.0\text{kg}} = (4.0\text{m/s}^2)\hat{\text{j}}.$$

(c) In this final situation, $\vec{a}$ is

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right) + \left((3.0\text{N})\hat{\text{i}} + (-4.0\text{N})\hat{\text{j}}\right)}{2.0\text{ kg}} = (3.0\text{m/s}^2)\hat{\text{i}}.$$

•3 If the 1 kg standard body has an acceleration of 2.00 m/s^2 at 20.0° to the positive direction of an x axis, what are (a) the x component and (b) the y component of the net force acting on the body, and (c) what is the net force in unit-vector notation?

3. We apply Newton's second law (specifically, Eq. 5-2).

(a) We find the x component of the force is

$$F_x = ma_x = ma\cos 20.0° = (1.00\,\text{kg})\left(2.00\,\text{m/s}^2\right)\cos 20.0° = 1.88\,\text{N}.$$

(b) The y component of the force is

$$F_y = ma_y = ma\sin 20.0° = (1.0\,\text{kg})\left(2.00\,\text{m/s}^2\right)\sin 20.0° = 0.684\,\text{N}$$

(c) In unit-vector notation, the force vector is

$$\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} = (1.88\text{ N})\hat{\text{i}} + (0.684\text{ N})\hat{\text{j}}\,.$$

••4 While two forces act on it, a particle is to move at the constant velocity $\vec{v} = (3\text{ m/s})\hat{\text{i}} - (4\text{ m/s})\hat{\text{j}}$. One of the forces is $\vec{F}_1 = (2\text{ N})\hat{\text{i}} + (-6\text{ N})\hat{\text{j}}$. What is the other force?

4. Since $\vec{v}$ = constant, we have $\vec{a} = 0$, which implies

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = m\vec{a} = 0 .$$

Thus, the other force must be

$$\vec{F}_2 = -\vec{F}_1 = (-2\text{ N})\,\hat{\text{i}} + (6\text{ N})\,\hat{\text{j}} .$$

••**53** In Fig. 5-48, three connected blocks are pulled to the right on a horizontal frictionless table by a force of magnitude $T_3 = 65.0$ N. If $m_1 = 12.0$ kg, $m_2 = 24.0$ kg, and $m_3 = 31.0$ kg, calculate (a) the magnitude of the system's acceleration, (b) the tension T_1, and (c) the tension T_2.

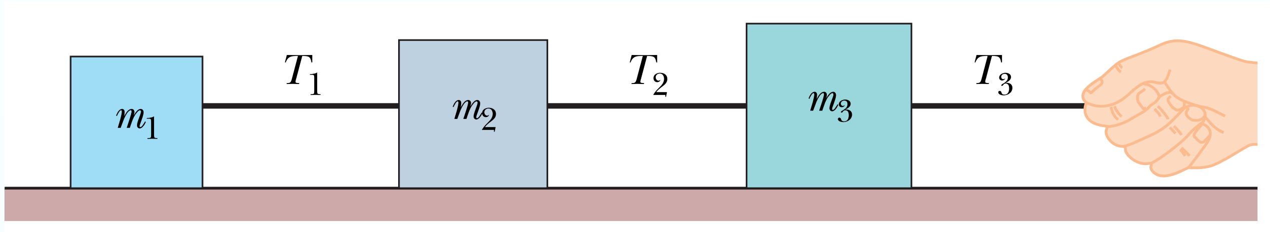


Fig. 5-48 Problem 53.

53. We apply Newton's second law first to the three blocks as a single system and then to the individual blocks. The $+x$ direction is to the right in Fig. 5-48.

(a) With $m_{sys} = m_1 + m_2 + m_3 = 67.0$ kg, we apply Eq. 5-2 to the x motion of the system, in which case, there is only one force $\vec{T}_3 = +T_3\ \hat{\mathrm{i}}$. Therefore,

$$T_3 = m_{\text{sys}} a \quad \Rightarrow \quad 65.0\,\text{N} = (67.0\,\text{kg})a$$

which yields $a = 0.970$ m/s^2 for the system (and for each of the blocks individually).

(b) Applying Eq. 5-2 to block 1, we find

$$T_1 = m_1 a = (12.0\,\text{kg})\left(0.970\,\text{m/s}^2\right) = 11.6\,\text{N}.$$

(c) In order to find T_2, we can either analyze the forces on block 3 or we can treat blocks 1 and 2 as a system and examine its forces. We choose the latter.

$$T_2 = \left(m_1 + m_2\right)a = (12.0\text{ kg} + 24.0\text{ kg})\left(0.970\text{ m/s}^2\right) = 34.9\text{ N}.$$

Physics 110
1435-1436 H

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Chapter 6
FORCE AND MOTION -II

Sections 6-2, 6-3

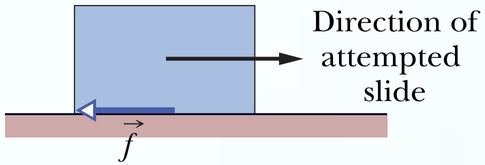
Friction

Properties of Friction

- Important skills from this lecture:
  1. Identify the friction force and its cause
  2. Identify the static friction force and the kinetic one
  3. Calculate the value of both kinds of friction force
  4. Applying Newton’s laws to solve problems including friction force

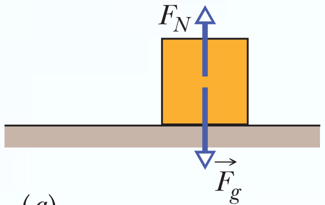
Friction

- **Frictional force:** a force that opposes motion, it is caused by rough surfaces of **all** materials
  - It is unavoidable force
  - If it was not counteracted
    → it would stop every moving & rotating objects
  - About 20% of the gasoline used in an automobile is needed to counteract friction in the engine



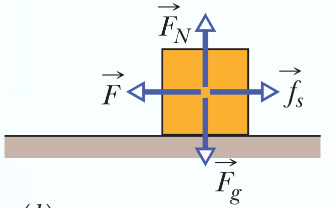
- If frictions were absent
  →we could not walk, hold a pencil, and, if we could, it would not write

- This chapter deals with the frictional forces that exist between dry solid surfaces, either stationary or moving



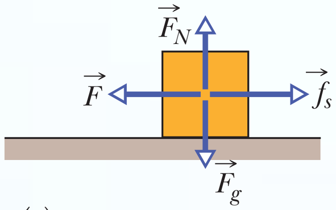
(a)

There is no attempt at sliding
→ no friction & no motion
$f_s = 0$



(b)

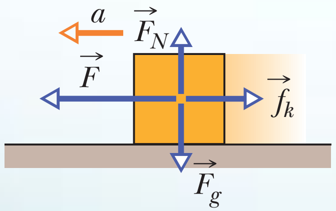
Force F attempts sliding but is balanced by the frictional force
→ no motion $F = f_s$



(c)

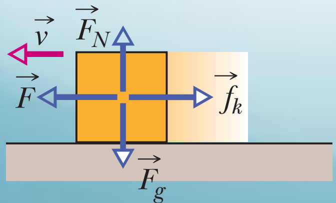
Force F is increased, but F is still balanced by the frictional
→ no motion $F = f_s$

The block is stationary →
$F = f_s$
f_s :static frictional force



(e)

Force F has overwhelmed the static frictional force
→block slides & accelerates



(f)

To maintain the speed, force F is weakened to match the weak f_k

The block is moving →
$f_k < F$
f_k: kinetic frictional force

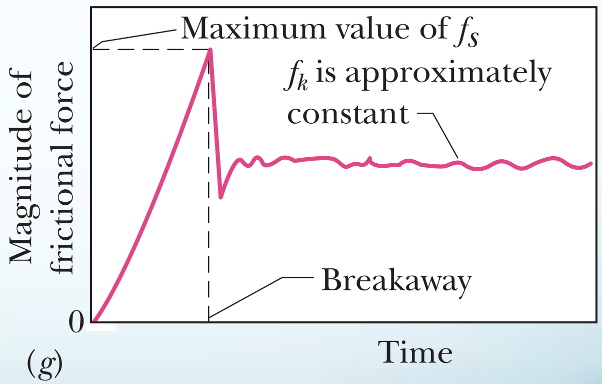
Kinds of Friction force

- **Static frictional force** $\vec{f}_s$: friction force between a **stationary** object and the surface
  - Its magnitude **increases** with increasing the applied force until it reaches a maximum

- **kinetic frictional force** $\vec{f}_k$ friction force between a **moving** object and the surface
  - Its magnitude is **constant**

- It is always true that $f_k < f_s$

- To maintain a speed of a block moving across a surface, the magnitude of the applied force has to be decreased once the block begins to move



Experimental results for the block situation from (a) to (f)

Properties of Friction

If a body presses against a surface and a force attempts to slide it
→ the frictional force has three properties:

1. If the body does not move
   → f_s and F component that is parallel to the surface balance each other (equal in magnitude and opposite in direction)
2. The magnitude of f_s has a maximum value $f_{s,\max}$:

   $$\boxed{f_{s,\max} = \mu_s F_N}$$ (not a vector equation)

   μ_s is the coefficient of static friction
   F_N is the magnitude of the normal force on the body from the surface
3. If the body begins to slide along the surface ($F > f_{s,\max}$)
   → the magnitude of the frictional force decreases to a value f_k :

   $$\boxed{f_k = \mu_k F_N}$$ (not a vector equation)

   μ_k is the coefficient of kinetic friction

Properties of Friction

- The strength of friction depends on:
  - How hard surfaces push together
  - Types of surfaces involved
- The direction of $\vec{f}_s$ or $\vec{f}_k$ is always **parallel** to the surface and **opposed** to the attempted sliding
- The coefficients μ_s & μ_k are **dimensionless** and must be determined experimentally
- The values of μ_s & μ_k depend on the properties of both the body & the surface
- It is assumed that the value of μ_k does not depend on the speed at which the body slides along the surface

CHECKPOINT 1

A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it? (c) If the maximum value $f_{s,\max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N? (d) If it is 12 N? (e) What is the magnitude of the frictional force in part (c)?

$$(a)\vec{f}_s = 0$$

$$(b)\vec{f}_s = -5N \Rightarrow f_s = 5N$$

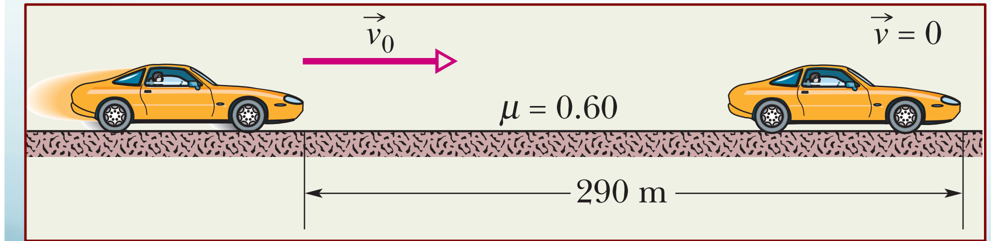
$$(c)\text{No, it will not move}$$

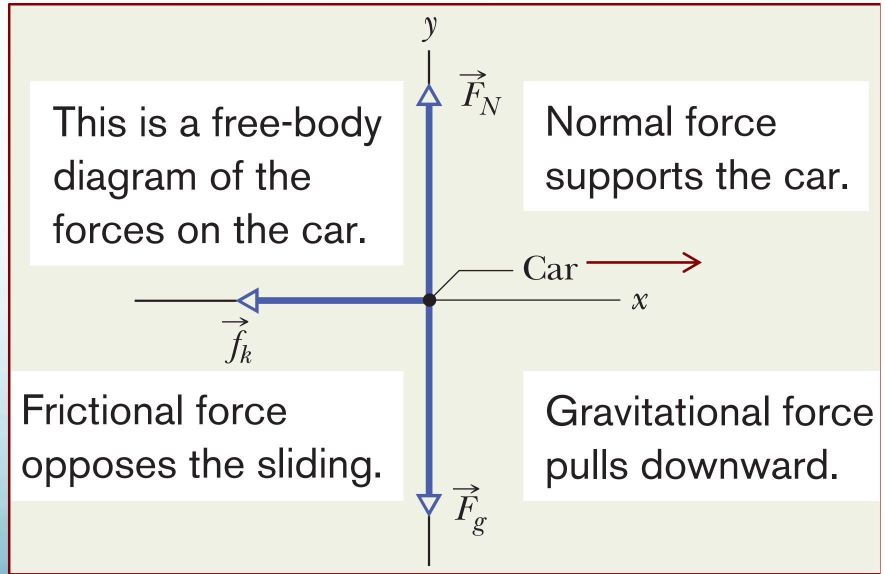
$$(d)\text{Yes, it will move}$$

$$(e)\vec{f}_s = -8N \Rightarrow f_s = 8N$$

Sample Problem

If a car's wheels are "locked" (kept from rolling) during emergency braking, the car slides along the road. Ripped-off bits of tire and small melted sections of road form the "skid marks" that reveal that cold-welding occurred during the slide. The record for the longest skid marks on a public road was reportedly set in 1960 by a Jaguar on the M1 highway in England (Fig. 6-3*a*)—the marks were 290 m long! Assuming that $\mu_k = 0.60$ and the car's acceleration was constant during the braking, how fast was the car going when the wheels became locked?





