طالبات - Girls

رياضيات ١١٠

MATH 110

الدوري النهائي

Sec 3.2,3.3,3.4,3.5,3.6

4.1,4.3

م. اشرف بركات

0504590132

١٥ ريال

ملاحظة: المذكرة لا ترد ولا تستبدل بعد الشراء

أكاديمية أشرف بركات
Ashraf Barakat Academy

لتدريس طلاب وطالبات جامعة الملك عبدالعزيز السنة التحضيرية كورس فيزياء ١١٠ و رياضيات ١١٠ وإحصاء ١١٠ و رياضيات ٢٠٢ ويحتوي الكورس على محاضرات فيديو مسجلة بالإضافة إلى مراجعات وحلول اختبارات للمهندس / أشرف بركات

للاشتراك أو الاستفسار يمكن الاتصال على جوال ٠٥٠٤٥٩٠١٣٢ أو برسالة على الواتس .

مع خالص الدعاء بالسداد والتوفيق.

أكاديمية أشرف بركات

The Product and quotient Rules
قاعدتي الضرب والقسمة

$f(x)$	$f'(x)$
① $f(x) = g(x)\cdot h(x)$	$f'(x) = g'(x)\cdot h(x) + g(x)\cdot h'(x)$ مشتقة حاصل الضرب = مشتقة الأول x الثاني + مشتقة الثاني x الأول
② $f(x) = \dfrac{g(x)}{h(x)}$	$f'(x) = \dfrac{g'(x)\, h(x) - g(x)\cdot h'(x)}{[h(x)]^2}$ مشتقة القسمة = $\dfrac{\text{مشتقة البسط x المقام - مشتقة المقام x البسط}}{(\text{المقام})^2}$

Ex: If $y = (x-3)(x-2)$, then y'

ⓐ $2x+1$ ⓑ $2x-1$ ⓒ $2x+5$ ⓓ $2x-5$

---

solution
$$y = (x-3)(x-2)$$

$$y' = 1(x-2) + (1)(x-3) = x-2+x-3$$
$$= 2x-5 \quad ⓓ$$

حل آخر

$$y = (x-3)(x-2) = x^2 - 2x - 3x + 6$$
$$= x^2 - 5x + 6$$

$$y' = 2x - 5 \quad ⓓ$$

Ex: If $y = \dfrac{x+3}{x-2}$, then $y' =$

ⓐ $-\dfrac{1}{(x-2)^2}$ ⓑ $-\dfrac{5}{(x-2)^2}$ ⓒ $\dfrac{5}{(x-2)^2}$

---

solution:
$$y = \frac{x+3}{x-2} \Rightarrow y' = \frac{(1)(x-2) - (1)(x+3)}{(x-2)^2}$$

$$y' = \frac{x-2-x-3}{(x-2)^2} = \frac{-5}{(x-2)^2} \quad ⓑ$$

Ex) If $f(x) = (x^2+3)(x^3-1)$, then $\frac{d}{dx}(f(x)) =$

ⓐ $5x^4 - 9x^2 + 2x$ ⓑ $5x^4 + 9x^2 - 2x$

<u>solution</u> $f(x) = (x^2+3)(x^3-1)$

$$\frac{df}{dx} = 2x(x^3-1) + 3x^2(x^2+3)$$

$$= 2x^4 - 2x + 3x^4 + 9x^2$$

$$= 5x^4 + 9x^2 - 2x \quad ⓑ$$

Ex) If $f(x) = x^2 e^x$, then $f'(x) =$

ⓐ $xe^x(x+2)$ ⓑ $e^x(x+2)$

ⓒ $x^2e^x(x+2)$ ⓓ $x^2e^x(x+1)$

<u>solution</u> $f(x) = x^2 e^x$

$$f'(x) = 2xe^x + x^2e^x$$

عامل مشترك

$$= xe^x(2+x)$$

$$= xe^x(x+2) \quad ⓐ$$

Ex If $g(x) = \dfrac{3x-1}{2x+1}$, then $g'(x) =$

(a) $\dfrac{5}{(2x+1)^2}$ (b) $\dfrac{-5}{(2x+1)^2}$

---

solution $g(x) = \dfrac{3x-1}{2x+1}$

$$g'(x) = \frac{3(2x+1) - 2(3x-1)}{(2x+1)^2}$$

$$= \frac{\cancel{6x} + 3 - \cancel{6x} + 2}{(2x+1)^2} = \frac{5}{(2x+1)^2}$$ (b)

Ex If $f(x) = \dfrac{x^3}{1-x^2}$, then $f'(x) =$

(a) $\dfrac{x^2(3-x^2)}{(1-x^2)^2}$ (b) $\dfrac{x^2(3+x^2)}{(1-x^2)^2}$

---

solution $f(x) = \dfrac{x^3}{1-x^2}$

$$f'(x) = \frac{3x^2(1-x^2) - (-2x)x^3}{(1-x^2)^2}$$

$$= \frac{3x^2 - 3x^4 + 2x^4}{(1-x^2)^2} = \frac{x^2(3-x^2)}{(1-x^2)^2}$$ (a)

Ex) If $f(x) = xe^x$, find $f'(x)$ and $f^{(n)}(x)$

Solution $\quad f(x) = xe^x$

$$f'(x) = 1 \cdot e^x + xe^x = (x+1)e^x$$

$$f''(x) = (1)e^x + (x+1)e^x = (x+2)e^x$$

$$f'''(x) = (1)e^x + (x+2)e^x = (x+3)e^x$$

$$f^{(4)}(x) = (1)e^x + (x+3)e^x = (x+4)e^x$$

$$\boxed{f^{(n)}(x) = (x+n)e^x}$$

Ex If $f(t) = \sqrt{t}\,(a+bt)$, find $f'(t)$

<u>solution</u> $f(t) = \sqrt{t}\,(a+bt)$

$$f'(t) = \frac{1}{2\sqrt{t}}(a+bt) + \sqrt{t}\,(b).$$

$$= \frac{1}{2\sqrt{t}}(a+bt) + b\sqrt{t} \cdot \frac{2\sqrt{t}}{2\sqrt{t}}$$

$$= \frac{a+bt+2bt}{2\sqrt{t}} = \frac{a+3bt}{2\sqrt{t}}$$

Ex If $f(x) = \sqrt{x}\, g(x)$ and $g(4) = 2$, $g'(4) = 3$, find $f'(4)$

<u>solution</u> $f(x) = \sqrt{x}\, g(x)$

$$f'(x) = \frac{1}{2\sqrt{x}} g(x) + \sqrt{x} \cdot g'(x)$$

$$f'(4) = \frac{1}{2\sqrt{4}} g(4) + \sqrt{4} \cdot g'(4)$$

$$= \frac{1}{4}(2) + \sqrt{4}\,(3)$$

$$= \frac{1}{2} + \frac{6}{1} = \frac{13}{2}$$

Ex: If $f(x) = e^x\, g(x)$, $g(0) = 2$, $g'(0) = 5$

find $f'(0)$

<u>Solution:</u> $f(x) = e^x\, g(x)$

$f'(x) = e^x\, g(x) + e^x\, g'(x)$

$f'(0) = e^0 \cdot g(0) + e^0 \cdot g'(0)$

$= 1 \cdot (2) + 1 \cdot (5) = 2 + 5 = 7$

Ex: If $f(x) = \dfrac{x^2}{1+x}$ find $f'(1)$

<u>Solution:</u> $f'(x) = \dfrac{2x(1+x) - (1)(x^2)}{(1+x)^2}$

$f'(x) = \dfrac{2x + 2x^2 - x^2}{(1+x)^2} = \dfrac{2x + x^2}{(1+x)^2}$

$f'(1) = \dfrac{2(1) + (1)^2}{(1+1)^2} = \dfrac{2+1}{4} = \dfrac{3}{4}$

Sec 3-2

8

Ex If f is a differentiable function, find an expression of the derivative of each function

ⓐ $y = x^2 f(x)$ ⓑ $y = \dfrac{x^2}{f(x)}$

Solution:

ⓐ $y = x^2 f(x)$

$$y' = 2x f(x) + x^2 f'(x)$$

ⓑ $y = \dfrac{x^2}{f(x)}$

$$y' = \frac{2x f(x) - x^2 f(x)}{(f(x))^2}$$

Ex. Find the equation of the bangent line to the cure $y = \frac{x}{x-1}$ at $(2,2)$

ⓐ $y - x + 4 = 0$ ⓑ $y - x - 4 = 0$
ⓒ $y + x - 4 = 0$ ⓓ $y + x + 4 = 0$

Solution $y = \frac{x}{x-1}$

$$y' = \frac{(1)(x-1)-(1)x}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$$m = y'\big|_{x=2} = \frac{-1}{(2-1)^2} = \frac{-1}{1} = -1$$

$m = -1$ $\quad \overset{x_1, y_1}{(2,2)}$

$$y = m(x - x_1) + y_1$$

$$y = (-1)(x-2) + 2 \Rightarrow y = -x + 2 + 2$$

$$y = -x + 4 \Rightarrow y + x - 4 = 0$$

ⓒ

Ex: If $f(x)$ and $g(x)$ are differentiable at $x=-2$ and $f(-2)=1$, $f'(-2)=3$, $g(-2)=7$, $g'(-2)=3$, then find

① $\frac{d}{dx}(f\cdot g)\Big|_{x=-2}$

$$\frac{d}{dx}(f\cdot g)\Big|_{x=-2} = f'(-2)\,g(-2) + f(-2)\,g'(-2)$$

$$=(3)(7)+(1)(3) = 24$$

② $\frac{d}{dx}\left(\frac{f}{g}\right)\Big|_{x=-2}$

$$\frac{d}{dx}\left(\frac{f}{g}\right)\Big|_{x=-2} = \frac{f'(-2)\,g(-2) - f(-2)\,g'(-2)}{(g(-2))^2}$$

$$= \frac{3(7)-(1)(-3)}{(7)^2}$$

$$= \frac{21-3}{49} = \frac{18}{49}$$

Sec 3-3 ①

Derivatives of trigonometric functions

مشتقات الدوال العكسية

$f(x)$	$f'(x)$
$f(x) = \sin x$ $f(x) = \cos x$	$f'(x) = \cos x$ $f'(x) = -\sin x$
$f(x) = \tan x$ $f(x) = \cot x$	$f'(x) = \sec^2 x$ $f'(x) = -\csc^2 x$
$f(x) = \sec x$ $f(x) = \csc x$	$f(x) = \sec x \tan x$ $f'(x) = -\csc x \cot x$

Ex If $y = x - 3\sin x$, then $y' =$

ⓐ $y' = 1 + 3\cos x$ ⓑ $y' = 1 - 3\cos x$

Solution $y = x - 3\sin x$

$y' = 1 - 3\cos x$ ⓑ

Ex If $y = \tan x - x$, then $y' =$

ⓐ $y' = \sec^2 x - 1$ ⓑ $y = \sec x - 1$

Solution: $y = \tan x - x$

$y' = \sec^2 x - 1$ ⓐ

Ex If $y = \sec x \tan x$, then $y' =$

ⓐ $y' = \sec x(\tan x - \sec x)$ ⓑ $y = \sec x(\tan x + \sec x)$

Solution $y = \sec x \tan x$

$y' = \sec x \tan x + \sec^2 x = \sec x(\tan x + \sec x)$

ⓑ

Ex. If $f(x) = \csc x \cdot \cot x$, then $f'(x) =$

ⓐ $-\csc x(\cot^2 x + \csc^2 x)$ ⓑ $\csc x(\cot^2 x - \csc^2 x)$

---

solution $f(x) = \csc x \cot x$

$$f'(x) = -\csc x \cot x \cdot \cot x + \csc x(-\csc^2 x)$$

$$= -\csc x(\cot^2 x + \csc^2 x) \quad ⓐ$$

Ex. If $y = \dfrac{x}{2-\tan x}$, then $y' =$

ⓐ $y' = \dfrac{2-\tan x + \sec^2 x}{(2-\tan x)^2}$ ⓑ $y' = \dfrac{2-\tan x + x\sec^2 x}{(2-\tan x)^2}$

