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# Viewing In 2D

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Windowing Concepts

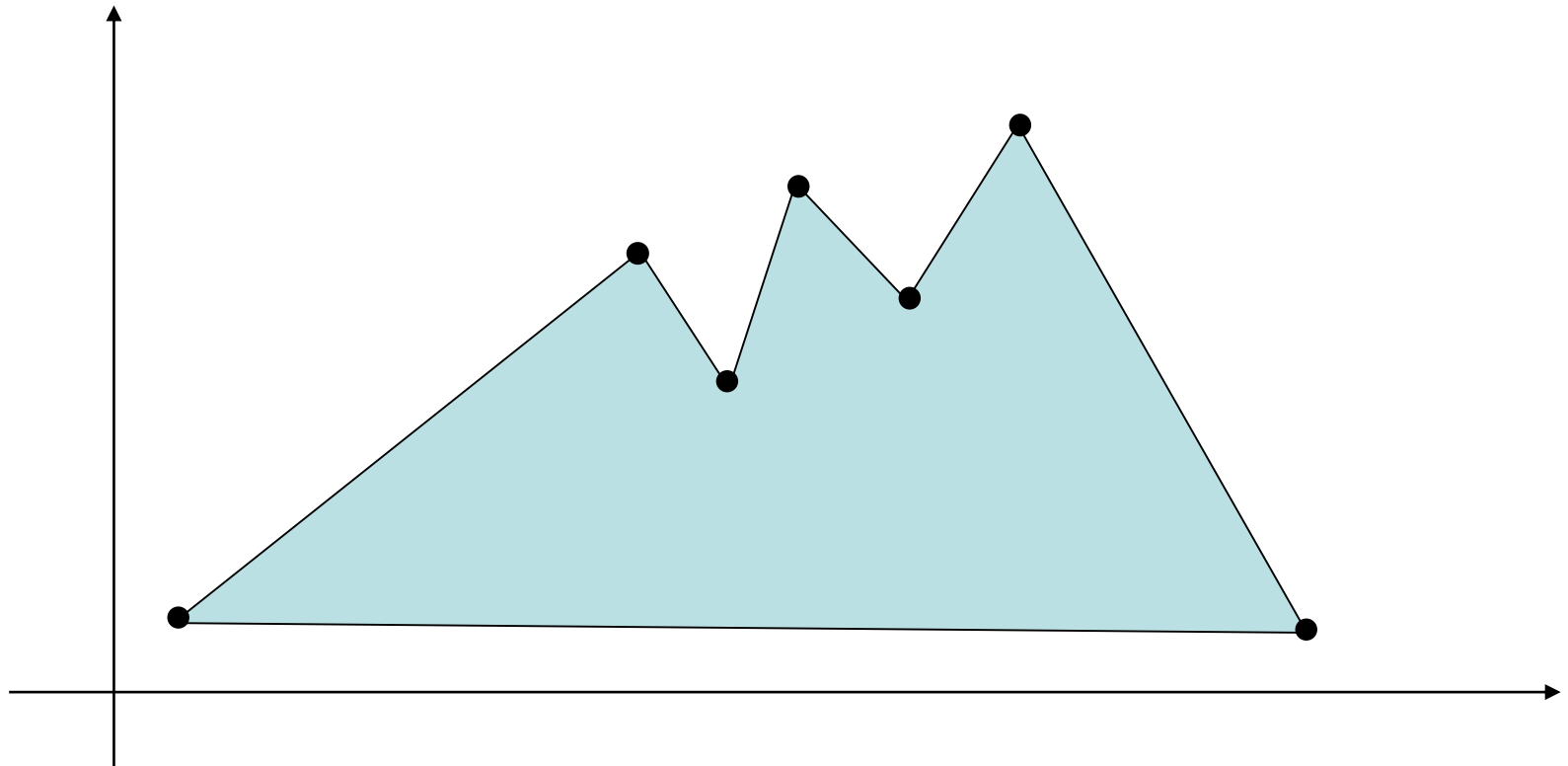
Clipping

- Introduction
- Brute Force
- Cohen-Sutherland Clipping Algorithm

Area Clipping

# Windowing I

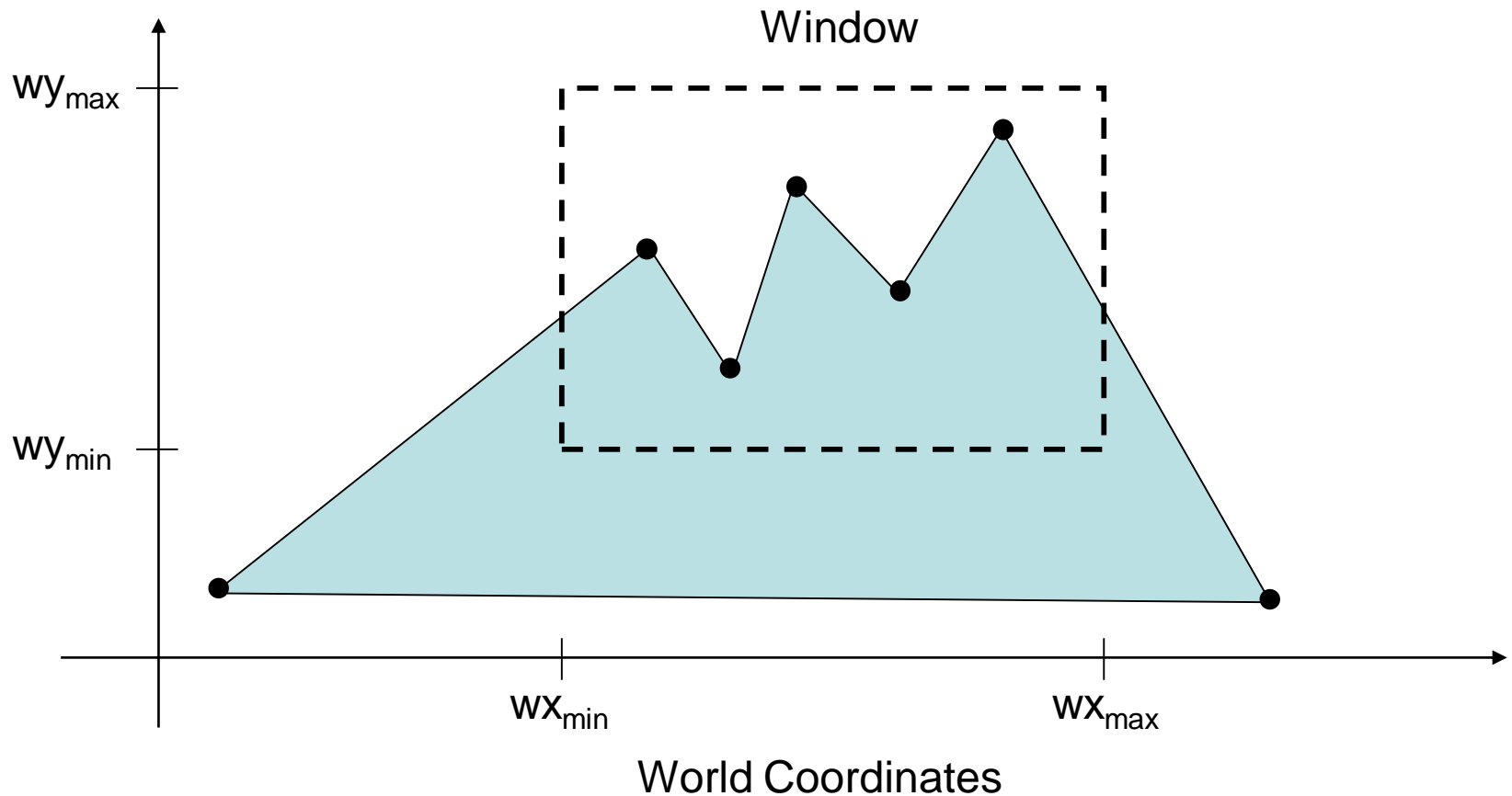
A scene is made up of a collection of objects specified in world coordinates



World Coordinates

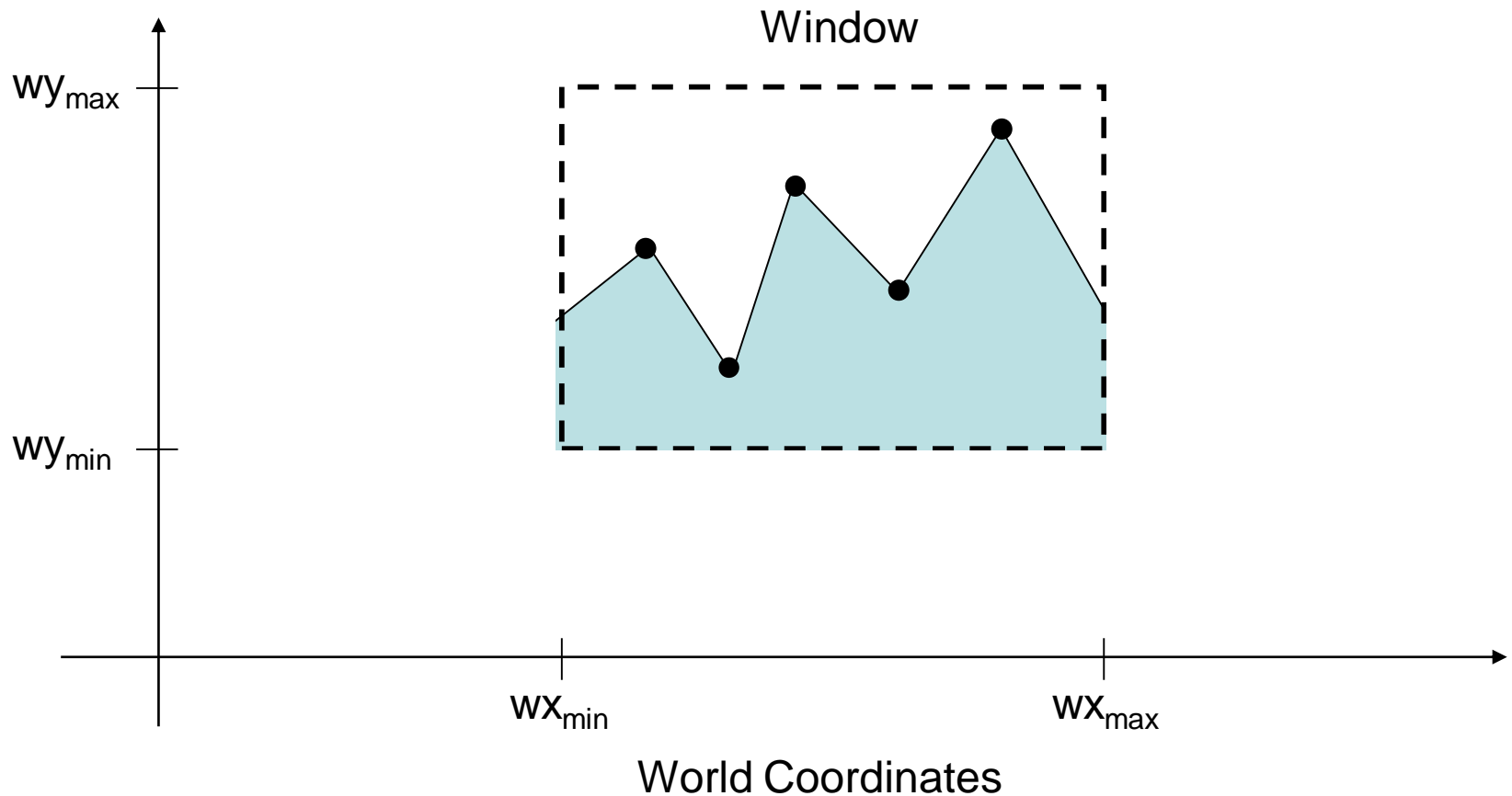
# Windowing II

When we display a scene only those objects within a particular window are displayed