- The acceleration is constant → we can apply equations of motion

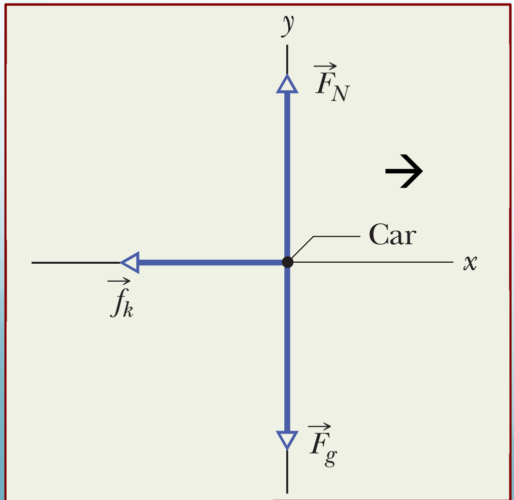
$$v^2 = v_0^2 + 2a(x - x_0)$$

- Because the car is moving along x-axes, there is no acceleration component along y-axes

- To find a_x:

$$F_{\text{net},x} = ma_x \rightarrow -f_k = ma \rightarrow a = -\frac{f_k}{m}$$

$$f_k = \mu_k F_N \qquad F_N = mg$$



$$\rightarrow \quad a = -\frac{f_k}{m} = -\frac{\mu_k mg}{m} = -\mu_k g$$

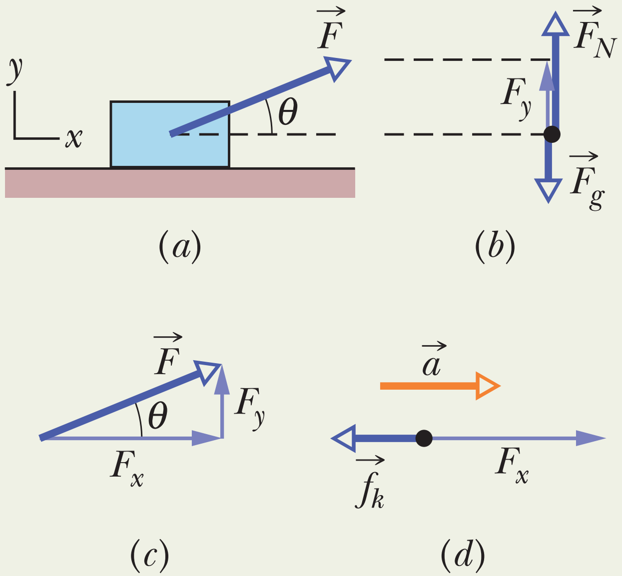
$$\rightarrow \quad \begin{aligned} v_0 &= \sqrt{2\mu_k g(x - x_0)} \\ &= \sqrt{(2)(0.60)(9.8\ \text{m/s}^2)(290\ \text{m})} \\ &= 58\ \text{m/s} = 210\ \text{km/h}. \end{aligned}$$

Sample Problem

In Fig. 6-4*a*, a block of mass $m = 3.0$ kg slides along a floor while a force $\vec{F}$ of magnitude 12.0 N is applied to it at an upward angle θ. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.40$. We can vary θ from 0 to 90° (the block remains on the floor). What θ gives the maximum value of the block's acceleration magnitude a?

This applied force accelerates block and helps support it.

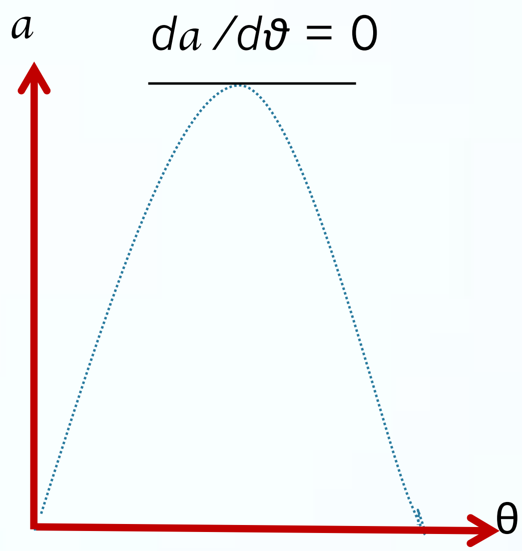
These vertical forces balance.



The applied force has these components.

These two horizontal forces determine the acceleration.

$$m = 3.0kg, \quad F = 12N, \quad \mu_k = 0.4, \quad \theta = ?$$



$$F_{net,x} = ma_x$$

$$F\cos\theta - f_k = ma_x$$

$$f_k = \mu_k F_N$$

$$F\cos\theta - \mu_k F_N = ma \quad (1)$$

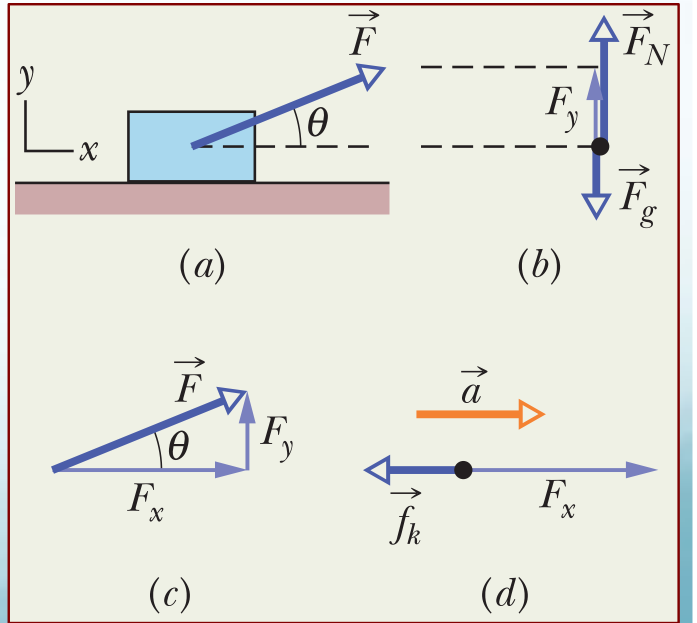
$$F_{net,y} = ma_y$$

$$F_N + F\sin\theta - F_g = ma_y$$

$$F_g = mg, \quad a_y = 0$$

$$F_N + F\sin\theta - mg = 0$$

$$F_N = mg - F\sin\theta \quad (2)$$



Substituting from 2 into 1 and solve for a:

$$F\cos\theta - \mu_k(mg - F\sin\theta) = ma$$

$$a = \frac{F}{m}\cos\theta - \mu_k g + \frac{F}{m}\mu_k \sin\theta$$

To find the value of θ that maximizes a, we take the derivative of a with respect to θ and set the result equal to zero

$$\frac{da}{d\theta} = -\frac{F}{m}\sin\theta - 0 + \frac{F}{m}\mu_k \cos\theta = 0$$

$$\Rightarrow \sin\theta = \mu_k \cos\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta = \mu_k$$

$$\Rightarrow \theta = \tan^{-1}\mu_k = \tan^{-1}(0.4) = 21.8^o$$

$$\boxed{\begin{aligned} \frac{d}{d\theta}\sin\theta &= \cos\theta \\ \frac{d}{d\theta}\cos\theta &= -\sin\theta \end{aligned}}$$

Chapter 6
FORCE AND MOTION -II

Section 6-5

Uniform Circular Motion

- Important skills from this lecture:
  1. Explain centripetal force and its direction
  2. Calculate the centripetal force

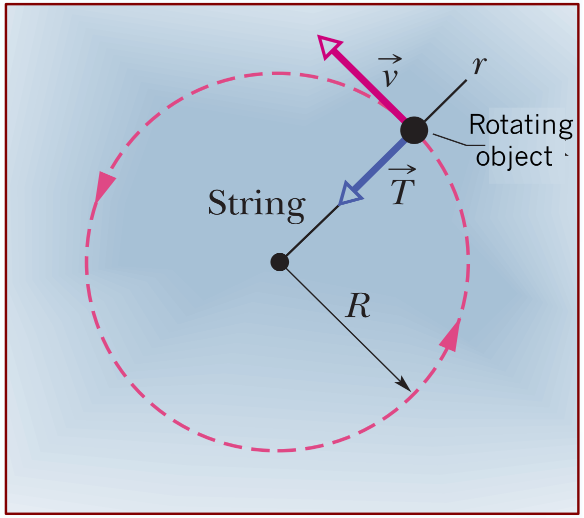
Uniform Circular Motion

- **Uniform circular motion:** motion of a body that moves in a circle (or a circular arc) at constant speed v
- The centripetal acceleration (has a constant magnitude given by (R is the radius of the circle):

$$\boxed{a = \frac{v^2}{R}}$$

  - a is directed toward the center of the circle
- **Centripetal force:** a force that causes the centripetal acceleration, and is directed toward the center of the circle

- Example for a centripetal force:
  An object is rotating around a circle at constant speed v while tied to a string looped around a central stone
  - The centripetal force is the radially inward pull on the object from the string
  - Without that force, the object would slide off in a straight line instead of moving in a circle



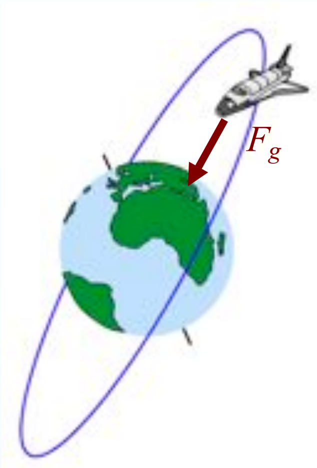
A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

- The magnitude of centripetal force:

$$\boxed{F = m\frac{v^2}{R}}$$

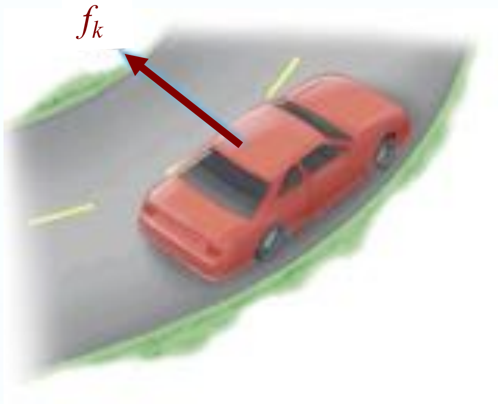
- Because v is constant → the magnitudes of the centripetal a & F are constant
  The directions of the centripetal a & F are not constant
  (always directed toward the center of curvature of the particle's path)

A centripetal force could be a gravitational force, a frictional force, a tension in a string and the force from a car wall or any other force



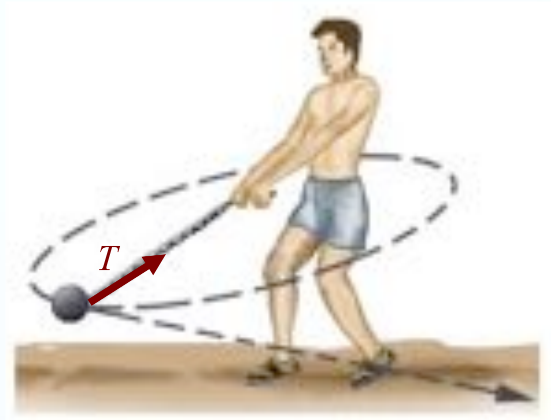
An object
is orbiting Earth

Gravitational force is a
centripetal
force



A car is moving
in a rounded road

Friction force is
a centripetal force

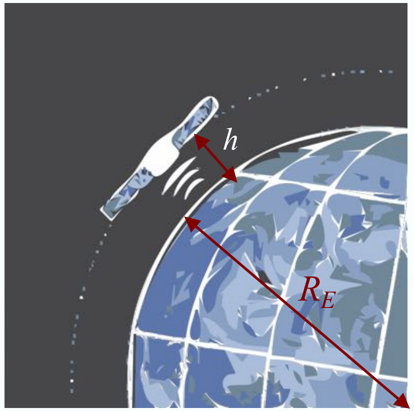


A man is rotating a
stone tied to a string

Tension force is
a centripetal force

Sample Problem

Igor is a cosmonaut on International Space Station, in a circular orbit around Earth, at an altitude *h* of 520 km and with a constant speed *v* of 7.6 km/s. Igor's mass m is 79 kg.
(a) What is his acceleration. (b) What force does Earth exert on Igor?