---

solution $y = \dfrac{x}{2-\tan x}$

$$y' = \frac{(1)(2-\tan x) - x(0-\sec^2 x)}{(2-\tan x)^2}$$

$$y' = \frac{2-\tan x + x\sec^2 x}{(2-\tan x)^2} \quad ⓑ$$

Ex If $f(x) = \sin x$, find

① $f^{(17)}(x)$ ② $f^{(34)}(x)$

③ $f^{(67)}(x)$ ④ $f^{(104)}(x)$

---

solution:

$$f(x) = \sin x$$
$$f'(x) = \cos x$$
$$f''(x) = -\sin x$$
$$f'''(x) = -\cos x$$
$$f^{(4)}(x) = \sin x$$

المشتقة تكرر كل ٤ مرات

$$\begin{array}{r} 4 \\ 4\,\overline{)\,17} \\ \underline{16} \\ 1 \end{array}$$

① $f^{(17)}(x) = \dfrac{d^{17}}{dx^{17}}(\sin x) = \dfrac{d}{dx}(\sin x) = \cos x$

$$\begin{array}{r} 8 \\ 4\,\overline{)\,34} \\ \underline{32} \\ 2 \end{array}$$

② $f^{(34)}(x) = f''(x) = -\sin x$

$$\begin{array}{r} 16 \\ 4\,\overline{)\,67} \\ \underline{4} \\ 27 \\ \underline{24} \\ 3 \end{array}$$

③ $f^{(67)}(x) = f'''(x) = -\cos x$

$$\begin{array}{r} 1 \\ 4\,\overline{)\,4} \\ \underline{4} \\ 0 \end{array}$$

④ $f^{(104)}(x) = f(x) = \sin x$

Ex Find $\dfrac{d^{127}}{dx^{127}}(\cos x)$

---

solution

$$f(x) = \cos x$$

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x$$

$$f'''(x) = \sin x$$

$$f^{4}(x) = \cos x$$

$$\begin{array}{r} 6 \\ 4\overline{)\,27} \\ 24 \\ \hline 3 \end{array}$$

$$\frac{d^{127}}{dx^{127}}(\cos x) = \frac{d^{3}}{dx^{3}}(\cos x) = \sin x$$

Ex Find $D^{137}(\cos x)$

$$\begin{array}{r} 9 \\ 4\overline{)\,37} \\ 36 \\ \hline 1 \end{array}$$

$$f(x) = \cos x$$

$$f'(x) = -\sin x$$

$$D^{137}(\cos x) = f'(x) = -\sin x$$

Math 110

Sec 3-4

1

2016-2017

1

The Chain Rule
قاعدة التسلسل

y	y'
① $y = (f(x))^n$	$y' = n\,(f(x))^{n-1} \cdot f'(x)$
Ex $y = (x^3+1)^4$	$y' = 4(x^3+1)^{4-1}(3x^2)$ $= 12x^2(x^3+1)^3$
② $y = \sqrt{f(x)}$	$y' = \dfrac{1}{2\sqrt{f(x)}} \cdot f'(x)$
Ex $y = \sqrt{x^2+1}$	$y' = \dfrac{2x}{2\sqrt{x^2+1}}$ $= \dfrac{x}{\sqrt{x^2+1}}$

خلاصة قاعدة التسلسل:

نعوض عن x في المشتقة العادية بـ $f(x)$ ونضرب في مشتقة $f(x)$

$x \longrightarrow f(x)$, $\cdot\,(f'(x))$

sec (3-4) 2

y	y'
3 $y = e^{f(x)}$	$y' = e^{f(x)} \cdot f'(x)$
Ex $y = e^{\sin x}$	$y' = e^{\sin x} \cdot \cos x$
4 $y = a^x$	$y' = a^x \ln a$
$y = a^{f(x)}$	$y' = a^{f(x)} \cdot f'(x) \cdot \ln a$
Ex $y = 3^{x^2}$	$y' = 3^{x^2} \cdot (2x) \cdot \ln 3$
5 $y = \sin(f(x))$	$y' = \cos(f(x)) \cdot f'(x)$
Ex $y = \sin 4x$	$y' = 4(\cos 4x) = 4\cos 4x$
6 $y = \cos(f(x))$	$y' = -\sin(f(x)) \cdot f'(x)$
Ex $y = \cos(a^3 + x^3)$	$y' = -\sin(a^3 + x^3) \cdot 3x^2$

Sec (3-4)

3

y	y'
(7) $y = \tan(f(x))$	$y' = \sec^2(f(x)) \cdot f'(x)$
Ex $y = \tan(\sin x)$	$y' = \sec^2(\sin x) \cdot \cos f(x)$
(8) $y = \underline{c}ot(f(x))$	$y' = -\csc^2(f(x)) \cdot f'(x)$
Ex $y = \underline{c}ot(e^x)$	$y' = -\csc^2(e^x) \cdot e^x$ $= -e^x \cdot \csc^2(e^x)$
(9) $y = \sec(f(x))$	$y' = \sec(f(x)) \cdot \tan(f(x)) \cdot f'(x)$
Ex $y = \sec(5x)$	$y' = \sec(5x) \tan(5x) \cdot 5$ $y' = 5 \sec(5x) \tan(5x)$
(10) $y = \csc(f(x))$	$y' = -\csc(f(x)) \cdot \cot(f(x)) \cdot f'(x)$
Ex $y = \csc(x^2)$	$y' = -\csc(x^2) \cot(x^2) \cdot 2x$ $y' = -2x \csc(x^2) \cot(x^2)$

Ex Find y'

1) $y = (x^4 - 5)^9$

a) $36x^2(x^4-5)^8$

b) $36x^2(x^4+5)^8$

c) $36x^3(x^4-5)^8$

d) $36x^3(x^4+5)^8$

$y = (x^4 - 5)^9$

$y' = 9(x^4 - 5)^{9-1}(4x^3) = 36x^3(x^4-5)^8$ c

---

2) $y = (x^3+2)^5$

a) $5x^2(x^3+2)^4$

b) $15x^2(x^3+2)^4$

c) $5x^2(x^3+2)^6$

d) $15x^2(x^3+2)^3$

$y = (x^3+2)^5$

$y' = 5(x^3+2)^{5-1}(3x^2) = 15x^2(x^3+2)^4$

b

Sec(3-4) 5

Ex) If $y = \dfrac{1}{(x^2+5x)^3}$, then $y' =$

(a) $-3(2x+5)^{-4}(x^2+5x)$

(b) $-3(2x+5)^4(x^2+5x)^{-4}$

(c) $-3(2x+5)(x^2+5x)^{-4}$

(d) $3(2x+5)(x^2+5x)^{-4}$

$$y = \frac{1}{(x^2+5x)^3} = (x^2+5x)^{-3}$$

$$y' = -3(x^2+5x)^{-3-1}(2x+5)$$

$$y' = -3(2x+5)(x^2+5x)^{-4}$$

c

Ex If $f(x) = (3x^2-3)^3$, then $\frac{df}{dx} =$

ⓐ $18x^2(3x^2-3)^2$ ⓑ $18x(3x^2-3)^2$

<u>solution</u>

$$f(x) = (3x^2-3)^3$$

$$f'(x) = 3(3x^2-3)^2(6x)$$

$$= 18x(3x^2-3)^2 \quad ⓑ$$

Ex If $f(x) = 3e^{3x}$, find $f''(0)$

ⓐ 9 ⓑ 3 ⓒ -27 ⓓ 27

<u>solution</u>

$$f(x) = 3e^{3x}$$

$$f'(x) = 3(3)e^{3x} = 9e^{3x}$$

$$f''(x) = 9(3)e^{3x} = 27e^{3x}$$

$$f''(0) = 27e^0 = 27(1)$$

$$= 27 \quad ⓓ$$

Ex $\frac{d}{dx}(3^x - 4e^x) =$

a $3^x - 4e^x$ b $3^x + 4e^x$ c $3^x . \ln 3 - 4e^x$

---

solution $\frac{d}{dx}(3^x - 4e^x) = 3^x . \ln 3 - 4e^x$

c

Ex If $f(x) = \sqrt{x^2 + 6x}$, then $f'(x) =$

a $\frac{x+3}{\sqrt{x^2+6x}}$ b $\frac{x+3}{2\sqrt{x^2+6x}}$

---

solution $f(x) = \sqrt{x^2 + 6x}$

$$f'(x) = \frac{2x+6}{2\sqrt{x^2+6x}}$$

$$= \frac{2(x+3)}{2\sqrt{x^2+6x}} = \frac{x+3}{\sqrt{x^2+6x}}$$

a

Sec(3-4) 8

Ex. If $y = \sqrt{x^4 - 4x^3}$, then $y' =$

(a) $\dfrac{2x^2(x+3)}{(x^4+4x^3)^{1/2}}$

(b) $\dfrac{2x^2(x-3)}{(x^4+4x^3)^{1/2}}$

(c) $\dfrac{2x^2(x-3)}{(x^4-4x^3)^{1/2}}$

(d) $\dfrac{2x^2(x+3)}{(x^4-4x^3)^{1/2}}$

$$y = \sqrt{x^4 - 4x^3}$$

$$y' = \frac{4x^3 - 12x^2}{2\sqrt{x^4 - 4x^3}} = \frac{\cancel{4}^{2}x^2(x-3)}{\cancel{2}\sqrt{x^4 - 4x^3}}$$

$$y' = \frac{2x^2(x-3)}{(x^4 - 4x^3)^{1/2}}$$

(c)

Sec (3-4)

Ex. If $y = \dfrac{6}{\sqrt{x^2+4}}$, then $y' =$

(a) $\dfrac{6x}{(x^2+4)^{3/2}}$

(b) $\dfrac{-6x}{(x^2+4)^{3/2}}$

(c) $\dfrac{6x}{(x^2-4)^{3/2}}$

(d) $\dfrac{-6x}{(x^2-4)^{3/2}}$

$$y = \frac{6}{\sqrt{x^2+4}} = \frac{6}{(x^2+4)^{1/2}}$$

$$y = 6(x^2+4)^{-1/2}$$

$$y' = 6\left(-\frac{1}{2}\right)(x^2+4)^{-\frac{1}{2}-1}(2x)$$

$$y' = -6x(x^2+4)^{-3/2}$$

$$y' = \frac{-6x}{(x^2+4)^{3/2}}$$

(b)

