(2 marks)

Question 1: Classify each variable as Qualitative or Quantitative.	The answer
The variable that records length of roads in the city.	Quan
The variable that records distance between countries.	Quan
The variable that records types of pens.	Qual
The variable that records colors of flowers.	Qual

(2 marks)

Question 2: Classify each variable as Continuous or Discrete.	The answer
The variable that records numbers of students in schools.	dis
The variable that records age of people in KSA.	cts
The variable that records heights of buildings in the university.	cts
The variable that records numbers of integer numbers in intervals of real numbers.	dis

(2 marks)

Question 3: Determine whether of the following statements is True or False.	The answer
$F_X(x) = P(X \le x)$ for some $x \in \mathbb{R}$. $\int_{-\infty}^{x}$	T
The mean of data is sensitive to extreme values.	T
If $\mathscr{A}$ is an algebra on Ω, then $\varnothing \in \mathscr{A}$.	T
The statistic $\bar{X}$ is an estimator for the variance σ^2 of a normal population.	F

$\bar{X} \longrightarrow \mu$ $\quad S \longrightarrow \sigma$ $\quad \hat{p} \longrightarrow p$

(2 marks)

Question 4: Put the right word or symbol in its proper position:
parameter, $\mathscr{A} \in \Omega$, statistic, $\Omega \in \mathscr{A}$, permutation, combination, independent, mutual exclusive.

Two events A and B are mutual................ if they have not common elementary events.
Selection r distinct objects from a set of n different objects, is called a ...~~[illegible]~~...comb
If a $\mathscr{A}$ is an algebra on Ω, then ......$\Omega \in A$........
The point estimation is an estimate of the population ...parameter.......by a single number.

(7 marks)

Question 5: Consider the following data:

7	16	7	12	13	2	12	21	17	4	2	16	11
16	7	11	10	8	9	8	6	22	17	7	14	22
12	6	6	1	15	17	11	11	25	1	12	13	16
12	18	23	17	8	13	22	21	21	15	18		

Then:

a) Complete the following frequency distribution table for the given data:

Class Limit	Class Boundaries	Midpoint	Frequency	Ascending Cumulative Frequency (ACF)
1 — 5	0.5 → 5.5	$\frac{1+5}{2} = 3$	5	5
6 — 10	5.5 → 10.5	$\frac{6+10}{2} = 8$	12	17
11 — 15	10.5 → 15.5	13	15	32
16 — 20	15.5 → 20.5	18	10	42
21 — 25	20.5 → 25.5	23	8	50
Sum			50	

b) Calculate the median for the data of above frequency distribution table.

$\frac{\sum f}{2} = \frac{50}{2} = 25$

$\tilde{X} = L + \frac{\frac{\sum f}{2} - (F - f)}{f} \times C =$

~~11~~ 18

10.5 – 15.5

$C = 15.5 - 10.5 = 5$

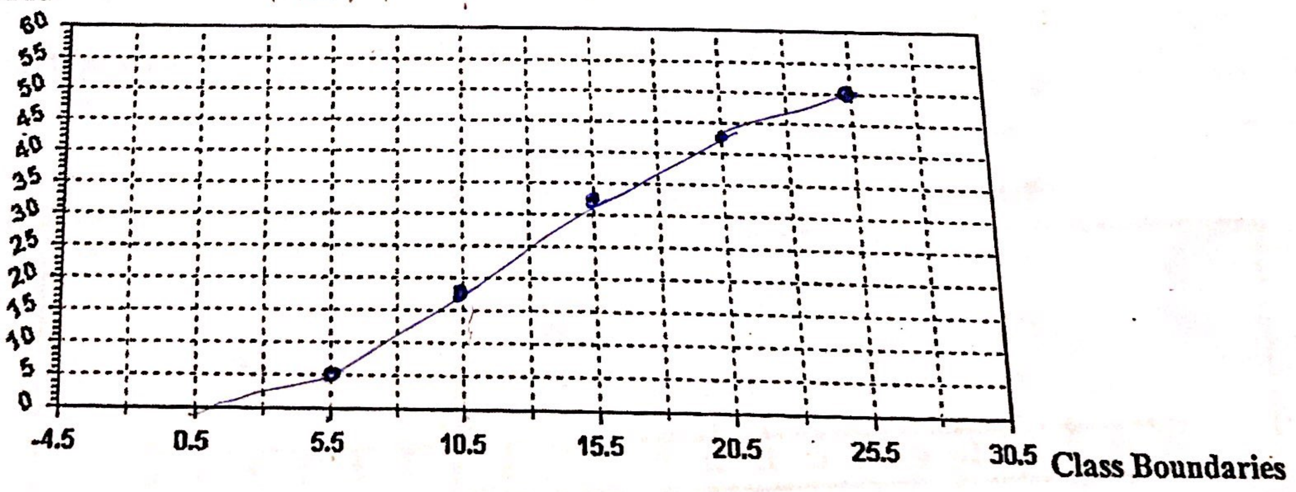
$10.5 + \frac{25 - (32 - 15)}{15} \times 5$

c) Calculate the range of data for the above frequency distribution table.