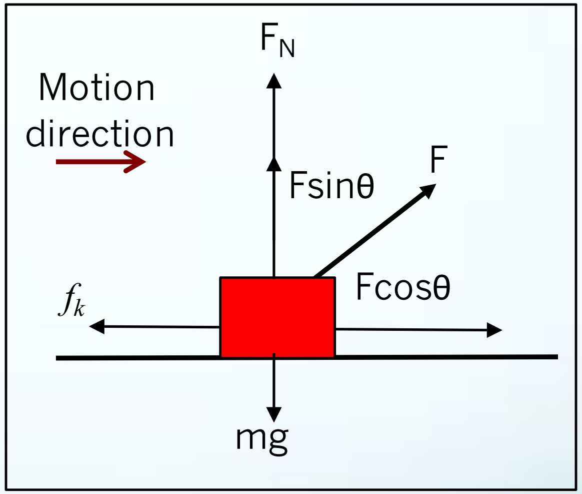
(a) $$a = \frac{v^2}{R} = \frac{v^2}{R_E + h} = \frac{\left(7.6\times10^3\right)^2}{6.37\times10^6 + 0.52\times10^6} = 8.38 m/s^2$$

(b) $$F = ma = (79)(8.38) = 662 N$$

Examples:

Q.1: A 3.5kg block is pulled at a constant velocity along a horizontal floor by a force F = 15N that makes an angle of 40° with the horizontal. The coefficient of kinetic friction is:

(a) 0.34 (b) zero (c) 0.47 (d) 0.1



Constant speed → $a = 0$

$$F_{net,y} = ma_y \Rightarrow F_N + F\sin\theta - mg = 0 \Rightarrow F_N = mg - F\sin\theta$$

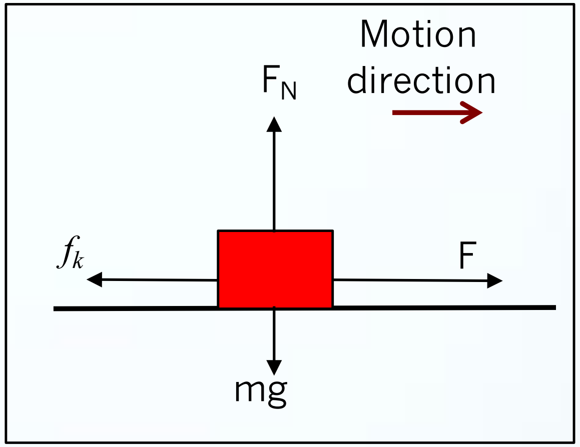
$$F_{net,x} = ma_x \Rightarrow F\cos\theta - f_k = 0 \Rightarrow f_k = F\cos\theta$$

$$f_k = \mu_k F_N$$

$$\mu_k = \frac{F\cos\theta}{mg - F\sin\theta} = \frac{15\cos 40^o}{(3.5\times 9.8 - 15\sin 40^o)} = 0.47$$

Q.2: A block of weight 5N moves with a constant speed by a force of 2N. The value of the coefficient of friction is:

(a) 0.3 **(b) 0.4** **(c) 0.5** **(d) 0.6**



Constant speed → $a = 0$

$$F_{net,y} = ma_y \Rightarrow F_N - mg = 0 \Rightarrow F_N = mg$$

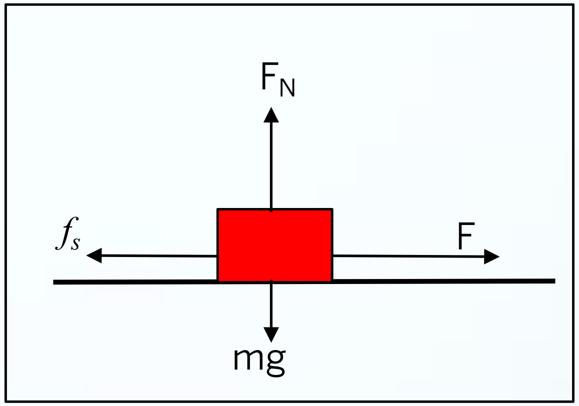
$$F_{net,x} = ma_x \Rightarrow F - f_k = 0 \Rightarrow f_k = F$$

$$f_k = \mu_k F_N$$

$$\mu_k = \frac{f_k}{F_N} = \frac{f_k}{mg} = \frac{2}{5} = 0.4$$

Q.3: The coefficient of static friction between a 5kg block and the horizontal surface is 0.1. The maximum horizontal force that can be applied to the block just before starting to move is:

(a) 19.6N (b) 24.5N (c) 4.9N (d) 9.8N



$$F_{net,y} = ma$$

$$\Rightarrow F_N - mg = 0$$

$$F_{net,x} = ma = 0$$

$$F - f_s = 0$$

$$\Rightarrow F = f_s = \mu_s F_N = 0.1 \times 5 \times 9.8 = 4.9N$$

•7 **SSM** **ILW** A person pushes horizontally with a force of 220 N on a 55 kg crate to move it across a level floor. The coefficient of kinetic friction is 0.35. What is the magnitude of (a) the frictional force and (b) the crate's acceleration?

•44 During an Olympic bobsled run, the Jamaican team makes a turn of radius 7.6 m at a speed of 96.6 km/h. What is their acceleration in terms of g?

•7 SSM ILW A person pushes horizontally with a force of 220 N on a 55 kg crate to move it across a level floor. The coefficient of kinetic friction is 0.35. What is the magnitude of (a) the frictional force and (b) the crate's acceleration?

$$f_k = \mu_k F_N .$$

Applying Newton's second law to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

(a) The second equation above yields the normal force $F_N = mg$, so that the friction is

$$f_k = \mu_k F_N = \mu_k mg = (0.35)(55\text{ kg})(9.8\text{ m/s}^2) = 1.9 \times 10^2\text{ N} .$$

(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with $F = 220$ N) we solve to find

$$a = \frac{F}{m} - \mu_k g = 0.56\text{ m/s}^2 .$$

•**44** During an Olympic bobsled run, the Jamaican team makes a turn of radius 7.6 m at a speed of 96.6 km/h. What is their acceleration in terms of g?

44. With v = 96.6 km/h = 26.8 m/s, Eq. 6-17 readily yields

$$a = \frac{v^2}{R} = \frac{(26.8\ \text{m/s})^2}{7.6\ \text{m}} = 94.7\ \text{m/s}^2$$

which we express as a multiple of g:

$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) g = 9.7g.$$

Physics 110
1435-1436 H

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Chapter 7
KINETIC ENERGY AND WORK

Sections 7-2, 7-3, 7-4, 7-5

What is Energy?
Kinetic Energy
Work
Work and Kinetic Energy

- Important skills from this lecture:
  1. Describe the kinetic energy and its relation with velocity
  2. Calculate kinetic energy
  3. Define the unit of kinetic energy
  4. Define work and its unit
  5. Evaluate the work done by a constant force
  6. Calculate the net work done by several forces
  7. Identify the work-kinetic energy theorem

What Is Energy?

- Energy is a scalar quantity associated with the state (or condition) of one or more objects
- Energy is the ability to make things change
- A system that has energy has the ability to do work
- Living organisms need energy for growing & moving
- **Some forms of energy:**
  - Thermal Energy
  - Electrical Energy
  - Chemical Energy
  - Nuclear Energy
  - Mechanical Energy (kinetic energy & potential energy)

- **Principle of energy conservation:**
  Energy can be transformed from one type to another, and transferred from one object to another, but the total amount is always the same (energy is conserved)
- Properties of Energy
  - Scalar quantity
  - Conserved
  - Transferred
  - Measured with Joule
- In this chapter we focus only on one type of energy (kinetic energy) and one way of transferring energy (work)

Kinetic Energy

- **Kinetic energy (K or K.E):** the energy associated with the state of motion of an object
  - The faster the object moves, the greater its K.E
  - When the object is stationary, K.E = 0
- For an object of mass m, and speed *v* (v < speed of light)

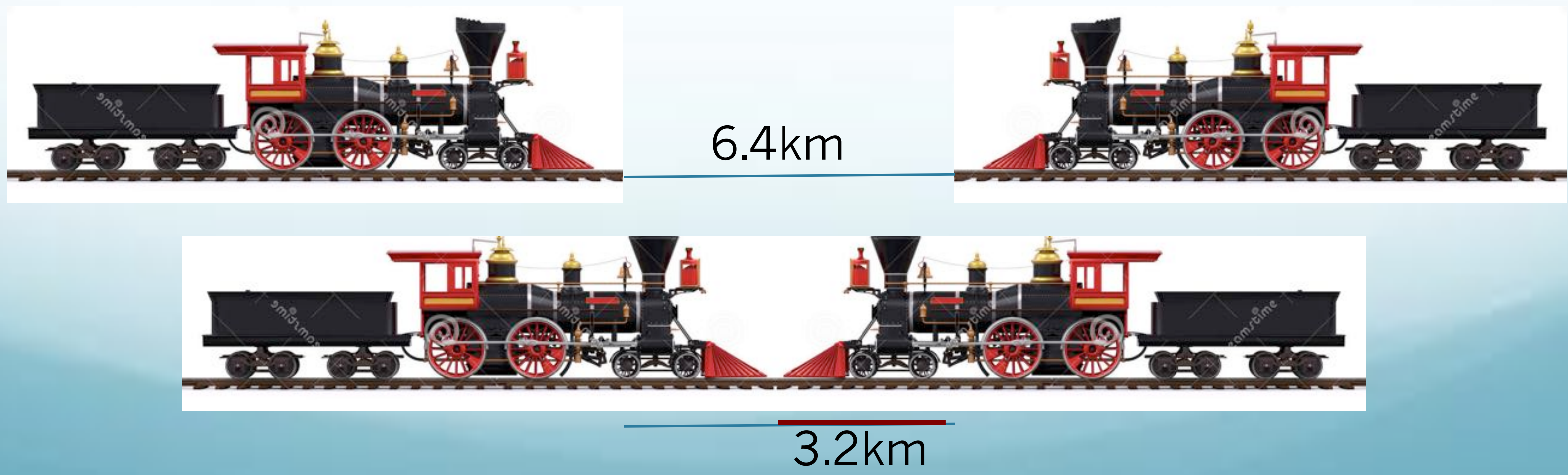
$$\boxed{K = \tfrac{1}{2}mv^2 \qquad \text{(kinetic energy)}}$$

- Energy unit in SI is joule (J)

$$\boxed{1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2}$$

Sample Problem

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7-1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant $0.26\ \text{m/s}^2$, what was the total kinetic energy of the two locomotives just before the collision?



$$K.E = 2\left(\frac{1}{2}mv^2\right)$$

$$v^2 = v_o^2 + 2a(x - x_0)$$

$$x - x_0 = 6.4 \div 2 = 3.2km$$

$$v^2 = 0 + 2(0.26m/s^2)(3.2\times10^3) = 40.8m/s$$

$$m = \frac{1.2\times10^6 N}{9.8m/s^2} = 1.22\times10^5 kg$$

$$K.E = 2\left(\frac{1}{2}mv^2\right) = (1.22\times10^5 kg)(40.8m/s)^2 = 2\times10^8 J$$

Work

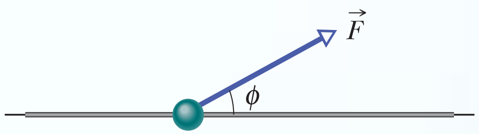
- If an object is accelerated → ↑ K.E
  - Energy is transferred _to_ the object
- If an object is decelerated → ↓ K.E
  - Energy is transferred _from_ the object
- Energy transfer means that there is a work (W) done on the object

> Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

- "Work": the transferred energy
  "doing work": the act of transferring the energy
- Work has the same units as energy
  It is a scalar quantity

Work & Kinetic Energy

- A bead of mass m slides along a frictionless wire along a horizontal *x* axis. A constant force $\vec{F}$, directed at an angle Φ to the wire, accelerates the bead along the wire