Ex: If $f(\theta) = \cos(3\theta)$ find $f'(\frac{\pi}{3})$

ⓐ -3 ⓑ 3 ⓒ 0 ⓓ

---

solution

$$f(\theta) = \cos(3\theta)$$

$$f'(\theta) = -3\sin(3\theta)$$

$$f'(\frac{\pi}{3}) = -3\sin(\cancel{3}\cdot\frac{\pi}{\cancel{3}})$$

$$= -3\sin\pi$$

$$= -3(0) = 0$$

Ex: If $f(\theta) = \sin(\frac{\theta}{2})$, then $f'(0) =$

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solution:

$$f(\theta) = \sin(\frac{\theta}{2}) = \sin(\frac{1}{2}\theta)$$

$$f'(\theta) = \frac{1}{2}\cos(\frac{1}{2}\theta)$$

$$f'(0) = \frac{1}{2}\cos(\frac{1}{2}(0))$$

$$= \frac{1}{2}\cos(0) = \frac{1}{2}(1) = \frac{1}{2}$$

Ex9 Find y'

(1) $y = (x+\sec x)^3$

(a) $3(x+\sec x)^2(1+\sec x\tan x)$

(b) $3(x+\sec x)^2(x+\sec x\tan x)$

(c) $3\sec x\tan x(x+\sec x)^2$

(d) $-3\sec x\tan x(x+\sec x)^2$

$y = (x+\sec x)^3$

$y' = 3(x+\sec x)^2.(1+\sec x\tan x)$

(a)

Sec (3-4) (12)

(2) $y = x \sin x^2$

(a) $\sin x^2 + 2x \cos x^2$

(b) $\sin x^2 + 2x^2 \cos x^2$

(c) $\sin x + 2x \cos x^2$

(d) $x \sin x^2 + 2x \cos x^2$

$$y = x \sin x^2$$

$$y' = 1 \cdot \sin x^2 + x \cdot 2x \cos x^2$$

$$y' = \sin x^2 + 2x^2 \cos x^2$$

(b)

Sec (3-4)

13

(3) $y = \sin(\tan x^2)$

(a) $2x \sec x^2 . \cos(\tan x^2)$

(b) $2x \cos(\tan x^2)$

(c) $2x \sec x \cos(\tan x^2)$

(d) $\underline{2x \sec^2 x^2 \cos(\tan x^2)}$

$y = \sin(\tan x^2)$

$y' = \cos(\tan x^2) . \sec^2 x^2 . 2x$

$y' = 2x \sec^2 x^2 . \cos(\tan x^2)$

(d)

Sec(3-4) (14)

(4) $y = \sin\sqrt{x^2+1}$

(a) $y' = \dfrac{x}{\sqrt{x^2+1}} \cos(x^2+1)$

(b) $y' = \dfrac{1}{\sqrt{x^2+1}} \cos(x^2+1)$

(c) $y' = \dfrac{x}{\sqrt{x^2+1}} \cos(\sqrt{x^2+1})$

(d) $y' = \dfrac{1}{\sqrt{x^2+1}} \cos(\sqrt{x^2+1})$

$y = \sin\sqrt{x^2+1}$

$y' = \cos(\sqrt{x^2+1}) \cdot \dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}$

$y' = \dfrac{x}{\sqrt{x^2+1}} \cos(\sqrt{x^2+1})$

(c)

sec (3-4)

15

(5) $y = \sin^3(x^2+5)$

(a) $y' = 6x\cos(x^2+5)$

(b) $y' = 6x^2\cos(x^2+5)\cdot\sin3(x^2+5)$

(c) $y' = 6x\cos(x^2+5)\cdot\sin^2(x^2+5)$

(d) $y' = 6x^2\cos(x^2+5)\cdot\sin^2(x^2+5)$

$y = [\sin(x^2+5)]^3$

$y' = 3(\sin(x^2+5))^2 \cos(x^2+5)\cdot 2x$

$y' = 6x\cos(x^2+5)\cdot\sin^2(x^2+5)$

(c)

Ex: If $y = 5^{\tan x}$, then $y' =$

(a) $5^{\tan x} \sec^2 x \ln 5$ (b) $5^{\tan x} \sec^2 x$ (c) $5^{\tan x} \ln 5$

Solution:

$$y = 5^{\tan x}$$

$$y' = 5^{\tan x} \sec^2 x \ln 5 \quad (a)$$

Ex: If $y = 3^x \cot x$, then $y' =$

(a) $3^x \ln 3 \cot x + 3^x \sec^2 x$ (b) $3^x \cot x + 3^x \sec^2 x$

(c) $3^x \cot x - 3^x \sec^2 x$ (d) $3^x \ln 3 \cot x - 3^x \csc^2 x$

Solution:

$$y = 3^x \cot x$$

$$y' = 3^x (\ln 3) \cot x + 3^x (-\csc^2 x)$$

$$= 3^x \ln 3 \cot x - 3^x \csc^2 x \quad (d)$$

Ex: If $y = \dfrac{5^x}{\cot x}$ then $y' =$

(a) $\dfrac{5^x(\cot x + \csc^2 x)}{\cot^2 x}$

(b) $\dfrac{5^x(\ln 5 + \csc^2 x)}{\cot^2 x}$

(c) $\dfrac{5^x(\cot x - \csc^2 x)}{\cot^2 x}$

(d) $\dfrac{5^x(\ln 5 \cot x + \csc^2 x)}{\cot^2 x}$

---

solution:

$$y = \frac{5^x}{\cot x}$$

$$y' = \frac{5^x \ln 5 \cot x - (-\csc^2 x) 5^x}{(\cot x)^2}$$

$$= \frac{5^x(\ln 5 \cot x + \csc^2 x)}{\cot^2 x}$$

(d)

Ex If $y = e^{2x}$, then $y^{(6)} =$

(a) $128e^{2x}$ (b) $16e^{2x}$ (c) $64e^{2x}$ (d) $32e^{2x}$

Solution: $y = e^{2x}$

$y' = 2e^{2x}$ $y'' = 2(2e^{2x}) = 4e^{2x}$

$y''' = 8e^{2x}$ $y^{(4)} = 16e^{2x}$ $y^{(5)} = 32e^{2x}$

$y^{(6)} = 64e^{2x}$ (c) $\boxed{y^{(n)} = 2^n e^{2x}}$ دحفظ

Ex If $y = x^{-2} e^{\sin x}$, then $y' =$

(a) $x^{-3} e^{\sin x}(x\cos x - 2)$ (b) $x^{-3} e^{\sin x}(\cos x - 2)$

(c) $x^{-3} e^{\sin x}(x\cos x - 1)$ (d) $x^{-2} e^{\sin x}(x\cos x - 2)$

Solution: $y = x^{-2} e^{\sin x}$

$y' = -2x^{-3} e^{\sin x} + x^{-2}(\cos x\, e^{\sin x})$

$= x^{-3} e^{\sin x}(-2 + x\cos x)$

$= x^{-3} e^{\sin x}(x\cos x - 2)$ (a)

Sec 3-5

1

الاشتقاق الضمني

Implicit Differentiation

الدالة	المشتقة
y	y'
y^2	$2y\,y'$
y^n	$n\,y^{n-1}\,y'$
x^2y^3	$2xy^3 + 3x^2y^2y'$
$\sin y$	$y'\cos y$
$\sin(xy)$	حاصل ضرب $(\cos xy)(1\cdot y + xy')$
$\sin(x+y)$	$(\cos(x+y))\,(1+y')$
e^y	$y'e^y$

Ex If $x^2 - y^2 = 4$, then $y' =$

(a) $-\frac{y}{x}$ (b) $-\frac{x}{y}$ (c) $\frac{x}{y}$ (d) $\frac{y}{x}$

Solution:

$$x^2 - y^2 = 4$$ نشتق الطرفين بالنسبة لـ x

$$2x - 2y\,y' = 0$$ نجعل حدود y' في طرف والباقي في طرف

$$\cancel{2}x = \cancel{2}y\,y' \xrightarrow[(\div y)]{} \boxed{y' = \frac{x}{y}}$$ (c)

Ex If $x^2 + y^2 = 3xy + 7$, then $y' =$

(a) $\frac{2x+y}{3x-2y}$ (b) $\frac{3y-2x}{2y-3x}$ (c) $\frac{2x}{3-2y}$ (d) $\frac{2x}{y}$

Solution:

$$x^2 + y^2 = 3xy + 7$$ نشتق

$$2x + 2yy' = 3y + 3xy' + 0$$

$$2yy' - 3xy' = 3y - 2x$$

$$y'(2y - 3x) = 3y - 2x$$

$$\boxed{y' = \frac{3y-2x}{2y-3x}}$$ (b)

3

Ex If $e^{2y} = x$, then $y' =$

(a) $y' = e^{2y}$ (b) $y' = 2e^{-2y}$ (c) $y' = e^{-2y}$

---

solution $e^{2y} = x$ نشتق بالنسبة لـ x

$2y' e^{2y} = 1$ $\div (2e^{2y})$

$y' = \frac{\cancel{2}}{\cancel{2} e^{2y}} \Rightarrow y' = \frac{1}{e^{2y}}$

$\boxed{y' = e^{-2y}}$ (c)

Ex If $x^3 + 3x \cdot y = 10$, then $y' =$

$y' = x + \frac{y}{x}$ (b) $y' = -(x + y/x)$

---

solution $x^3 + 3xy = 10$

$3x^2 + 3(y) + 3xy' = 0$ $\div 3$

$x^2 + y + xy' = 0 \Rightarrow xy' = -x^2 - y = -(x^2 + y)$

$y' = -\left(\frac{x^2}{x} + \frac{y}{x}\right) \Rightarrow y' = -(x + y/x)$ (b)

Sec (3-5)

4

Ex If $x^2 - 5y^2 + \sin y = 0$, then $y' =$

(a) $\dfrac{2}{10y + \cos y}$ (b) $\dfrac{2x}{10y - \cos y}$ (c) $\dfrac{2x}{10y + \cos y}$

Solution:

$$x^2 - 5y^2 + \sin y = 0$$

نشتق

$$2x - 10y\,y' + y'\cos y = 0$$

$$2x = 10y\,y' - y'\cos y$$

$$2x = y'(10y - \cos y)$$

$$\boxed{y' = \frac{2x}{10y - \cos y}}$$

(b)

Sec (3-5)

5

Ex: Find y'

1. $x^2y^3 + 3x = 5 + \cot y$

$$2xy^3 + 3x^2y^2y' + 3 = 0 - y'\csc^2 y$$ نشتق 1

$$3x^2y^2y' + y'\csc^2 y = -3 - 2xy^3$$

$$y'(3x^2y^2 + \csc^2 y) = -3 - 2xy^3$$

$$\boxed{y' = \frac{-3 - 2xy^3}{3x^2y^2 + \csc^2 y}}$$

2. $y = x^2 + \sin xy$

$$y' = 2x + (\cos xy)\cdot(1\cdot y + xy')$$ نشتق

$$y' = 2x + y\cos xy + xy'\cos xy$$

$$y' - xy'\cos xy = 2x + y\cos xy$$

$$y'(1 - x\cos xy) = 2x + y\cos xy$$

$$\boxed{y' = \frac{2x + y\cos xy}{1 - x\cos xy}}$$

Sec (3-5) 6

3 $y = \dfrac{x}{x+y^2}$ أوجد y'