# Windowing III

Because drawing things to a display takes time we *clip* everything outside the window



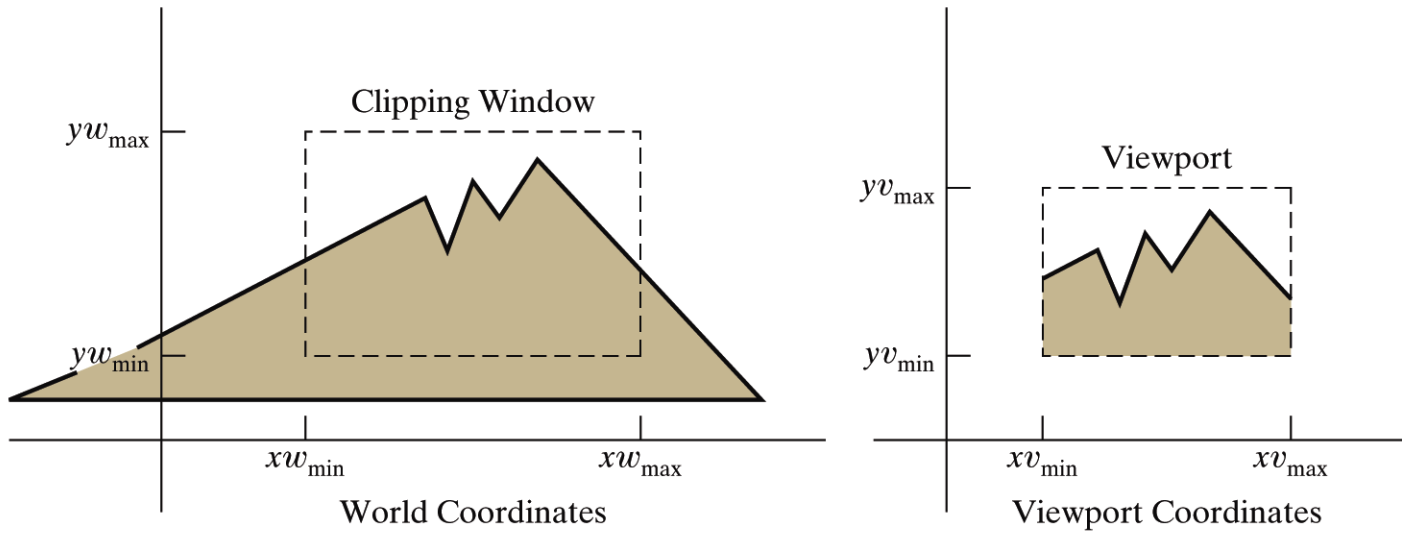


Figure 6-2

A clipping window and associated viewport, specified as rectangles aligned with the coordinate axes.

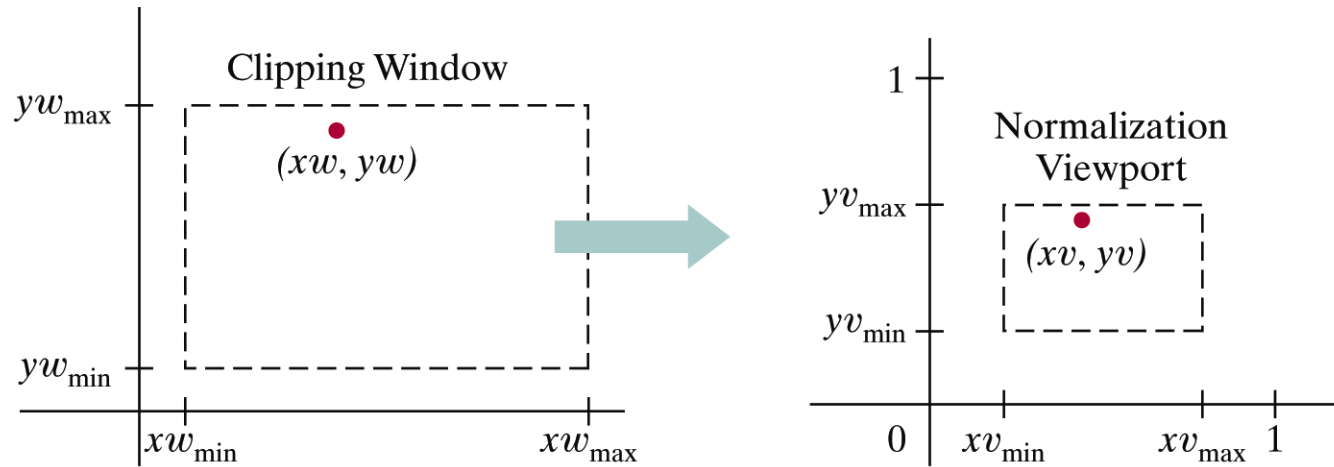


Figure 6-7

A point  $(xw, yw)$  in a world-coordinate clipping window is mapped to viewport coordinates  $(xv, yv)$ , within a unit square, so that the relative positions of the two points in their respective rectangles are the same.



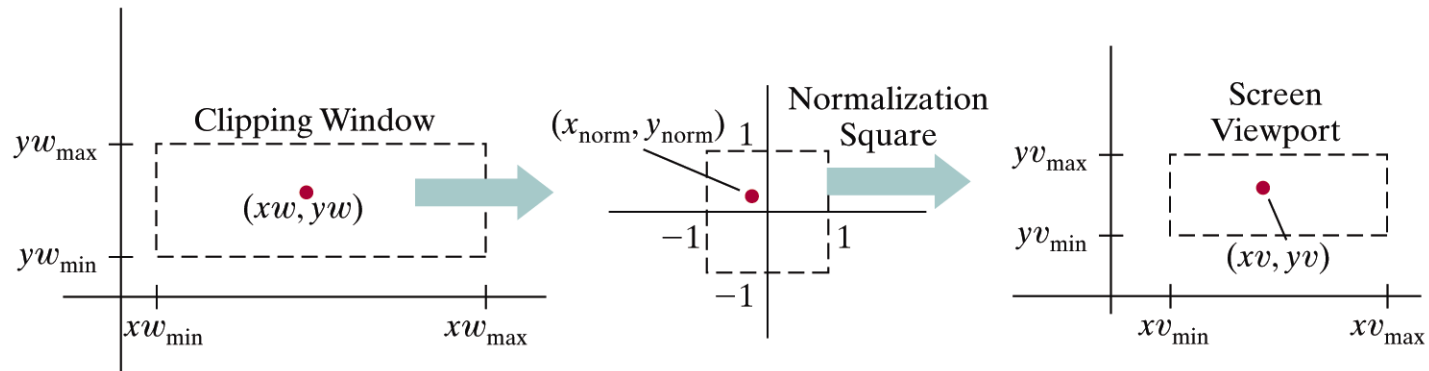


Figure 6-8

A point  $(xw, yw)$  in a clipping window is mapped to a normalized coordinate position  $(x_{\text{norm}}, y_{\text{norm}})$ , then to a screen-coordinate position  $(xv, yv)$  in a viewport. Objects are clipped against the normalization square before the transformation to viewport coordinates.

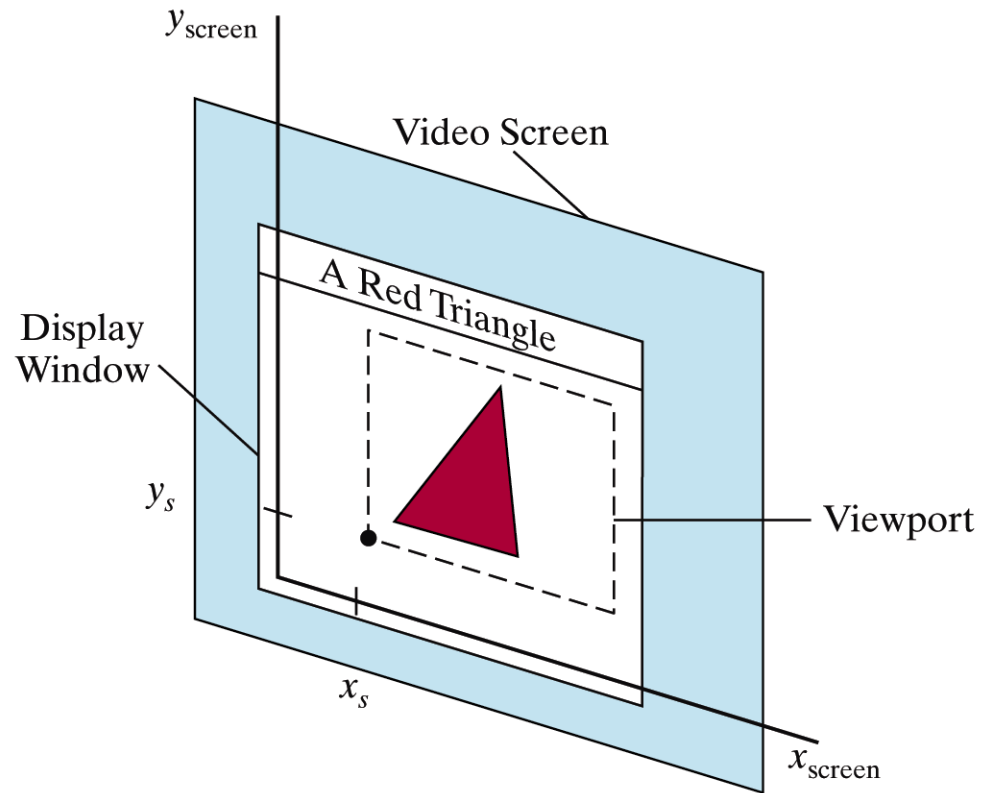


Figure 6-9

A viewport at coordinate position  $(x_s, y_s)$  within a display window.