$R = X_k - X_1 = 23 - 3 = 20.$

d) Draw the ~~ogive~~ (ascending cumulative polygon) of the above frequency distribution table.

ACFP ?



(10 marks)

Question 6: Consider the data: 9, ~~5~~, ~~3~~, 9, ~~5~~, ~~7~~, ~~1~~, ~~7~~, ~~6~~, 16, 9. Then:

a) Calculate the mean for the given data.

$\bar{X} = \frac{\sum X}{n} = \frac{9+5+3+9+5+7+1+7+6+16+9}{11} = \frac{77}{11} = 7$

b) Calculate the median for the given data.

1, 3, 5, 5, 6, 7, 7, 9, 9, 9, 16 (X_1, X_{11})

$\frac{n+1}{2} = \frac{11+1}{2} = 6$

$n = 11$ odd $\quad X_{\frac{n+1}{2}} = X_6 = 7$

c) Find the mode of the given data.

$\hat{X} = 9$

d) If the standard deviation of the given data is $S = 3.92$, then calculate the standard score for the value 6.

$S^2 = 10$

$S = \sqrt{10}$

$Z_x = \frac{X - \bar{X}}{S} = \frac{6-7}{3.92} = -0.25$

e) Calculate the coefficient of variation for the given data.

$C.V = \frac{S}{\bar{X}} \times 100\% = \cancel{\frac{7}{3.92} \times 100}\ \frac{3.92}{7} \times 100\%$

f) Calculate Q_1, Q_3, LF and HF for the given data.

For Q_1: $= X_k + S(X_{k+1} - X_k)$ $\quad q_1 = \frac{1(n+1)}{4} = \frac{1(11+1)}{4} = 3$ $\quad k = 3 \quad S = 0$

$X_3 + 0(X_4 - X_3) = 5 + 0 = 5$

For Q_3: $X_k + S(X_{k+1} - X_k)$ $\quad q_3 = \frac{3(n+1)}{4} = 9$ $\quad k = 9 \quad S = 0$

$= X_9 + 0(X_{10} - X_9) = 9$ $\quad Q_3 = 9$

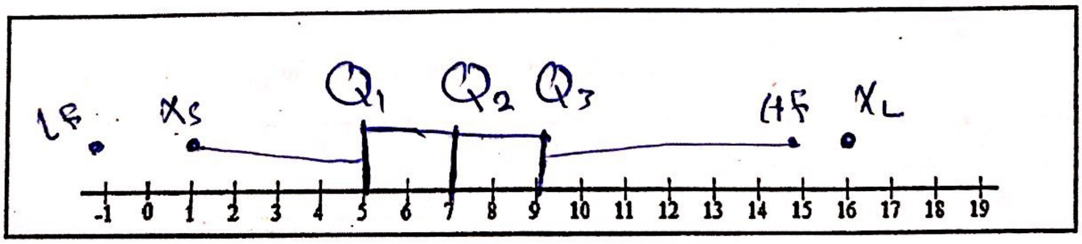
For LF: $Q_1 - 1.5(Q_3 - Q_1) = 5 - 1.5(9-5) = -1$

For HF: $Q_3 + 1.5(Q_3 - Q_1) = 9 + 1.5(9-5) = 15$

g) Check if the given data have outliers.

$X = 16$ is outlier

h) Draw the box plot for the given data and determine the five numbers on the graph:



$X_S = 1$

$X_L = 16$

$Q_1 = 5$

$Q_2 = 7 = \tilde{X}$

$Q_3 = 9$

(3.5 marks)

Question 7: The following data represent the grades of students:

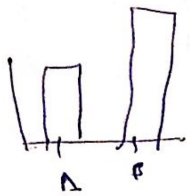
A	C	C	A	B	C	B	A	D	B
D	B	C	D	B	A	C	A	D	C
A	B	A	D	C	B	B	C	D	B
B	B	B	A	A	B	C	B	A	A

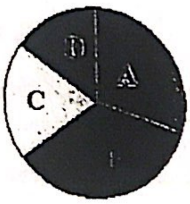
Then:

a) Complete the frequency table for the above data.