$y(x+y^2) = x$

$xy + y^3 = x$ نشتق

$1\cdot y + xy' + 3y^2y' = 1$

$xy' + 3y^2y' = 1-y$

$y'(x+3y^2) = 1-y$

$$\boxed{y' = \frac{1-y}{x+3y^2}}$$

Ex. If $x^3+y^3=6xy$

1. find y'
2. find the tangent to the curve $x^3+y^3=6xy$ at the point $(3,3)$

Solution ①

$$x^3+y^3=6xy$$

$$3x^2+3y^2y'=6y+6xy'$$

$$3y^2y'-6xy'=6y-3x^2$$

$$y'(3y^2-6x)=6y-3x^2$$

$$y'=\frac{6y-3x^2}{3y^2-6x}\xrightarrow{\text{÷ ٣ بسطاً ومقاماً}}y'=\frac{2y-x^2}{y^2-2x}$$

②

$$\text{at }(3,3)\Rightarrow m=\frac{2(3)-(3)^2}{(3)^2-2(3)}=\frac{6-9}{9-6}=\frac{-3}{3}=-1$$

$$y=m(x-x_1)+y_1\Rightarrow y=-1(x-3)+3$$

$$y=-x+3+3\Rightarrow\boxed{y+x=6}$$

Sec (3-5)

8

Derivatives of inverse trigometric functions

مشتقات الدوال المثلثية العكسية

y	y'
1) $y = \sin^{-1} x$	$y' = \dfrac{1}{\sqrt{1 - x^2}}$
$y = \sin^{-1}(f(x))$	$y' = \dfrac{f'(x)}{\sqrt{1 - (f(x))^2}}$
Ex) $y = \sin^{-1}(x^2)$	$y' = \dfrac{2x}{\sqrt{1 - (x^2)^2}}$
2) $y = \cos^{-1} x$	$y' = -\dfrac{1}{\sqrt{1 - x^2}}$
$y = \cos^{-1}(f(x))$	$y' = -\dfrac{f'(x)}{\sqrt{1 - (f(x))^2}}$
Ex) $y = \cos^{-1}(x^2)$	$y' = -\dfrac{2x}{\sqrt{1 - (x^2)^2}}$

Sec (3-5) 9

y	y'
3 $y = \tan^{-1} x$	$y' = \frac{1}{1+x^2}$
$y = \tan^{-1}(f(x))$	$y' = \frac{f'(x)}{1+(f(x))^2}$
Ex $y = \tan^{-1}(x^2)$	$y' = \frac{2x}{1+(x^2)^2}$
4 $y = \underline{\underline{c}}ot^{-1} x$	$y' = -\frac{1}{1+x^2}$
$y = \underline{\underline{c}}ot^{-1}(f(x))$	$y' = -\frac{f'(x)}{1+(f(x))^2}$
Ex $y = \underline{\underline{c}}ot^{-1}(x^2)$	$y' = -\frac{2x}{1+(x^2)^2}$

Sec (3-5)

10

y	y'
5 $y = \sec^{-1} x$	$y' = \dfrac{1}{x\sqrt{x^2-1}}$
$y = \sec^{-1}(f(x))$	$y' = \dfrac{f'(x)}{f(x)\sqrt{(f(x))^2-1}}$
Ex $y = \sec^{-1}(x^2)$	$y' = \dfrac{2x}{x^2\sqrt{(x^2)^2-1}}$
5 $y = \underline{\underline{c}}sc^{-1} x$	$y' = -\dfrac{1}{x\sqrt{x^2-1}}$
$y = \underline{\underline{c}}sc^{-1}(f(x))$	$y' = -\dfrac{f'(x)}{(f(x))\sqrt{(f(x))^2-1}}$
Ex $y = \underline{\underline{c}}sc^{-1}(x^2)$	$y' = -\dfrac{2x}{x^2\sqrt{(x^2)^2-1}}$

Ex: If $y = \sin^{-1}(x^4)$, then $y' =$

(a) $\dfrac{x^3}{\sqrt{1-x^8}}$ (b) $\dfrac{-x^3}{\sqrt{1-x^8}}$ (c) $\dfrac{3x^2}{\sqrt{1-x^8}}$ (d) $\dfrac{4x^3}{\sqrt{1+x^8}}$

---

solution $y = \sin^{-1}(x^4) \Rightarrow y' = \dfrac{4x^3}{\sqrt{1+(x^4)^2}}$

$$y' = \frac{4x^3}{\sqrt{1+x^8}} \quad (d)$$

Ex: If $y = \sec^{-1}(x^3)$, the $y' =$

(a) $\dfrac{3}{x\sqrt{x^5-1}}$ (b) $-\dfrac{3}{x\sqrt{x^5-1}}$ (c) $\dfrac{-3}{x\sqrt{x^6-1}}$ (d) $\dfrac{3}{x\sqrt{x^6-1}}$

---

solution $y = \sec^{-1}(x^3)$

$$y' = \frac{3x^2}{x^3\sqrt{(x^3)^2-1}} = \frac{3}{x\sqrt{x^6-1}} \quad (d)$$

Ex: If $y = \cos^{-1}(e^x)$, then $y' =$

a) $\frac{-1}{\sqrt{1-e^{2x}}}$ b) $\frac{e^x}{\sqrt{1-e^{2x}}}$ c) $-\frac{e^x}{\sqrt{1-e^{2x}}}$ d) $\frac{1}{\sqrt{1-e^{2x}}}$

Solution:

$$y = \cos^{-1}(e^x)$$

$$y' = -\frac{e^x}{\sqrt{1-(e^x)^2}} = \frac{-e^x}{\sqrt{1-e^{2x}}} \quad \text{c}$$

Ex: If $y = \cos^{-1}(x^3)$, then $y' =$

a) $\frac{3x^2}{\sqrt{1-x^6}}$ b) $-\frac{3x^2}{\sqrt{1-x^6}}$ c) $\frac{-3x^2}{\sqrt{1+x^6}}$ d) $\frac{3x^2}{\sqrt{1-x^5}}$

Solution:

$$y = \cos^{-1}(x^3)$$

$$y' = -\frac{3x^2}{\sqrt{1-(x^3)^2}} = -\frac{3x^2}{\sqrt{1-x^6}} \quad \text{b}$$

Ex: If $y = \tan^{-1}(x^3)$, then $y' =$

(a) $-\dfrac{3x^2}{1+x^6}$ (b) $-\dfrac{3x^2}{1+x^5}$ (c) $\dfrac{3x^2}{1+x^5}$ (d) $\dfrac{3x^2}{1+x^6}$

---

solution:

$$y = \tan^{-1}(x^3)$$

$$y' = \frac{3x^2}{1+(x^3)^2} = \frac{3x^2}{1+x^6} \quad (d)$$

Ex: If $y = \tan^{-1}(e^x)$, then $y' =$

(a) $-\dfrac{e^x}{1+e^{2x}}$ (b) $\dfrac{1}{1+e^{2x}}$ (c) $-\dfrac{1}{1+e^{2x}}$ (d) $\dfrac{e^x}{1+e^{2x}}$

Solution:

$$y = \tan^{-1}(e^x)$$

$$y' = \frac{e^x}{1+(e^x)^2} = \frac{e^x}{1+e^{2x}} \quad (d)$$

Sec 3-5

14

ملاحظة هامة

If $y = \sin^{-1}\left(\frac{x}{a}\right) \Rightarrow y' = \frac{1}{\sqrt{a^2-x^2}}$

$y = \cos^{-1}\left(\frac{x}{a}\right) \Rightarrow y' = \frac{-1}{\sqrt{a^2-x^2}}$

Ex If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' =$

(a) $\frac{3}{\sqrt{9+x^2}}$ (b) $\frac{x}{\sqrt{9-x^2}}$ (c) $\frac{1}{\sqrt{9-x^2}}$ (d) $\frac{1}{\sqrt{9-x^2}}$

---

solution $y = \sin^{-1}\left(\frac{x}{3}\right) \longrightarrow a = 3$

$y' = \frac{1}{\sqrt{a^2-x^2}} = \frac{1}{\sqrt{(3)^2-x^2}} = \frac{1}{\sqrt{9-x^2}}$ (d)

Ex If $y = \cos^{-1}\left(\frac{x}{2}\right)$ then $y' =$

---

solution $y = \cos^{-1}\left(\frac{x}{2}\right) \longrightarrow a = 2$

$y' = \frac{-1}{\sqrt{a^2-x^2}} = \frac{-1}{\sqrt{(2)^2-x^2}} = \frac{-1}{\sqrt{4-x^2}}$

ملاحظة هامة

If $y = \tan^{-1}\left(\frac{x}{a}\right) \Longrightarrow y' = \frac{a}{a^2+x^2}$

$y = \cot^{-1}\left(\frac{x}{a}\right) \Longrightarrow y' = \frac{-a}{a^2+x^2}$

Ex) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' =$

ⓐ $\frac{4}{4+x^2}$ ⓑ $\frac{2}{4+x^2}$ ⓒ $\frac{-2}{4+x^2}$ ⓓ $\frac{1}{\sqrt{4-x^2}}$

---

solution $y = \tan^{-1}\left(\frac{x}{2}\right) \longrightarrow a = 2$

$y' = \frac{a}{a^2+x^2} = \frac{2}{(2)^2+x^2} = \frac{2}{4+x^2}$ ⓑ

Ex) If $y = \cot^{-1}\left(\frac{x}{5}\right)$, then $y' =$

ⓐ $\frac{25}{25+x^2}$ ⓑ $\frac{5}{25+x^2}$ ⓒ $\frac{-5}{25+x^2}$

---

solution: $y = \cot^{-1}\left(\frac{x}{5}\right) \longrightarrow a = 5$

$y' = \frac{-a}{a^2+x^2} = \frac{-5}{(5)^2+x^2} = \frac{-5}{25+x^2}$ ⓒ

أكاديمية أشرف بركات
Ashraf Barakat Academy

لتدريس طلاب وطالبات جامعة الملك عبدالعزيز السنة التحضيرية كورس فيزياء ١١٠ و رياضيات ١١٠ وإحصاء ١١٠ و رياضيات ٢٠٢ ويحتوي الكورس على محاضرات فيديو مسجلة بالإضافة إلى مراجعات وحلول اختبارات للمهندس / أشرف بركات

للاشتراك أو الاستفسار يمكن الاتصال على جوال ٠٥٠٤٥٩٠١٣٢ أو برسالة على الواتس .

مع خالص الدعاء بالسداد والتوفيق.

أكاديمية أشرف بركات

Derivatives of logarithmic functions

مشتقات الدوال اللوغارتمية

$f(x)$	$f'(x)$
① $y = \log_a x$	$y' = \dfrac{1}{x \ln a}$
$y = \log_a (f(x))$	$y' = \dfrac{f'(x)}{f(x) \ln a}$
② $y = \ln x$	$y' = \dfrac{1}{x}$
$y = \ln (f(x))$	$y' = \dfrac{f'(x)}{f(x)}$

① $\ln (xy) = \ln x + \ln y$

② $\ln \left(\dfrac{x}{y}\right) = \ln x - \ln y$

③ $\ln x^n = n \ln x$ تذكر

Ex If $y = \log_5 \sqrt{x}$, then $y' =$

(a) $\frac{1}{2\sqrt{x}\ln 5}$ (b) $\frac{1}{2x\ln 5}$ (c) $\frac{1}{2\sqrt{x}}$.