- Applying Newton's 2nd law along the x axis: $F_x = ma_x$ (1)
- The bead's velocity changes from $\vec{v}_0$ to $\vec{v}$ as the bead moves through a displacement $\vec{d}$.
- Because F is constant → a is constant → we can use the equation of motion:

$$v^2 = v_0^2 + 2a_x d \quad (2)$$

- Sub 1 into 2 gives:

$$\boxed{\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d}$$

  - The 1st term on the left side gives the final kinetic energy K_f
  - The 2nd term on the left gives the initial kinetic energy K_i
  - → the left side gives the change ΔK.E by the force
  - The right side of the equation shows that the change is equal to $F_x d$
  - The work W done on the bead by the force: $\boxed{W = F_x d}$

$$\boxed{W = F_x d}$$

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

$$F_x = F \cos \phi$$

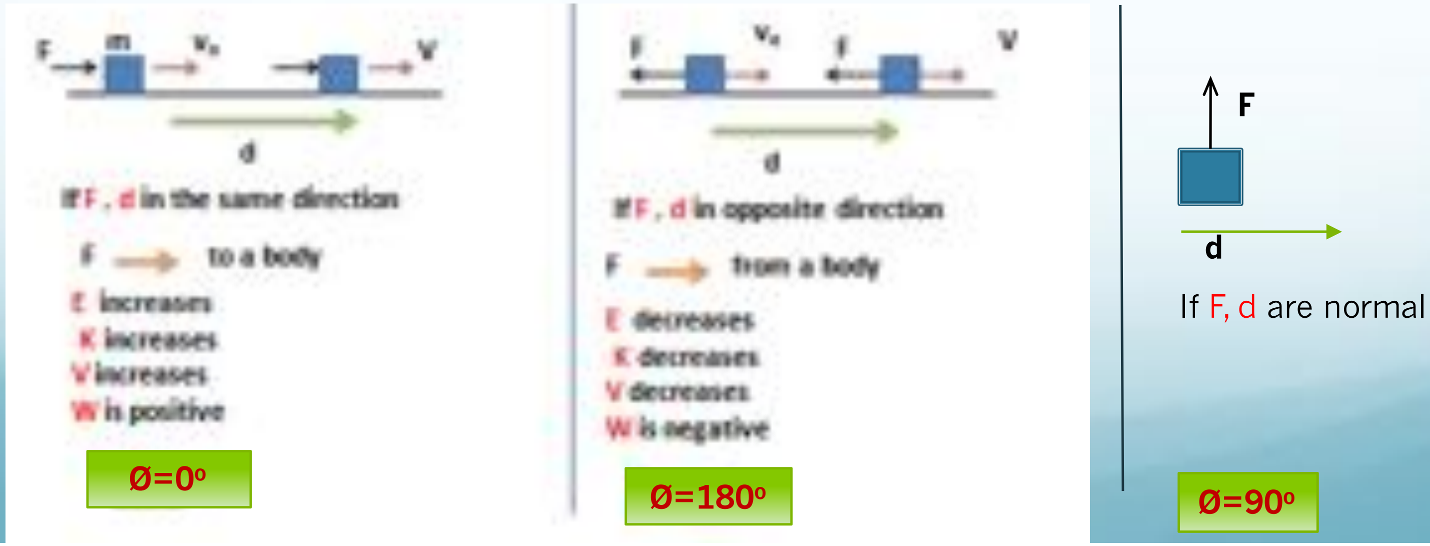
$$\boxed{W = Fd \cos \phi \qquad \text{(work done by a constant force)}} \quad (3)$$

$$\boxed{W = \vec{F} \cdot \vec{d} \qquad \text{(work done by a constant force)}} \quad (4)$$

- For using the above equations to calculate work done on an object by a force
  - First, the force must be a constant force (no change in magnitude or direction as the object moves)
  - Second, the object must be rigid (all its parts must move together, in the same direction)

- **Signs for work:**
  To find the sign of the work done by a force, consider the force vector component that is parallel to the displacement (Eq. 3)
  - If $0 < \Phi < 90°$ → $\cos \Phi$ is +ve → W is + ve
  - If $90° < \Phi < 180°$ → $\cos \Phi$ is – ve → W is – ve
  - If $\Phi = 90^{o}$ → $W=0$

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.



- **Units for work:**
  - Work has the SI unit of the joule,
  - From Eq.3, an equivalent unit for joule is the newton.meter (N.m)
  - The corresponding unit in the British system is the foot.pound (ft.lb).

$$\boxed{1\ \text{J} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 1\ \text{N}\cdot\text{m} = 0.738\ \text{ft}\cdot\text{lb}.}$$

- **Net work done by several forces:**

  When two or more forces act on an object, $W_{net} = \Sigma W$

  W_{net} is calculated in two ways:

  1. Find the work done by each force and then sum those works
  2. Find F_{net} of forces. Then use Eq. 3, substituting the magnitude F_{net} for F and also the angle between the directions of F_{net} and d for Φ

Work–Kinetic Energy Theorem

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d$$

$$\Delta K = K_f - K_i = W,$$

$$\boxed{\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}}$$

$$K_f = K_i + W,$$

$$\boxed{\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}}$$

- These statements are known as the work–kinetic energy theorem for particles
  - These statements are true for both +ve & –ve work
  - If W_{net} is +ve → ↑ K.E of the particle's by the work
  - If W_{net} is –ve → ↓ K.E of the particle's by the work
  - e.g., if k_i = 5 J, and there is a net transfer of 2 J to the particle → (+ve W_{net})→ k_f = 7 J.
    If there is a net transfer of 2 J from the particle → (–ve W_{net})→ k_f = 3 J

CHECKPOINT 1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from −3 m/s to −2 m/s and (b) from −2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

(a) $K_i = \frac{1}{2} m V_i^2 = \frac{1}{2} m(-3)^2 = 4.5m$

$K_f = \frac{1}{2} m V_f^2 = \frac{1}{2} m(2)^2 = 2m$

$K_i > K_f$

(c) $W = K_f - K_i = 2 - 4.5 = -2.5$

(b) $K_i = \frac{1}{2} m V_i^2 = \frac{1}{2} m(-2)^2 = 2m$

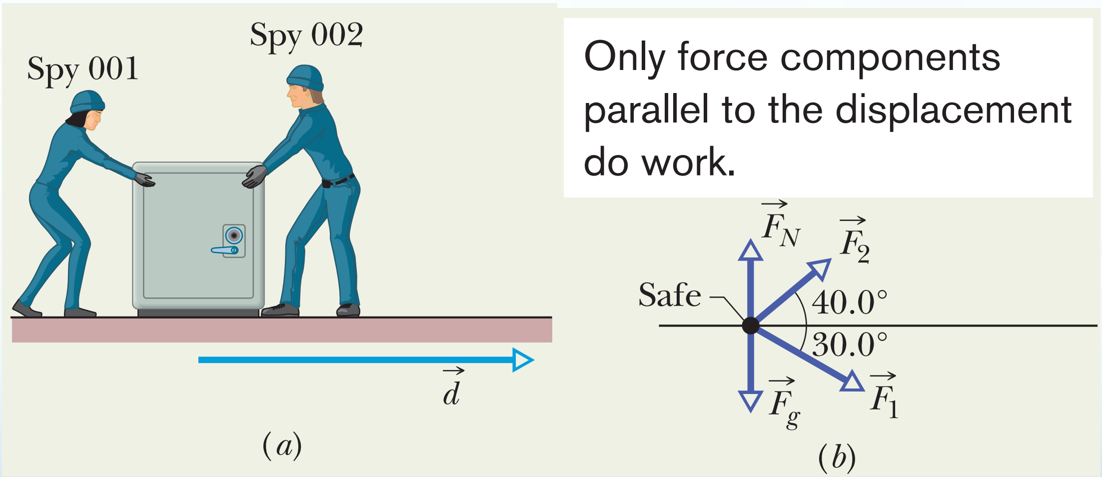
$K_f = \frac{1}{2} m V_f^2 = \frac{1}{2} m(2)^2 = 2m$

$\Rightarrow K_i = K_f$

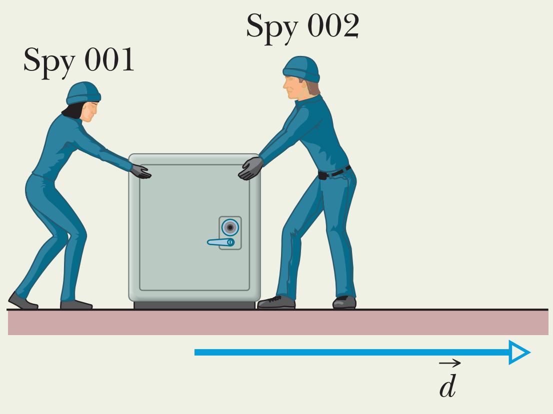
(c) $W = K_f - K_i = 2 - 2 = 0$

Sample Problem

Figure 7-4*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?



(a)

the work done by $\vec{F}_1$ is

$$W_1 = F_1 d \cos \phi_1 = (12.0\text{ N})(8.50\text{ m})(\cos 30.0°)$$
$$= 88.33\text{ J},$$

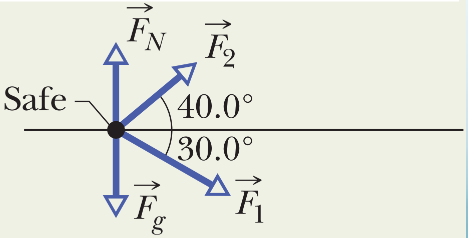
and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d \cos \phi_2 = (10.0\text{ N})(8.50\text{ m})(\cos 40.0°)$$
$$= 65.11\text{ J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33\text{ J} + 65.11\text{ J}$$
$$= 153.4\text{ J} \approx 153\text{ J}. \qquad \text{(Answer)}$$

Only force components parallel to the displacement do work.



(b)

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

$$W_g = mgd\cos 90^\circ = mgd(0) = 0 \qquad \text{(Answer)}$$

and

$$W_N = F_N d\cos 90^\circ = F_N d(0) = 0. \qquad \text{(Answer)}$$

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\text{ J})}{225\text{ kg}}}$$

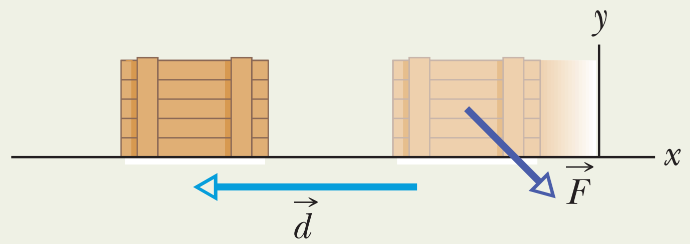
$$= 1.17\text{ m/s}. \qquad \text{(Answer)}$$

Sample Problem

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\text{ m})\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

The parallel force component does *negative* work, slowing the crate.



$$W = \vec{F}\cdot\vec{d} = [(2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}]\cdot[(-3.0\text{ m})\hat{i}].$$

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{i}\cdot\hat{i} + (-6.0\text{ N})(-3.0\text{ m})\hat{j}\cdot\hat{i}$$

$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \qquad \text{(Answer)}$$

The force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}.$$

Examples:

Q.1: 5kg block moves with a speed of 72km/h. Its kinetic energy is:
(a) 900kg.m²/s² (b) 1000kg.m²/s² (c) 1200kg.m²/s² (d) 50kg.m²/s²

$$v = 72km/h = \frac{72 \times 10^3}{3600} = 20m/s$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(5)(20)^2 = 1000J$$

Q.2: A 5kg block moves with velocity v=6i + 8j m/s. Its kinetic energy is:
(a) 250J (b) 400J (c) 540J (d) 180J

$$v = \sqrt{6^2 + 8^2} = 10m/s$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(5)(10)^2 = 250J$$

Q.3: 1 joule is equal to:
(a) kg.m²/s (b) kg.m/s³ (c) kg.m/s² (d) kg.m²/s²

Q.4: A particle moves 5m in the positive x-direction while being acted upon by a constant force F = 2i + 2j. The work done on the particle by this force is:

(a) 20J (b) 10J (c) 30J (d) – 15J

$$W = \vec{F} \cdot \vec{d} = (2\hat{i} + 2\hat{j}) \cdot 5\hat{i} = 10J$$

Q.5: A force acts on a 3kg particle in such away that the position of the object is x = 3t – 4t^2 + t^3, where x in meters and t in second. Find the work done on the object by the force from t = 0 to t = 4s

(a) 528J (b) 10J (c) 50J (d) 180J

$$v(t) = \frac{dx}{dt} = 3 - 8t + 3t^2$$

$$v(0) = 3m/s, \quad v(4) = 3 - 8(4) + 3(16) = 19m/s$$

$$W = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{3}{2}(361 - 9) = 528J$$

Q.6: Force F acts on a particle of mass m making a displacement D. If F = 7i + 3j – 1.5k N, and D = 2i – 3j + 2.5k m. The work done by the force is:

(a) 9.25J (b) 7.25J (c) 5.25J (d) 1.25J

$$W = \vec{F} \cdot \vec{d} = (7\hat{i} + 3\hat{j} - 1.5\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 2.5\hat{k}) = 14 - 9 - 3.75 = 1.25J$$

Q.7: A 5kg cart is moving horizontally at 6m/s. In order to change its speed to 10m/s, the net work done on the cart must be:

(a) 40J (b) 90J (c) 160J (d) 400J

$$v_1 = 6m/s, \quad v_2 = 10m/s$$

$$W = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{5}{2}(100 - 36) = 160J$$

Chapter 7
KINETIC ENERGY AND WORK

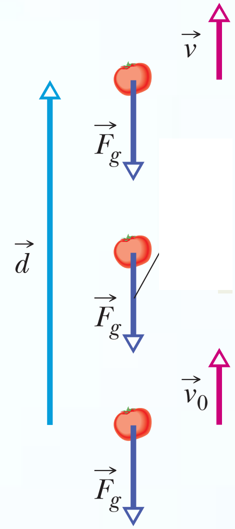
Sections 7-6, 7-7, 7-9

Work Done by the Gravitational Force
The Work Done by a Spring Force
Power

- Important skills from this lecture:
  1. Calculate the amount of work done by gravitational force in both raising and falling object
  2. Define the spring force and its relationship with displacement of the spring
  3. Calculate the spring force from Hook's law
  4. Define the power and its unit
  5. Calculate the average and instantaneous power
  6. Calculate the power in terms of force exerted on a body and its velocity

Work Done by the Gravitational Force

If tomato of mass m is thrown upward with initial speed v_o



- Its speed is slowed to v by the gravitational force F_g
  → $\vec{F}_g$ does work on the tomato
  → The tomato's K.E decreases from $K_i\,(=\frac{1}{2}mv_0^2)$ to $K_f\,(=\frac{1}{2}mv^2)$.

