

1.

a.

X	2	3	4	5	6	7	8	9	10	11	12
b. $P_x = P(X=x)$	$\frac{1}{21}$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{3}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{2}{21}$	$\frac{1}{21}$	$\frac{1}{21}$
c. $F_x(x)$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{4}{21}$	$\frac{6}{21}$	$\frac{9}{21}$	$\frac{12}{21}$	$\frac{15}{21}$	$\frac{17}{21}$	$\frac{19}{21}$	$\frac{20}{21}$	$\frac{21}{21}$

d. $mean = \mu = \sum x \cdot P(x)$

$$(2 \cdot \frac{1}{21}) + (3 \cdot \frac{1}{21}) + (4 \cdot \frac{2}{21}) + (5 \cdot \frac{2}{21}) + (6 \cdot \frac{3}{21}) + (7 \cdot \frac{3}{21}) + (8 \cdot \frac{3}{21}) + (9 \cdot \frac{2}{21}) + (10 \cdot \frac{2}{21}) + (11 \cdot \frac{1}{21}) + (12 \cdot \frac{1}{21}) = 7$$

$$Variance = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \mu^2$$

X	2	3	4	5	6	7	8	9	10	11	12
x^2	4	9	16	25	36	49	64	81	100	121	144

$$E(x^2) = \sum x^2 \cdot P(x) = 2^2 \cdot \frac{1}{21} + 3^2 \cdot \frac{1}{21} + 4^2 \cdot \frac{2}{21} + 5^2 \cdot \frac{2}{21} + 6^2 \cdot \frac{3}{21} + 7^2 \cdot \frac{3}{21} + 8^2 \cdot \frac{3}{21} + 9^2 \cdot \frac{2}{21} + 10^2 \cdot \frac{2}{21} + 11^2 \cdot \frac{1}{21} + 12^2 \cdot \frac{1}{21} = 55.66$$

$$55.66 - 7^2 = 6.66$$

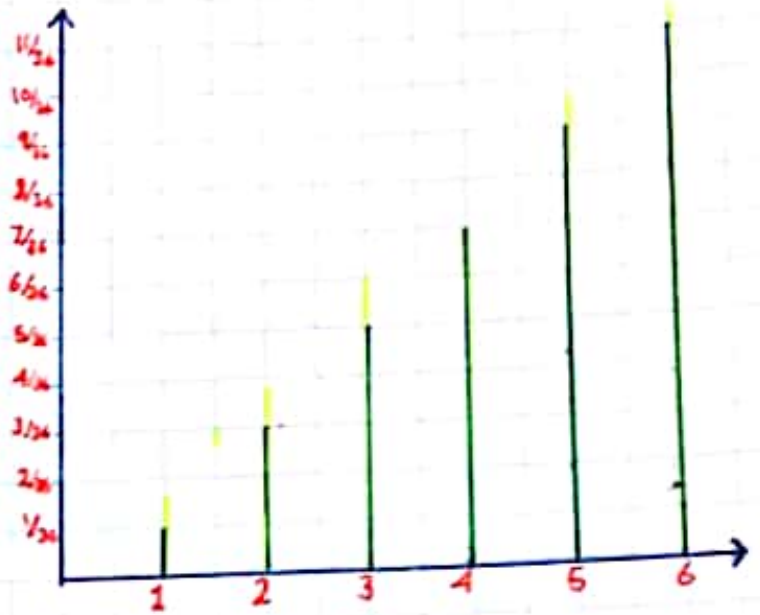
2.

a.

