Section 1.1 Solution of Ex.4 Ex. 8 Ex.13 Ex.20 Ex.30 Ex.31 Page 6

Ex.4

Is the set finite or infinite? Is 10 element of the set?

- The set $\{x|x\text{is an even natural number}\}$ is infinite:
  The even natural numbers are $2, 4, 6, 8, 10, \ldots, 102, \ldots$ infinitely many numbers.
- The number 10 is an element of the $\{x|x\text{is an even natural number}\}$: $10 = 2 \cdot 5$, therefore 10 is even. (The even natural numbers are $2 \cdot 1, 2 \cdot 2, 2 \cdot 3, 2 \cdot 4, 2 \cdot 5, \ldots.)$

Ex.8

List the element of the set?
The elements of the set x|x is a natural number not greater than 4 are 1, 2, 3, 4
The statement not greater than is equivalent to less than or equal.

Ex.13

Which statement is true? $\{2\}$ is a set, which contains one elements and the set $\{2, 4, 6, 8\}$ is a set of elements so

- $\{2\} \notin \{2, 4, 6, 8\}$ is not true and $\{2\} \in \{2, 4, 6, 8\}$ is not true
- The true statement is $\{2\} \subset \{2, 4, 6, 8\}$.

Ex.20

Is the statement true? $\{x|x\text{is a natural number greater than}10\} = \{11, 12, 13, ...\}$
YES, the statement above is true. The natural numbers greater than 10 are 11, 12,...

Ex.30

Is the statement true? $\{8, 11, 15\} \cap \{8, 11, 19, 20\} = \{8, 11\}$
YES, the statement above is true. The elements 8, 11 are in both sets. Therefore the intersection of the sets $\{8, 11, 15\}$ and $\{8, 11, 19, 20\}$ is the set $\{8, 11\}$.

Ex.31

Is the statement true? $\{6, 12, 14, 16\} \cup \{6, 14, 19\} = \{6, 14\}$
NO, the statement above is false.
The union of two sets consists of the element, which are either in the first one or in the second one. The number 19 is not ele- ment of the resulting set. The statement will be true if $\{6, 12, 14, 16\} \cup \{6, 14, 19\} = \{6, 12, 14, 16, 19\}$.

Mini-Test

1. Using set notation, the elements belonging to the set: $\{x|\ x$ is a natural number less than $2\}$ is

A) ϕ

B) $\{\phi\}$

C) $\{0\}$

D) $\{1\}$

---

2. Let $U = \{-2, -1, 1, 2, 3, 4\}$, $A = \{-1, 2, 4\}$ and $B = \{-2, -1, 3\}$, then $U' \cap B$

A) ϕ

B) $\{-2, 3\}$

C) $\{3\}$

D) $\{-2, -1, 3\}$

---

3. Which elements in the set below are irrational numbers?

$$\{-7, -\sqrt{5}, -2, -\frac{1}{6}, 0, 1, 2\frac{1}{3}, \sqrt{25}, \frac{17}{2}\}$$

A) $\{-7, -2\}$

B) $\{-\sqrt{5}\}$

C) $\{-\sqrt{5}, -\frac{1}{6}, 0, 2\frac{1}{3}, \sqrt{25}, \frac{17}{2}\}$

D) $\{-\sqrt{5}, \sqrt{25}\}$

---

4. The number $0.6666\cdots$ is

A) natural number

B) rational number

C) irrational number

D) none of the above

Section 1.2 Solutions of Ex.11- 24, 49, 51, 53, 57, 83, 85 and 87Pages 13-15

Ex. 11 - 16

Let the set $A = \{-6, -\frac{12}{4}, -\frac{5}{8}, -\sqrt{3}, 0, \frac{1}{4}, 1, 2\pi, 3, \sqrt{12}\}$. *List all the elements of A that belong to each set.*

11. Natural numbers
12. Whole numbers
13. Integers
14. Rational numbers
15. Irrational numbers
16. Real numbers

Solution

11. $\{1,,3\}$
12. $\{0, 1, 3\}$
13. $\{-6, -\frac{12}{4}, 0, 1, 3\}$
14. $\{-6, -\frac{12}{4}, -\frac{5}{8}, 0, \frac{1}{4}, 1, 3\}$
15. $\{-\sqrt{3}, 2\pi, \sqrt{12}\}$
16. $A = \{-6, -\frac{12}{4}, -\frac{5}{8}, -\sqrt{3}, 0, \frac{1}{4}, 1, 2\pi, 3, \sqrt{12}\}$

Ex. 17 – 24

Evaluate each expression.

17. $-2^4 = -(2^4) = -16$
18. $-3^5 = -(3^5) = -243$
19. $(-2)^4 = 2^4 = 16$
20. $(-2)^6 = 2^6 = 64$
21. $(-3)^5 = -(3^5) = -243$
22. $(-2)^5 = -2^5 = -32$
23. $-2 . 3^4 = -2 . 81 = -162$
24. $-4 . 5^3 = -4 . 125 = -500$

Ex.49, 51, 53 and 57

Identify the property illustrated in the statement. Assume all variables represent real numbers.

49) $6 \cdot 12 + 6 \cdot 15 = 6\ 12 + 15$
51) $(t-6) \cdot \left(\frac{1}{t-6}\right) = 1, t - 6 \neq 0$
53) $(7.5 - y) + 0 = 7.5 - y$
57) $(5x)\left(\frac{1}{x}\right) = 5\left(x \cdot \frac{1}{x}\right)$

Solution

49) Distributive property
51) Inverse property
53) Identity property
57) Inverse property

Ex.83, 85 and 88

Evaluate the expression

83) $|-10|$
85) $-\left|\frac{4}{7}\right|$
88) $-|-12|$

Solution

83) $|-10| = 10$
85) $-\left|\frac{4}{7}\right| = -\frac{4}{7}$
88) $-|-12| = -12$

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Mini – Test

***Choose the correct answer (a, b, c or d)*:**

1. Simplify the expression $(5-2)^2 - 16 \div 4 \cdot 2 - 1$

   $(a)\ 0$ $(b)\ 1$

   $(c) - 1$ $(d)\ 9$

2. Use the commutative and associative properties to simplify

$$\left(\frac{3}{4} \cdot r\right)(-12)$$

   $(a) - 9\,r$ $(b)\ 9\,r$

   $(c) - 3\,r$ $(d)\ 3\,r$

3. Use the distributive property to simplify

$$\frac{1}{3}\left(\frac{12}{5}\,m - \frac{3}{2}\,n - 27\right)$$

   $(a)\ 12\,m - 3n - 9$ $(b)\ \frac{4}{5}\,m - \frac{n}{2} - 9$

   $(c)\ \frac{m}{15} - \frac{n}{6}\ 9$ $(d)\ 27$

4. Let $x = -3\ \ and\ y = 2$. Then the value of the expression

$$|2x - 3y| \text{ is}$$

   $(a)\ 0$ $(b) 12$

   $(c) - 12$ $(d)\ 5$

5. The distance between 5 and -5 is

   $(a)\ 10$ $(b) - 10$

   $(c)\ 25$ $(d) - 25$

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Section 1.3: Polynomials

Solution of the proposed exercises **<u>Page 21-22</u>:** Ex.5 Ex.14 Ex.21 Ex.45

Ex.5 Simplify each expression.

- $9^3 \cdot 9^5$

Solution. $9^3 \cdot 9^5 = 9^8$

Ex.14 Simplify the expression. Assume variables represent nonzero real numbers.

- $(-2x^5)^5$

Solution. $(-2x^5)^5 = -23x^{25}$

Ex.21 Simplify the expression. Assume variables represent nonzero real numbers.

- $-\left(\frac{x^3y^5}{z}\right)^0$

Solution. $-\left(\frac{x^3y^5}{z}\right)^0 = -1$

Ex.45 Find the product.

- $(4r-1)(7r+2)$

Solution. $(4r-1)(7r+2) = 28r^2 + r - 2$

Mini Test (1.4)

1]- Factor the following polynomial: $6mp^3 + qmn - 6nm - mqp^3$.

a) $m(p^3 - n)(6 + q)$.
b) $m(p^3 - n)(6 - q)$.
c) $(p^3 - n)(6m - qm)$.
d) $(p^3 + n)(6\text{m} + qm)$.