# Coordinate Systems

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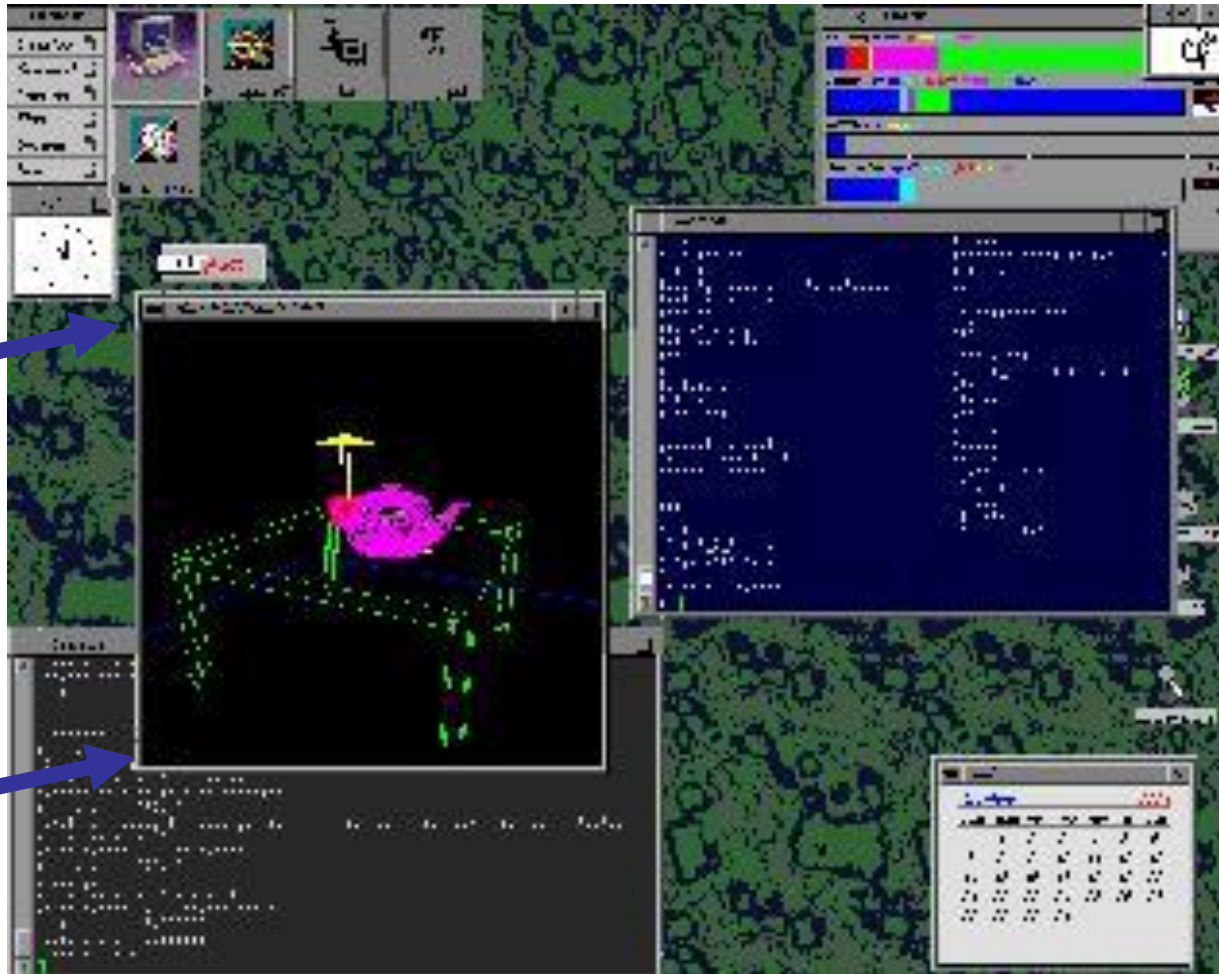
- Screen Coordinate system
- World Coordinate system
- World window
- Viewport
- Window to viewport mapping

# Screen Coordinate System

Glut

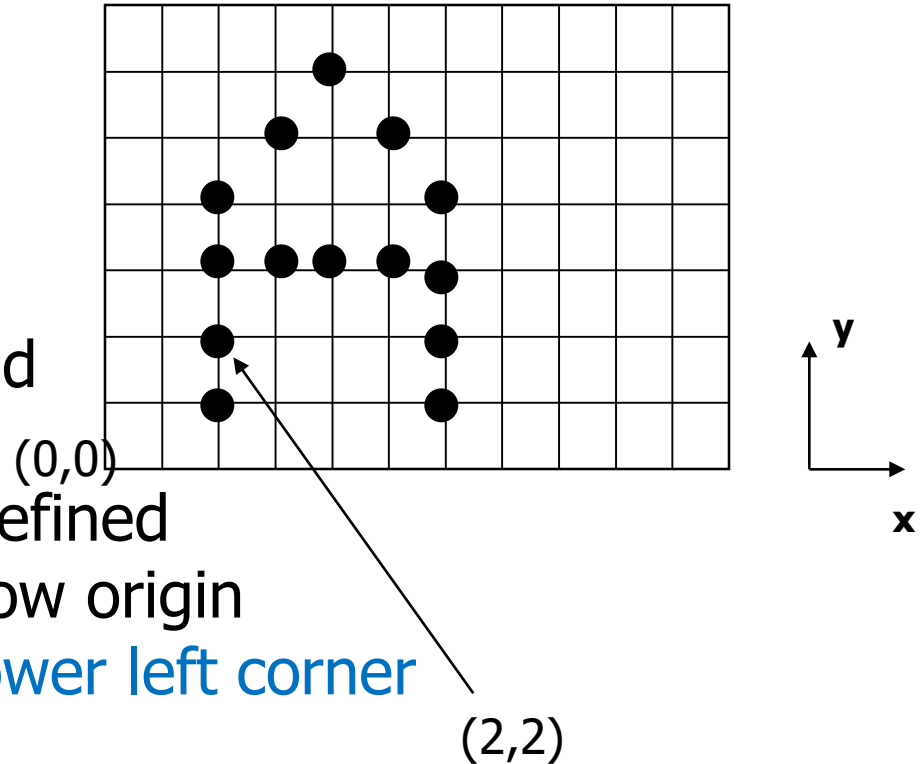


OpenGL  
(0,0)



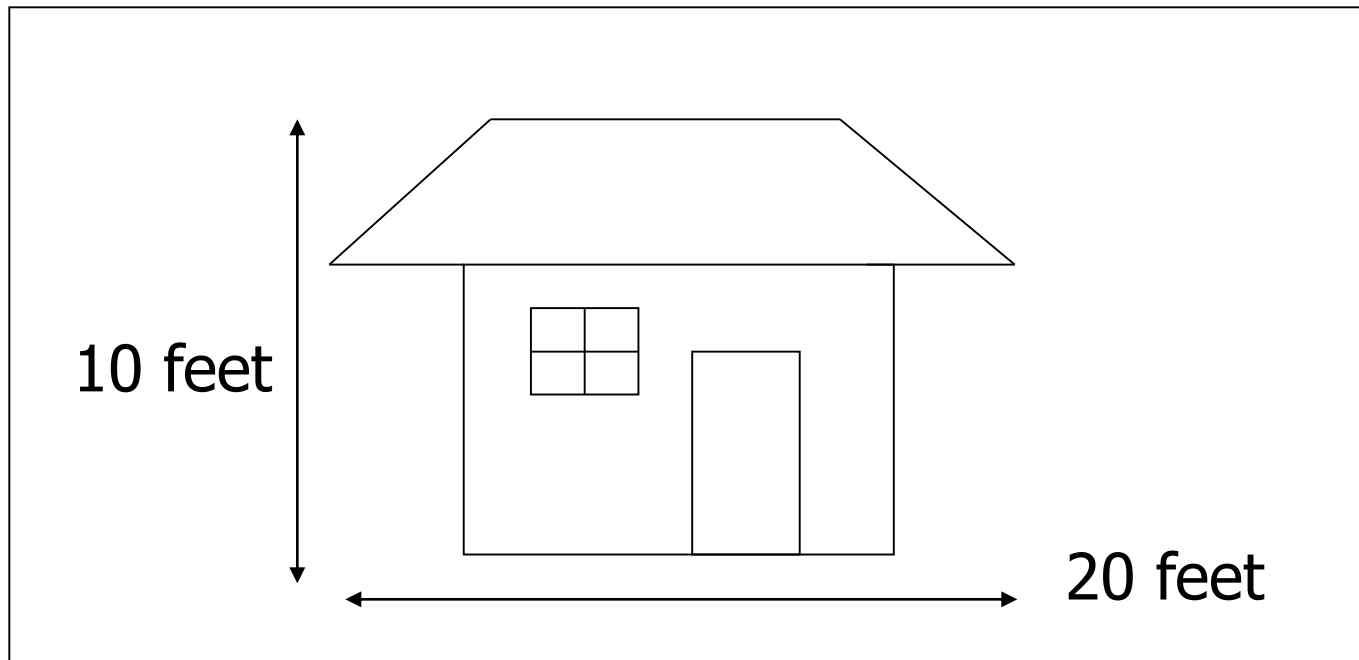
# Screen Coordinate System

- 2D Regular Cartesian Grid
- Origin  $(0,0)$  at lower left corner
- Horizontal axis –  $x$   
Vertical axis –  $y$
- Pixels are defined at the grid intersections
- This coordinate system is defined relative to the display window origin (OpenGL convention: the lower left corner of the window)



# World Coordinate System

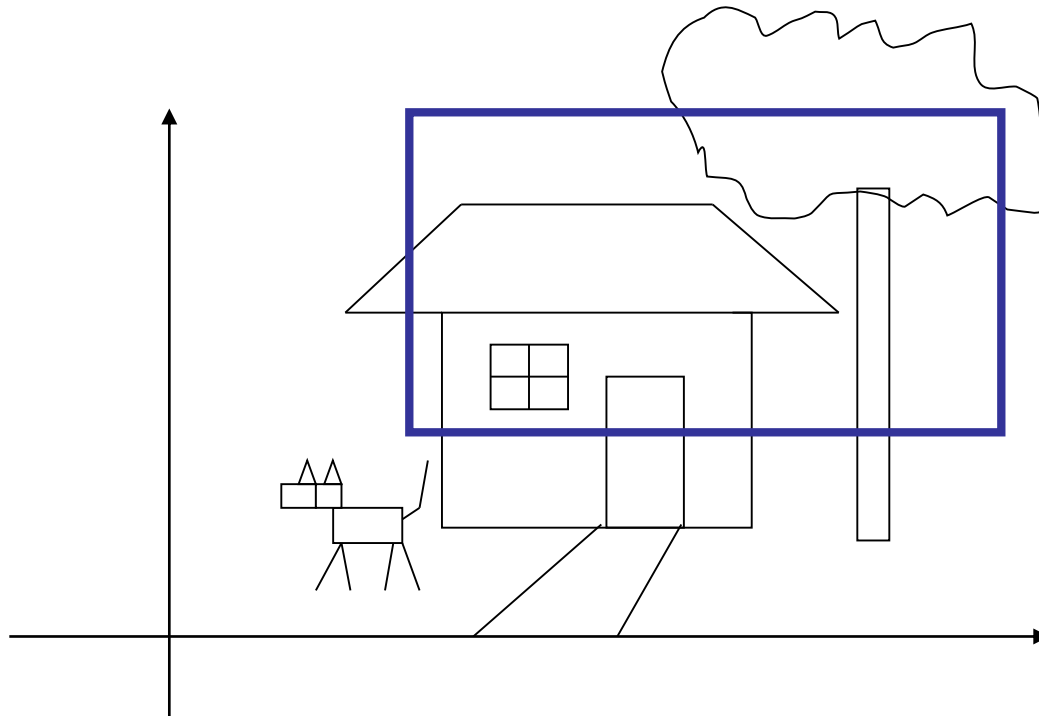
- Screen coordinate system is not easy to use