X	1	2	3	4	5	6
$P_x = P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$
$F_x(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{15}{36}$	1

2-
b.

$$F_x(x) = \begin{cases} 0 & \rightarrow x < 1 \\ 1/36 & \rightarrow 1 \leq x < 2 \\ 4/36 & \rightarrow 2 \leq x < 3 \\ 9/36 & \rightarrow 3 \leq x < 4 \\ 16/36 & \rightarrow 4 \leq x < 5 \\ 25/36 & \rightarrow 5 \leq x < 6 \\ 1 & \rightarrow x \geq 6 \end{cases}$$



3-

a.

$$C \frac{2}{7} + C \frac{3}{7} + C \frac{4}{7} + C \frac{5}{7} = 1$$

$$\frac{2C}{2} = \frac{1}{2} \rightarrow C = \frac{1}{2}$$

b.

$$f(k) = \begin{cases} 1/7 & \text{for } k=2 \\ 3/14 & \text{for } k=3 \\ 2/7 & \text{for } k=4 \\ 5/14 & \text{for } k=5 \end{cases}$$

$$F_x(x) = P(X=k) = \begin{cases} 0 & x < 2 \\ 1/7 & 2 \leq x < 3 \\ 5/14 & 3 \leq x < 4 \\ 9/14 & 4 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$$

c. $\text{var} = E(x)^2 - \sum x^2 \cdot P(x)$
 $4 \cdot 1/7 + 9 \cdot 3/14 + 16 \cdot 2/7 + 25 \cdot 5/14 = 16$

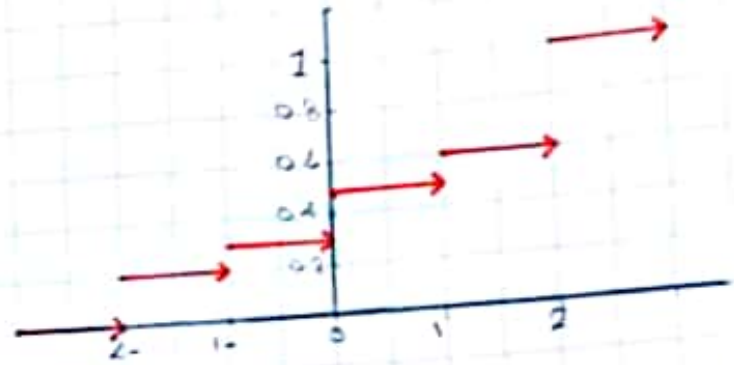
$$\sigma^2 = E(x^2) = 4^2 - 16(3.85)^2 = 1.775$$

$$E(3x-5) = 3E(x) - 5 = (3 \cdot 3.85) - 5 = 6.55$$

$$\text{var}(3x-5) = 3^2 \text{var}(x) - \text{var}(5) \\ = (9 \times 1.775) = 10.60$$

4.

x	-2	-1	0	1	2
$P_x = P(X=x)$	0.20	0.15	0.15	0.1	0.4
$F_x(x)$	0.20	0.35	0.5	0.6	1

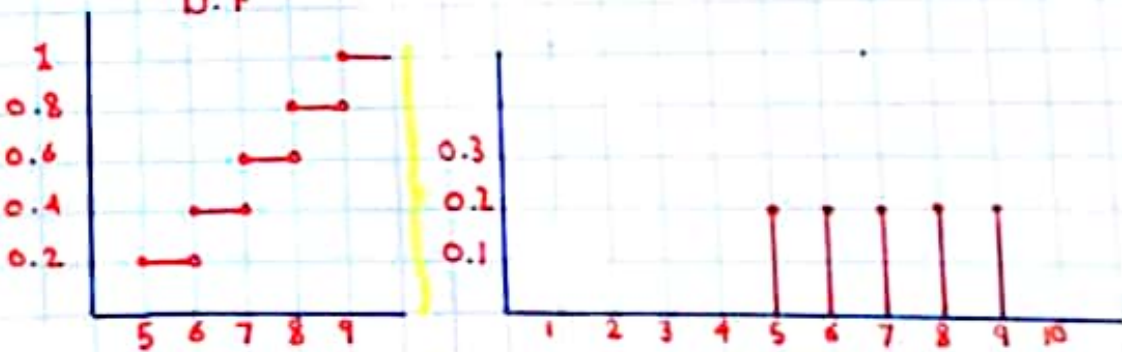


$$F_x(x) = \begin{cases} 0 & x < -2 \\ 0.20 & -2 \leq x < -1 \\ 0.35 & -1 \leq x < 0 \\ 0.5 & 0 \leq x < 1 \\ 0.6 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