2]- Factor by grouping : $x^3 - x^2 + 2x - 2$.

a) $(x^2 + 1)(x + 2)$.
b) $(x^2 - 2)(x + 1)$.
c) $(x^2 + 2)(x - 1)$.
d) $(x^2 - 1)(x - 2)$.

3]- Factor the following polynomial: $x^2y - 10y + xy^2 - 10x$.

a) $(xy + 10)(x - y)$.
b) $(xy + 10)(x + y)$.
c) $(xy - 10)(x + y)$.
d) $(xy - 10)(x - y)$.

4]- Write the expression in simplest form: $\frac{t^2-1}{t^2-3t+2}$.

a) $\frac{t+1}{t-2}$.

b) $\frac{t-1}{t-2}$.

c) $\frac{t+1}{t+2}$.

d) $\frac{t+2}{t-1}$.

5]- Factor : $27\, m^3 + 8$.

a) $(3\, m – 2)(9\, m^2 + 4)$.
b) $(3\, m – 2)(9\, m^2 + 6\, m + 4)$.
c) $(3\, m + 2)(9\, m^2 – 6\, m + 4)$.
d) $(3\, m – 2)(9\, m^2 – 6\, m + 4)$.

6]- Factor out the greatest common factor: $– 12\, x^4 – 42\, x^3 + 24\, x^2$.

a) $– 12\, x^2 (x^2 – 42\, x + 24)$.
b) $– 12\, x (x^3 – 42\, x^2 + 24x)$.
c) $– 6\, x^2 (2\, x^2 – 7x + 4)$.
d) $– 6\, x^2 (2\, x^2 + 7x – 4)$.

7]- Factor the following polynomial: $5\, xu – 20\, x + 10\, u – 40$.

a) $5(x + 2)(u – 4)$.
b) $5(x + 2)(u + 4)$.
c) $5(x – 2)(u + 4)$.
d) $5(x – 2)(u – 4)$.

8]- Factor the trinomial completely: $7b +12b^2 – 12$.

a) $(3x + 4)(4x – 3)$.
b) $(3x – 4)(4x + 3)$.
c) $(3x – 4)(4x – 3)$.
d) $(3x + 4)(4x + 3)$.

9]- Factor using u-substitution: $x^4 – 13\, x^2 + 36$.

a) $(x^2 – 4)(x^2 – 9)$.
b) $(x^2 – 4)(x^2 + 9)$.
c) $(x^2 – 6)(x^2 + 6)$.
d) $(x^2 + 12)(x^2 – 3)$.

Section 1.5 : Rational Expressions

Solution of the proposed exercises Page 35: Ex.1 Ex.3 Ex.7 Ex.14.

Ex.1 Find the domain of the rational expression: $\frac{2x-4}{x+7}$?

- $x + 7 = 0$

  $x = -7$

The domain is the set of real number not equal to –7, written

$\{ x \mid x \in R, x \neq -7 \}$

Ex.3 Find the domain of the rational expression: $\frac{3}{x^2 - 5x - 6}$?

- $x^2 - 5x - 6 = 0$

  $(x - 6)(x + 1) = 0$

$x - 6 = 0$ or $x + 1 = 0$

$x = 6$ or $x = -1$

The domain is the set of real number not equal to –1, 6 written

$\{ x \mid x \in R, x \neq -1, 6 \}$

Ex.7 Write of the rational expression in lowest terms: $\frac{-8(4-y)}{(y+2)(y-4)}$?

- $\frac{-8(4-y)}{(y+2)(y-4)} = \frac{8(-4+y)}{(y+2)(y-4)}$

$= \frac{8(y-4)}{(y+2)(y-4)}$

$= \frac{8}{(y+2)}, \quad y \neq -2$

Ex.14 Find each product or quotient: $\frac{y^3+y^2}{7} \cdot \frac{49}{y^4+y^3}$?

- $$\frac{y^3+y^2}{7} \cdot \frac{49}{y^4+y^3} = \frac{y^2(y+1)}{7} \cdot \frac{7 \cdot 7}{y^3\,(y+1)}$$

$$= \frac{7\ y^2}{y^3}$$

$$= 7y^{2-3}$$

$$= 7y^{-1}$$

$$= \frac{7}{y}$$

Section 1.6 : Rational Exponents

Solution of the proposed exercises **Page 41**: Ex.1 Ex.8 Ex.20 .

Ex.1 Match each expression in column *I* with its equivalent expression in column *II* ?

(a) 5^{-3}	---	A. 125
(b) -5^{-3}	---	B. – 125
(c) $(-5)^{-3}$	a & d	C. $\frac{1}{125}$
(d) $-(-5)^{-3}$	b & c	D. $-\frac{1}{125}$

- $(a)\quad 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$.

  $(b)\ -5^{-3} = -\frac{1}{5^3} = -\frac{1}{125}$.

  $(c)\ (-5)^{-3} = \frac{1}{(-5)^3} = \frac{1}{-125} = -\frac{1}{125}$.

  $(d) -(-5)^{-3} = -\frac{1}{(-5)^3} = -\frac{1}{(-125)} = \frac{1}{125}$.

Ex.8 Write the expression with only positive exponents and evaluate if possible. Assume all variables represent nonzero real numbers:$\frac{5^9}{5^7}$?

- $\frac{5^9}{5^7} = 5^{9-7} = 5^2 = 25$.

Ex.20 Evaluate the expression :$\mathbf{121^{\frac{1}{2}}}$?

- $\mathbf{121^{\frac{1}{2}}} = (11^2)^{\frac{1}{2}} = 11^{\frac{2}{2}} = 11^1 = 11$

2.1 EXERCISES

EXERCISE 2. Decide whether the statement " the equation $5(x-8) = 5x-40$ is an example of an identity" is true or false.

SOLUTION

$5(x-8) = 5x-40$

$5x-40 = 5x-40$

$0 = 0$

Hence, the statement is true.

EXERCISE 8. In solving the equation $3(2x-8) = 6x-24$, a student obtains the result $0 = 0$ and gives the solution set $\{0\}$. Is this correct ? Explain why.

SOLUTION

This is not correct since the solution set is all real numbers.

EXERCISE 9. Solve the equation $5x+4 = 3x-4$.

SOLUTION

$5x+4 = 3x-4$

$2x = -8$

$x = -4$

So, the solution set is $\{-4\}$.

EXERCISE 20. Solve the equation $\frac{1}{15}(2x+5) = \frac{x+2}{9}$.

SOLUTION

$$\frac{1}{15}(2x+5) = \frac{x+2}{9}$$

$$9 \cdot \frac{1}{15}(2x+5) = 9 \cdot \frac{x+2}{9}$$

$$\frac{3}{5}(2x+5) = x+2$$

$3(2x+5) = 5(x+2)$

$6x+15 = 5x+10$

$6x-5x = -15+10$

$x = -5$

Thus, the solution set is $\{-5\}$.

EXERCISE 25. Solve the equation $0.5x + \frac{4}{3}x = x + 10$.

SOLUTION

$$\frac{1}{2}x + \frac{4}{3}x = x + 10$$

$$6 \cdot \frac{1}{2}x + 6 \cdot \frac{4}{3}x = 6 \cdot x + 6 \cdot 10$$

$$3x + 8x = 6x + 60$$

$$5x = 60$$

$$x = 12$$

The solution set is $\{12\}$.

2.2 EXERCISES

EXERCISE 9. Identify the number $5 + i$ as real, complex, pure imaginary or nonreal complex.(More than one of these descriptions will apply)

SOLUTION

The number $5 + i$ is a complex and nonreal.

EXERCISE 25. Find the sum $(3 + 2i) + (9 - 3i)$. Write the answer in standard form.

SOLUTION

$(3 + 2i) + (9 - 3i)$

$= 3 + 2i + 9 - 3i$

$= (3 + 9) + (2i - 3i)$

$= 12 - i.$

EXERCISE 47. Simplify i^{22}.

SOLUTION

$i^{22} = i^{20} \cdot i^{2} = (i^{4})^{5} \cdot i^{2}$

$= (1)^{5} \cdot i^{2} = 1 \cdot i^{2}$

$= i^{2} = -1$

2.3 EXERCISES

EXERCISE 7. Solve the equation $x^2 - 5x + 6 = 0$ by the zero-factor property.