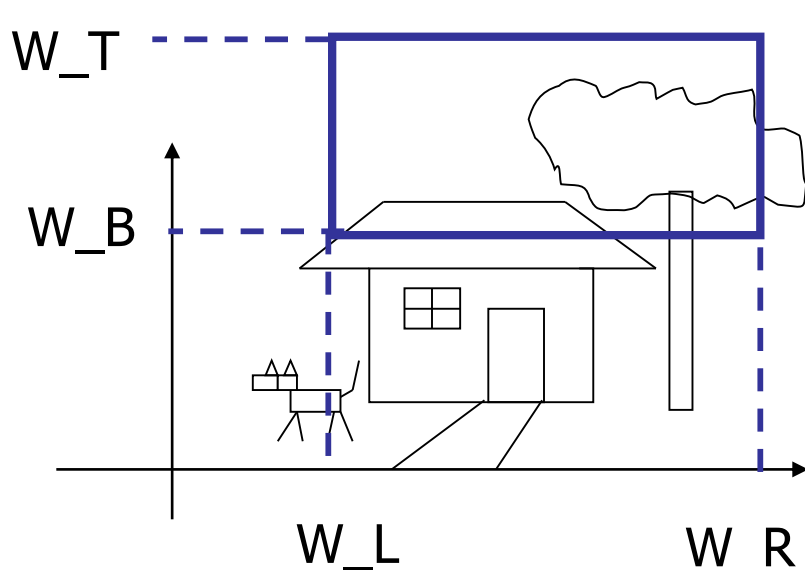
# Define a world window

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# World Window

- World window – a rectangular region in the world that limits our view



Define by

$W_L, W_R, W_B, W_T$

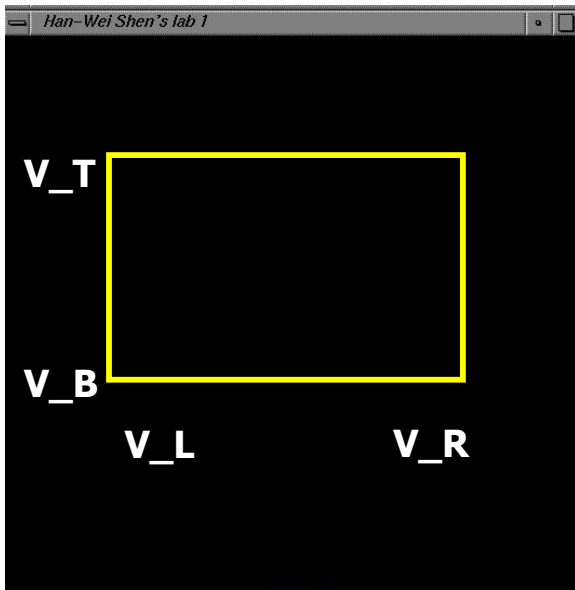
Use OpenGL command:

```
gluOrtho2D(left, right, bottom,  
            top)
```



# Viewport

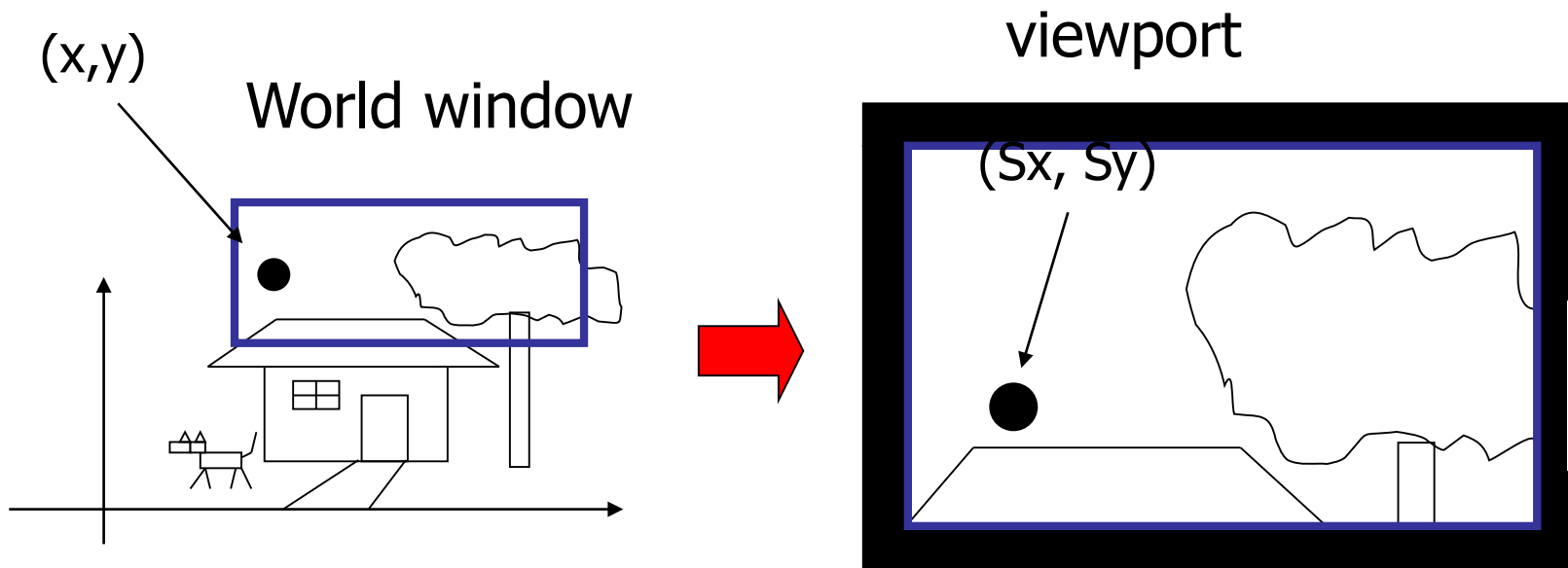
- The rectangular region in the screen that maps to our world window
- Defined in the window's (or control's) coordinate system



```
glViewport(int left, int bottom,  
           int (right-left),  
           int (top-bottom));
```

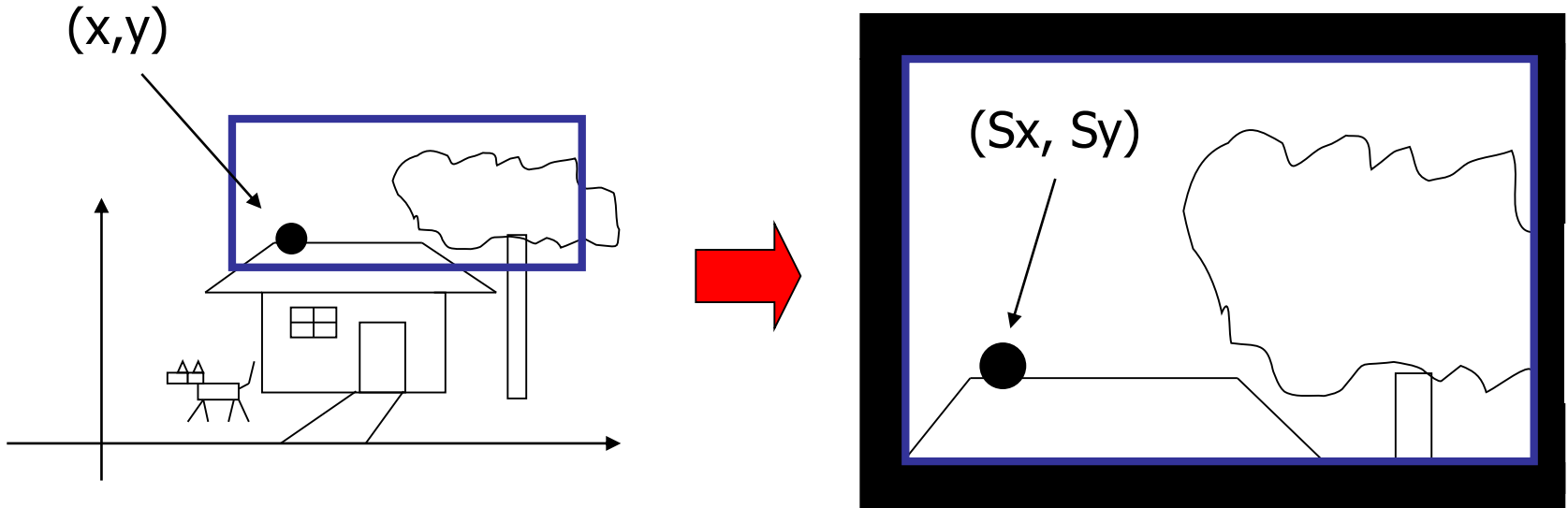
# Window to viewport mapping

- The objects in the world window will then be drawn onto the viewport



# Window to viewport mapping

- How to calculate  $(sx, sy)$  from  $(x, y)$ ?





# Window to viewport mapping

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- First thing to remember – you don't need to do it by yourself. OpenGL will do it for you
  - You just need to define the viewport (with `glViewport()`), and the world window (with `gluOrtho2D()`)
- But we will look 'under the hood'



## Also, one thing to remember ...

---

- A practical OpenGL issue
  - Before calling `gluOrtho2D()`, you need to have the following two lines of code –

```
glMatrixMode(GL_PROJECTION);  
glLoadIdentity();  
gluOrtho2D(Left, Right, Bottom, Top);
```



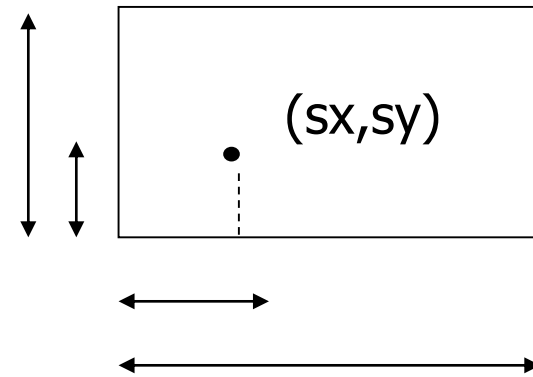
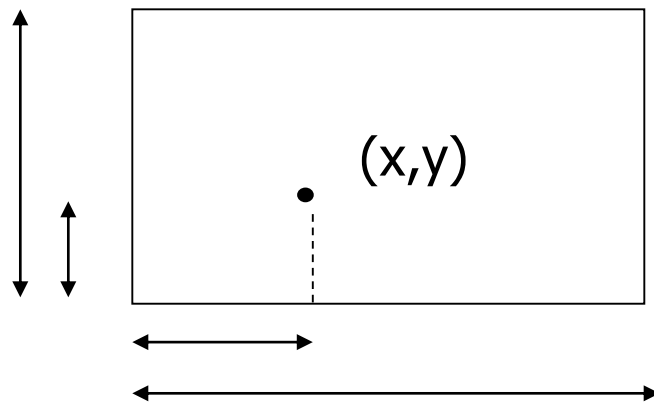
# Window to viewport mapping

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- Things that are given:
  - The world window ( $W_L, W_R, W_B, W_T$ )
  - The viewport ( $V_L, V_R, V_B, V_T$ )
  - A point  $(x, y)$  in the world coordinate system
- Calculate the corresponding point  $(s_x, s_y)$  in the screen coordinate system