- The gravitational work is:

$$\boxed{W_g = mgd\cos\phi \qquad \text{(work done by gravitational force).}}$$

- For a rising object, the direction of $\vec{F}_g$ is opposite to the displacement
  →Φ = 180° →

$$W_g = mgd\cos 180° = mgd(-1) = -mgd.$$

  - The – sign means: $\vec{F}_g$ transfers energy in the amount mgd *from* the object
    → –ve work → slowing of the object as it rises

- For a falling object, $\vec{F}_g$ is in the same direction of the displacement
  →Φ = 0° → $W_g = mgd\cos 0° = mgd(+1) = +mgd.$

  - The + sign means: $\vec{F}_g$ transfers energy in the amount mgd *to* the object
    → +ve work → speeding up of the object as it falls

Work Done by the Gravitational Force

- **Rising:**
  1. Object's kinetic energy decreases
  2. Object deceleration
  3. The gravitational force dose –ve work
  4. $\Phi = 180°$
  5. $W_g = mgd \cos 180° = mgd(-1) = -mgd.$

- **Falling:**
  1. Object kinetic energy increases
  2. Object acceleration
  3. The gravitational force dose positive work on it
  4. $\Phi = 0°$
  5. $W_g = mgd \cos 0° = mgd(+1) = +mgd.$

Sample Problem

One of the lifts of Paul Anderson in the 1950s remains a record: Anderson stooped beneath a reinforced wood platform, placed his hands on a short stool to brace himself, and then pushed upward on the platform with his back, lifting the platform straight up by 1 cm. The platform held automobile parts and safe filled with lead, with a total weight of 27900N.

(a) As Anderson lifted the load, how much work was done on it by the gravitational force F_g?

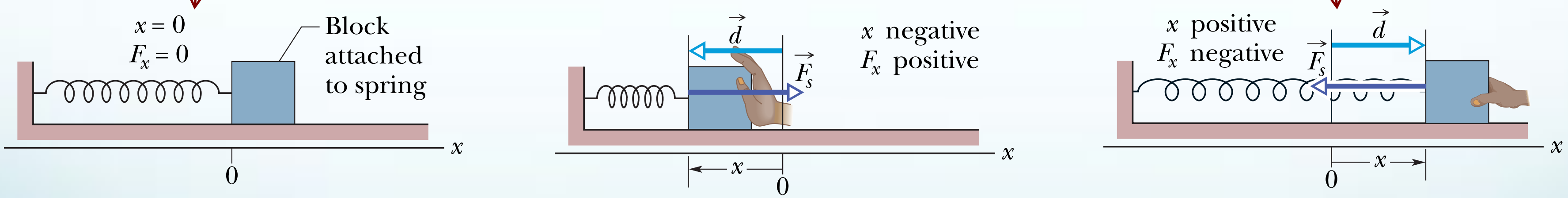
$$mg = 27900N, \quad d = 1cm = 0.01m, \quad \phi = 180$$

$$W_g = mgd\cos\phi = (27900)(0.01)\cos 180 = -289J$$

The Spring Force

- **A spring force:** is the force from a spring,
  - It is variable force
  - Many forces in nature have the same mathematical form as the spring force

- When an object (block) is attached to the spring free end, and a force acts on it → 3 states of a spring:



Relaxed	Compressed	stretched
(neither compressed nor extended) x = 0 is called **Equilibrium position**	By pushing the block to the left, the spring now pushes on the block toward the right	By pulling the block to the right, the spring pulls on the block to the left (**Restoring force**)

- The spring force $\vec{F}_s$ is proportional to the displacement $\vec{d}$ of the free end from its equilibrium position
- The spring force is given by $\boxed{\vec{F}_s = -k\vec{d} \qquad \text{(Hooke's law)}}$

$$\boxed{F_x = -kx \qquad \text{(Hooke's law)},}$$

- The –sign means: $\vec{F}_s$ is in the opposite direction of $\vec{d}$
- The constant k is called the **spring constant** (or **force constant**)
  - k measures the stiffness of the spring
  - The larger k is, the stiffer the spring
    → the stronger the spring's pull or push for a given
  - The SI unit for k is the N/m
  - If the spring is stretched toward the right
    → x is +ve → F_x is –ve (it is a pull toward the left)
  - If the spring is compressed toward the left
    → x is –ve → F_x is +ve (it is a push toward the right)
  - A spring force is a variable force because it is a function of x,
    → F_x is wrote as $F(x)$
  - Hooke's law is a linear relationship between F_x and x

The Work Done by a Spring Force

- To find the work done by the spring force we assume:
  - The spring is massless
  - The spring is an ideal spring (obeys Hooke's law)
  - The contact between the block and the floor is frictionless

- It is not possible to find the work by using $W = Fd \cos \Phi$ **?**
  Because F_x is not constant (variable force)

- **To find the work done by the spring, we use calculus:**
  1. If the block's initial position is x_i & later position is x_f
  2. The distance between those two positions is Divided into many segments of tiny length Δx
  3. The force magnitude (F_{x1} in the 1st segment, F_{x2} in the 2ed segment, and so on) is constant within each segment
  4. → the work done within each segment is found using the relation $W = Fd \cos \Phi$, Φ = 180° → cos Φ = -1

5. The work done is $-F_{x1}\,\Delta x$ in segment 1, $-F_{x2}\,\Delta x$ in segment 2, and so on

6. The net work W_s done by the spring, from x_i to x_f, is the sum of all these works:

$$W_s = \sum -F_{xj}\,\Delta x,$$

7. When $\Delta x \rightarrow 0 \rightarrow$ $W_s = \int_{x_i}^{x_f} -F_x\,dx.$

$$W_s = \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx$$

$$= (-\tfrac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\tfrac{1}{2}k)(x_f^2 - x_i^2).$$

$$\boxed{W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force).}}$$

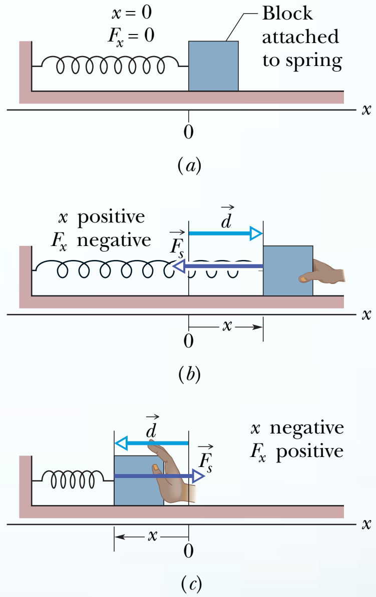
Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

8. If $x_i = 0$ and if we call the final position x,

$$\boxed{W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force).}}$$

CHECKPOINT 2

For three situations, the initial and final positions, respectively, along the x axis for the block in Fig. 7-9 are (a) -3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) -2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?



(a)
$$W = \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2$$
$$W = \frac{1}{2}K\left[(-3)^2 - (2)^2\right]$$
$$W = \frac{1}{2}K[9-4]$$
$$W = +ve$$

(b)
$$W = \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2$$
$$W = \frac{1}{2}K\left[(2)^2 - (3)^2\right]$$
$$W = \frac{1}{2}K[4-9]$$
$$W = -ve$$

(c)
$$W = \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2$$
$$W = \frac{1}{2}K\left[-(2)^2 - (2)^2\right]$$
$$W = \frac{1}{2}K[4-4]$$
$$W = 0$$

Sample Problem

A package of spicy Cajun pralines lies on a frictionless floor, attached to the free end of a spring in the arrangement of Fig.7-9a. A rightward applied force of magnitude F_a = 4.9 N would be needed to hold the package at x_1= 12mm.

(a) How much work does the spring force do on the package if the package is pulled rightward from x_0 = 0 to x_2 = 17 mm?

$$F_S = -k\,x \Rightarrow k = -\frac{F_S}{x_1} = -\frac{-4.9\;N}{12\times 10^{-3}\;m} = 408\;N/m$$

$$W_s = -\tfrac{1}{2}kx_2^2 = -\tfrac{1}{2}(408\;N/m)(17\;x\,10^{-3}\;m)^2$$

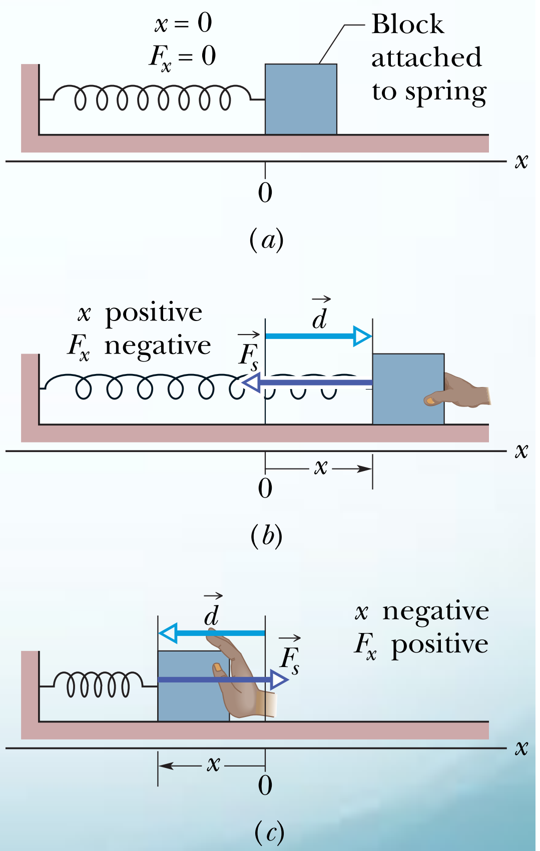
$$= -0.059\;J$$

(b) Next, the package is moved leftward to x_3 = –12 mm. How much work does the spring force do on the package during this displacement? Explain the sign of this work.

$$W_s = -\tfrac{1}{2}kx_f{}^2 + \tfrac{1}{2}kx_i{}^2$$

$$= \tfrac{1}{2}(408\;N/m)\left[-(-12\times 10^{-3}\;m)^2 + (17\times 10^{-3}\,m)^2\right]$$

$$= 0.030\;J = 30\;mJ$$



Power

- **Power (*P*):** the time rate at which work is done by a force
  - If a force does a work W in a time interval Δt
    → the average power due to the force during that time interval is:

$$\boxed{P_{\text{avg}} = \frac{W}{\Delta t} \qquad \text{(average power).}}$$

  - The instantaneous power *P* is:

$$\boxed{P = \frac{dW}{dt} \qquad \text{(instantaneous power).}}$$

  - The SI unit of power is: J/s, and called watt (W)

$$\boxed{1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}}$$

$$\boxed{1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W.}}$$

- **Work** can be expressed as power multiplied by time, as in the unit kilowatt-hour

$$\boxed{\begin{aligned} 1 \text{ kilowatt-hour} &= 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) \\ &= 3.60 \times 10^6 \text{ J} = 3.60 \text{ MJ.} \end{aligned}}$$

- For a particle moves along a straight line on an x axis, and a constant force F acted on it with an angle Φ

$$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right),$$

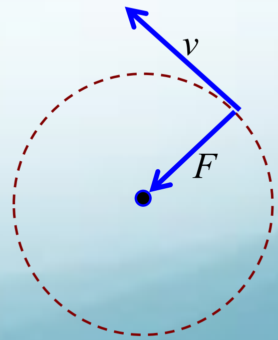
$$P = Fv\cos\phi.$$

$$\boxed{P = \vec{F}\cdot\vec{v} \qquad \text{(instantaneous power).}}$$

CHECKPOINT 3

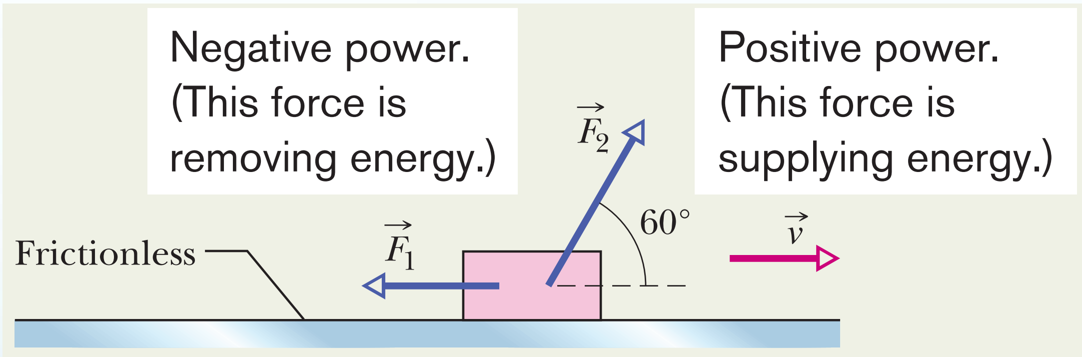
A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

$$\phi = 90 \quad \Rightarrow P = F.v = Fv\cos 90 = 0$$



Sample Problem

Figure 7-14 shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?