SOLUTION

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x - 2 = 0$ or $x - 3 = 0$

$x = 2$ or $x = 3$

The solution set is $\{2, 3\}$.

EXERCISE 26. Solve the equation $x^2 - x - 1 = 0$ using the quadratic formula.

SOLUTION

$x^2 - x - 1 = 0$

$a = 1, b = -1, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

The solution set is $\left\{\frac{1}{2} \pm \frac{\sqrt{5}}{2}\right\}$.

EXERCISE 40. For the equation $4x^2 - 2xy + 3y^2 = 2$,

(a) solve for x in terms of y, and

(b) solve for y in terms of x.

SOLUTION

(a) $4x^2 - 2xy + 3y^2 = 2$

$4x^2 - 2xy + 3y^2 - 2 = 0$

$4x^2 + (-2y)x + (3y^2 - 2) = 0$

$a = 4, b = -2y, c = 3y^2 - 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2y) \pm \sqrt{(-2y)^2 - 4(4)(3y^2 - 2)}}{2(4)}$$

$$x = \frac{2y \pm \sqrt{4y^2 - 4(4)(3y^2 - 2)}}{2(4)}$$

$$x = \frac{2y \pm 2\sqrt{y^2 - (4)(3y^2 - 2)}}{2(4)}$$

$$x = \frac{y \pm \sqrt{y^2 - (4)(3y^2 - 2)}}{4}$$

$$x = \frac{y \pm \sqrt{y^2 - 12y^2 + 8}}{4}$$

$$x = \frac{y \pm \sqrt{-11y^2 + 8}}{4}$$

$$x = \frac{y \pm \sqrt{8 - 11y^2}}{4}$$

(b) $4x^2 - 2xy + 3y^2 = 2$

$4x^2 - 2xy + 3y^2 - 2 = 0$

$3y^2 + (-2x)y + (4x^2 - 2) = 0$

$a = 3, b = -2x, c = 4x^2 - 2$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4(3)(4x^2 - 2)}}{2(3)}$$

$$y = \frac{2x \pm \sqrt{4x^2 - 4(3)(4x^2 - 2)}}{2(3)}$$

$$y = \frac{2x \pm 2\sqrt{x^2 - (3)(4x^2 - 2)}}{2(3)}$$

$$y = \frac{x \pm \sqrt{x^2 - (3)(4x^2 - 2)}}{(3)}$$

$$y = \frac{x \pm \sqrt{x^2 - 12x^2 + 6}}{3}$$

$$y = \frac{x \pm \sqrt{-11x^2 + 6}}{3}$$

$$y = \frac{x \pm \sqrt{6 - 11x^2}}{3}$$

EXERCISE 44. Evaluate the discriminant of the equation

$4x^2 = -6x + 3$. Then use it to predict the number of distinct solutions, and whether they are rational, irrational, or nonreal complex. Do not solve the equation.

SOLUTION

$4x^2 = -6x + 3$

$4x^2 + 6x - 3 = 0$

$a = 4, b = 6, c = -3$

$b^2 - 4ac = (6)^2 - 4(4)(-3) = 36 + 48 = 84$

The discriminant 84 is positive and not a perfect square, so there are two distinct irrational solutions.

2.4 EXERCISES

The solutions of Exercises 1,2,3,4, and 5 are

$1 \longrightarrow F$, $2 \longrightarrow A$, $3 \longrightarrow I$, $4 \longrightarrow B$, $5 \longrightarrow E$.

EXERCISE 7. Solve the inequality $-2x + 8 \leq 16$. Write the solution set in interval notation.

SOLUTION

$-2x + 8 \leq 16$

$-2x \leq -8 + 16$

$-2x \leq 8$

$\frac{-2x}{-2} \geq \frac{8}{-2}$

$x \geq -4$

The solution set is $[-4, \infty)$.

EXERCISE 11. Solve the inequality $\frac{4x+7}{-3} \leq 2x + 5$. Write the solution set in interval notation.

SOLUTION

$\frac{4x + 7}{-3} \leq 2x + 5$

$-3 \cdot \frac{4x + 7}{-3} \geq -3 \cdot (2x + 5)$

$4x + 7 \geq -6x - 15$

$6x + 4x \geq -7 - 15$

$10x \geq -22$

$x \geq -\frac{22}{10}$

$x \geq -\frac{11}{5}$

The solution set is $[-\frac{11}{5}, \infty)$.

EXERCISE 19. Solve the inequality $-3 \le \frac{x-4}{-5} < 4$. Write the solution set in interval notation.

SOLUTION

$-3 \le \frac{x-4}{-5} < 4$

$-5 \cdot (-3) \ge -5 \cdot \frac{x-4}{-5} > -5 \cdot 4$

$15 \ge x - 4 > -20$

$15 + 4 \ge x - 4 + 4 > -20 + 4$

$19 \ge x > -16$

$-16 < x \le 19$

The solution set is $(-16, 19]$.

EXERCISE 26. Solve the inequality $x^2 - 2x \le 1$. Write the solution set in interval notation.

SOLUTION

$x^2 - 2x \le 1$

$x^2 - 2x - 1 \le 0$

Step 1 We find the values of x that satisfy the equation

$x^2 - 2x - 1 = 0$ using the quadratic formula.

$a = 1, b = -2, c = -1$

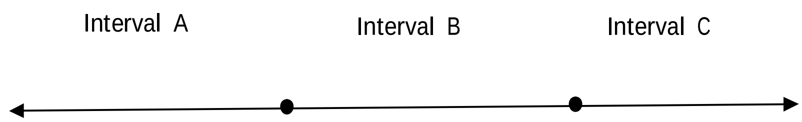
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{1+1}}{2} = 1 \pm \sqrt{2}$$

$x = 1 \pm \sqrt{2}$

Step 2 The two numbers $1-\sqrt{2}$ and $1+\sqrt{2}$ cause the expression x^2-2x-1 to equal zero and divide the number line into three intervals as shown in the figure. We used closed circles at $1-\sqrt{2}$ and $1+\sqrt{2}$ to indicate that they are included in the solution set.



Step 3 Choose a test values in each interval to see whether it satisfies the inequality $x^2-2x-1 \le 0$. If the test value makes the statement true, then the entire interval belongs to the solution set.

Interval	Test value	Is $\boldsymbol{x^2-2x-1\le 0}$ True or False
A: $(-\infty, \mathbf{1}-\sqrt{\mathbf{2}})$	$\mathbf{-2}$	$(\mathbf{-2})^2-\mathbf{2}(\mathbf{-2})-\mathbf{1}\le\mathbf{0}$ $\mathbf{7}\le\mathbf{0}$ False
B: $[\,\mathbf{1}-\sqrt{\mathbf{2}}, \mathbf{1}+\sqrt{\mathbf{2}}\,]$	$\mathbf{1}$	$(\mathbf{1})^2-\mathbf{2}(\mathbf{1})-\mathbf{1}\le\mathbf{0}$ $\mathbf{-2}\le\mathbf{0}$ True
C: $(\mathbf{1}+\sqrt{\mathbf{2}}, \infty)$	$\mathbf{3}$	$(\mathbf{3})^2-\mathbf{2}(\mathbf{3})-\mathbf{1}\le\mathbf{0}$ $\mathbf{2}\le\mathbf{0}$ False

The solution set is $[\,1-\sqrt{2}, 1+\sqrt{2}\,]$.

ex2_5 Absolute Value Equations and Inequalities

ex2_5(1)-(8), Page 89

Math equations or inequality in column I with graph of its solution set in column II:

Answer

$(1) \rightarrow \mathrm{F}$
$(2) \rightarrow \mathrm{B}$
$(3) \rightarrow \mathrm{D}$
$(4) \rightarrow \mathrm{E}$
$(5) \rightarrow \mathrm{G}$
$(6) \rightarrow \mathrm{A}$
$(7) \rightarrow \mathrm{C}$
$(8) \rightarrow \mathrm{H}$

ex2_5(18),(26),(37) Page 89

Solve each inequality(or equation) and give the solution set using interval notation:

$(\mathbf{18})\ |\mathbf{2}x + \mathbf{5}| < 3$

Solution

$$|\mathbf{2}x + \mathbf{5}| < 3 \Rightarrow -\mathbf{3} < 2x + \mathbf{5} < 3$$

$-\mathbf{3} - \mathbf{5} < 2x + \mathbf{5} - \mathbf{5} < 3 - \mathbf{5}$, *By adding* – **5** *to three sides*
$\Rightarrow -\mathbf{8} < 2x < -2$

$\Rightarrow \frac{-\mathbf{8}}{\mathbf{2}} < \frac{\mathbf{2}x}{\mathbf{2}} < \frac{-\mathbf{2}}{\mathbf{2}}$, *Divided by* **2**

$\Rightarrow -\mathbf{4} < x < -1$

Solution set $= (-4, -1)$.