Grade	Frequency	Relative Frequency	Percentage
A	11	$\frac{11}{40} = 0.275$	27.5 %
B	14	$\frac{14}{40} = 0.35$	35 %
C	9	$\frac{9}{40} = 0.225$	22.5 %
D	6	$\frac{6}{40} = 0.15$	15 %
sum	40	1	100 %

b) For the above data, calculate the measure angles of categories of the pie chart.



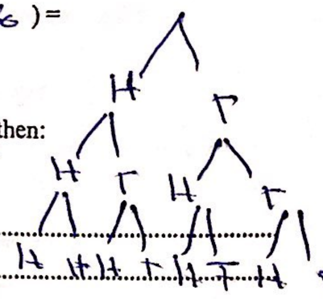


For category (A) the measure angle = (0.275)(360) =

For category (B) the measure angle = (0.35)(360) =

For category (C) the measure angle = (0.225)(360) =

For category (D) the measure angle = (0.15)(360) =



(2.5 marks)

Question 8: If $[\Omega, \mathscr{A}, P]$ is a probability space of tossing a fair coin three times, then:

a) Determine Ω, $\mathscr{A}$ and P for this random experiment.

$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\mathscr{A} = 2^{\Omega}$ $\quad P(A) = \frac{|A|}{|\Omega|}$

b) Calculate the probability of getting at most one tails.

$B = \{HHH, HHT, HTH, THH\}$

$P(B) = \frac{|B|}{|\Omega|} = \frac{4}{8}$

c) Calculate the probability of getting three heads or three tails.

$C = \{HHH, TTT\}$ $\quad P(C) = \frac{|C|}{|\Omega|} = \frac{2}{8}$

nPr one after other / order / arrang

nCr same time / un order /

(3 marks)

Question 9: We select **three** balls randomly and at the same time of a box contains **4 black** and **3 green** balls. If all balls have the same chance at selecting. Now:

a) If A is the event that the selected balls are black, then calculate $P(A)$.

$\Omega = 7C3$ $\quad\quad$ $\boxed{\begin{matrix} B & G \\ 4 & 3 \end{matrix}}$

$P(A) = \dfrac{4C3}{7C3} \approx 0.11$

$A = 4C3$

b) If B is the event that the selected balls have the same colors, then calculate $P(B)$.

$\dfrac{(4C3) + (3C3)}{7C3}$

c) What is the probability that the selected balls have different colors?

Impossible $\quad\quad P(\phi) = 0$

$f(x) = ?x \quad (0 \le x \le 1)$

(3.5 marks)

Question 10: Suppose that $\Omega = \{1,2,3,4,5,6\}$, $\mathscr{A} = 2^{\Omega}$ and $P(A) = \dfrac{|A|}{|\Omega|}$. Now, let $X : \Omega \longrightarrow \mathbb{R}$ be a random variable on the probability space $[\Omega, \mathscr{A}, P]$ defined by $X(\omega) = \begin{cases} 0 & for\ \omega = 1,3,5 \\ 1 & for\ \omega = 2,4,6 \end{cases}$

Then:

a) What is the name of this random variable and calculate the probability $P(X = 0)$?

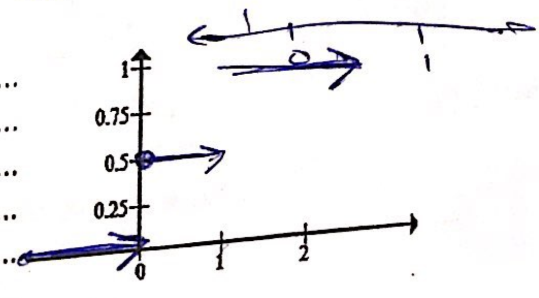
descrete r.v $\quad\quad P(X=0) = \dfrac{3}{6} = \dfrac{1}{2}$

Binomial

b) Determine the event in the relation: $\{\omega \in \Omega\ ; X(\omega) \le x\} = \begin{cases} \phi & for\ x < 0 \\ \{1,3,5\} & for\ 0 \le x < 1 \quad x=0 \\ \{1,2,3,4,5,6\} = \Omega & for\ x \ge 1 \quad x \le 1 \end{cases}$

c) Determine the distribution function F_X and draw it.