Solution

$$y = \log_{\sqrt{5}} \sqrt{x} = \log_5 x^{1/2}$$

$$y = \frac{1}{2}\log_5 x$$

$$y' = \left(\frac{1}{2}\right)\frac{1}{x\ln 3} = \frac{1}{2x\ln 5}$$

(b)

Ex If $y = \ln\left(\frac{1}{x^{-4}}\right)$, then $y' =$

(a) $\frac{4}{x}$ (b) $4x$ (c) $\frac{1}{x^4}$

Solution

$$y = \ln\frac{1}{x^{-4}} = \ln x^4$$

$$y = 4\ln x$$

$$y' = 4\left(\frac{1}{x}\right) = \frac{4}{x}$$

(a)

Ex: If $y = \ln(\sin x)$, then $y' =$

(a) $\tan x$ (b) $-\tan x$ (c) $\cot x$ (d) $-\cot x$

<u>solution:</u>

$$y = \ln(\sin x)$$

$$y' = \frac{\cos x}{\sin x} = \cot x$$ (c)

Ex: If $y = \ln\sqrt{3x^2+5x}$, then $y' =$

(a) $\dfrac{6x+5}{3x^2+5x}$ (b) $\dfrac{6x+5}{2(3x^2+5x)}$

(c) $\dfrac{6x+5}{(3x^2+5x)\ln 5}$ (d) $\dfrac{6x}{2(3x^2+5x)}$

<u>Solution:</u>

$$y = \ln\sqrt{3x^2+5x} = \ln(3x^2+5x)^{1/2}$$

$$= \frac{1}{2}\ln(3x^2+5x)$$

$$y' = \frac{1}{2}\,\frac{6x+5}{3x^2+5x} = \frac{6x+5}{2(3x^2+5x)}$$ (b)

Sec(3-6)

4

Ex: If $y = \log_3(3x^2 - 5\cos x)$, find y'

Solution:

$$y = \log_3(3x^2 - 5\cos x)$$

$$y' = \frac{6x - 5(-\sin x)}{(3x^2 - 5\cos x)\ln 3}$$

$$\boxed{y' = \frac{6x + 5\sin x}{(3x^2 - 5\cos x)\ln 3}}$$

Ex: If $y = \ln\sqrt{x^2 + 4x}$, find y'

Solution:

$$y = \ln(x^2 + 4x)^{1/2} = \frac{1}{2}\ln(x^2 + 4x)$$

$$y' = \frac{1}{2}\,\frac{2x + 4}{x^2 + 4x} = \frac{1}{\cancel{2}}\,\frac{\cancel{2}(x+2)}{x^2 + 4x}$$

$$\boxed{y' = \frac{x + 2}{x^2 + 4x}}$$

Sec (3-6) 5

Ex) If $y = \log_7(x^3-2)$, then $y' =$

(a) $\dfrac{x^2}{(x^3-2)\ln 7}$ (b) $\dfrac{3x^2}{x^3-2}$ (c) $\dfrac{3x^2}{(x^3-2)\ln 7}$

<u>Solution:</u>

$$y = \log_7(x^3-2)$$

$$y' = \frac{3x^2}{(x^3-2)\ln 7} \quad \text{(c)}$$

Ex) If $y = \log_5(x^3-2\csc x)$, then $y' =$

(a) $\dfrac{3x^2+2\csc x\cot x}{x^3-2\csc x\ln 5}$ (b) $\dfrac{3x^2-2\csc x\cot x}{(x^3-2\csc x)\ln 5}$

(c) $\dfrac{3x^2+2\csc x\cot x}{x^3-2\csc x}$ (d) $\dfrac{3x^2+2\csc x\cot x}{(x^3-2\csc x)\ln 5}$

<u>Solution:</u>

$$y = \log_5(x^3-2\csc x)$$

$$y' = \frac{3x^2-2(-\csc x\cot x)}{(x^3-2\csc x)\ln 5} = \frac{3x^2+2\csc x\cot x}{(x^3-2\csc x)\ln 5} \quad \text{(d)}$$

Sec (3-6) 6

Ex) If $f(x) = \frac{\ln x}{x^2}$, then $f'(1) =$

a) 1 b) 4 c) 0 d) 2

Solution: $f(x) = \frac{\ln x}{x^2} = x^{-2} \ln x$

$f'(x) = -2x^{-3} \ln x + x^{-2} \cdot \frac{1}{x}$

$= \frac{-2 \ln x}{x^3} + \frac{1}{x^3}$

$f'(1) = \frac{-2 \ln 1}{(1)^3} + \frac{1}{(1)^3} = 0 + \frac{1}{1} = 1$ a

---

Ex) If $y = \ln(\cos x)$, then y'

a) $\tan x$ b) $-\tan x$ c) $\cot x$ d) $-\cot x$

Solution: $y = \ln(\cos x)$

$y' = \frac{-\sin x}{\cos x} = -\tan x$ b

Ex. If $y = 9^{\tan x} + \ln(e^{-2x^3})$, then $y' =$

(a) $9^{\tan x} \sec^2 x \ln 9 - 6x^2$ (b) $-9^{\tan x} \sec^2 x \ln 9 + 6x^2$

Solution: $y = 9^{\tan x} + \ln(e^{-2x^3}) = 9^{\tan x} - 2x^3$

$$y' = 9^{\tan x} \cdot \sec^2 x \cdot \ln 9 - 6x^2 \quad (a)$$

Ex. If $y = \log_5 (x^3 - e^x)$, then $y' =$

(a) $\dfrac{3x^2 - e^x}{x^3 - e^x}$ (b) $\dfrac{1}{(x^3 - e^x)\ln 5}$ (c) $\dfrac{3x^2 - e^x}{(x^3 - e^x)\ln 5}$

Solution

$$y = \log_5 (x^3 - e^x)$$

$$y' = \frac{3x^2 - e^x}{(x^3 - e^x)\ln 5} \quad (c)$$

Sec (3-6) 8

Ex: If $y = \ln \frac{x-1}{\sqrt{x+2}}$, then y'

ⓐ $\frac{x+5}{2(x-1)(x+2)}$ ⓑ $\frac{x+5}{(x-1)(x+2)}$

ⓒ $\frac{x-5}{2(x-1)(x+2)}$ ⓓ $\frac{x}{2(x-1)(x+2)}$

Solution:

$$y = \ln \frac{x-1}{\sqrt{x+2}}$$

$$y = \ln(x-1) - \ln(x+2)^{1/2} = \ln(x-1) - \frac{1}{2}\ln(x+2)$$

$$y' = \frac{1}{x-1} - \frac{1}{2}\,\frac{1}{x+2} = \frac{2(x+2)-(x-1)}{2(x-1)(x+2)}$$

$$y' = \frac{2x+4-x+1}{2(x-1)(x+2)} = \frac{x+5}{2(x-1)(x+2)}$$

ⓐ

Sec (3-6)   9

Logarithmic Differentiation   الاشتقاق اللوغاريتمي

نستخدمه عندما تكون

$y = (f(x))^{g(x)}$ $y = (\text{دالة})^{\text{دالة}}$

تذكر

$$\frac{d}{dx}(\ln y) = \frac{y'}{y}$$

Ex If $y = x^x$, then $y' =$

ⓐ $x^x(\ln x+1)$ ⓑ $x^x(\ln x-1)$

ⓒ $(\ln x+1)$ ⓓ $\ln(x-1)$

Solution:

$y = x^x$ ln للطرفين

$\ln y = \ln x^x = x\ln x$ نشتق الطرفين بالنسبة لـ x

$\frac{y'}{y} = 1\cdot\ln x + x\cdot\frac{1}{x}$ نضرب y

$y' = y(\ln x+1) = x^x(\ln x+1)$ نعوض عن y

ⓐ

Sec (3-6) 10

Ex: If $y = x^{\sin x}$, then $y' =$

(a) $x^{\sin x}[\cos x \ln x + \sin x]$

(b) $x^{\sin x}[\cos x \ln x - \frac{\sin x}{x}]$

(c) $x^{\sin x}[\cos x \ln x + \frac{\sin x}{x}]$

solution:

$$y = x^{\sin x}$$

ln بطرفيه

$$\ln y = \ln(x^{\sin x}) = \sin x \ln x$$

نشتق الطرفيه بالنسبة لـ x

$$\frac{y'}{y} = \cos x \ln x + \frac{\sin x}{1} \cdot \frac{1}{x}$$

نضرب في y

$$y' = y[\cos x \ln x + \frac{\sin x}{x}]$$

نعوض عن y

$$y' = x^{\sin x}[\cos x \ln x + \frac{\sin x}{x}]$$ (c)

Sec (3-6) 11

Ex. If $y = x^{\cos x}$, then y'

(a) $x^{\cos x}\left[\frac{\sin x}{x} + \cos x \ln x\right]$ (b) $\frac{\cos x}{x} - \sin x \ln x$

(c) $x^{\cos x}\left[\frac{\cos x}{x} + \sin x \ln x\right]$ (d) $x^{\cos x}\left[\frac{\cos x}{x} - \sin x \ln x\right]$

---

solution

$$y = x^{\cos x}$$

ln

$$\ln y = \ln x^{\cos x} = \cos x \ln x$$

نشتق

$$\frac{y'}{y} = -\sin x \ln x + \cos x \cdot \frac{1}{x}$$

نضرب في y

$$y' = y\left[\frac{\cos x}{x} - \sin x \ln x\right]$$

نعوض عن y

$$y' = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \ln x\right] \quad (d)$$

(12)

Ex: If $y = x^{7x}$, then $y' =$

(a) $7(1+\ln x)$ (b) $7^x(1+\ln x)$.

(c) $7^x(1-\ln x)$ (d) $7 \cdot 7^x(1+\ln x)$

---

solution $y = x^{7x}$

ln

$$\ln y = \ln x^{7x}$$

$$\ln y = 7x \cdot \ln x$$

مشتق

$$\frac{y'}{y} = 7 \cdot \ln x + 7\cancel{x} \cdot \frac{1}{\cancel{x}}$$

$$y' = y\,[7\ln x + 7]$$

مشترك

$$y' = 7^x \cdot 7\,(\ln x + 1)$$

or $\boxed{y' = 7 \cdot 7^x\,(1+\ln x)}$ (d)

or $\boxed{y' = 7^{x+1}\,(1+\ln x)}$

Sec (3-6) (13)

Ex: If $y = x^{\sqrt{x}}$, then $y' =$

(a) $x^{\sqrt{x}}\left(\frac{1+\ln x}{\sqrt{x}}\right)$ (b) $x^{\sqrt{x}}\left(\frac{2+\ln x}{2\sqrt{x}}\right)$

(c) $\frac{2+\ln x}{2\sqrt{x}}$ (d) $x\left(\frac{2+\ln x}{2\sqrt{x}}\right)$

---

solution:

$$y = x^{\sqrt{x}}$$ ln

$$\ln y = \ln x^{\sqrt{x}} = \sqrt{x}\,\ln x$$ نشتق

$$\frac{y'}{y} = \frac{1}{2\sqrt{x}}\ln x + \sqrt{x}\cdot\frac{1}{x}$$

نضرب في y

$$y' = y\left[\frac{\ln x}{2\sqrt{x}} + \frac{\cancel{\sqrt{x}}}{(\sqrt{x})^{\cancel{2}}}\right]$$

نعوض عن y

$$y' = x^{\sqrt{x}}\left[\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\cdot\frac{2}{2}\right]$$

$$\boxed{y' = x^{\sqrt{x}}\left[\frac{\ln x + 2}{2\sqrt{x}}\right]}$$ (b)