(26) $|4x+3|-2=-1$

Solution

$|4x+3|-2=-1 \quad \Rightarrow |4x+3| = -1+2$

$\Rightarrow |4x+3| = 1 \quad \Rightarrow 4x+3 = \pm 1$

$\Rightarrow \; 4x+3 = 1 \; or \; 4x+3=-1$

$\Rightarrow \; 4x+3-3 = 1-3 \; or \; 4x+3-3=-1-3$

$\Rightarrow \; 4x = -2 \; or \; 4x = -4$, *Divided by* **4**

$\Rightarrow \frac{4x}{4} = \frac{-2}{4} \; or \; \frac{4x}{4} = \frac{-4}{4}$

$\Rightarrow x = \frac{-1}{2} \; or \; x = -1$

Solution set$= \left\{\frac{-1}{2}, -1\right\}$.

----

(37) $|3x+2| > 0$

Solution

Since $|3x+2| > 0$ **, then we get**

$\Rightarrow 3x+2 > 0 \quad or \; 3x+2 < 0$ *By adding* -2 to both sides

$\Rightarrow 3x+2-2 > 0-2 \quad or \; 3x+2-2 < 0-2$

$\Rightarrow 3x > -2 \quad or \quad 3x < -2$ *Divided by* 3

$\Rightarrow \frac{3x}{3} > \frac{-2}{3} \quad or \quad \frac{3x}{3} < \frac{-2}{3}$

$\Rightarrow x > \frac{-2}{3} \quad or \quad x < \frac{-2}{3}$

Solution set $= \left(\frac{-2}{3}, \infty\right) \cup \left(-\infty, \frac{-2}{3}\right)$.

Exercises Section (3.1)

Decide whether each relation defines a function and give the domain and range.

5. Is not a function.
because there are two ordered pairs $(3,7),(3,9)$ **have the same x-value paired with two different y-values.**

Domain:{2,3,5}

Range:{5,7,9,11}

---

6. Is a function.

Because for each different x-value there is exactly one y-value.

Domain:{1,2,3,5}

Range:{10,15,19,27}

---

7. Is a function.

Because for each different x-value there is exactly one y-value.

Domain:{0,1,2}

Range:{0,-1,-2}

---

8. Is a function.

Because for each different x-value there is exactly one y-value.

Domain:{2006,2007,2008,2009}

Range:{10 878 322,11 120 822, 11 160 293, 11 134 738}

---

9. Is a function.

Because every vertical line intersects the graph of a relation in one point.

Domain: $(-\infty,\infty)$

Range: $(-\infty,4]$

---

10. Is not a function.

Because every vertical line intersects the graph of a relation in two points.

Domain: $[-4,4]$

Range: $[-3,3]$

---

11. Is a function.

Because every vertical line intersects the graph of a relation in one point.

Domain: $[-2, 2]$

Range: $[0, 4]$

Let $f(x) = -3x + 4$**. Find and simplify each of the following.**

21. $f(-3)$

<u>**The solution**</u>**:**

$f(-3) = -3(-3) + 4 = 9 + 4 = 13$

For each function, find (a)$f(2)$ **and (b)**$f(-1)$

29. $f(2) = 5$ **and** $f(-1) = 11$

30. $f(2) = 1$ **and** $f(-1) = 7$

31. $f(2) = -3$ **and** $f(-1) = 2$

Dr.Gehan Ashry

37: $\left|3x+2\right|>0$

Solution:

$$\left|3x+2\right|>0$$

$$3x+2>0 \quad \text{or} \quad 3x+2<0$$

$$3x>-2 \quad \text{or} \quad 3x<-2$$

$$x>\frac{-2}{3} \quad \text{or} \quad x<\frac{-2}{3}$$

$$\frac{-2}{3}<x<\frac{-2}{3}$$

$$\text{solution interval} = (-\infty,\frac{-2}{3})\cup(\frac{-2}{3},\infty)$$

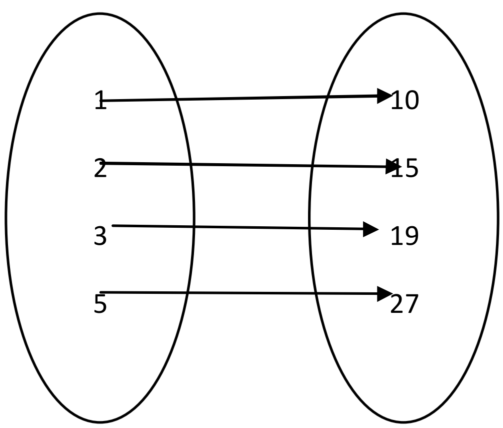
Section 3.1:

Decide whether each relation defines a function and give the domain and range.

$5-\{(2,5),(3,7),(3,9),(5,11)\}$

Answer: This is not function .The domain= $\{2,3,5\}$ and Range= $\{5,7,9,11\}$

6-



Answer: This is a function .The domain= $\{1,2,3,5\}$ and Range= $\{10,15,19,27\}$

7-

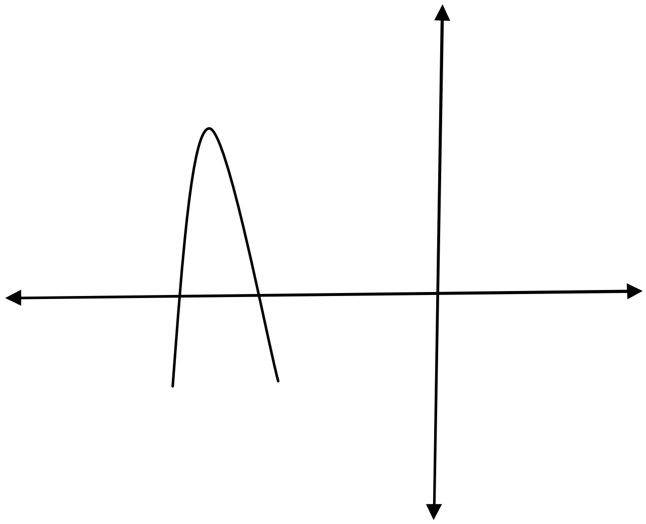
x	y
0	0
1	-1
2	-2

Answer: This is a function .The domain=$\{0,1,2\}$ and Range=$\{0,-1,-2\}$

8- Attendance at NCAA Women's College Basketball Games:

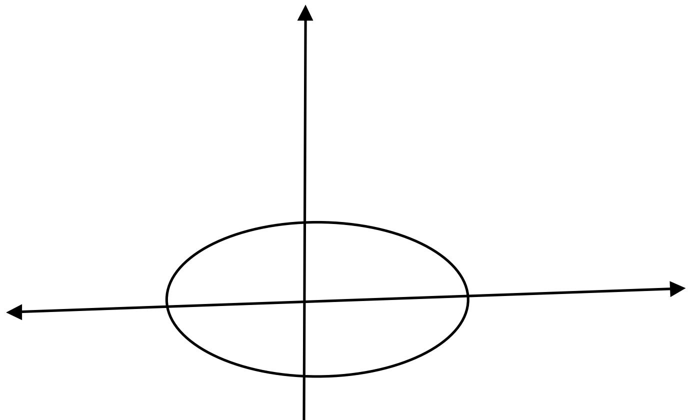
Season (x)	Attendance (Y)
2006	10.878.322
2007	11.120.822
2008	11.160.293
2009	11.134.738

Answer: This is a function .The domain=$\{2006,2007,2008,2009\}$ and Range=$\{10.878.322, 11.120.822, 11.160.293, 11.134.738\}$