For force $\vec{F}_1$, at angle $\phi_1 = 180°$ to velocity $\vec{v}$, we have

$$P_1 = F_1 v \cos \phi_1 = (2.0 \text{ N})(3.0 \text{ m/s}) \cos 180°$$
$$= -6.0 \text{ W.} \qquad \text{(Answer)}$$

This negative result tells us that force $\vec{F}_1$ is transferring energy *from* the box at the rate of 6.0 J/s.

For force $\vec{F}_2$, at angle $\phi_2 = 60°$ to velocity $\vec{v}$, we have

$$P_2 = F_2 v \cos \phi_2 = (4.0 \text{ N})(3.0 \text{ m/s}) \cos 60°$$
$$= 6.0 \text{ W.} \qquad \text{(Answer)}$$

This positive result tells us that force $\vec{F}_2$ is transferring energy *to* the box at the rate of 6.0 J/s.

The net power is the sum of the individual powers:

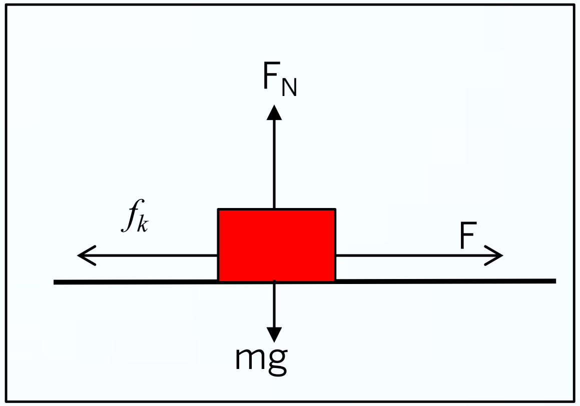
$$P_{\text{net}} = P_1 + P_2$$
$$= -6.0 \text{ W} + 6.0 \text{ W} = 0, \qquad \text{(Answer)}$$

which tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy ($K = \frac{1}{2}mv^2$) of the box is not changing, and so the speed of the box will remain at 3.0 m/s.

Examples:

Q.1: A horizontal force of 180N used to pull a 50kg box on a rough horizontal surface to the right with a distance of 8m. If the box moves at a constant speed, find:

(a) The work done by the horizontal force F



$$W = Fd\cos\phi = (180)(8)\cos 0 = 1440J$$

(b) The work done by the friction force

Constant speed → $a_x = 0$

$$F_{net,x} = ma_x = 0 \Rightarrow F - f_k = 0 \Rightarrow F = f_k = 180N$$

$$W = f_k d\cos 180 = 180(8)(-1) = -1440N$$

(b) The work done by the force of gravity

$$W_g = mgd\cos 90 = 0$$

(c) The work don by the normal force

$$W_N = mgd\cos 90 = 0$$

Q.2 A watt is equal to:

(a) $kg.m^2/s^2$ (b) $kg.m^2/s^3$ (c) $kg.m/s^3$ (d) $kg.m^3/s^2$

$$P = \frac{W}{t} = \frac{kg \cdot m^2 \cdot s^{-2}}{s} = \frac{kg \cdot m^2}{s^3}$$

Q.3: Horse-power (hp)=

(a) 1000W (b) 100W (c) 749W (d) 476W

Q.4: Which of the following group does not contain a scalar quantity?

(a) velocity, force, power

(b) displacement, acceleration, force

(c) acceleration, speed, work

(d) energy, work, distance

Q.5: An object of mass 1kg moves in a horizontal circle of radius 0.5m at a constant speed of 2m/s. The power done on the object during on revolution is:

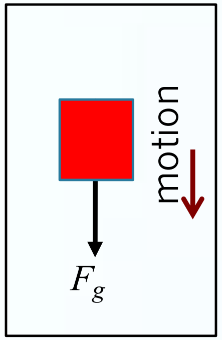
(a) 1J (b) 2J (c) 4J (d) zero

$$\phi = 90 \quad \Rightarrow P = F.v = Fv\cos 90 = 0$$

Q.6: A block of mass 0.5kg is dropped from a height of 45m above the ground. The work done by the gravitational force is:

(a) 5J (b) 40J (c) 10J (d) 220.5J

$$W_g = mgd\cos\phi = 0.5(9.8)(45)\cos 0 = 220J$$



Q.7: In the previous question, the average power during the time interval of 10s is:

(a) 20W (b) 22W (c) 10W (d) –5W

$$P = \frac{W}{\Delta t} = \frac{220}{10} = 22W$$

Q.8: A spring of k = 300N/m initially at x =0, and forced to move to x = 10cm. The work done by the spring force is:

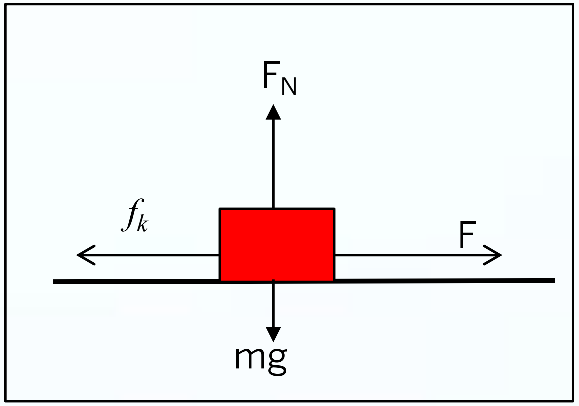
(a) –1.5 J (b) –5.5 J (c) –1 J (d) 1.5 J

$$W = -\frac{1}{2}kx^2 = -\frac{1}{2}(300)(0.1)^2 = -1.5J$$

Q.9: A mass of 100kg is pushed by a horizontal force across rough horizontal floor at a constant speed of 5m/s. If μ_k =0.2, at what rate is work being done by the horizontal force F?

(a) 50W **(b) 980W** **(c) 392W** **(d) 400W**

Constant speed → $a_x = 0$



$$F_{net,x} = ma_x = 0 \Rightarrow F - f_k = 0$$

$$\Rightarrow F = f_k = \mu_k F_N = \mu_k mg$$

$$\Rightarrow F = 0.2(100)(9.8) = 196N$$

$$P = Fv\cos\phi = 196(5)(1) = 980W$$

Problems:

••15 GO Figure 7-27 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0°$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?

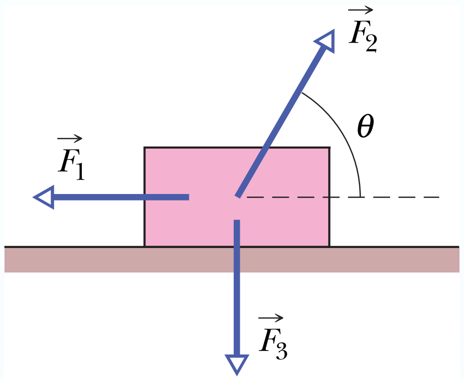
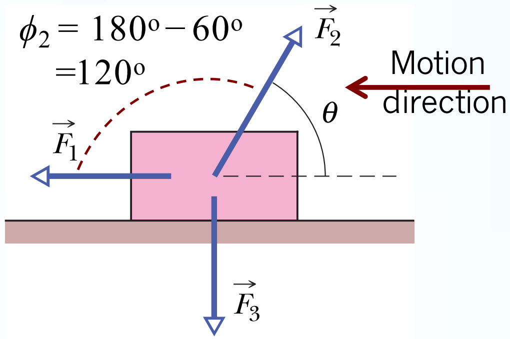


Fig. 7-27 Problem 15.

•45 SSM ILW A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which the force does work on the block?

Problems for chapter 7

••15 GO Figure 7-27 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0°$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?



15. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d \cos\phi_1 = (5.00\,\text{N})(3.00\,\text{m})\cos 0° = 15.0\ \text{J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d \cos\phi_2 = (9.00\,\text{N})(3.00\,\text{m})\cos 120° = -13.5\ \text{J}.$$

Force $\vec{F}_3$ is perpendicular to the displacement, so

$$W_3 = F_3 d \cos\phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\ \text{J} - 13.5\ \text{J} + 0 = +1.50\ \text{J}.$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.

•45 SSM ILW A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which the force does work on the block?

45. The power associated with force $\vec{F}$ is given by $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi = (122\ \text{N})(5.0\ \text{m/s})\cos 37° = 4.9 \times 10^2\ \text{W}.$$

Physics 110
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Chapter 9
CENTER OF MASS AND LINEAR MOMENTUM

Sections 9-2, 9-3, 9-4, 9-5, 9-7

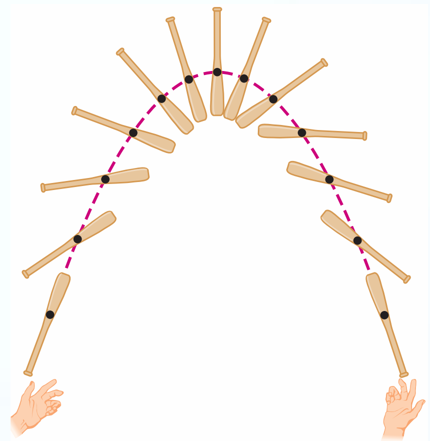
The Center of Mass
Newton's Second Law for a System of Particles
Linear Momentum
The Linear Momentum of a System of Particles
Conservation of Linear Momentum

- Important skills from this lecture:

1. Define the center of mass of a system of particles
2. Calculate the center of mass for two particle in one dimension
3. Calculate the center of mass for many particles in one dimension
4. Calculate the center of mass for many particles in two and three dimensions
5. Identify Newton's 2nd law for a system of particles
6. Apply Newton's 2nd law to a system of particles to calculate the acceleration of the center of mass
7. Define the linear momentum and its unit
8. Derive Newton's 2nd law in terms of momentum
9. Explain the conservation of linear momentum
10. Apply the conservation of momentum to solve problems

The Center of Mass

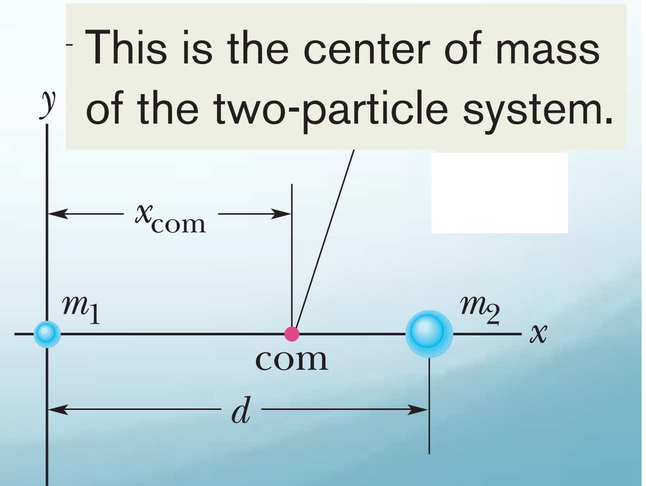
- **The center of mass (com) of a system of particles:** is the point that moves as though
  (1) all of the system's mass were concentrated there
  (2) all external forces were applied there



- For 2 particles of masses m_1 & m_2 separated by distance d, the *position* of the com of this system is (the origin of an x axis was chosen to be at m_1):

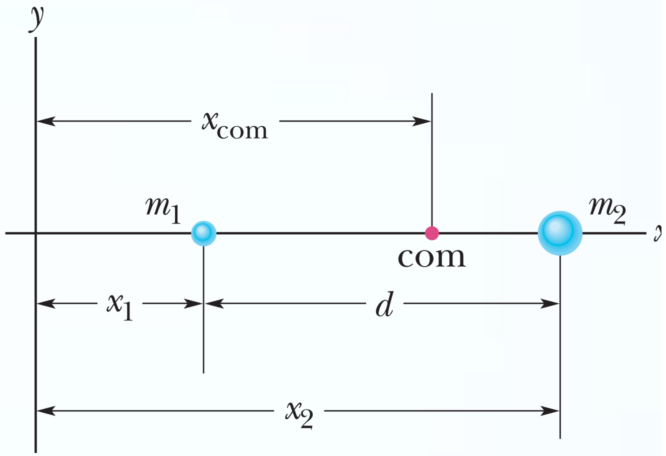
$$x_{\text{com}} = \frac{m_2}{m_1 + m_2} d. \quad (1)$$