① ملخص المشتقات

y	y'
$y = c$ عدد ثابت	$y' = 0$
$y = x$	$y' = 1$
$y = x^n$	$y' = n x^{n-1}$
$y = e^x$	$y' = e^x$
$y = \sqrt{x}$	$y' = \frac{1}{2\sqrt{x}}$
$y = a^x$	$y' = a^x \ln a$
$y = f(x) \cdot g(x)$	$y' = f'(x)\, g(x) + f(x) \cdot g'(x)$
$y = \frac{f(x)}{g(x)}$	$y' = \frac{f'(x) \cdot g(x) - f(x)\, g'(x)}{(g(x))^2}$

y	y'
$y = \sin x$	$y' = \cos x$
$y = \cos x$	$y' = -\sin x$
$y = \tan x$	$y' = \sec^2 x$
$y = \cot x$	$y' = -\csc^2 x$
$y = \sec x$	$y' = \sec x \tan x$
$y = \csc x$	$y' = -\csc x \cot x$
$y = \sin^{-1} x$	$y' = \frac{1}{\sqrt{1-x^2}}$
$y = \cos^{-1} x$	$y' = \frac{-1}{\sqrt{1-x^2}}$
$y = \tan^{-1} x$	$y' = \frac{1}{1+x^2}$
$y = \cot^{-1} x$	$y' = \frac{-1}{1+x^2}$

③

y	y'
$y = \sec^{-1} x$	$y' = \dfrac{1}{x\sqrt{x^2-1}}$
$y = \csc^{-1} x$	$y' = \dfrac{-1}{x\sqrt{x^2-1}}$
$y = \ln x$	$y' = \dfrac{1}{x}$
$y = \log_a x$	$y' = \dfrac{1}{x \ln a}$
y^2	$2y\,y'$
e^y	$y' e^y$
$\sin y$	$y' \cos y$
$\sin(xy)$	$\cos(xy)\,(1 \cdot y + x y')$

أكاديمية أشرف بركات
Ashraf Barakat Academy

لتدريس طلاب وطالبات جامعة الملك عبدالعزيز السنة التحضيرية كورس فيزياء ١١٠ و رياضيات ١١٠ وإحصاء ١١٠ و رياضيات ٢٠٢ ويحتوي الكورس على محاضرات فيديو مسجلة بالإضافة إلى مراجعات وحلول اختبارات للمهندس / أشرف بركات

للاشتراك أو الاستفسار يمكن الاتصال على جوال ٠٥٠٤٥٩٠١٣٢ أو برسالة على الواتس .

مع خالص الدعاء بالسداد والتوفيق.

أكاديمية أشرف بركات

Maximum and Minimum values

القيم العظمى والصغرى

Def 1: Let c be a number in the domain D of a function f.

Then $f(c)$ is the

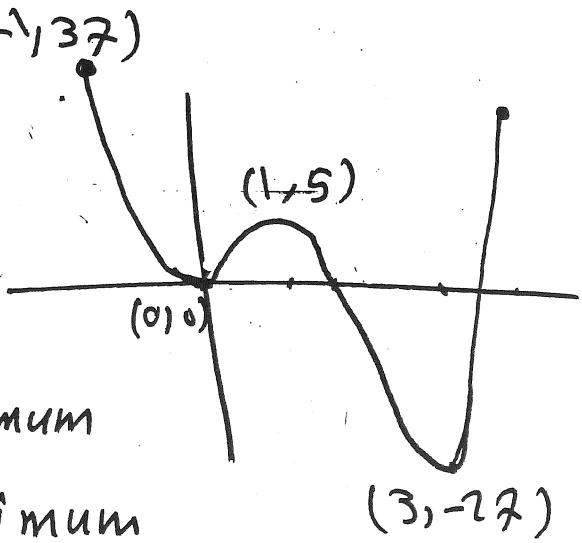
- absolute maximum value of f on D if $f(c) \geq f(x)$ for all x in D.
- absolute minimum value of f on D if $f(c) \leq f(x)$ for all x in D.

Def 2: The number $f(c)$ is

- **local maximum** value of f if
  $f'(c) \geq f(x)$ when x is near c.
- **local minimum** value of f if
  $f'(c) \leq f(x)$ when x is near c.

$\boxed{\text{EX 4}}$ The graph of the function

$$f(x) = 3x^4 - 16x^3 + 18x^2 \qquad -1 \leq x \leq 4$$



$f(-1) = 37$ is absolute maximum
not local maximum

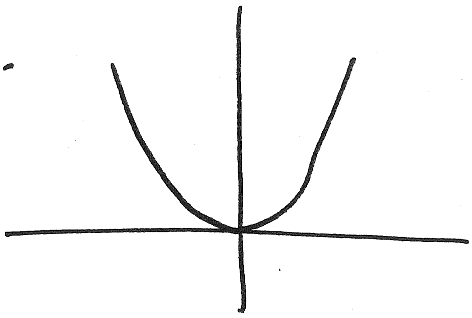
$f(1) = 5$ local maximum

$f(0) = 0$ local minimum

$f(3) = -27$ local and absolute minimum

ملاحظات هامة

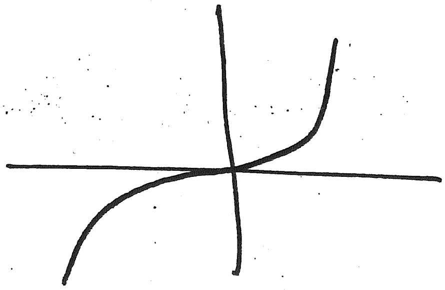
① $f(x) = x^2$ or x^4, or x^6, ...



$f(0) = 0$ is the absolute and local minimum value and has no maximum value

---

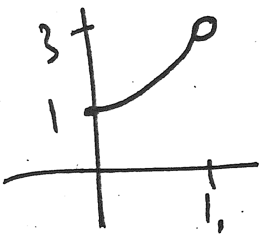
② $f(x) = x^3, x^5, x^7, ...$



has neither an absolute maximum value nor an absolute minimum value and it has no local extrem values either

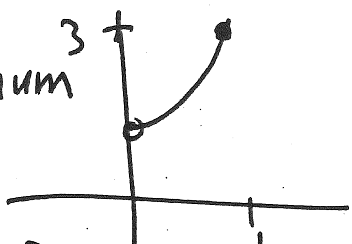
---

③



$f(0) = 1$ is minimum value ; no maximum value

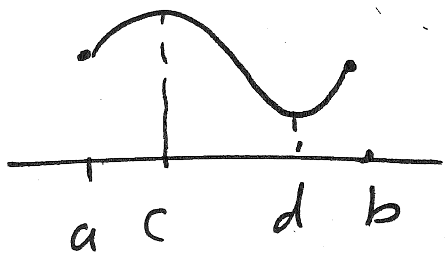
no minimum value

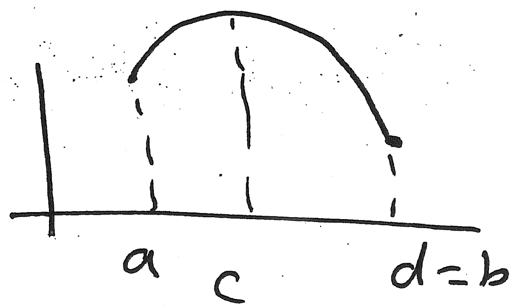


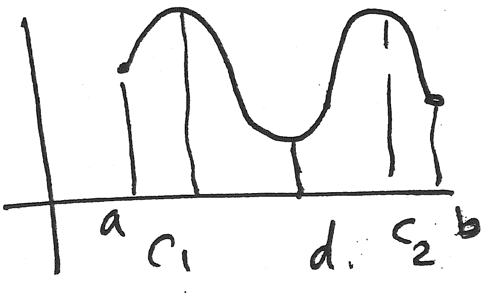
$f(1) = 3$ is maximum value

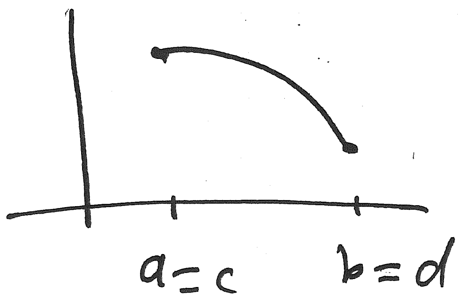
The Extreme Value Theorem

If f is continuous on a closed interval $[a,b]$, then f attains an absolute maximum value $f(c)$ and absolute minimum value $f(d)$ at some numbers c and d in $[a,b]$









Def 6 A critical number of a function f is a number in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist

---

$\boxed{\text{Ex 7}}$ Find the critical number of

$$f(x) = x^{3/5}(4-x)$$

Solution:
$$f'(x) = \frac{3}{5}x^{-2/5}(4-x) + x^{3/5}(-1)$$

$$= \frac{3(4-x)}{5x^{2/5}} - x^{3/5}$$

$$= \frac{3(4-x) - 5x^{3/5} \cdot x^{2/5}}{5x^{2/5}}$$

$$= \frac{12-8x}{5x^{2/5}}$$

$f'(x) = 0 \Longrightarrow 12 - 8x = 0$

البسط = صفر

$-8x = -12 \quad \div(-8)$

$x = \frac{-12}{-8} = \frac{3}{2}$

$f'(x)$ does not exist

المقام = صفر $\Rightarrow 5x^{2/5} = 0$

$\boxed{x = 0}$

The critical numbers are $\frac{3}{2}, 0$

ملاحظة هامة

If f has a local maximum or minimum at c, then c is a critical number of f.

$\frac{29}{283}$ Find the critical numbers of the function $f(x) = 4 + \frac{1}{3}x - \frac{1}{2}x^2$

Solution $f(x) = 4 + \frac{1}{3}x - \frac{1}{2}x^2$

$f'(x) = \frac{1}{3} - \frac{1}{2}(2x)$

$= \frac{1}{3} - x$

$f'(x) = 0 \Longrightarrow \frac{1}{3} - x = 0$

$-x = -\frac{1}{3}$

$\boxed{x = \frac{1}{3}}$ is a critical number

The closed Interval Method To find the absolute maximum and minimum values of a continuous function f on a closed interval [a,b]

1. نوجد critical numbers للدالة f في الفترة [a,b]
2. نوجد f(a) , f(b)
3. أكبر قيمة من الخطوتين 1 , 2 هي القيمة العظمى المطلقة absolute maximum value

وأصغر قيمة هي القيمة الصغرى المطلقة absolute minimum value

$\boxed{\text{Ex 8}}$ Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1 \qquad -\frac{1}{2} \leq x \leq 4$$