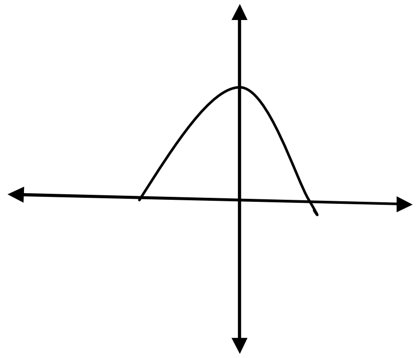
Answer: This is a function .The domain=$[-5,-2]$ and Range=$(-\infty,4]$

10:



Answer: This is not function .The domain=$\mathbf{[-4,4]}$ and Range=$\mathbf{[-3,3]}$

11-



11- **Answer:** This is a function .The domain=$\mathbf{[-2,2]}$ and Range=$\mathbf{[0,4]}$

Let $\mathbf{f(x)=-3x+4,\ and\ g(x)=-x^2+4x+1}$ find and simplify each of the following

21- $\mathbf{f(-3)=-3(-3)+4=9+4=13}$

For each function find (a) f(2), (b) f(-1)

29. $\mathbf{f=\{(2,5),(3,9),(-1,11),(5,3)\}}$

Solution:

$\mathbf{a)f(2)=5,\quad b)f(-1)=11,}$

31-Solution:

$\mathbf{a)f(2)=-2,\quad b)f(-1)=1}$

Section 3.3:

7- Write an equation for the line described. Give answers in standard form for Exercises 3-7 and in slope –intercept form (if possible) for Exercises 3-13.

7- through (-1,3) and (3,4)

Solution:

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-3}{x-(-1)}=\frac{4-3}{3-(-1)}$$

$$\frac{y-3}{x+1}=\frac{1}{4}$$

$$4(y-3)=x+1$$

$$4y-12=x+1$$

$$4y-x=12+1$$

$$4y-x=13$$

Give the slope and y-intercept of each line and graph it.

15. y=3x-1

Solution : m=3, b=-1

19.

$$y-\frac{3}{2}x-1=0$$

$$y=\frac{3}{2}x+1$$

$$m=\frac{3}{2},b=1$$

In exercises 23-26, write an equation (a) in standard form and (b) in slope –intercept form for the line describe.

26- through (-5,6), perpendicular to x=-2

Solution: Since the line x=-2 is a vertical line then the perpendicular line must be horizontal and the slope is m=0, therefore

y=mx+b, using the point (-5,6) in this equation it yield

$$y = mx + b$$

$$y = 0(-5) + 6$$

$$y = 6$$

Prepared by Dr. Bahaa Gaber Mohamed

Homework 3

Given that $f(x) = \sqrt{x}$ and $g(x) = 4x + 2$, find each of the following.

(a) $(f \circ g)(x)$ and its domain **(b)** $(g \circ f)(x)$ and its domain

Solution
For each composition, we follow the procedure given in the definition.
(a) $(f \circ g)(x) = f\big(g(x)\big) = \sqrt{g(x)} = \sqrt{4x+2}$
For $\sqrt{4x+2}$ to be a real number, we must have $4x + 2 \geqslant 0$. The domain of $f \circ g$ is

$$\mathcal{D}_{f\circ g} = [-\frac{1}{2}, \infty).$$

(b) $(g \circ f)(x) = g\big(f(x)\big) = 4f(x) + 2 = 4\sqrt{x} + 2$ which is defined when $x \geqslant 0$.
The composite functions and also their domains are different since we have

$$\mathcal{D}_{g\circ f} = [0, \infty).$$

Exercises 1,2,3,4

Let $f(x) = x^2 + 3$ and $g(x) = -2x + 6$, find each of the following.

1. $(f + g)(3)$ **2.** $(f - g)(-1)$ **3.** $(fg)(4)$ **4.** $(\frac{f}{g})(-1)$

Solution
1. $(f + g)(3) = f(3) + g(3) = (3^2 + 3) + (-2 \cdot 3 + 6) = 12 + 0 = 12$
2. $(f - g)(-1) = f(-1) - g(-1) = ((-1)^2 + 3) - (-2 \cdot (-1) + 6) = (1+3) - (2+6) = -4$
3. $(fg)(4) = f(4) \cdot g(4) = (4^2 + 3) \cdot (-2 \cdot 4 + 6) = (16+3) \cdot (-8+6) = 19 \cdot (-2) = -38$
4. Already we have found in **2.** the values of our functions at -1 which are $f(-1) = 4$ and $g(-1) = 8$. **Since $\mathbf{g(-1) \neq 0}$**, the quotient of f by g is defined at -1 and it gives

$$(\frac{f}{g})(-1) = \frac{f(-1)}{g(-1)} = \frac{4}{8} = \frac{1}{2}$$

Exercises 5,6,7

For the pair of functions defined, find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$ and $(\frac{f}{g})(x)$. Give the domain of each.

5. $f(x) = 3x+4$, $g(x) = 2x-5$

6. $f(x) = 2x^2 - 3x$, $g(x) = x^2 - x + 3$

7. $f(x) = \sqrt{4x-1}$, $g(x) = \dfrac{1}{x}$

Solution

Recall that the domains of the sum $(f+g)$, the difference $(f-g)$ and the product (fg) are the same and correspond to the intersection of the domain of f with the domain of g. We must exclude from this intersection the real numbers which make $g(x) = 0$ to obtain the domain of $\frac{f}{g}$ as illustrated for the following pairs of functions.

5. When $f(x) = 3x+4$ and $g(x) = 2x-5$, we have:

- $(f+g)(x) = f(x) + g(x) = (3x+4) + (2x-5) = 5x-1$
- $(f-g)(x) = f(x) - g(x) = (3x+4) - (2x-5) = x+9$
- $(fg)(x) = f(x).g(x) = (3x+4).(2x-5) = 6x^2 - 7x - 20$

While both f and g have domains that consist of all real numbers, the domain of $(f+g), (f-g)$ and (fg) is $\mathbb{R}$ too. Finally, for the quotient, write that:

- $(\dfrac{f}{g})(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x+4}{2x-5}$, where $x \in \mathbb{R}$ such that $2x - 5 \neq 0$. Its domain is then $\{x \in \mathbb{R}, x \neq \dfrac{5}{2}\}$.

6. When $f(x) = 2x^2 - 3x$ and $g(x) = x^2 - x + 3$, we have:

- $(f+g)(x) = f(x) + g(x) = (2x^2 - 3x) + (x^2 - x + 3) = 3x^2 - 4x + 3$
- $(f-g)(x) = f(x) - g(x) = (2x^2 - 3x) - (x^2 - x + 3) = x^2 - 2x - 3$
- $(fg)(x) = f(x).g(x) = (2x^2 - 3x).(x^2 - x + 3) = 2x^4 - 5x^3 + 9x^2 - 9x$

Again these functions are defined on $\mathbb{R}$ which is also the domain of $\dfrac{f}{g}$ because the quadratic g has no real zeros. In fact, the discriminant of $g(x) = x^2 - x + 3$ is

$$(-1)^2 - 4 \cdot 1 \cdot 3 = 1 - 12 < 0$$

- $(\dfrac{f}{g})(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x^2 - 3x}{x^2 - x + 3}$, defined on $\mathbb{R}$. ($x^2 - x + 3 \neq 0$ for every real number).

7. When $f(x)=\sqrt{4x-1}$ and $g(x)=\dfrac{1}{x}$, the domains are respectively:

$$\mathcal{D}_f=\{x\in\mathbb{R},4x-1\geqslant 0\}=[\frac{1}{4},\infty) \qquad \text{and} \qquad \mathcal{D}_g=\{x\in\mathbb{R},x\neq 0\}$$

On their intersection which is the interval $[\frac{1}{4},\infty)$, are defined the functions:

- $(f+g)(x)=f(x)+g(x)=\sqrt{4x-1}+\dfrac{1}{x}$
- $(f-g)(x)=f(x)-g(x)=\sqrt{4x-1}-\dfrac{1}{x}$
- $(fg)(x)=f(x).g(x)=\sqrt{4x-1}\cdot\dfrac{1}{x}=\dfrac{\sqrt{4x-1}}{x}$

  No real we exclude from this interval to define $\dfrac{f}{g}$ because $g(x)=\dfrac{1}{x}$ is always $\neq 0$.

- $(\dfrac{f}{g})(x)=\dfrac{\sqrt{4x-1}}{\frac{1}{x}}=x\sqrt{4x-1}$, defined on $[\frac{1}{4},\infty)$.