5-a.

$$F_x(x) = \begin{cases} 0 & x < 5 \\ 0.2 & 5 \leq x < 6 \\ 0.4 & 6 \leq x < 7 \\ 0.6 & 7 \leq x < 8 \\ 0.8 & 8 \leq x < 9 \\ 1 & x \geq 9 \end{cases}$$

D.F



x	5	6	7	8	9
$P(X=x)$	0.2	0.2	0.2	0.2	0.2

6.

$$\begin{aligned} a. &= \int_0^1 0 + \int_1^x 6(x-1)(2-x) \\ &= \int_1^x (6x-6)(2-x) \\ &= \int_1^x 12x - 6x^2 - 12 + 6x = \frac{18x^2}{2} - \frac{6x^3}{3} - 12x + 6x \Big|_1^x = (9x^2 - 2x^3 - 12x) - (9 - 2 - 12) \\ &= (9x^2 - 2x^3 - 12x) - 9 + 2 + 12 = 9x^2 - 2x^3 - 12x + 5 \end{aligned}$$

$x \geq 2$

$$\begin{aligned} &= \int_1^2 -6x^2 + 12x + 6x - 12 + \int_2^x 0 = \int_1^2 -6x^2 + 18x - 12 \\ &= \frac{-6x^3}{3} + 18 \frac{x^2}{2} - 12x = -2x^3 + 9x^2 - 12x \Big|_1^2 \\ &= (-2(2)^3 + 9(2)^2 - 12(2)) - (-2(1)^3 + 9(1)^2 - 12(1)) = 1 \end{aligned}$$

$$F_x(x) \begin{cases} 0 & x < 1 \\ 9x^2 - 2x^3 - 12x + 5 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

6-

$$b. \mu = E(x) = \int_{-\infty}^{\infty} x f_x(x) dx = \int_1^2 x f_x(x) dx$$

$$= \int_1^2 x(6(x-1)(2-x)) dx = \int_1^2 x(16x-6)(2-x) = \int_1^2 x(12x-6x^2-12+6x)$$

$$= \int_1^2 x(18-16x^2-12) = \int_1^2 18x^2-6x^3-2x$$

$$= \left. \frac{18x^3}{3} - \frac{6x^4}{4} - \frac{2x^2}{2} \right|_1^2 = 6x^3 - \frac{6x^4}{4} - 6x^2 \Big|_1^2$$

$$= (6(2)^3 - \frac{6(2)^4}{4} - 6(2)^2) - (6(1)^3 - \frac{6(1)^4}{4} - 6(1)^2)$$

$$= 6 - (-1.5) = 1.5$$

$$E(x)^2 = \int_1^2 x^2(6(x-1)(2-x)) dx$$

$$= \int_1^2 x^2(6x-x)(2-x) = \int_1^2 x^2(12x-6x^2-12+6x)$$

$$= \int_1^2 x^2(18x-6x^2-12) = \int_1^2 18x^3-6x^4-12x^2$$

$$= \left. \frac{18x^4}{4} - \frac{6x^5}{5} - \frac{12x^3}{3} \right|_1^2 = \frac{18x^4}{4} - \frac{6x^5}{5} - 4x^3 \Big|_1^2$$

$$= \left(\frac{18(2)^4}{4} - \frac{6(2)^5}{5} - 4(2)^3 \right) - \left(\frac{18(1)^4}{4} - \frac{6(1)^5}{5} - 4(1)^3 \right)$$

$$= 1.6 - (-0.4) = 1.6 + 0.4 = 2.0$$

$$\text{Var}(x) = E(x^2) - (E(x))^2 = 2.0 - (1.5)^2 = 0.25$$

c. finish the test in 90 min = 1.5 hours

$$P(X \leq 1.5) = F(1.5) \quad 1 \leq 1.5 < 2$$

$$F_x(x) = -2x^3 + 9x^2 - 12x + 5$$

$$= -2(1.5)^3 + 9(1.5)^2 - 12(1.5) + 5$$

$$= \frac{1}{2} = 0.5$$