<u>solution</u> $f(x) = x^3 - 3x^2 + 1 \qquad [-\frac{1}{2}, 4]$

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Longrightarrow 3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$x = 0 \in [-\tfrac{1}{2}, 4] \qquad x - 2 = 0$$

$$x = 2 \in [-\tfrac{1}{2}, 4]$$

$$f(0) = 0 - 0 + 1 = 1$$

$$f(4) = 64 - 3(16) + 1 = 17$$

$$f(2) = 8 - 3(4) + 1 = -3$$

$$f(-\tfrac{1}{2}) = -\tfrac{1}{8} - 3(\tfrac{1}{4}) + 1 = \tfrac{1}{8}$$

The absolue maximum value is $f(4) = 17$

The absolute minimum value is $f(2) = -3$

47
284

Find the absolute maximum and minimum values of

$$f(x) = 12 + 4x - x^2 \quad \text{on } [0, 5]$$

<u>solution</u>

$$f(x) = 12 + 4x - x^2$$

$$f'(x) = 4 - 2x$$

$$f'(x) = 0 \Longrightarrow 4 - 2x = 0$$

$$-2x = -4$$

$$x = 2 \in [0, 5]$$

$$f(2) = 12 + 4(2) - (2)^2 = 16$$

$$f(0) = 12 + 0 - 0 = 12$$

$$f(5) = 12 + 4(5) - (5)^2 = 7$$

The absolute maximum value is

$$f(2) = 16$$

The absolute minimum value is

$$f(5) = 7$$

$\boxed{\frac{53}{284}}$ Find the absolute maximum and minimum values of

$$f(x) = x + \frac{1}{x} \quad \text{on} \quad [0.2, 4]$$

<u>solution</u> $f(x) = x + x^{-1}$ $\quad [0.2, 4]$

$$f'(x) = 1 - x^{-2}$$

$$= 1 - \frac{1}{x^2}$$

$$= \frac{x^2 - 1}{x^2}$$

$f'(x) = 0 \implies x^2 - 1 = 0 \implies x^2 = 1$

$x = -1 \notin [0.2, 4]$ $x = 1 \in [0.2, 4]$

$f'(x)$ does not exist at $x = 0 \notin [0.2, 4]$

$f(0.2) = 0.2 + \frac{1}{0.2} = 0.2 + 5 = 5.2$

$f(4) = \frac{4}{1} + \frac{1}{4} = \frac{16+1}{4} = \frac{17}{4}$

12

$$f(1) = 1 + \frac{1}{1} = 2$$

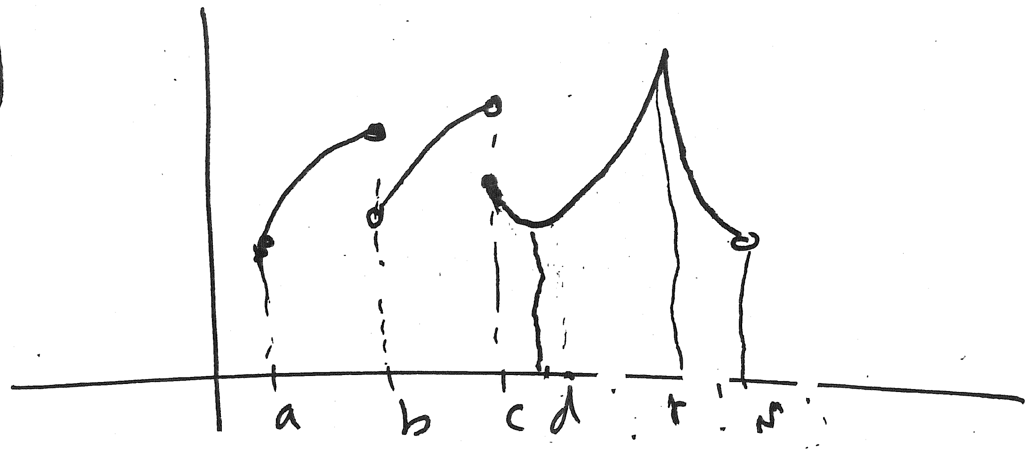
The absolute maximum value is

$$f(0.2) = 5.2$$

The absolute minimum value is

$$f(1) = 2$$

4
283



For each of the numbers a, b, c, d, r, and s, state whether the function whose graph is shown has an absolute maximum or minimum, a local maximum or minimum or neither a maximum nor a minimum.

Solution

f(a) is the absolute minimum
f(r) is the absolute maximum
f(d) is a local minimum
f(r) is a local maximum

Sec 4-3 ①

How Derivatives Affect the Shape of a Graph

Increasing / Decreasing Test

ⓐ If $f'(x) > 0$ on an interval, then f is increasing on that interval

ⓑ If $f'(x) < 0$ on an interval, then f is decreasing on that interval

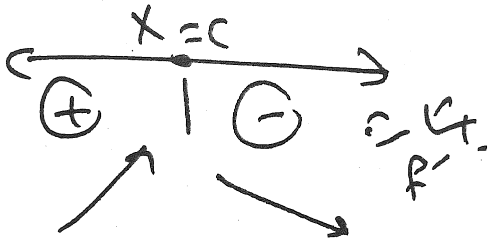
---

The First Derivative Test: Suppose c is a critical number of a continuous function f

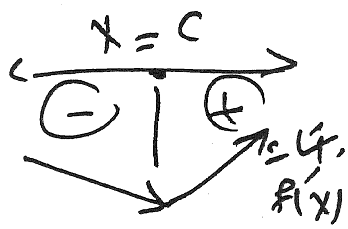
① If f' changes from positive to negative at c, then f has a local

②

maximum at c

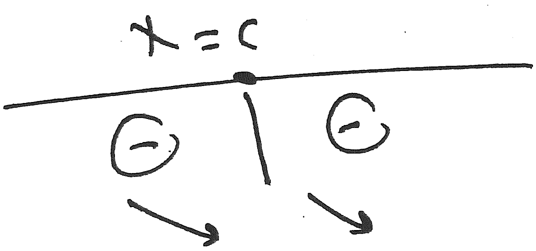


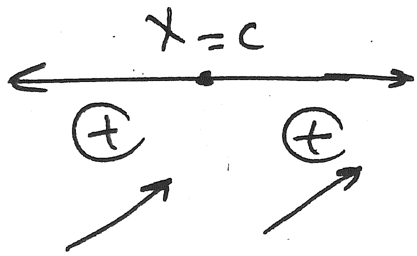
② If f' changes from negative to positive at c, then f has a local minimum at c

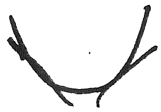


③ إذا كانت اشارة f'(x) لا تتغير قبل وبعد x = c

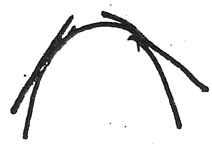
∴ f has no local maximum or minimum at c





Def If the graph of f lies :
above all of its tangents on
an interval I, then it is called
Concave upward on I 

If the graph of f lies below
all of its tangent on I, it is
called concave downward on I



Concavity Test

ⓐ If $f''(x) > 0$ for all x in I, then
the graph of f is concave upward on I.

ⓑ If $f''(x) < 0$ for all x in I, then
the graph of f is concave downward
on I.

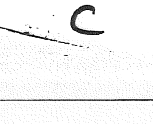
Def A point p on a curve $y = f(x)$ is called an inflection point if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at p

نقطة انقلاب

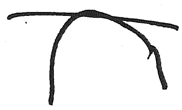
نقطة انقلاب

The Second Derivative Test suppose f'' is continuous near c

ⓐ $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c



(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c



الخلاصة

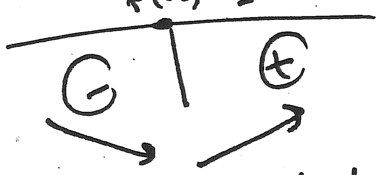
خطوات دراسة منحنى الدالة

* نوجد $f'(x)$

* ندرس اشارة $f'(x)$

$f'(x) < 0$	$f'(x) = 0$	$f'(x) > 0$
(سالبة)	نقطة قيمة عظمى محلية أو صغرى محلية بشرط تغير التزايد والتناقص قبل وبعد النقطة	موجبة
f is decreasing		f is increasing

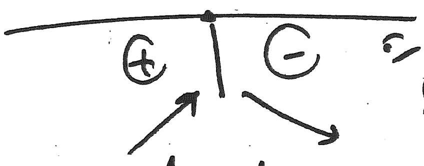
$f(a) = 0$



اشارة $f'(x)$

local minimum

$f'(a) = 0$



اشارة $f'(x)$

local maximum

(6)

- نوجد $f'(x)$
- ندرس اشارة $f'(x)$

$f'(x) < 0$	$f''(x) = 0$	$f''(x) > 0$
(سالبة)	نقطة انقلاب	موجبة
Concave downward	بشرط تغير التقعر قبل وبعد النقطة	Concave upward
$\cap$		$\cup$

اختبار المشتقة الثانية

① If $f'(a) = 0$ and $f''(a) = \oplus \Longrightarrow$

local minimum $\cup$

② If $f'(a) = 0$ and $f''(a) = \ominus \Longrightarrow$

local maximum $\cap$

Ex If $f(x) = 2x^2 - 2$, then f is increasing on

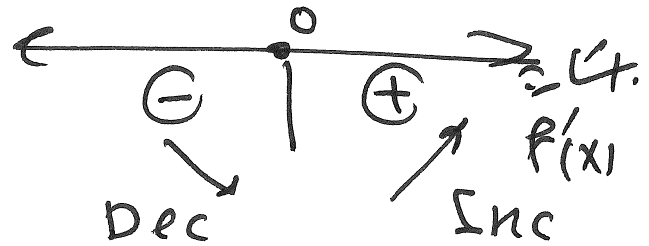
ⓐ $\mathbb{R}$ ⓑ $(-\infty, 0)$ ⓒ $(0, \infty)$ ⓓ $(4, \infty)$

Solution $f(x) = 2x^2 - 2$

$f'(x) = 4x$

$f'(x) = 0 \Rightarrow 4x = 0 \xRightarrow{\div 4} x = 0$

f is increasing on $(0, \infty)$ ⓒ



Ex If $f(x) = -x^2 - 4x + 5$, then f is decreasing on

ⓐ $(-2, \infty)$ ⓑ $(-\infty, -2)$ ⓒ $\mathbb{R}$ ⓓ $(0, \infty)$

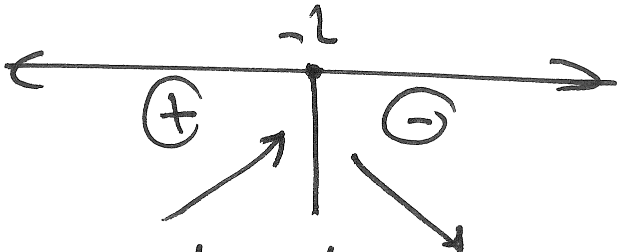
Solution $f(x) = -x^2 - 4x + 5$

$f'(x) = -2x - 4$

$f'(x) = 0$

$-2x - 4 = 0$

$-2x = 4 \Rightarrow \boxed{x = -2}$



f is decreasing on $(-2, \infty)$ ⓐ

Ex If $f(x) = x^2 + 4x$, then f has a local minimum at $x =$

ⓐ 1 ⓑ 2 ⓒ -1 ⓓ -2

Solution

$f(x) = x^2 + 4x$

$f'(x) = 2x + 2$

$f'(x) = 0 \Longrightarrow 2x + 2 = 0 \Longrightarrow 2x = -2$

$\boxed{x = -1}$

-1 ⊖ ⊕

f has a local minimum at $x = -1$ ⓒ

Ex If $f = 3x - x^2$, then f has a local minimum at $x = \frac{3}{2}$

ⓐ True ⓑ False

Solution $f(x) = 3x - x^2 \Longrightarrow f'(x) = 3 - 2x$

$f'(x) = 0 \Longrightarrow 3 - 2x = 0 \Longrightarrow x = \frac{3}{2}$

$3/2$ ⊕ ⊖

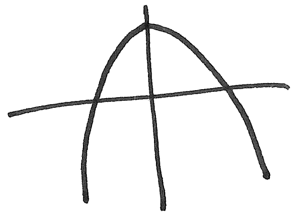
f has local maximum at $x = \frac{3}{2}$