Exercise 41

Show that $(f\circ g)(x)$ is not equivalent to $(g\circ f)(x)$ for

$$f(x)=3x-2\ ,\ g(x)=2x-3$$

Solution

$$\begin{aligned}(f\circ g)(x) &= f\big(g(x)\big)\\ &= 3g(x)-2\\ &= 3(2x-3)-2\\ &= 6x-11\end{aligned}$$

but

$$\begin{aligned}(g\circ f)(x) &= g\big(f(x)\big)\\ &= 2f(x)-3\\ &= 2(3x-2)-3\\ &= 6x-7\end{aligned}$$

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In this section, we will use the vertex formula summarized below:

The quadratic function defined by $f(x) = ax^2 + bx + c$ can be written as

$$\boldsymbol{y = f(x) = a(x - h)^2 + k}, \quad a \neq 0,$$

where $\qquad h = -\dfrac{b}{2a} \quad$ and $\quad k = f(h)$. Vertex formula

The graph of f has the following characteristics.

1. It is a parabola with vertex (h, k) and the vertical line $x = h$ as axis.
2. It opens up if $a > 0$ and down if $a < 0$.
3. It is wider than the graph of $y = x^2$ if $|a| < 1$ and narrower if $|a| > 1$.
4. The y-intercept is $f(0) = c$.
5. The x-intercepts are found by solving the equation $ax^2 + bx + c = 0$.

Homework 2

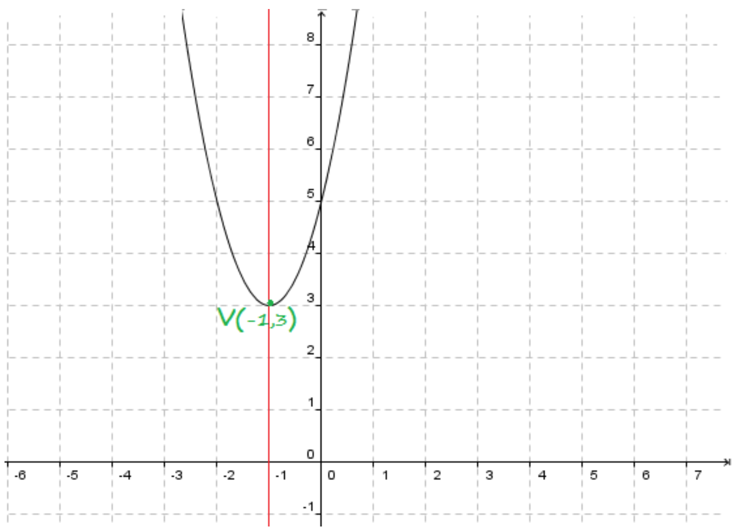
Find the axis and vertex of the parabola having equation $f(x) = 2x^2 + 4x + 5$

Solution The vertex form of $f(x) = 2x^2 + 4x + 5$ is

$$f(x) = a(x - h)^2 + k \;=\; 2(x + 1)^2 + 3$$

In fact: $\quad a = 2$, $\quad h = -\frac{b}{2a} = -\frac{4}{2\times 2} = -1$, $\quad k = f(-1) = 2(-1)^2 + 4(-1) + 5 = 3$

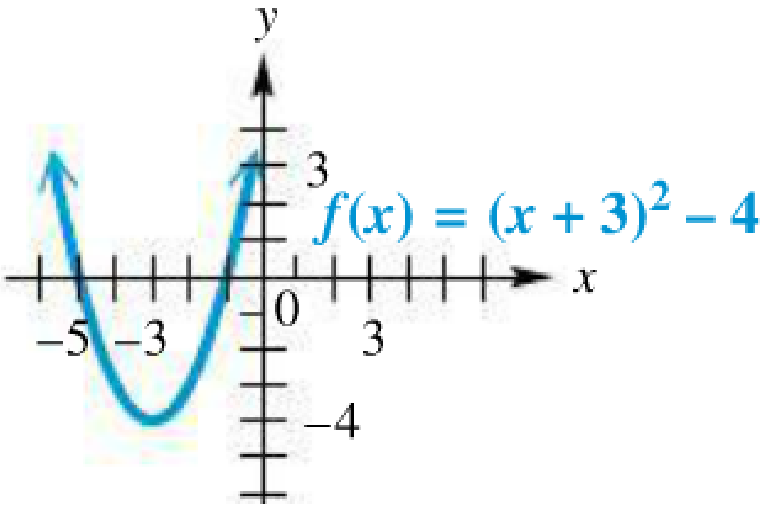
We deduce that its axis is the vertical line "$x = -1$" and its vertex is the point $V(-1, 3)$.



Exercises 1,2

For each of the following, find	**(a)** the domain and range
(b) the coordinates of the vertex	**(c)** the equation of the axis
(d) the x-intercepts	**(e)** the y-intercept

Solution
1.



(a) Obviously, the domain is the set of all real numbers $\mathbb{R} = (-\infty, \infty)$.
For the range, here our function takes its values on $[-4, \infty)$

(b) The vertex is $V_{(-3,-4)}$ and **(c)** the equation of the axis is "$x = -3$"

(d) We expect at most two zeros for any quadratic function. Here, by solving the equation

$$f(x) = 0,$$

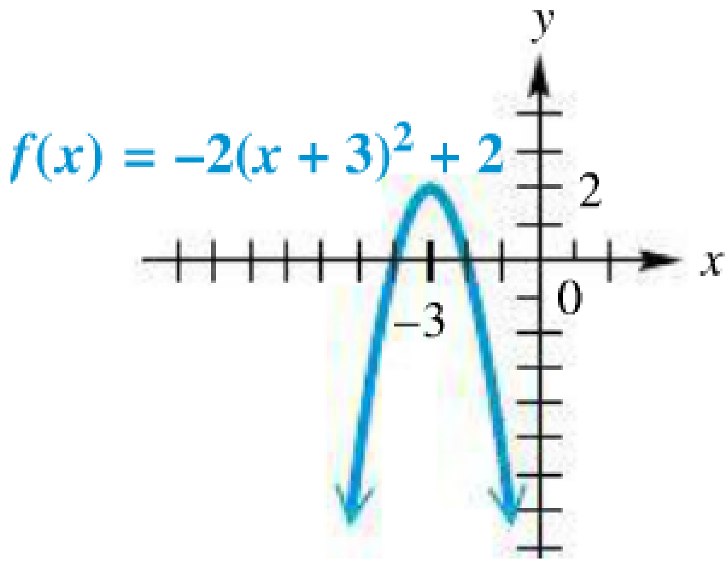
we obtain

$$(x+3)^2 - 4 = 0 \iff (x+3)^2 = 4 \iff x+3 = \pm 2 \iff x = \pm 2 - 3$$

Then, we deduce that the zeros of f are -5 and -1 which correspond to the x-intercepts.

(e) The y-intercept is always a unique point, it is given by $f(0) = (0+3)^2 - 4 = 9 - 4 = 5$

2.



(a) The domain is the set of all real numbers $\mathbb{R} = (-\infty, \infty)$.
The range is $(-\infty, 2]$ which is due to the negative sign of a.

(b) The coordinates of the vertex are $(-3, 2)$ and **(c)** the equation of the axis is "$x = -3$"

(d) By solving the equation $f(x) = 0$, we obtain

$$-2(x+3)^2 + 2 = 0 \iff (x+3)^2 = 1 \iff x + 3 = \pm 2 \iff x = \pm 1 - 3$$

Then, we deduce that the zeros of f are -4 and -2 which correspond to the x-intercepts.

(e) The y-intercept is $f(0) = -2(0+3)^2 + 2 = -2 \times 9 + 2 = -16$

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Homework 1

Let $f(x) = -x^4 + 3x^2 - 4x - 5$. Use the reminder theorem to find $f(-3)$.

Solution

-3	-1	0	3	-4	-5
		3	-9	18	-42
	-1	3	-6	14	-47

This is the remainder which is equal to $f(-3)$.

Exercise 3

Use synthetic division to perform the division $\dfrac{x^4 + 5x^3 + 4x^2 - 3x + 9}{x + 3}$.