$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{3}{6} = \dfrac{1}{2} & 0 \le x < 1 \\ 1 & 1 \le x \end{cases}$



Scanned with CamScanner

$P(A\cup B)=0.75= P(A)+P(B)-P(A\cap B)$

$P(\bar{A}) = 1-P(A)=0.65$

$P(A)=1-0.65=0.35$

(3 marks)

Question 11: If we have Ω a space of elementary evens, A and $B \in 2^{\Omega}$ with $P(A\cap B)=0.10$, $P(A\cup B)=0.75$ and $P(\bar{A})=0.65$. Then calculate the following probabilities:

a) $P(B)=$ $P(A\cup B)= P(A)+P(B)-P(A\cap B)$

$0.75 = 0.35 + P(B) - 0.10$ $\quad 0.75=0.25+P(B)$ $\quad P(B)=0.5$

b) $P(A\setminus B)=$ $P(A)-P(A\cap B) = 0.35-0.1 = 0.25$

c) $P(\bar{A}\cap\bar{B})=$ $P(\overline{A\cup B}) = 1-P(A\cup B) = 1-0.75 = 0.25$

d) $P(A|B)=$ $\dfrac{P(A\cap B)}{P(B)} = \dfrac{0.1}{0.5} = 0.2$

e) Are the events A and B independent? and why?

$P(A)\cdot P(B) \neq P(A\cap B)$ not indep.

$0.35\cdot 0.5 = 0.175 \neq 0.1\ P(A\cap B)$

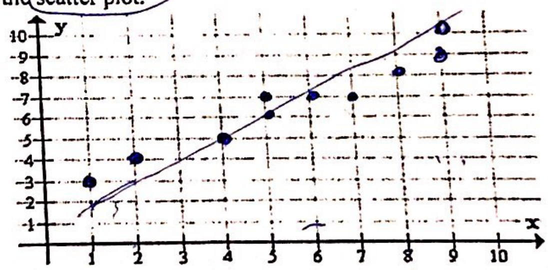
(3 marks)

Question 12: The results of 10 students on Stat quiz marks (X) and Math quiz marks (Y) are as follows:

X	2	4	5	5	6	7	7	8	1	9
Y	4	5	6	7	7	7	8	9	3	10

a) Represent this data on the scatter plot.

X	Y	XY	X^2	Y^2



b) If you know that $\sum_{i=1}^{10} x_i = 54$, $\sum_{i=1}^{10} y_i = 66$, $\sum_{i=1}^{10} x_i^2 = 350$, $\sum_{i=1}^{10} y_i^2 = 478$, $\sum_{i=1}^{10} x_i y_i = 405$, $\sum_{i=1}^{10}(x_i-\bar{x})^2 = 58.4$, $\sum_{i=1}^{10}(y_i-\bar{y})^2 = 42.4$ and $\sum_{i=1}^{10}(x_i-\bar{x})(y_i-\bar{y}) = 48.6$. Then calculate the coefficient of correlation (r).

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2-(\sum x)^2}\sqrt{n\sum y^2+(\sum y)^2}} = \frac{10(405)-(54)(66)}{\sqrt{10(350)-(54)^2}\sqrt{10(478)-(66)^2}}$$

$=0.97$

c) Is the relationship between X and Y linear? Positive or negative? Strong or weak?

Yes.

$r=0.97$ Positive Strong

(1.5 marks)

Question 13: We suppose that the number of defects in newly manufactured bulbs in each working shift is a normal distributed random variable X with a mean $\mu = 5$ and standard deviation $\sigma = 1.25$. Now, consider that in one of the working shifts a random sample of new bulbs is tested. What is the sampling distribution of $\bar{X}$ based on sample of size 144.

4.1 / C.L.T

$n = 144 > 30$ normale sample distribution

(5 marks)

Question 14: A simple random sample of 81 students from a university yields mean GPA (Grade Point Average) 3.4 with standard deviation 0.75. Then:

a) Determine 95% confidence interval for the mean GPA of all students at the university.

$1 - 0.95 = \alpha$

$n = 81 \quad \bar{X} = 3.4 \quad \sigma = 0.75 \qquad 0.05 = \alpha$

$\bar{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \qquad 1 - \frac{\alpha}{2} = 1 - \frac{0.05}{2}$

$= 0.975$

$3.4 - 1.96\frac{(0.75)}{\sqrt{81}} < \mu < 3.4 + 1.96\left(\frac{0.75}{\sqrt{81}}\right) \quad Z = 1.96$

b) Using the significance level $\alpha = 0.0099$ for testing the null hypothesis $H_0 : \mu = 3$ versus the alternative hypothesis $H_1 : \mu > 3$.

$H_0 = \mu = 3$

$H_1 : \mu > 3$

$Z_{1-\alpha}$

$Z_0 = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{3.4 - 3}{0.75/\sqrt{81}} = 4.8$

$P(Z > 4.8)$

$1 - P(Z < 4.8)$

$Z_0 > Z_{1-\alpha}$

$\alpha = 0.0099$

reject H_0

$1 - 0.0099 = .9901$

$Z_{1-\alpha}$

$Z_{1-\alpha} = 2.33$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964

End of Exam