# Window to viewport mapping

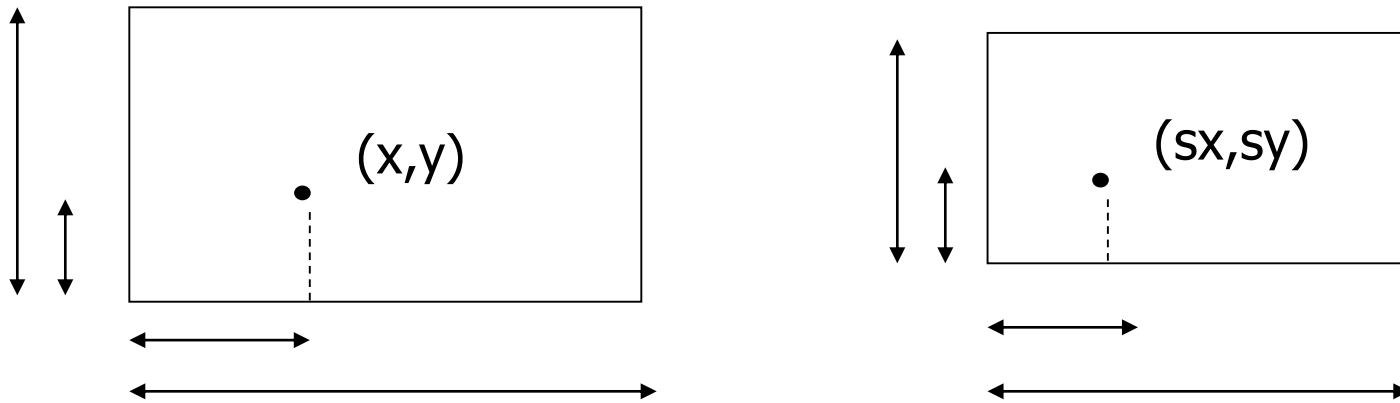
- Basic principle: the mapping should be proportional



$$(x - W_L) / (W_R - W_L) = (sx - V_L) / (V_R - V_L)$$

$$(y - W_B) / (W_T - W_B) = (sy - V_B) / (V_T - V_B)$$

# Window to viewport mapping



$$(x - W_L) / (W_R - W_L) = (sx - V_L) / (V_R - V_L)$$

$$(y - W_B) / (W_T - W_B) = (sy - V_B) / (V_T - V_B)$$


$$sx = (x - W_L) * (V_R - V_L) / (W_R - W_L) + V_L$$

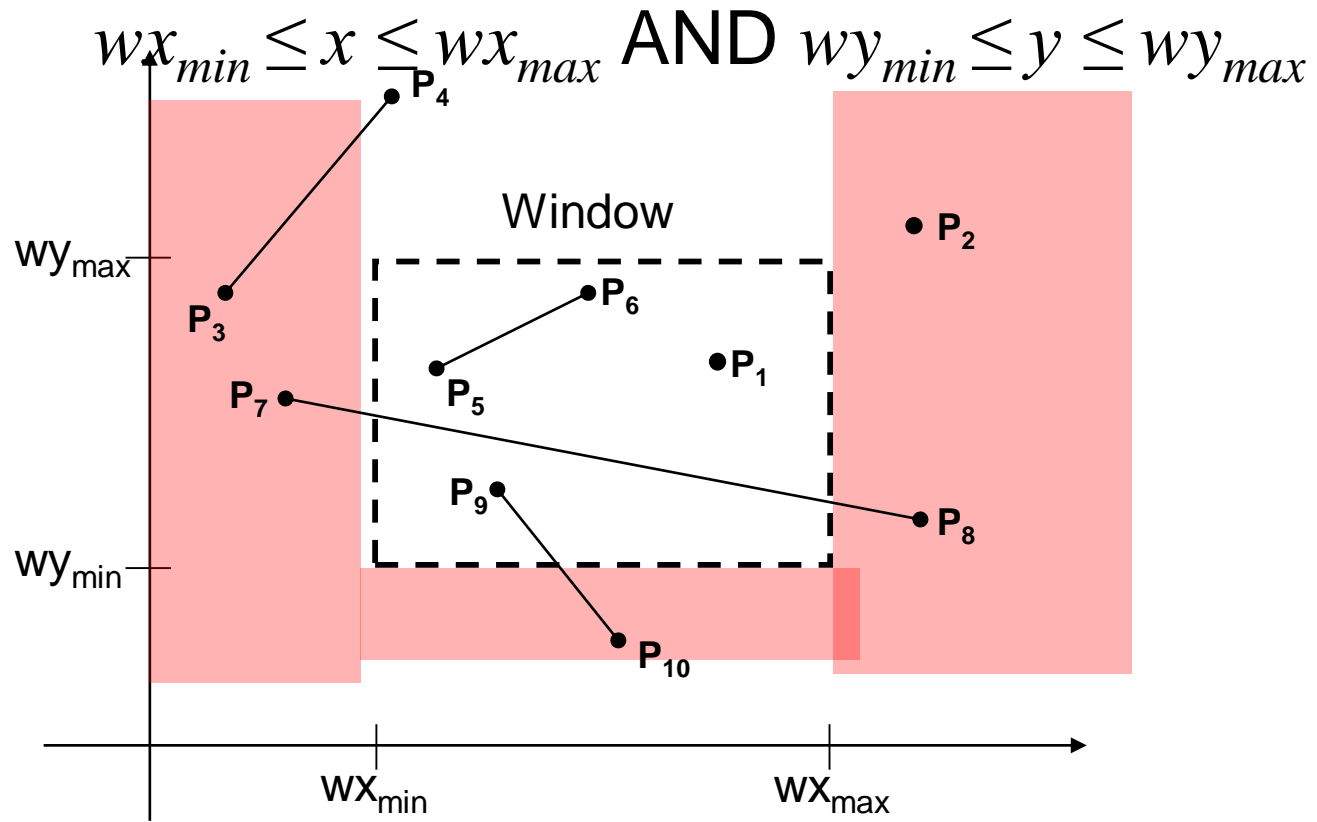
$$sy = (y - W_B) * (V_T - V_B) / (W_T - W_B) + V_B$$



# Clipping

Point clipping is easy:

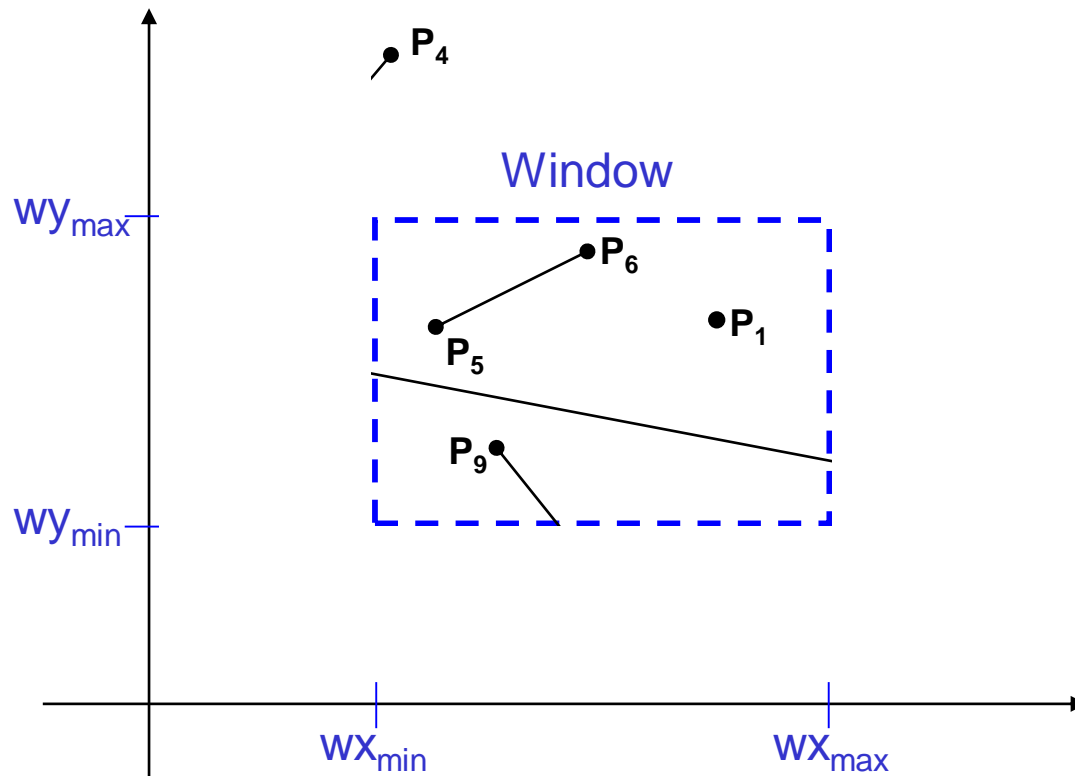
- For point  $(x,y)$  the point is not clipped if



Before Clipping

# Clipping

For the image below consider which lines and points should be kept and which ones should be clipped

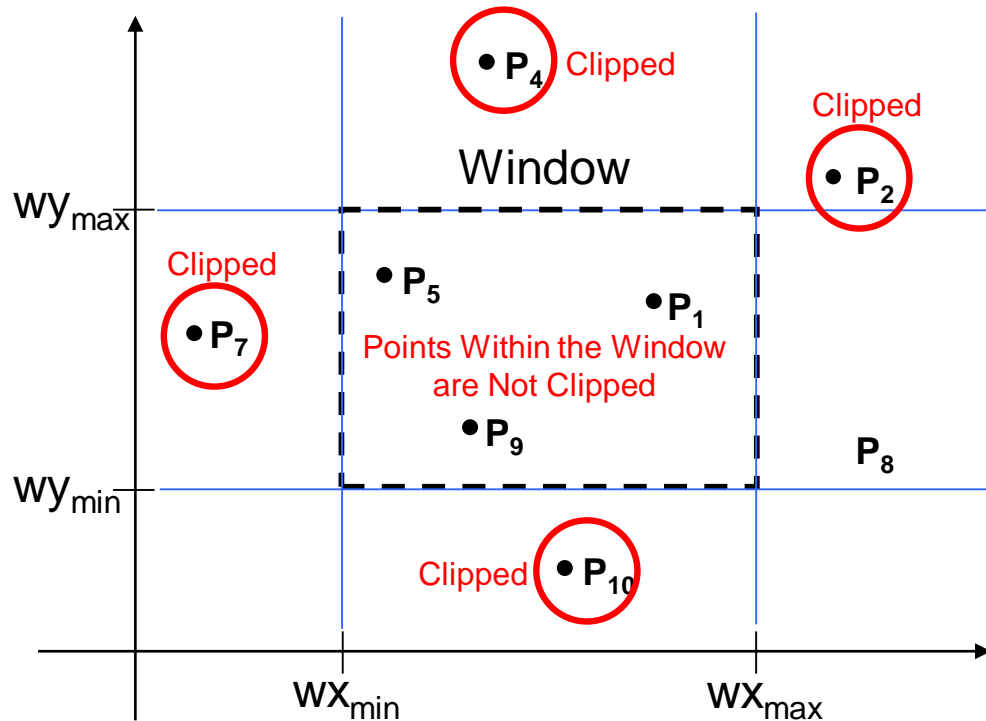


# Point Clipping

Easy - a point  $(x,y)$  is not clipped if:

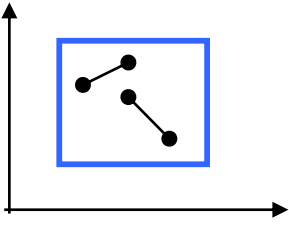
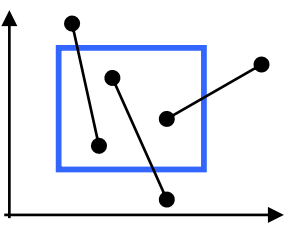
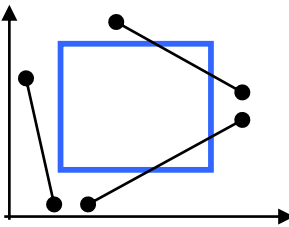
$$wx_{min} \leq x \leq wx_{max} \text{ AND } wy_{min} \leq y \leq wy_{max}$$

otherwise it is clipped



# Line Clipping

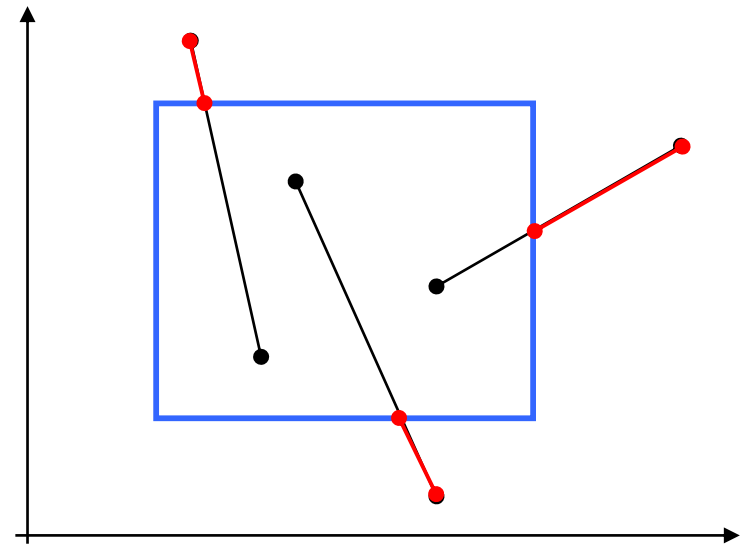
Harder - examine the end-points of each line to see if they are in the window or not

<b>Situation</b>	<b>Solution</b>	<b>Example</b>
Both end-points inside the window	Don't clip	 A 2D coordinate system with x and y axes. A blue square window is drawn. A line segment with two black dots at its endpoints is entirely contained within the square window.
One end-point inside the window, one outside	Must clip	 A 2D coordinate system with x and y axes. A blue square window is drawn. A line segment with two black dots at its endpoints is shown. One dot is inside the square window, and the other dot is outside the window to the right.
Both end-points outside the window	Don't know!	 A 2D coordinate system with x and y axes. A blue square window is drawn. A line segment with two black dots at its endpoints is shown. Both dots are outside the square window, one to the left and one to the right.

# Brute Force Line Clipping

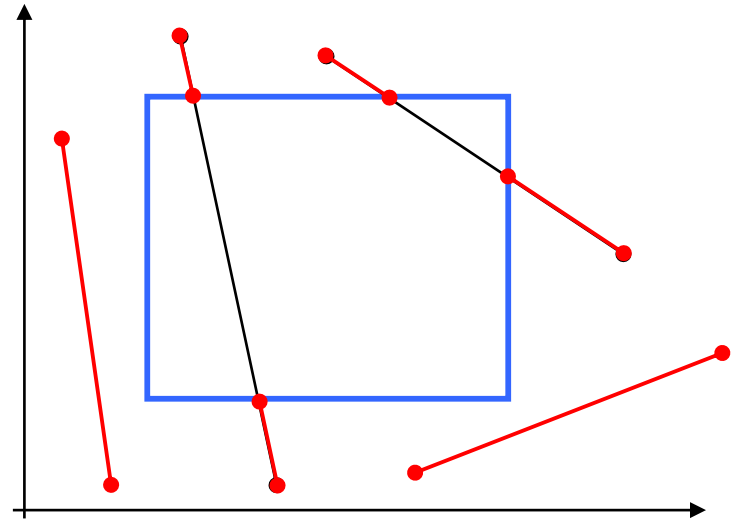
Brute force line clipping can be performed as follows:

- Don't clip lines with both end-points within the window
- For lines with one end-point inside the window and one end-point outside, calculate the intersection point (using the equation of the line) and clip from this point out



# Brute Force Line Clipping (cont...)

- For lines with both endpoints outside the window test the line for intersection with all of the window boundaries, and clip appropriately



However, calculating line intersections is computationally expensive

Because a scene can contain so many lines, the brute force approach to clipping is much too slow

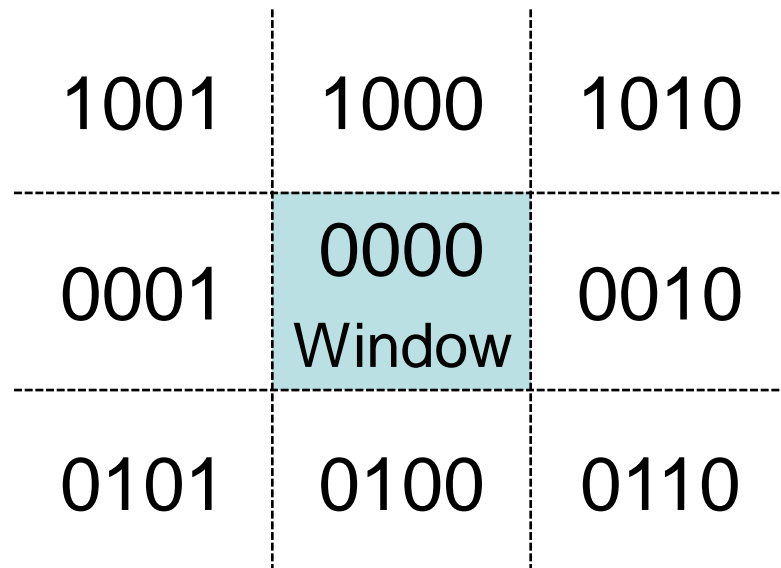
# Cohen-Sutherland: World Division

World space is divided into regions based on the window boundaries

- Each region has a unique four bit region code
- Region codes indicate the position of the regions with respect to the window

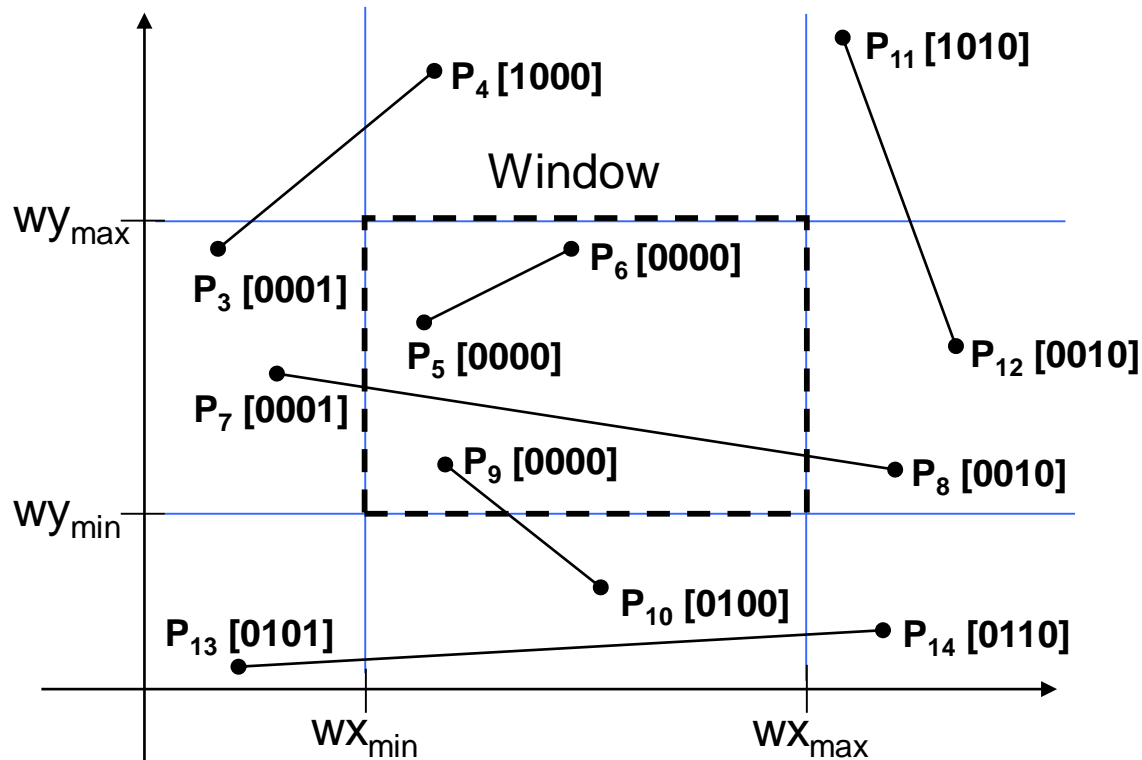
3	2	1	0
above	below	right	left

Region Code Legend



# Cohen-Sutherland: Labelling

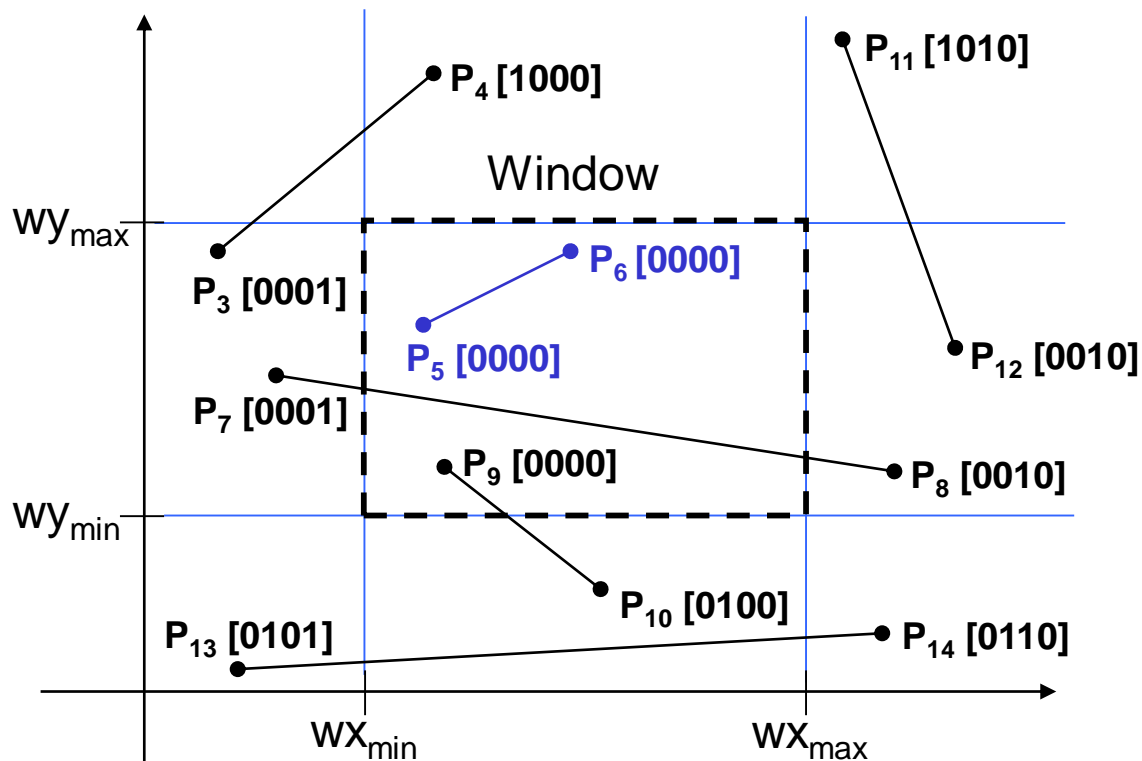
Every end-point is labelled with the appropriate region code