Solution

-3	1	5	4	-3	9
		-3	-6	6	-9
	1	2	-2	3	0

Here the remainder is 0, which means that $(x + 3)$ is a factor, so

$$\frac{x^4 + 5x^3 + 4x^2 - 3x + 9}{x + 3} = x^3 + 2x^2 - 2x + 3$$

or $$x^4 + 5x^3 + 4x^2 - 3x + 9 = (x + 3)(x^3 + 2x^2 - 2x + 3)$$

Exercise 10

Express $f(x)$ in the form $f(x) = (x-k)q(x) + r$, where

$$f(x) = 2x^3 + 3x^2 - 16x + 10 \; ; \; k = -4$$

Solution

-4	2	3	-16	10
		-8	20	-16
	2	-5	4	-6

The quotient is $q(x) = 2x^2 - 5x + 4$

and the remainder r is $-6 = f(-4)$. Finally we have

$$2x^3 + 3x^2 - 16x + 10 \;=\; (x+4)(2x^2 - 5x + 4) - 6$$

Exercise 23

Use synthetic division to decide wether the number $k = 1$ is a zero of the function

$$f(x) = 2x^3 + 9x^2 - 16x + 12$$

Solution

Certainly the answer is **negative** because $f(1) = 2+9-16+12 \neq 0$, but unfortunately we must use the synthetic division as required:

$k = 1$	2	9	-16	12
		2	11	-5
	2	11	-5	7

which confirms the previous calculation $f(\;1\;) = 7$. Finally while

$$f(1) \neq 0 \; , \; 1 \text{ is not a zero of } f.$$

The main theorem of this section is:

Factor Theorem

For any polynomial function $f(x)$, $x - k$ is a factor of the polynomial if and only if $f(k) = 0$.

Homework 1

Factor $f(x) = 6x^3 + 19x^2 + 2x - 3$ into linear factors if -3 is a zero of f .

Solution

-3	6	19	2	-3
		-18	-3	3
	6	1	-1	0

The quotient is $q(x) = 6x^2 + x - 1$

By the remainder theorem, $f(-3) = 0$, which confirms that

-3 is a zero, or simply that $x + 3$ is a factor. Finally,

$$\begin{aligned} 6x^3 + 19x^2 + 2x - 3 &= (x+3)(6x^2 + x - 1) \\ &= (x+3)(2x+1)(3x-1) \end{aligned}$$

Exercises 3, 6, 8

Use the factor theorem and synthetic division to decide wether the polynomial $f(x)$ is a factor of the polynomial $g(x)$.

3. $f(x) = x^3 - 5x^2 + 3x + 1$, $g(x) = x - 1$

6. $f(x) = 4x^2 + 2x + 54$, $g(x) = x - 4$

8. $f(x) = 2x^4 + 5x^3 - 2x^2 + 5x + 6$, $g(x) = x + 3$

Solution

3. Let $f(x) = x^3 - 5x^2 + 3x + 1$ and $g(x) = x - 1$. Then by synthetic division, one get a null reminder as detailed below:

$k = 1$	1	-5	3	1
		1	-4	-1
	1	-4	-1	0

It follows that $f(\ 1\ ) = 0$, and then $x - 1$ is a factor of $f(x) = x^3 - 5x^2 + 3x + 1$

6. Write now the synthetic division of $f(x) = 4x^2 + 2x + 54$ by $g(x) = x - 4$.

4	4	2	54
		16	72
	4	18	126

The reminder is $126 = f(4)$ not zero and $x - 4$ can't be a factor of $f(x) = 4x^2 + 2x + 54$

6. Finally, when $f(x) = 2x^4 + 5x^3 - 2x^2 + 5x + 6$ and $g(x) = x + 3$, we write:

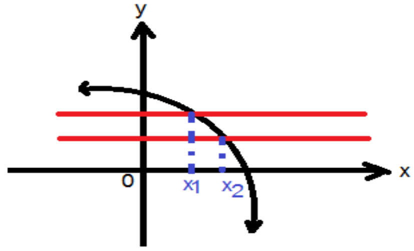
-3	2	5	-2	5	6
		-6	3	-3	-6
	2	-1	1	2	0

The remainder is zero : $x + 3$ is a factor.

5.1 Exercises (Page: 215)

Decide whether each function as graphed or defined is one-to-one.

(3)

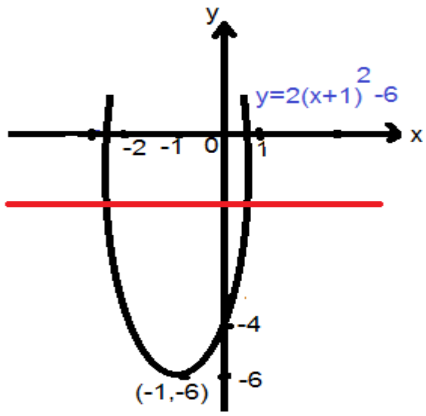


Solution

The graph of the function is one-to-one, as shown in the Figure. Because every horizontal line intersects the graph in one point.

(8) $y = 2(x+1)^2 - 6$

Solution



The graph of the function is a parabola, as shown in the Figure. Because there is at least on horizontal line that intersects the graph in more than one point, this function is not one-to-one.

(21) Use the definition of the inverse to determine whether f and g are inverse.

$f(x) = -3x + 12$, $g(x) = -\frac{1}{3}x - 12$

Solution

The function $g(x)$ is the inverse function of $f(x)$ if $(fog)(x) = (gof)(x) = x$ (Inverse Definition)

$$(fog)(x) = f(g(x)) = f(-\frac{1}{3}x - 12) = -3(-\frac{1}{3}x - 12) + 12 = x + 36 + 12 = x + 48$$

$$(gof)(x) = g(f(x)) = g(-3x + 12) = -\frac{1}{3}(-3x + 12) - 12 = x - 4 - 12 = x - 16$$

Since $(fog)(x) \neq x$ and $(gof)(x) \neq x$, function g is not the inverse of function f and also function f is not the inverse of function g .

5.2 Exercises (Page: 226)

For $f(x)=3^x$ **and** $g(x)=\left(\frac{1}{4}\right)^x$ **, find each of the following:**

(3) $g(2)$

Solution

$$g(2)=\left(\frac{1}{4}\right)^2=\frac{1}{16}$$ **(Replace** x **with** 2 **)**

(4) $g(-2)$

Solution

$$g(-2)=\left(\frac{1}{4}\right)^{-2}=\frac{1}{\left(\frac{1}{4}\right)^2}=\frac{1}{\frac{1}{16}}=16$$ **(Replace** x **with** -2 **)**

Solve each equation:

(34) $e^{4x-1}=(e^2)^x$

Solution **Write each side of the equation using a common base.**

$e^{4x-1}=(e^2)^x=e^{2x}$ **:** $\left[(a^m)^n=a^{mn}\right]$

$4x-1=2x$ **(Set exponents equal)**

$2x=1$ **(Subtract** $2x$**, Add** 1**)**

$x=\frac{1}{2}$ **(Divide by 2)**

Check by substituting $\frac{1}{2}$ **for** x **in original equation. The solution set is** $\left\{\frac{1}{2}\right\}$**.**

(38) $\left(\sqrt{2}\right)^{x+4}=4^x$

Solution **Write each side of the equation using a common base.**

$\left(\sqrt{2}\right)^{x+4}=4^x$

$\left(2^{\frac{1}{2}}\right)^{x+4}=\left(2^2\right)^x$ **(Write** $\sqrt{2}=2^{\frac{1}{2}}$**,** 4 **as a power of** 2 **)**

$(2)^{\frac{1}{2}(x+4)}=(2)^{2x}$

$\frac{1}{2}x+2=2x$ **(Set exponents equal)**

$-\frac{3}{2}x=-2$ **(Subtract $2x$ and 2)**

$x=\frac{4}{3}$ **(Divide by $-\frac{3}{2}$)**

Check by substituting $\frac{4}{3}$ for x in original equation. The solution set is $\left\{\frac{4}{3}\right\}$.

5.3 Exercises (Page: 234)

If the statement is in exponential form, write it in an equivalent logarithmic form. If the statement is in logarithmic form, write it in an exponential form.

(3) $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$ **(Statement is in exponential form)**

Solution **Logarithmic form is** $\log_{\frac{2}{3}} \frac{27}{8} = -3$.