False ⓑ

Ex) $f(x) = -2x^2 + 9$ is concave up on $\mathbb{R}$

(a) True (b) False

<u>solution</u> $f(x) = -2x^2 + 9$



$f'(x) = -4x$

$f''(x) = -4 < 0$ (سالب)

$f(x)$ is concave <u>down</u>

(b)

Ex) If $f(x) = -x^2 - 2x$, then f has an inflection point at $x = 1$

(a) True (b) False

<u>solution</u> $f(x) = -x^2 - 2x$ $f'(x) = -2x - 2$

$f''(x) = -2 < 0$ دائماً

Concave down $\cap$ دائماً

no inflection point

(b)

Ex) If $f(x) = x^3$, then f is increasing on

(a) R (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(-1, 1)$

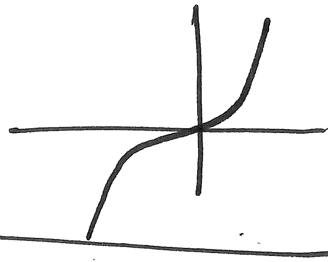
Solution $f(x) = x^3 \Longrightarrow f'(x) = 3x^2$

دائماً موجب

$f(x)$ is increasing on $R = (-\infty, \infty)$

(a)

no local max or min values ملاحظة



Ex) If $f(x) = x^3$, the f is concave up on

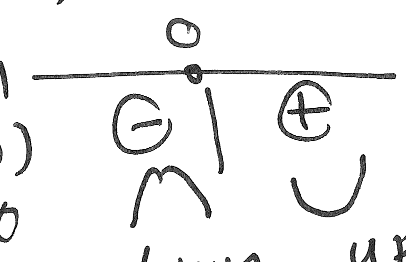
Solution $f(x) = x^3 \longrightarrow f'(x) = 3x^2$

$f'(x) = 6x \longrightarrow f'(x) = 0 \Rightarrow 6x = 0$

$x = 0$

f is concave up on $(0, \infty)$

f is concave down on $(-\infty, 0)$



f has inflection point at $x = 0$

Ex: For the function $f(x) = 12x - x^3$

1. $f(x)$ has a critical number at

ⓐ $x = -1,\ x = 3$ ⓑ $y = -1$

ⓒ $x = -2,\ x = 3$ ⓓ $x = \pm 2$

2. $f(x)$ is increasing on

ⓐ $(-\infty, -2) \cup (2, \infty)$ ⓑ $(-1, 1)$

ⓒ $(-3, 0)$ ⓓ $(-2, 2)$

3. $f(x)$ is decreasing on

ⓐ $(-\infty, -2) \cup (2, \infty)$ ⓑ $(-3, 0)$

ⓒ $(-2, 2)$ ⓓ $(-1, 1)$

4. $f(x)$ has a local maximum at

ⓐ $x = -1$ ⓑ $x = 3$

ⓒ $x = -2$ ⓓ $x = 2$

5. $f(x)$ has a local minimum at

(a) $x=-1$ (b) $x=2$

(c) $x=3$ (d) $x=-2$

6. $f(x)$ is concave up on

(a) $(0,\infty)$ (b) $(-\infty,0)$

(c) $(-\infty,-1)$ (d) R

7. $f(x)$ is concave down on

(a) $(0,\infty)$ (b) $(-\infty,0)$

(c) $(-\infty,-1)$ (d) R

8. The inflection point for $f(x)$ is

(a) $(1,0)$ (b) $(0,1)$

(c) $(0,0)$ (d) $(0,-1)$

(13)

<u>solution</u> $f(x) = 12x - x^3$

① $f'(x) = 12 - 3x^2$

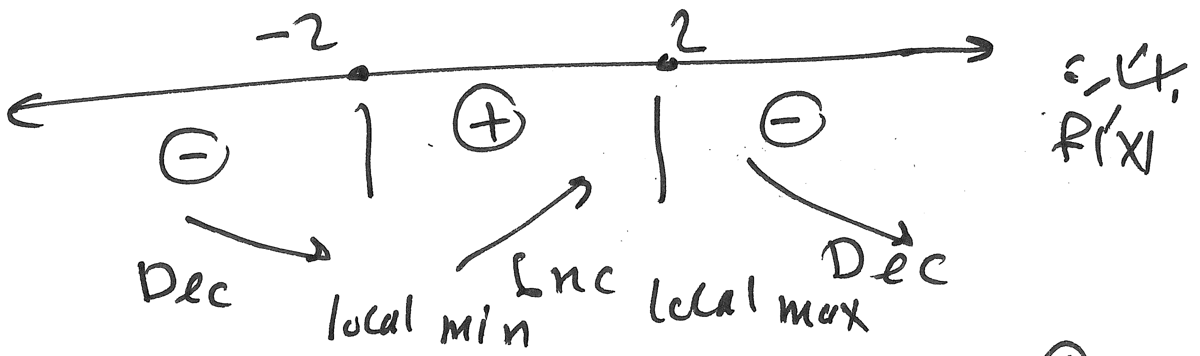
$f'(x) = 0 \Rightarrow 12 - 3x^2 = 0$

$12 = 3x^2 \Rightarrow x^2 = 4$ "$\sqrt{\ }$"

$x = \pm 2$ (d)

---

② $f'(x) = 0$ at $x = \pm 2$



$f(x)$ is increasing on $(-2, 2)$ (d)

③ $f(x)$ is decreasing on $(-\infty, -2) \cup (2, \infty)$ (a)

④ local maximum at $x = 2$ (d)

⑤ local minimum at $x = -2$

(14)

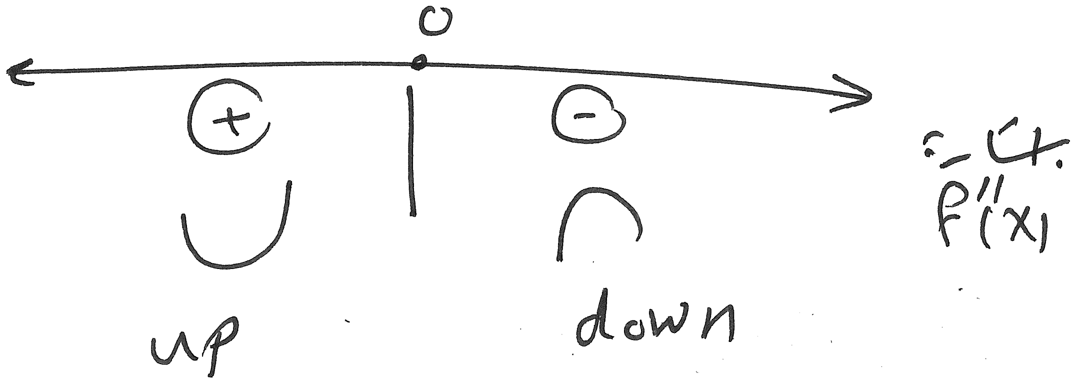
(6) $f'(x) = 12 - 3x^2$

$f''(x) = -6x$

$f''(x) = 0$

$-6x = 0$ $\quad (\div 6)$

$x = 0$



$f(x)$ is concave up on $(-\infty, 0)$ (b)

(7) $f(x)$ is concave down on $(0, \infty)$ (a)

(8) $f(0) = 12(0) - (0)^3 = 0 - 0 = 0$

$(0, 0)$ is the inflection point

(c)

(15)

$\boxed{\frac{1}{300}}$ Use the given graph of f to find the following

(a) The open intervals on which f is increasing

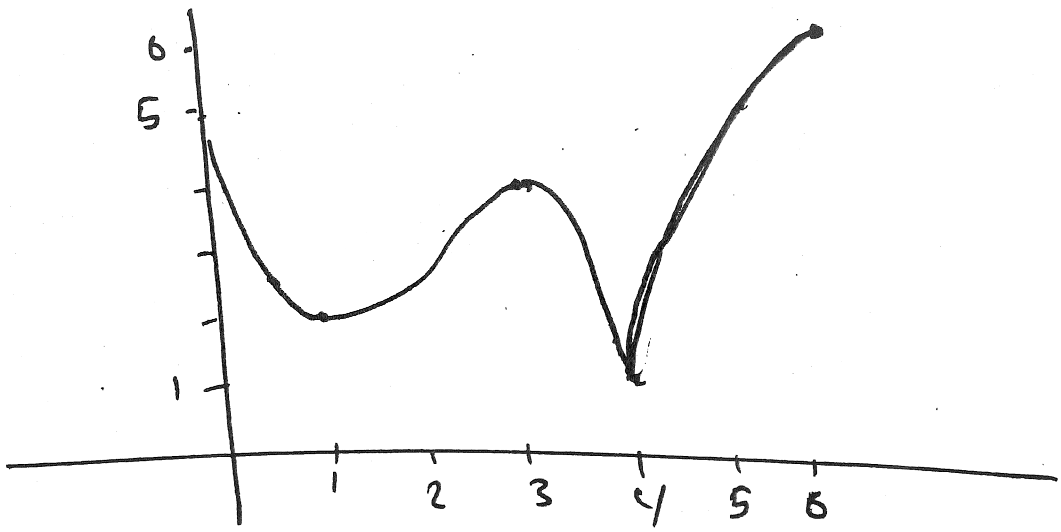
(b) The open intervals on which f is decreasing.

(c) The open intervals on which f is concave upward.

(d) The open intervals on which f is concave downward.

(e) The coordinates of the points of inflection.

(16)



solution

(a) $f(x)$ is increasing on $(1,3)\cup(4,6)$

(b) $f(x)$ is decreasing on $(0,1)\cup(3,4)$

(c) $f(x)$ is concave upward on $(0,2)$

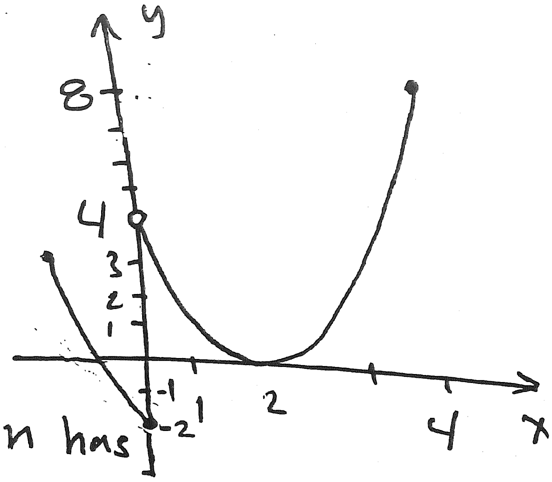
(d) $f(x)$ is concave downward on $(2,4)\cup(4,6)$

(e) $(2,3)$ is an inflection point

Sec (4-3) (17)

Ex

The function $g(x)$ whose graph is shown has



(1) absolute maximum at ....

(2) absolute minimum at ....

(3) local extremum at .....

Solution

- أكبر قيمة للدالة على فترة التعريف $[-2,4]$

هي عند $x=4$ والقيمة العظمى المطلقة هي $f(4)=8$

(1) absolute maximum at $x=4$

- أقل قيمة للدالة على فترة التعريف $[-2,4]$ هي عند

$x=0$ والقيمة الصغرى المطلقة للدالة هي $f(0)=-2$

(2) absolute minimum at $x=0$

- القيمة العظمى والصغرى المحلية للدالة عند $x=2$

وهي قيمة صغرى محلية

2

(3) local extremum (minimum at $x=2$)

local minimum value is $f(2)=0$