# Cohen-Sutherland: Lines In The Window

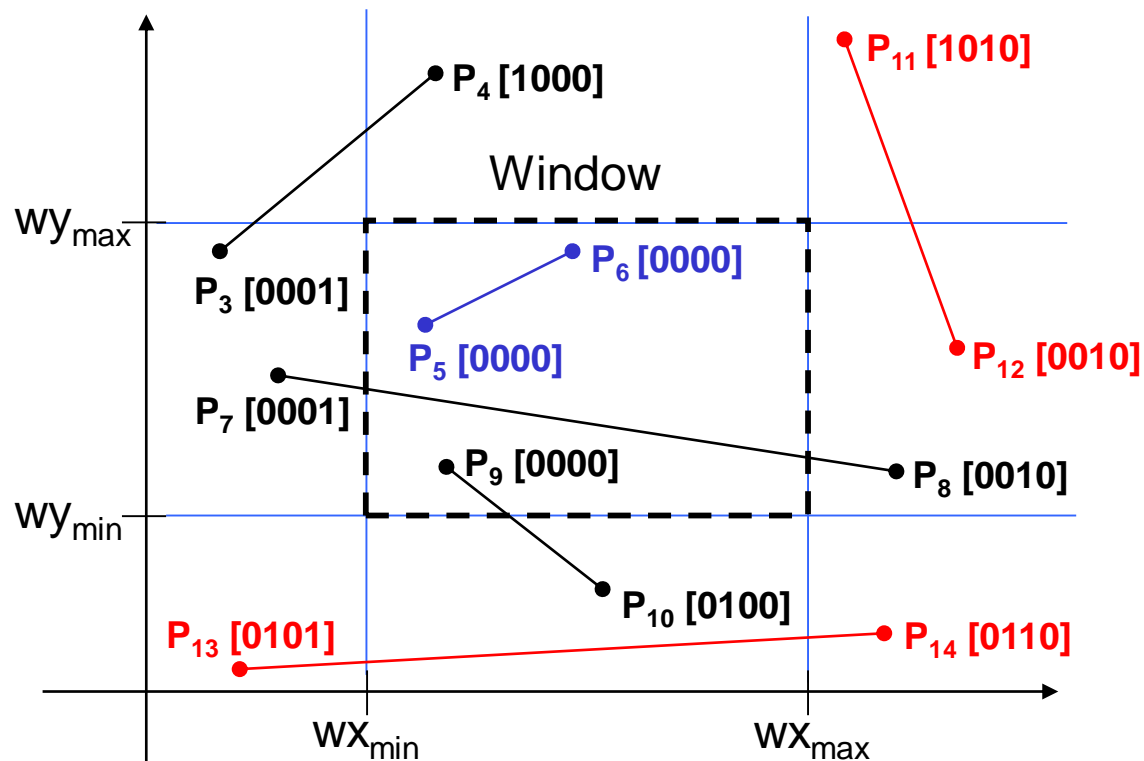
Lines completely contained within the window boundaries have region code [0000] for both end-points so are not clipped



# Cohen-Sutherland: Lines Outside The Window

Any lines with a common set bit in the region codes of both end-points can be clipped

- The AND operation can efficiently check this



# Cohen-Sutherland: Other Lines

Lines that cannot be identified as completely inside or outside the window may or may not cross the window interior

These lines are processed as follows:

- Compare an end-point outside the window to a boundary (choose any order in which to consider boundaries e.g. left, right, bottom, top) and determine how much can be discarded
- If the remainder of the line is entirely inside or outside the window, retain it or clip it respectively

# Cohen-Sutherland: Other Lines (cont...)

- Otherwise, compare the remainder of the line against the other window boundaries
- Continue until the line is either discarded or a segment inside the window is found

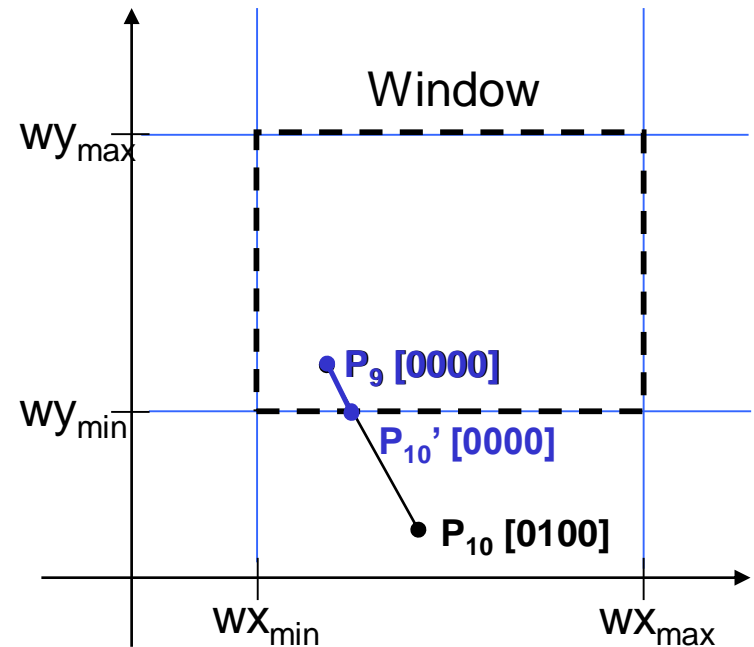
We can use the region codes to determine which window boundaries should be considered for intersection

- To check if a line crosses a particular boundary we compare the appropriate bits in the region codes of its end-points
- If one of these is a 1 and the other is a 0 then the line crosses the boundary

# Cohen-Sutherland Examples

Consider the line  $P_9$  to  $P_{10}$  below

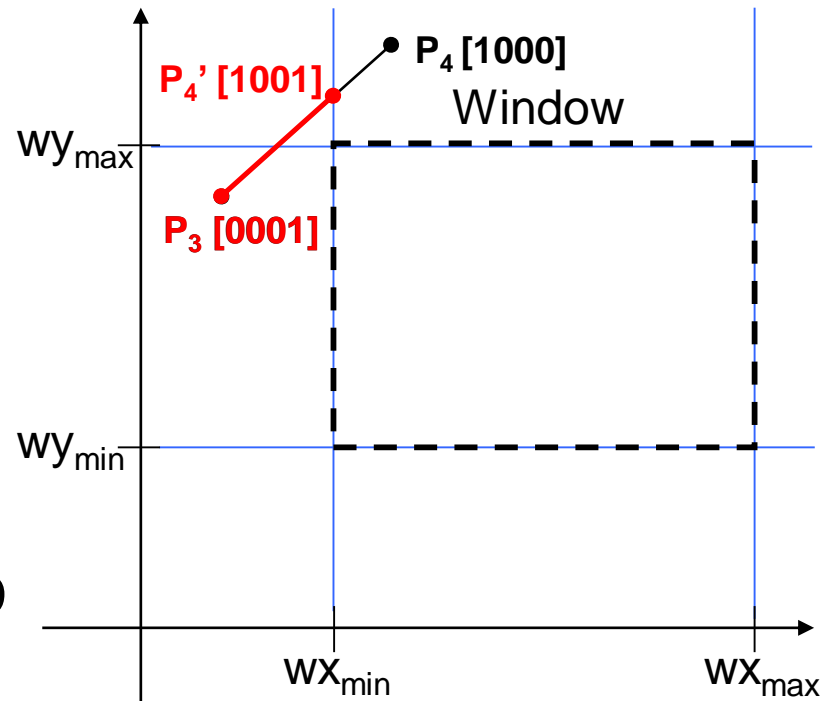
- Start at  $P_{10}$
- From the region codes of the two end-points we know the line doesn't cross the left or right boundary
- Calculate the intersection of the line with the bottom boundary to generate point  $P_{10}'$
- The line  $P_9$  to  $P_{10}'$  is completely inside the window so is retained



# Cohen-Sutherland Examples (cont...)

Consider the line  $P_3$  to  $P_4$  below

- Start at  $P_4$
- From the region codes of the two end-points we know the line crosses the left boundary so calculate the intersection point to generate  $P_4'$