(9) $x = \log_8 \sqrt[4]{8}$ **(Statement is in logarithmic form)**

Solution **Exponential form is** $8^x = \sqrt[4]{8}$ **or** $8^x = 8^{\frac{1}{4}}$.

(37) Use the properties of logarithms to rewrite each expression. Simplify the result if possible. Assume all variables represent positive real numbers.

$$\log_3 \frac{\sqrt{x}.\sqrt[3]{y}}{w^2\sqrt{z}}$$

Solution

$$\log_3 \frac{\sqrt{x}.\sqrt[3]{y}}{w^2\sqrt{z}} = \log_3 \sqrt{x} + \log_3 \sqrt[3]{y} - \left(\log_3 w^2 + \log_3 \sqrt{z}\right)$$

$$= \log_3 x^{\frac{1}{2}} + \log_3 y^{\frac{1}{3}} - (\log_3 w^2 + \log_3 z^{\frac{1}{2}})$$

$$= \frac{1}{2}\log_3 x + \frac{1}{3}\log_3 y - (2\log_3 w + \frac{1}{2}\log_3 z)$$

(42) Write each expression as a single logarithm with coefficient 1. Assume all variables represent positive real numbers.

$$-\frac{2}{3}\log_5 5m^2 + \frac{1}{2}\log_5 25m^2$$

Solution

$$-\frac{2}{3}\log_5 5m^2 + \frac{1}{2}\log_5 25m^2 = \log_5 \left(5m^2\right)^{-\frac{2}{3}} + \log_5 \left(25m^2\right)^{\frac{1}{2}}$$

$$= \log_5 \left(5m^2\right)^{-\frac{2}{3}} \left(25m^2\right)^{\frac{1}{2}} = \log_5 \frac{5m}{\left(5m^2\right)^{\frac{2}{3}}}$$

(46) Given the approximations $\log_{10} 3 = 0.4771$ **find each logarithm without using a calculator.**

$$\log_{10}\sqrt{30}$$

Solution

$$\log_{10}\sqrt{30} = \log_{10}(30)^{\frac{1}{2}} = \frac{1}{2}\log_{10} 30 = \frac{1}{2}\log_{10}(10\times 3) = \frac{1}{2}[\log_{10} 10 + \log_{10} 3]$$

$$= \frac{1}{2}[1 + 0.4771] = \frac{1}{2}\times(1.4771) = 0.73855.$$

Ex.8 Solve the system by substitution:

$$-2x = 6y + 18$$
$$-29 = 5y - 3x$$

Step 1 Transform the system in the standard form:

(1) $-2x - 6y = 18$
(2) $-3x + 5y = -29$

From (1) $-2x = 18 + 6y$

$$x = \frac{18+6y}{-2} = -9 - 3y \quad (3)$$

in (2) replace x by $-9-3y$

$$-3(-9-3y) + 5y = -29$$
$$+27 + 9y + 5y = -29$$
$$14y = -29 - 27 = -56$$
$$y = -56/14 = -4$$

Using (1) or (2) or (3) we compute $x = -9 - 3y$
$$= -9 + 12$$
$$= 3$$

The solution is the pair $(x,y) = (3,-4)$

M. ZAHRI

Ex. 14 (page 254)

Solve the system by elimination

(1) $\frac{x}{2} + \frac{y}{3} = 4$

(2) $\frac{3}{2}x + \frac{3}{2}y = 15$

<u>First</u>, we write the system in a simple form

we multiply (1) by (6) and (2) by $\frac{2}{3}$

(1)' $3x + 2y = 24$

(2)' $x + y = 10$

we multiply (2)' by -3

$$
\begin{array}{rl}
+ & 3x + 2y = 24 \\
 & -3x - 3y = -30 \\
\hline
 & 0 - y = -6
\end{array}
$$

so $y = +6$

from (2)' : $x = 10 - y$
$= 10 - 6$
$= 4$

$(x, y) = (4, 6)$

<u>Test</u> $\frac{4}{2} + \frac{6}{3} = 4$ ✓

$\frac{3}{2} \cdot 4 + \frac{3}{2} 6 = 6 + 9 = 15$ ✓

The solution is $(x, y) = (4, 6)$

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EX. 35

$$\frac{2}{x} + \frac{1}{y} = \frac{3}{2} \quad (1)$$

$$\frac{3}{x} - \frac{1}{y} = 1 \quad (2)$$

(1) + (2) Using a simple addition of (1) and (2)

$$\frac{2}{x} + \frac{3}{x} = \frac{3}{2} + 1 = \frac{5}{2}$$

$$\frac{5}{x} = \frac{5}{2}$$

so $\frac{x}{5} = \frac{2}{5} \Rightarrow \underline{x = 2}$

in (1) $\frac{2}{2} + \frac{1}{y} = \frac{3}{2}$

$$\frac{1}{y} = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$$

then $\underline{y = 2}$

The solution is $(x, y) = (2, 2)$

The solution is $(x, y) = (2, 2)$

M. ZAHRI

Section 7.1 Ex. 6, 9, 16, 20 p. 287-288

EX.6

$$\lim_{x\to 1} \frac{x^2-1}{x+1} = \frac{1-1}{1+1} = 0$$

لسنا بحاجة إلى
التفكير في الاختزال
لأن $\frac{x^2-1}{x+1}$ معرفة عند $x=1$

EX.9

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{(\sqrt{x}-3)}{(\sqrt{x}-3)(\sqrt{x}+3)}$$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3}$$

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

نجب الاختزال
لأن تعويض 9
في المقام يعطي صفر
وهذا غير مقبول

EX.16

$$\lim_{x\to 2} \frac{x^4-16}{x^3-8} = \lim_{x\to 2} \frac{(x^2-4)(x^2+4)}{(x-2)(x^2+2x+4)}$$

$$= \lim_{x\to 2} \frac{(x-2)(x+2)(x^2+4)}{(x-2)(x^2+2x+4)}$$

$$= \lim_{x\to 2} \frac{(x+2)(x^2+4)}{(x^2+2x+4)}$$

$$= \frac{(2+2)(4+4)}{4+4+4} = \frac{32}{12}$$

استعمال Perfect Trinomials and Binomials
مهم جداً في النهايات

EX.20

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x-x-h}{h(x+h)x}$$

$$= \lim_{h\to 0} -\frac{1}{(x+h)x}$$

$$= -\frac{1}{x^2}$$

نختزل x من البسط ثم
نختزل h من البسط والمقام.

بالتوفيق د/ مصطفى زهيري

Section 7.2
MATH101

7.2 p293

2) $\lim_{x\to\infty} \frac{3x^3-5x^2+7}{8+2x-5x^3} = \lim_{x\to\infty} \frac{3x^3}{-5x^3} = -\frac{3}{5}$

5) $\lim_{x\to-\infty} \frac{2x-1}{\sqrt{3x^2+x+1}} = \lim_{x\to-\infty} \frac{2x}{\sqrt{3x^2}} = \lim_{x\to-\infty} \frac{2x}{\sqrt{3}|x|} = \frac{-2}{\sqrt{3}}$

15) $\lim_{x\to\infty} \frac{1}{\sqrt{x^2-2x}-x} \cdot \frac{\sqrt{x^2-2x}+x}{\sqrt{x^2-2x}+x} = \lim_{x\to\infty} \frac{\sqrt{x^2-2x}+x}{x^2-2x-x^2} = \lim_{x\to\infty} \frac{\sqrt{x^2-2x}}{-2x} - \frac{x}{2x}$

$= -\frac{1}{2} - \frac{1}{2} = -1$

Section 7.3
MATH 101

7.3 p302

4) $f(x) = \begin{cases} x & , x < 0 \\ x^2 & , x \geq 0 \end{cases}$

f(0)=0

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x = 0$

$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} x^2 = 0$

$\lim_{x \to 0} f(x) = 0 = f(0)$

f is continuous

6) $f(x) = \frac{x^2-4}{x-2} \quad at\ x = 2$

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x + 2)(x - 2)}{(x - 2)} = \lim_{x \to 2} x + 2 = 4$$

The continuous function should be :

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & x \neq 2 \\ 4 & x = 2 \end{cases}$$

8) $f(x) = \begin{cases} x^2 & , \ x \leq 2 \\ k - x^2 & , \ x > 2 \end{cases}$

$\lim_{x \to 2} x^2 = \lim_{x \to 2} k - x^2$

4=k-4

K=8