- If $m_2 = 0 \rightarrow x_{\text{com}} = 0$
  If $m_1 = 0 \rightarrow x_{\text{com}} = d$
  If $m_1 = m_2 \rightarrow x_{\text{com}} = \frac{1}{2} d$
  If neither m_1 or $m_2 \neq 0$
  $\rightarrow 0 < x_{\text{com}} < d$



- If the coordinate system is shifted leftward:

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad (2)$$



  - If $x_1 = 0 \rightarrow x_2 = d$ (Eq.2 reduces to Eq.1)
  - In spite of shifting the coordinate system, x_{com} still has the same distance from each particle

- Eq.2 could be written as:

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{M},$$

in which M: the total mass of the system ($M = m_1 + m_2$)

- For a more general situation with n particles
  $\rightarrow M = m_1 + m_2 + \ldots + m_n$, and x_{com} will be:

$$\boxed{\begin{aligned} x_{com} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{M} \\ &= \frac{1}{M} \sum_{i=1}^{n} m_i x_i. \end{aligned}}$$

- In 3D system, the center of mass is identified by three coordinates as:

$$\boxed{x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.}$$

- We can also define the com using vectors:
  if the position of a particle at coordinates x_i, y_i, and z_i is:

$$\vec{r}_i = x_i\hat{\mathrm{i}} + y_i\hat{\mathrm{j}} + z_i\hat{\mathrm{k}}.$$

→ the position of the center of mass of a system of particles is given by a position vector:

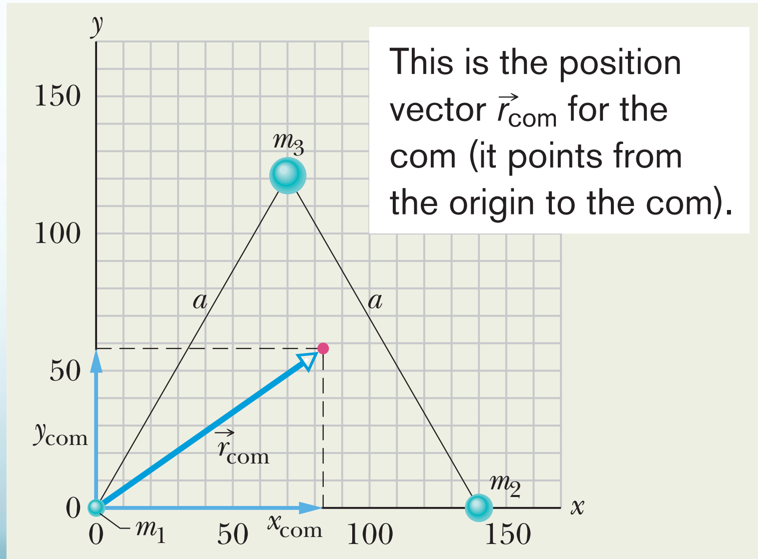
$$\vec{r}_{\text{com}} = x_{\text{com}}\hat{\mathrm{i}} + y_{\text{com}}\hat{\mathrm{j}} + z_{\text{com}}\hat{\mathrm{k}}.$$

$$\boxed{\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i,}$$

Sample Problem

Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system?

Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120



$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}}$$

$$= 83\text{ cm} \qquad \text{(Answer)}$$

and $$y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(120\text{ cm})}{7.1\text{ kg}}$$

$$= 58\text{ cm}. \qquad \text{(Answer)}$$

Newton's 2nd Law for a System of Particles

- Here we discuss how external forces can move a center of mass
- For a system of two billiard balls:
  - If a 1st ball is rolled at a 2nd billiard ball that is at rest
    → the two-ball system will continue to have forward motion after the impact
  - If the motion is not affected by another collision, what continues to move forward is the com of the system

- The vector equation that governs the motion of the com of a system of particles is:

$$\vec{F}_{net} = M\vec{a}_{com} \quad \text{Newton's 2nd law for (system of particles).} \qquad (3)$$

$$\vec{F}_{net} = M\vec{a}_{com} \quad \text{Newton's 2}^{nd}\text{ law for (system of particles).} \qquad (3)$$

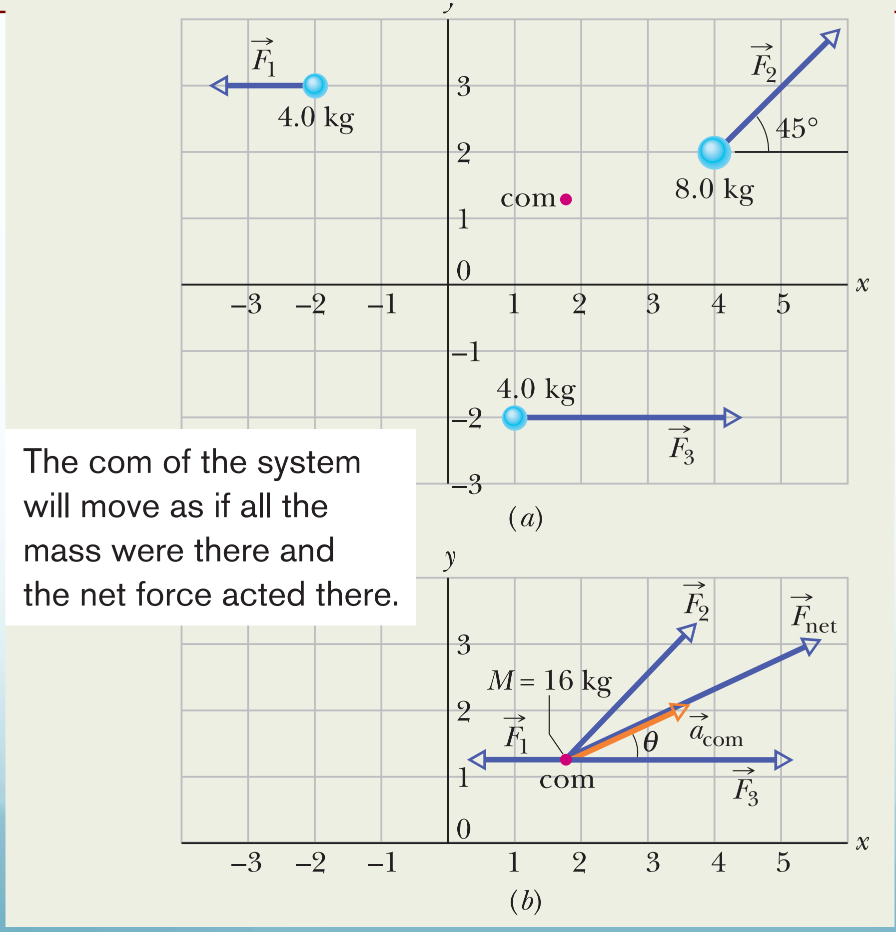
- F_{net}: the net force of all external forces that act on the system (the internal forces are not included)
- M: the total mass of the system
  We assume that no mass enters or leaves the system as it moves (constant M → **closed system**)
- a_{com}: the acceleration of the com of the system

- Equation 3 is equivalent to 3 equations involving the components of F & a along the 3 coordinate axes:

$$\boxed{F_{net,x} = Ma_{com,x} \qquad F_{net,y} = Ma_{com,y} \qquad F_{net,z} = Ma_{com,z}.}$$

Sample Problem

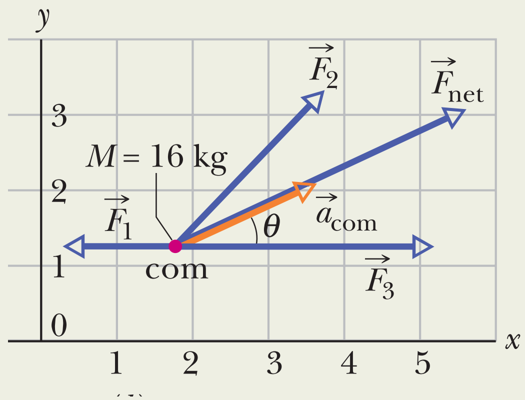
The three particles in Fig. 9-7*a* are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move?



$$\vec{F}_{net} = M\vec{a}_{com}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = M\vec{a}_{com}$$

$$\vec{a}_{com} = \frac{\vec{F}_1 + \vec{F}_2 + \vec{F}_3}{M}.$$



$$a_{com,x} = \frac{F_{1x} + F_{2x} + F_{3x}}{M}$$

$$= \frac{-6.0\text{ N} + (12\text{ N})\cos 45° + 14\text{ N}}{16\text{ kg}} = 1.03\text{ m/s}^2.$$

$$a_{com,y} = \frac{F_{1y} + F_{2y} + F_{3y}}{M}$$

$$= \frac{0 + (12\text{ N})\sin 45° + 0}{16\text{ kg}} = 0.530\text{ m/s}^2.$$

$$a_{com} = \sqrt{(a_{com,x})^2 + (a_{com,y})^2}$$

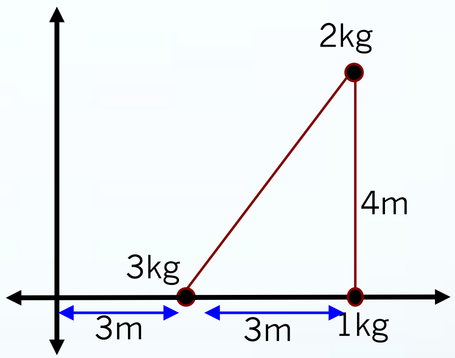
$$= 1.16\text{ m/s}^2 \approx 1.2\text{ m/s}^2$$

$$\theta = \tan^{-1}\frac{a_{com,y}}{a_{com,x}} = 27°.$$

Examples:

Q.1: A system consists of 3 particles as shown. The center of mass is:
(a) (2, 1) (b) (4.5, 1.3) (c) (2.1, 1.2) (d) (3, 6)

m	x	y
1	6	0
2	6	4
3	3	0



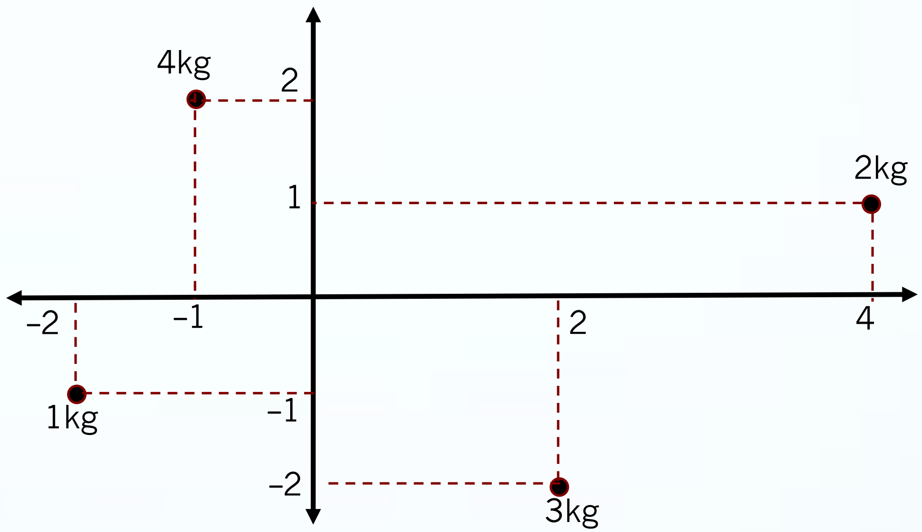
$$x_{com} = \frac{(1\times 6)+(2\times 6)+(3\times 3)}{1+2+3} = 4.5$$

$$y_{com} = \frac{(1\times 0)+(2\times 4)+(3\times 0)}{1+2+3} = 1.3$$

The com is located at (4.5, 1.3)

Q.2: A system consists of 4 particles as shown. The center of mass is:
(a) (0.8, 0.3) **(b) (4.5, 1.3)** **(c) (2.1, 1.2)** **(d) (3, 6)**

m	x	y
1	-2	-1
2	4	1
3	2	-2
4	-1	2



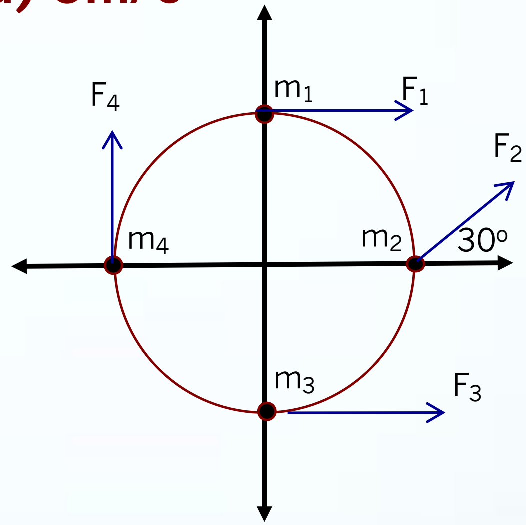
$$x_{com} = \frac{(1\times -2)+(2\times 4)+(3\times 2)+(4\times -1)}{1+2+3+4} = 0.8$$

$$y_{com} = \frac{(1\times -1)+(2\times 1)+(3\times -2)+(4\times 2)}{1+2+3+4} = 0.3$$