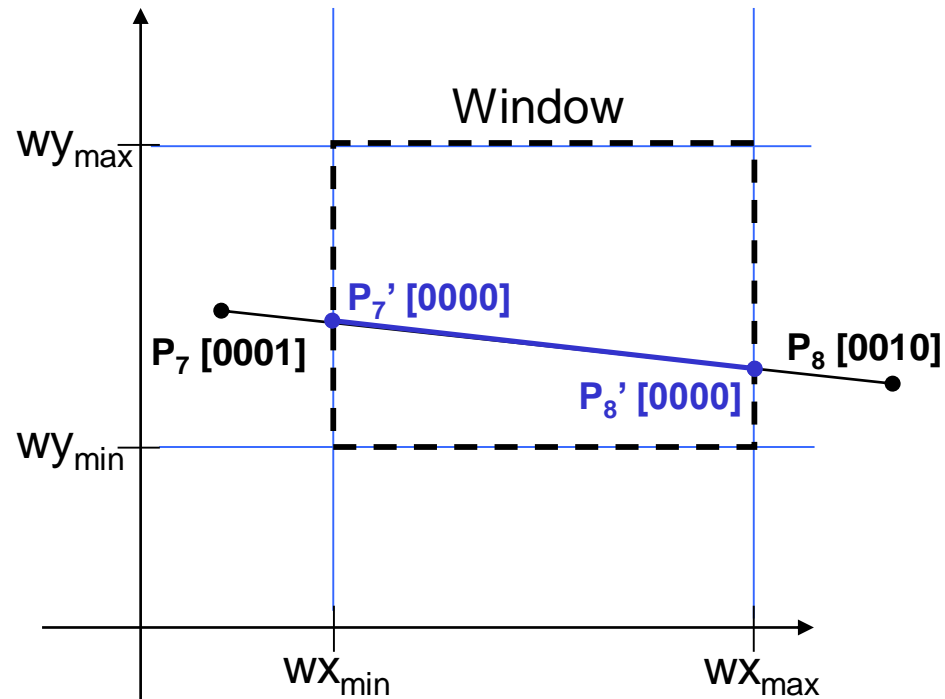


- The line  $P_3$  to  $P_4'$  is completely outside the window so is clipped

# Cohen-Sutherland Examples (cont...)

Consider the line  $P_7$  to  $P_8$  below

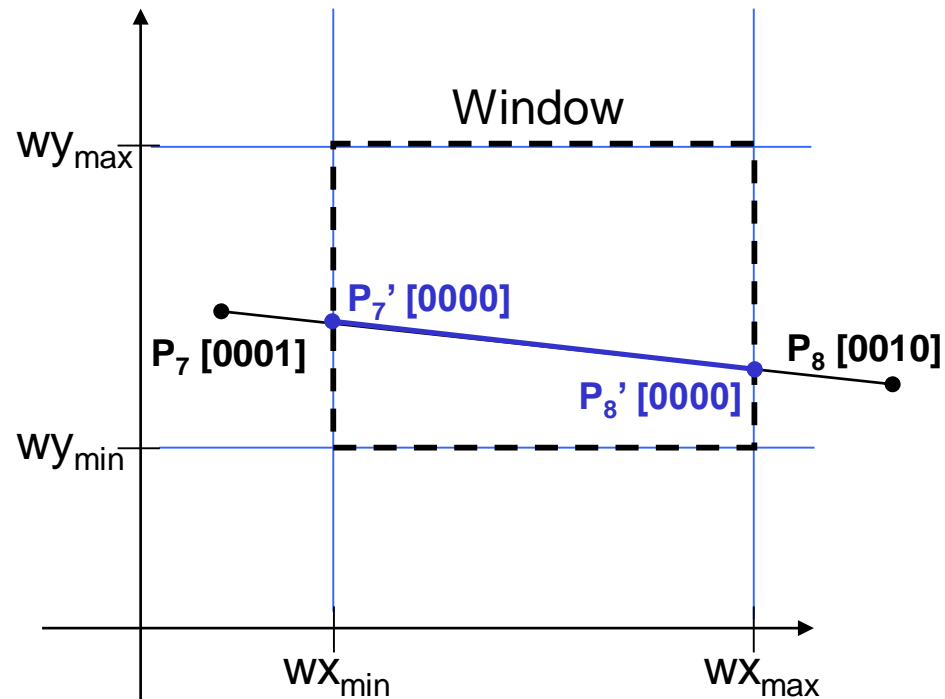
- Start at  $P_7$
- From the two region codes of the two end-points we know the line crosses the left boundary so calculate the intersection point to generate  $P_7'$



# Cohen-Sutherland Examples (cont...)

Consider the line  $P_7'$  to  $P_8$

- Start at  $P_8$
- Calculate the intersection with the right boundary to generate  $P_8'$
- $P_7'$  to  $P_8'$  is inside the window so is retained





# Calculating Line Intersections

Intersection points with the window boundaries are calculated using the line-equation parameters

- Consider a line with the end-points  $(x_1, y_1)$  and  $(x_2, y_2)$
- The y-coordinate of an intersection with a vertical window boundary can be calculated using:

$$y = y_1 + m (x_{boundary} - x_1)$$

where  $x_{boundary}$  can be set to either  $wx_{min}$  or  $wx_{max}$

# Calculating Line Intersections (cont...)

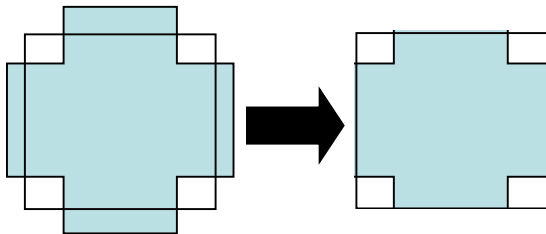
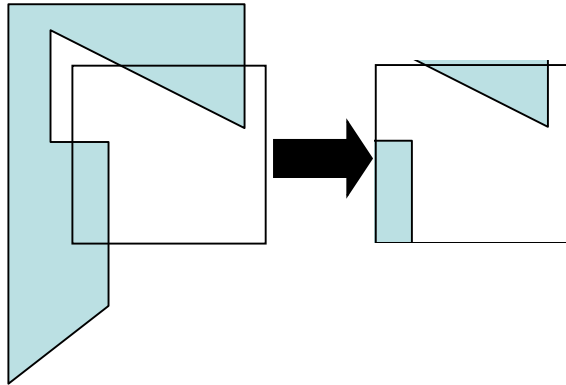
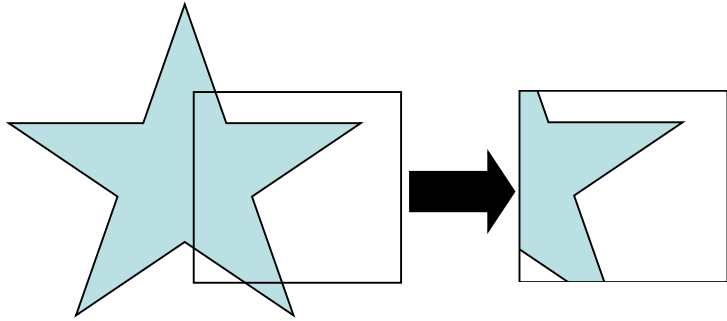
- The x-coordinate of an intersection with a horizontal window boundary can be calculated using:

$$x = x_1 + (y_{\text{boundary}} - y_1) / m$$

where  $y_{\text{boundary}}$  can be set to either  $wy_{\text{min}}$  or  $wy_{\text{max}}$

- $m$  is the slope of the line in question and can be calculated as  $m = (y_2 - y_1) / (x_2 - x_1)$

# Area Clipping



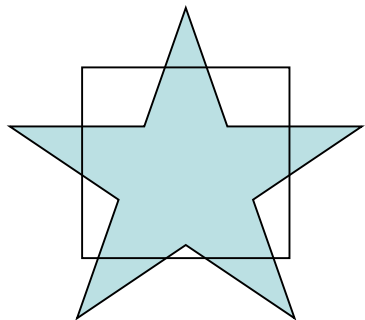
Similarly to lines, areas must be clipped to a window boundary

Consideration must be taken as to which portions of the area must be clipped

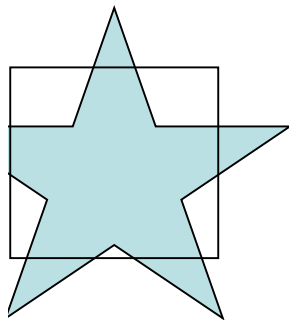
# Sutherland-Hodgman Area Clipping Algorithm

A technique for clipping areas developed by Sutherland & Hodgman

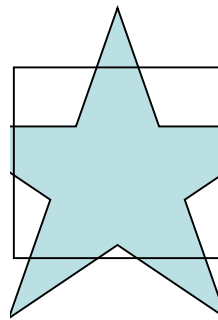
Put simply the polygon is clipped by comparing it against each boundary in turn



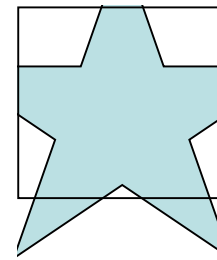
Original Area



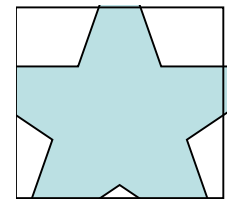
Clip Left



Clip Right



Clip Top



Clip Bottom

# Sutherland-Hodgman Area Clipping Algorithm (cont...)

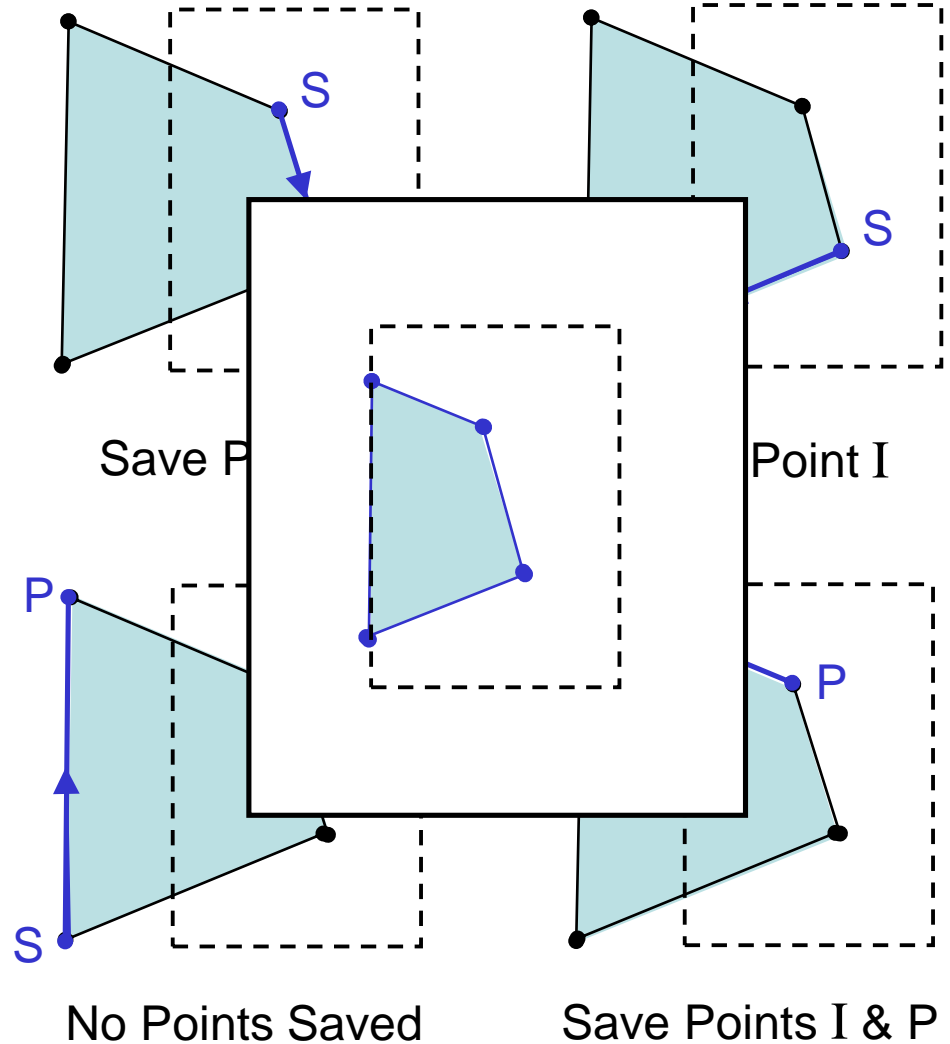
To clip an area against an individual boundary:

- Consider each vertex in turn against the boundary
- Vertices inside the boundary are saved for clipping against the next boundary
- Vertices outside the boundary are clipped
- If we proceed from a point inside the boundary to one outside, the intersection of the line with the boundary is saved
- If we cross from the outside to the inside intersection point and the vertex are saved

# Sutherland-Hodgman Example

Each example shows the point being processed (P) and the previous point (S)

Saved points define area clipped to the boundary in question



# Summary

Objects within a scene must be clipped to display the scene in a window

Because there are can be so many objects clipping must be extremely efficient

The Cohen-Sutherland algorithm can be used for line clipping

The Sutherland-Hodgman algorithm can be used for area clipping