The com is located at (0.8, 0.3)

Q.3: A s shown, if the radius of the circle is 1m, and the masses are $m_1=m_2=m_3=m_4=3kg$, and if $F_1=2N$, $F_2=5N$, $F_3=1N$ and $F_4=8N$. The magnitude of the acceleration of the center of mass is:

(a) 0.6i+0.9j (b) $1.1m/s^2$ (c) $2.2m/s^2$ (d) $3m/s^2$



$$\vec{F}_1 = 2\cos 0\hat{i} + 2\sin 0\hat{j} + 0\hat{k} = 2\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{F}_2 = 5\cos 30\hat{i} + 5\sin 30\hat{j} + 0\hat{k} = 4.3\hat{i} + 2.5\hat{j} + 0\hat{k}$$

$$\vec{F}_3 = 1\cos 0\hat{i} + 1\sin 0\hat{j} + 0\hat{k} = 1\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{F}_4 = 8\cos 90\hat{i} + 8\sin 90\hat{j} + 0\hat{k} = 0\hat{i} + 8\hat{j} + 0\hat{k}$$

$$\sum \vec{F} = 7.3N\hat{i} + 0.5N\hat{j}$$

$$\sum m = 3(4) = 12kg$$

$$\vec{a}_{com} = \frac{\sum \vec{F}}{\sum m} = \frac{7.3N\hat{i} + 0.5N\hat{j}}{12kg} = 0.6\hat{i} + 0.9\hat{j}$$

$$a_{com} = \sqrt{0.6^2 + 0.9^2} = 1.1m/s^2$$

Q.4: In the previous question, the angle between x-axis and $\vec{a}_{com}$ is:

(a) 60° **(b) 56°** **(c) 30°** **(d) 100°**

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{0.9}{0.6} = 56^o$$

Q.5 In the previous question, the acceleration of the center of mass at the direction of x-axis is:

(a) 0.7m/s² **(b) 3m/s²** **(c) 0.6m/s²** **(d) 5m/s²**

$$\sum \vec{F}_x = 7.3N\hat{i}$$

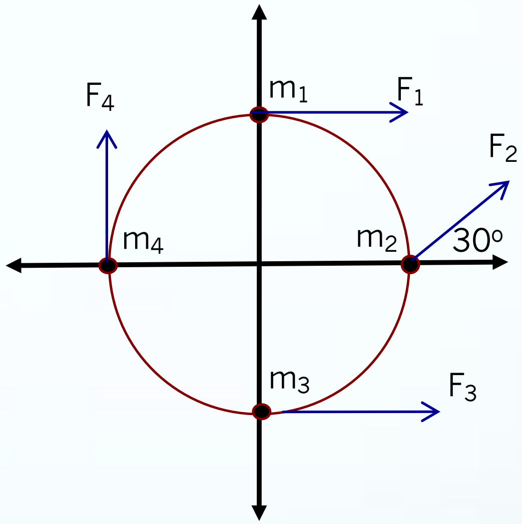
$$\sum m = 3(4) = 12kg$$

$$\vec{a}_{com,x} = \frac{\sum \vec{F}_x}{\sum m} = \frac{7.3N\hat{i}}{12kg} = 0.6\hat{i}$$

$$a_{com,x} = 0.6m/s^2$$

Q.6 In the previous question, the coordinate of the center of mass is:
(a) (0, 0) **(b) (1, 1)** **(c) (2, 1)** **(d) (–1, 1)**

m	x	y
3	0	1
3	1	0
3	0	-1
3	-1	0



$$x_{com} = \frac{(3\times 0)+(3\times 1)+(3\times 0)+(3\times -1)}{3(4)} = 0$$

$$y_{com} = \frac{(3\times 1)+(3\times 0)+(3\times -1)+(3\times 0)}{3(4)} = 0$$

The com is located at (0, 0)

•1 A 2.00 kg particle has the xy coordinates $(-1.20\text{ m}, 0.500\text{ m})$, and a 4.00 kg particle has the xy coordinates $(0.600\text{ m}, -0.750\text{ m})$. Both lie on a horizontal plane. At what (a) x and (b) y coordinates must you place a 3.00 kg particle such that the center of mass of the three-particle system has the coordinates $(-0.500\text{ m}, -0.700\text{ m})$?

(a) The x coordinate of the system's center of mass is:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(-1.20\text{ m}) + (4.00\text{ kg})(0.600\text{ m}) + (3.00\text{ kg})x_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.500\text{ m}.$$

Solving the equation yields $x_3 = -1.50$ m.

(b) The y coordinate of the system's center of mass is:

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(0.500\text{ m}) + (4.00\text{ kg})(-0.750\text{ m}) + (3.00\text{ kg})y_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.700\text{ m}.$$

Solving the equation yields $y_3 = -1.43$ m.

Linear Momentum

- **The linear momentum of a particle (p):** is a vector quantity $\vec{p}$ that is defined as:

$$\boxed{\vec{p} = m\vec{v} \qquad \text{(linear momentum of a particle)},} \quad (1)$$

in which m is the mass of the particle, $\vec{v}$ is its velocity

  - Because m is +ve & scalar quantity → $\vec{p}$ & $\vec{v}$ have the same direction
  - The SI unit for momentum is (kg.m/s)

- Newton 2nd law in terms of momentum:

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

$$\boxed{\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.} \quad (2)$$

→
  - The net external force on a particle changes its linear momentum
  - The linear momentum can be changed *only* by a net external force
  - If there is no net external force, $\vec{F}_{net} = \frac{d\vec{p}}{dt} = 0$ → $\vec{p}$ cannot change (constant)

- From Eqs.1 & 2: $\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$. (→ Newton's 2nd law)

The Linear Momentum of a System of Particles

- For a system of n particles, each has its own mass, velocity, & linear momentum, the total linear momentum of the system is the vector sum of the individual particles' linear momenta:

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n \\ &= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n.\end{aligned}$$

$$\boxed{\vec{P} = M\vec{v}_{\text{com}} \qquad \text{(linear momentum, system of particles),}} \quad (3)$$

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

- By taking the time derivative of Eq.3

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{\text{com}}}{dt} = M\vec{a}_{\text{com}} = \vec{F}_{net} \Rightarrow \boxed{\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \qquad \text{(system of particles),}} \quad (4)$$

- The net external force on a particle changes its linear momentum
- The linear momentum can be changed *only* by a net external force
- If there is no net external force $\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0 \rightarrow \vec{P}$ cannot change (constant)

Conservation of Linear Momentum

- If $\vec{F}_{net} = 0$ for a system of particles (**isolated** system) & no particles leave or enter the system (**closed** system), Eq. 4 becomes:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0 \Rightarrow \boxed{\vec{P} = \text{constant} \qquad \text{(closed, isolated system).}}$$

  - Note: momentum unit is also N.s

> If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

- The **law of conservation of linear momentum**:
  for a closed & isolated system,

$$\vec{P}_i = \vec{P}_f \quad (5)$$

$$\begin{pmatrix}\text{total linear momentum} \\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{total linear momentum} \\ \text{at some later time } t_f\end{pmatrix}$$

  - Note: the energy is not always conserved when the momentum is conserved

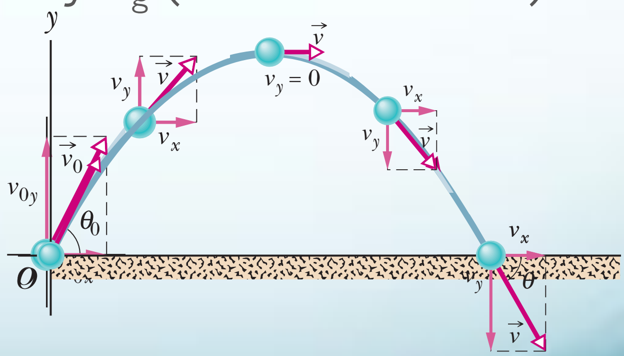
$$\vec{P}_i = \vec{P}_f \qquad (5)$$

- Eq.5 is a *vector* equation → could be written in to 3 equations in xyz coordinates

- The linear momentum might be conserved in one or two directions but not in all directions

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

- e.g., a tossed grapefruit (projectile motion):
  The only external force acting on it is the gravity F_g (downward force)
  - The vertical component of the tossed grapefruit linear momentum change
  - The horizontal component of the tossed grapefruit linear momentum is constant



Sample Problem

A 6kg ballot box slides with a speed of 4m/s across a frictionless floor in the positive direction of an x axis. It suddenly exploded into two pieces. One piece of mass 2kg moves with 8m/s in the positive x axis. What is the velocity of the second one with mass m_2?

$$\vec{p} = \vec{p}_1 + \vec{p}_2$$

$$m\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$m_2 = m - m_1 = 4$$

$$6(4\hat{i}) = 2(8\hat{i}) + 4\vec{v}_2$$

$$24\hat{i} = 16\hat{i} + 4\vec{v}_2$$

$$8\hat{i} = 4\vec{v}_2$$

$$\vec{v}_2 = 2\hat{i} \Rightarrow |\vec{v}_2| = 2m/s$$ in the +ve direction of x-axis

Examples:

Q.1: A 2kg object is moving leftward with 20m/s. The object hits a wall then returns back in the opposite direction with the same velocity. The initial momentum of the ball is:

(a) –40 kg.m/s **(b) –50 kg.m/s** **(c) 60 kg.m/s**

$$\vec{p}_i = m\vec{v}_i = 2\times -20 = -40 kg.m/s$$

Q.2: The final momentum of the ball in the previous question is:

(a) 40 kg.m/s **(b) –50 kg.m/s** **(c) 60 kg.m/s**

Q.3: The change in the momentum in the previous question is:

(a) 40 kg.m/s **(b) 80 kg.m/s** **(c) 60 kg.m/s**

$$\Delta\vec{p}_i = \vec{p}_f - \vec{p}_i = 40 + 40 = 80 kg.m/s$$

Q.4. A body of 2kg mass is moving with a kinetic energy of 25J. The momentum of the body is:

(a) 10 kg.m/s **(b) 15kg.m/s** **(c) 18 kg.m/s**

$$k = \frac{1}{2}mv^2 \Rightarrow 25 = v^2 \Rightarrow v = 5$$

$$p = mv = 2 \times 5 = 10 kg.m/s$$

Q.5: A 5kg body has a momentum of 20kg.m/s. The kinetic energy of the body is:

(a) 50J **(b) 40J** **(c) 20J**

$$p = mv \Rightarrow 20 = 5v \Rightarrow v = 4 m/s$$

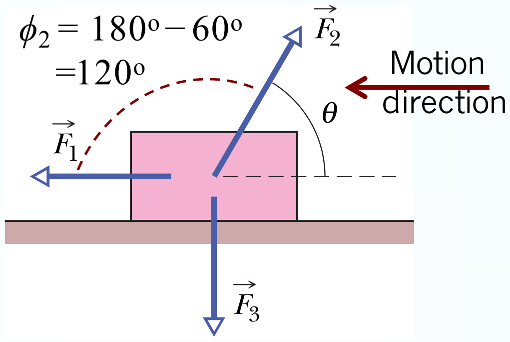
$$k = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2}(5)(4^2) = 40J$$

•18 A 0.70 kg ball moving horizontally at 5.0 m/s strikes a vertical wall and rebounds with speed 2.0 m/s. What is the magnitude of the change in its linear momentum?

$$\Delta p = m\left|v_f - v_i\right| = 0.7\left|-2-5\right| = 0.7\left|-7\right| = 4.9 kg.m/s$$

Problems for chapter 7

••15 GO Figure 7-27 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0°$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?



15. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d \cos\phi_1 = (5.00\,\text{N})(3.00\,\text{m})\cos 0° = 15.0\ \text{J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d \cos\phi_2 = (9.00\,\text{N})(3.00\,\text{m})\cos 120° = -13.5\ \text{J}.$$

Force $\vec{F}_3$ is perpendicular to the displacement, so

$$W_3 = F_3 d \cos \phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\ \text{J} - 13.5\ \text{J} + 0 = +1.50\ \text{J}.$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.

•45 SSM ILW A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which the force does work on the block?

45. The power associated with force $\vec{F}$ is given by $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi = (122\ \text{N})(5.0\ \text{m/s})\cos 37° = 4.9\times 10^2\ \text{W}.$$